
WELCOME
TO
15-213
14-513
18-213
18-613
15-513

Thread-Level Parallelism

15-213/15-513/14-513: Introduction to Computer Systems
24th Lecture, December 2, 2025

Logistics

- **Written #10 peer review due Wed Dec 3**

- **SFS due Thur Dec 4 at 11:59pm**
  - NO late submissions beyond Fri Dec 5 (CMU policy)
  - Any extensions beyond this require taking an incomplete for the course

- **Final Exam on Mon Dec 8 at 8:30-11:30 am**
  - We have multiple rooms. No pre-assigned rooms, pick your own room
  - You can bring two 8.5”x11” / A4 cheat sheets, written or printed
  - Make-up final will be on Mon Dec 15 at 9:30 am – 12:30 pm. Room TBD. Only three permitted reasons for taking the makeup final:
    - It is physically impossible for you to take the exam at scheduled time
    - You discover you are too sick to take an exam on the morning of Dec 8
    - You have so many exams that you cannot take the exam on the 8th without violating the university exam conflict policy

Exam Reviews

- **In Recitation on Fri Dec 5**

- **In Bootcamp #6 on Sun Dec 7**
  - Time and Room TBD. We will post to Ed.

Hours and FCEs

- **Please fill out your FCEs (faculty course evaluations)**
  - Response rates have been declining in recent semesters
  - Please accurately estimate the number of hours spent on the course
    - University looks at closely (so do the instructors)

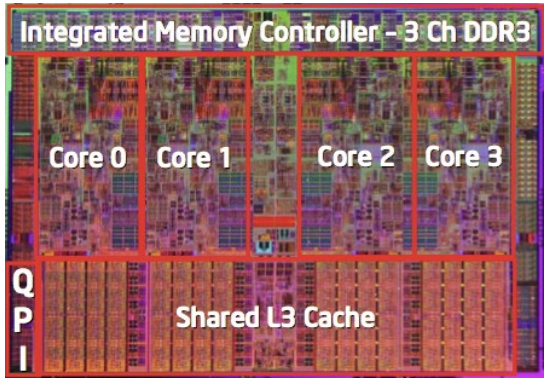
Today's Lecture Disclaimer

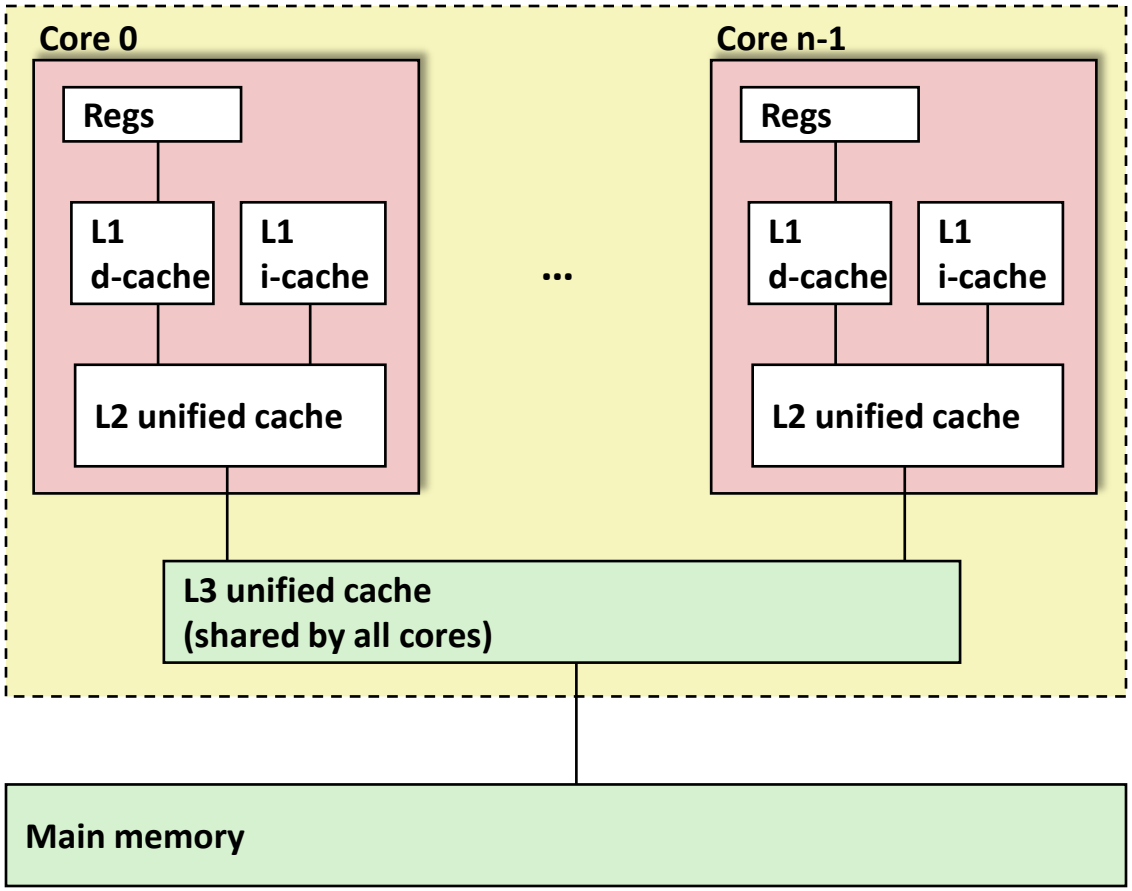
- **We do not have time to fully cover today's content**
  - Take -346, -410, -418 …
- **Valuable to know as you start writing parallel programs**

Today

- **Parallel Computing Hardware** CSAPP 12.6
  - Multicore
    - Multiple separate processors on single chip
  - Hyperthreading
    - Efficient execution of multiple threads on single core
- **Consistency Models** CSAPP 12.6
  - What happens when multiple threads are reading & writing shared state
- **Thread-Level Parallelism** CSAPP 12.6
  - Splitting program into independent tasks
    - Example: Parallel summation
    - Examine some performance artifacts
  - Divide-and conquer parallelism
    - Example: Parallel quicksort

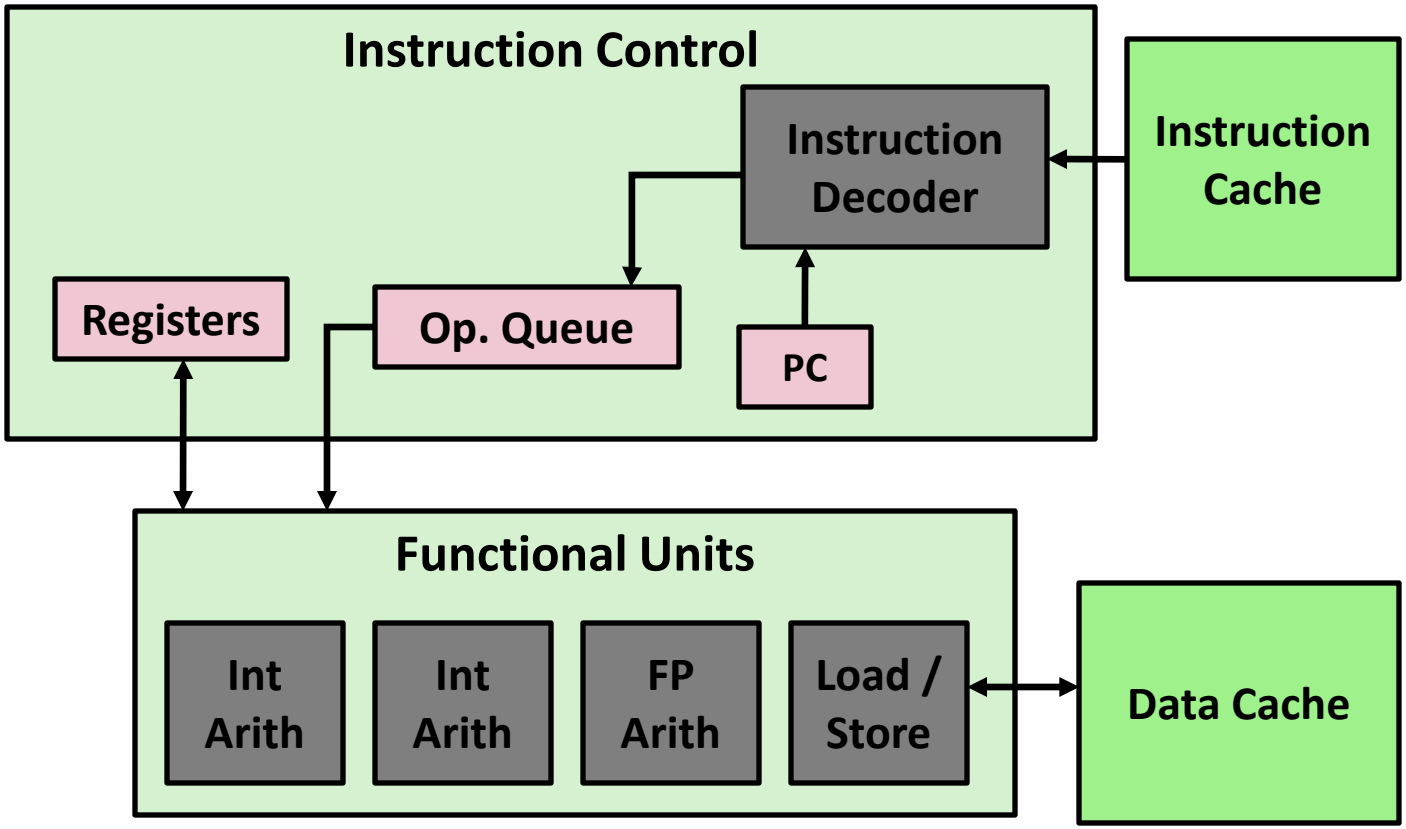
Typical Multicore Processor





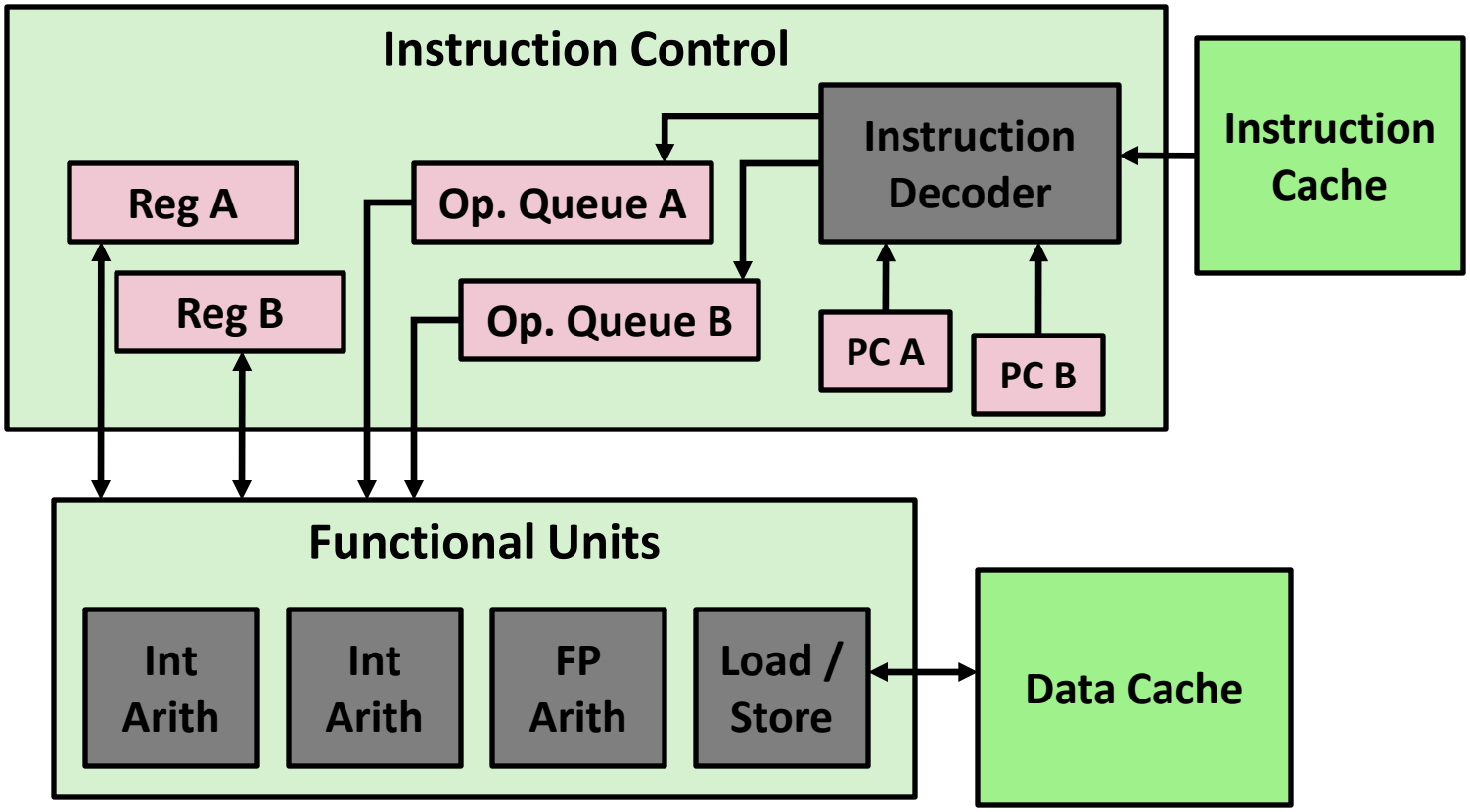
- Multiple processors operating with coherent view of memory

Out-of-Order Processor Structure



- Instruction control dynamically converts program into stream of operations
- Operations mapped onto functional units to execute in parallel

Hyperthreading Implementation



- Replicate instruction control to process K instruction streams. K copies of all registers. (Typically K=2)
- Share functional units

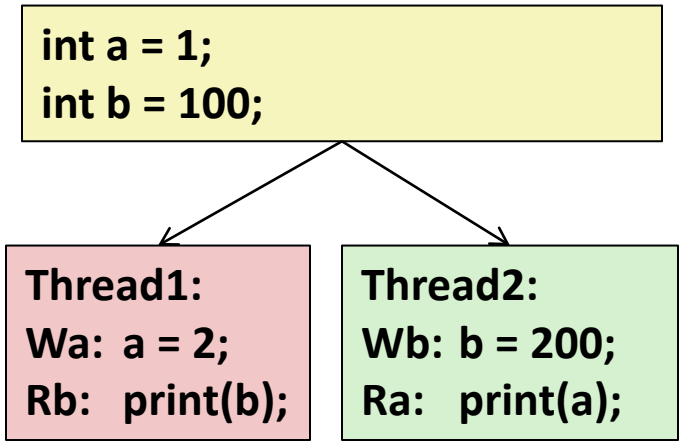
Benchmark Machine

- **Get data about machine from /proc/cpuinfo**
- **Shark Machines**
  - Intel Xeon E5520 @ 2.27 GHz
  - Nehalem, ca. 2010
  - 8 Cores
  - Each can do 2-way hyperthreading

Exploiting parallel execution

- **So far, we've used threads to deal with I/O delays**
  - e.g., one thread per client to prevent one from delaying another
- **Multi-core CPUs offer another opportunity**
  - Spread work over threads executing in parallel on P cores
  - Happens automatically, if many independent tasks
    - e.g., running many applications or serving many clients
  - Can also write code to make one big task go faster
    - by organizing it as multiple parallel sub-tasks
- **Shark machines can execute 16 threads at once**
  - 8 cores, each with 2-way hyperthreading
  - Theoretical speedup of 16X
    - never achieved in our benchmarks

Memory Coherence / Consistency



Thread consistency constraints

Wa → Rb

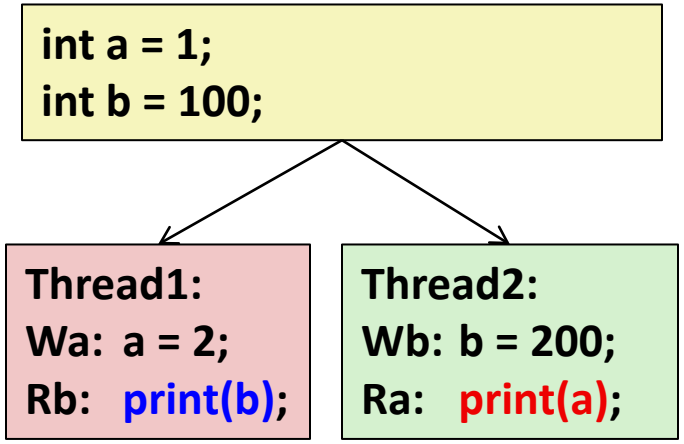
Wb → Ra

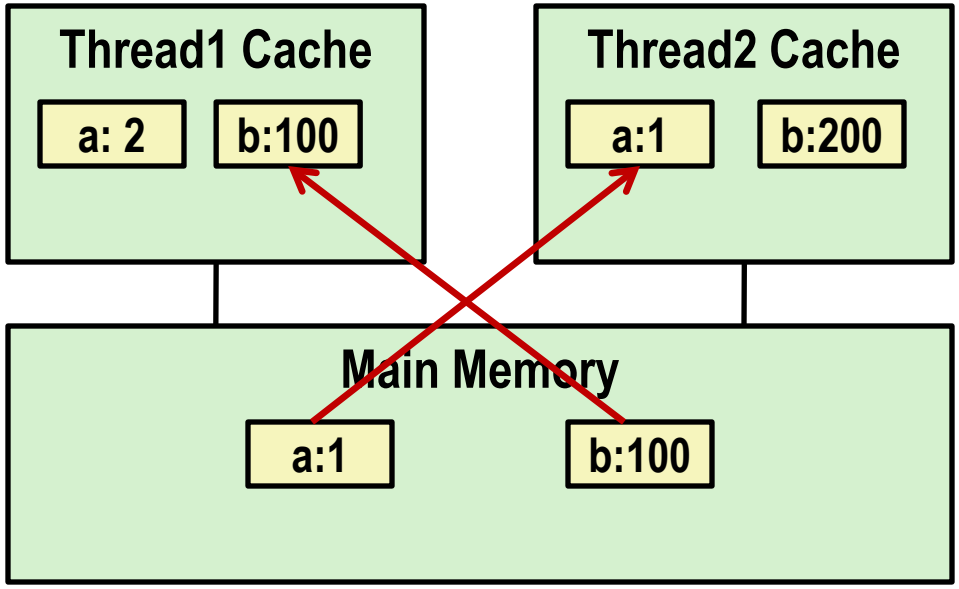
- **What are the possible values printed?**
  - Depends on memory consistency model
  - Abstract model of how hardware handles concurrent accesses

- **How do the two threads really see the writes?**

Non-Coherent Cache Scenario

- Write-back caches, without coordination between them





print 1

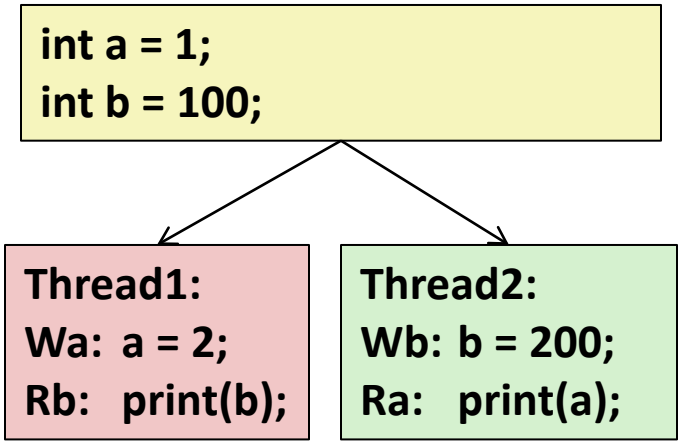
print 100

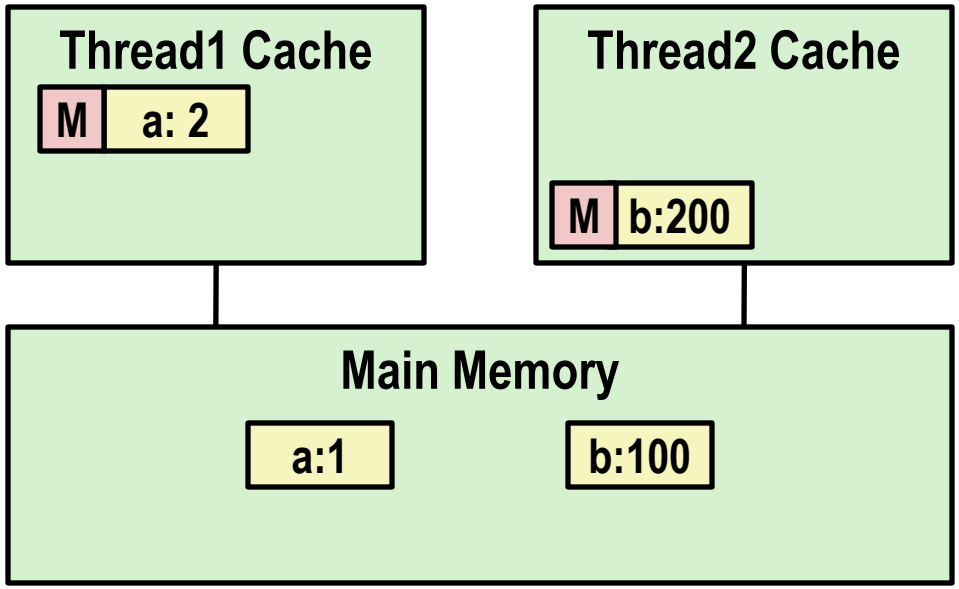
At later points, a=2 and b=200 are written back to main memory

Snoopy Caches

- **Tag each cache block with state**

Invalid	Cannot use value
Shared	Readable copy
Modified	Writeable copy





Snoopy Caches

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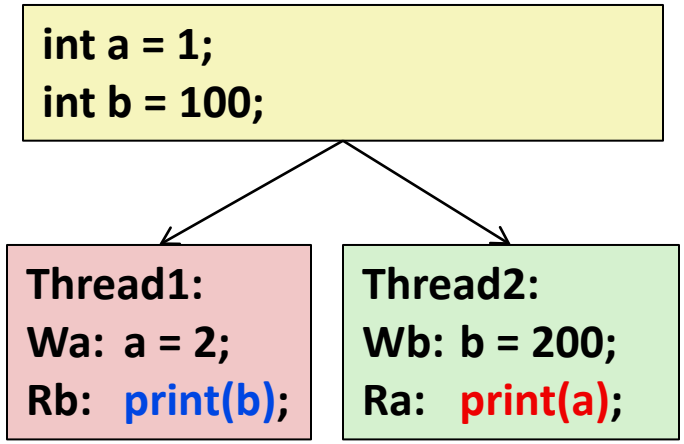
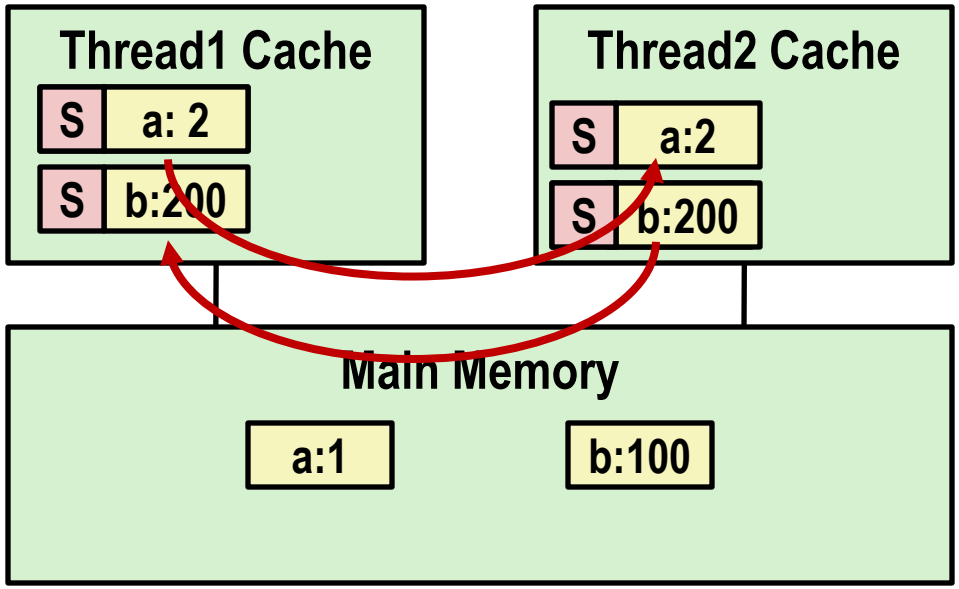


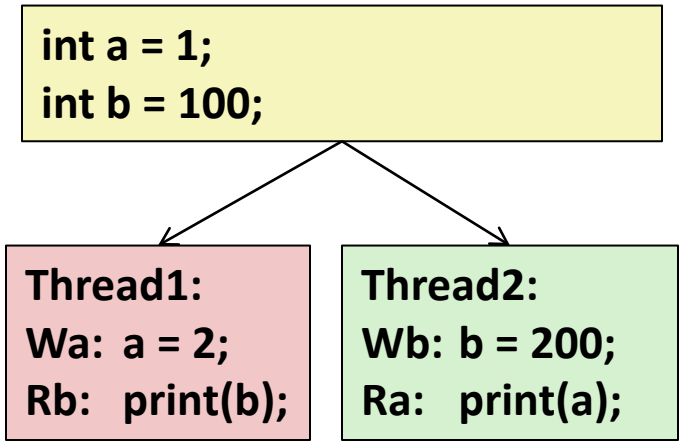


print 2

print 200

- When cache sees request for one of its M-tagged blocks
  - Supply value from cache (Note: value in memory may be stale)
  - Set tag to S

Memory Consistency



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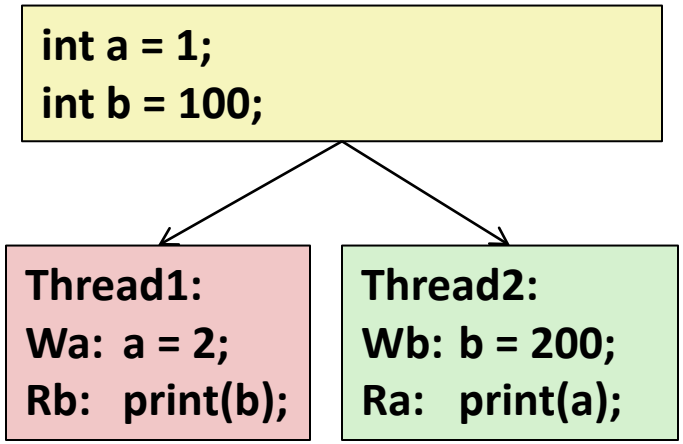
Memory Consistency



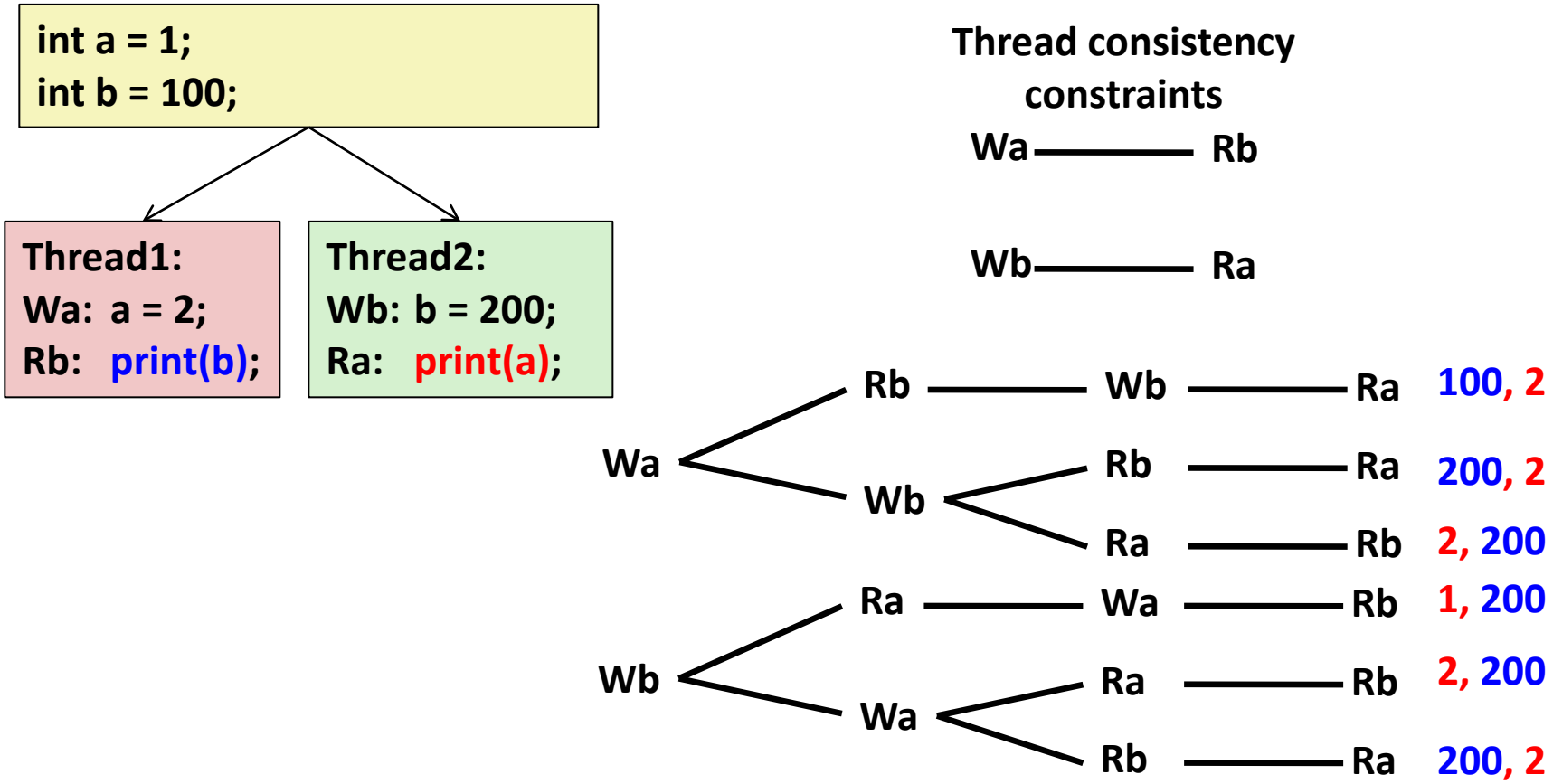
Thread consistency constraints

Wa → Rb

Wb → Ra

- **What are the possible values printed?**
  - Depends on memory consistency model
  - Abstract model of how hardware handles concurrent accesses
- **Sequential consistency**
  - As if only one operation at a time, in an order consistent with the order of operations within each thread
  - Thus, overall effect consistent with each individual thread but otherwise allows an arbitrary interleaving

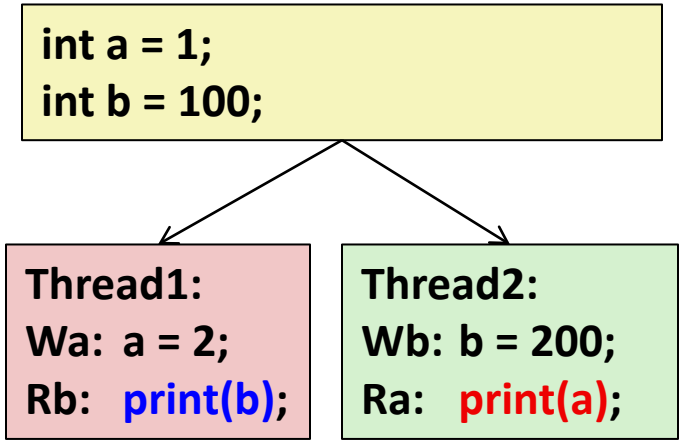
Sequential Consistency Example

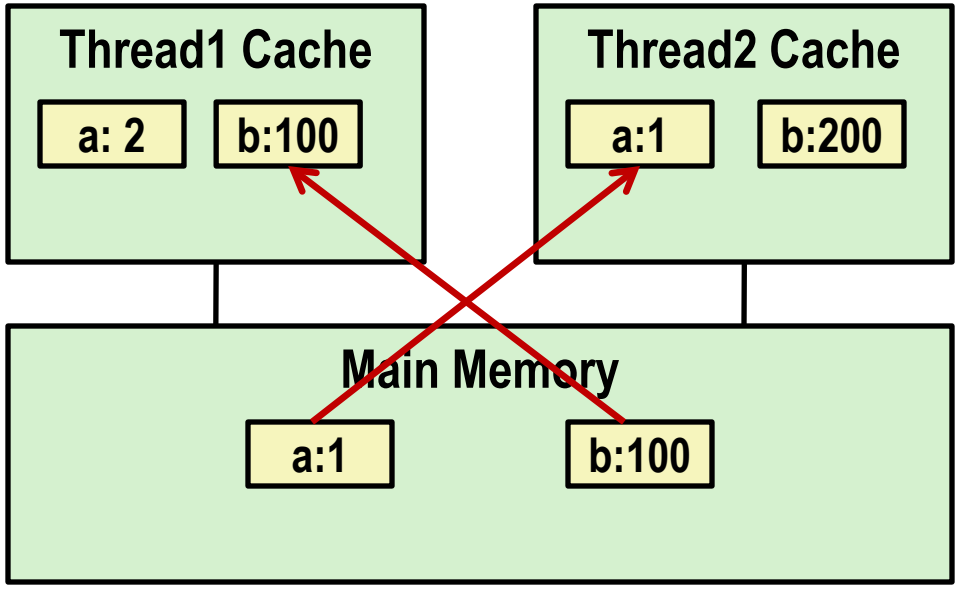


- **Impossible outputs**
  - 100, 1 and 1, 100
  - Would require reaching *both* Ra and Rb before *either* Wa or Wb

Non-Coherent Cache Scenario

- Write-back caches, without coordination between them





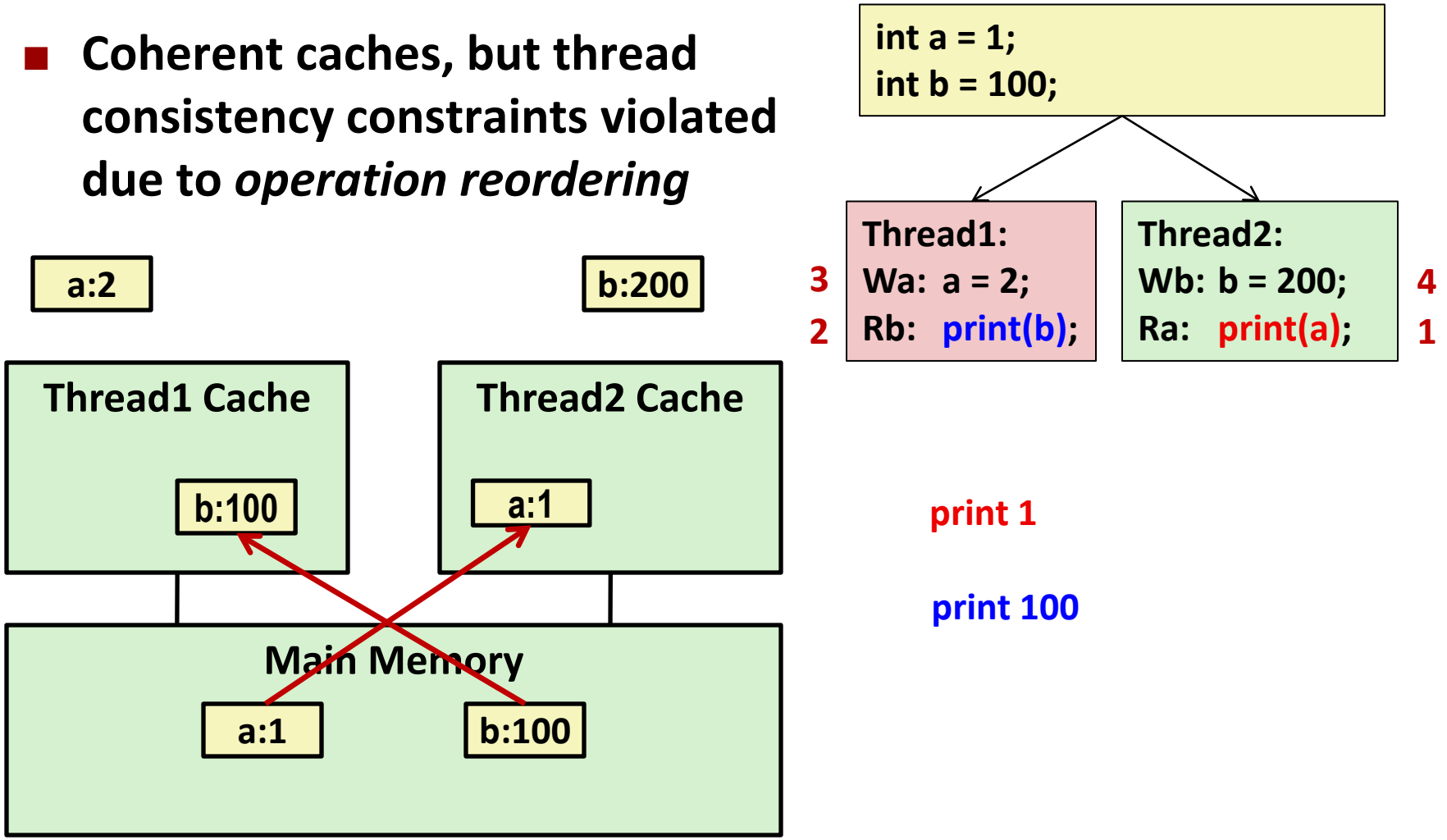
print 1

print 100

Sequentially consistent? No!

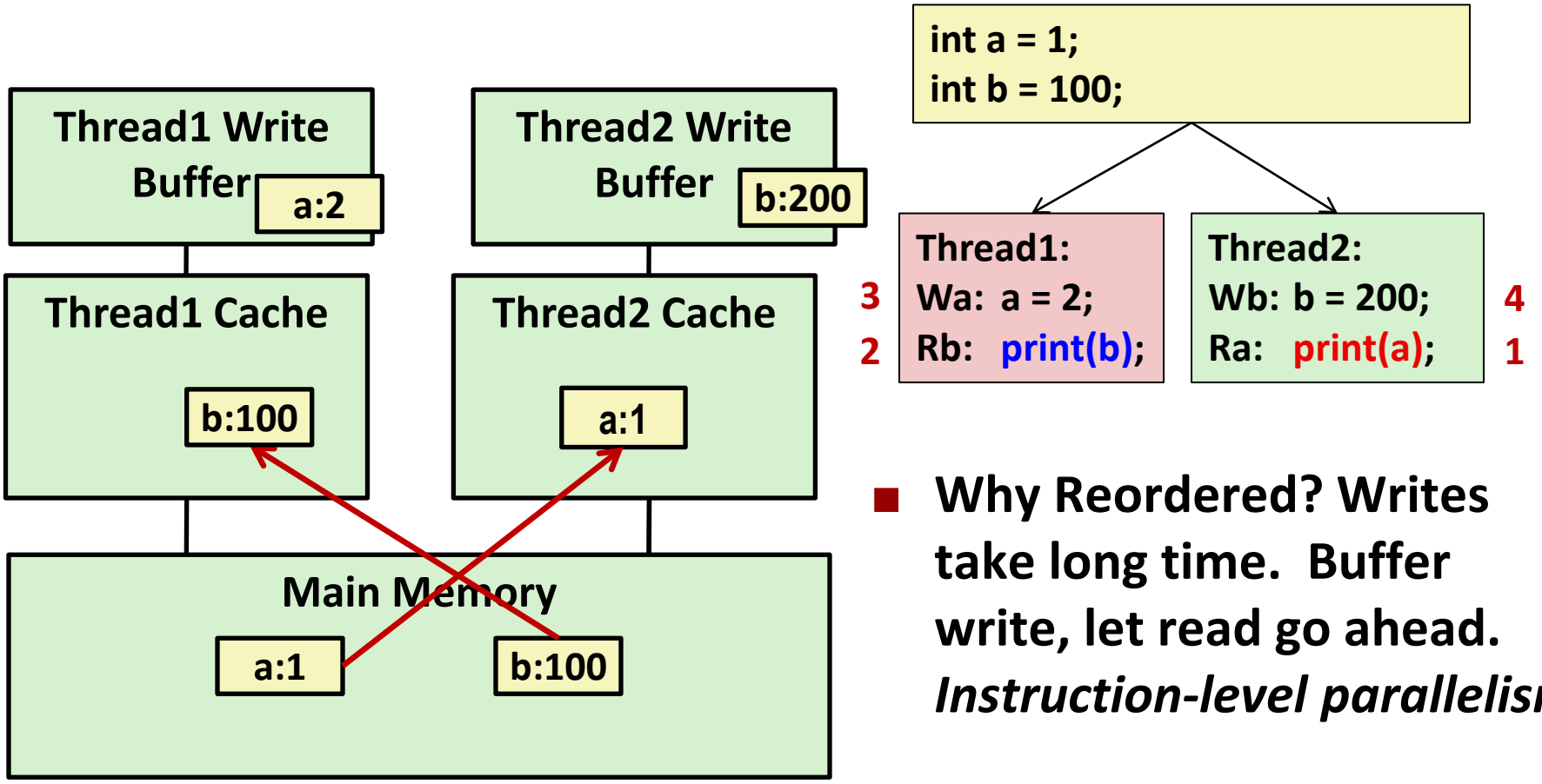
Non-Sequentially Consistent Scenario

- Coherent caches, but thread consistency constraints violated due to *operation reordering*



- Architecture allows reads to finish before writes because single thread accesses different memory locations

Non-Sequentially Consistent Scenario



- Why Reordered? Writes take long time. Buffer write, let read go ahead. *Instruction-level parallelism*

- Fix: Add `SFENCE` instructions between Wa & Rb and Wb & Ra
- Fix: Use synchronization (properly written, it fences)

Memory Models

- **Sequentially Consistent:**
  - Each thread executes in proper order, any interleaving
- **To ensure, requires**
  - Proper cache/memory behavior
  - Proper intra-thread ordering constraints
- **Thread ordering constraints**
  - Use synchronization to ensure the program is free of data races

Today

- **Parallel Computing Hardware**
  - Multicore
    - Multiple separate processors on single chip
  - Hyperthreading
    - Efficient execution of multiple threads on single core
- **Consistency Models**
  - What happens when multiple threads are reading & writing shared state
- **Thread-Level Parallelism**
  - Splitting program into independent tasks
    - Example: Parallel summation
    - Examine some performance artifacts
  - Divide-and conquer parallelism
    - Example: Parallel quicksort

Summation Example

- **Sum numbers 0, …, N-1**
  - Should add up to (N-1)*N/2
- **Partition into K ranges**
  - ⌊N/K⌋ values each
  - Each of the *t* threads processes 1 range
  - Accumulate leftover values serially
- **Method #1: All threads update single global variable**
  - 1A: No synchronization
  - 1B: Synchronize with pthread semaphore
  - 1C: Synchronize with pthread mutex
    - "Binary" semaphore. Only values 0 & 1

Accumulating in Single Global Variable: Declarations

```
typedef unsigned long data_t;
/* Single accumulator */
volatile data_t global_sum;
```

Accumulating in Single Global Variable: Declarations

```
typedef unsigned long data_t;
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volatile data_t global_sum;

/* Mutex & semaphore for global sum */
sem_t semaphore;
pthread_mutex_t mutex;
```

Accumulating in Single Global Variable: Declarations

```
typedef unsigned long data_t;
/* Single accumulator */
volatile data_t global_sum;

/* Mutex & semaphore for global sum */
sem_t semaphore;
pthread_mutex_t mutex;

/* Number of elements summed by each thread */
size_t nelems_per_thread;

/* Keep track of thread IDs */
pthread_t tid[MAXTHREADS];

/* Identify each thread */
int myid[MAXTHREADS];
```

Accumulating in Single Global Variable: Operation

```
nelems_per_thread = nelems / nthreads;

/* Set global value */
global_sum = 0;

/* Create threads and wait for them to finish */
for (i = 0; i < nthreads; i++) {
    myid[i] = i;
    Pthread_create(&tid[i], NULL, thread_fun, &myid[i]);
}
for (i = 0; i < nthreads; i++)
    Pthread_join(tid[i], NULL);

result = global_sum;

/* Add leftover elements */
for (e = nthreads * nelems_per_thread; e < nelems; e++)
    result += e;
```

Thread ID

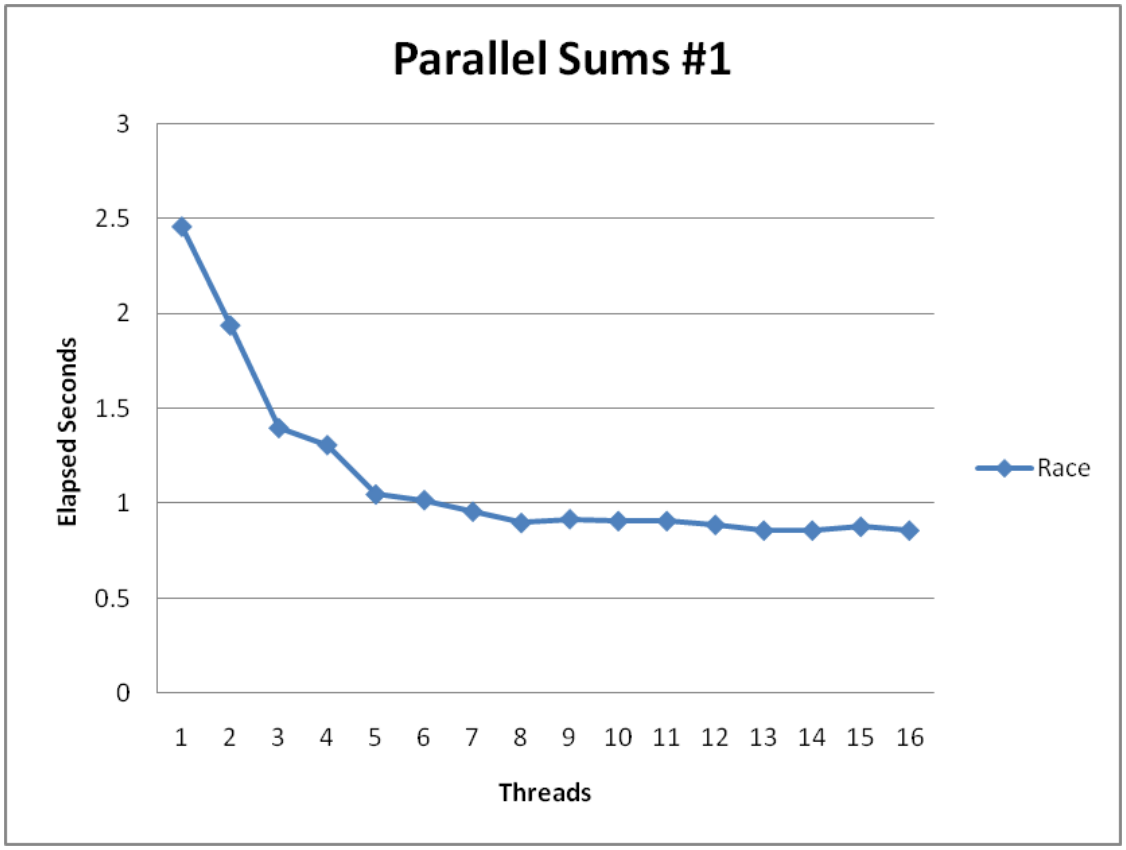
Thread routine

*Thread arguments (void *p)*

Thread Function: No Synchronization

```
void *sum_race(void *vargp)
{
    int myid = *((int *)vargp);
    size_t start = myid * nelems_per_thread;
    size_t end = start + nelems_per_thread;
    size_t i;

    for (i = start; i < end; i++) {
        global_sum += i;
    }
    return NULL;
}
```

Unsynchronized Performance



- N = 2^{30}
- Best speedup = 2.86X
- Gets wrong answer when > 1 thread! Why?

Thread Function: Semaphore / Mutex

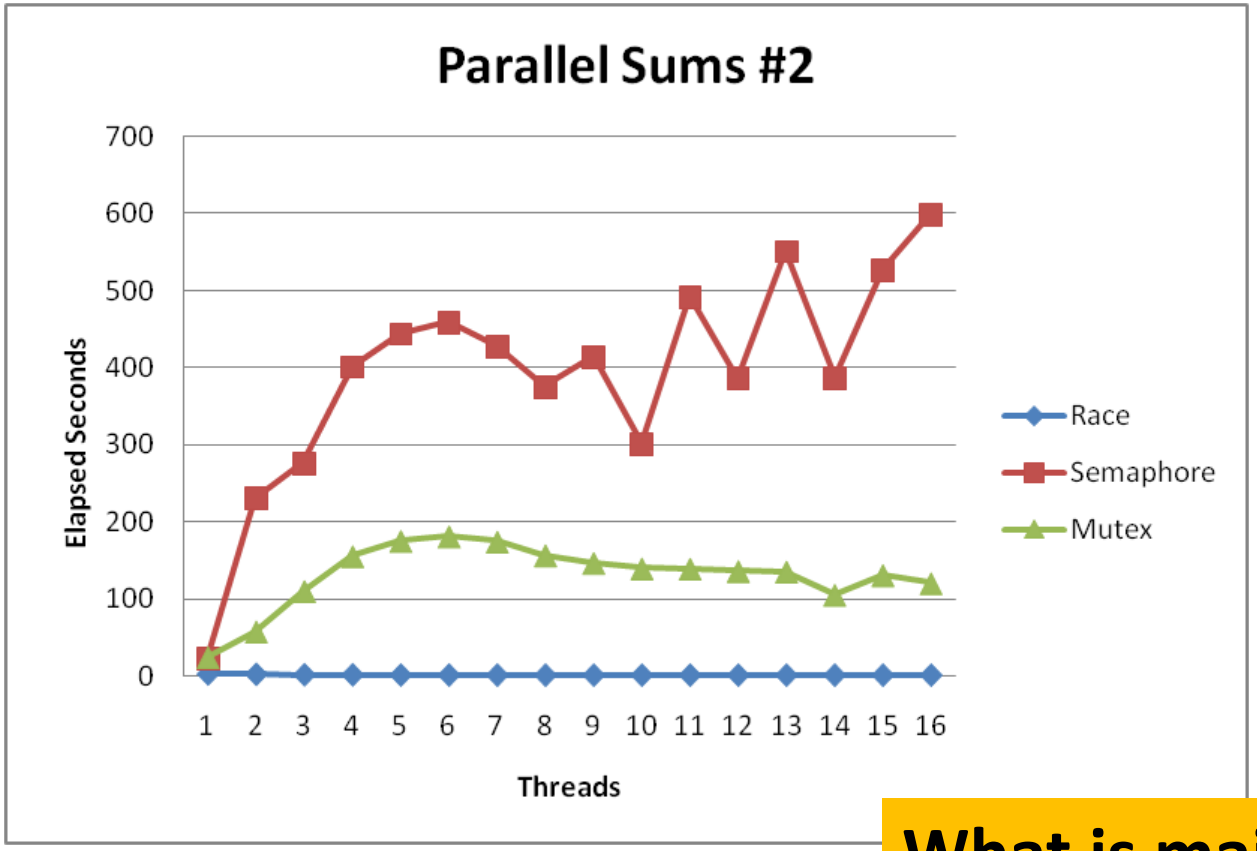
Semaphore

```
void *sum_sem(void *vargp)
{
    int myid = *((int *)vargp);
    size_t start = myid * nelems_per_thread;
    size_t end = start + nelems_per_thread;
    size_t i;

    for (i = start; i < end; i++) {
        sem_wait(&semaphore);
        global_sum += i;
        sem_post(&semaphore);
    }
    return NULL;
}
```

Mutex

```
pthread_mutex_lock(&mutex);
global_sum += i;
pthread_mutex_unlock(&mutex);
```

Semaphore / Mutex Performance



- **Terrible Performance**
  - 2.5 seconds ➔ ~10 minutes
- **Mutex 3X faster than semaphore**
- **Clearly, neither is successful**

What is main reason for poor performance?

Separate Accumulation

- **Method #2: Each thread accumulates into separate variable**
  - 2A: Accumulate in contiguous array elements
  - 2B: Accumulate in spaced-apart array elements
  - 2C: Accumulate in registers

```
/* Partial sum computed by each thread */
data_t psum[MAXTHREADS*MAXSPACING];

/* Spacing between accumulators */
size_t spacing = 1;
```

Separate Accumulation: Operation

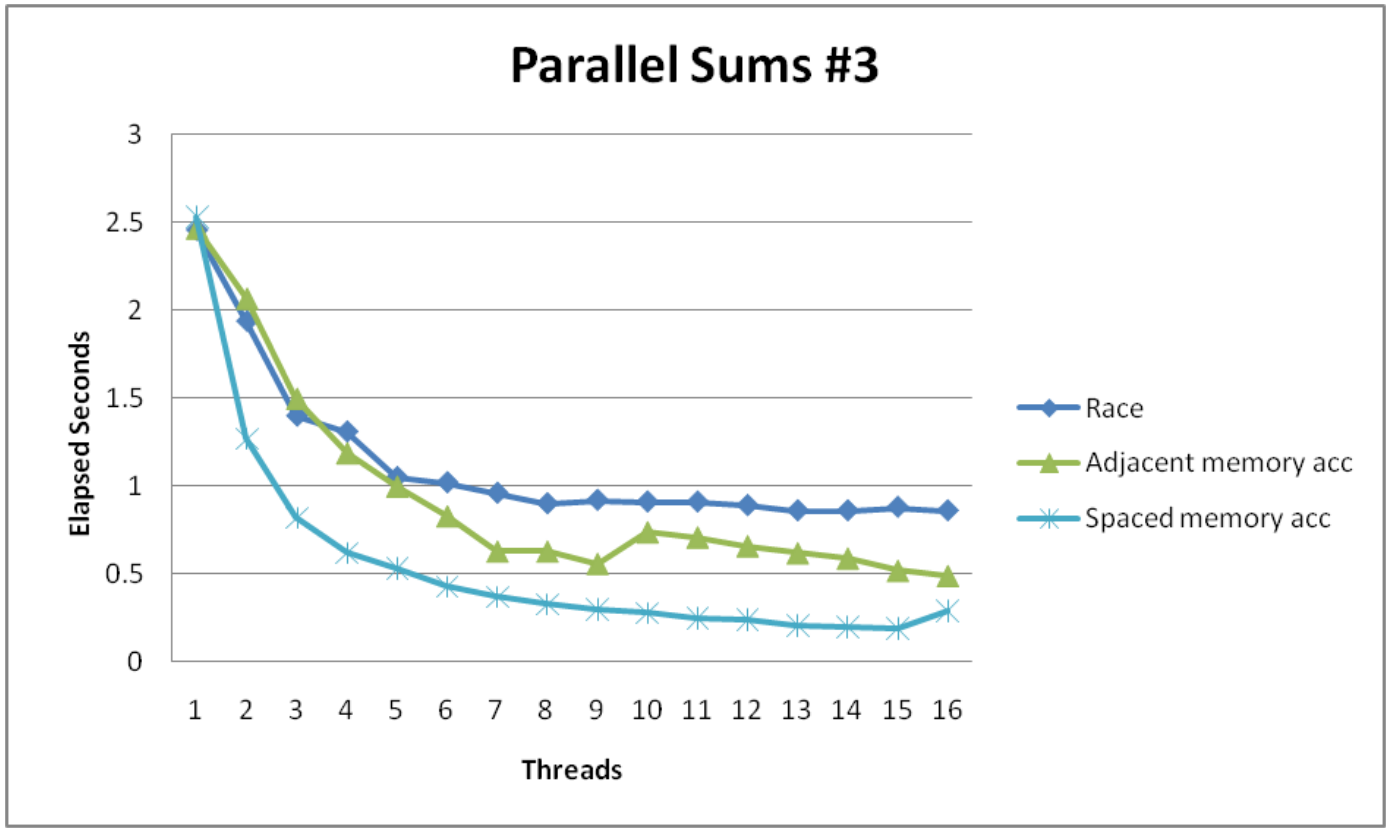
```
nelems_per_thread = nelems / nthreads;

/* Create threads and wait for them to finish */
for (i = 0; i < nthreads; i++) {
    myid[i] = i;
    psum[i*spacing] = 0;
    Pthread_create(&tid[i], NULL, thread_fun, &myid[i]);
}
for (i = 0; i < nthreads; i++)
    Pthread_join(tid[i], NULL);

result = 0;

/* Add up the partial sums computed by each thread */
for (i = 0; i < nthreads; i++)
    result += psum[i*spacing];

/* Add leftover elements */
for (e = nthreads * nelems_per_thread; e < nelems; e++)
    result += e;
```

Thread Function: Memory Accumulation

Where is the mutex?

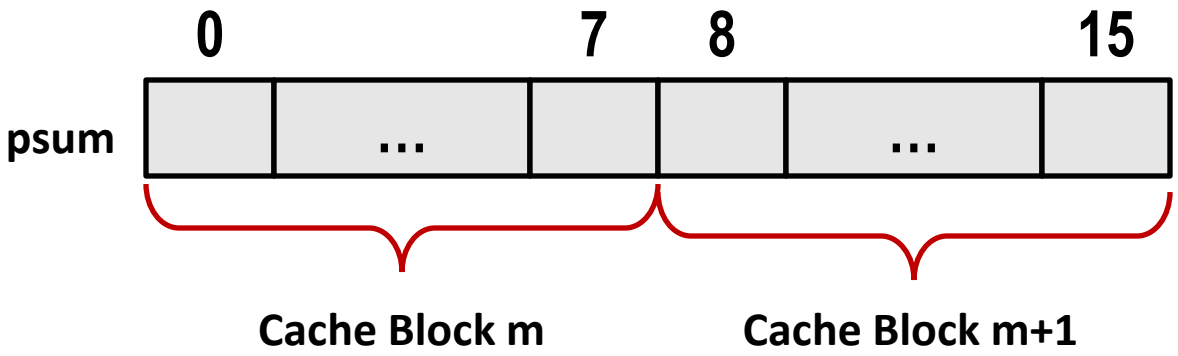
```
void *sum_global(void *vargp)
{
    int myid = *((int *)vargp);
    size_t start = myid * nelems_per_thread;
    size_t end = start + nelems_per_thread;
    size_t i;

    size_t index = myid*spacing;
    psum[index] = 0;
    for (i = start; i < end; i++) {
        psum[index] += i;
    }
    return NULL;
}
```

Memory Accumulation Performance



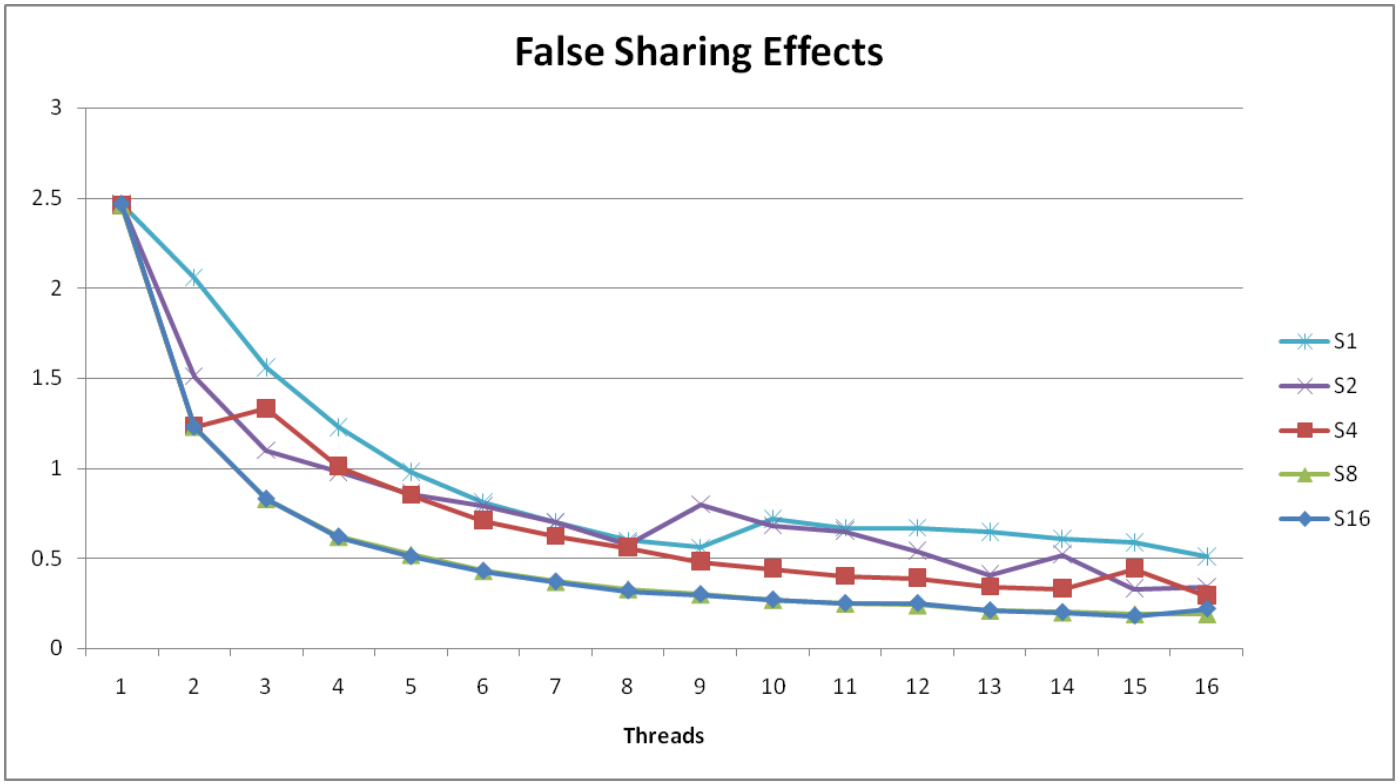
- **Clear threading advantage**
  - Adjacent speedup: 5 X
  - Spaced-apart speedup: 13.3 X (Only observed speedup > 8)
- **Why does spacing the accumulators apart matter?**

False Sharing: Different Bytes but Same Cache Block



- **Coherence maintained on cache blocks**
- **To update psum[i], thread i must have exclusive access**
  - Threads sharing common cache block will keep fighting each other for access to block
- **Contrasts with True Sharing = same bytes being shared**

False Sharing Performance



- Best spaced-apart performance 2.8 X better than best adjacent

- **Demonstrates cache block size = 64**
  - 8-byte values
  - No benefit increasing spacing beyond 8

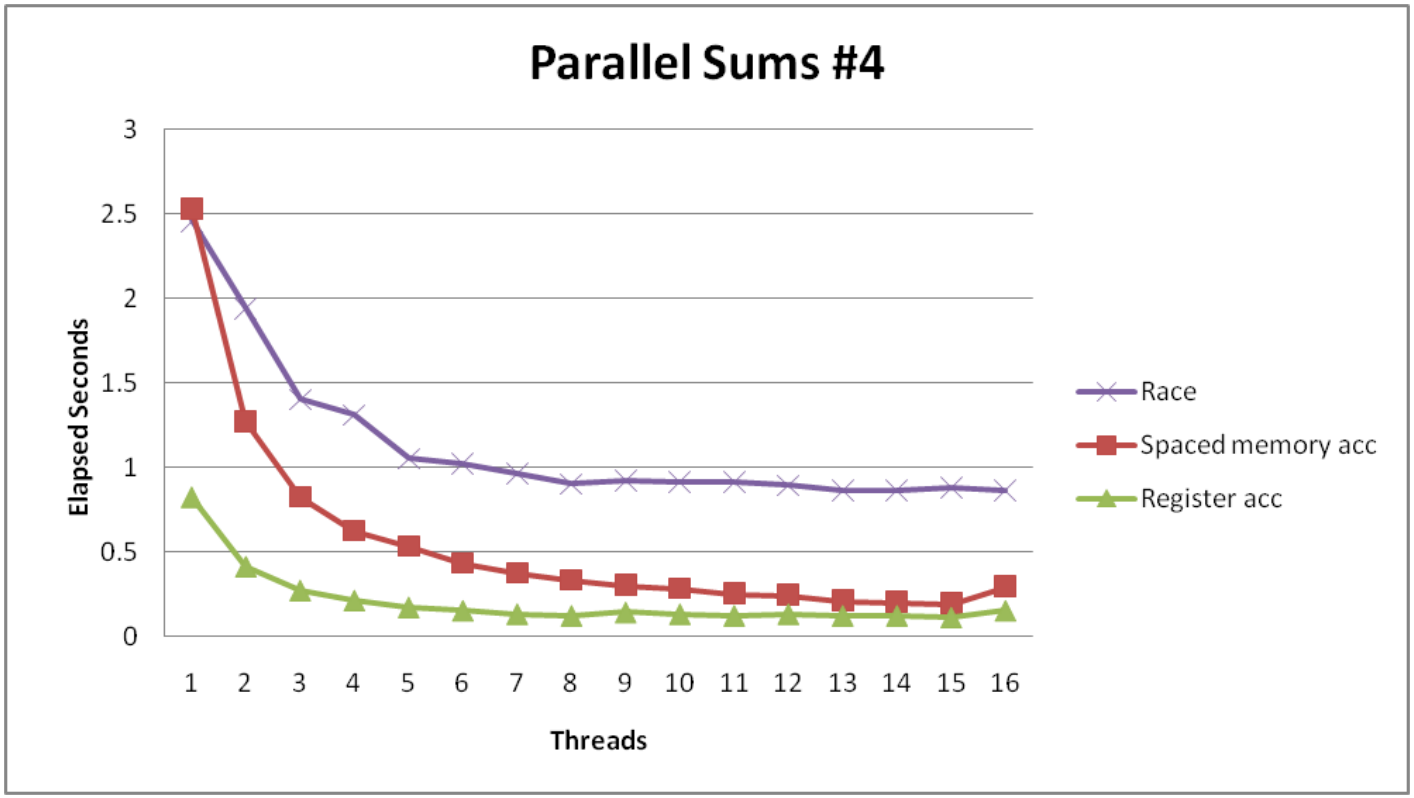
Quiz time!

https://canvas.cmu.edu/courses/49105/quizzes/150035

Thread Function: Register Accumulation

```
void *sum_local(void *vargp)
{
    int myid = *((int *)vargp);
    size_t start = myid * nelems_per_thread;
    size_t end = start + nelems_per_thread;
    size_t i;
    size_t index = myid*spacing;
    data_t sum = 0;
    for (i = start; i < end; i++) {
        sum += i;
    }
    psum[index] = sum;
    return NULL;
}
```

Register Accumulation Performance



- Clear threading advantage
  - Speedup = 7.5 X
- 2X better than fastest memory accumulation

Beware the speedup metric!

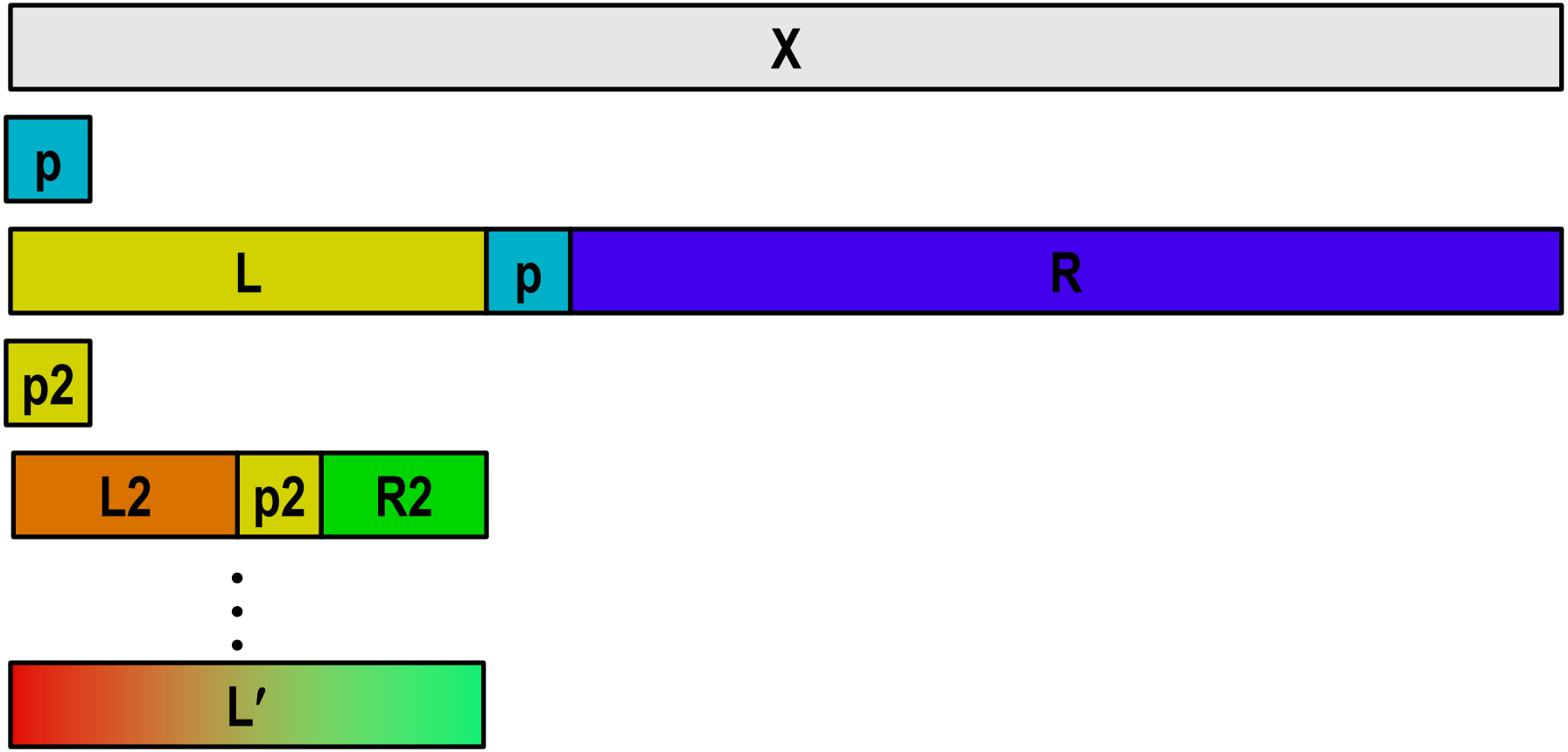
Lessons learned

- **Sharing memory can be expensive**
  - Pay attention to true sharing
  - Pay attention to false sharing
- **Use registers whenever possible**
  - (Remember cachelab)
  - Use local cache whenever possible
- **Deal with leftovers**
- **When examining performance, compare to best possible sequential implementation**

A More Substantial Example: Sort

- **Sort set of N random numbers**
- **Multiple possible algorithms**
  - Use parallel version of quicksort
- **Sequential quicksort of set of values X**
  - Choose “pivot” p from X
  - Rearrange X into
    - L: Values $\leq$ p
    - R: Values $\geq$ p
  - Recursively sort L to get L′
  - Recursively sort R to get R′
  - Return L′ : p : R′

Sequential Quicksort Visualized



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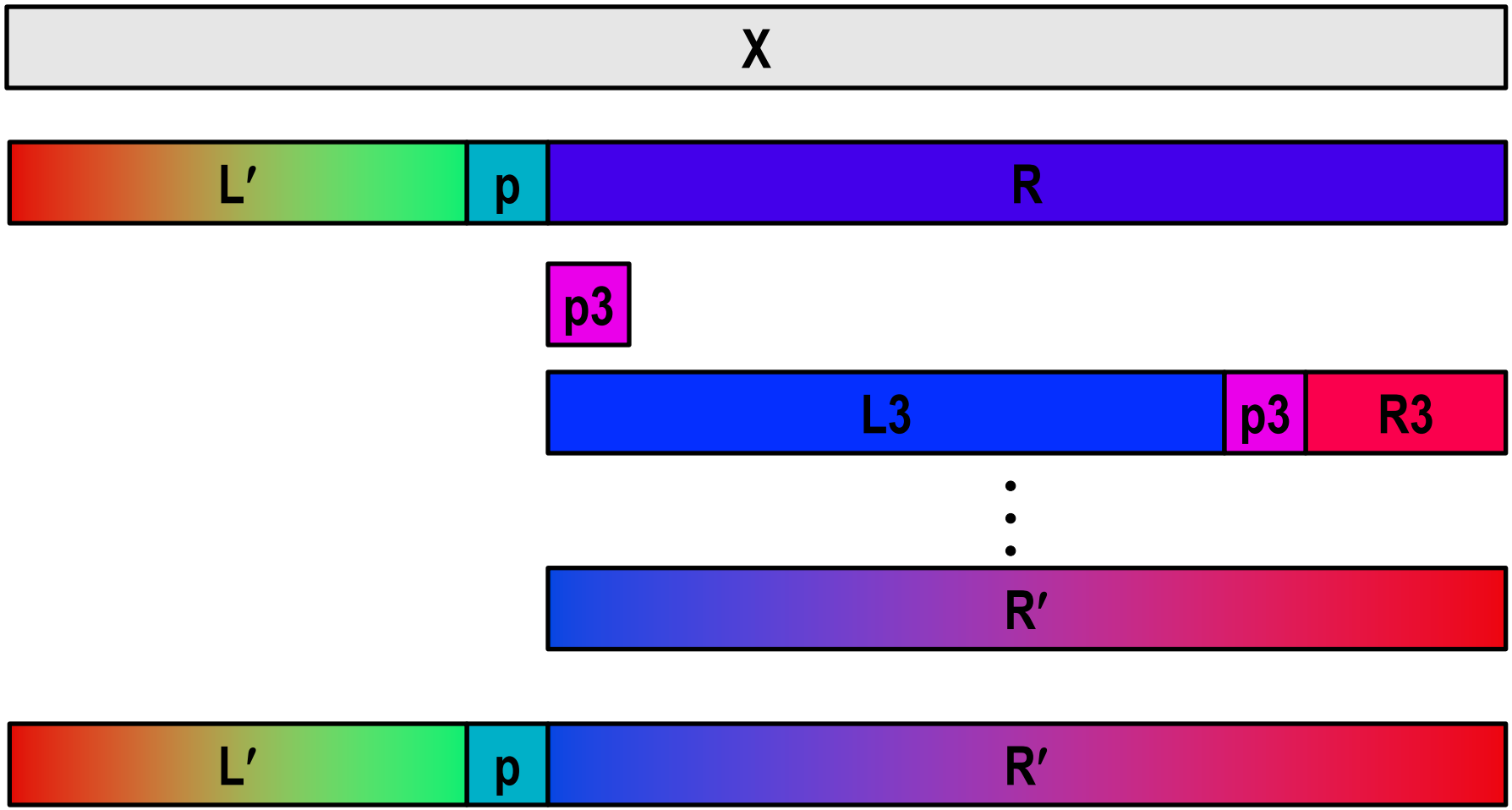


Sequential Quicksort Code

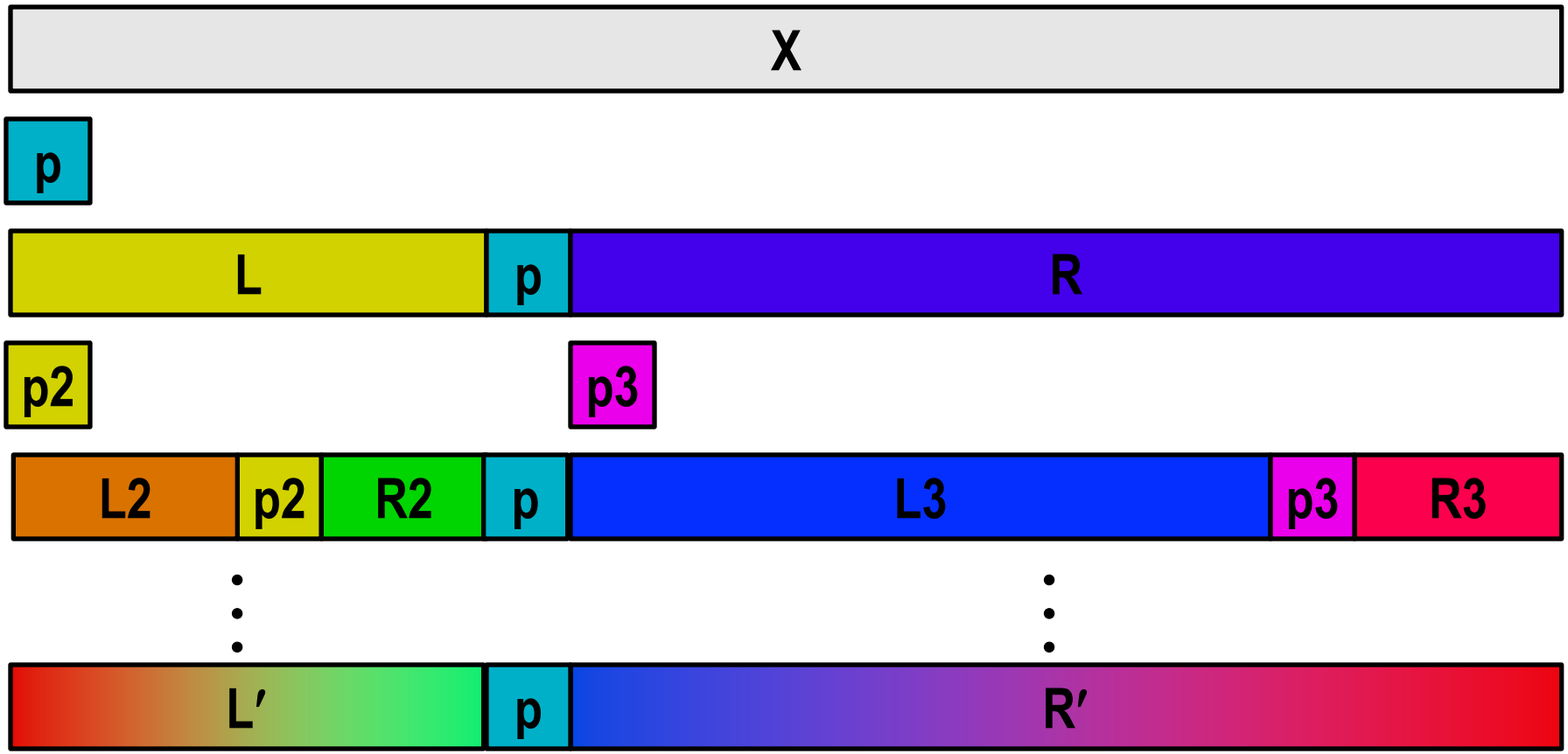
```
void qsort_serial(data_t *base, size_t nele) {
  if (nele <= 1)
    return;
  if (nele == 2) {
    if (base[0] > base[1])
      swap(base, base+1);
    return;
  }

  /* Partition returns index of pivot */
  size_t m = partition(base, nele);
  if (m > 1)
    qsort_serial(base, m);
  if (nele-1 > m+1)
    qsort_serial(base+m+1, nele-m-1);
}
```

- **Sort nele elements starting at base**
  - Recursively sort L or R if has more than one element

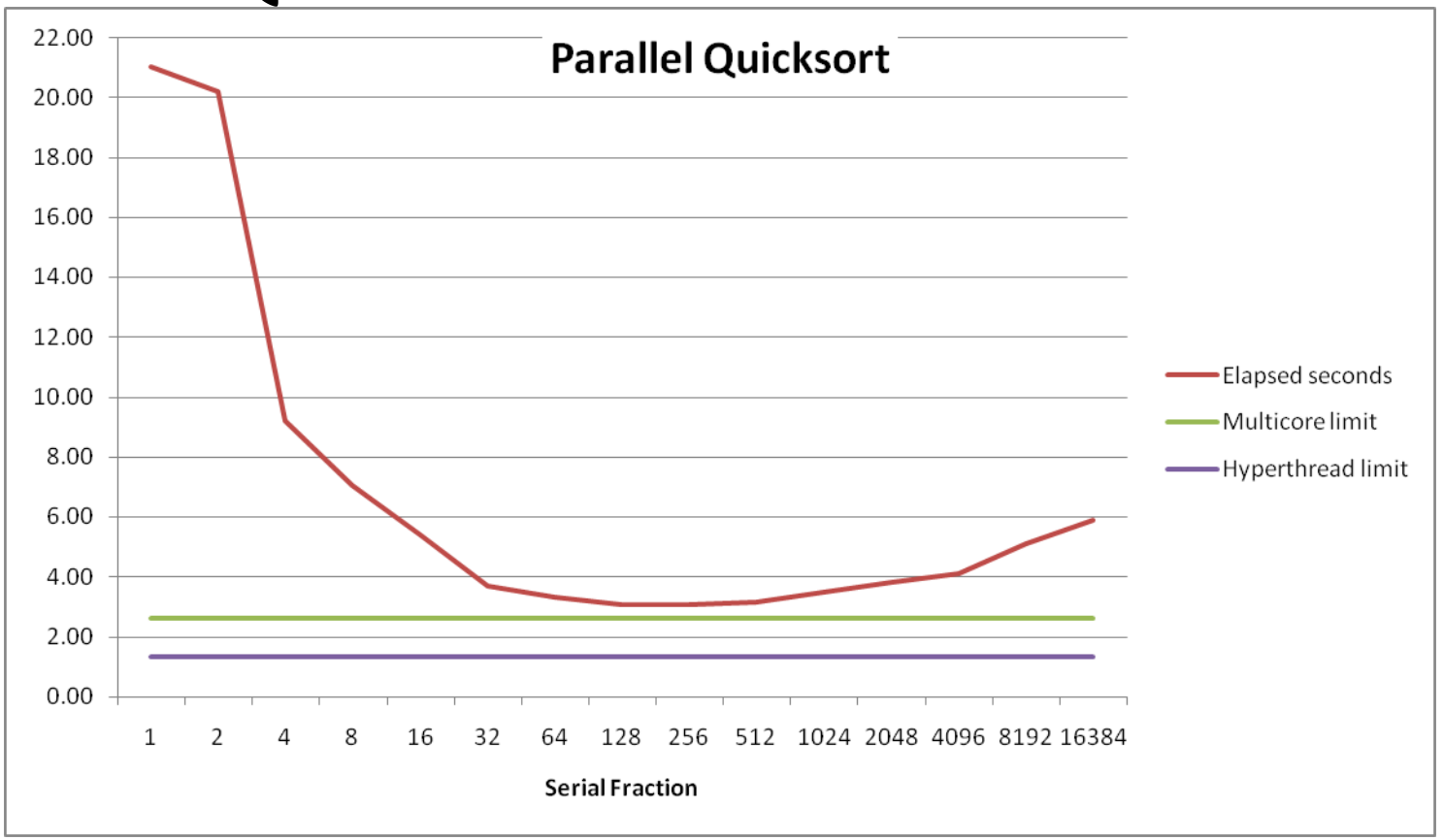
Parallel Quicksort

- **Parallel quicksort of set of values X**
  - If N ≤ Nthresh, do sequential quicksort
  - Else
    - Choose “pivot” p from X
    - Rearrange X into
      - L: Values ≤ p
      - R: Values ≥ p
    - Recursively spawn separate threads
      - Sort L to get L′
      - Sort R to get R′
    - Return L′ : p : R′

Parallel Quicksort Visualized

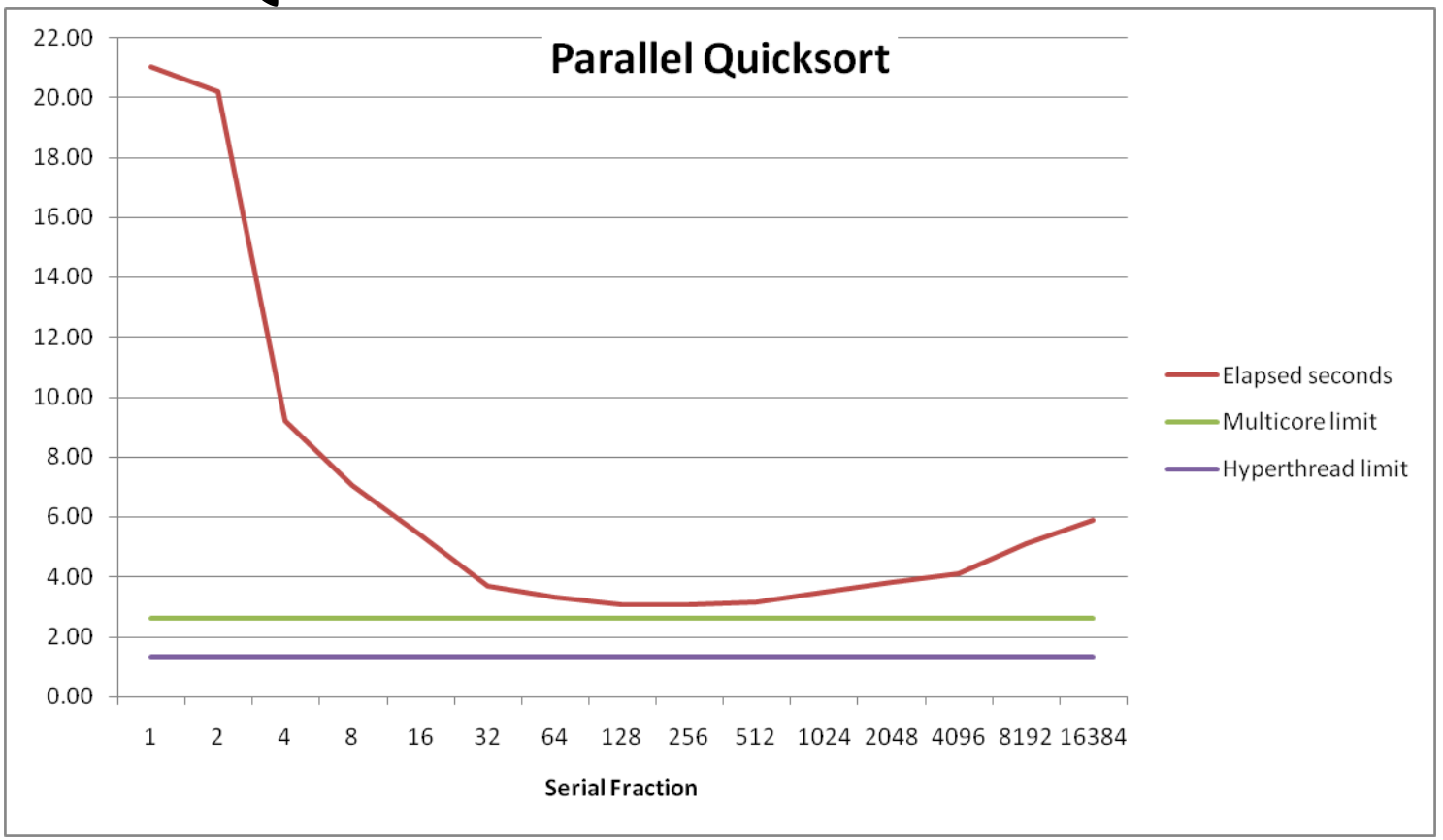


Parallel Quicksort Performance



- Serial fraction: Fraction of input at which do serial sort
- Sort 2^{27} (134,217,728) random values
- Best speedup = 6.84X

Parallel Quicksort Performance



- **Good performance over wide range of fraction values**
  - F too small: Not enough parallelism
  - F too large: Thread overhead too high

Amdahl's Law (Travel Analogy)

		Speed-Up
Flying jet non-stop from PIT -> LHR:	**7.5 Hours**	**1**
Or, old fashioned SST way:		
Fly jet from PIT -> JFK: 1.5 Hours		
Fly SST from JFK -> LHR: 3.5 Hours	**5 Hours**	**1.5x**
Or, Using FTL:		
Fly jet from PIT -> JFK: 1.5 Hours		
Fly FTL from JFK -> LHR: .01 Hours	**1.51 Hours**	**~5x**

- **Best possible speed up is 5X, even with FTL because have to get to New York.**

Amdahl's Law

- **Overall problem**
  - T Total sequential time required
  - p Fraction of total that can be sped up ($0 \leq p \leq 1$)
  - k Speedup factor
- **Resulting Performance**
  - $T_k = pT/k + (1-p)T$
    - Portion which can be sped up runs k times faster
    - Portion which cannot be sped up stays the same
  - Maximum possible speedup
    - $k = \infty$
    - $T_\infty = (1-p)T$

Amdahl's Law (Travel Analogy)

		Speed-Up
Flying jet non-stop from PIT -> LHR:	**7.5 Hours**	**1**
Or, old fashioned SST way:		
Fly jet from PIT -> JFK: 1.5 Hours		
Fly SST from JFK -> LHR: 3.5 Hours	**5 Hours**	**1.5x**
Or, Using FTL:		
Fly jet from PIT -> JFK: 1.5 Hours		
Fly FTL from JFK -> LHR: .01 Hours	**1.51 Hours**	**~5x**

- **Best possible speed up is 5X, even with FTL because have to get to New York.**
  - T=7.5, p=6/7.5=.8, k= $\infty \Rightarrow T_\infty = (1-p)T=1.5$ max speed-up =5x

Amdahl's Law Example

- **Overall problem**
  - T = 10 Total time required
  - p = 0.9 Fraction of total which can be sped up
  - k = 9 Speedup factor
- **Resulting Performance**
  - $T_9 = 0.9 * 10/9 + 0.1 * 10 = 1.0 + 1.0 = 2.0$ (a 5x speedup)
- **Maximum possible speedup**
  - $T_\infty = 0.1 * 10.0 = 1.0$ (a 10x speedup)
    - With **infinite** parallel computing resources!
  - Limit speedup shows **algorithmic** limitation

Amdahl's Law & Parallel Quicksort

- **Sequential bottleneck**
  - Top-level partition: No speedup
  - Second level: $\leq$ 2X speedup
  - k^{th} level: $\leq 2^{k-1}$X speedup
- **Implications**
  - Good performance for small-scale parallelism
  - Would need to parallelize partitioning step to get large-scale parallelism
    - Parallel Sorting by Regular Sampling
      - H. Shi & J. Schaeffer, J. Parallel & Distributed Computing, 1992

Lessons Learned

- **Must have parallelization strategy**
  - Partition into K independent parts
  - Divide-and-conquer
- **Inner loops must be synchronization free**
  - Synchronization operations very expensive
- **Watch out for hardware artifacts**
  - Need to understand processor & memory structure
  - True sharing and false sharing of global data
- **Beware of Amdahl's Law**
  - Serial code can become bottleneck
- **You can do it!**
  - Achieving modest levels of parallelism is not difficult
  - Set up experimental framework and test multiple strategies

Thursday's Lecture: Frontiers of Computing

- **Valerie Choung: Designing malloc to better serve programmers**
- **Kaiyang Zhao: Speeding up virtual memory address translation**
- **Deepanjali Mishra: Sustainability in computer systems**

  - Not recorded, not on the final
  - No separate 14513 lecture
  - 15513 and 14513 students are encouraged to attend, in GHC 4401

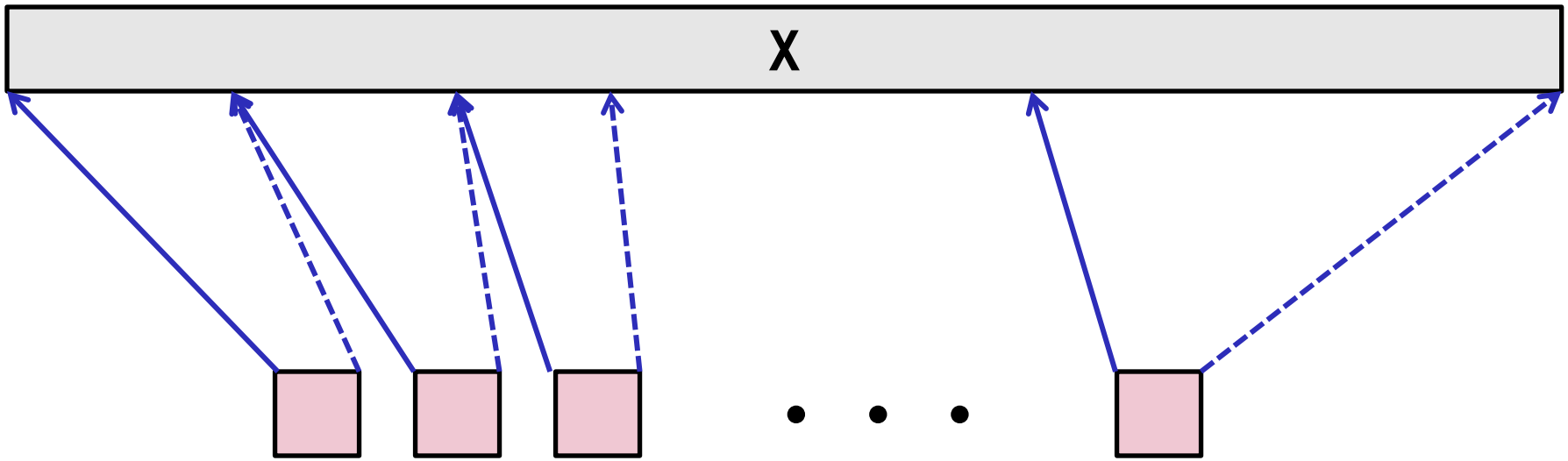
Supplemental slides

Powerful, Parallel Computing Is

- Two threads, with X and Y initialized to 0

```
X = 1                          Y = 1
if (Y == 0) print Hello        if (X == 0) print World
```

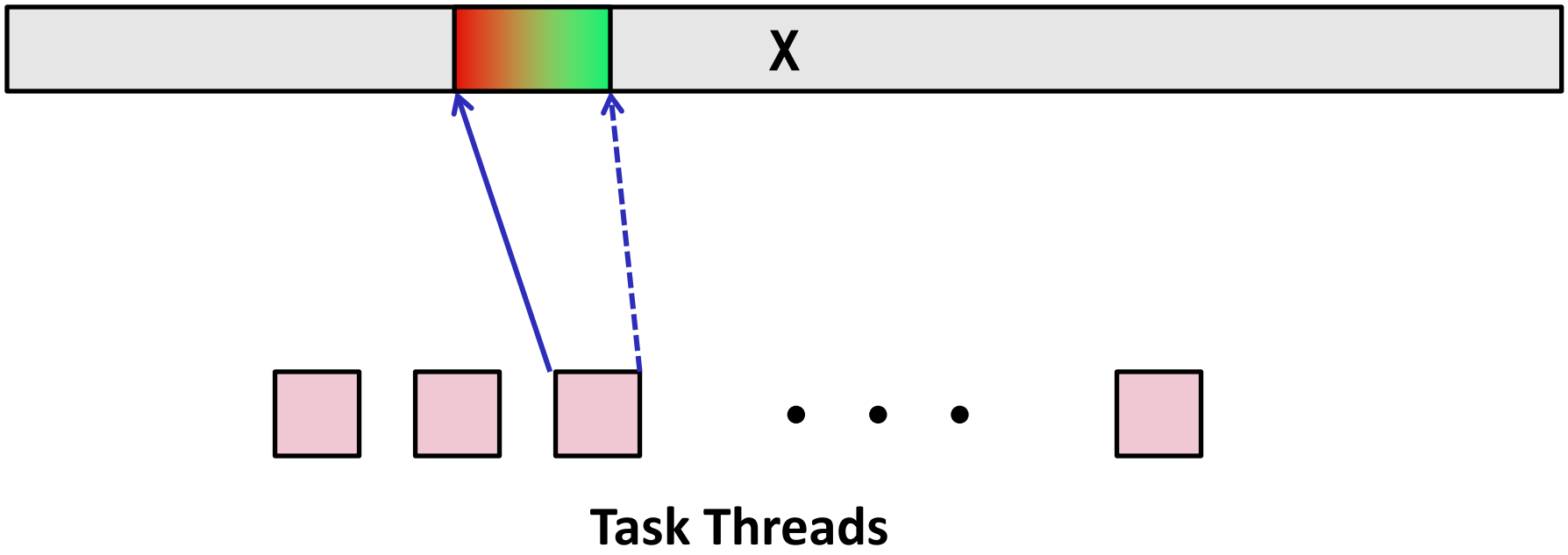
Thread Structure: Sorting Tasks



Task Threads

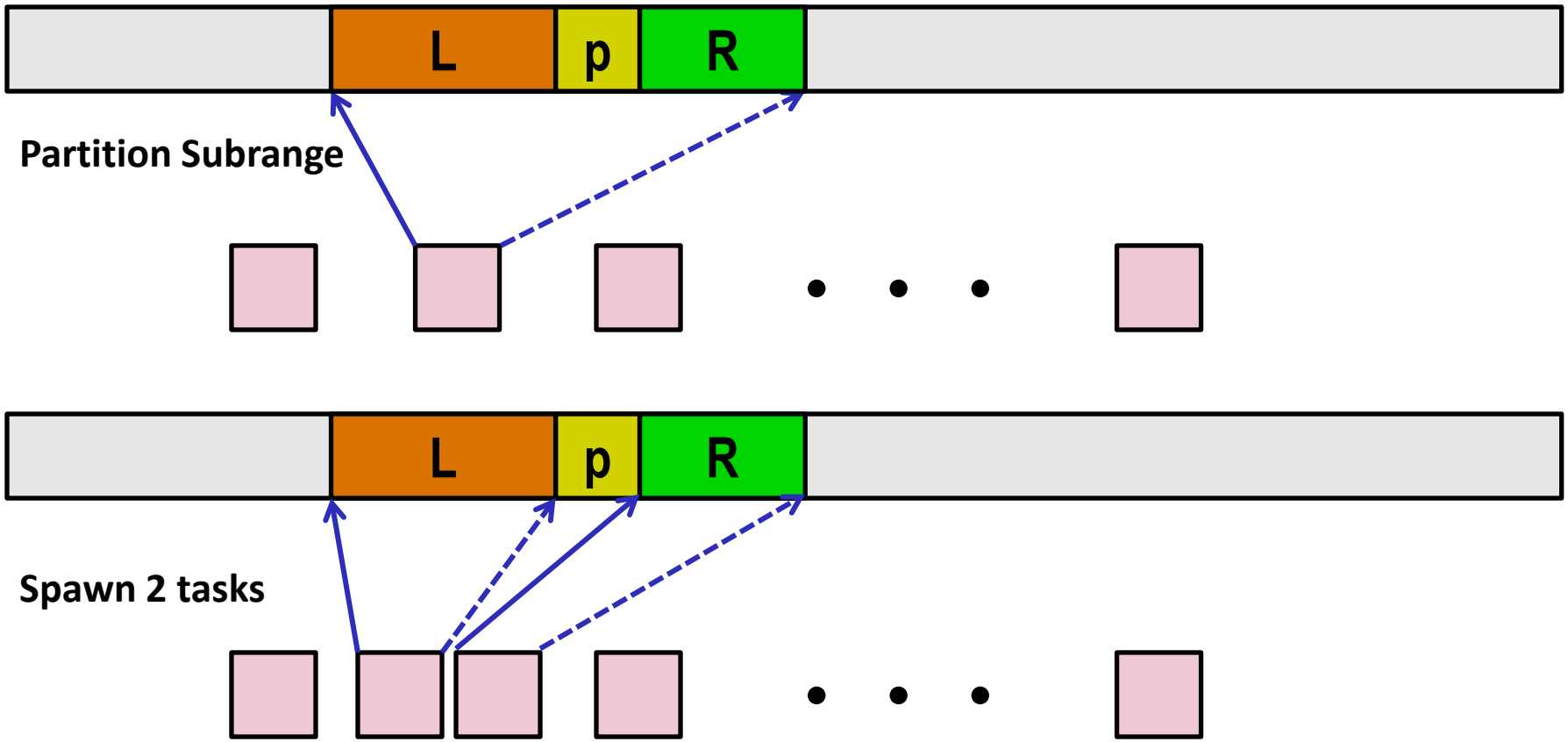
- **Task: Sort subrange of data**
  - Specify as:
    - **`base`**: Starting address
    - **`nele`**: Number of elements in subrange
- **Run as separate thread**

Small Sort Task Operation



- Sort subrange using serial quicksort

Large Sort Task Operation



Top-Level Function (Simplified)

```
void tqsort(data_t *base, size_t nele) {
    init_task(nele);
    global_base = base;
    global_end = global_base + nele - 1;
    task_queue_ptr tq = new_task_queue();
    tqsort_helper(base, nele, tq);
    join_tasks(tq);
    free_task_queue(tq);
}
```

- Sets up data structures
- Calls recursive sort routine
- Keeps joining threads until none left
- Frees data structures

Recursive sort routine (Simplified)

```
/* Multi-threaded quicksort */
static void tqsort_helper(data_t *base, size_t nele,
                          task_queue_ptr tq) {
    if (nele <= nele_max_sort_serial) {
        /* Use sequential sort */
        qsort_serial(base, nele);
        return;
    }
    sort_task_t *t = new_task(base, nele, tq);
    spawn_task(tq, sort_thread, (void *) t);
}
```

- Small partition: Sort serially
- Large partition: Spawn new sort task

Sort task thread (Simplified)

```
/* Thread routine for many-threaded quicksort */
static void *sort_thread(void *vargp) {
    sort_task_t *t = (sort_task_t *) vargp;
    data_t *base = t->base;
    size_t nele = t->nele;
    task_queue_ptr tq = t->tq;
    free(vargp);
    size_t m = partition(base, nele);
    if (m > 1)
        tqsort_helper(base, m, tq);
    if (nele-1 > m+1)
        tqsort_helper(base+m+1, nele-m-1, tq);
    return NULL;
}
```

- Get task parameters
- Perform partitioning step
- Call recursive sort routine on each partition (if size of part > 1)