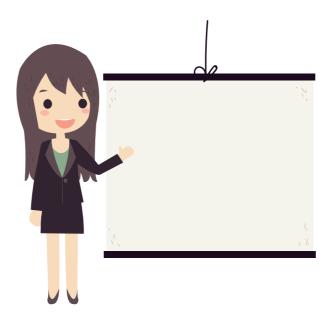
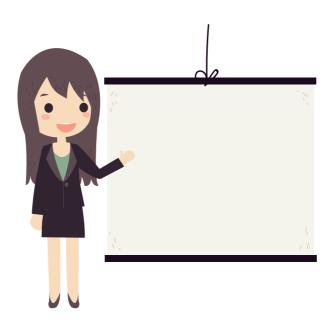
Shared Session Types for Safe, Practical Concurrency¹

Stephanie Balzer Carnegie Mellon University

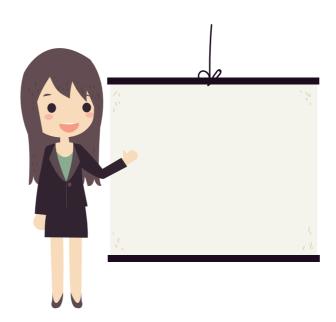
Typelevel Summit Philadelphia 2019





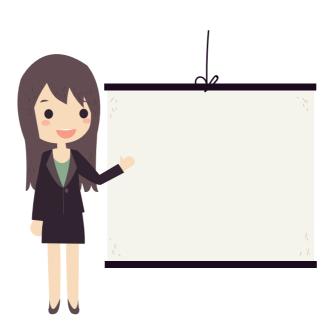








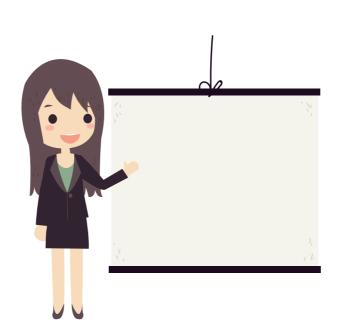






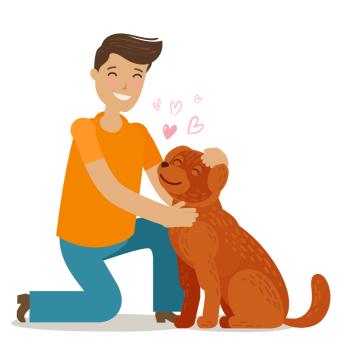






















The world surrounding us is inherently concurrent











Many programming problems demand concurrency

• Flight booking system, online store, search engines, etc.

The world surrounding us is inherently concurrent











Many programming problems demand concurrency

Flight booking system, online store, search engines, etc.

Computing devices themselves are concurrent

Run various apps concurrently

The world surrounding us is inherently concurrent









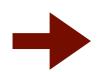


Many programming problems demand concurrency

Flight booking system, online store, search engines, etc.

Computing devices themselves are concurrent

Run various apps concurrently



programming languages must support concurrency

The world surrounding us is inherently concurrent











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Flight booking system, online store, search engines, etc.

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Run various apps concurrently

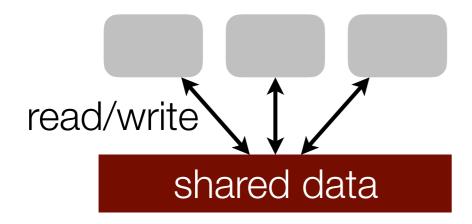


programming languages must support concurrency



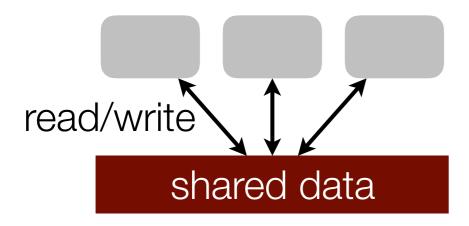
concurrent programming is notoriously difficult and error-prone

Shared memory

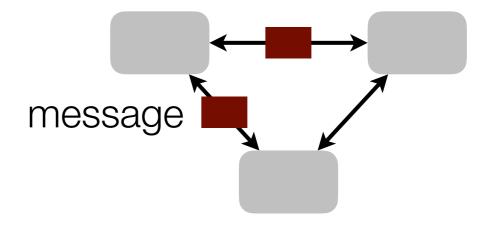




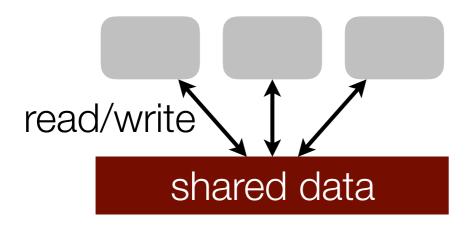
Shared memory



Message-passing

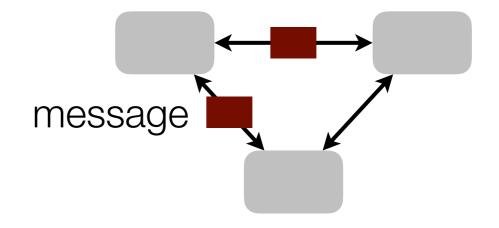


Shared memory



 computation by reading from and writing to shared data

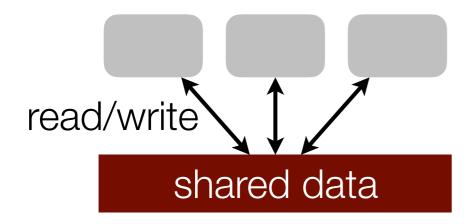
Message-passing



Legend:

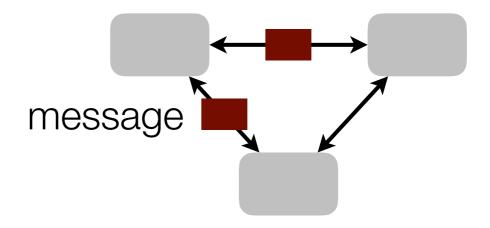


Shared memory



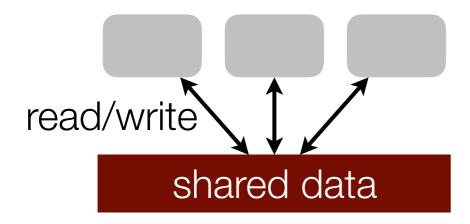
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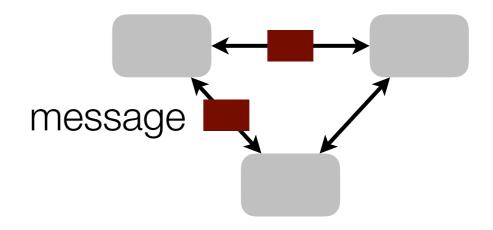
 computation by exchange of messages

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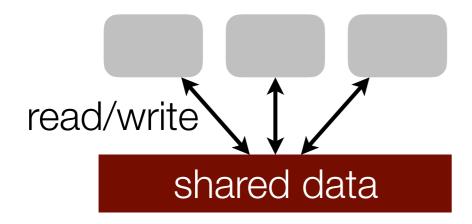


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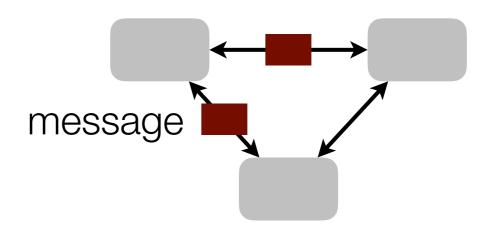
message-passing offers higher-level of abstraction

Shared memory



 computation by reading from and writing to shared data

Message-passing



 computation by exchange of messages



message-passing offers higher-level of abstraction



message-passing adopted by practical languages such as Erlang, Go, and Rust.

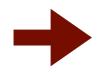
Legend:



concurrently executing component

Goal: make concurrent programming safe and practical

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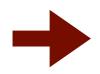


message-passing model



session types to express protocols of message exchange and reason sequentially about communicating parties

Goal: make concurrent programming safe and practical



message-passing model



session types to express protocols of message exchange and reason sequentially about communicating parties

Contributions:

Goal: make concurrent programming safe and practical



message-passing model



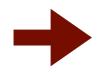
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Contributions:



shared session types

Goal: make concurrent programming safe and practical



message-passing model



session types to express protocols of message exchange and reason sequentially about communicating parties

Contributions:



shared session types



accommodate real-world programming scenarios



guarantee protocol adherence, data-race-freedom, and deadlock-freedom

Session types, what are they? Why do we need them in practice?

- Servo is Mozilla's next-generation browser engine under development and implemented in Rust.
- Servo uses message-passing concurrency for maximal parallelization of tasks, such as loading and rendering of webpage elements.

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Image loader:

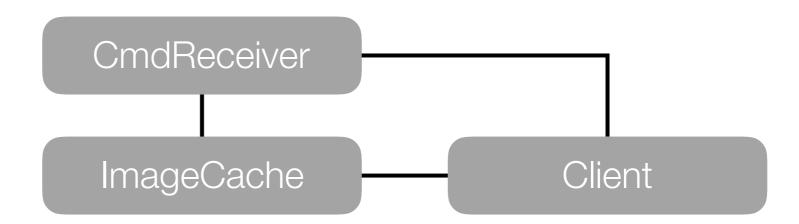
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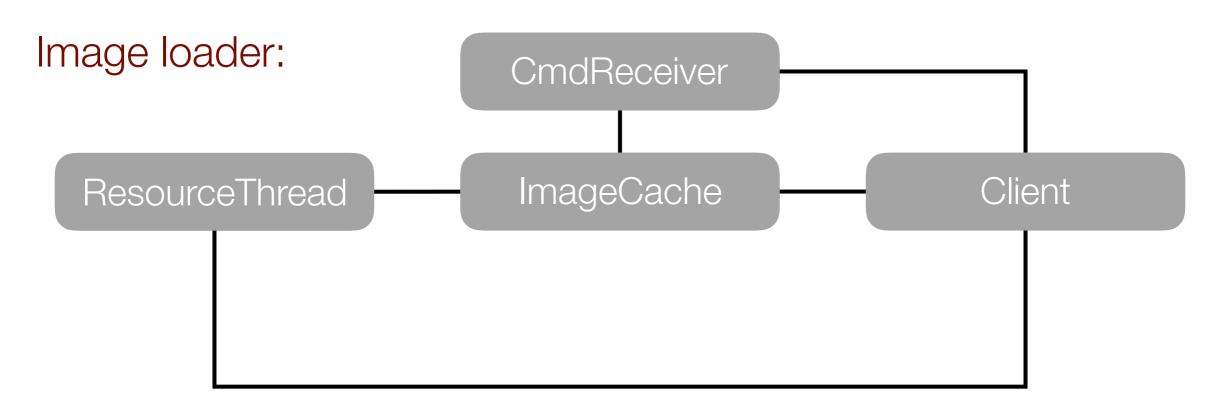
ImageCache

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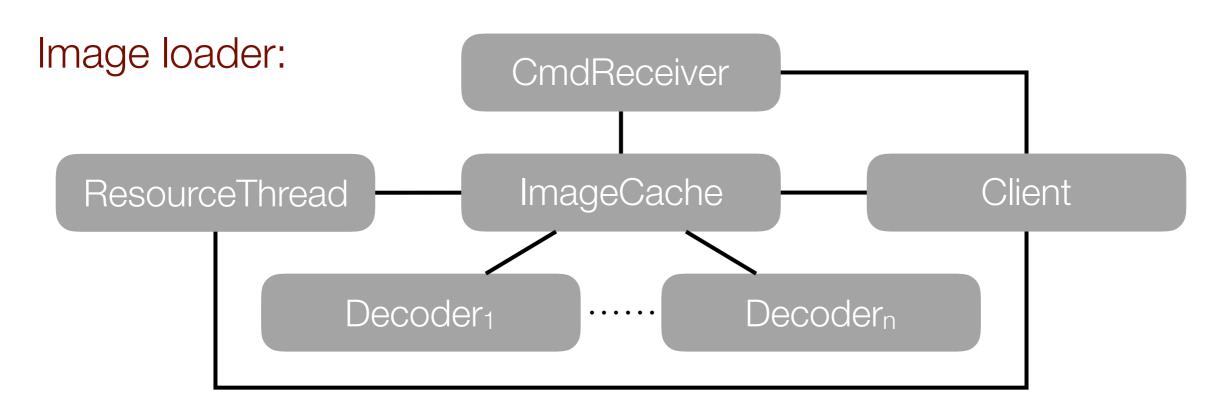
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Legend:

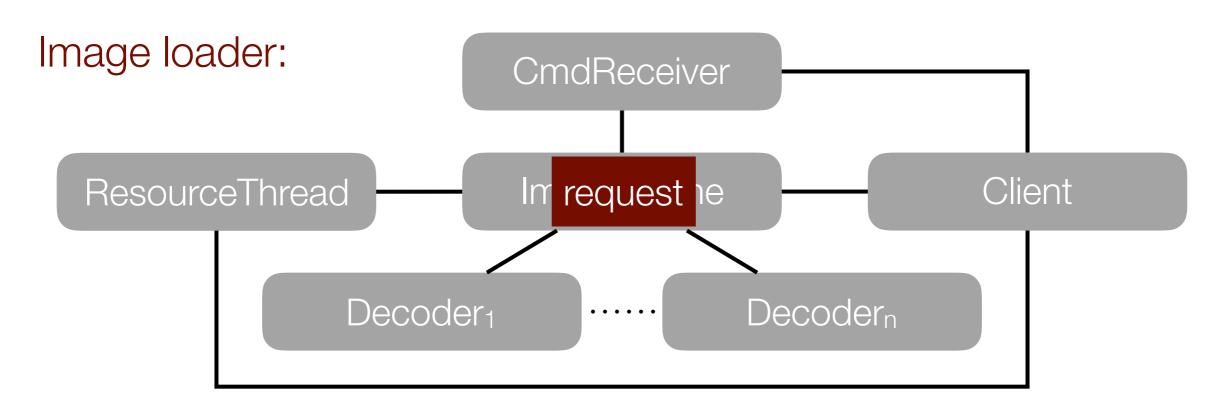
component, runs in separate thread — channel

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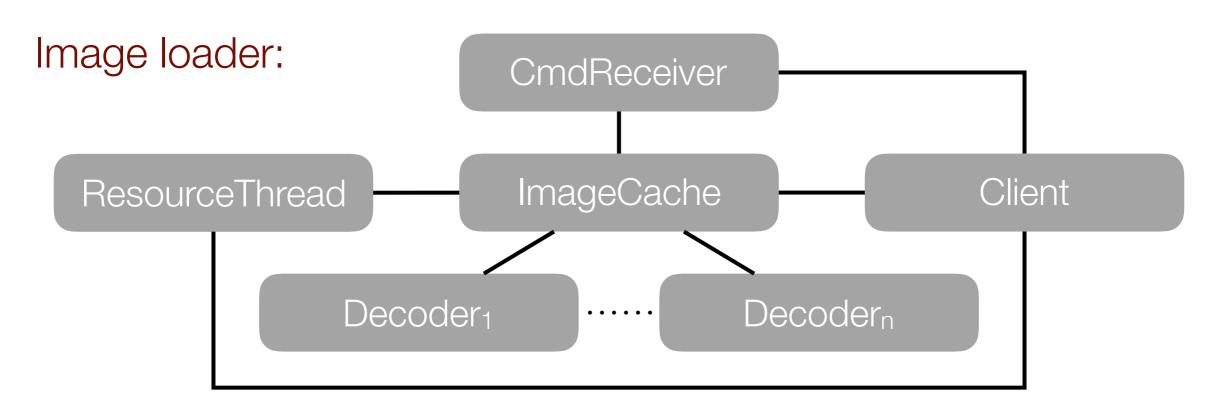
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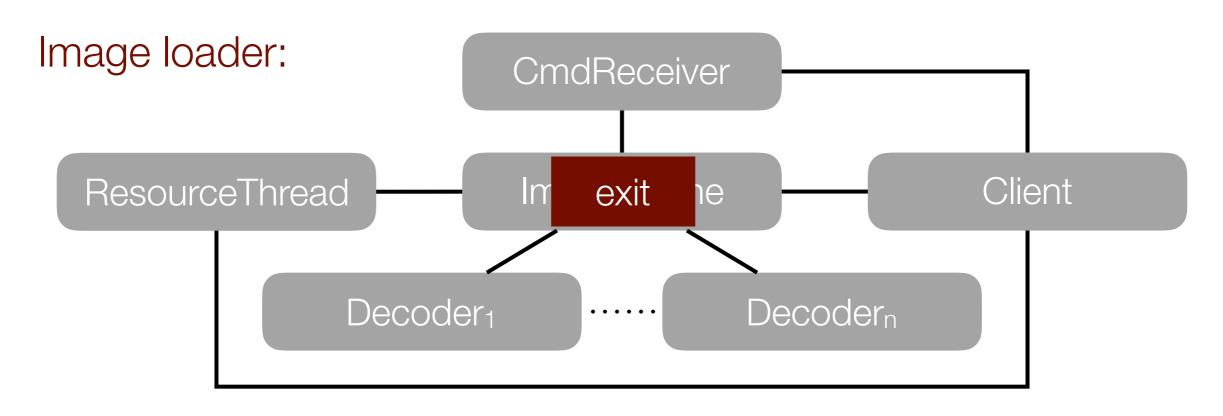
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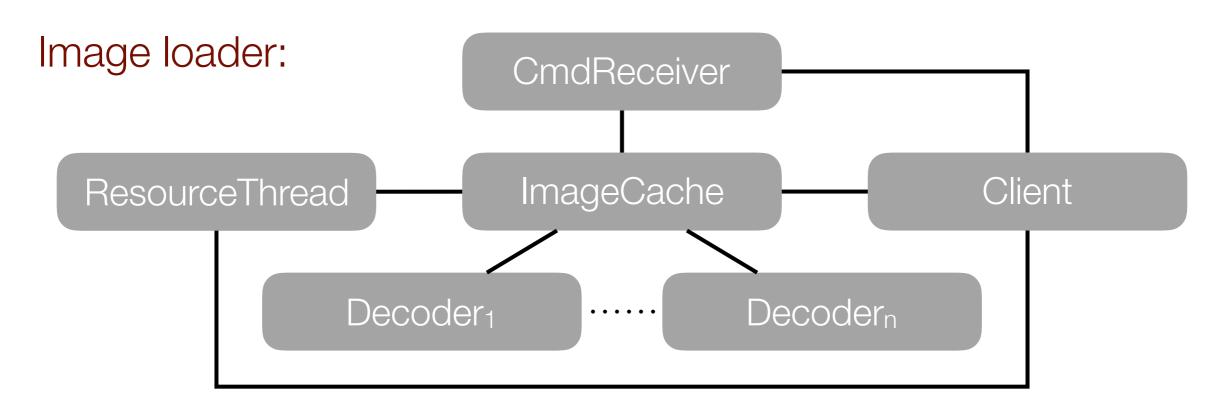


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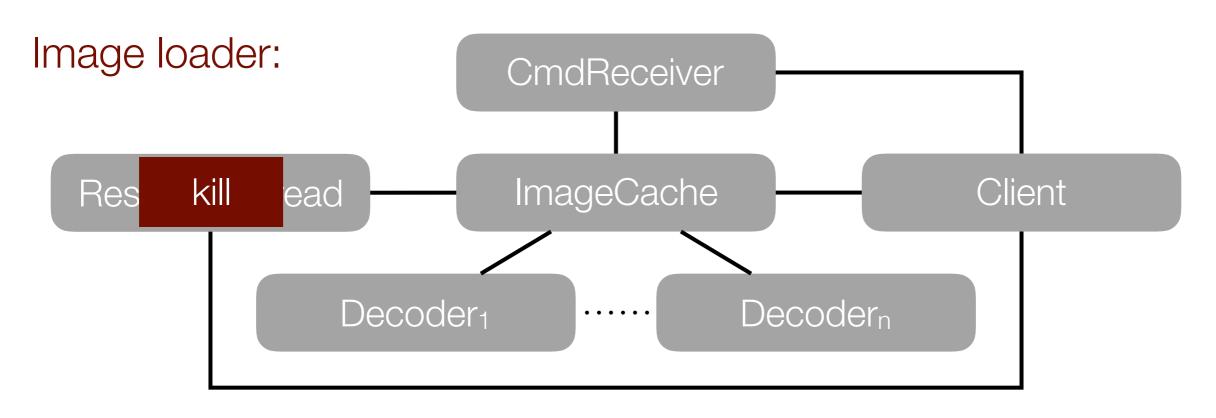
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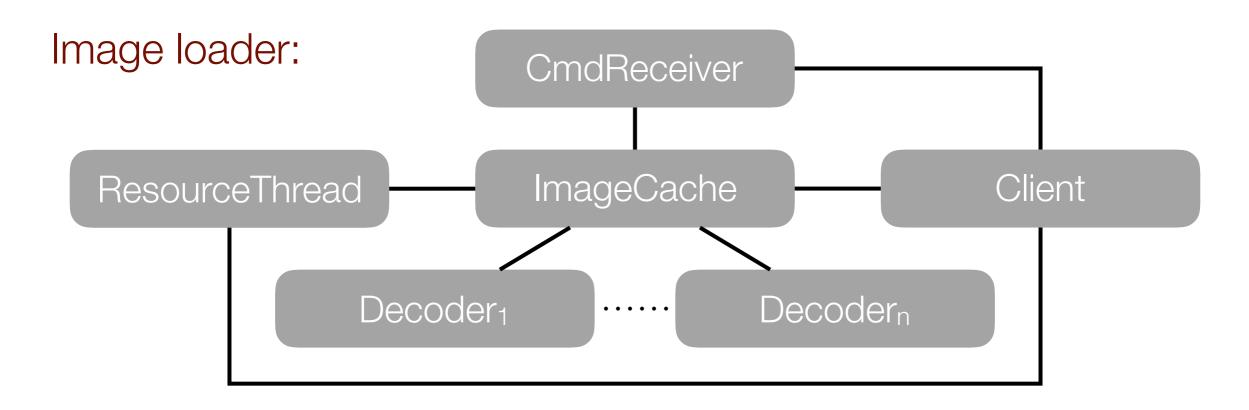


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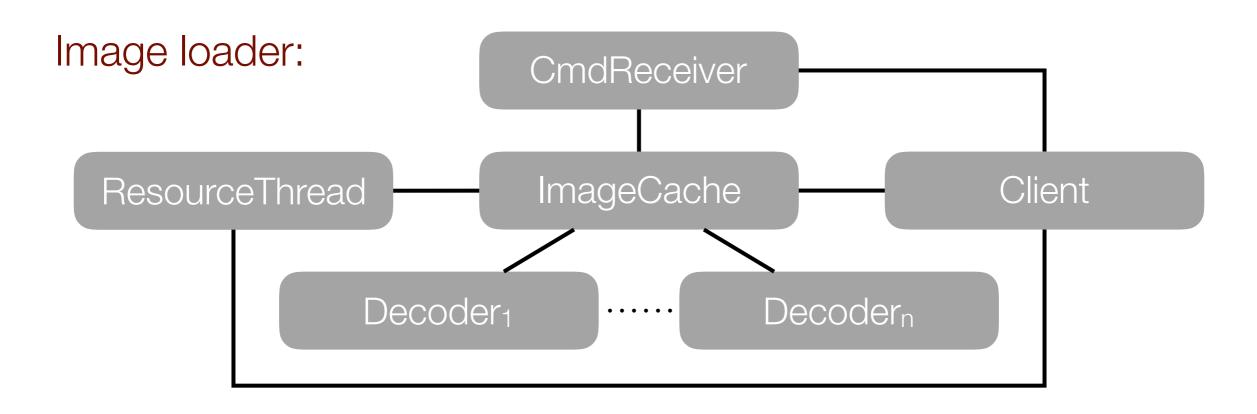
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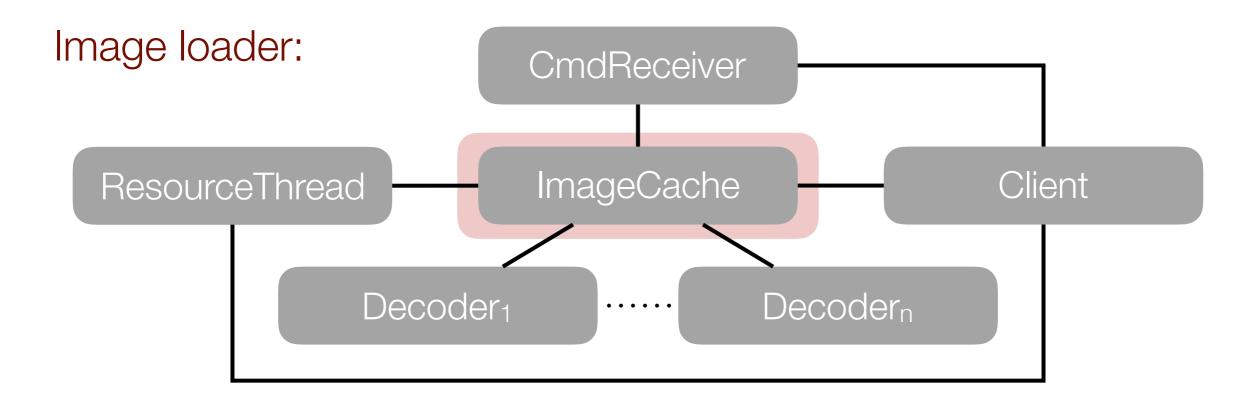


To restrict the kinds of messages that can be sent over a channel, Rust channels are typed with enumeration types.

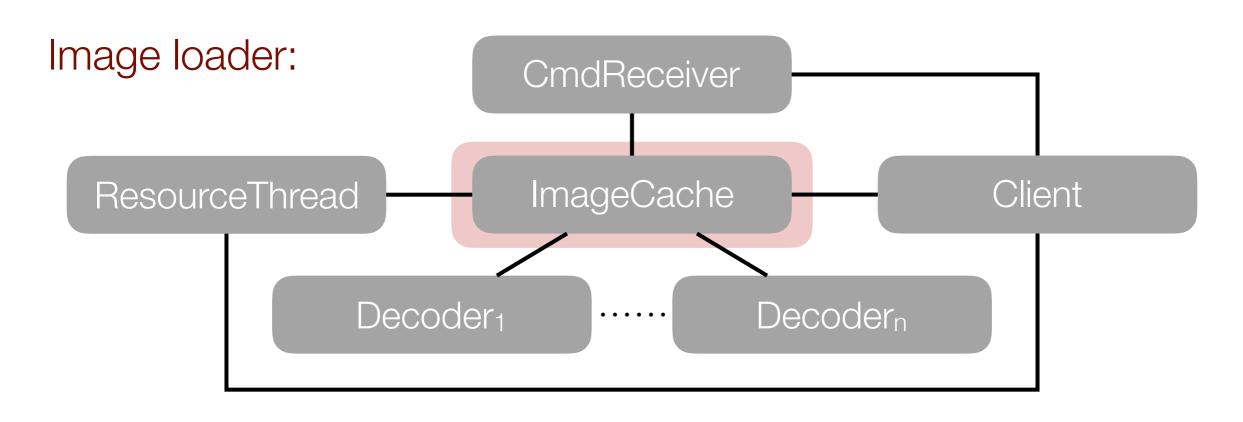


Legend: compo

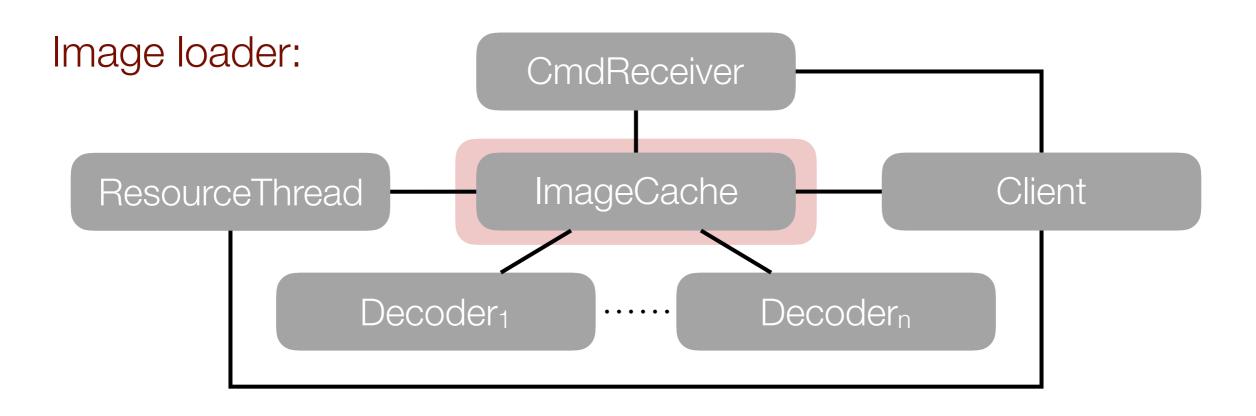
- **→**
- To restrict the kinds of messages that can be sent over a channel, Rust channels are typed with enumeration types.
- Example: enumeration for ImageCache



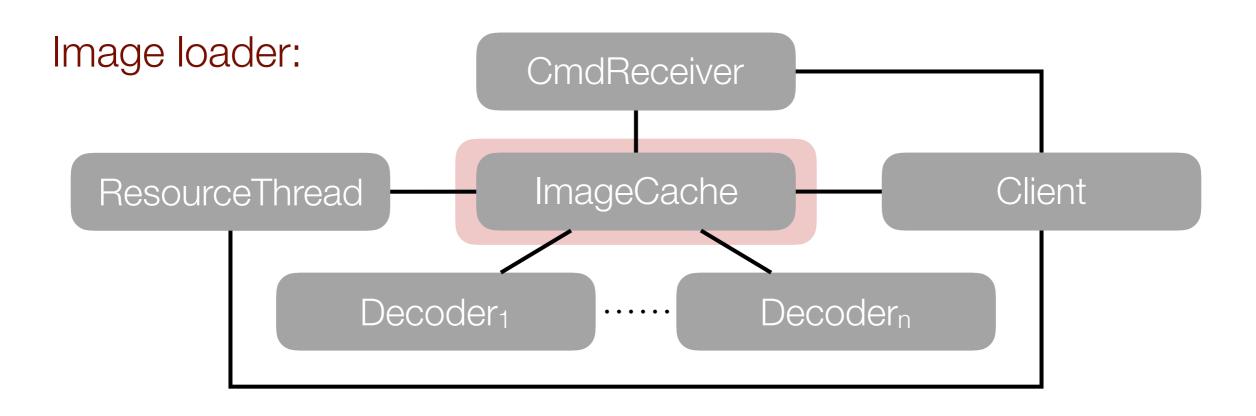
Legend:

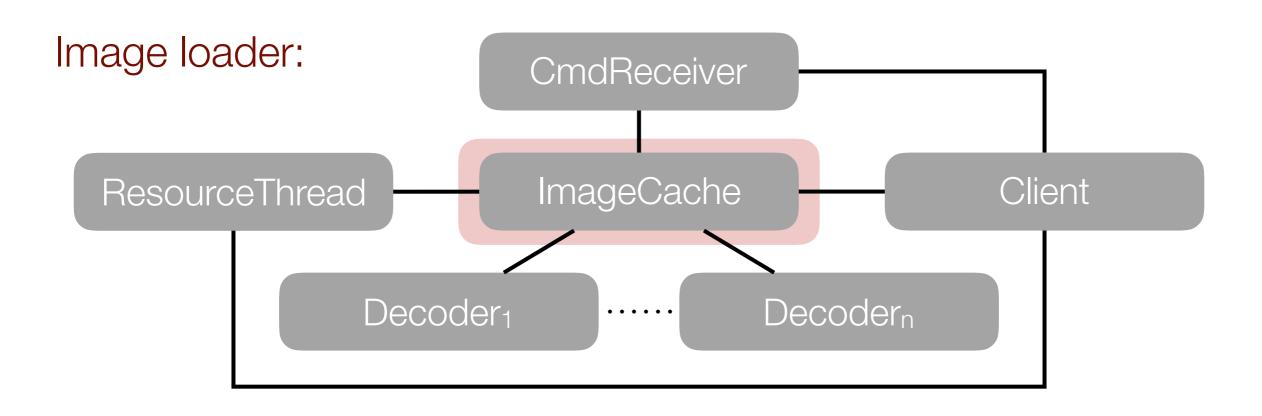


```
pub enum ImageCacheCommand {
   RequestImage (Url, ImageCacheChan, Option<ImageResponder>),
   GetImageIfAvailable (Url, UsePlaceholder, IpcSender<Result<Arc<Image>, ImageState>>),
   StoreDecodeImage (Url, Vec<u8>),
   ...
   // Clients must wait for a response before shutting down ResourceThread
   Exit ()
}
```

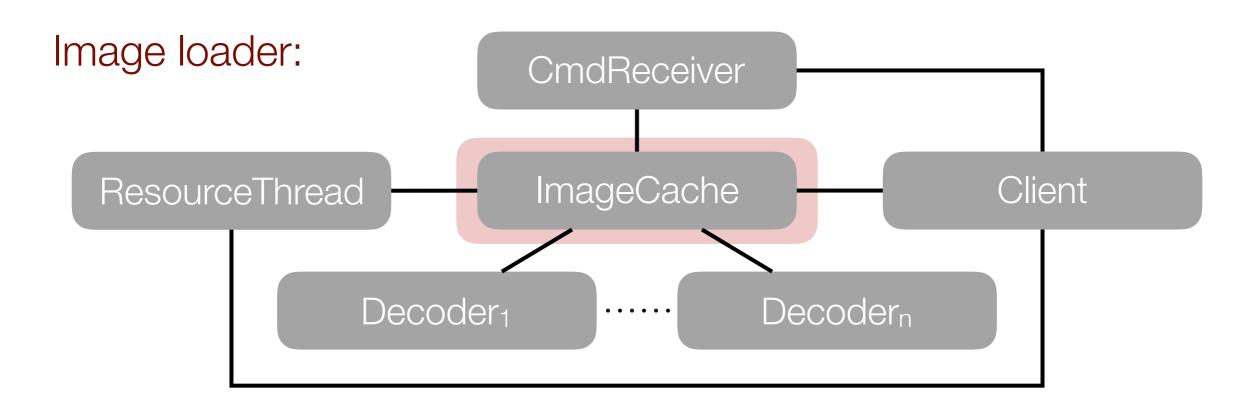


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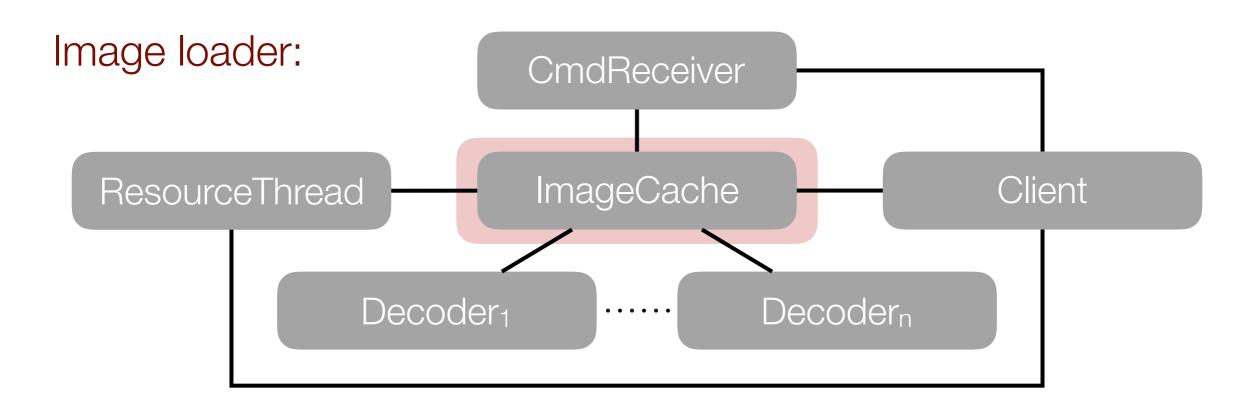




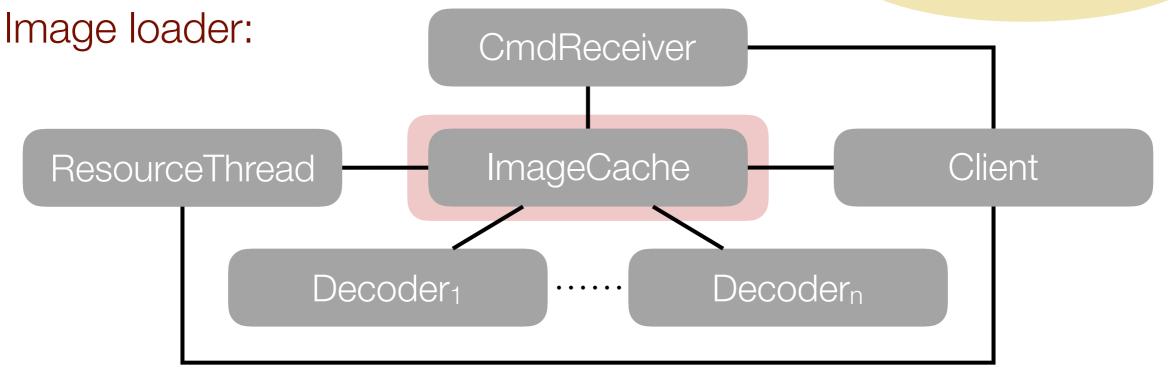
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 // Clients must wait for a response before shutting down ResourceThread
 Exit()
                                                              implicit protocol
   Image loader:
                                   CmdReceiver
                                   ImageCache
      ResourceThread
                                                                  Client
                                                  Decodern
                        Decoder<sub>1</sub>
```





enumeration types ensure that only defined messages can be communicated along a channel

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- enumeration types fail to ensure that messages are sent according to the intended protocol

- enumeration types ensure that only defined messages can be communicated along a channel
- enumeration types fail to ensure that messages are sent according to the intended protocol
- let's use session types!



Session types define protocols of message exchange.



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$$A, B \triangleq \&\{\overline{l_i : A_i}\}$$
 external choice $\oplus\{\overline{l_i : A_i}\}$ internal choice $A \multimap B$ channel input $A \otimes B$ channel output $\mathbf{1}$



Session types define protocols of message exchange.



"protocol = sequence of actions"

$$A,B riangleq riangleq riangle riangle \{ \overline{l_i : A_i} \} \ A \multimap B \ A \otimes B \ 1$$

external choice
internal choice
channel input
channel output
termination

client chooses among sending one of the labels *l_i*



Session types define protocols of message exchange.



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provider chooses among sending one of the labels *l*_i



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client sends channel reference of type A



Session types define protocols of message exchange.



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provider terminates



Session types define protocols of message exchange.



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Session types define protocols of message exchange.



$$A, B \triangleq \&\{\overline{l_i : A_i}\}$$
 external choice $\oplus \{\overline{l_i : A_i}\}$ internal choice $A \multimap B$ channel input $A \otimes B$ channel output $T \to A$ value input $T \times A$ value output $T \Leftrightarrow A$ internal choice internal choice $A \to B$ channel input $A \otimes B$ channel output $A \otimes B$ channel output $A \otimes B$ value input $A \otimes B$ value output $A \otimes B$ value output $A \otimes B$ value output $A \otimes B$ internal choice inte



Session types define protocols of message exchange.



	A, B	\triangleq	$\&\{\overline{l_i:A_i}\}$	external choice
support communicati values			$\oplus \{\overline{l_i:A_i}\}$	internal choice
			$A \multimap B$	channel input
			$A\otimes B$	channel output
			1	termination
			$T \to A$	value input
			$T \times A$	value output
	T	\triangleq	int string	

Session type for image loader

A, B	\triangle	$\&\{\overline{l_i:A_i}\}$	external choice
		$\oplus \{\overline{l_i:A_i}\}$	internal choice
		$A \multimap B$	channel input
		$A\otimes B$	channel output
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Session type for image loader

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ImgCacheCmd =

```
A, B \triangleq \&\{\overline{l_i : A_i}\} external choice \oplus \{\overline{l_i : A_i}\} internal choice A \multimap B channel input A \otimes B channel output T \to A value input T \times A value output T \triangleq \inf |\operatorname{string}| \dots
```

 $ImgCacheCmd = \& \{$

}

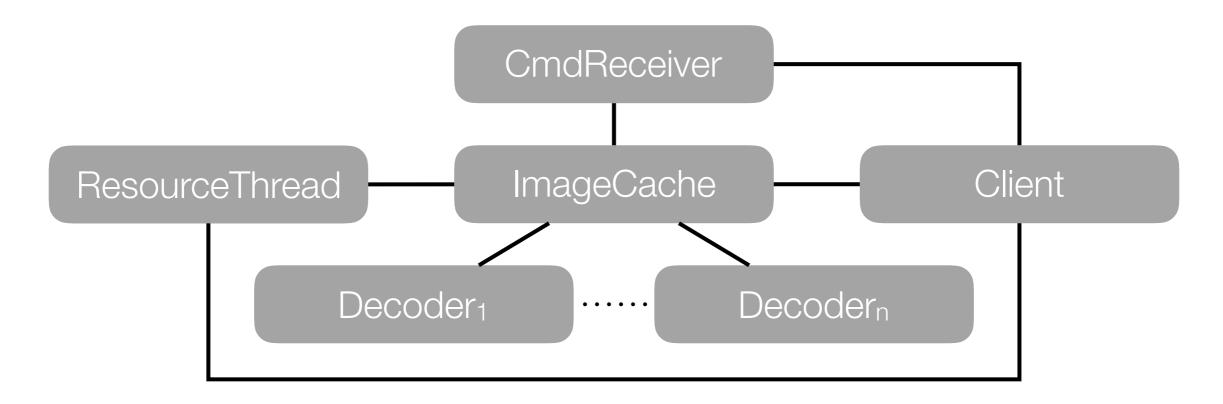
```
\label{eq:local_local_local_local} Img Cache Cmd = \& \; \{ Request Image : string \rightarrow Requester \multimap Img Cache Cmd, \\ \dots \\ Exit : \\ \  \  \, \}
```

```
A, B \triangleq \&\{l_i : A_i\}
                             \oplus \{\overline{l_i:A_i}\}
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                             A \multimap B
                                                 channel input
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                                                 value output
                        \triangleq int | string | . . .
ImgCacheCmd = \& \{RequestImage : string \rightarrow Requester \rightarrow ImgCacheCmd, \}
                       Exit : ⊕ {Running : ImgCacheCmd,
                                  Done : ResourceThread \otimes 1
```

external choice

```
\label{eq:local_cond} ImgCacheCmd = \& \; \{ RequestImage : string \rightarrow Requester \longrightarrow ImgCacheCmd, \\ \dots \\ Exit : \oplus \; \{ Running : ImgCacheCmd, \\ Done : ResourceThread \otimes \mathbf{1} \} \\ \}
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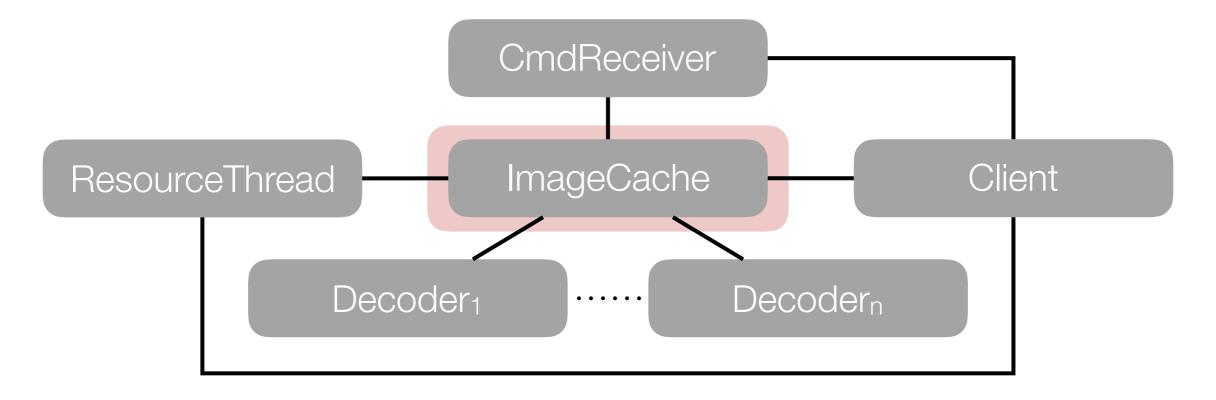
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\label{eq:local_cond} ImgCacheCmd = \& \; \{ RequestImage : string \rightarrow Requester \longrightarrow ImgCacheCmd, \\ \dots \\ Exit : \oplus \; \{ Running : ImgCacheCmd, \\ Done : ResourceThread \otimes \mathbf{1} \} \\ \}
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ImgCacheCmd = \& \{RequestImage : string \rightarrow Requester \rightarrow ImgCacheCmd, \}
                          Exit : \( \operatorname{\text{Running}} \) Running : ImgCacheCmd,
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ImageCache:
                                      CmdReceiver
                                       ImageCache
     ResourceThread
                                                                          Client
                         Decoder<sub>1</sub>
                                                        Decodern
```

```
\label{eq:lmgCacheCmd} ImgCacheCmd = \& \; \{ RequestImage : string \rightarrow Requester \longrightarrow ImgCacheCmd, \\ \dots \\ Exit : \oplus \; \{ Running : ImgCacheCmd, \\ Done : ResourceThread \otimes \mathbf{1} \} \\ \}
```

ImageCache: ImgCacheCmd



Decoder₁

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ImgCacheCmd = \& \{RequestImage : string \rightarrow Requester \rightarrow ImgCacheCmd,
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ImageCache: ImgCacheCmd
                                 CmdReceiver
                                RequestImage
     ResourceThread
                                                                Client
```

Decodern

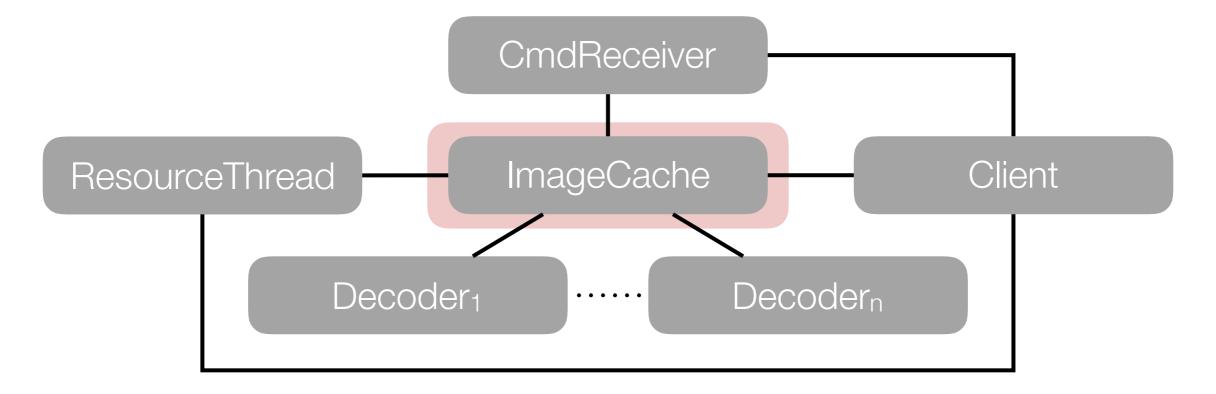
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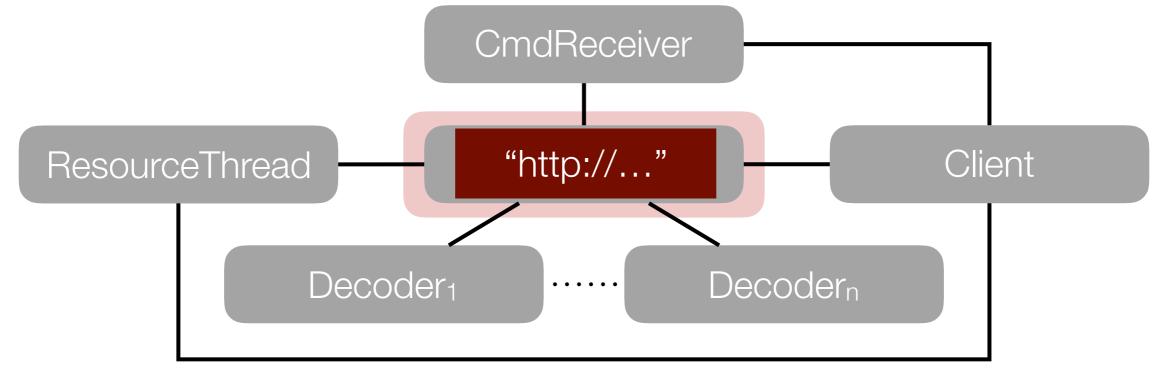
Decodern

```
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```

ImageCache: string → Requester → ImgCacheCmd



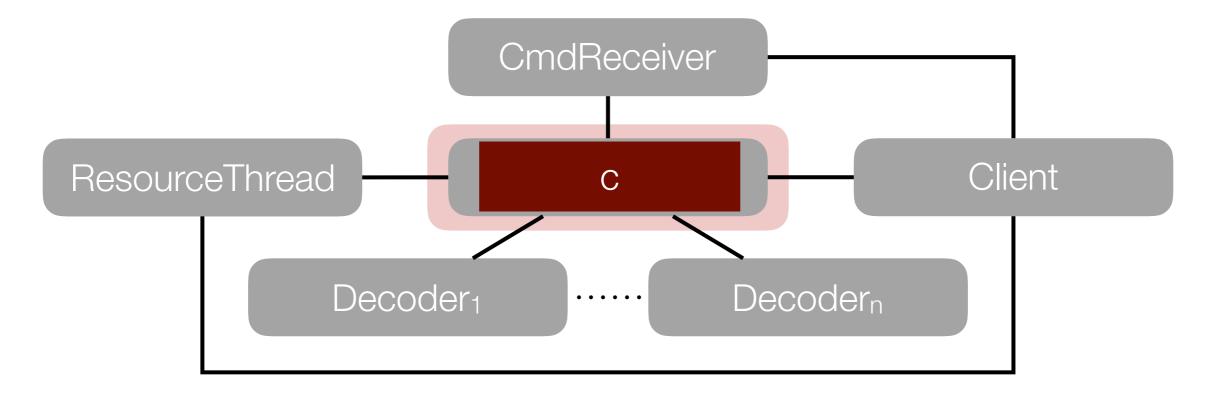
```
\label{eq:local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_local_
```



```
ImgCacheCmd = \& \{RequestImage : string \rightarrow Requester \rightarrow ImgCacheCmd,
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                                  Done : ResourceThread \otimes 1
ImageCache: Requester — ImgCacheCmd
                                 CmdReceiver
                                  ImageCache
     ResourceThread
                                                                 Client
                      Decoder<sub>1</sub>
                                                 Decodern
```

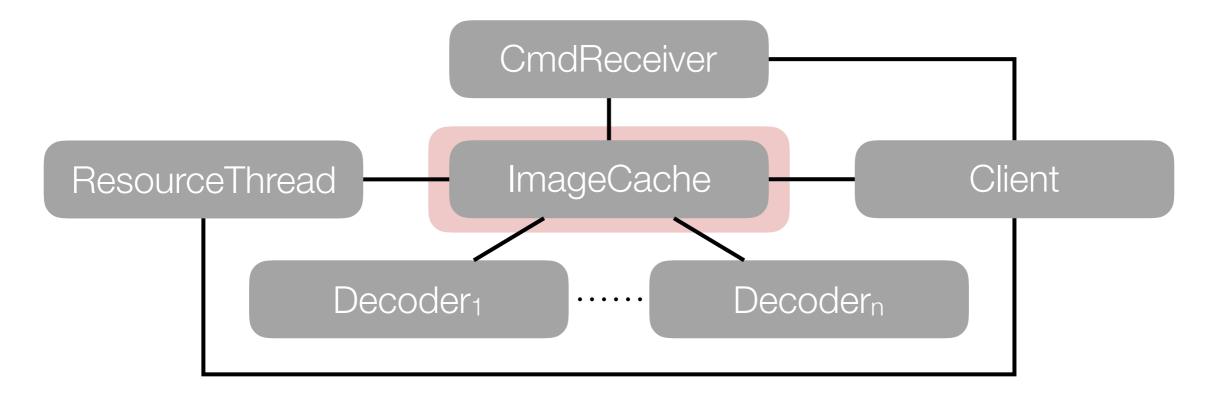
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\label{eq:lmgCacheCmd} ImgCacheCmd = \& \; \{ RequestImage : string \rightarrow Requester \longrightarrow ImgCacheCmd, \\ \qquad \qquad \qquad \qquad \\ Exit : \oplus \; \{ Running : ImgCacheCmd, \\ \qquad \qquad \qquad \qquad \\ Done : ResourceThread \otimes \mathbf{1} \} \\ \qquad \qquad \}
```

ImageCache: Requester — ImgCacheCmd



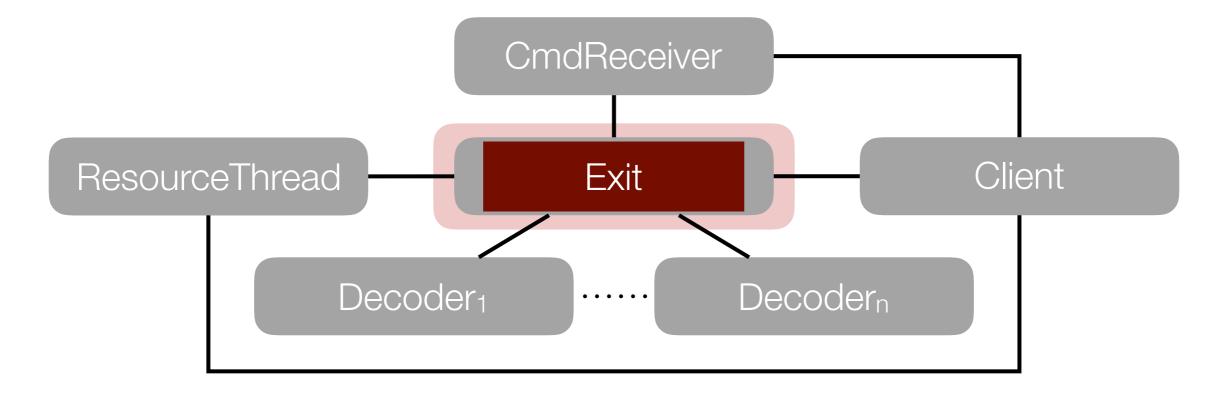
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\label{eq:local_cond} ImgCacheCmd = \& \; \{ RequestImage : string \rightarrow Requester \longrightarrow ImgCacheCmd, \\ \dots \\ Exit : \oplus \; \{ Running : ImgCacheCmd, \\ Done : ResourceThread \otimes \mathbf{1} \} \\ \}
```

ImageCache: ImgCacheCmd



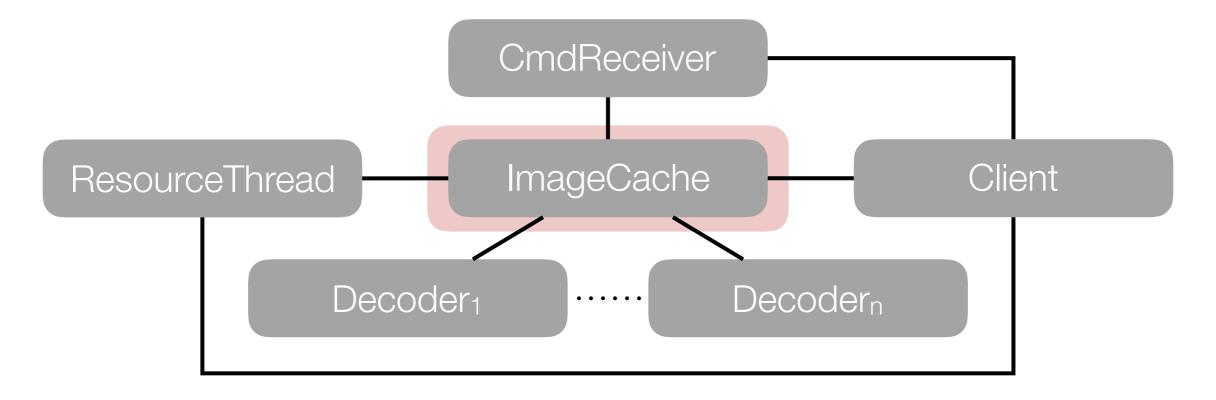
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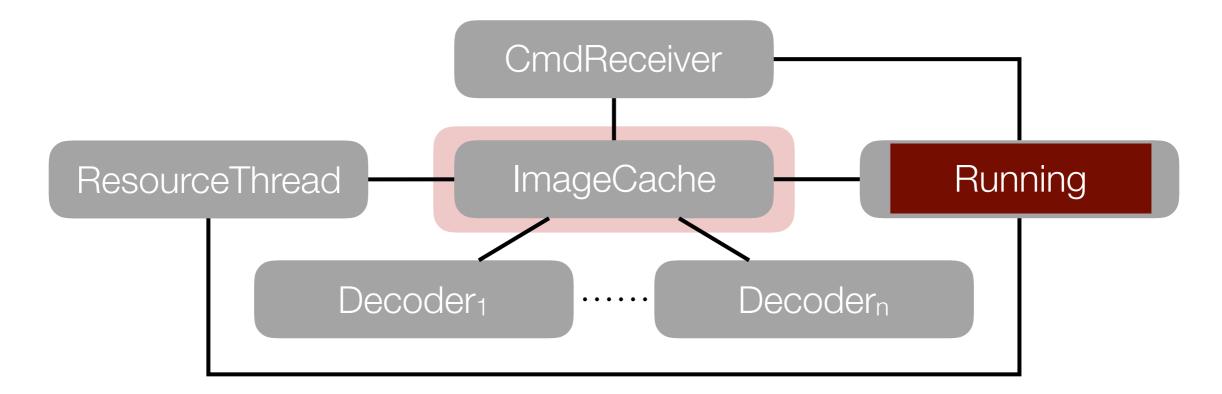
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```

 $ImageCache: \ \oplus \{Running: ImgCacheCmd, Done: ResourceThread \otimes \mathbf{1}\}$



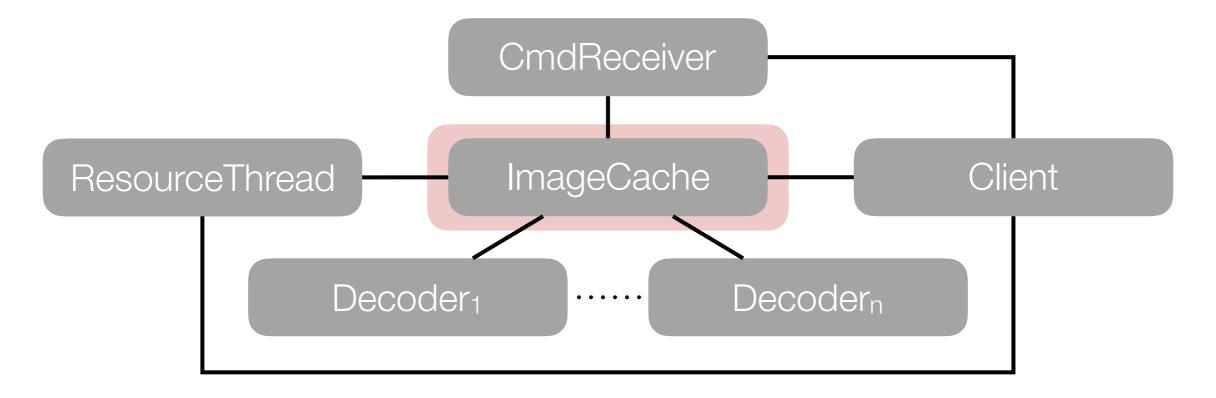
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\label{eq:lmgCacheCmd} ImgCacheCmd = \& \; \{ RequestImage : string \rightarrow Requester \longrightarrow ImgCacheCmd, \\ \qquad \qquad \qquad \qquad \\ Exit : \oplus \; \{ Running : ImgCacheCmd, \\ \qquad \qquad \qquad Done : ResourceThread \otimes \mathbf{1} \} \\ \qquad \qquad \}
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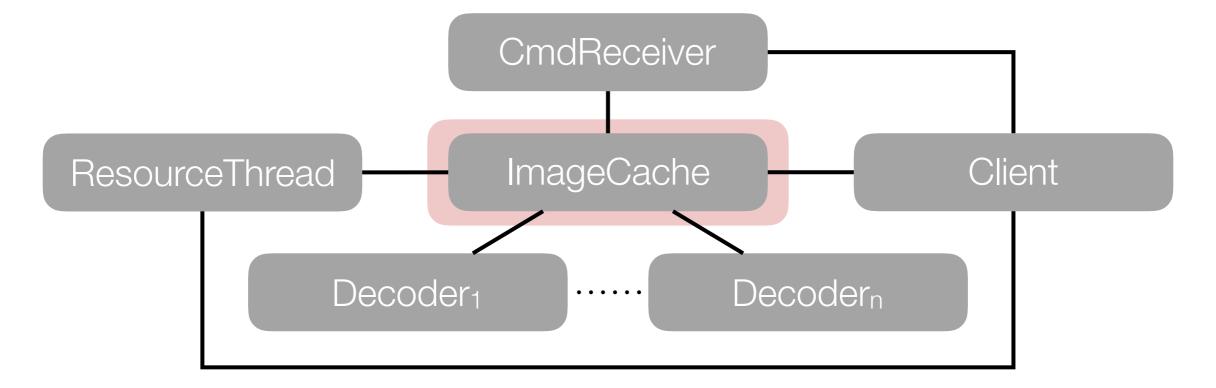


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\label{eq:lmgCacheCmd} ImgCacheCmd = \& \; \{ RequestImage : string \rightarrow Requester \longrightarrow ImgCacheCmd, \\ \dots \\ Exit : \oplus \; \{ Running : ImgCacheCmd, \\ Done : ResourceThread \otimes \mathbf{1} \} \\ \}
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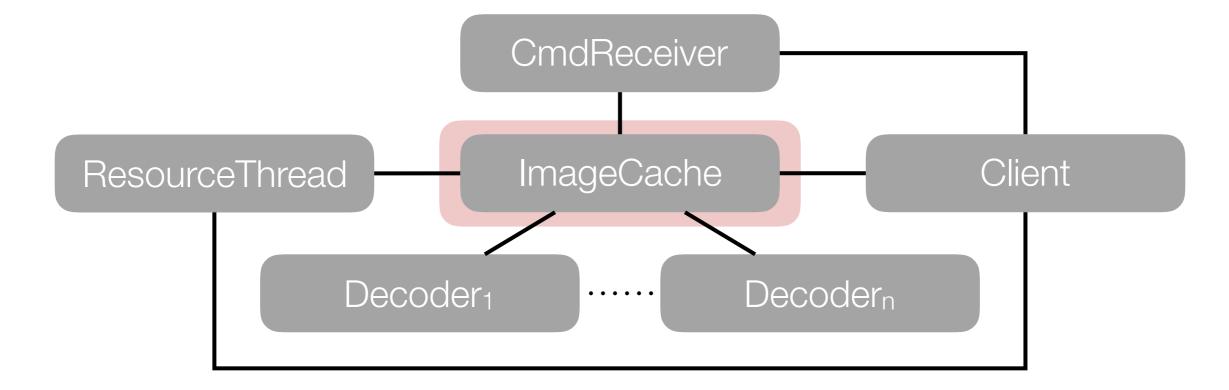






components change their session type along with message exchange

ImageCache: ImgCacheCmd



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Session types are the types of message-passing concurrency.

Research

- active research area since inception in 90s [Honda 1993]
- logical reconstruction based on linear logic, providing strong guarantees [Caires & Pfenning 2010, Wadler 2012]
- extension of logical session types to sharing [Balzer & Pfenning ICFP 2017, Balzer et al. CONCUR 2018, Balzer et al. ESOP 2019]

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Logic-based shared session types

Provide strong guarantees:

- Data-race-freedom
- Protocol adherence (a.k.a. session fidelity, preservation)
- Deadlock-freedom (a.k.a. progress)

exactly one client

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processes graph forms a tree at run-time

Provide strong guarantees

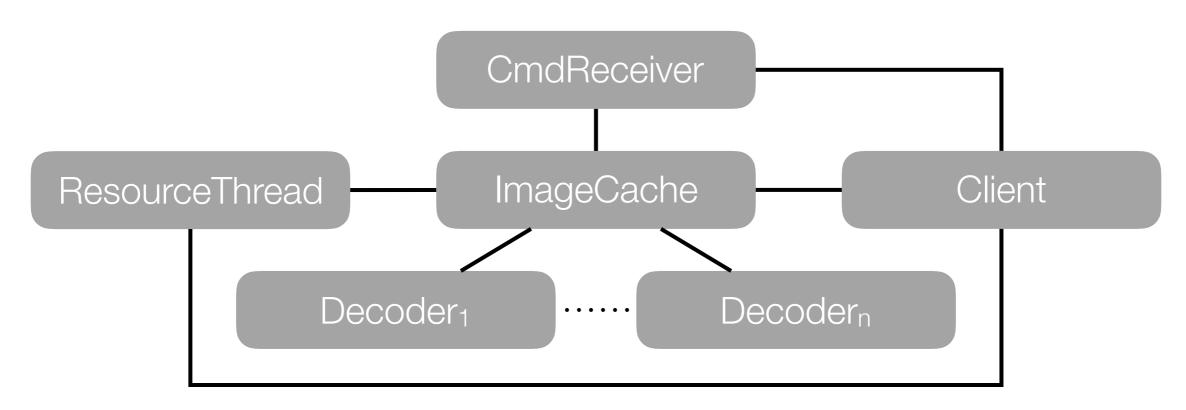
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Linear logic session types cannot accommodate certain practical programming scenarios.

Provide strong guarantees

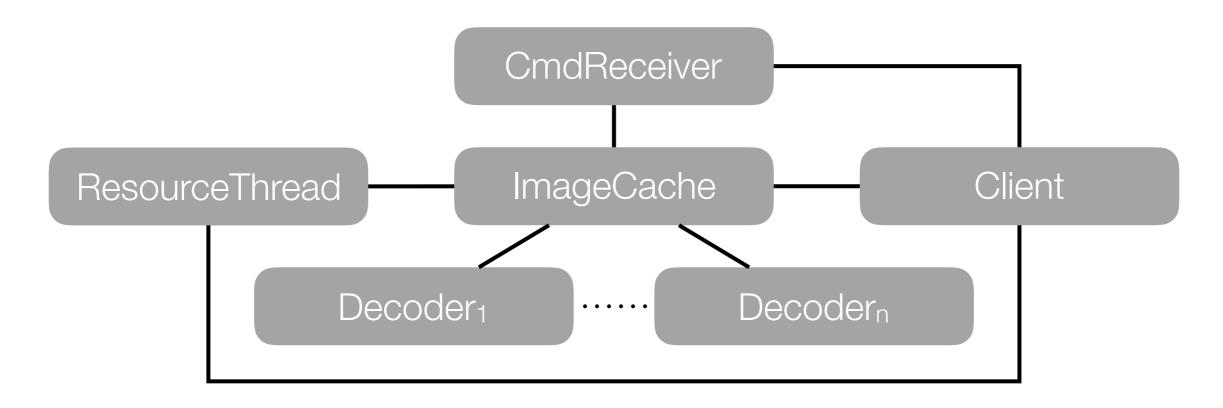
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- -

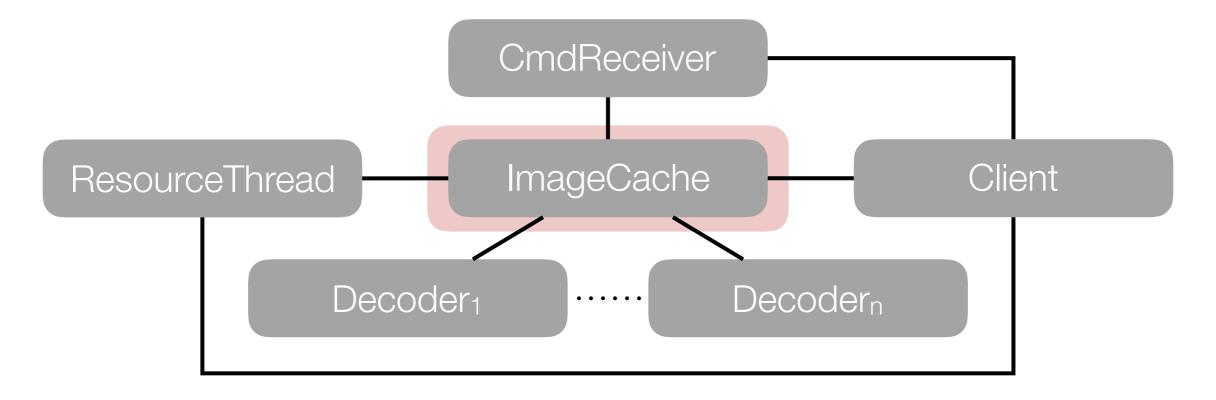
Let's introduce sharing while maintaining above guarantees.

```
\label{eq:local_cond} ImgCacheCmd = \& \; \{ RequestImage : string \rightarrow Requester \longrightarrow ImgCacheCmd, \\ \dots \\ Exit : \oplus \; \{ Running : ImgCacheCmd, \\ Done : ResourceThread \otimes \mathbf{1} \} \\ \}
```



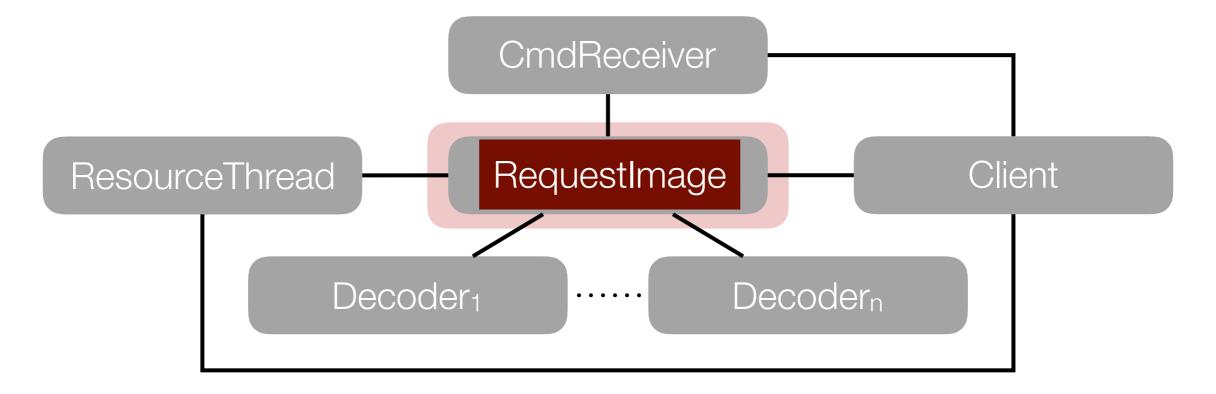
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```

ImageCache: ImgCacheCmd



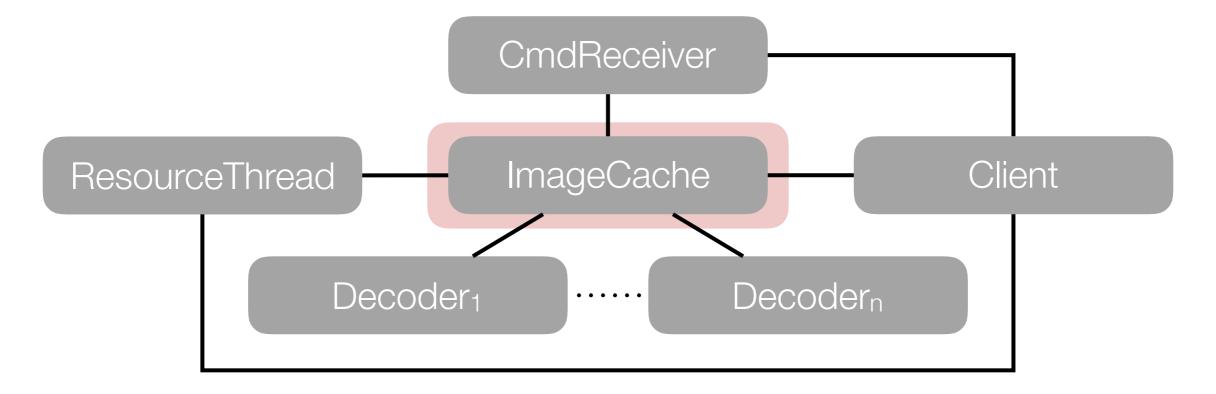
```
\label{eq:lmgCacheCmd} \mbox{ImgCacheCmd} : \mbox{RequestImage} : \mbox{string} \rightarrow \mbox{Requester} \longrightarrow \mbox{ImgCacheCmd}, \\ \mbox{Exit} : \oplus \{\mbox{Running} : \mbox{ImgCacheCmd}, \\ \mbox{Done} : \mbox{ResourceThread} \otimes \mathbf{1} \} \\ \mbox{} \}
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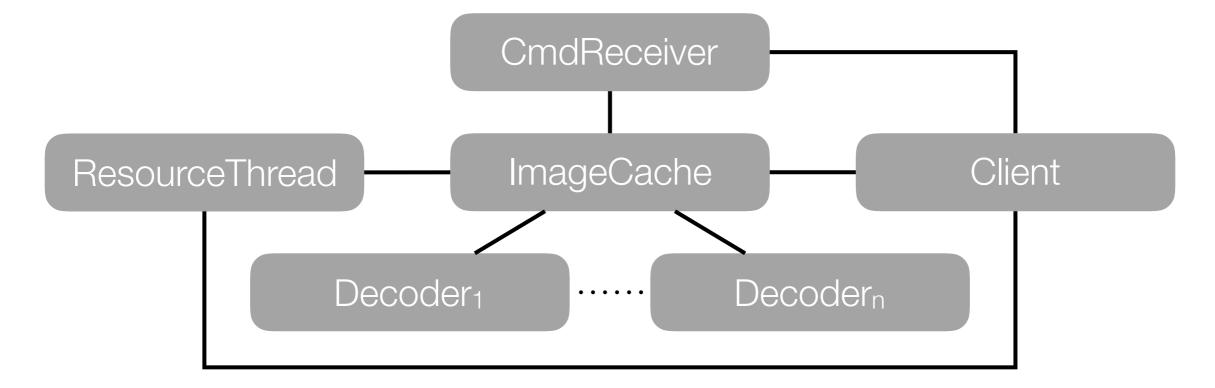
ImageCache: string → Requester → ImgCacheCmd



```
ImgCacheCmd = \& \{RequestImage : string \rightarrow Requester \multimap ImgCacheCmd, \}
                        Exit : \( \operatorname{\text{Running}} : ImgCacheCmd, \)
                                   Done : ResourceThread \otimes 1
ImageCache: string → Requester → ImgCacheCmd
                                                            protocol violated!
                                   CmdReceiver
     ResourceThread
                                         Exit
                                                                    Client
                       Decoder<sub>1</sub>
                                                    Decodern
```

```
\label{eq:lmgCacheCmd} ImgCacheCmd = \& \; \{ RequestImage : string \rightarrow Requester \longrightarrow ImgCacheCmd, \\ \qquad \qquad \dots \\ \qquad \qquad \qquad Exit : \oplus \; \{ Running : ImgCacheCmd, \\ \qquad \qquad \qquad Done : ResourceThread \otimes \mathbf{1} \} \\ \qquad \qquad \}
```

How to restore protocol adherence in the presence of sharing (a.k.a. aliasing)?





Clients of shared channels must communicate along that channel in mutual exclusion from each other.

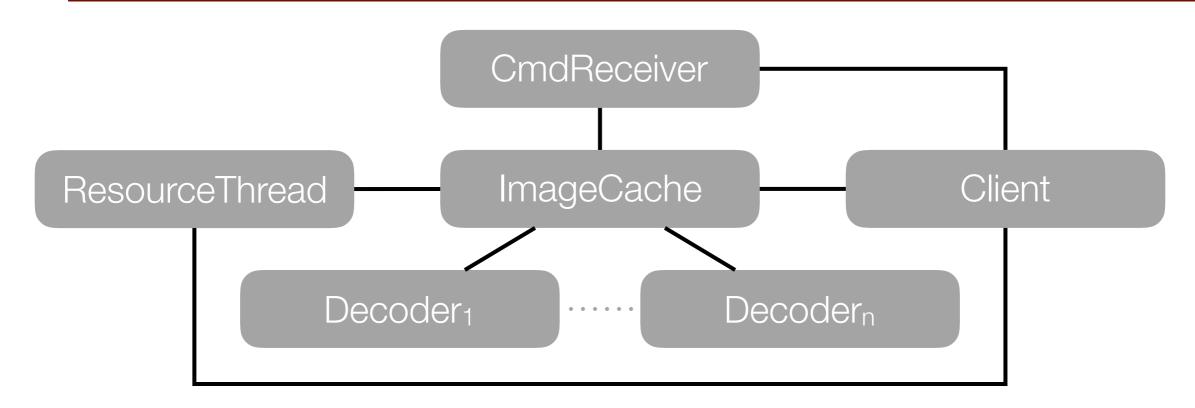


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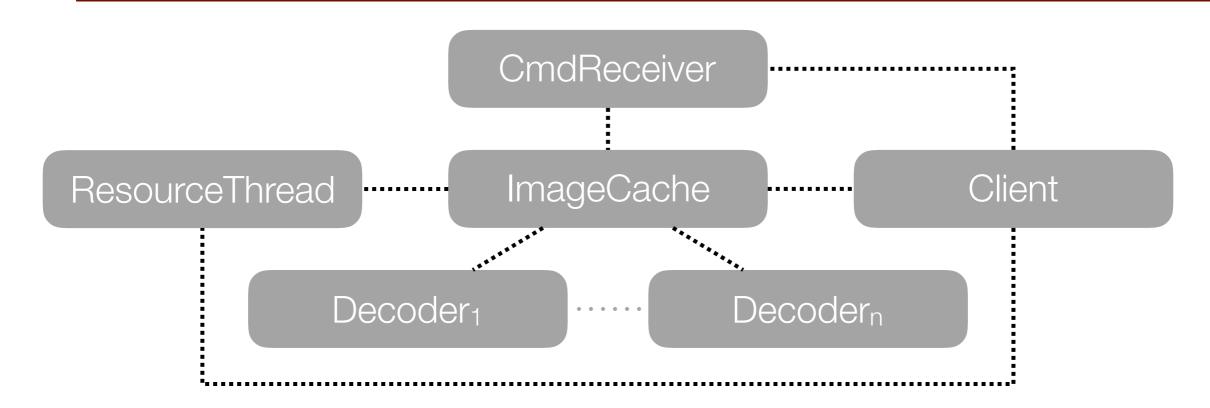
Acquiring a shared channel gives exclusive access, releasing an acquired channel relinquishes exclusive access.

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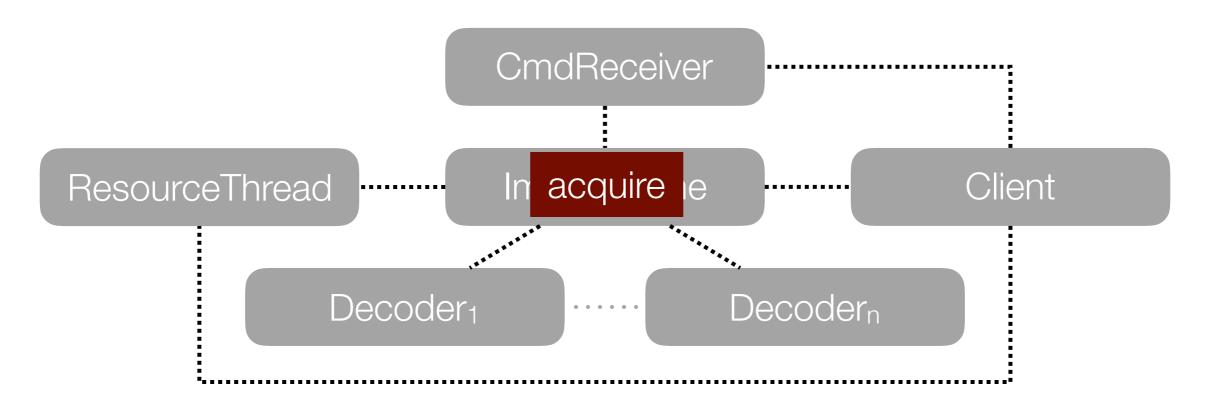
- **-**
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- **→**

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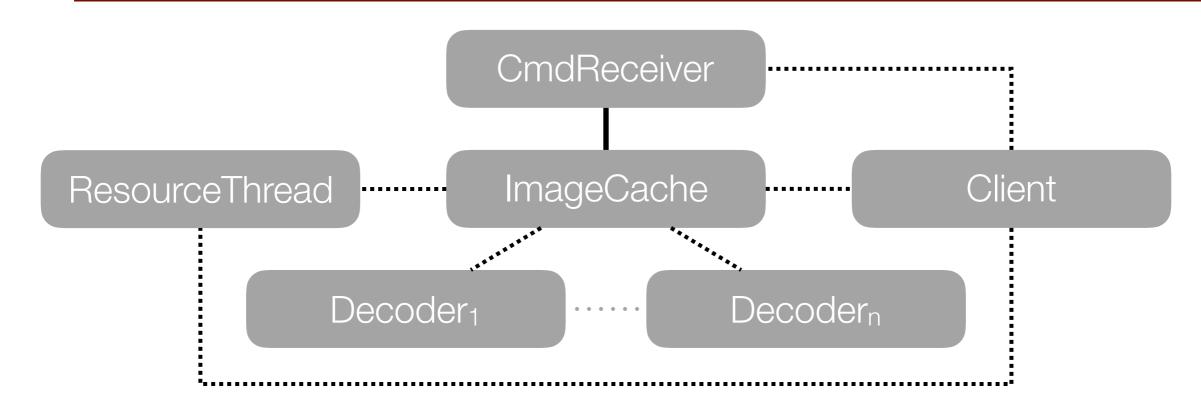
Legend: shared channel

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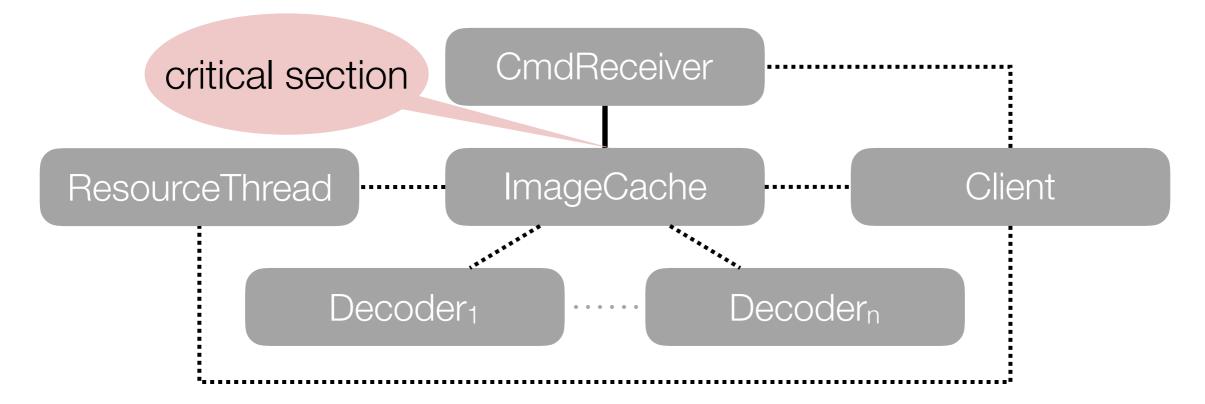


Legend: shared channel

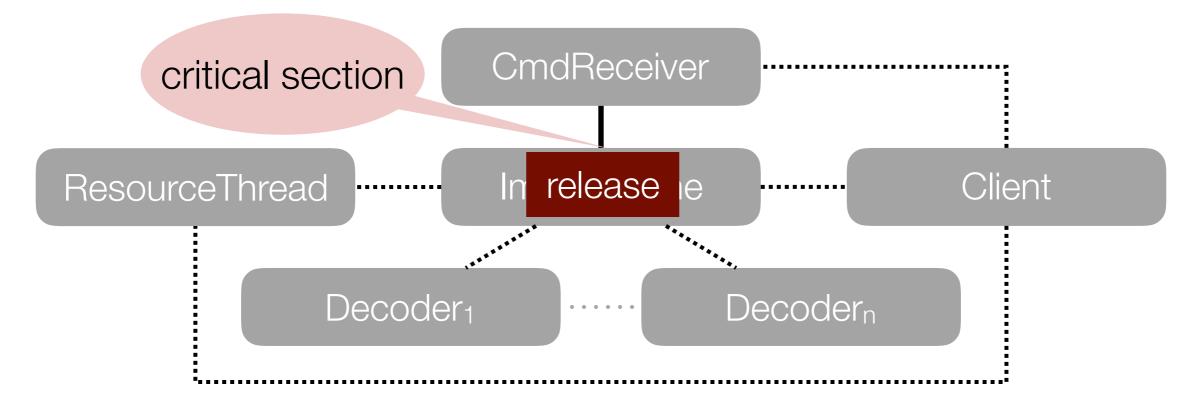
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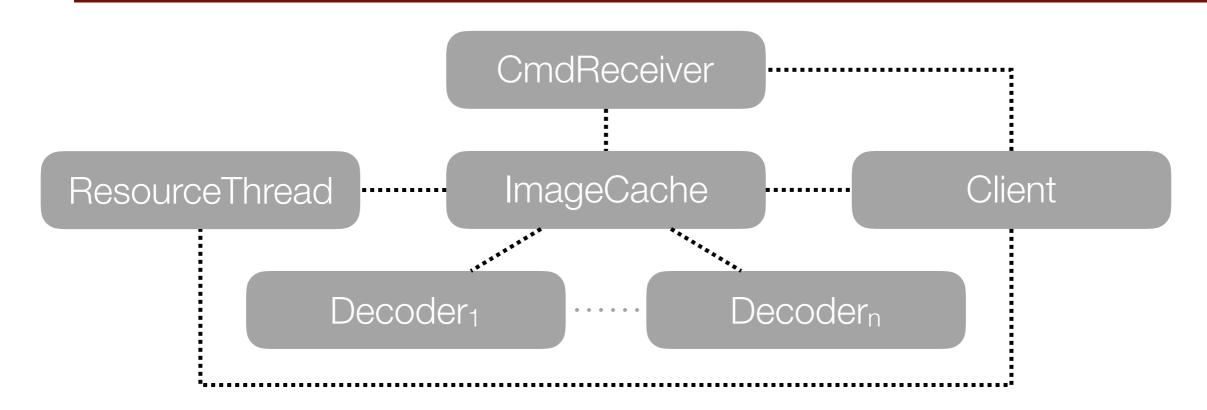


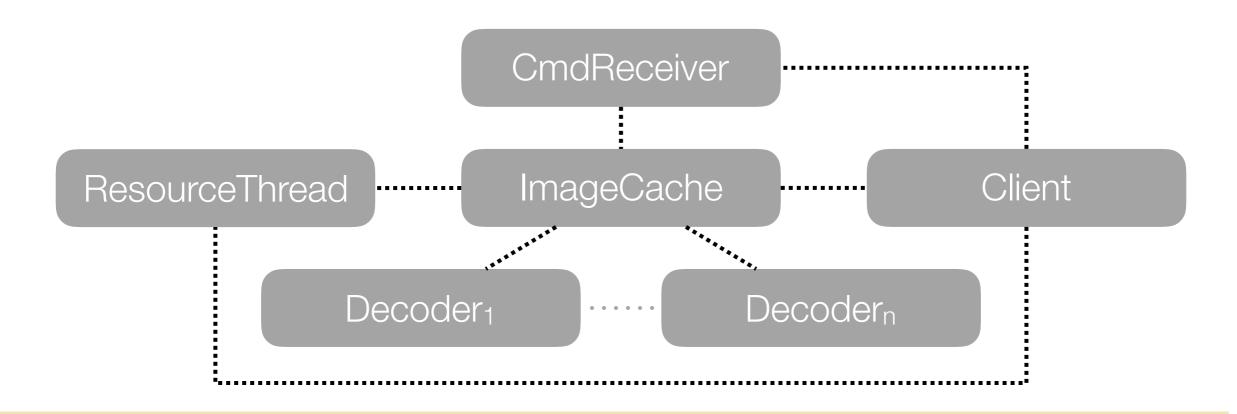
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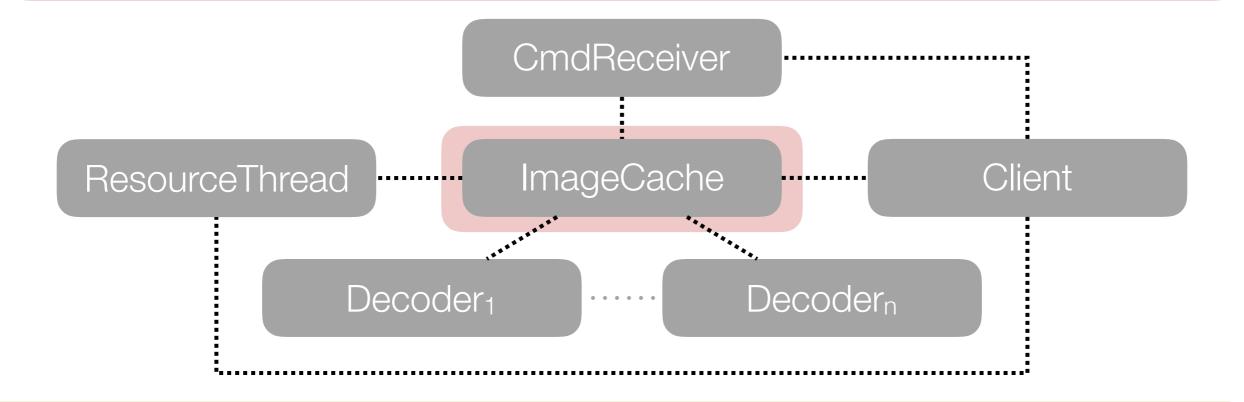
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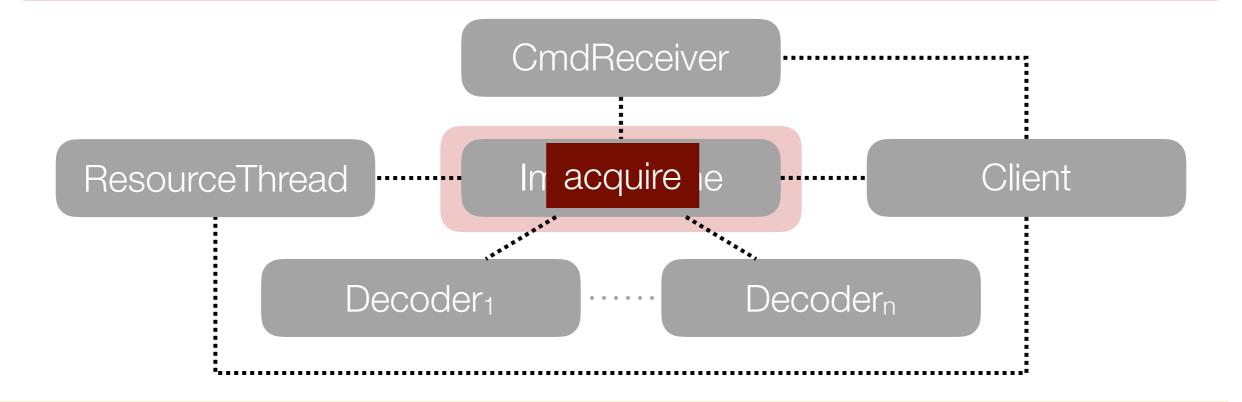
```
\begin{split} \mathsf{ImgCacheCmd} &= \& \{ \mathsf{RequestImage} : \mathsf{string} \to \mathsf{Requester} \multimap \mathsf{ImgCacheCmd}, \\ &\mathsf{Exit} : \oplus \; \{ \mathsf{Running} : \mathsf{ImgCacheCmd}, \\ &\mathsf{Done} : \mathsf{ResourceThread} \otimes \mathbf{1} \} \} \end{split}
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ImageCache: ImgCacheCmd



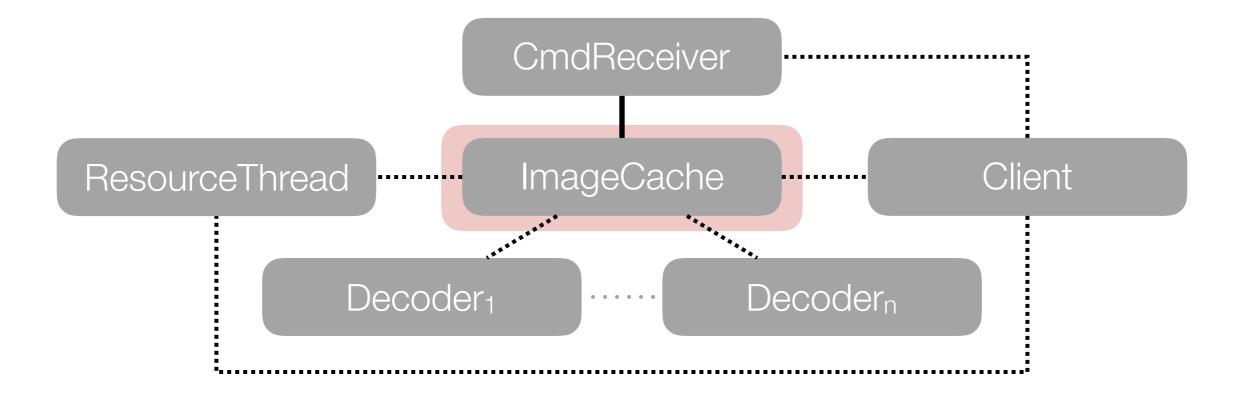
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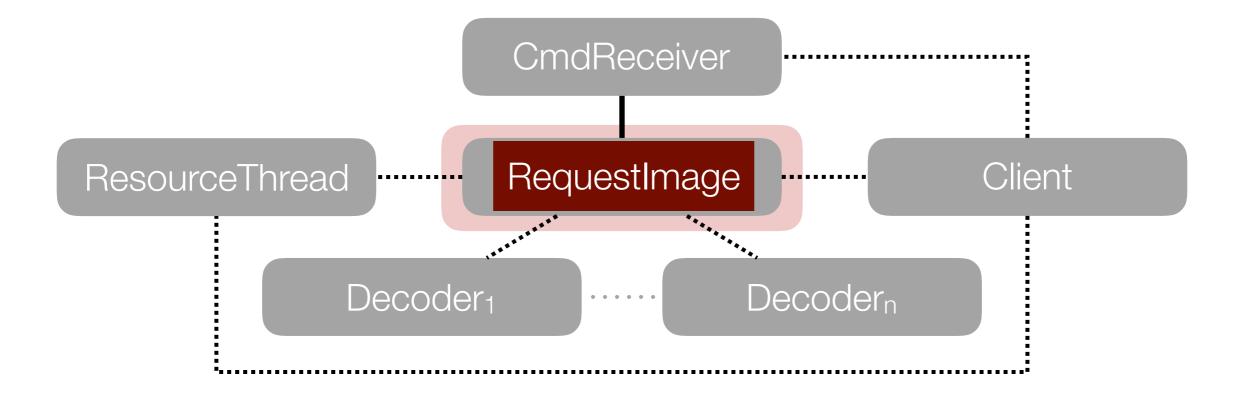
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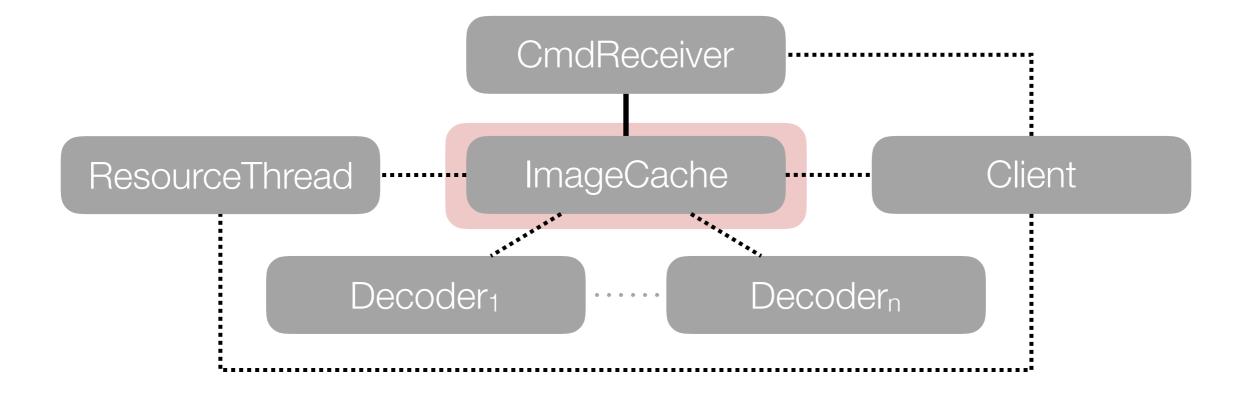
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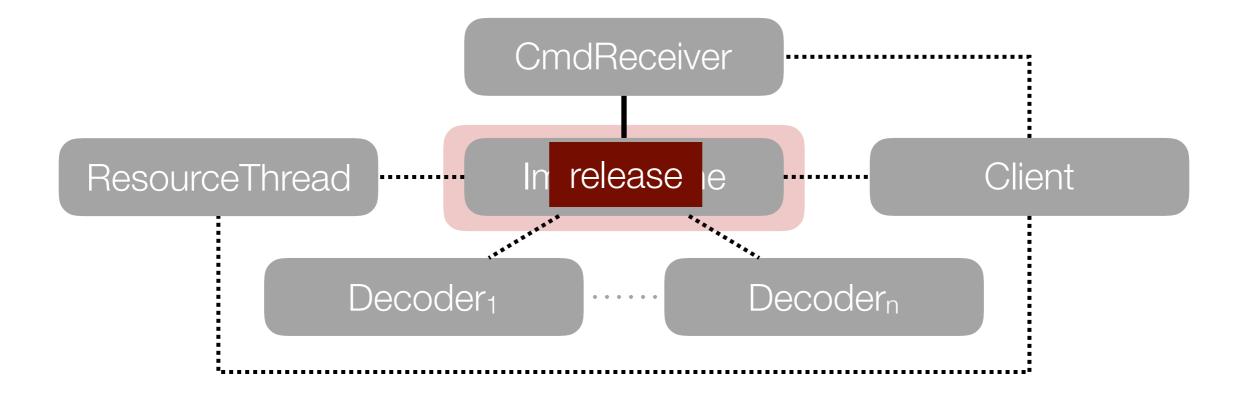
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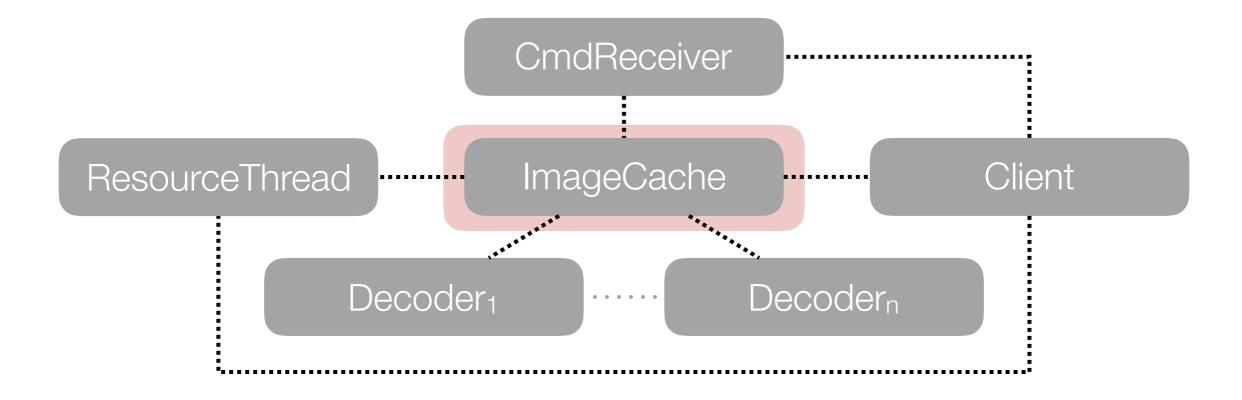
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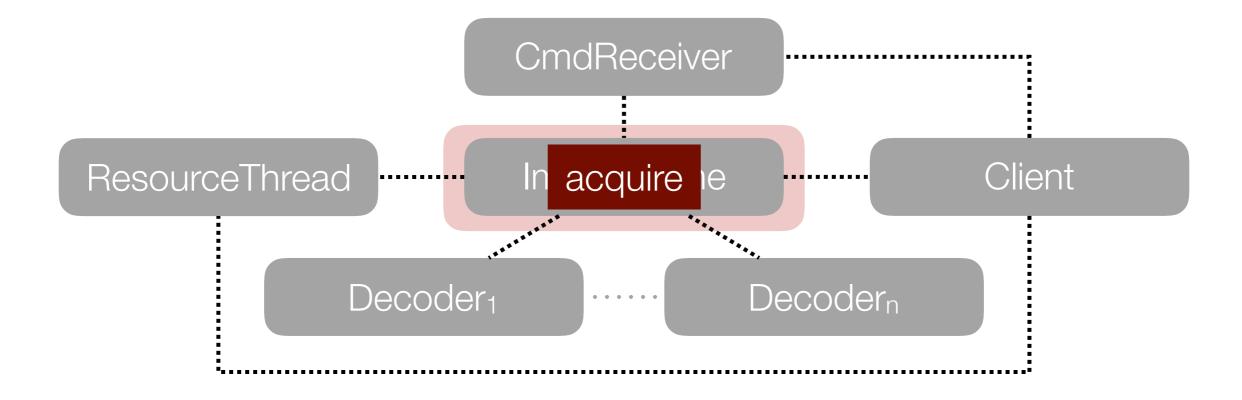
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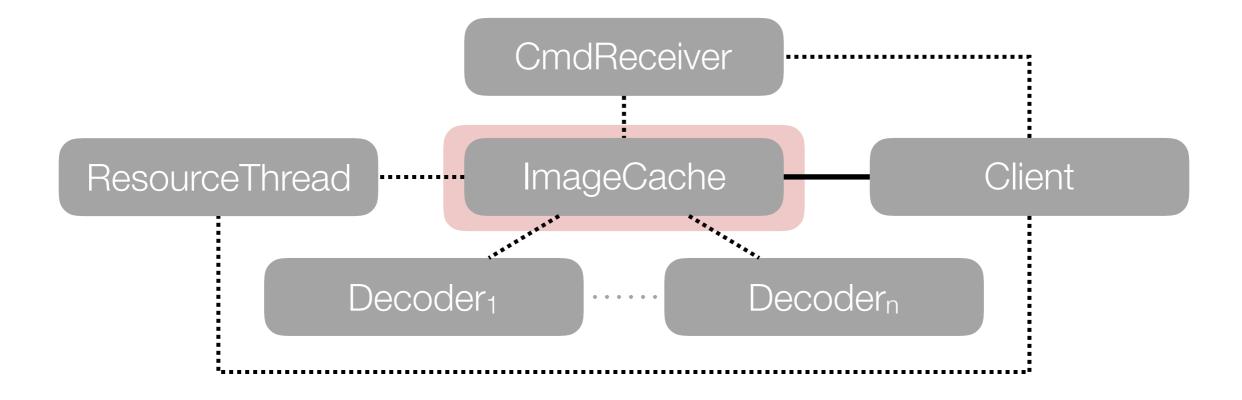
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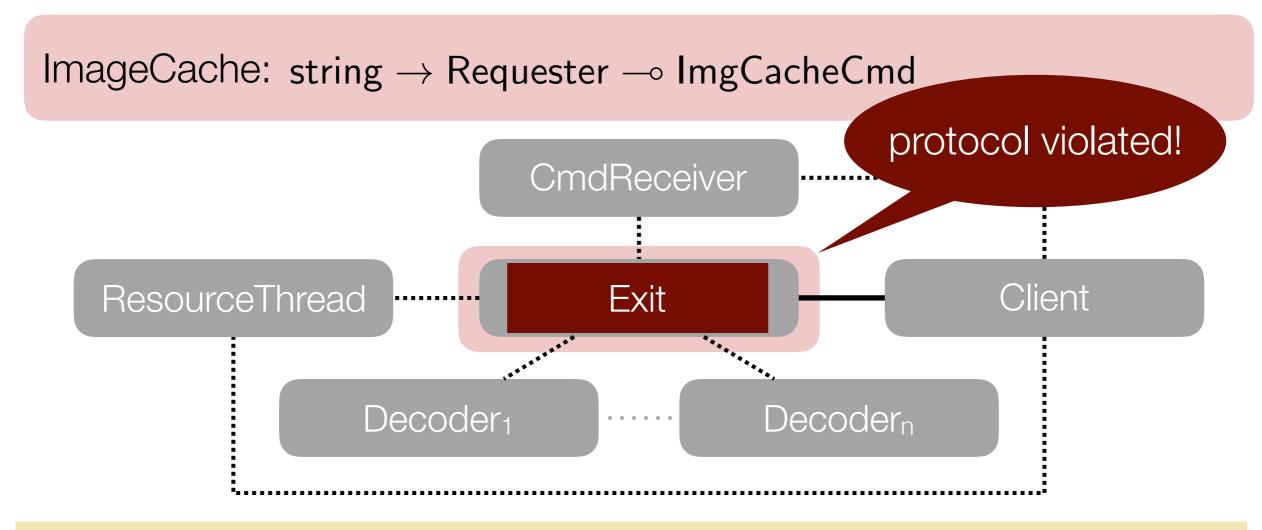


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Idea: equi-synchronizing

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In addition to imposing acquire-release on shared channels, shared channels must be equi-synchronizing:

Idea: equi-synchronizing



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i.e., shared channels must be released back to the same type at which they were acquired, if released.



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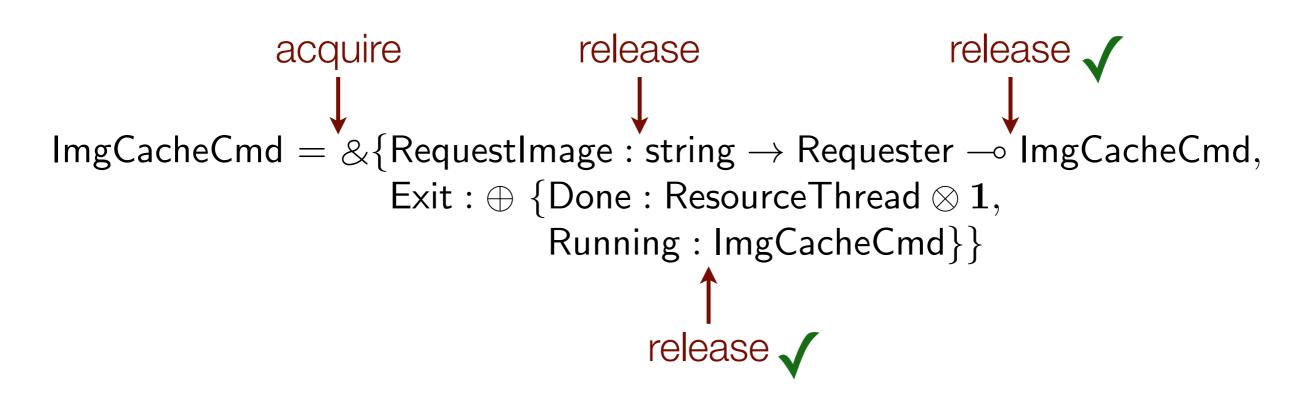


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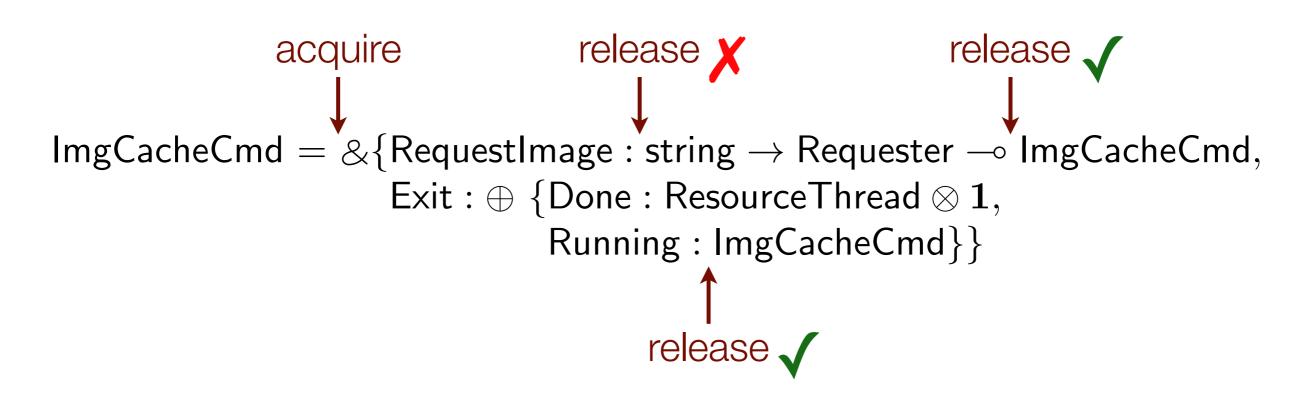






In addition to imposing acquire-release on shared channels, shared channels must be equi-synchronizing:







Acquire-release + equi-synchronizing:



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restore protocol adherence;



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guarantee freedom of (high-level) data races because execution between acquire-release is atomic.



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We could state the policy of acquire-release and equisynchronizing as a programming methodology.



Acquire-release + equi-synchronizing:



restore protocol adherence;



guarantee freedom of (high-level) data races because execution between acquire-release is atomic.



We could state the policy of acquire-release and equisynchronizing as a programming methodology.



But, why not lift this policy to the type level and have it enforced statically?





```
\label{eq:localized} \begin{array}{c} \text{acquire} & \text{release} \\ \downarrow & \downarrow \\ \text{ImgCacheCmd} = \&\{\text{RequestImage}: \text{string} \rightarrow \text{Requester} \multimap \text{ImgCacheCmd}, \\ & \text{Exit}: \oplus \{\text{Done}: \text{ResourceThread} \otimes \mathbf{1}, \\ & \text{Running}: \text{ImgCacheCmd}\}\} \\ & \uparrow \\ & \text{release} \end{array}
```



Legend: shared phase linear phase

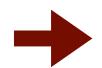






Stratify session types into a linear and shared layer

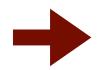




Stratify session types into a linear and shared layer

$$A_{\mathsf{S}} \triangleq \\ A_{\mathsf{L}}, B_{\mathsf{L}} \triangleq \bigoplus \{\overline{l} : A_{\mathsf{L}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \\ & \& \{\overline{l} : A_{\mathsf{L}}\} \mid A_{\mathsf{L}} \multimap B_{\mathsf{L}}$$





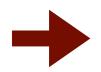
Stratify session types into a linear and shared layer



Connect layers with modalities going back and forth

$$A_{\mathsf{S}} \triangleq \\ A_{\mathsf{L}}, B_{\mathsf{L}} \triangleq \bigoplus \{\overline{l} : A_{\mathsf{L}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \\ & \& \{\overline{l} : A_{\mathsf{L}}\} \mid A_{\mathsf{L}} \multimap B_{\mathsf{L}}$$





Stratify session types into a linear and shared layer



Connect layers with modalities going back and forth

$$A_{S} \triangleq \uparrow_{L}^{S} A_{L}$$

$$A_{L}, B_{L} \triangleq \bigoplus \{\overline{l} : A_{L}\} \mid A_{L} \otimes B_{L} \mid \mathbf{1} \mid$$

$$\& \{\overline{l} : A_{L}\} \mid A_{L} \multimap B_{L} \mid \downarrow_{L}^{S} A_{S}$$





Stratify session types into a linear and shared layer



Connect layers with modalities going back and forth



Support communication of shared channels





Stratify session types into a linear and shared layer



Connect layers with modalities going back and forth

$$A_{\mathsf{S}} \triangleq \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}$$

$$A_{\mathsf{L}}, B_{\mathsf{L}} \triangleq \bigoplus \{\overline{l} : A_{\mathsf{L}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \exists x : A_{\mathsf{S}} . B_{\mathsf{L}} \mid$$

$$\& \{\overline{l} : A_{\mathsf{L}}\} \mid A_{\mathsf{L}} \multimap B_{\mathsf{L}} \mid \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \mid \Pi x : A_{\mathsf{S}} . B_{\mathsf{L}}$$



Support communication of shared channels



$$A_{\mathsf{S}} \triangleq \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}$$

$$A_{\mathsf{L}}, B_{\mathsf{L}} \triangleq \bigoplus \{\overline{l} : A_{\mathsf{L}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \exists x : A_{\mathsf{S}} . B_{\mathsf{L}} \mid$$

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A_{\mathsf{S}} \triangleq \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}
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```

```
\label{eq:local_equation} \begin{array}{ll} ImgCacheCmd = & \& \{RequestImage : string \rightarrow Requester \longrightarrow & ImgCacheCmd, \\ & Exit : \oplus \{Done : ResourceThread \otimes \mathbf{1}, \\ & Running : & ImgCacheCmd\} \} \end{array}
```



```
A_{\mathsf{S}} \triangleq \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}
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```

```
\begin{array}{ll} ImgCacheCmd = \uparrow^s_L \& \{RequestImage : string \rightarrow Requester \rightarrow \\ & Exit : \oplus \{Done : ResourceThread \otimes \mathbf{1}, \\ & Running : & ImgCacheCmd\} \} \end{array}
```



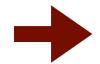
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A_{\mathsf{S}} \triangleq \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}
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```

```
\begin{split} ImgCacheCmd &= \uparrow_L^s \& \{RequestImage: string \rightarrow Requester \rightarrow \downarrow_L^s | ImgCacheCmd, \\ &Exit: \oplus \{Done: ResourceThread \otimes \mathbf{1}, \\ &Running: \downarrow_L^s | ImgCacheCmd \} \} \end{split}
```



```
A_{\mathsf{S}} \triangleq \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}
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\begin{split} ImgCacheCmd &= \uparrow_{L}^{s} \& \{RequestImage : string \rightarrow Requester \rightarrow \downarrow_{L}^{s} ImgCacheCmd, \\ &Exit : \oplus \{Done : ResourceThread \otimes \mathbf{1}, \\ &Running : \downarrow_{L}^{s} ImgCacheCmd\} \} \end{split}
```



Up and down shifts denote acquire and release, resp.



$$A_{\mathsf{S}} \triangleq \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}$$

$$A_{\mathsf{L}}, B_{\mathsf{L}} \triangleq \bigoplus \{\overline{l} : A_{\mathsf{L}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \exists x : A_{\mathsf{S}} . B_{\mathsf{L}} \mid$$

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Based on correspondence between intuitionistic linear logic and session-typed pi-calculus:



$$A_{\mathsf{S}} \triangleq \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}$$

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Based on correspondence between intuitionistic linear logic and session-typed pi-calculus:

$$\Gamma \vdash_{\Sigma} P :: (x_{\mathsf{S}} : A_{\mathsf{S}})$$

$$\Gamma; \ \Delta \vdash_{\Sigma} P :: (x_{\mathsf{L}} : A_{\mathsf{L}})$$



$$A_{\mathsf{S}} \triangleq \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}$$

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"Process P provides session of type A_m along x_m using channels in $(\Gamma \text{ and}) \Delta$."



$$A_{\mathsf{S}} \triangleq \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}$$

$$A_{\mathsf{L}}, B_{\mathsf{L}} \triangleq \bigoplus \{\overline{l} : A_{\mathsf{L}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \exists x : A_{\mathsf{S}} . B_{\mathsf{L}} \mid$$

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Based on correspondence between intuitionistic linear logic and session-typed pi-calculus:

shared (structural)
$$\Gamma \vdash_{\Sigma} P :: (x_{\mathsf{S}} : A_{\mathsf{S}})$$
 context
$$\Gamma; \ \Delta \vdash_{\Sigma} P :: (x_{\mathsf{L}} : A_{\mathsf{L}})$$

"Process P provides session of type A_m along x_m using channels in $(\Gamma \text{ and}) \Delta$."



$$A_{\mathsf{S}} \triangleq \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}$$

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Based on correspondence between intuitionistic linear logic and session-typed pi-calculus:

$$\Gamma \vdash_{\Sigma} P :: (x_{\mathsf{S}} : A_{\mathsf{S}})$$

$$\Gamma; \ \Delta \vdash_{\Sigma} P :: (x_{\mathsf{L}} : A_{\mathsf{L}})$$

"Process P provides session of type A_m along x_m using channels in $(\Gamma \text{ and}) \Delta$."



Typing judgments

$$A_{\mathsf{S}} \triangleq \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}$$

$$A_{\mathsf{L}}, B_{\mathsf{L}} \triangleq \bigoplus \{\overline{l} : \overline{A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \exists x : A_{\mathsf{S}} . B_{\mathsf{L}} \mid$$

$$\& \{\overline{l} : \overline{A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \multimap B_{\mathsf{L}} \mid \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \mid \Pi x : A_{\mathsf{S}} . B_{\mathsf{L}}$$

Based on correspondence between intuitionistic linear logic and session-typed pi-calculus:

linear (substructural)
$$\Gamma \vdash_{\Sigma} P :: (x_{\mathsf{S}} : A_{\mathsf{S}})$$
 context
$$\Gamma; \ \Delta \vdash_{\Sigma} P :: (x_{\mathsf{L}} : A_{\mathsf{L}})$$

"Process P provides session of type A_m along x_m using channels in $(\Gamma \text{ and}) \Delta$."





$$\frac{\Gamma, x_{\mathsf{S}} : \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}; \ \Delta, x_{\mathsf{L}} : A_{\mathsf{L}} \vdash_{\Sigma} Q_{x_{\mathsf{L}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}{\Gamma, x_{\mathsf{S}} : \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}; \ \Delta \vdash_{\Sigma} x_{\mathsf{L}} \leftarrow \mathsf{acquire} \ x_{\mathsf{S}} : Q_{x_{\mathsf{L}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}$$

$$(T-\uparrow_{\mathsf{L}}^{\mathsf{S}})$$



$$\frac{\Gamma, x_{\mathsf{S}} : \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}; \ \Delta, x_{\mathsf{L}} : A_{\mathsf{L}} \vdash_{\Sigma} Q_{x_{\mathsf{L}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}{\Gamma, x_{\mathsf{S}} : \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}; \ \Delta \vdash_{\Sigma} x_{\mathsf{L}} \leftarrow \text{acquire } x_{\mathsf{S}} :; Q_{x_{\mathsf{L}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}$$

$$(T-\uparrow_{\mathsf{LL}}^{\mathsf{S}})$$



$$\frac{\Gamma, x_{\mathsf{S}} : \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}; \ \Delta, x_{\mathsf{L}} : A_{\mathsf{L}} \vdash_{\Sigma} Q_{x_{\mathsf{L}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}{\Gamma, x_{\mathsf{S}} : \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}; \ \Delta \vdash_{\Sigma} x_{\mathsf{L}} \leftarrow \text{acquire } x_{\mathsf{S}} : Q_{x_{\mathsf{L}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}$$

$$(T-\uparrow_{\mathsf{L}}^{\mathsf{S}})$$

$$\frac{\Gamma; \cdot \vdash_{\Sigma} P_{x_{\mathsf{L}}} :: (x_{\mathsf{L}} : A_{\mathsf{L}})}{\Gamma \vdash_{\Sigma} x_{\mathsf{L}} \leftarrow \mathsf{accept} \ x_{\mathsf{S}} \ ; P_{x_{\mathsf{L}}} :: (x_{\mathsf{S}} : \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}})} \ (\mathsf{T} - \uparrow_{\mathsf{LR}}^{\mathsf{S}})$$



$$\frac{\Gamma, x_{\mathsf{S}} : \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}; \ \Delta, x_{\mathsf{L}} : A_{\mathsf{L}} \vdash_{\Sigma} Q_{x_{\mathsf{L}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}{\Gamma, x_{\mathsf{S}} : \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}; \ \Delta \vdash_{\Sigma} x_{\mathsf{L}} \leftarrow \text{acquire } x_{\mathsf{S}} : Q_{x_{\mathsf{L}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}$$

$$(T-\uparrow_{\mathsf{L}}^{\mathsf{S}})$$

$$\frac{\Gamma; \vdash_{\Sigma} P_{x_{\mathsf{L}}} :: (x_{\mathsf{L}} : A_{\mathsf{L}})}{\Gamma \vdash_{\Sigma} x_{\mathsf{L}} \leftarrow \mathsf{accept} \ x_{\mathsf{S}} \ ; P_{x_{\mathsf{L}}} :: (x_{\mathsf{S}} : \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}})} \ (\mathsf{T} - \uparrow_{\mathsf{LR}}^{\mathsf{S}})$$





$$\frac{\Gamma, x_{\mathsf{S}} : A_{\mathsf{S}}; \ \Delta \vdash_{\Sigma} Q_{x_{\mathsf{S}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}{\Gamma; \ \Delta, x_{\mathsf{L}} : \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \vdash_{\Sigma} x_{\mathsf{S}} \leftarrow \mathsf{release} \ x_{\mathsf{L}} \ ; Q_{x_{\mathsf{S}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}$$



$$\frac{\Gamma, x_{\mathsf{S}} : A_{\mathsf{S}}; \ \Delta \vdash_{\Sigma} Q_{x_{\mathsf{S}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}{\Gamma; \ \Delta, x_{\mathsf{L}} : \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \vdash_{\Sigma} x_{\mathsf{S}} \leftarrow \mathsf{release} \ x_{\mathsf{L}} \ ; Q_{x_{\mathsf{S}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})} \ (\mathsf{T} - \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{L})$$



$$\frac{\Gamma, x_{\mathsf{S}} : A_{\mathsf{S}}; \ \Delta \vdash_{\Sigma} Q_{x_{\mathsf{S}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}{\Gamma; \ \Delta, x_{\mathsf{L}} : \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \vdash_{\Sigma} x_{\mathsf{S}} \leftarrow \mathsf{release} \ x_{\mathsf{L}} \ ; Q_{x_{\mathsf{S}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})} \ (\mathsf{T} - \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{L})$$

$$\frac{\Gamma \vdash_{\Sigma} P_{x_{S}} :: (x_{S} : A_{S})}{\Gamma; \cdot \vdash_{\Sigma} x_{S} \leftarrow \operatorname{detach} x_{L} ; P_{x_{S}} :: (x_{L} : \downarrow_{L}^{S} A_{S})} (T - \downarrow_{LR}^{S})$$



$$\frac{\Gamma, x_{\mathsf{S}} : A_{\mathsf{S}}; \ \Delta \vdash_{\Sigma} Q_{x_{\mathsf{S}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}{\Gamma; \ \Delta, x_{\mathsf{L}} : \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \vdash_{\Sigma} x_{\mathsf{S}} \leftarrow \mathsf{release} \ x_{\mathsf{L}} \ ; Q_{x_{\mathsf{S}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})} \ (\mathsf{T} - \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{L})$$

$$\frac{\Gamma \vdash_{\Sigma} P_{x_{\mathsf{S}}} :: (x_{\mathsf{S}} :: A_{\mathsf{S}})}{\Gamma; \vdash_{\Sigma} x_{\mathsf{S}} \leftarrow \operatorname{detach} x_{\mathsf{L}} ; P_{x_{\mathsf{S}}} :: (x_{\mathsf{L}} :\downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}})} (T - \downarrow_{\mathsf{LR}}^{\mathsf{S}})$$

Taking stock

Taking stock



We have a session type system that allows shared and linear channels to coexist and guarantees:



data-race-freedom (low-level and high-level)



protocol adherence

Taking stock



We have a session type system that allows shared and linear channels to coexist and guarantees:



data-race-freedom (low-level and high-level)



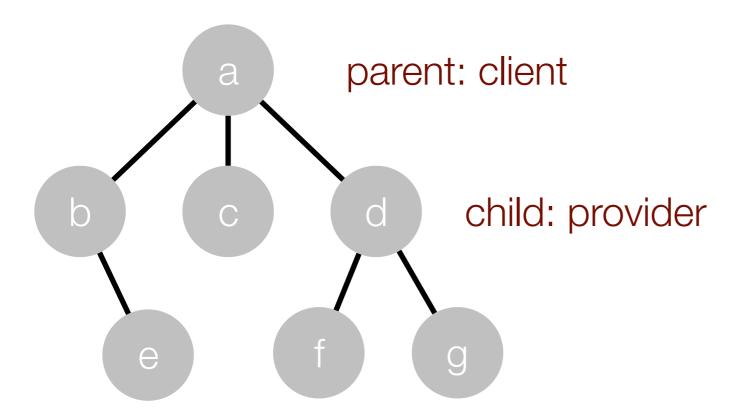
protocol adherence



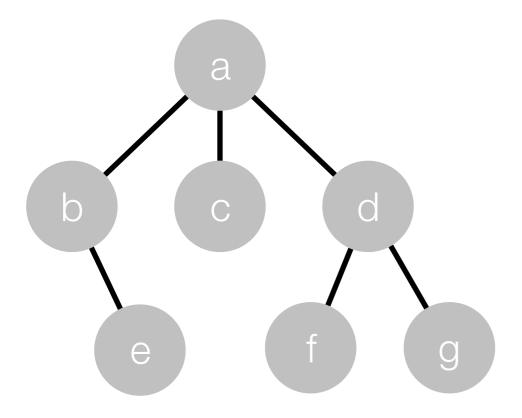
What about deadlock-freedom?

Linearity ("exactly one client") turns process graph into a tree.

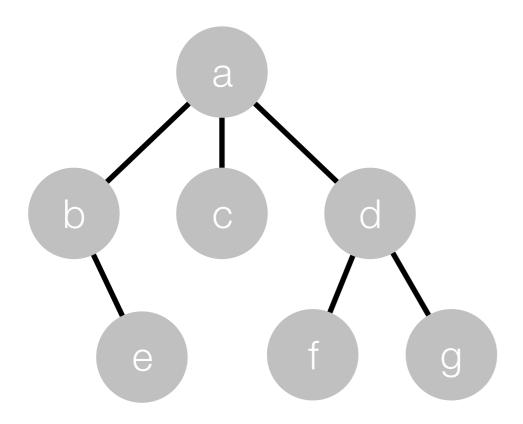
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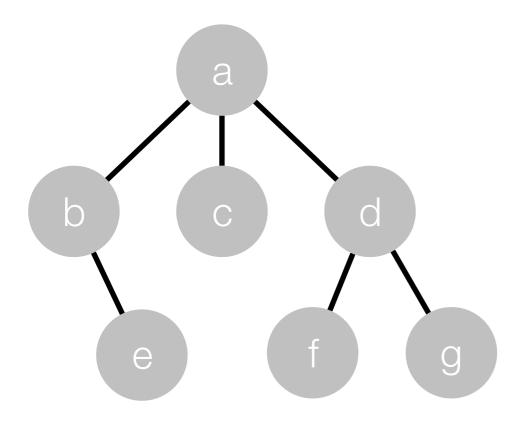


Linearity ("exactly one client") turns process graph into a tree.



What are the threats to progress?

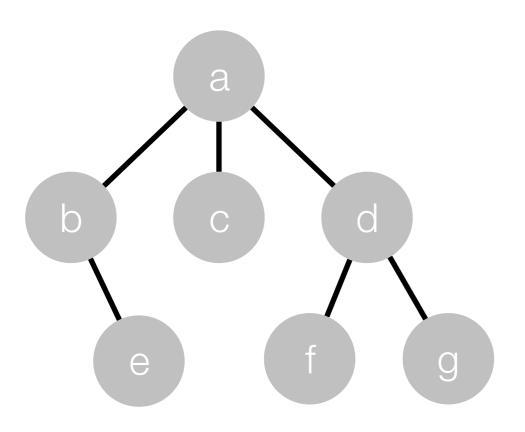
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What are the threats to progress?

Two scenarios:

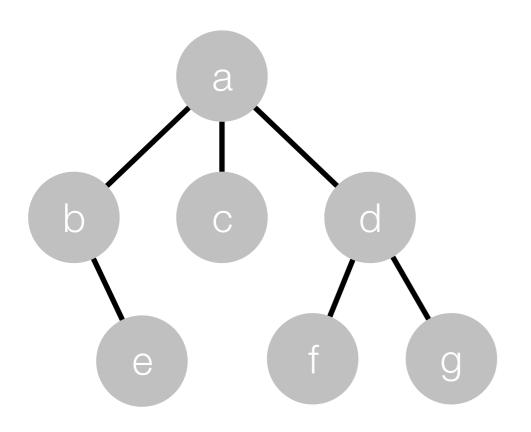
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What are the threats to progress?

- Two scenarios:
 - provider ready to synchronize, client not

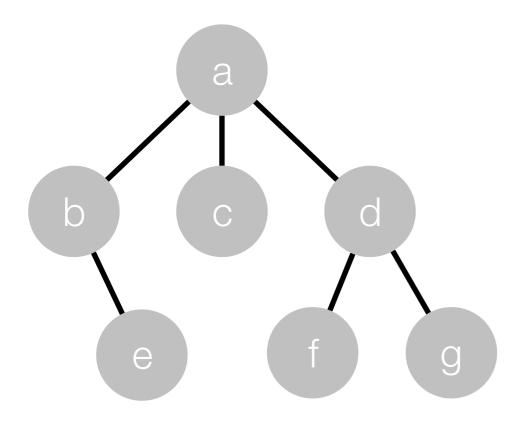
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What are the threats to progress?

- Two scenarios:
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 - client ready to synchronize, provider not

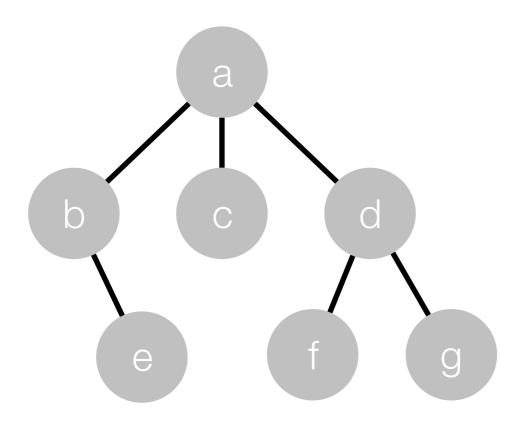
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What are the threats to progress?

- Two scenarios:
 - provider ready to synchronize, client not
 - client ready to synchronize, provider not
- Let's visualize this waiting dependency with a green arrow

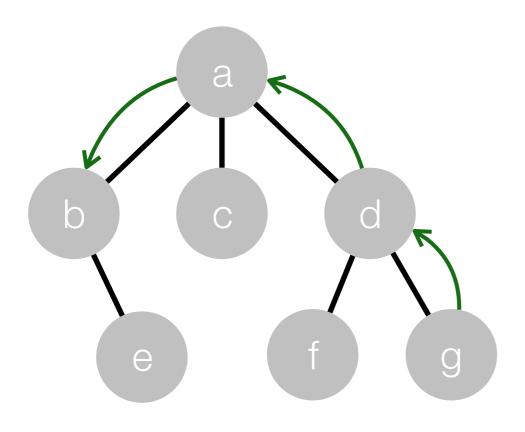
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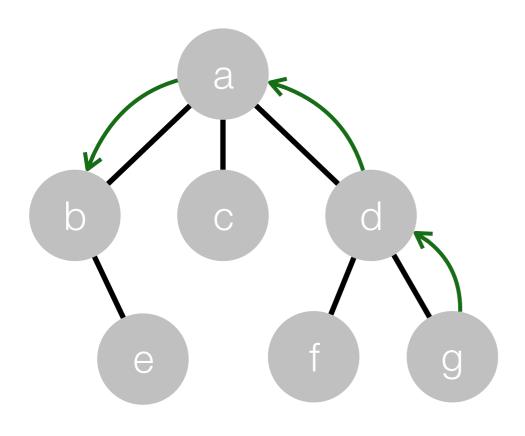
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Linearity ("exactly one client") turns process graph into a tree.



What are the threats to progress?

- Two scenarios:
 - provider ready to synchronize, client not
 - client ready to synchronize, provider not
- Let's visualize this waiting dependency with a green arrow



No green cycles: green arrows can only go along linear channels, and client and provider cannot both be waiting for each other.

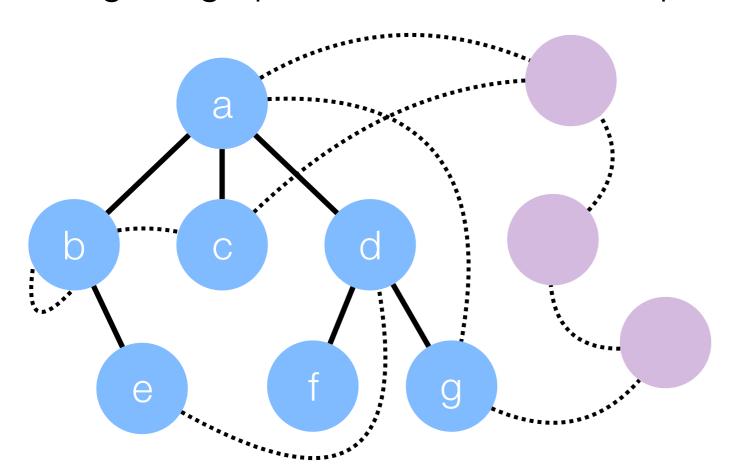
Legend: — linear channel

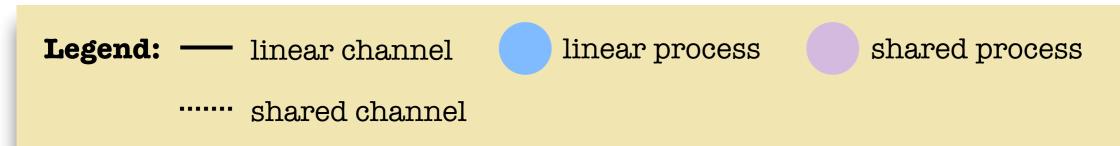


"a waits for b"

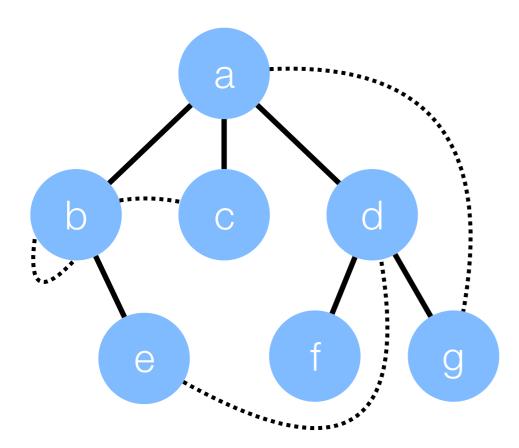
We get a graph of linear and shared processes, with a linear tree inside.

We get a graph of linear and shared processes, with a linear tree inside.

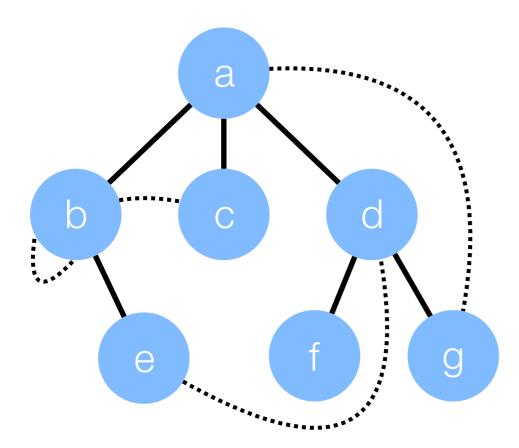




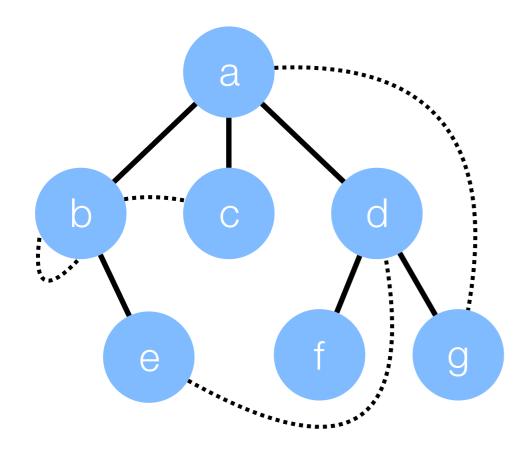
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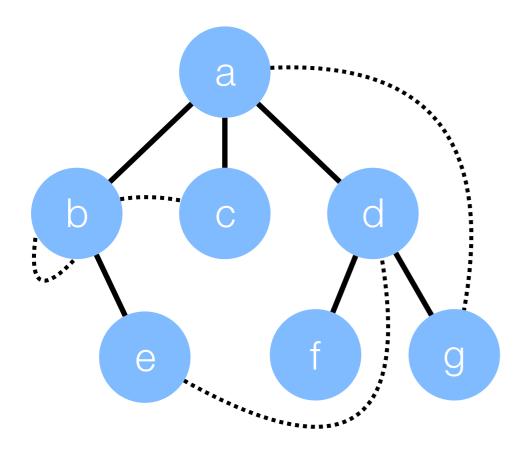


We get a graph of linear and shared processes, with a linear tree inside.



Acquire-release amounts to "locking"

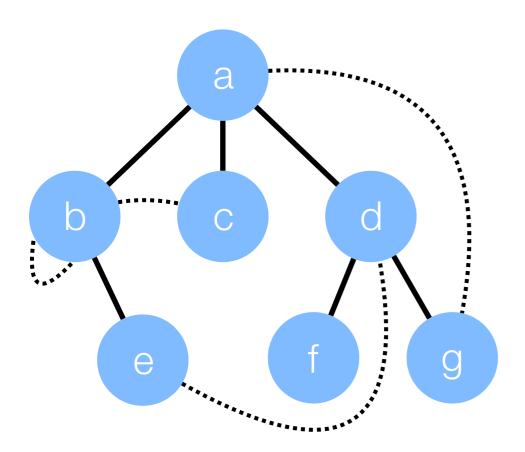
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Acquire-release amounts to "locking"

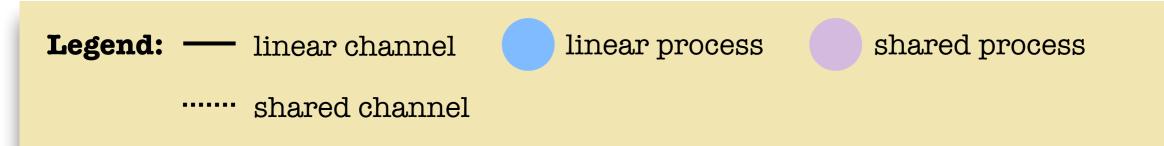
Possibility of cyclic dependencies

We get a graph of linear and shared processes, with a linear tree inside.

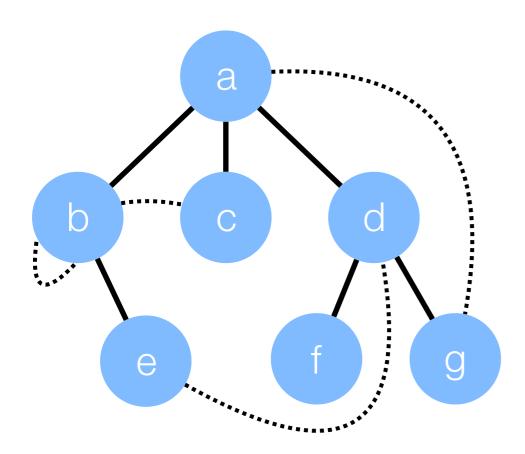


Acquire-release amounts to "locking"

- Possibility of cyclic dependencies
- Let's visualize this waiting dependency with a red arrow

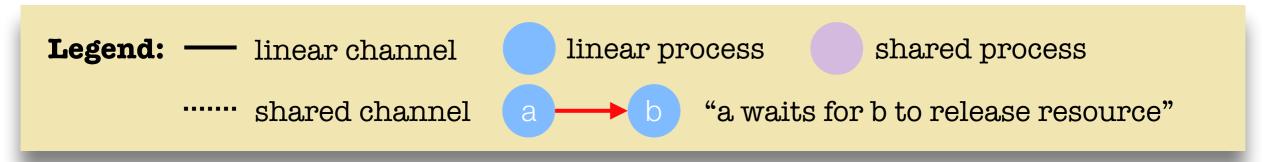


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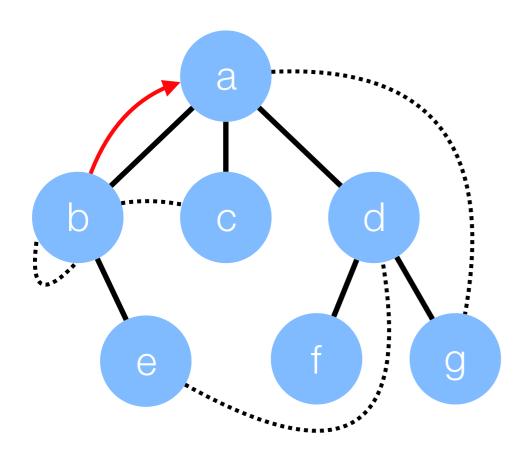


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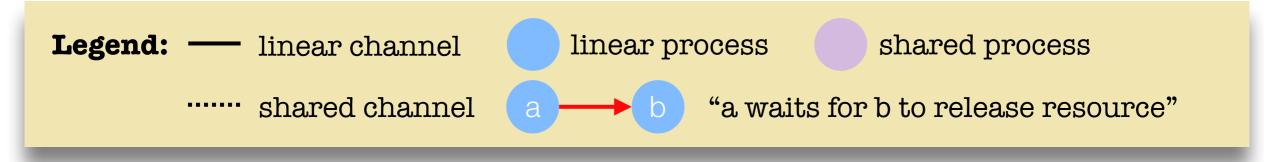


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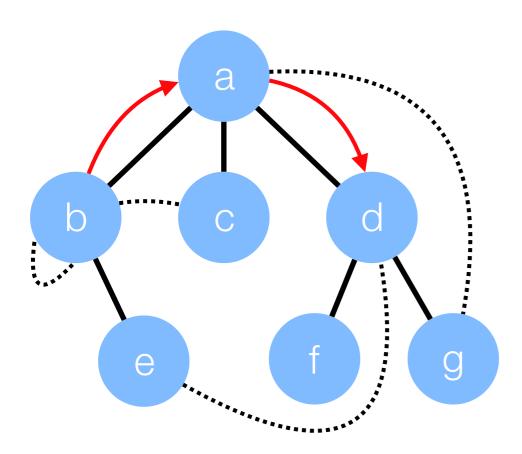
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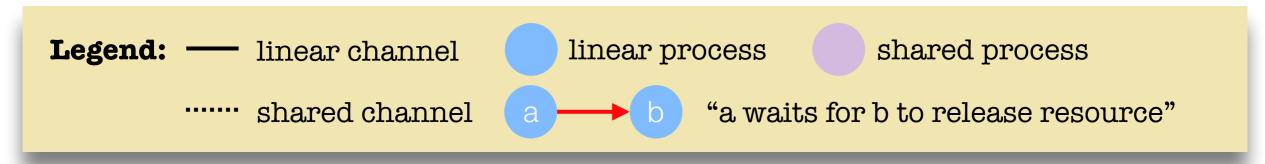
Let's add sharing

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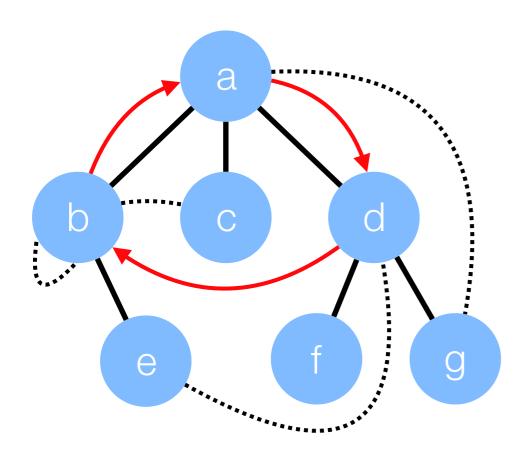
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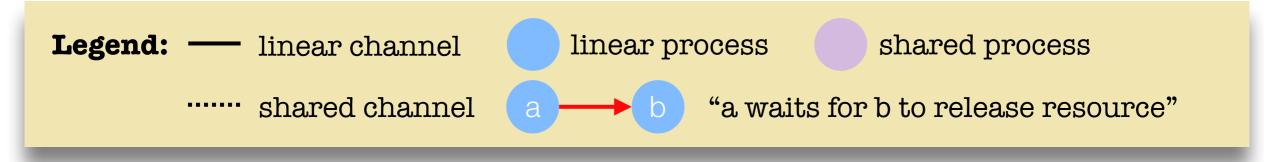
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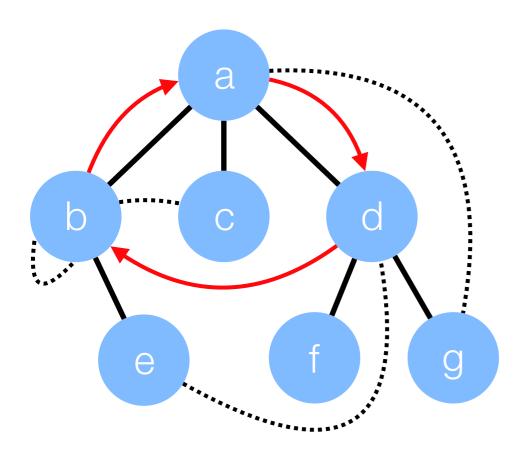
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Let's add sharing

We get a graph of linear and shared processes, with a linear tree inside.



Acquire-release amounts to "locking"

- Possibility of cyclic dependencies
- Let's visualize this waiting dependency with a red arrow
- Note: red arrows can connect arbitrary nodes

Legend: — linear channel linear process shared process shared channel a — b "a waits for b to release resource"

An enticing solution: "locking up"

- Impose a partial order on resources.
- Ensure that resources are acquired ("locked") in increasing order.

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Two Forms of waiting dependencies:

waiting to synchronize:
 a -> b
 "a waits for b to synchronize"

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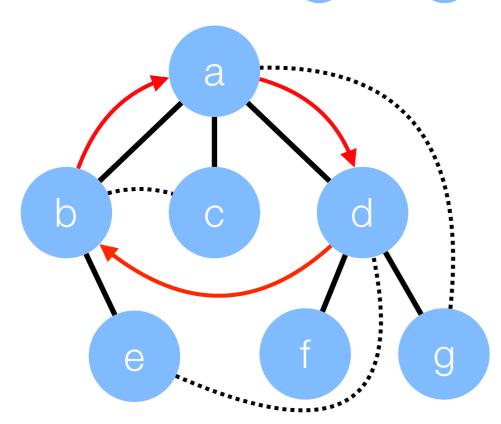


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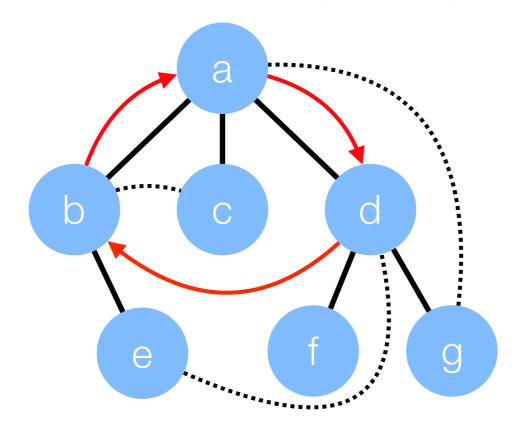
- waiting to synchronize:
 a -> b
 "a waits for b to synchronize"
- waiting to release: (a) (b) "a waits for b to release resource"

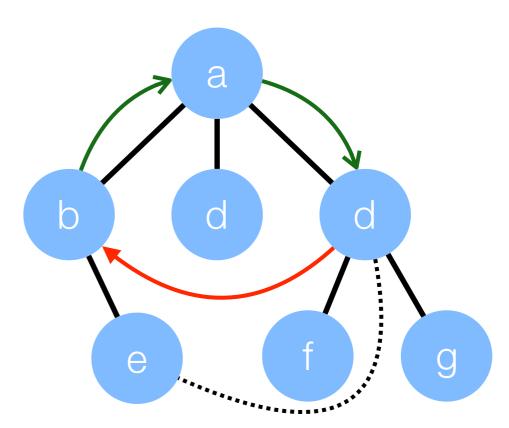
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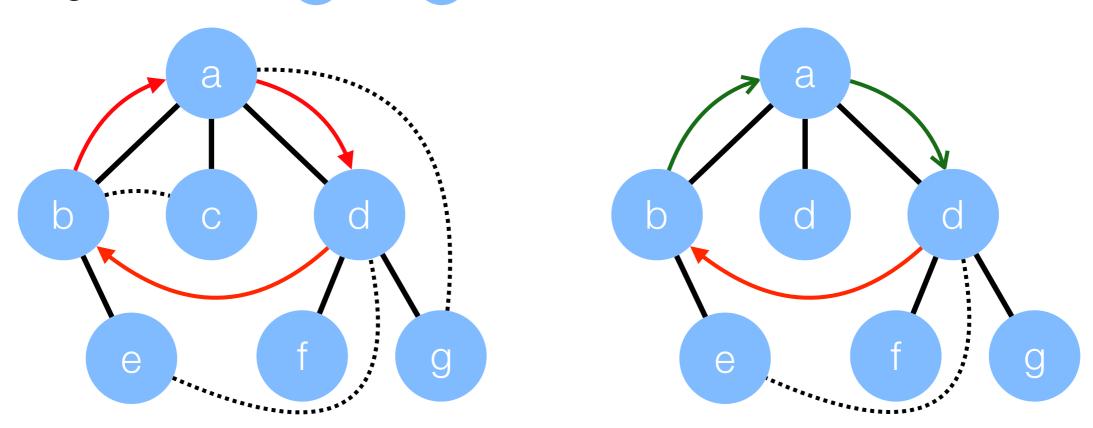
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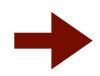




Two Forms of waiting dependencies:

- waiting to synchronize:
 a -> b
 "a waits for b to synchronize"





Cycles can consist of red arrows only or a combination of red and green arrows.



Competitors: overlap in set of resources acquired



Competitors: overlap in set of resources acquired



Collaborators: do not overlap in set of resources acquired



Competitors: overlap in set of resources acquired

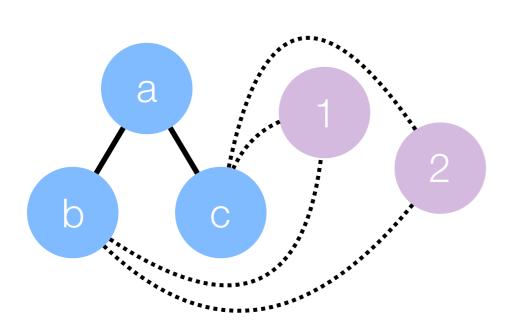


Collaborators: do not overlap in set of resources acquired

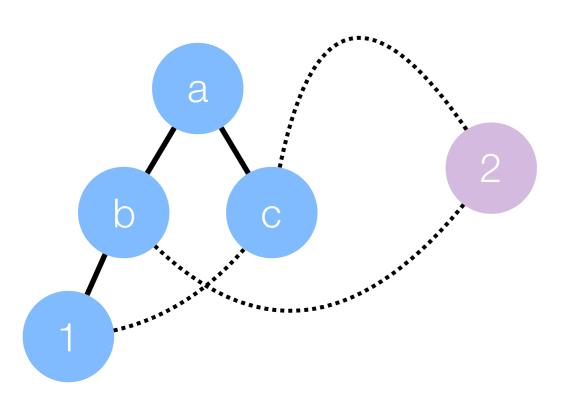


Notice: a resource acquired becomes a child of the acquirer

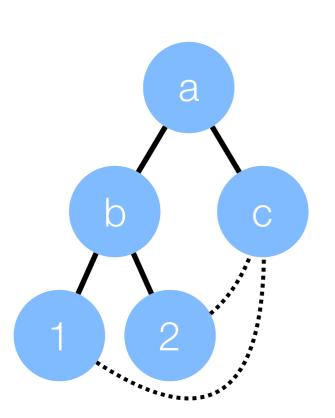
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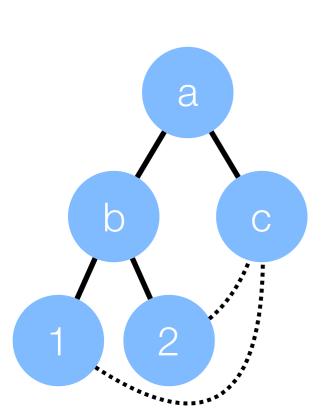
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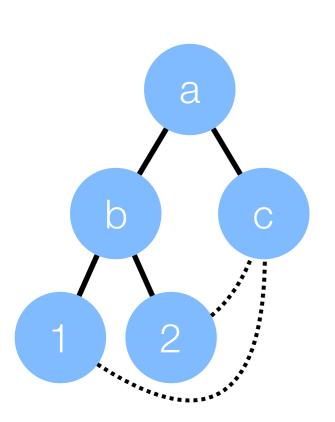




competitors tend to be siblings

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- Competitors: overlap in set of resources acquired
- -
- Collaborators: do not overlap in set of resources acquired
- **→**

Notice: a resource acquired becomes a child of the acquirer



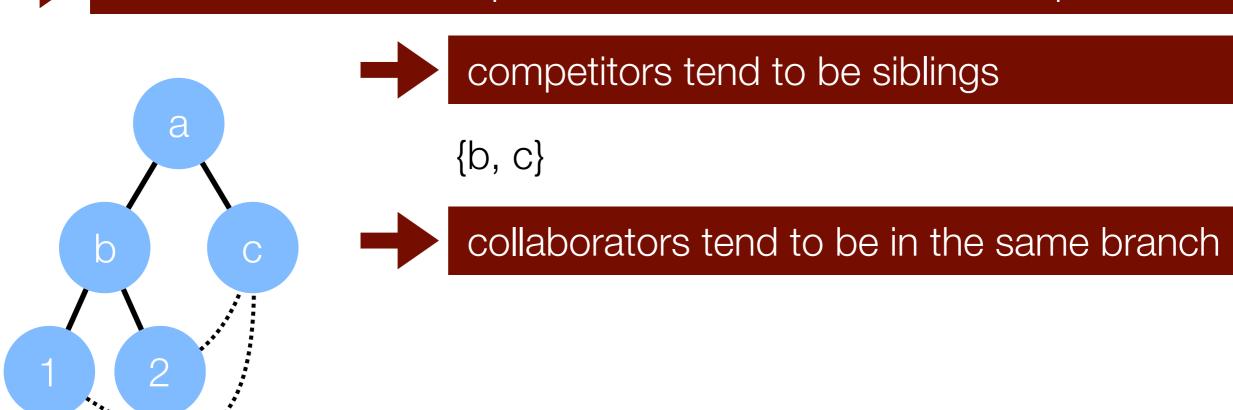


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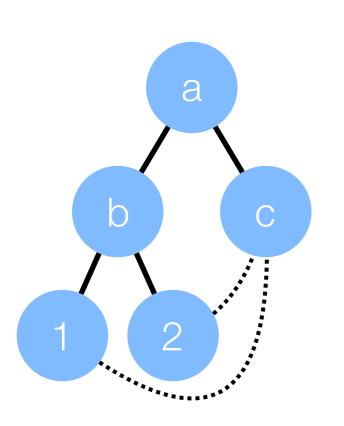
{b, c}

- Competitors: overlap in set of resources acquired

 Collaborators: do not overlap in set of resources acquired
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competitors tend to be siblings

{b, c}



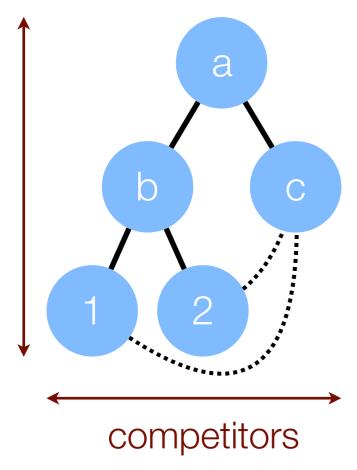
collaborators tend to be in the same branch

{a, b, 1} {a, b, 2} {a, c}



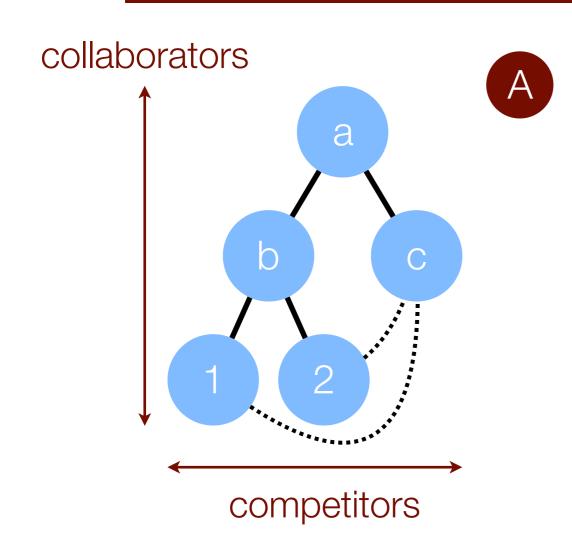
Define type system enforcing the following invariants:

collaborators





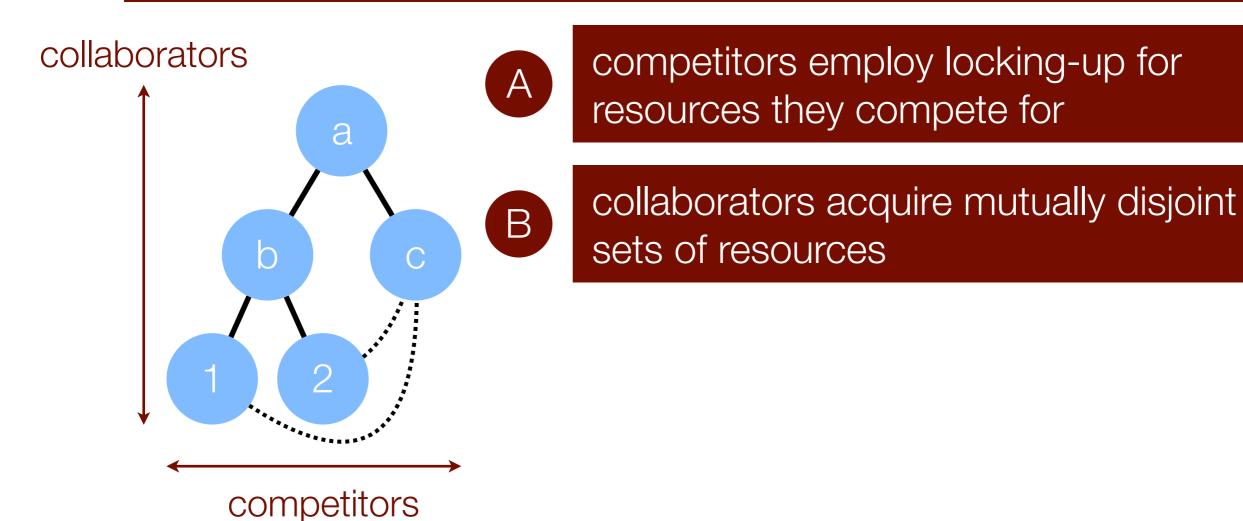
Define type system enforcing the following invariants:



competitors employ locking-up for resources they compete for

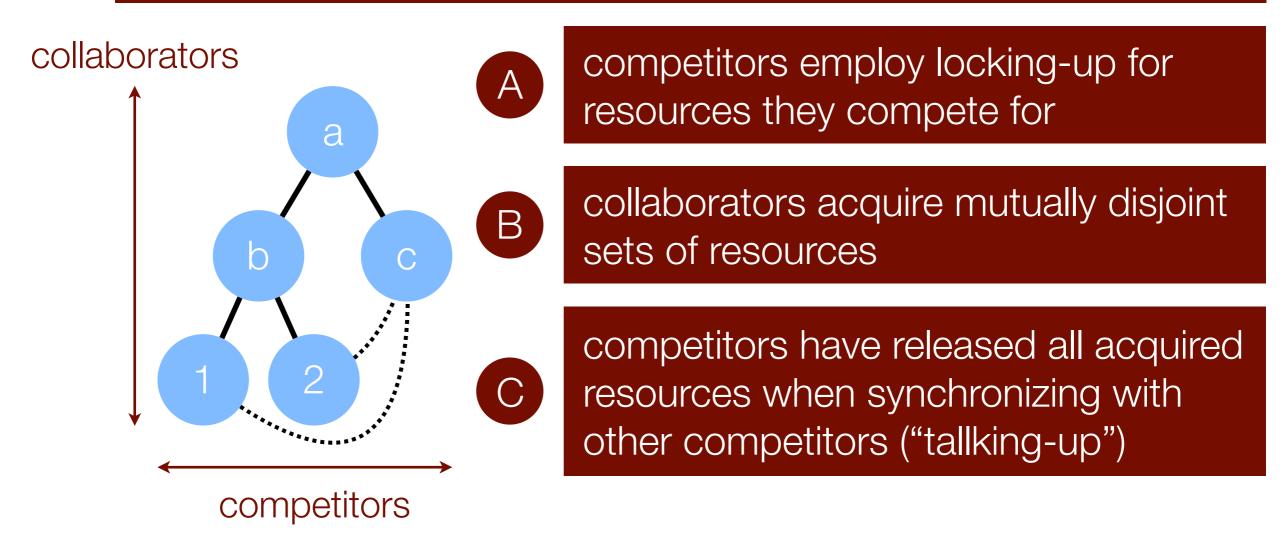


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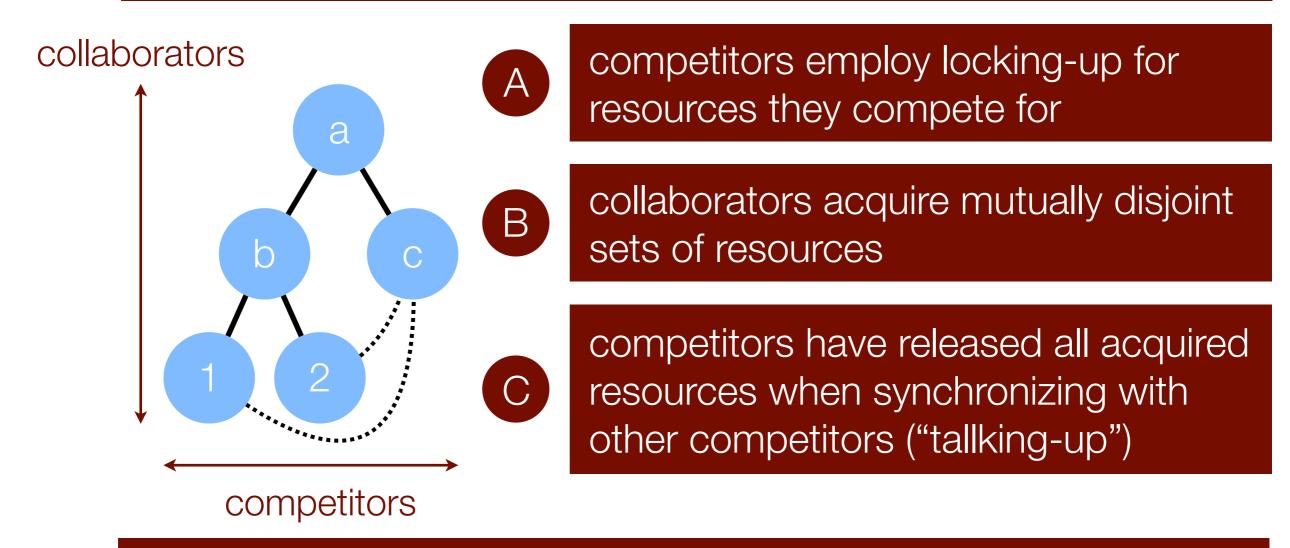


Define type system enforcing the following invariants:





Define type system enforcing the following invariants:





A rules out red-arrow cycles, B and C rule out red-green-arrow cycles.







Introduce a world, an abstract value equipped with a partial order.





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Every process invariantly resides at a world.





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Every process indicates the range of worlds it may acquire.





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worlds associated with process





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self-world:
world at which process
resides





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min-world:
world of minimal resource
to be acquired





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max-world: world of maximal resource to be acquired





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world order





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possibly "aliased" linear channels



```
\Psi; \Gamma \vdash_{\Sigma} P :: (x_{\mathsf{S}} : A_{\mathsf{S}}[\omega_k \updownarrow_{\omega_l}^{\omega_n}]) (where \Psi^+ irreflexive)

\Psi; \Gamma; \Phi; \Delta \vdash_{\Sigma} P :: (x_{\mathsf{L}} : A_{\mathsf{L}}[\omega_k \updownarrow_{\omega_l}^{\omega_n}]) (where \Psi^+ irreflexive)
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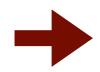


Express invariants A, B, and C in terms of:



```
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```



Express invariants A, B, and C in terms of:



 $min(parent) \le self(acquired_child) \le max(parent)$



```
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```

- -
- Express invariants A, B, and C in terms of:
- min(parent) ≤ self(acquired_child) ≤ max(parent)
- max(parent) < min(child)



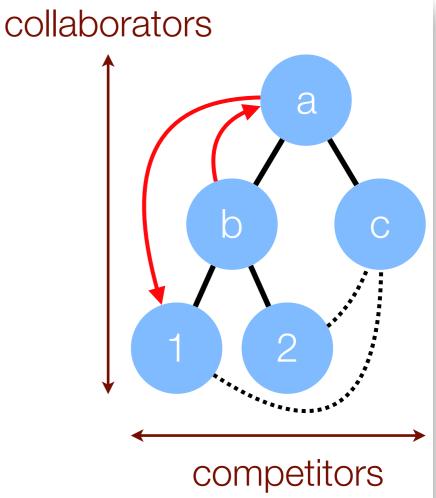
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$$\Psi; \Gamma; \Phi; \Delta \vdash_{\Sigma} P :: (x_{\mathsf{L}} : A_{\mathsf{L}}[\omega_k \updownarrow_{\omega_l}^{\omega_n}]) \quad (\mathsf{w}$$









no vertical red arrows



```
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- Express invariants A, B, and C in terms of:
- **→**

min(parent) ≤ self(acquired_child) ≤ max(parent)



max(parent) < min(child)



```
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```

- Express invariants A, B, and C in terms of:
 - min(parent) ≤ self(acquired_child) ≤ max(parent)
 - max(parent) < min(child)
 - for an acquire: lock-up

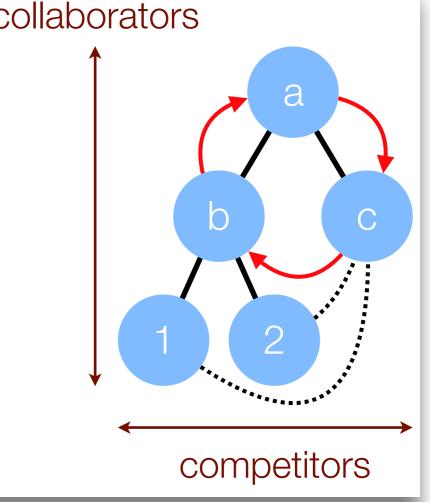


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$$\Psi; \Gamma; \Phi; \Delta \vdash_{\Sigma} P :: (x_{\mathsf{L}} : A_{\mathsf{L}}[\omega_k \downarrow_{\omega_l}^{\omega_n}]) \quad (\mathsf{v}$$



- min(parent) ≤ self(acquired_child) ≤
- max(parent) < min(child)
- for an acquire: lock-up



no red cycles



```
\Psi; \Gamma \vdash_{\Sigma} P :: (x_{\mathsf{S}} : A_{\mathsf{S}}[\omega_k \updownarrow_{\omega_l}^{\omega_n}]) (where \Psi^+ irreflexive)

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- Express invariants A, B, and C in terms of:
 - min(parent) ≤ self(acquired_child) ≤ max(parent)
 - max(parent) < min(child)
 - for an acquire: lock-up
 - for right-rule: Φ must be empty

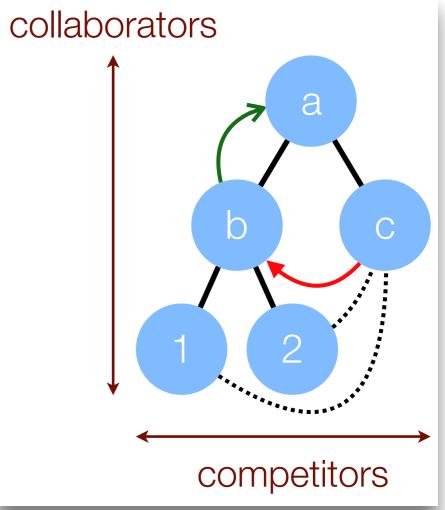


$$\Psi$$
; $\Gamma \vdash_{\Sigma} P :: (x_{\mathsf{S}} : A_{\mathsf{S}}[\omega_k \updownarrow_{\omega_l}^{\omega_n}])$ (where Ψ^+ irreflexive)

$$\Psi; \Gamma; \Phi; \Delta \vdash_{\Sigma} P :: (x_{\mathsf{L}} : A_{\mathsf{L}}[\omega_k \updownarrow_{\omega_l}^{\omega_n}]) \quad (\mathbf{w})$$



- min(parent) ≤ self(acquired_child) ≤
- max(parent) < min(child)
- for an acquire: lock-up
- for right-rule: Φ must be empty



no ingoing red and up-going green arrow



```
\Psi; \Gamma \vdash_{\Sigma} P :: (x_{\mathsf{S}} : A_{\mathsf{S}}[\omega_k \updownarrow_{\omega_l}^{\omega_n}]) (where \Psi^+ irreflexive)

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```

- Express invariants A, B, and C in terms of:
 - min(parent) ≤ self(acquired_child) ≤ max(parent)
 - max(parent) < min(child)
 - for an acquire: lock-up
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- Express invariants A, B, and C in terms of:
 - min(parent) ≤ self(acquired_child) ≤ max(parent)
 - max(parent) < min(child)
 - for an acquire: lock-up
 - for right-rule: Φ must be empty
- These low-level invariants are enforced by typing.

Taking stock

Taking stock



We have a session type system that allows shared and linear channels to coexist and guarantees:



data-race-freedom (low-level and high-level)



protocol adherence



deadlock-freedom

Taking stock



We have a session type system that allows shared and linear channels to coexist and guarantees:



data-race-freedom (low-level and high-level)



protocol adherence



deadlock-freedom



We have increased practicality of linear session types while maintaining their guarantees.

Current & future work



Unique application field for shared session types:



Unique application field for shared session types:

```
  \text{auction} = \uparrow_{\text{L}}^{\text{S}} \oplus \{\text{running}: \&\{\text{bid}: \text{id} \rightarrow \text{money} \multimap \downarrow_{\text{L}}^{\text{S}} \text{auction}, \\  \quad \text{cancel}: \downarrow_{\text{L}}^{\text{S}} \text{auction}\}, \\  \quad \text{ended}: \text{id} \rightarrow \oplus \{\text{won}: \text{lot} \otimes \downarrow_{\text{L}}^{\text{S}} \text{auction}, \\  \quad \text{lost}: \text{money} \otimes \downarrow_{\text{L}}^{\text{S}} \text{auction}\}\}
```



Unique application field for shared session types:

```
 \text{auction} = \uparrow_{\text{L}}^{\text{S}} \oplus \{ \text{running} : \& \{ \text{bid} : \text{id} \rightarrow \text{money} \multimap \downarrow_{\text{L}}^{\text{S}} \text{auction}, \\ \text{cancel} : \downarrow_{\text{L}}^{\text{S}} \text{auction} \}, \\ \text{ended} : \text{id} \rightarrow \oplus \{ \text{won} : \text{lot} \otimes \downarrow_{\text{L}}^{\text{S}} \text{auction}, \\ \text{lost} : \text{money} \otimes \downarrow_{\text{L}}^{\text{S}} \text{auction} \} \}
```



Resource analysis for static prediction of execution cost.



Unique application field for shared session types:

```
\label{eq:auction} \text{auction} = \uparrow_{\text{L}}^{\text{S}} \oplus \{\text{running} : \&\{\text{bid} : \text{id} \rightarrow \text{money} \multimap \downarrow_{\text{L}}^{\text{S}} \text{auction},\\ \quad \text{cancel} : \downarrow_{\text{L}}^{\text{S}} \text{auction}\},\\ \quad \text{ended} : \text{id} \rightarrow \oplus \{\text{won} : \text{lot} \otimes \downarrow_{\text{L}}^{\text{S}} \text{auction},\\ \quad \text{lost} : \text{money} \otimes \downarrow_{\text{L}}^{\text{S}} \text{auction}\}\}
```

- Resource analysis for static prediction of execution cost.
- Under development: Nomos, a digital contract language based on resource-aware shared session types.



Shared session types recover expressiveness of untyped asynchronous pi-calculus [Balzer et al. CONCUR 2018]



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introduce nondeterminism



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linear logic session types are deterministic



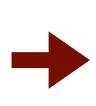
Shared session types recover expressiveness of untyped asynchronous pi-calculus [Balzer et al. CONCUR 2018]



introduce nondeterminism



linear logic session types are deterministic



Opportunity for unifying framework that combines both deterministic (parallel) and nondeterministic (concurrent) computation.

Thank you for your attention!

Papers for this talk:

- Stephanie Balzer and Frank Pfenning: Manifest Sharing with Session Types. ICFP 2017.
- Stephanie Balzer, Bernardo Toninho, and Frank Pfenning: Manifest Deadlock-Freedom for Shared Session Types. ESOP 2019.
- Stephanie Balzer, Frank Pfenning, and Bernardo Toninho: A Universal Session Type for Untyped Asynchronous Communication. CONCUR 2019.