Stochastic Packing-Market Planning

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The Problem

- Clever mechanisms for combinatorial auctions (CAs) have many nice properties, but require the bidders to know their preferences when submitting bids.
- Some markets have agents with probabilistic demand. (Preferences depend on future circumstances).
- How can we model and design good mechanisms for such markets?

Modeling the Problem: Bidder's view

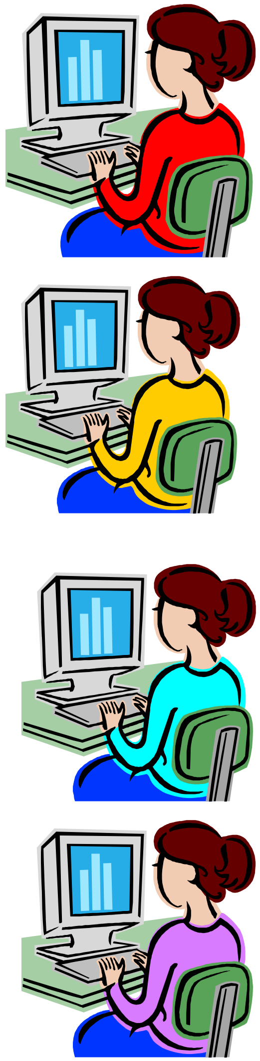
1

- Bidders bid on sets of goods, and provide probability estimates for each bid.
- Auctioneer collects bids, decides on winners, and *notifies* winning bidders.

2

- Bidders probabilistically commit to bids they've won.
- Auctioneer either executes a committed winning bid, or "buys it back" at a predetermined *compensation cost*.

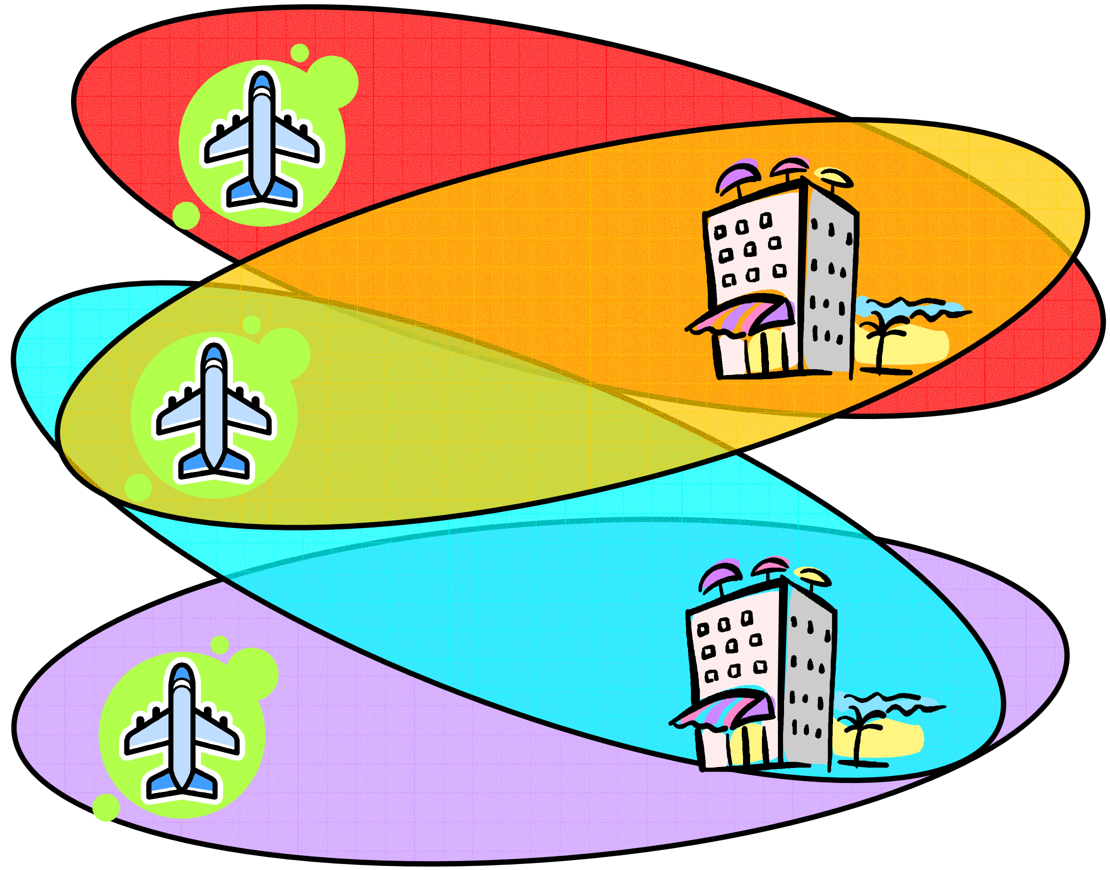
Bidders



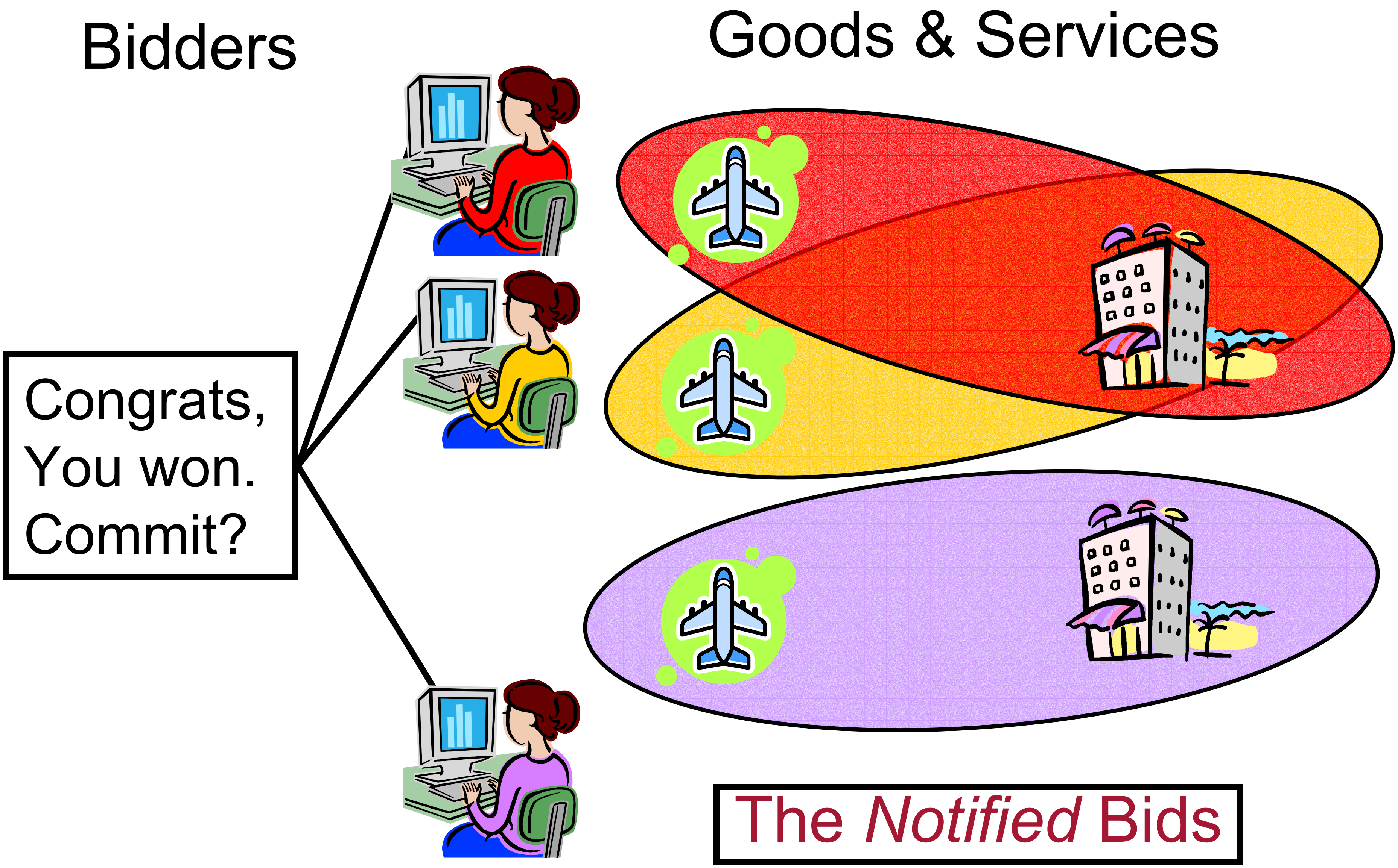
$400
50%

$500
30%

Goods & Services

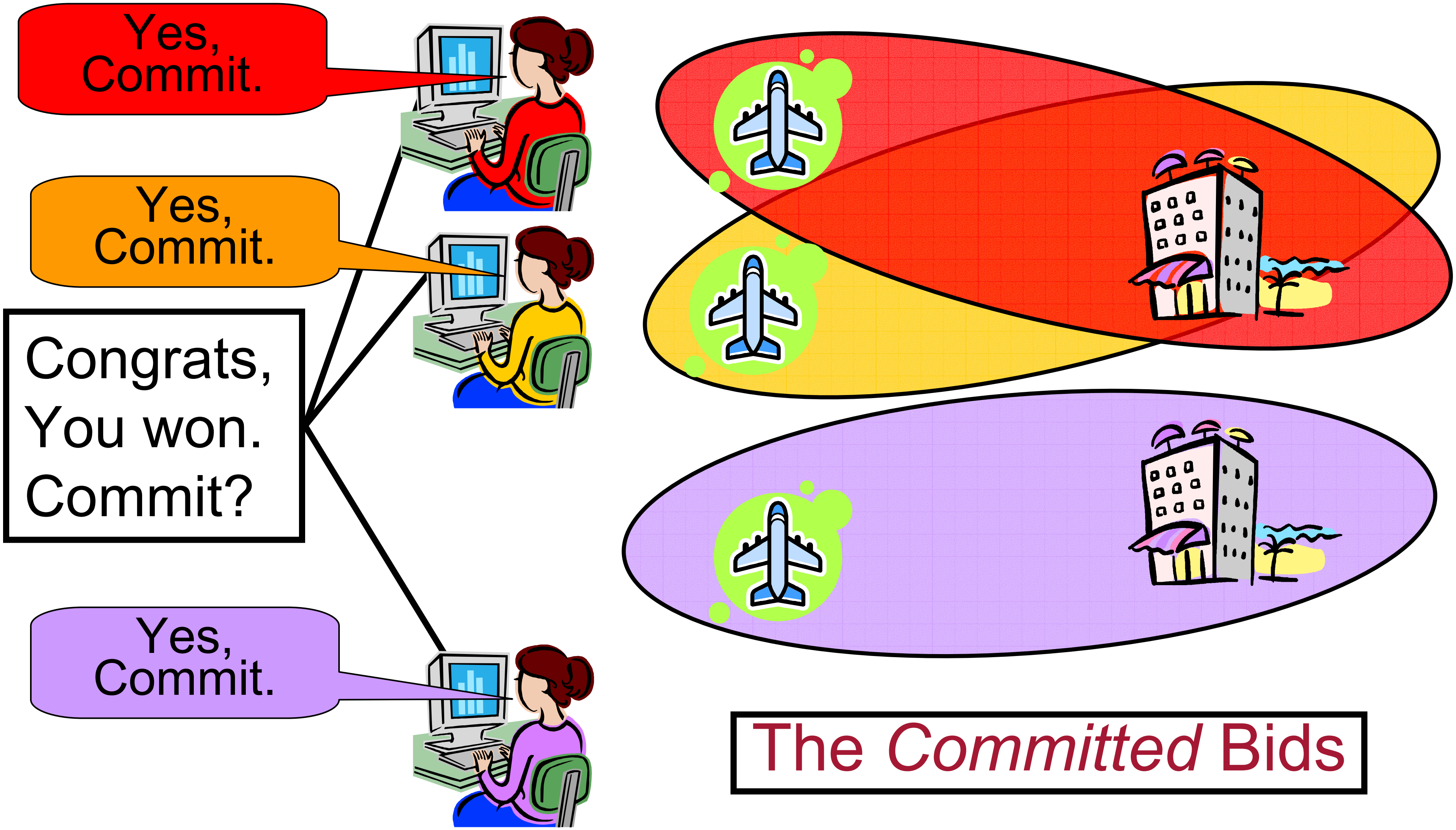


The Bids


Bidders
Goods & Services
Congrats,
You won.
Commit?
The *Notified* Bids

Bidders

Goods & Services



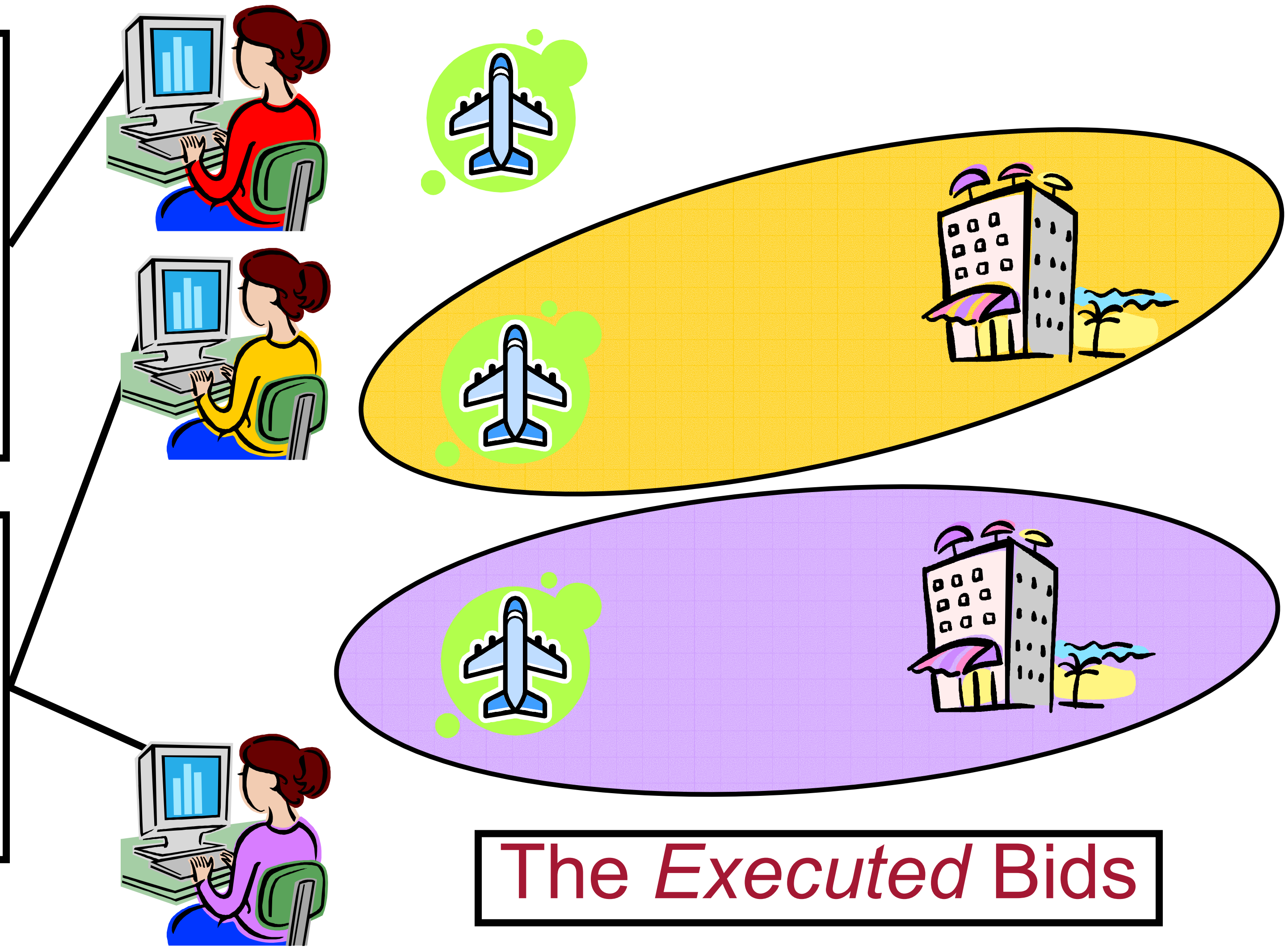
The *Committed* Bids

Bidders

Goods & Services

Sorry, here's $100 off your next trip.

Here's the bill. Have a good trip.



The *Executed* Bids

Why Bother?

More expressive bidding language

⬇

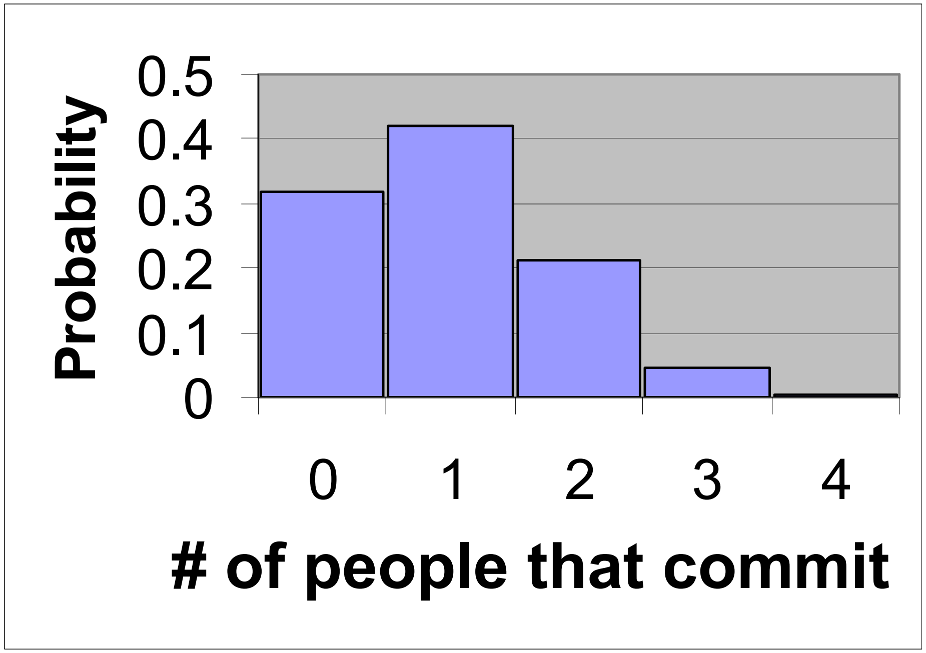
greater social welfare

Example:

“Last-minute” vacation packages:

What if many people have a small probabilistic demand for them?

- 4 people with a 25% chance to take the vacation package. One spot left.
- (Everyone bids, everyone notified, one bid executed if someone commits)



68% chance the spot is sold.

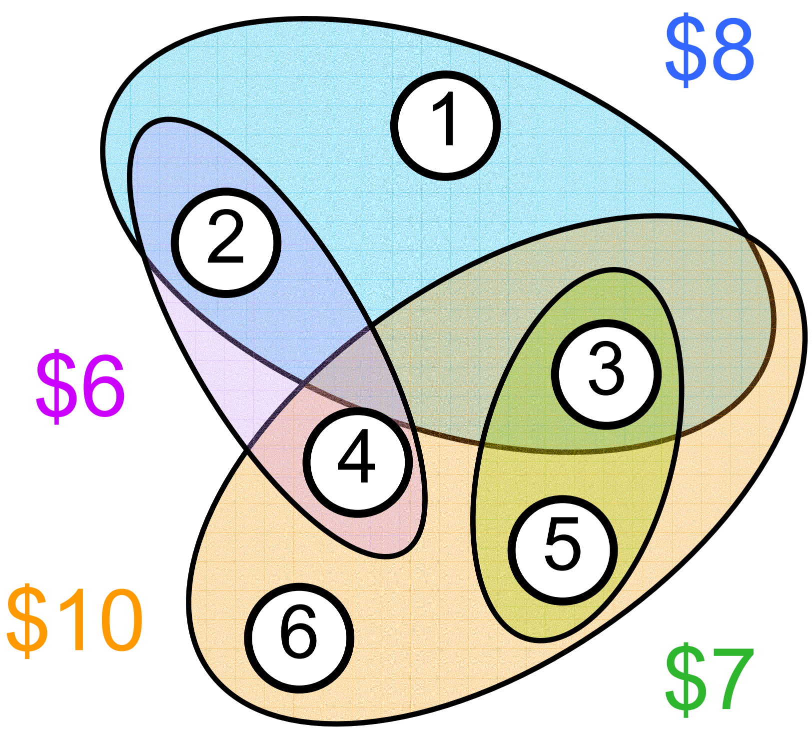
0.32 ticket “buy backs” in expectation.

The Auctioneer’s view of things

- Auctioneer selects:
  - which bids to notify.
  - which of the committed bids are executed.
- Goals:
  - (Approximately) maximize expected welfare (factoring in “buy back”/compensation costs)
  - Mechanism should be truthful.

Background

- Combinatorial Auctions as Set Packing



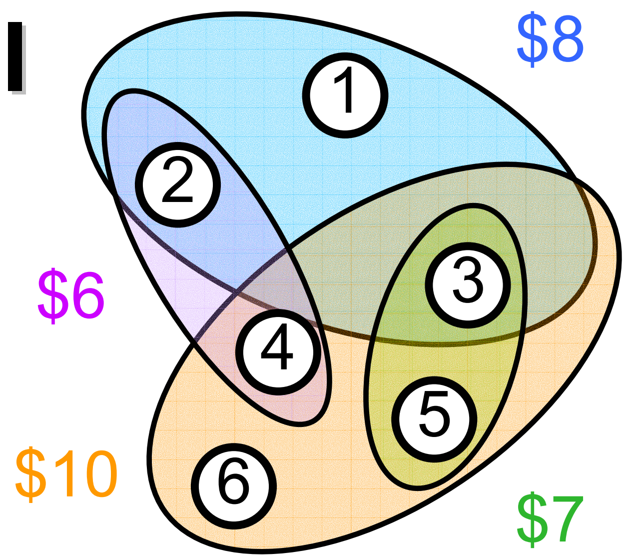
Previous Work:
k ≡ Max set size (k = 4 here)

Hurkens, Schrijver ('89)
k/2 Approx (unweighted)

Chandra, Halldórsson ('01)
2(k+1)/3 Approx (weighted)

Hazan, Safra, Schwartz ('06)
Ω(k/log(k)) Hardness

Background II

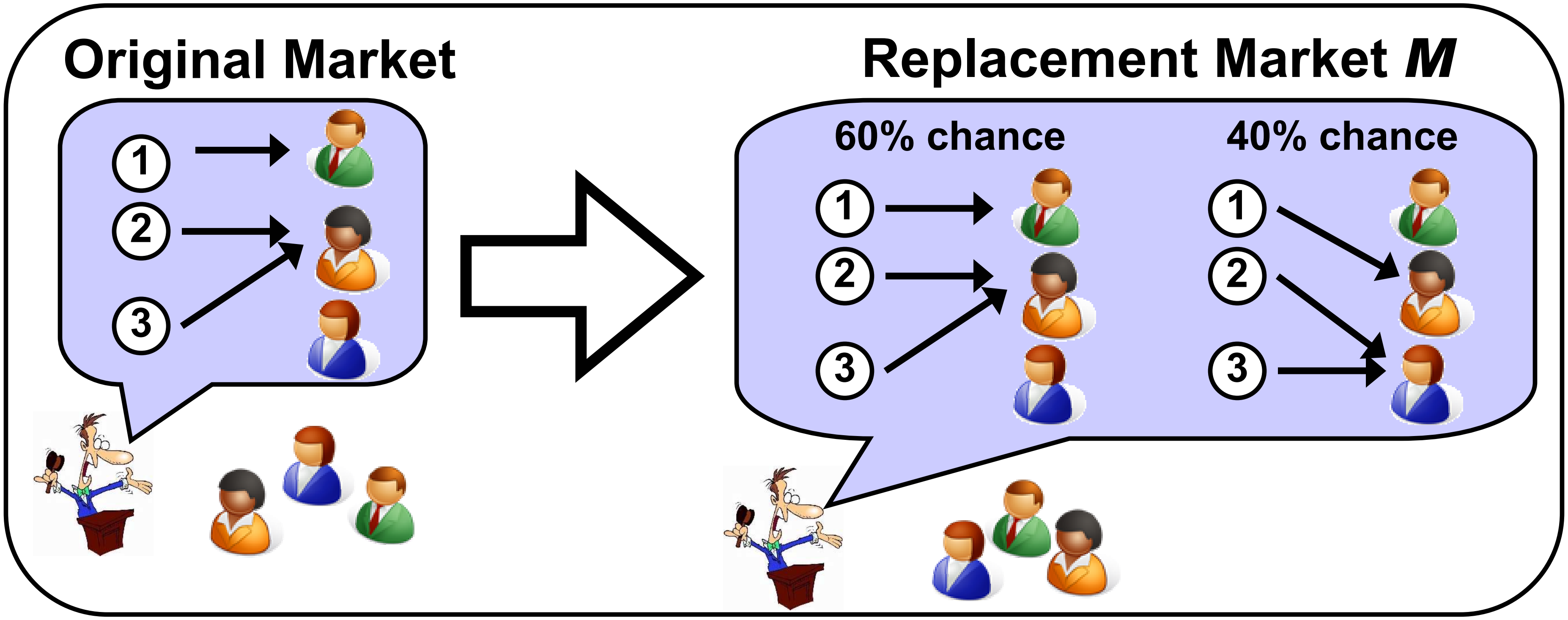


- Mechanism Design
  - Welfare Maximizing
  - Truthful
- Previous Work:
  - Vickrey-Clarke-Groves (‘61, ‘71, ‘73)
  - Lehmann, O’Callaghan, Shoham (’02)
  - Lavi & Swamy (‘05)
  - Dobzinski, Nisan, Schapira (‘06)
  - Feige (‘06)
  - …

Our Contributions

- New model for probabilistic markets
- Approx Algorithm nearly matching best k-Set Packing approx (8k vs. 2k/3)
- O(k)-approx mechanism that is truthful in expectation (with a caveat: the bidders must be exactly compensated for their disappointment)

Obtaining the Mechanism



Lavi & Swamy ('05):

- Efficient truthful mechanism for ***M*** (via LP relaxation and very clever rounding)
- Run that mechanism, sample from its outcome distrib to obtain the actual outcome. (truthful in expectation)

A Key Observation

Original Market ⇨ **Replacement Market *M***

Many suitable possibilities!

Sufficient conditions for ***M***:

- Outcomes are distributions over allocations and prices.
- Efficient truthful mechanism for ***M***
- Random sampling from any outcome of ***M*** is possible.

Resulting mech. is truthful in expectation

Must pick ***M*** carefully to ensure good welfare approx factor

The Original Market IP

- Exponentially many variables & constraints
- No time to go into details now…

maximize	$\sum_{b \in \text{Bids}} (w(b)s(b) - c(b)f(b))$	
subject to	$s(b) = \sum_{B \subseteq \text{Bids}} \mathbf{Pr}[B] \cdot y(B,b)$	$\forall\, b \in \text{Bids}$
	$f(b) = p(b)x(b) - s(b)$	$\forall\, b \in \text{Bids}$
	$0 \leq y(B,b) \leq x(b) \leq 1$	$\forall B \subseteq \text{Bids}, b \in \text{Bids}$
	$\sum_{b : b \in B, i \in b} y(B,b) \leq 1$	$\forall B \subseteq \text{Bids}, i \in \text{Items}$
	$y(B,b), x(b) \in \{0,1\}$	$\forall B \subseteq \text{Bids}, b \in \text{Bids}$

The Replacement Market LP

- Bidders bid on probability of notification (via w(*b*))
- We ensure: $\mathbf{Pr}[b \text{ executed}] \;\propto\; \mathbf{Pr}[b \text{ notified and committed}]$
- LP ⇨ VCG mech. is feasible
- Use VCG mech w/randomized rounding

$$\begin{aligned} \text{maximize} \quad & \textstyle\sum_{b \in \text{Bids}} w(b) x(b) & \\ \text{subject to} \quad & 0 \le x(b) \le p(b) & \forall\, b \in \text{Bids} \\ & \textstyle\sum_{b : i \in b} x(b) \le 1/2k & \forall\, i \in \text{Items} \end{aligned}$$

w(*b*) := Value of bid *b*.

Variable x(*b*): "Should we notify bid *b*?"

p(*b*) := Probability *b* is committed to.

Conclusions

- First Model to represent CAs with probabilistic supply/demand
- Evidence that probabilistic supply/demand doesn't make the problem much harder in theory (from an approximation standpoint)

Open Problems

- Estimate bid probabilities in a way that's robust against strategic manipulation.
- Handle more complex distributions over bid commitments.
- Obtain full truthfulness (not "in expectation"), remove the caveat.
- Online variants.

Thank You

Questions?