Algorithms: Assignment 1
Due date: September 7 (Thursday)

Problem 1 (3 points)
Let $A[1..n]$ be an array of n distinct numbers. If $i < j$ and $A[i] > A[j]$, then the pair (i, j) is called an *inversion*. For example, the array $\langle 2, 3, 8, 6, 1 \rangle$ contains five inversions. Write an algorithm $\text{Inversions}(A, n)$ that determines the number of inversions in $A[1..n]$.

Problem 2 (3 points)
Let $A[1..n]$ be a *sorted* array of n distinct numbers. Write an efficient algorithm $\text{Binary-Search}(A, n, k)$ that finds a given value k in the array $A[1..n]$. The algorithm should return the index of the found element; for example, if $A = \langle 1, 3, 4, 6, 9 \rangle$ and $k = 6$, then the returned index is 4, which means that $k = A[4]$. If the array does not include the value k, the algorithm should return 0.

Problem 3 (4 points)
Prove the following equalities:

(a) $1 + x + x^2 + x^3 + ... + x^n = \frac{x^{n+1}-1}{x-1}$ (where $x \neq 1$).

(b) $(1 + 2 + 3 + 4 + ... + n)^2 = 1^3 + 2^3 + 3^3 + 4^3 ... + n^3$.