The Logical Meeting Point of Multiset Rewriting and Process Algebra

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Motivations

- Security protocol specifications
    - Transition-based
    - Process-based
  - ➢ Different languages and techniques
  - ➢ Ad-hoc translations
- Attempt at a unified approach
  - ➢ Rewriting re-interpretation of logic
    - Open derivations
    - Left rule semantics
  - ➢ Foundation of multiset rewriting
  - ➢ Bridge to process algebra
  - ➢ Effective protocol specification language

- logic
- system ω
- rewriting
- processes
- security

Linear Logic

- Formulas

$$A, B ::= a \mid 1 \mid A \otimes B \mid A \multimap B \mid !A \mid \top \mid A \,\&\, B \mid \forall x.\, A \mid \exists x.\, A$$

- LV sequents

> Constructor: ","
> Empty: "•"

$$\Gamma ; \Delta \dashrightarrow_{\Sigma} C$$

Unrestricted context

Linear context

Signature

Goal formula

Some LV Rules

- logic
- system ω
- rewriting
- processes
- security

Left rules

$$\frac{\Gamma; \Delta, A, B \longrightarrow_{\Sigma} C}{\Gamma; \Delta, A \otimes B \longrightarrow_{\Sigma} C}$$

$$\frac{\Gamma; \Delta' \longrightarrow_{\Sigma} A \qquad \Gamma; \Delta, B \longrightarrow_{\Sigma} C}{\Gamma; \Delta, \Delta', A \multimap B \longrightarrow_{\Sigma} C}$$

$$\frac{\Sigma \vdash t \qquad \Gamma; \Delta, [t/x]A \longrightarrow_{\Sigma} C}{\Gamma; \Delta, \forall x.A \longrightarrow_{\Sigma} C}$$

$$\frac{\Gamma; \Delta, A \longrightarrow_{\Sigma,x} C}{\Gamma; \Delta, \exists x.A \longrightarrow_{\Sigma} C}$$

$$\frac{\Gamma, A; \Delta \longrightarrow_{\Sigma} C}{\Gamma; \Delta, !A \longrightarrow_{\Sigma} C}$$

...

Structural rules

$$\frac{}{\Gamma; A \longrightarrow_{\Sigma} A}$$

$$\frac{\Gamma, A; \Delta, A \longrightarrow_{\Sigma} C}{\Gamma, A; \Delta \longrightarrow_{\Sigma} C}$$

Cut rules

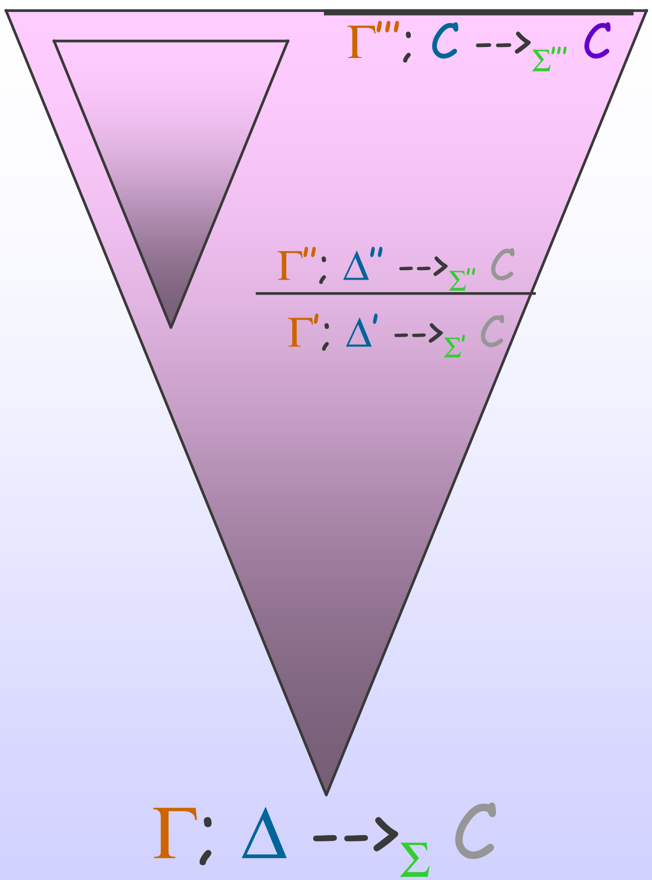
$$\frac{\Gamma; \Delta' \longrightarrow_{\Sigma} A \qquad \Gamma; \Delta, A \longrightarrow_{\Sigma} C}{\Gamma; \Delta, \Delta' \longrightarrow_{\Sigma} C}$$

$$\frac{\Gamma; \bullet \longrightarrow_{\Sigma} A \qquad \Gamma, A; \Delta \longrightarrow_{\Sigma} C}{\Gamma; \Delta \longrightarrow_{\Sigma} C}$$

Right rules

...

Logical Derivations



- Proof of C from Δ and Γ
  - Emphasis on C
    - C is input
- Finite
  - Closed
- Rules shown
  - Major premise
    - Preserves C
  - Minor premise
    - Starts subderivation

- logic
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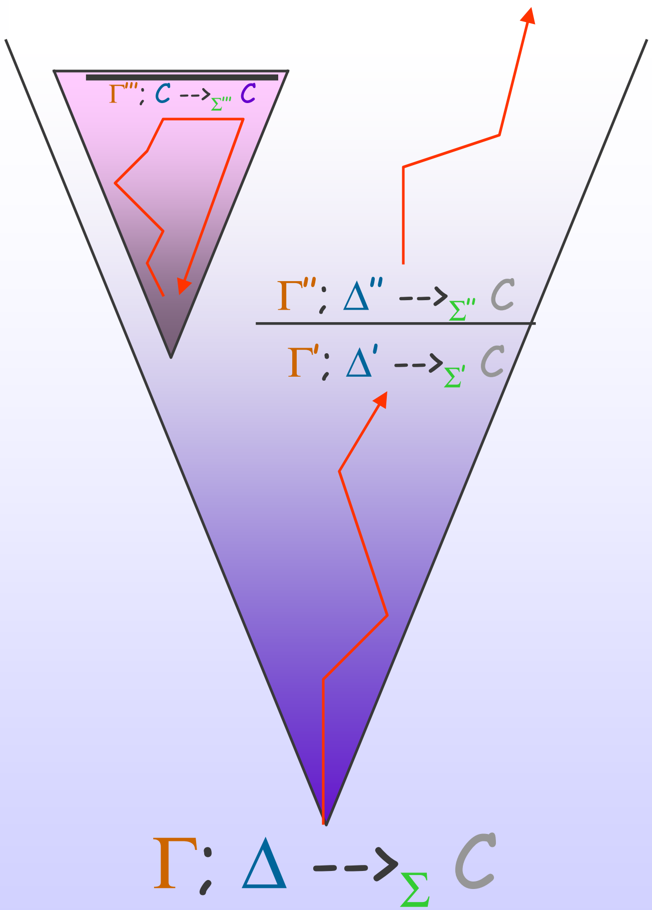
A Rewriting Re-Interpretation



- Transition
  - From conclusion
  - To major premise
  - Emphasis on Γ, Δ and Σ
  - C is output, at best
    - Does not change
- Possibly infinite
  - Open
- Minor premise
  - Auxiliary rewrite chain
    - Finite
  - Topped with axiom

State and Transitions

- States

$$\Sigma \quad ; \quad \Gamma \quad ; \quad \Delta$$

  - Σ is a list
  - Γ and Δ are commutative monoids
  - No *C*
    - Does not change

> - Constructor: ","
> - Empty: "•"

- Transitions

$$\Sigma; \Gamma; \Delta \rightarrow \Sigma'; \Gamma'; \Delta'$$

  - $\rightarrow^*$ for reflexive and transitive closure

Interpreting Unary Rules

- logic
- system ω
- rewriting
- processes
- security

$$\frac{\Gamma;\ \Delta, A, B \dashrightarrow_{\Sigma} C}{\Gamma;\ \Delta, A \otimes B \dashrightarrow_{\Sigma} C} \qquad \Sigma;\ \Gamma;\ (\Delta, A \otimes B) \rightarrow \Sigma;\ \Gamma;\ (\Delta, A, B)$$

$$\frac{\boxed{\Sigma \vdash t} \quad \Gamma;\ \Delta, [t/x]A \dashrightarrow_{\Sigma} C}{\Gamma;\ \Delta, \forall x.A \dashrightarrow_{\Sigma} C} \qquad \Sigma;\ \Gamma;\ (\Delta, \forall x.\ A) \rightarrow \Sigma;\ \Gamma;\ (\Delta, [t/x]A) \quad \boxed{\text{if } \Sigma \vdash t}$$

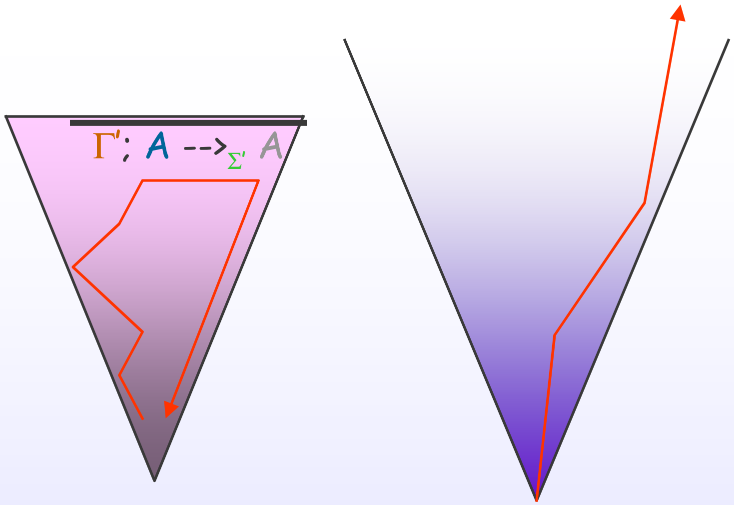
$$\frac{\Gamma;\ \Delta, A \dashrightarrow_{\Sigma,x} C}{\Gamma;\ \Delta, \exists x.A \dashrightarrow_{\Sigma} C} \qquad \Sigma;\ \Gamma;\ (\Delta, \exists x.\ A) \rightarrow (\Sigma, x);\ \Gamma;\ (\Delta, A)$$

$$\frac{\Gamma, A;\ \Delta \dashrightarrow_{\Sigma} C}{\Gamma;\ \Delta, !A \dashrightarrow_{\Sigma} C} \qquad \Sigma;\ \Gamma;\ (\Delta, !A) \rightarrow \Sigma;\ (\Gamma, A);\ \Delta$$

... ...

Binary Rules and Axiom

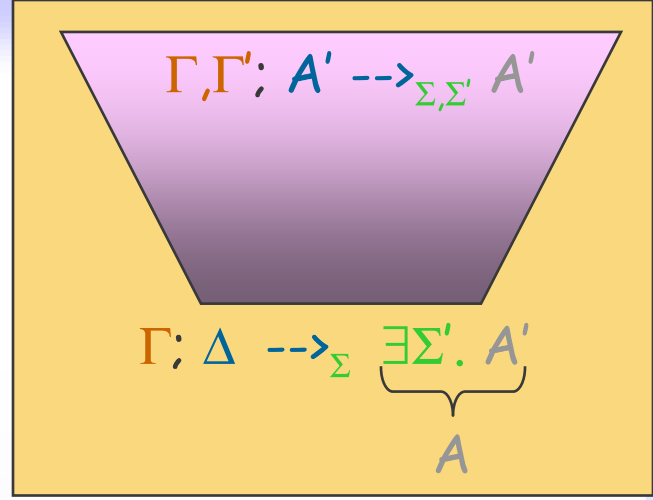
- logic
- system ω
- rewriting
- processes
- security



$$\frac{\Gamma; \Delta' \longrightarrow_{\Sigma} A \qquad \Gamma; \Delta, B \longrightarrow_{\Sigma} C}{\Gamma; \Delta, \Delta', A \multimap B \longrightarrow_{\Sigma} C}$$

- Minor premise
  - Auxiliary rewrite chain
- Top of tree
  - Focus shifts to RHS
    - Axiom rule
  - Observation

Observations



- Observation states

$$\Sigma \quad ; \quad \Delta$$

  - In Δ, we identify
    - , with $\otimes$
    - • with 1

    Categorical semantics

  $\Delta = \otimes\Delta$

  - Identified with $\exists x_1. \ldots \exists x_n. \Delta$
    - For $\Sigma = x_1, \ldots, x_n$

    De Bruijn's telescopes

  $\Sigma; \Delta = \exists\Sigma. \otimes\Delta$

- Observation transitions

$$\Sigma; \Gamma; \Delta \rightarrow^* \Sigma'; \Delta'$$

Interpreting Binary Rules

$$\frac{}{\Gamma;\, A \dashrightarrow_{\Sigma} A}$$

$$\Sigma;\, \Gamma;\, \Delta \rightarrow^{*} \Sigma;\, \Delta$$

$$\Sigma;\, \Gamma;\, \Delta \rightarrow^{*} \Sigma'';\, \Delta''$$

if $\quad \Sigma;\, \Gamma;\, \Delta \rightarrow \Sigma';\, \Gamma';\, \Delta'$

and $\quad \Sigma';\, \Gamma';\, \Delta' \rightarrow^{*} \Sigma'';\, \Delta''$

$$\frac{\boxed{\Gamma;\, \Delta' \dashrightarrow_{\Sigma} A} \qquad \Gamma;\, \Delta, B \dashrightarrow_{\Sigma} C}{\Gamma;\, \Delta, \Delta', A \multimap B \dashrightarrow_{\Sigma} C}$$

$$\Sigma;\, \Gamma;\, (\Delta, \Delta', A \multimap B) \rightarrow \Sigma;\, \Gamma;\, (\Delta, B)$$

$\boxed{\text{if } \Sigma;\, \Gamma;\, \Delta' \rightarrow^{*} \Sigma;\, A}$

$$\frac{\boxed{\Gamma;\, \Delta' \dashrightarrow_{\Sigma} A} \qquad \Gamma;\, \Delta, A \dashrightarrow_{\Sigma} C}{\Gamma;\, \Delta, \Delta' \dashrightarrow_{\Sigma} C}$$

$$\Sigma;\, \Gamma;\, \Delta, \Delta' \rightarrow \Sigma;\, \Gamma;\, (A, \Delta)$$

$\boxed{\text{if } \Sigma;\, \Gamma;\, \Delta' \rightarrow^{*} \Sigma;\, A}$

… …

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Formal Correspondence

- Soundness

  If $\Sigma ; \Gamma ; \Delta \to^{*} \Sigma, \Sigma' ; \Delta'$

  then $\Gamma ; \Delta \dashrightarrow_{\Sigma} \exists \Sigma' . \otimes \Delta'$

- Completeness?
  - No! We have only crippled right rules

    $\bullet ; \bullet ; a \multimap b, b \multimap c \not\to^{*} \bullet ; a \multimap c$

System ω

- With cut, rule for $\multimap$ can be simplified to

$$\Sigma; \Gamma; (\Delta, A, A \multimap B) \rightarrow \Sigma; \Gamma; (\Delta, B)$$

- Cut elimination holds

  = in-lining of auxiliary rewrite chains

  - But …
    - Careful with extra signature symbols
    - Careful with extra persistent objects

- No rule for $\rightarrow$ needs a premise
  - $\rightarrow$ does not depend on $\rightarrow^*$

Multiset Rewriting

- Multiset: set with repetitions allowed

$$\underline{a} ::= \bullet \mid a, \underline{a}$$

  - Commutative monoid

- Multiset rewriting (a.k.a. Petri nets)
  - Rewriting within the monoid
  - Fundamental model of distributed computing
    - Alternative: Process Algebras
  - Basis for security protocol spec. languages
    - MSR family
    - … several others
  - Many extensions, more or less ad hoc

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First-Order Multiset Rewriting

- logic
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- Multiset elements are FO atomic formulas
- Rules have the form

$$\forall x_1 \ldots x_n.\ \underline{a}(\mathbf{x}) \rightarrow \exists y_1 \ldots y_k.\ \underline{b}(\mathbf{x},\mathbf{y})$$

- Semantics

$$\boxed{\Sigma\ ;\ \underline{a}(\mathbf{t}),\ \underline{s} \ \rightarrow_{R,\ (\underline{a}(\mathbf{x}) \rightarrow \exists \mathbf{y}.\ \underline{b}(\mathbf{x},\mathbf{y}))}\ \Sigma, \mathbf{y}\ ;\ \underline{b}(\mathbf{t},\mathbf{y}),\ \underline{s} \qquad \text{if } \Sigma \vdash \mathbf{t}}$$

- Several encodings into linear logic
  - [Martí-Oliet, Meseguer, 91]

ω-Multisets vs. Multiset Rewriting

- MSR 1 is an instance of ω-multisets
  - Uses only ⊗, 1, ∀, ∃, and ⊸
  - ⊸ never nested, always persistent
  - ➢ $\Sigma ; \underline{s} \rightarrow_R \Sigma' ; \underline{s}'$

    iff $\Sigma ; \text{“}R\text{”} ; \text{“}\underline{s}\text{”} \rightarrow^* \Sigma' ; \text{“}\underline{s}'\text{”}$

- Interpretation of MSR **as** linear logic
  - ➢ Logical explanation of multiset rewriting
    - MSR is logic
  - ➢ Guideline to design rewrite systems

The Asynchronous π-Calculus

Another fundamental model of distributed computing

- Language

  $P \ ::= \ 0 \ | \ P||Q \ | \ \nu\, x.\, P \ | \ !P \ | \ x(y).P \ | \ \underline{x}\langle y\rangle$

- Semantics
  - Structural equivalence
    - Comm. monoidal congruence of || and 0
    - Binder mobility congruence of ν
      - $\nu\, x.\, \nu\, y.\, P \equiv \nu\, y.\, \nu\, x.\, P$
      - $0 \equiv \nu\, x.\, 0$
      - $P \,||\, \nu\, x.\, Q \equiv \nu\, x.\, (P \,||\, Q)$ if $x \notin FN(P)$
    - $!P \equiv \ !P \,||\, P$
  - Reaction law
    - $\underline{x}\langle y\rangle \,||\, x(z).\, P \,||\, Q \Rightarrow [y/z]P \,||\, Q$

Properties

- If $P \Rightarrow^* Q$
  then $\bullet; \bullet;$ "P" $\rightarrow^* \Sigma; \Gamma; \Delta$
  where "Q" $= \exists\Sigma.\ !\Gamma \otimes \Delta$ mod $!A = !A \otimes A$
  - Note: with $!P \rightarrow\ !P \mid\mid P$ as a transition
    - If $P \Rightarrow^* Q$
      then $\bullet; \bullet;$ "P" $\rightarrow^* \Sigma; \Gamma; \Delta$
      where "Q" $= \exists\Sigma.\ !\Gamma \otimes \Delta$

ω-Multisets vs. Process Algebra

- Simple encoding of asynchronous π-calculus into ω-multisets
  - Doesn't show that π-calculus is logic
  - Uses only a fraction of ω-multiset syntax
  - Inverse encoding?
    - As hard as going from multiset rewriting to π-calculus
- Other languages
  - Join calculus
  - Strand spaces
  - To do: Synchronous π-calculus

MSR 3

- Instance of ω-multisets for cryptographic protocol specification
  - Security-relevant signature
  - Typing infrastructure
  - Modules, equations, …
- 3rd generation
  - MSR 1: First-order multiset rewriting with ∃
    - Undecidability of protocol analysis
  - MSR 2: MSR 1 + typing
    - Actual specification language
    - More theoretical results
    - Implementation underway

Example

Needham-Schroeder public-key protocol

1. $A \rightarrow B: \{n_A, A\}_{kB}$
2. $B \rightarrow A: \{n_A, n_B\}_{kA}$
3. $A \rightarrow B: \{n_B\}_{kB}$

- Can be expressed in several ways
  - State-based
    - Explicit local state
    - As in MSR 2
  - Process-based: embedded $\rightarrow$
    - Continuation-passing style
    - As in process algebra
  - (Intermediate approaches)

State-Based

$A \to B: \{n_A, A\}_{kB}$
$B \to A: \{n_A, n_B\}_{kA}$
$A \to B: \{n_B\}_{kB}$

MSR 2 spec.

$\forall A$: princ.

$\{\ \exists L$: princ $\times\ \Sigma B$:princ.pubK B $\times$ nonce $\to$ mset.

$\forall B$: princ. $\forall k_B$: pubK B.

$\bullet$

$\to\ \exists n_A$: nonce.

net $(\{n_A, A\}_{kB})$, $L\ (A, B, k_B, n_A)$

$\forall B$: princ. $\forall k_B$: pubK B.
$\forall k_A$: pubK A. $\forall k_A'$: prvK k_A.
$\forall n_A$: nonce. $\forall n_B$: nonce.

net $(\{n_A, n_B\}_{kA})$, $L\ (A, B, k_B, n_A)$
$\to$ net $(\{n_B\}_{kB})$

$\}$

Interpretation of L
- Rule invocation
  - Implementation detail
  - Control flow
- Local state of role
  - Explicit view
  - Important for DOS

- logic
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Process-Based

$\forall A$:princ.

$\forall B$: princ. $\forall k_B$: pubK B.

$\bullet \to \exists n_A$: nonce.

$net\ (\{n_A, A\}_{kB}),$

$(\forall k_A$: pubK A. $\forall k_A'$: prvK k_A. $\forall n_B$: nonce.

$net\ (\{n_A, n_B\}_{kA}) \to net\ (\{n_B\}_{kB}))$

- logic
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- Succinct
- Continuation-passing style
  - Rule asserts what to do next
  - Lexical control flow
- State is implicit
  - Abstract

NSPK in Process Algebra

$A \rightarrow B$: $\{n_A, A\}_{kB}$
$B \rightarrow A$: $\{n_A, n_B\}_{kA}$
$A \rightarrow B$: $\{n_B\}_{kB}$

$\forall A$:princ.
$\forall B$: princ. $\forall k_B$: pubK B.
$\forall k_A$: pubK A. $\forall k_A'$: prvK k_A. $\forall n_B$: nonce.

νn_A: nonce.
  net $(\{n_A, A\}_{kB})$.
    $\underline{\text{net}}$ $\langle\{n_A, n_B\}_{kA}\rangle$.
      net $(\{n_B\}_{kB})$. 0

Same structure !
- Not a coincidence
- MSR 3 very close to Process Algebra
  - ω-multiset encodings of π-calculus and Join Calculus

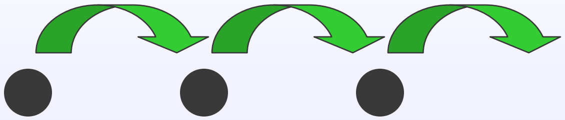
- MSR 3 is promising middle-ground for relating
  - State-based
  - Process-based

  representations of a problem

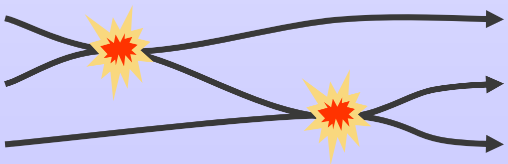
- logic
- system ω
- rewriting
- processes
- security

State-Based vs. Process-Based

- State-based languages
  - Multiset Rewriting
  - NRL Prot. Analyzer, CAPSL/CIL, Paulson's approach, …
  - ➢ State transition semantics

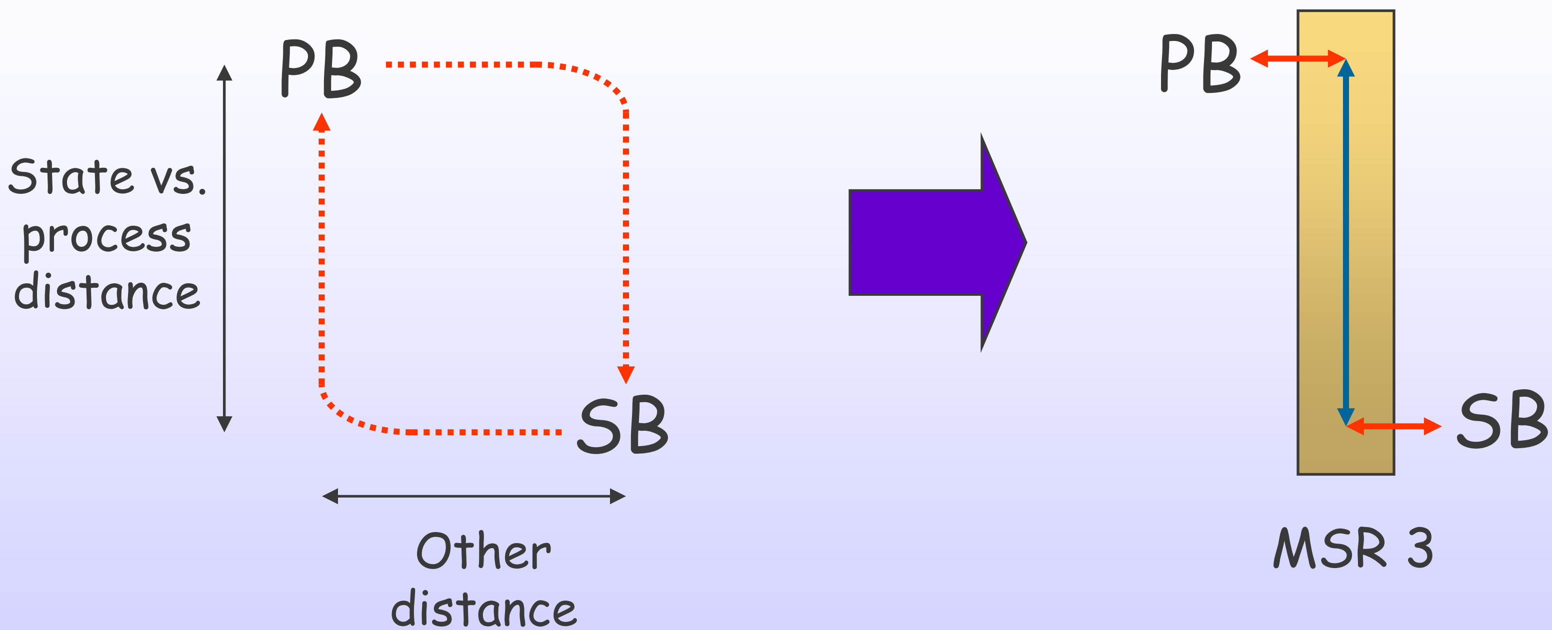


- Process-based languages
  - Process Algebra
  - Strand spaces, spi-calculus, …
  - ➢ Independent communicating threads



MSR 3 Bridges the Gap

- Difficult to go from one to the other
  - Different paradigms



State ↔ Process translation done once and for all in MSR 3

- logic
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Conclusions

- ω-multisets
  - Logical foundation of multiset rewriting
  - Relationship with process algebras
  - Unified logical view
    - Better understanding of where we are
    - Hint about where to go next
- MSR 3.0
  - Language for security protocol specification
  - Succinct representations
    - Simpler specifications
    - Economy of reasoning
  - Bridge between
    - State-based representation
    - Process-based representation