Readings:

K&F: 4.1, 4.2, 4.3, 4.4, 8.4, 8.5, 8.6

"Recursive Conditioning", Adnan Darwiche. In Artificial Intelligence Journal, 125:1, pp. 5-41

Context-specific independence

CSI-BNs

Graphical Models – 10708

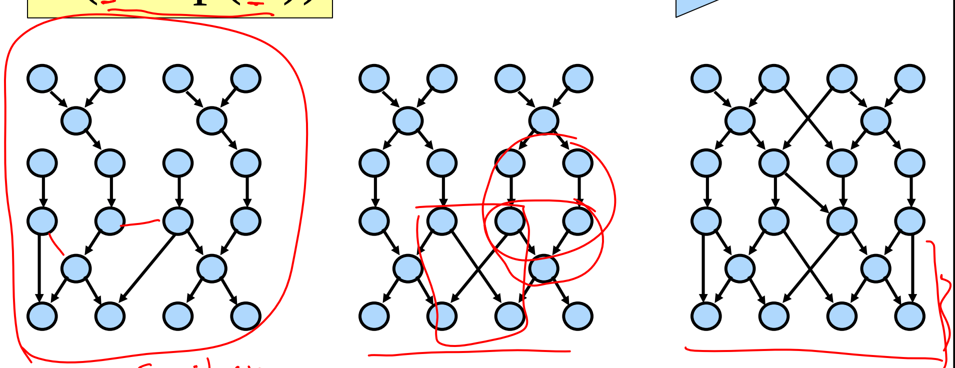
Carlos Guestrin

Carnegie Mellon University

October 16th, 2006

Global Structure: Treewidth w

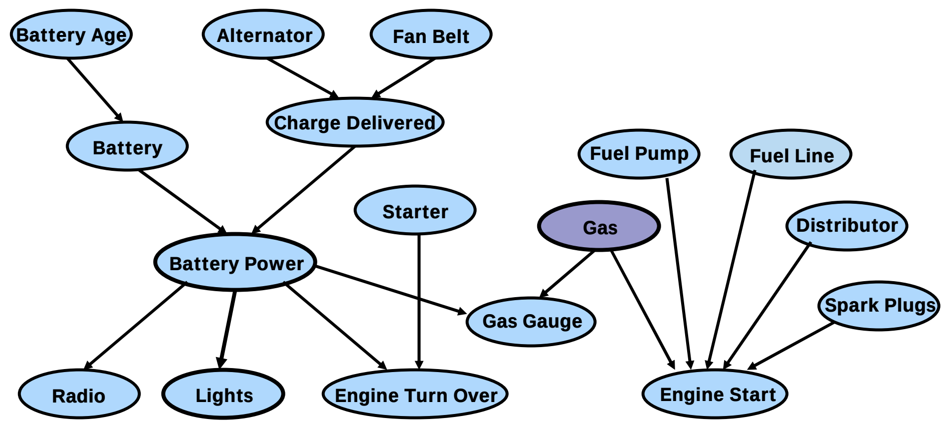
$O(n\exp(w))$



w = Family size +1

Local Structure 1: Context specific indepencence

CSI



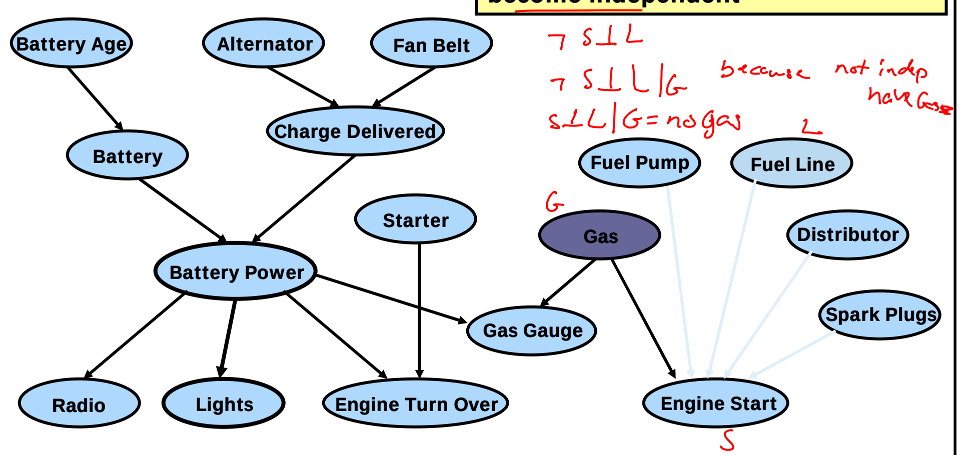
Local Structure 1: Context specific indepencence

Context Specific Independence (CSI)
After observing a variable, some vars become independent

$\neg\, S \perp L$

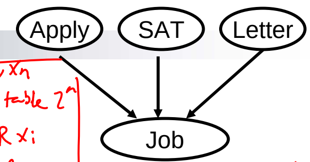
$\neg\, S \perp L \mid G$ because not indep have Gas

$S \perp L \mid G = \text{no gas}$



Representation

CSI example: Tree CPD



- Represent $P(X_i|\mathbf{Pa}_{X_i})$ using a decision tree
  - Path to leaf is an assignment to (a subset of) $\mathbf{Pa}_{X_i}$
  - Leaves are distributions over X_i given assignment of $\mathbf{Pa}_{X_i}$ on path to leaf
- **Interpretation of leaf**:
  - For specific assignment of $\mathbf{Pa}_{X_i}$ on path to this leaf – X_i is independent of other parents
- Representation can be exponentially smaller than equivalent table

$X_1 \ldots X_n$, Y, table 2^n

$Y = OR\ X_i$

$O(n)$

$P(J \mid A, S, L)$ ← table

binary: 8 or 16 table

$P(J=t \mid A=f, S, L) = 0$

SL irrelevant

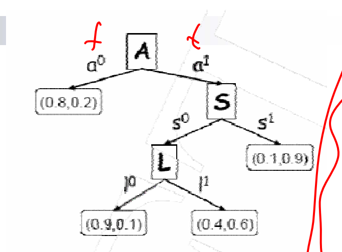
$P(J=t \mid A=t, S=low, L) = \epsilon = 0.1$

$J \perp \{S, L\} \mid A=f$

4 parents

Tabular VE with Tree CPDs

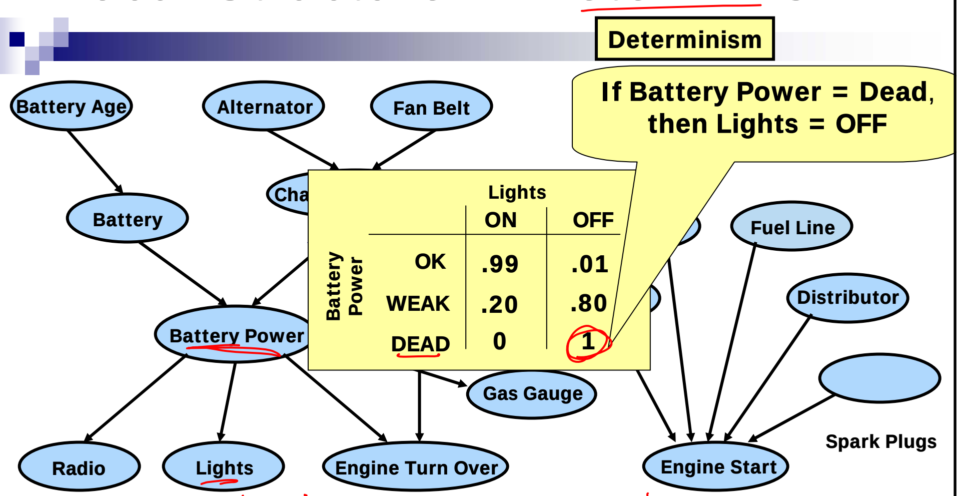
- If we turn a **tree CPD into table**
  - **“Sparsity” lost**!
- Need inference approach that **deals with tree CPD directly**!



$P(J \mid A, S, L)$

ASL	ttt	ttf	tft	tff	...
J=t	.9	.9			
J=f					

Local Structure 2: Determinism



$P(L|B)$ $P(L=t|B=dead)=0$

Determinism and inference

$P(x_i | Pa\,x_i)$ exp in # parents

- Determinism gives a little sparsity in table, but much bigger impact on inference
- Multiplying deterministic factor with other factor introduces many new zeros
  - Operations related to theorem proving, e.g., unit resolution

Battery Power	Lights ON	Lights OFF
OK	.99	.01
WEAK	.20	.80
DEAD	0	1

$\prod_i f_i$

$g(y)$

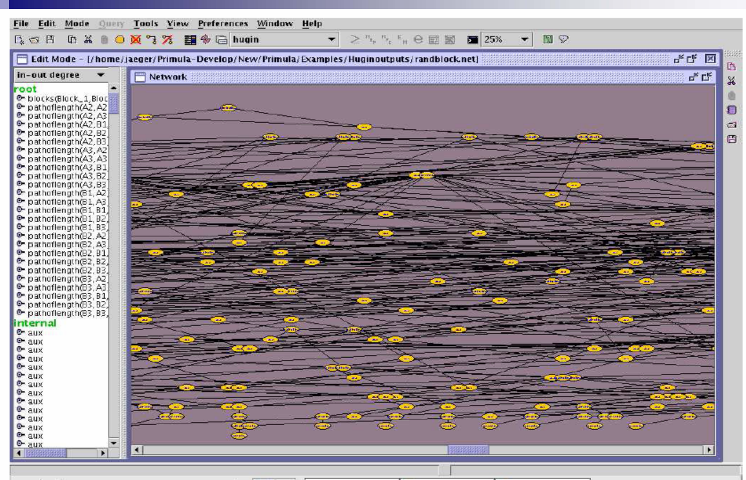
$f_i(x_i)=0$

all values of the $g(y)$ table where x_i will be zero

Today's Models …

- **Often characterized by:**
  - Richness in local structure (determinism, CSI)
  - Massiveness in size (10,000's variables)
  - High connectivity (treewidth) high
- **Enabled by:**
  - High level modeling tools: relational, first order
  - Advances in machine learning
  - New application areas (synthesis):
    - Bioinformatics (e.g. linkage analysis)
    - Sensor networks
- **Exploiting local structure a must!**

Exact inference in large models is possible…

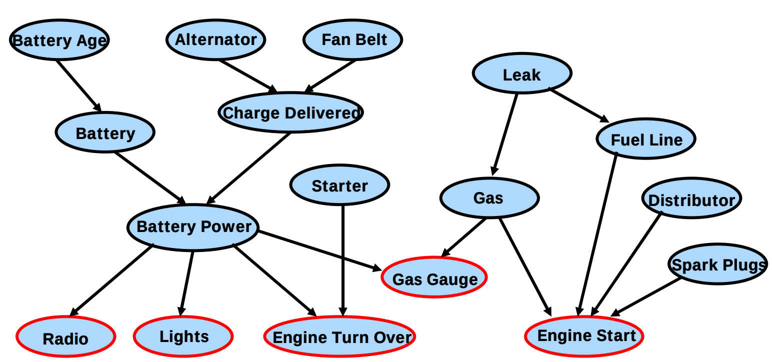


- BN from a relational model

Recursive Conditioning

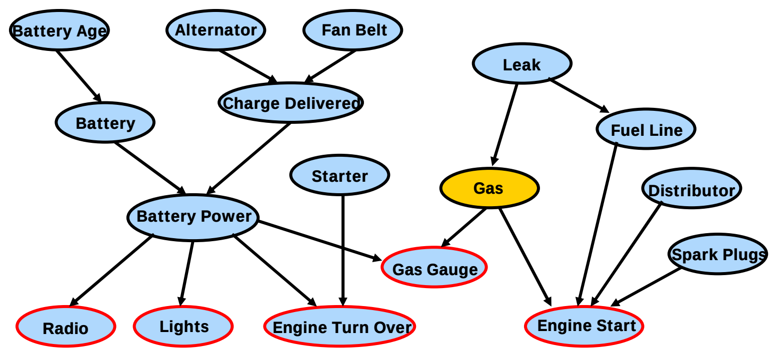
- Treewidth complexity (worst case)
- Better than treewidth complexity with local structure
- Provides a framework for time-space tradeoffs
- Only quick intuition today, details in readings

The Computational Power of Assumptions



A. Darwiche

The Computational Power of Assumptions



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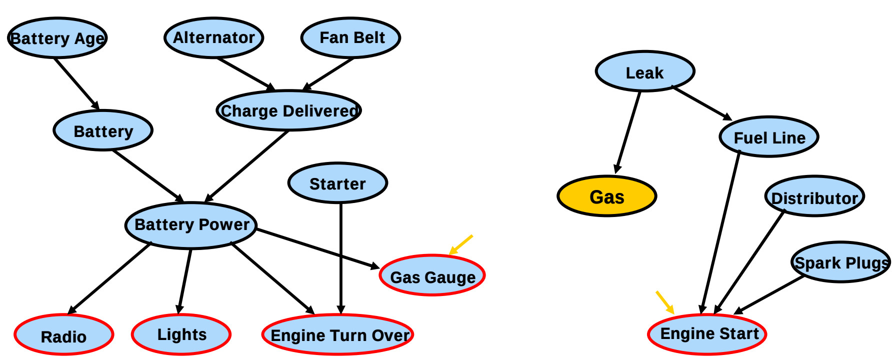
Decomposition

VE exploits this



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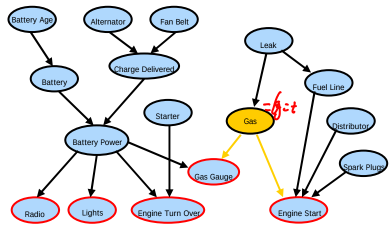
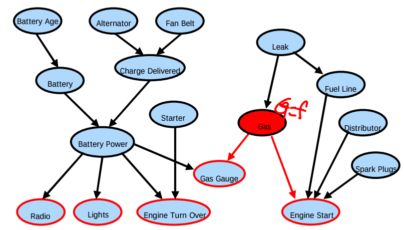
Case Analysis

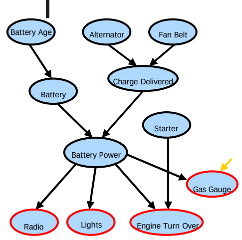


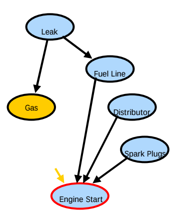


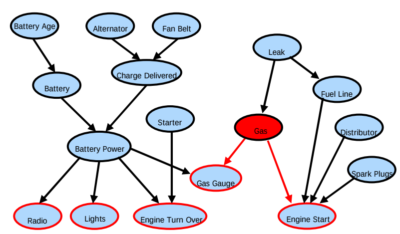
p + p

A. Darwiche

Case Analysis







p_l * p_r + p

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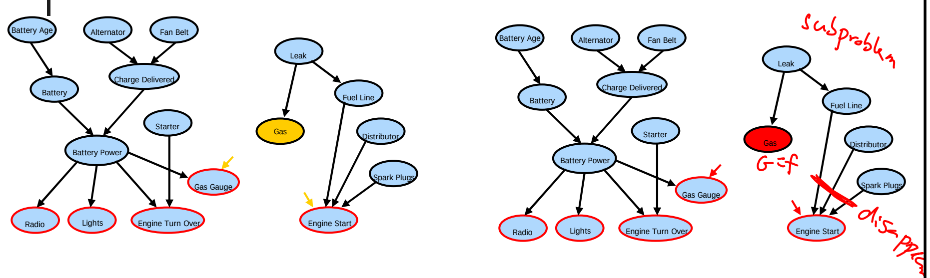
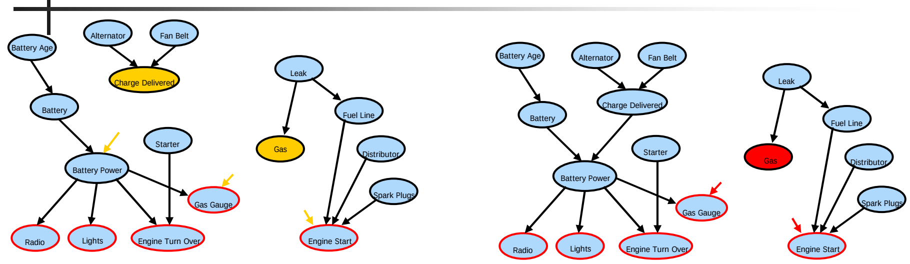
Case Analysis



p_l * p_r + p_l * p_r

A. Darwiche

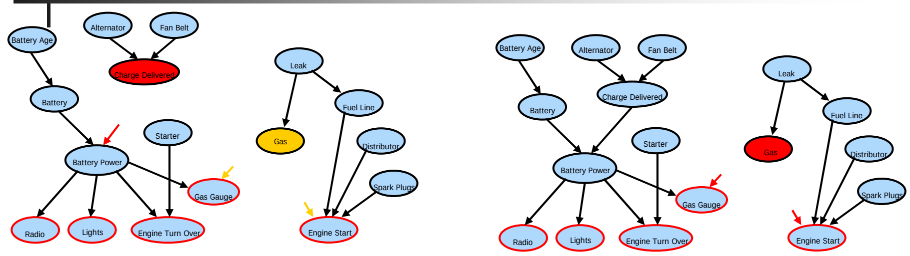
Case Analysis



p_l * p_r + p_l * p_r

A. Darwiche

Case Analysis



p_l * p_r + p_l * p_r

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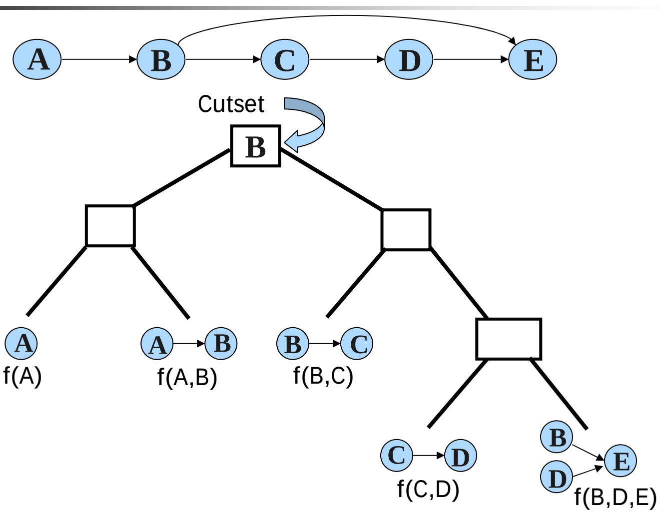
Decomposition Tree



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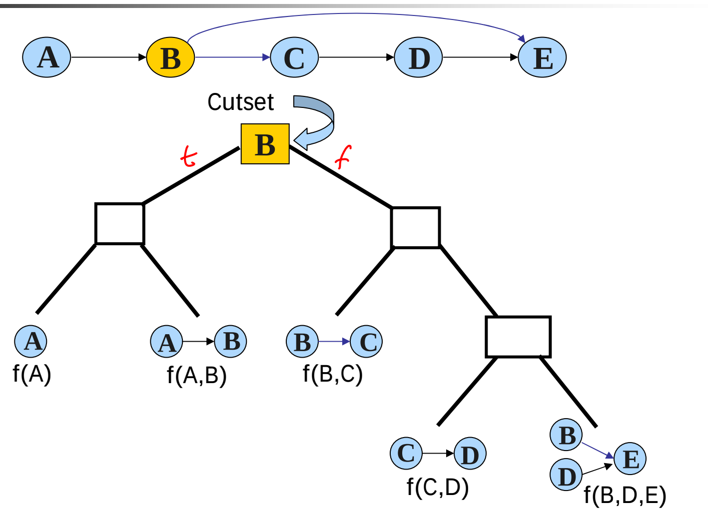
Decomposition Tree



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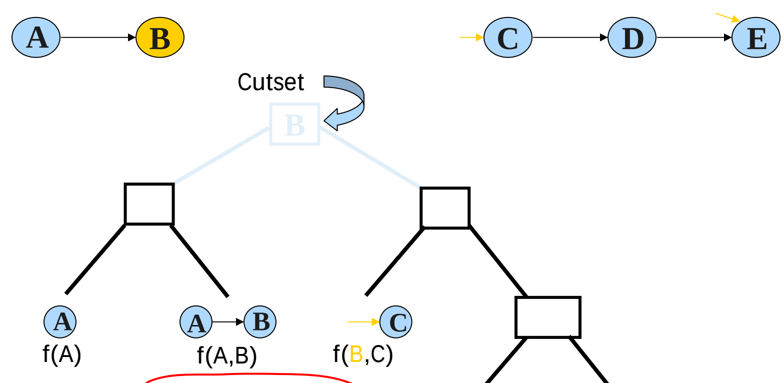
Decomposition Tree



Time: O(n exp(w log n))
Space: Linear
(using appropriate dtree)

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RC1

choosing T related to choosing VE order

```
RC1(T,e)
  // compute probability of evidence e on dtree T

  If T is a leaf node
  Return Lookup(T,e)
  Else
     p := 0
     for each instantiation c of cutset(T)-E do
        p := p + RC1(Tl,ec) RC1(Tr,ec)
     return p
```

look up prob. from CPTs

A. Darwiche

Lookup(T,**e**)

$\Theta_{X|\mathbf{U}}$: CPT associated with leaf T

If X is instantiated in **e**, then

x: value of X in **e**

u: value of **U** in **e**

Return $\theta_{x|\mathbf{u}}$

Else return 1 = $\Sigma_x\ \theta_{x|\mathbf{u}}$

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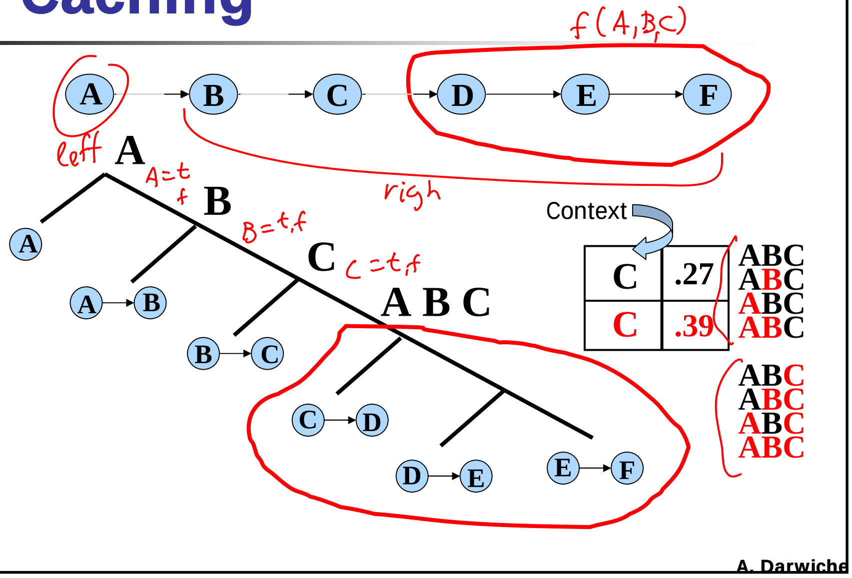
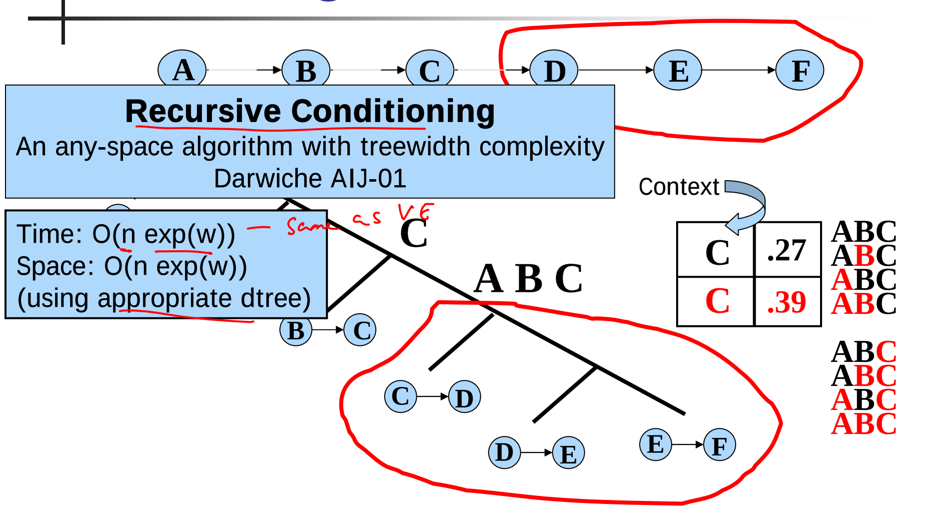
Caching



Caching



Recursive Conditioning
An any-space algorithm with treewidth complexity
Darwiche AIJ-01

Time: O(n exp(w)) — same as VE
Space: O(n exp(w))
(using appropriate dtree)

RC2

RC2(T,**e**)

at leaf look at CPT

- If T is a leaf node, return Lookup(T,e)
- **y** := instantiation of context(T)
- If cache$_T$[**y**] <> nil, return cache$_T$[**y**]

or check cache

compute

- p := 0
- For each instantiation **c** of cutset(T)-**E** do
  - p := p + RC2(T^l,**ec**) RC2(T^r,**ec**)
- cache$_T$[**y**] := p
- Return p

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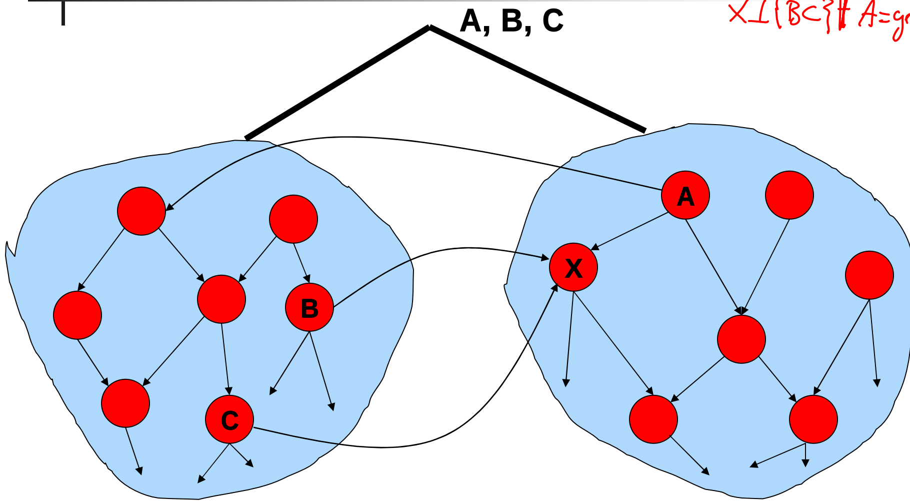
Decomposition with Local Structure

X Independent of B, C given $\boxed{A}$

$X \perp \{BC\} \mid A$ = gelb



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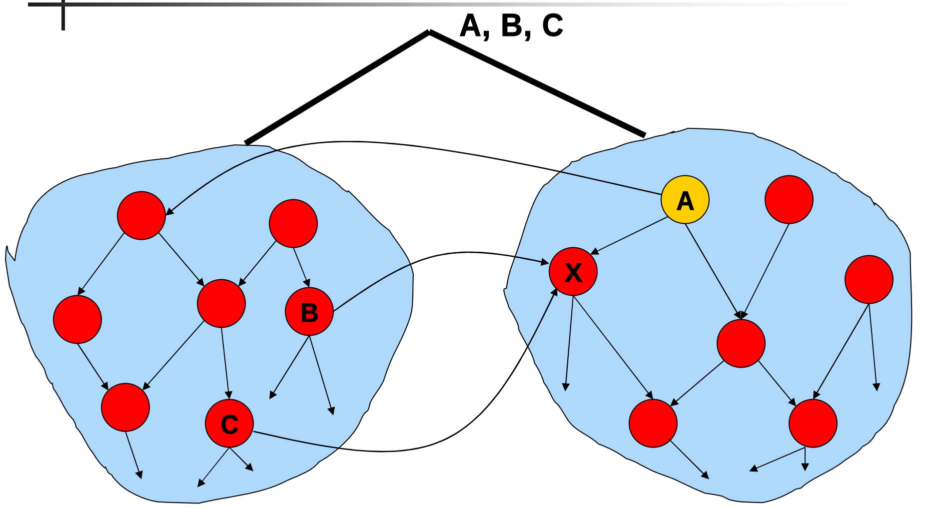
Decomposition with Local Structure

X Independent of B, C given A



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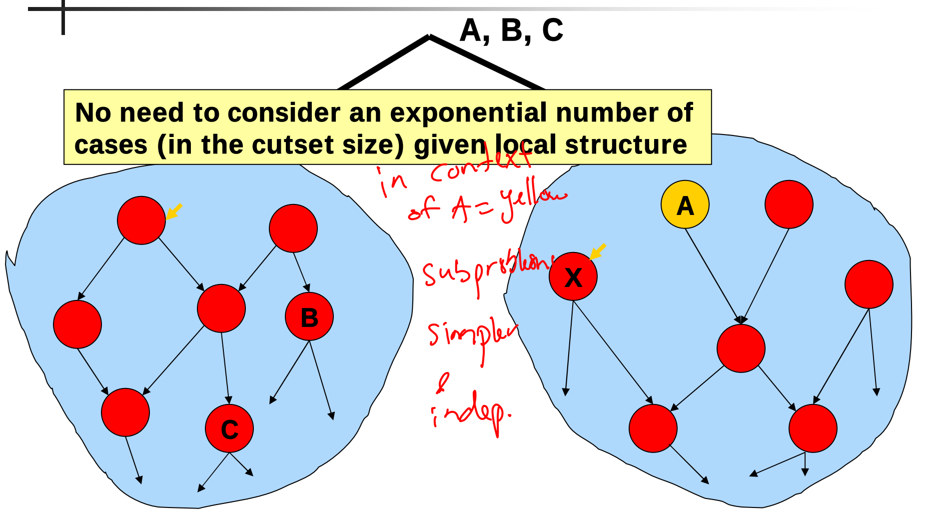
Decomposition with Local Structure

X Independent of B, C given A

No need to consider an exponential number of cases (in the cutset size) given local structure



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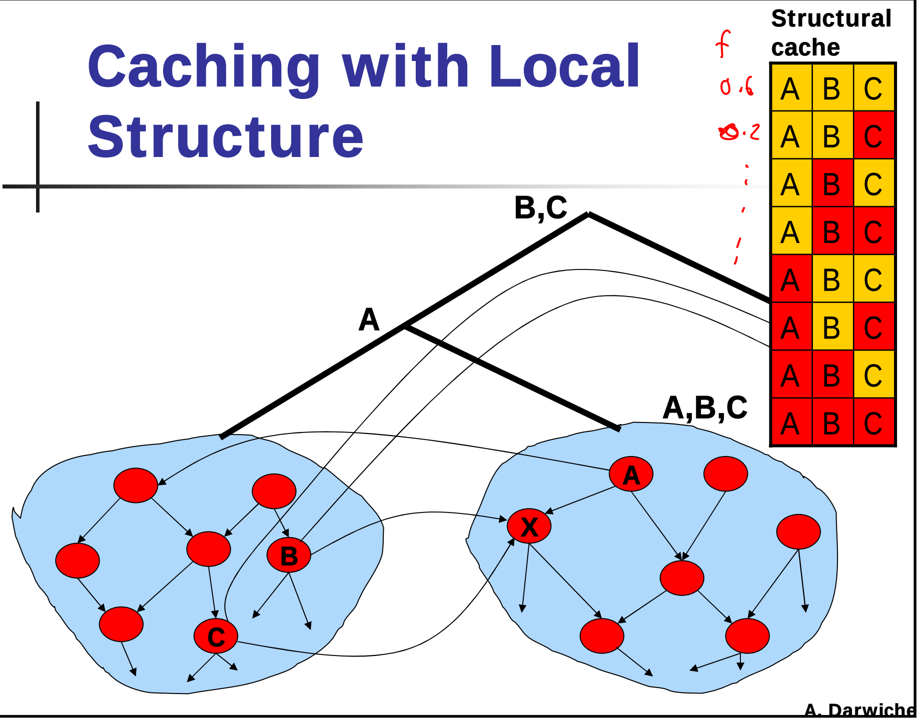
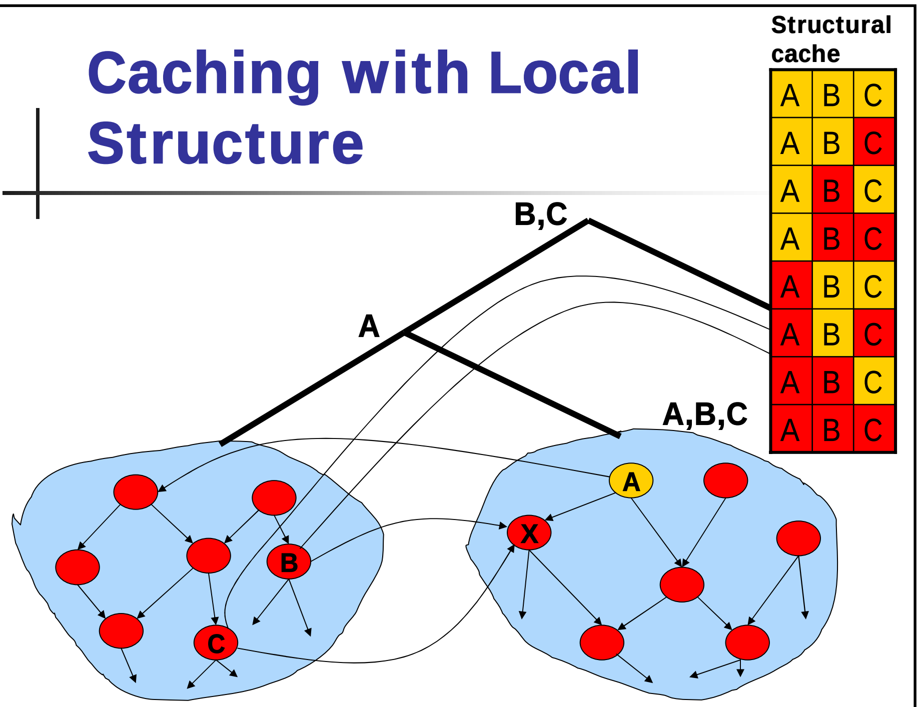
Caching with Local Structure

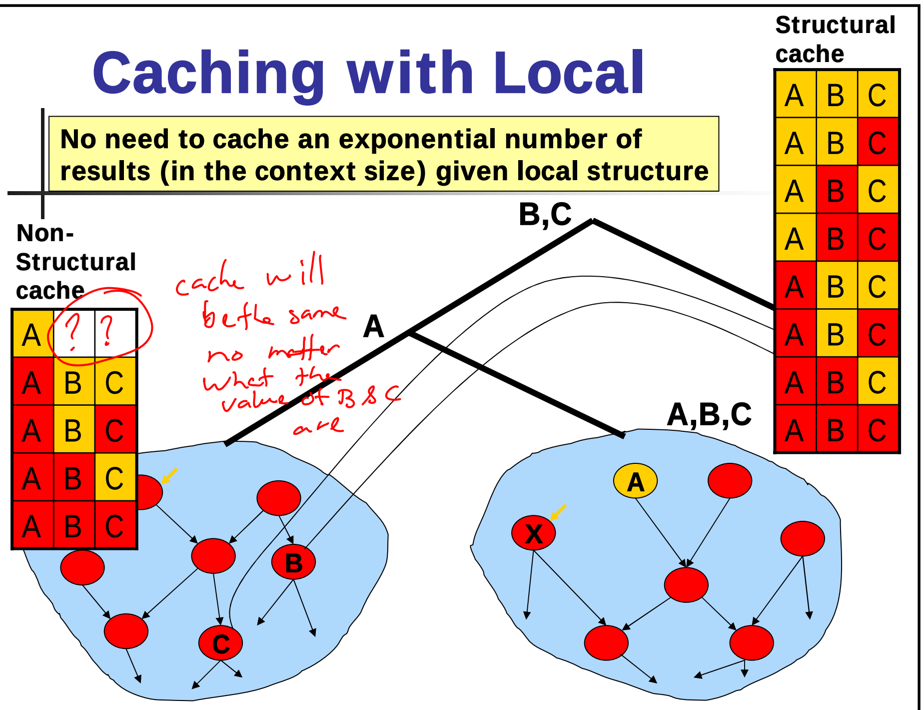


Caching with Local Structure



Caching with Local

No need to cache an exponential number of results (in the context size) given local structure



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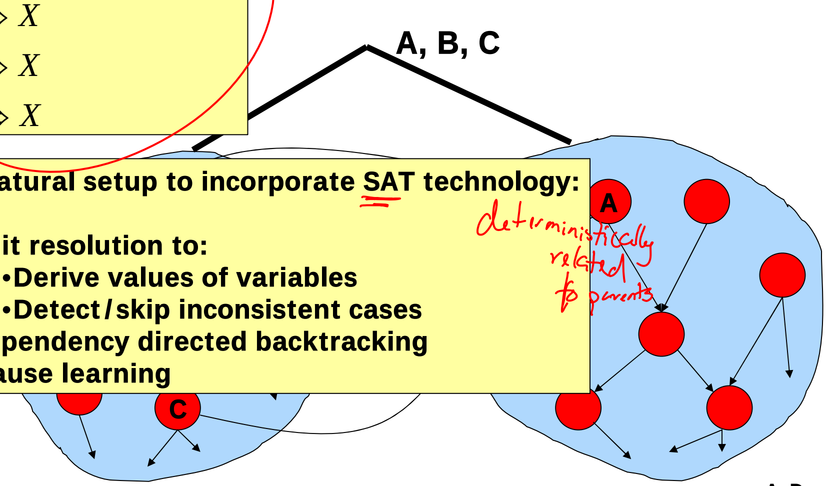
Determinism...

$$\neg A \wedge \neg B \wedge \neg C \Rightarrow \neg X$$
$$A \Rightarrow X$$
$$B \Rightarrow X$$
$$C \Rightarrow X$$



A natural setup to incorporate SAT technology:

- Unit resolution to:
  - Derive values of variables
  - Detect/skip inconsistent cases
- Dependency directed backtracking
- Clause learning

A. Darwiche

CSI Summary

- Exploit local structure inside CPT
  - Context-specific independence
  - Determinism
- Significantly speed-up inference
  - Tackle problems with tree-width in the thousands
- Acknowledgements
  - Recursive conditioning slides courtesy of Adnan Darwiche
  - Implementation available:
    - http://reasoning.cs.ucla.edu/ace