15-453

FORMAL LANGUAGES, AUTOMATA AND COMPUTABILITY

A_{TM} = { (M,w) | M is a TM that accepts string w }

$HALT_{TM}$ = { (M,w) | M is a TM that halts on string w }

E_{TM} = { M | M is a TM and L(M) = ∅ }

REG_{TM} = { M | M is a TM and L(M) is regular}

EQ_{TM} = {(M, N) | M, N are TMs and L(M) =L(N)}

ALL_{PDA} = { P | P is a PDA and L(P) = Σ* }

ALL UNDECIDABLE

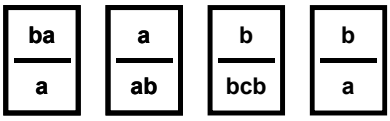
Use Reductions to Prove

Which are SEMI-DECIDABLE?

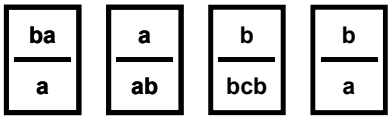
THE POST CORRESPONDENCE PROBLEM

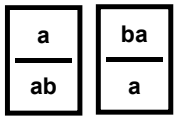
TUESDAY FEB 25

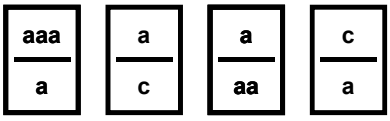
THE PCP GAME

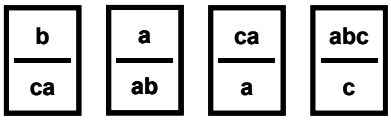


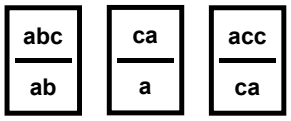
THE PCP GAME

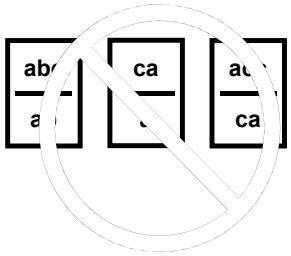






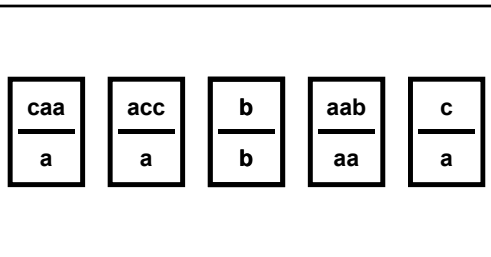






GENERAL RULE #1

If every top string is longer than the corresponding bottom one, there can't be a match



GENERAL RULE #2

If there is a domino with the same string on the top and on the bottom, there is a match

POST CORRESPONDENCE PROBLEM

Given a collection of dominos, is there a match?

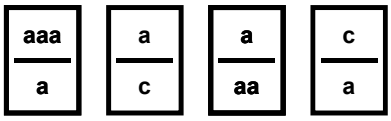
PCP = { P | P is a set of dominos with a match }

PCP is *undecidable*!

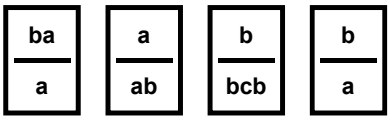
THE FPCP GAME

… is just like the PCP game except that a match has to start with the first domino

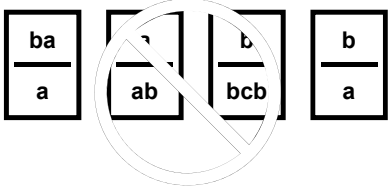
FPCP



FPCP



FPCP



Theorem: FPCP is undecidable

Proof: Assume machine C decides FPCP

We will show how to use C to decide A_{TM}

Given (M,w)

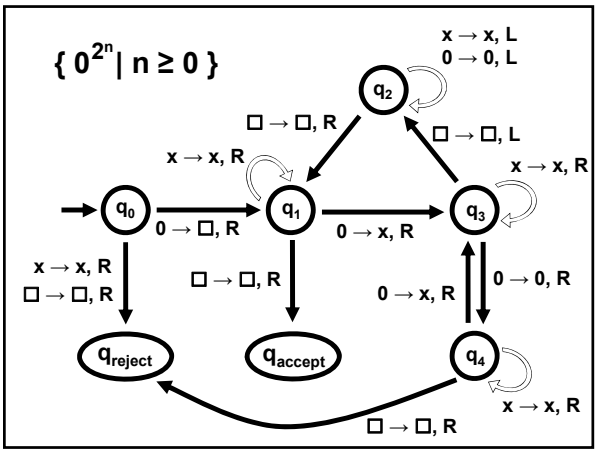
we will construct a set of dominos $P_{M,w}$ where a match is an accepting computation history for M on w

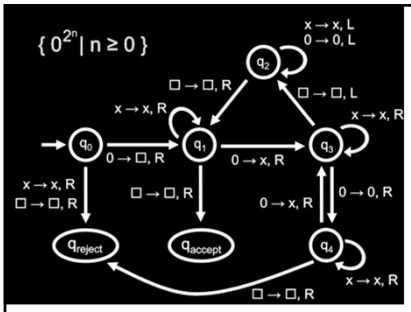
$P_{M,w} = \frac{caa}{c}\ \frac{aba}{bb}\ \cdots\ \frac{a}{d}$

So, $(M, w) \in A_{TM} \Leftrightarrow P_{M,w} \in PCP$

C

$P_{M,w}$ has a match?





$q_0 0000$
$\square q_1 000$
$\square x q_3 00$
$\square x 0 q_4 0$
$\square x 0 x q_3$
$\square x 0 q_2 x$
$\square x q_2 0 x$
$\square q_2 x 0 x$
$q_2 \square x 0 x$
:
$\#q_0 0000\#\square q_1 000\#\square x q_3 00\#\square x 0 q_4 0\#\square x 0 x q_3\# \ldots \#$

Given (M,w), we will construct an instance P of FPCP in 7 steps

Assume M on w never attempts to move off left hand edge of tape

STEP 1

Put $\frac{\#}{\#q_0 w_1 w_2 \ldots w_n \#}$ **into P**

For start configuration

START

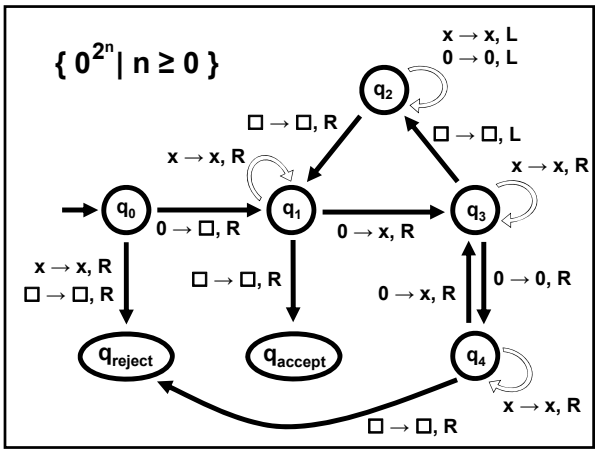
STEP 2

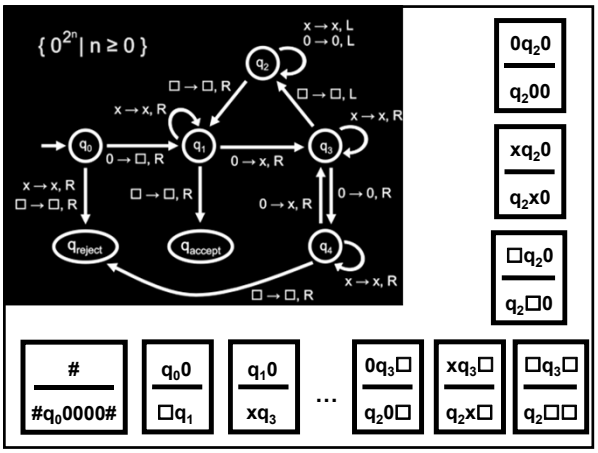
If $\delta(q,a) = (p,b,R)$ then add $\frac{qa}{bp}$

STEP 3

If $\delta(q,a) = (p,b,L)$ then add $\frac{cqa}{pcb}$ **for all $c \in \Gamma$**

RULES





STEP 4

add $\frac{a}{a}$ for all $a \in \Gamma$

For tape cells not adjacent to head

STEP 5

add $\frac{\#}{\#}$ $\frac{\#}{\square\#}$

For configuration separator

To simulate the blanks on the right hand side of tape

CONTINUE

STEP 4

add $\frac{a}{a}$ for all $a \in \Gamma$

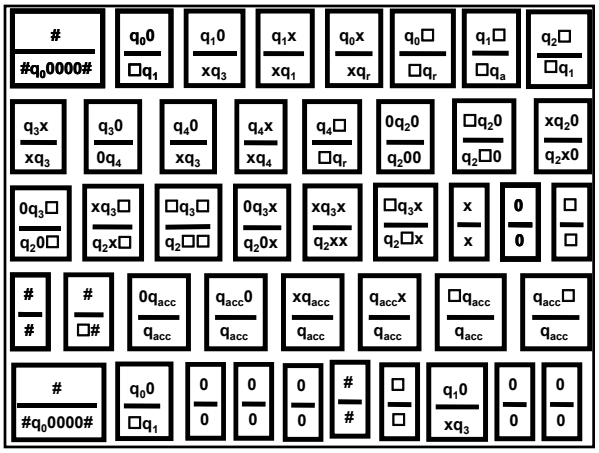
STEP 5

add $\frac{\#}{\#}$ $\frac{\#}{\square\#}$

Adds pseudo-steps after TM halts (catch up)

STEP 6

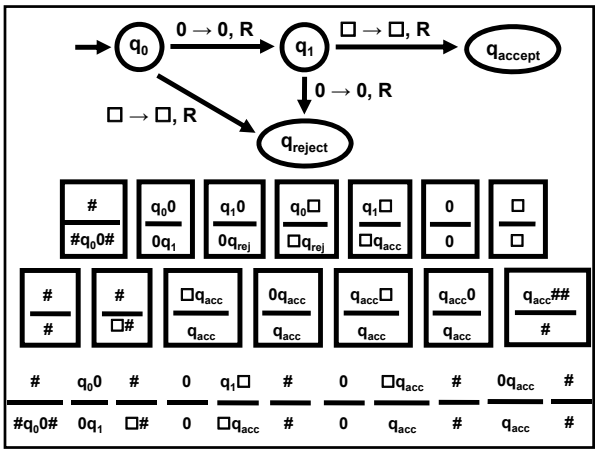
add $\frac{aq_{acc}}{q_{acc}}$ $\frac{q_{acc}a}{q_{acc}}$ for all $a \in \Gamma$



STEP 7

add $\frac{q_{acc}\#\#}{\#}$

END



Given (M,w), we can construct an instance of FPCP that has a match if and only if M accepts w

Can convert an instance of FPCP into one of PCP:

Let $u = u_1u_2\ldots u_n$, define:

$\star u = * u_1 * u_2 * u_3 * \ldots * u_n$

$u\star = u_1 * u_2 * u_3 * \ldots * u_n *$

$\star u\star = * u_1 * u_2 * u_3 * \ldots * u_n *$

FPCP: $\frac{t_1}{b_1}$ $\frac{t_2}{b_2}$ … $\frac{t_k}{b_k}$

PCP: $\frac{\star t_1}{\star b_1\star}$ $\frac{\star t_1}{b_1\star}$ $\frac{\star t_2}{b_2\star}$ … $\frac{\star t_k}{b_k\star}$ $\frac{*\blacklozenge}{\blacklozenge}$

Given (M,w), we can construct an instance of PCP that has a match if and only if M accepts w

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Read Chapters 5.2 and 5.3 of the book for next time