

15-751*, Fall 2016:

A Theorist's Toolkit

Lecture 1



Venkat Guruswami



Ryan O'Donnell

This will be the only lecture with slides.

Today's Lecture

Part 1: What this course is about.
(Answer: CS theory and math.)

Part 2: How to **present** mathematics.

Part 3: How to **do** mathematics.

Part 1: What this course is about.

What this course is about

A Theorist's Toolkit

What this course is about

A Theorist's Toolkit

The indefinite article.
Moving on.

What this course is about

A Theorist's Toolkit

Theoretical Computer Science (TCS)
= Algorithms and Computational Complexity

What this course is about

A Theorist's Toolkit

A brief introduction to
a number of topics and tools
– especially mathematical ones –
that arise in TCS research.

What this course is about

An approximate syllabus

Asymptotics

Central Limit Theorem

Chernoff Bounds

Computational Models

Spectral Graph Theory

Fields and polynomials

Error-correcting codes

Derandomization

Linear Programming

Semidefinite Programming

Constraint Satisfaction Problems

Communication Complexity

Information Theory

High-dimensional Geometry

Streaming & Property Testing

Analysis of Boolean Functions

Proof Complexity

Quantum Computation

Cryptography

Probabilistically Checkable Proofs

What this course is about

More breadth than depth...

The “background knowledge”
that it helps to know for...

- Reading TCS papers
- Attending TCS talks
- Doing TCS research

Who is this class for?

- Anyone who might be interested in doing TCS research.
- Prime target audience member (?):
First-year graduate student with some interest in CS theory.
- “Mathematical maturity” is a must.

Logistics

<http://www.cs.cmu.edu/~15751>

for links to homework, Piazza (sign up!), schedule

HOMEWORK will be the main form of evaluation.

Approximately 8, typically due/out on Mondays.

LaTeX solutions mandatory.

Homework 1 is released, due Sept. 19!

Grades: 80% homework

15% take-home final

5% participation

Part 2: How to present mathematics.

Writing math: LaTeX

You must become fluent in LaTeX.

If you are very unfamiliar with LaTeX:

- Start with [Overleaf.com](https://overleaf.com) or [ShareLaTeX.com](https://sharelatex.com)
- Use Google & tex.stackexchange.com to find out how to do anything & everything
- Maybe get “Short Math Guide for LaTeX”

LaTeX workflow

Select a good text editor that understands you are writing in LaTeX.

Essential features....

- syntax highlighting
- hotkey to compile/display
- “synchronization”/“roundtripping”
- text/reference autocompletion
- block commenting

Produce **pdf** output (command: PDF_Latex).

Nobody uses .dvi or .ps these days!

Popular LaTeX editors

Browser-based: (good for collaboration... or solo!)

- Overleaf.com, ShareLaTeX.com

Windows: (typically also need MiKTeX)

- WinEdt, TeXnicCenter

Mac:

- Texpad, TeXShop

Linux:

- Kile

All three:

- Texmaker, TeXstudio
- Sublime + LaTeXing (or LaTeXTools) plugin
- Emacs + AUCTeX + RefTeX

LaTeX workflow


**Create a stub .tex file,
a lifetime .sty file,
and a lifetime .bib file.**

LaTeX — stub .tex file example

```
\documentclass[11pt]{article}
```

```
\usepackage{odonnell}
```

refers to odonnell.sty,
my lifetime .sty file



```
\begin{document}
```

```
\title{}
```

```
\author{Ryan O'Donnell\thanks{odonnell@cs.cmu.edu}}
```

```
\date{\today}
```

```
\maketitle
```

```
%\begin{abstract}
```

```
%\end{abstract}
```

refers to odonnell.bib,
my lifetime .bib file



```
%\section{}
```

```
%\bibliographystyle{alpha}
```

```
%\bibliography{odonnell}
```

```
\end{document}
```

LaTeX — lifetime .sty file

```
\usepackage{fixltx2e,amsmath,amssymb,amsthm,amsfonts,bbm,graphicx,fullpage}
\usepackage[colorlinks,citecolor=blue,bookmarks=true]{hyperref}

\theoremstyle{plain}
\newtheorem{theorem}{Theorem}[section]
\newtheorem{lemma}[theorem]{Lemma}
\newtheorem{corollary}[theorem]{Corollary}
\newtheorem{proposition}[theorem]{Proposition}

\theoremstyle{definition}
\newtheorem{definition}[theorem]{Definition}
\newtheorem{remark}[theorem]{Remark}

% for ``commenting out'' chunks of text
\newcommand{\ignore}[1]{}

% for notes on the text
\newcommand{\ryansays}[1]{{\color{red}}{\tiny [Ryan: #1]}}

% macros
\newcommand{\R}{{\mathbbm R}}
\newcommand{\eps}{{\epsilon}}
% ...
```

LaTeX — lifetime .bib file

**Start yours
today!**

**Use .bib file
management
software:**

e.g., JabRef, Zotero,
Mendeley, Papers

```
@STRING{ann = {Annual}}
@STRING{proc = {Proceedings of the}}
@STRING{focs = {IEEE Symposium on Foundations of Computer Science}}
@STRING{focs12 = proc # { 53rd } # ann # { } # focs}
@STRING{focs13 = proc # { 54th } # ann # { } # focs}
@STRING{stoc = {ACM Symposium on Theory of Computing}}
@STRING{stoc12 = proc # { 44th } # ann # { } # stoc}
@STRING{stoc13 = proc # { 45th } # ann # { } # stoc}

@ARTICLE{AGHP92,
  author = {Alon, Noga and Goldreich, Oded and H{\aa}stad, Johan and Peralta, Ren{\`e}},
  title = {Simple constructions of almost {\$k\$}-wise independent random variables},
  journal = {Random Structures \& Algorithms},
  volume = {3},
  year = {1992},
  number = {3},
  pages = {289--304},
  doi = {10.1002/rsa.3240030308},
  url = {http://dx.doi.org/10.1002/rsa.3240030308},
}

@ONLINE{CLRS13,
  author = {Chan, Siu On and Lee, James and Raghavendra, Prasad and Steurer, David},
  title = {Approximate constraint satisfaction requires large {LP} relaxations},
  version = {1},
  date = {2013-09-03},
  note = {arXiv:1309.0563},
  eprinttype = {arXiv},
  eprintclass = {cs.CC},
  eprint = {1309.0563}
}

@INCOLLECTION{MR12,
  author = {Mossel, Elchanan and R{\`a}cz, Mikl{\`o}s},
  title = {A quantitative {G}ibbard--{S}atterthwaite theorem without neutrality},
  booktitle = stoc12,
  pages = {1041--1060},
  publisher = {ACM},
  year = {2012},
  doi = {10.1145/2213977.2214071},
  url = {http://dx.doi.org/10.1145/2213977.2214071},
}
```

LaTeX — lifetime .bib file

Please make your .bib entries high quality!

References

- [AKK⁺08] Sanjeev Arora, Subhash Khot, Alexandra Kolla, David Steurer, Madhur Tulsiani, and Nisheeth K. Vishnoi, *Unique games on expanding constraint graphs are easy: extended abstract*, STOC (Richard E. Ladner and Cynthia Dwork, eds.), ACM, 2008, pp. 21–28.
- [Alo86] Noga Alon, *Eigenvalues and expanders*, Combinatorica **6** (1986), no. 2, 83–96.
- [AM85] N. Alon and V. D. Milman, λ_1 , *isoperimetric inequalities for graphs, and superconcentrators*, Journal of Combinatorial Theory. Series B **38** (1985), 73–88.
- [AR98] Yonatan Aumann and Yuval Rabani, *An $O(\log k)$ approximate min-cut max-flow theorem and approximation algorithm*, SIAM Journal on Computing **27** (1998), no. 1, 291–301.
- [ARV04] Sanjeev Arora, Satish Rao, and Umesh Vazirani, *Expander flows, geometric embeddings and graph partitioning*, Proceedings of the thirty-sixth annual ACM Symposium on Theory of Computing (STOC-04) (New York), ACM Press, June 13–15 2004, pp. 222–231.
- [Che70] J. Cheeger, *A lower bound on smallest eigenvalue of a laplacian*, Problems in Analysis (Papers dedicated to Salomon Bochner) (1970), 195–199.
- [DKSV06] Nikhil R. Devanur, Subhash Khot, Rishi Saket, and Nisheeth K. Vishnoi, *Integrality gaps for sparsest cut and minimum linear arrangement problems*, STOC, 2006, pp. 537–546.
- [FS02] Feige and Schechtman, *On the optimality of the random hyperplane rounding technique for MAX CUT*, RSA: Random Structures Algorithms **20** (2002).
- [GMPT07] Konstantinos Georgiou, Avner Magen, Toniann Pitassi, and Iannis Tzoulas, *Integrality gaps of $2 - o(1)$ for vertex cover SDPs in the lovász-schrijver hierarchy*, FOCS, IEEE Computer Society, 2007, pp. 702–712.
- [Kho02] Subhash Khot, *On the power of unique 2-prover 1-round games*, STOC, 2002, pp. 767–775.

This is gross.
You may as well
← have a spelling
mistake in your
title.

LaTeX — lifetime .bib file

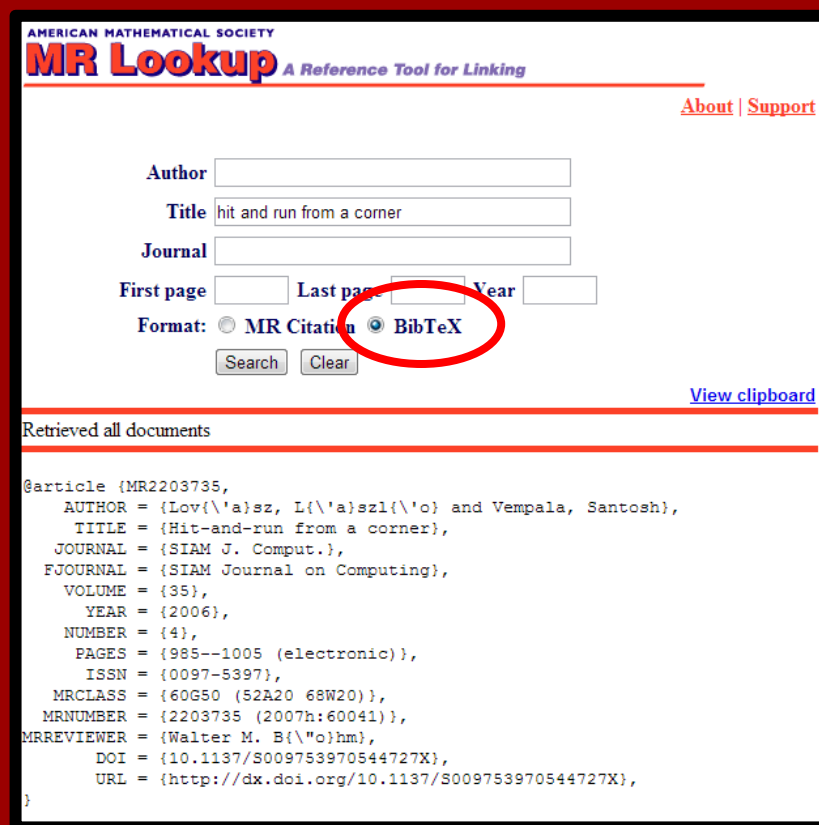
Please make your .bib entries high quality!

- Have a standard for citing proceedings (FOCS, STOC, etc.) and arXiv and ECCC.
- Get capitalization correct: {B}races needed
- Put in people's first and last names—with the diacritics!
- Use math mode for math parts of titles

LaTeX — lifetime .bib file

Where to get .bib entries:

1. Always try **www.ams.org/mrlookup** first.
(The **best** entries, and you **don't** need ams.org access.)



The screenshot shows the MR Lookup interface. At the top, it says "AMERICAN MATHEMATICAL SOCIETY MR Lookup A Reference Tool for Linking". There are links for "About" and "Support". The search form includes fields for "Author", "Title" (filled with "hit and run from a corner"), "Journal", "First page", "Last page", and "Year". The "Format" section has two radio buttons: "MR Citation" and "BibTeX", with "BibTeX" being selected and circled in red. There are "Search" and "Clear" buttons. A "View clipboard" link is also present. Below the search form, it says "Retrieved all documents". The main content area displays a BibTeX entry for a document.

```
@article {MR2203735,  
  AUTHOR = {Lov{\a}sz, L{\a}szl{\o} and Vempala, Santosh},  
  TITLE = {Hit-and-run from a corner},  
  JOURNAL = {SIAM J. Comput.},  
  FJOURNAL = {SIAM Journal on Computing},  
  VOLUME = {35},  
  YEAR = {2006},  
  NUMBER = {4},  
  PAGES = {985--1005 (electronic)},  
  ISSN = {0097-5397},  
  MRCLASS = {60G50 (52A20 68W20)},  
  MRNUMBER = {2203735 (2007h:60041)},  
  MRREVIEWER = {Walter M. B{\o}hm},  
  DOI = {10.1137/S009753970544727X},  
  URL = {http://dx.doi.org/10.1137/S009753970544727X},  
}
```

LaTeX — lifetime .bib file

Where to get .bib entries:

2. Failing that, **scholar.google.com**.

(DBLP is also decent.)

The image shows a screenshot of the Google Scholar interface. On the left, the 'Scholar Settings' panel is open, displaying options for search results, collections, results per page, where results open, and bibliography manager. The 'Bibliography manager' section at the bottom is circled in red, showing two radio buttons: 'Don't show any citation import links.' and 'Show links to import citations into BibTeX', with the latter being selected. The main search results area shows a search for 'wigderson' with about 12,200 results. Two articles are listed: 'How to play any mental game' by O. Goldreich, S. Micali, and A. Wigderson, and 'Completeness theorems for non-cryptographic fault-tolerant distributed computation' by M. Ben-Or, S. Goldwasser, and A. Wigderson. Both articles have a red circle around the 'Import into BibTeX' link. The 'Scholar Settings' panel also has a red circle around the 'Bibliography manager' section.

Scholar Settings

Search results
Languages
Library links

Collections

☒ Search articles (☒ include legal documents)
☐ Search legal documents.

Results per page

10 Google's default

Where results open

☐ Open each selected result in a new window

Bibliography manager

☐ Don't show any citation import links.
☒ Show links to import citations into BibTeX

Google wigderson

Scholar About 12,200 results (0.05 sec)

Articles

Legal documents

Any time

Since 2013
Since 2012
Since 2009
Custom range...

Sort by relevance

How to play any mental game
O Goldreich, S Micali, A Wigderson - Proceedings of the nineteenth ..., 1987 - dl.acm.org
Abstract We present a polynomial-time algorithm that, given as a input the description of a game with incomplete information and any number of players, produces a protocol for playing the game that leaks no partial information, provided the majority of the players is ...
Cited by 2450 Related articles All 10 versions Import into BibTeX More ▾

Completeness theorems for non-cryptographic fault-tolerant distributed computation
M Ben-Or, S Goldwasser, A Wigderson - Proceedings of the twentieth ..., 1988 - dl.acm.org
Abstract Every function of n inputs can be efficiently computed by a complete network of n processors in such a way that: If no faults occur, no set of size $t < n/2$ of players gets any additional information (other than the function value), Even if Byzantine faults are allowed, ...
Cited by 1573 Related articles All 13 versions Import into BibTeX More ▾

LaTeX — version control

Version control software a must: will save you heartache and help you collaborate with others.

Browser editors like Overleaf & ShareLaTeX have some built in.

Else, Dropbox or similar are the bare minimum.

If sophisticated & can convince your coauthors:
Git (bitbucket, SourceTree, gitobox)

LaTeX — my top peeves

DON'T

$\$ < U, V > \$$

"quotes"

$\$ f(x) = x^2 \$$

$\$ \log(1+x) \$$

$\left[\left(\frac{ax+b}{cy} \right)^2 \right]$

DO

$\$ \langle U, V \rangle \$$

``"quotes"'`

$$f(x) = x^2$$

$\$ \backslash \log(1+x) \$$

$\left[\left(\frac{ax+b}{cy} \right)^2 \right]$

LaTeX — my top peeves

DON'T

```
\begin{eqnarray}
y &=& (x+1)^2 \\
&=& x^2+2x+1
\end{eqnarray}
```

If A is a matrix, then

Lemma `\ref{lem:big}` is
due to Blum `\cite{Blu99}`

assuming- as we do - that
the Birch-Swinnerton-Dyer
Conjecture holds

DO

```
\begin{align}
y &= (x+1)^2 \\
&= x^2+2x+1
\end{align}
```

If A is a matrix, then

Lemma~`\ref{lem:big}` is
due to Blum~`\cite{Blu99}`

assuming--as we do---that
the Birch--Swinnerton-Dyer
Conjecture holds

LaTeX — my top peeves

DON'T

one party, e.g. Alice, is

```
\[
  a-b=2 ~~~~~~ a+b=4
\]
```

we execute $\$ALG(x)\$$

```
\begin{proof}
\[
  x=1 \implies x^2=1.
\]
\end{proof}
```

DO

one party, e.g. \ Alice, is

```
\[
  a-b=2 \quad a+b=4
\]
```

we execute $\$\textnormal{ALG}(x)\$$

```
\begin{proof}
\[
  x=1 \implies x^2=1. \quad \text{\texttt{\textbackslash qedhere}}
\]
\end{proof}
```

LaTeX — my top peeves

I could go on. When in doubt, look up the correct thing to do at **tex.stackexchange.com** !

Writing mathematics well

This is a challenging, lifelong skill.

If I had to give two pieces of advice...

1. This is math, so it has to be 100% correct.

that said,

2. Take pity on your poor reader; help them out.

LaTeX — drawing

DON'T BE LAZY: include figures to help the reader.

```
\usepackage{graphicx}  
...  
\includegraphics{mypicture.png}
```

Was that so hard?

Works with .jpg, .png, .pdf, .eps

To draw figs: *Inkscape, TikZ, ...*

but there's a learning curve.

Recommendation: draw figs with your presentation software (PowerPoint, Keynote, ...), since you have to learn it anyway...

Presentation software

If you write a paper, you'll have to make a talk.

To make a talk, you'll need PowerPoint/Keynote/Beamer.

Any of these is fine, but you'll still suffer the
“drawing figures” challenge with Beamer.

It's not “hip”, but **become a hacker** in one of these.

Learn to integrate **beautiful math equations**:

PowerPoint: Built-in Equation Editor is now good!

Alternative: IguanaTex

Keynote: LaTeXiT (I'm told)

Beamer: Automatic

Presenting math well



I like Kayvon Fatahalian's tips:

<http://www.cs.cmu.edu/~kayvonf/misc/cleartalktips.pdf>

Part 3: How to **do** mathematics/TCS.

Finding papers

Use **Google Scholar**.

- Use CMU credentials (VPN) to get journal issues online.
- Alternative: check the author's home page.
- Books/some older journal articles can be found in the actual physical science library, in Wean.
- For books, first use Google Books / Amazon's "Read Inside" feature to try to find what you want.
- There are other methods.
- All else fails: Interlibrary loan is not too slow
(<https://illiad.library.cmu.edu/illiad/illiad.dll>)

Finding papers

If you look at a paper, even briefly:

1. Check its “cited by” link on Google Scholar
- 2. Save a local copy.**

Maintain a giant folder of saved papers!

Even better: Add to Jabref/Mendeley/Zotero...

Use a consistent naming convention.

E.g., `nisan-wigderson-log-rank-conj.pdf`

This will save you 100's of hours, lifetime.

How to find papers to read

Papers citing / cited by the paper you're reading

Proceedings of recent FOCS/STOC/SODA/CCC

Google Scholar Alerts

Recent posts to ECCC (<http://eccc.hpi-web.de/>)
or arXiv (<http://arxiv.org/corr/home>)

<http://scirate.com>

Stay au courant

Read TCS blogs: **<http://feedworld.net/toc>**

Follow your favorite researchers on Twitter

Attend talks at CMU:

Theory Lunch, every Wed. noon, GHC6115

ACO Seminar, every Thu. 3:30, Wean 8220

CMU's theory-announce mailing list:

<https://mailman.srv.cs.cmu.edu/mailman/listinfo/theory-announce>

CMU's TCS YouTube channel:

<https://www.youtube.com/channel/UCWFp4UWNiOv71j0sPbdNiqw>

Stay au courant

Watch videos of talks from elsewhere:

- <https://sites.google.com/site/plustcs/>
- <https://video.ias.edu/csdm>
- <https://simons.berkeley.edu/videos>
- <http://www.birs.ca/videos/2016>
- (e.g., Venkat's talk there from 2 days ago...)
- <https://www.youtube.com/user/MicrosoftResearch>
- <http://theory.cs.princeton.edu/videos.html>
- Many conferences (e.g., some STOC/FOCS)
have videos of the talks available online
- Some researchers' home pages have links to their talks

Streetfighting Mathematics

(title stolen from Sanjoy Mahajan)

Q: What is the next number in the series?

1, 2, 5, 20, 125, 1070, ???

A: Just look it up at oeis.org
(Online Encyclopedia of Integer Sequences)

1, 2, 5, 20, 125, 1070

Search

[Hints](#)

(Greetings from [The On-Line Encyclopedia of Integer Sequences!](#))

Search: **seq:1,2,5,20,125,1070**

Displaying 1-1 of 1 result found.

page 1

Sort: [relevance](#) | [references](#) | [number](#) | [modified](#) | [created](#) Format: [long](#) | [short](#) | [data](#)

[A076795](#) Partial sums of $(2n-1)!!$. +20
3

0, 1, 2, 5, 20, 125, 1070, 11465, 146600, 2173625, 36633050, 691362125, 14440672700, 330674815925, 8236528396550, 221694575073425, 6411977928702800, 198310761891213425, 6530970632654064050 ([list](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

OFFSET 0,3

LINKS Vincenzo Librandi, [Table of n, a\(n\) for n = 0..300](#)

FORMULA E.g.f.: $\exp(x) * \text{Integral}_{t=0, x} \exp(-t)/\sqrt{1-2*t} \, dt$.
 $a(n)=a(n-1)(2n-2)-a(n-2)(2n-3) \dots a(n) \sim 1/\sqrt{2}/n * 2^n * (n/e)^n$.
G.f.: $A(x)=x/(1-x)*(1 + x/(U(0)-x))$, where $U(k) = (2*k+1)*x + 1 - (2*k+3)*x/U(k+1)$; (continued fraction Euler's 1st kind, 1-step) . -
Sergei N. Gladkovskii, Jun 27 2012
G.f.: $x/(1-x)/Q(0)$, where $Q(k) = 1 - x*(k+1)/Q(k+1)$; (continued fraction) . -
[Sergei N. Gladkovskii](#), May 19 2013
G.f.: $G(0)*x/(1-x)$, where $G(k) = 1 - x*(k+1)/(x*(k+1) - 1/G(k+1))$;
(continued fraction) . - [Sergei N. Gladkovskii](#), Aug 04 2013

MATHEMATICA Join[{0}, Accumulate[Table[(2n-1)!!, {n, 0, 20}]]] (* [Harvey P. Dale](#), Jan 27 2013 *)

PROG (PARI) a(n)=if(n<0, 0, sum(k=0, n-1, (2*k)!/k!/2^k))

CROSSREFS Cf. [A001147](#).

KEYWORD nonn

AUTHOR Michael Somos, Nov 16 2002

Q: What are the
Stirling numbers of the second kind ?

What is the explicit formula for them?

A: Look it up on Wikipedia

Stirling numbers of the second kind

From Wikipedia, the free encyclopedia

In [mathematics](#), particularly in [combinatorics](#), a **Stirling number of the second kind** is the number of ways to [partition a set](#) of n objects into k non-empty subsets and is denoted by $S(n, k)$ or $\left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\}$.^[1] Stirling numbers of the second kind occur in the field of [mathematics](#) called [combinatorics](#) and the study of [partitions](#).

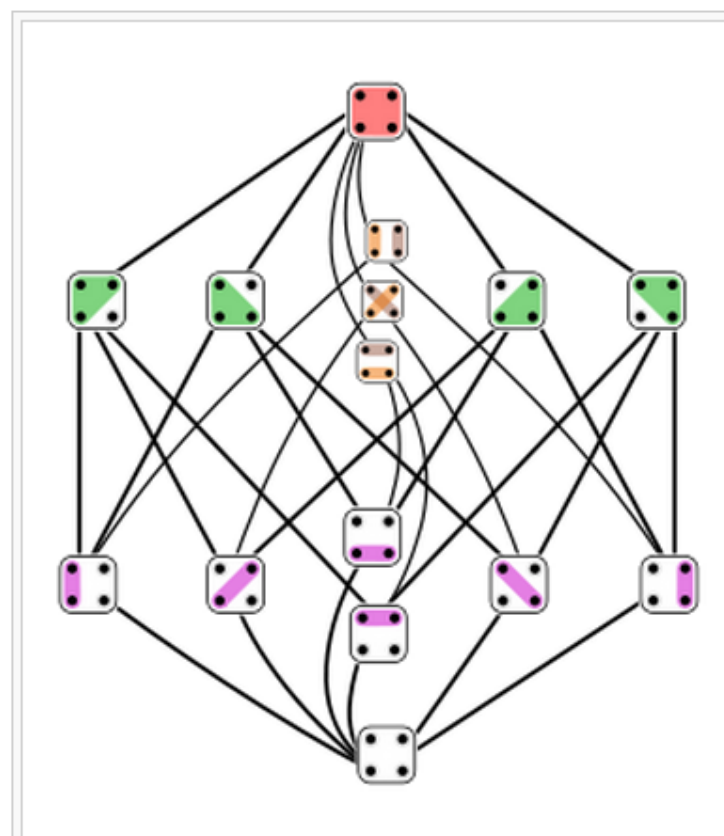
Stirling numbers of the second kind are one of two kinds of [Stirling numbers](#), the other kind being called [Stirling numbers of the first kind](#). Mutually inverse (finite or infinite) [triangular matrices](#) can be formed by arranging the Stirling numbers of the first respectively second kind according to the parameters n, k .

Contents [\[show\]](#)

Definition [\[edit source\]](#) [\[edit beta\]](#)

The Stirling numbers of the second kind $\left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\}$ count the number of ways to partition a set of n labelled objects into k nonempty unlabelled subsets. Equivalently, they count the number of different [equivalence relations](#) with precisely k equivalence classes that can be defined on an n element set. Obviously,

$$\left\{ \begin{smallmatrix} n \\ 1 \end{smallmatrix} \right\} = \left\{ \begin{smallmatrix} n \\ n \end{smallmatrix} \right\} = 1.$$



The 15 partitions of a 4-element set ordered in a [Hasse diagram](#)

Q: What is 0.601907230197?

(This question based on a true story.)



A: Look it up at Inverse Symbolic Calculator

Standard lookup results for 0.601907230197

Best guess: BesK(1,1)

BesK(1,1)-exp(-Pi)^GAM(1/12)	6019072301972343
BesK(1,1)	6019072301972345
BesselK(1,1)	
BesK(1,1)+exp(-Pi)^GAM(1/12)	6019072301972347

Q: What is the Bessel K function?

A: Look it up on Wikipedia

An anecdote



Ryan Williams had an awesome CMU PhD thesis.
I read the first draft. Its #1 theorem was:

Theorem: Any alg. for SAT using $n^{o(1)}$ space requires
time $\tilde{\Omega}(n^c)$, where c is the largest root of
 $c^3 - c^2 - 2c + 1 = 0$; i.e., $c \approx 1.801$.

I had my computer calculate a few more digits:

$$c \approx 1.801937736.$$

Plugged it into Inverse Symbolic Calculator...

Standard lookup results for 1.801937736

Best guess: $1+2*x-x^2-x^3$

$\text{rootof } x^3-x^2-2*x+1;$	1801937735804838
$F(2/7, 5/7; 1/2; 3/4)$	
$-3-3*x+6*x^2-6*x^3+3*x^5$	
$1+2*x-x^2-x^3$	
$\cos(\text{Pi}*1/7)/\cos(\text{Pi}*1/3)$	
$\cos(\text{Pi}*1/7)+\cos(\text{Pi}*1/7)$	

An anecdote

I let him know, and now his famous theorem reads:

Theorem: Any alg. for SAT using $n^{o(1)}$ space requires time $\tilde{\Omega}(n^{2 \cos(\pi/7)})$.

Q: Let $K, L \subseteq \mathbf{R}^n$ be closed, bounded, convex sets with smooth boundary.
Does $K \cup L$ have piecewise-smooth boundary?

A: Well, I didn't know, but it's the kind of question where you just **know** that some expert in analysis knows the answer.

Ask on mathoverflow.net.

If K and L are compact convex sets with smooth boundary, does their union have piecewise-smooth boundary?



Clarification: by "piecewise", I mean a *finite* number of pieces.

2



I'm sure this must be true, but my search for a citation was in vain (although I did learn the new term "polyconvex").



Thanks!

[integral-geometry](#)
[dg.differential-geometry](#)
[analytic-geometry](#)
[convexity](#)
[smoothness](#)
[share](#) [edit](#) [close](#) [delete](#) [flag](#)

asked Nov 10 '10 at 16:57



Ryan O'Donnell

2,269 ● 8 ● 19

[add comment](#)
[start a bounty](#)

2 Answers

[active](#)
[oldest](#)
[votes](#)


9



I don't think this is true. Suppose one of the sets is essentially $\{(x,y) : y \geq x^2\}$ in the plane (cut off in some smooth way at the top, to make it compact). And suppose the other one is the same except that the parabolic lower boundary has been replaced by the graph of something like $y = x^2 + e^{-1/x^2} \sin(1/x)$ In other words add a fierce oscillation but damped so strongly that the region above the curve is still convex (i.e., d^2y/dx^2 remains positive). (I haven't done the arithmetic to make sure my e^{-1/x^2} damping is sufficient; if it isn't, then replace it by a more vigorous damping.) The union of the two convex sets will have infinitely many corners, where $\sin(1/x)$ is 0.

[share](#) [edit](#) [flag](#)

answered Nov 10 '10 at 17:16



Andreas Blass

35.4k ● 3 ● 62 ● 126

Tagged

[dg.differential-geometry](#) × 2347

[convexity](#) × 194

[smoothness](#) × 34

[analytic-geometry](#) × 16

[integral-geometry](#) × 13

Asked 2 Years Ago

Viewed 293 Times

Active 2 Years Ago

StackExchange

We're 5 years old!

Share your story.

Community Bulletin

meta [Who gets the reputation?](#)

meta [Best of MathOverflow](#)

meta [Long-term archiving of MathOverflow](#)

meta [Two tags for partially ordered sets](#)

Love this site?

Stackexchange sites

[Mathoverflow.net](https://mathoverflow.net):

For **research-level** questions about math.

[CSTheory.stackexchange.com](https://cstheory.stackexchange.com),

[CS.stackexchange.com](https://cs.stackexchange.com),

Two places for questions about TCS.

math.stackexchange.com:

For help with math questions at any level.

(Do not post your homework here!!!)

tex.stackexchange.com:

For any questions about LaTeX.

Q: What's the 4th-order Taylor series for $\arcsin(x)$?

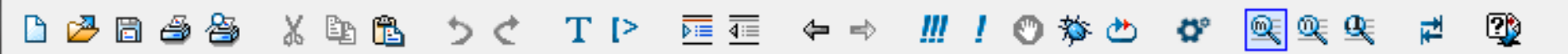
A: Ask Maple/Mathematica/Sage.

(The first two are equally awesome.

Sage is free, and is based around python.

Maple/Mathematica freely accessible at CMU.

For quick things, use wolframalpha.com.)




> series(arcsin(x), x, 4);

$$x + \frac{1}{6}x^3 + O(x^4)$$

(1)

> |

 **WolframAlpha** computational... knowledge engine

Examples Random

Input interpretation:

series	$\sin^{-1}(x)$	point	$x = 0$
--------	----------------	-------	---------

$\sin^{-1}(x)$ is the inverse sine function »

Series expansion at x=0:

$$x + \frac{x^3}{6} + \frac{3x^5}{40} + O(x^7)$$

[More terms](#)

What else are Maple/Mathematica good for?

A: Everything. Use liberally.

Plotting functions	Finding eigenvalues, SVDs
Testing numerical conjectures	Writing code
Solving linear progs (symbolically, too)	Gröbner bases
Simplifying complicated expressions	Finite field arithmetic
Generating random numbers	Solving differential equations
Integrating (symbolically/numerically)	Explicit computations
Finding roots of equations	Visualizing graphs
Empirically checking inequalities	Solving systems of equations
Maximizing/minimizing expressions	Quadratic programming
Outputting LaTeX of expressions	Curve fitting
Inverting matrices (symbolically, too)	Asymptotics and Taylor series
Testing primality/irreducibility	Differentiating

What else are Maple/Mathematica good for?

A: Everything. Use liberally.

Basically, if it's a math problem, and you think someone in history ever thought of using a computer to do it, then Maple/Mathematica can do it.

PS: You should also learn Matlab.
Often better for numerical things.

Streetfighting Mathematics

an example

Q: Suppose $p(x)$ is a polynomial of degree $\leq k$ which is bounded in $[-1, +1]$ for $x \in [-1, +1]$.
What is the largest $p'(0)$ can be?

Remark: This question actually comes up from time to time in analysis of boolean functions.

You can *probably* solve it with judicious Googling.

Also appropriate for math.stackexchange.com,
if you put in a reasonable effort first.

Let's solve it using streetfighting mathematics.

Q: Suppose $p(x)$ is a polynomial of degree $\leq k$ which is bounded in $[-1, +1]$ for $x \in [-1, +1]$.
What is the largest $p'(0)$ can be?

Let's think about $k = 3$, say, so

$$p(x) = a + bx + cx^2 + dx^3$$

For each value of x , e.g. $x = .2$,
we have a **constraint**:

$$-1 \leq a + .2b + .04c + .008d \leq +1$$

We want to maximize **b**

Q: Suppose $p(x)$ is bounded
which is bounded
What is the

We have infinitely many constraints, but probably not much changes if we just take some random 5000 of them.

Let's think about $k = 3$, say, so

$$p(x) = a + bx + cx^2 + dx^3$$

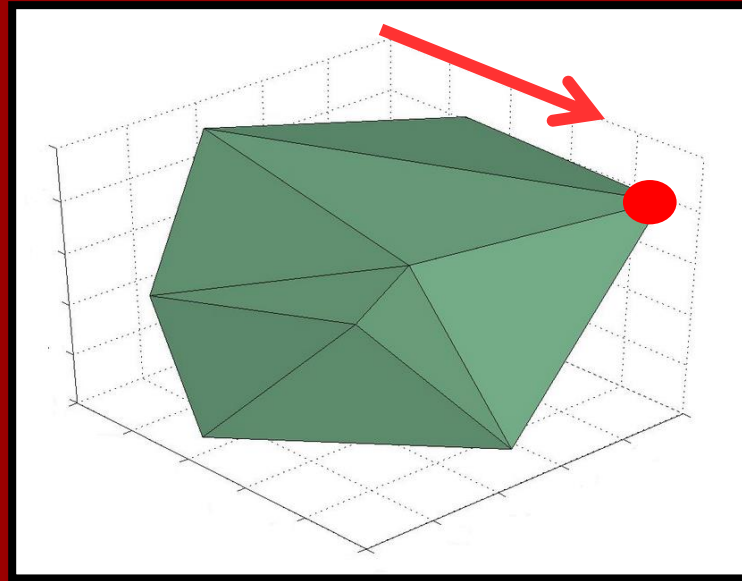
For each value of x , e.g. $x = .2$,
we have a **constraint**:

$$-1 \leq a + .2b + .04c + .008d \leq +1$$

We want to maximize **b**

So say we have 10,000 linear inequalities
over the variables a, b, c, d ;
they form some polytope in \mathbf{R}^4 .

We want to maximize b .



This is a “**Linear Program**” (a topic we’ll study).
Maple/Mathematica/Matlab can solve it.

```
> with(Optimization): with(RandomTools):  
> deg := 3; numPts := 5000:  
> P := x -> add(c[i]*x^i, i = 0..deg):  
> ConstrPts := [seq(2*Generate(float(range = 0..1, method = uniform))-1, i=1..numPts)]:  
> Constrs := map(y -> (P(y) >= -1, P(y) <= 1), ConstrPts):  
> Soln := LPSolve(c[1], Constrs, maximize):  
> BestDeriv := Soln[1];  
> BestPoly := subs(op(Soln[2]), P(x));  
> plot(BestPoly, x = -1..1);
```

deg := 3

(1)

```
> P := x -> add(c[i]*x^i, i = 0..deg):  
> ConstrPts := [seq(2*Generate(float(range = 0..1, method = uniform))-1, i=1..numPts)]:  
> Constrs := map(y -> (P(y) >= -1, P(y) <= 1), ConstrPts):  
> Soln := LPSolve(c[1], Constrs, maximize):  
> BestDeriv := Soln[1];
```

BestDeriv := 3.00056057358036

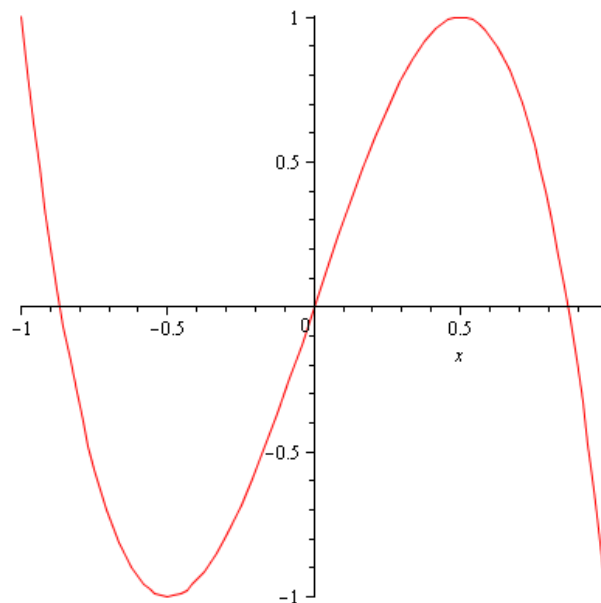
(2)

```
> BestPoly := subs(op(Soln[2]), P(x));
```

BestPoly := -0.000426544117962384708 + 3.00056057358035799 x + 0.00170592447557217143 x² - 4.00223867084451790 x³

(3)

```
> plot(BestPoly, x = -1..1);
```



deg = 3: looks like maximizer is $p(x) = \mathbf{3}x - 4x^3$

$deg := 1$

(1)

```
> P := x -> add(c[i]*x^i, i = 0..deg):  
> ConstrPts := [seq(2*Generate(float(range = 0..1, method = uniform))-1, i=1..numPts)]:  
> Constrs := map(y -> (P(y) >= -1, P(y) <= 1), ConstrPts):  
> Soln := LPSolve(c[1], Constrs, maximize):  
> BestDeriv := Soln[1];
```

$BestDeriv := 1.00041785758203$

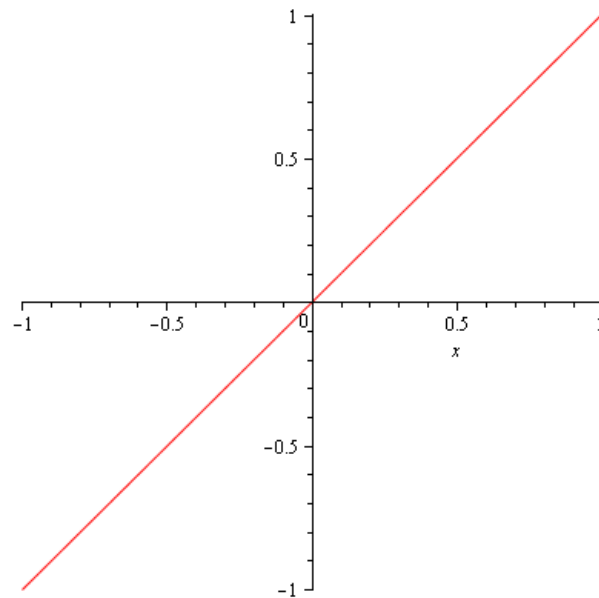
(2)

```
> BestPoly := subs(op(Soln[2]), P(x));
```

$BestPoly := 0.000239437959177035980 + 1.00041785758202950 x$

(3)

```
> plot(BestPoly, x = -1..1);
```



$deg = 1: p(x) = \mathbf{1}x$

deg := 2

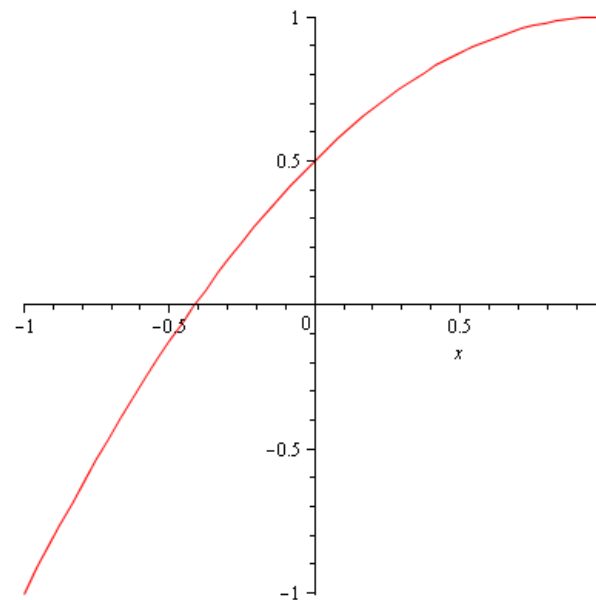
```
> P := x -> add(c[i]*x^i, i = 0..deg):  
> ConstrPts := [seq(2*Generate(float(range = 0..1, method = uniform))-1, i=1..numPts)]:  
> Constrs := map(y -> (P(y) >= -1, P(y) <= 1), ConstrPts):  
> Soln := LPSolve(c[1], Constrs, maximize):  
> BestDeriv := Soln[1];
```

BestDeriv := 1.00154659899863

```
> BestPoly := subs(op(Soln[2]), P(x));
```

BestPoly := 0.500012915154317117 + 1.00154659899863407 x - 0.501560741214689410 x²

```
> plot(BestPoly, x = -1..1);
```



deg = 2: $p(x) = .5 + \mathbf{1}x - .5x^2$

deg := 3

(1)

```
> P := x -> add(c[i]*x^i, i = 0..deg):  
> ConstrPts := [seq(2*Generate(float(range = 0..1, method = uniform))-1, i=1..numPts)]:  
> Constrs := map(y -> (P(y) >= -1, P(y) <= 1), ConstrPts):  
> Soln := LPSolve(c[1], Constrs, maximize):  
> BestDeriv := Soln[1];
```

BestDeriv := 3.00056057358036

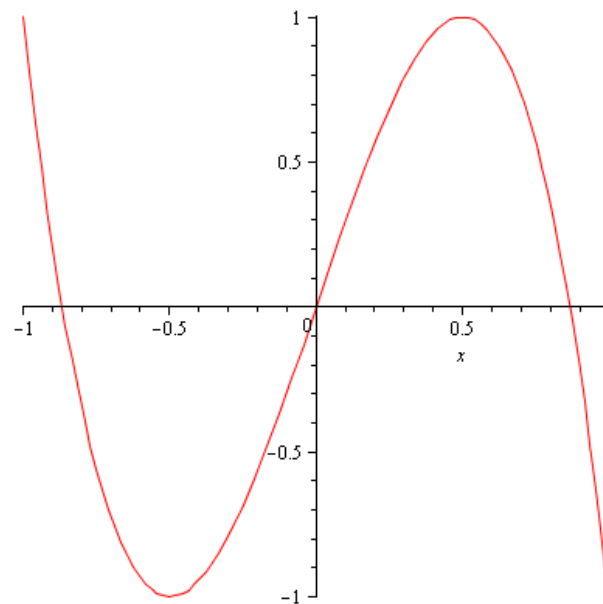
(2)

```
> BestPoly := subs(op(Soln[2]), P(x));
```

BestPoly := -0.000426544117962384708 + 3.00056057358035799 x + 0.00170592447557217143 x² - 4.00223867084451790 x³

(3)

```
> plot(BestPoly, x = -1..1);
```



deg = 3: $p(x) = 3x - 4x^3$

deg := 4

(1)

```
> P := x -> add(c[i]*x^i, i = 0..deg):  
> ConstrPts := [seq(2*Generate(float(range = 0..1, method = uniform))-1, i=1..numPts)]:  
> Constrs := map(y -> (P(y) >= -1, P(y) <= 1), ConstrPts):  
> Soln := LPSolve(c[1], Constrs, maximize):  
> BestDeriv := Soln[1];
```

BestDeriv := 3.00028125652306

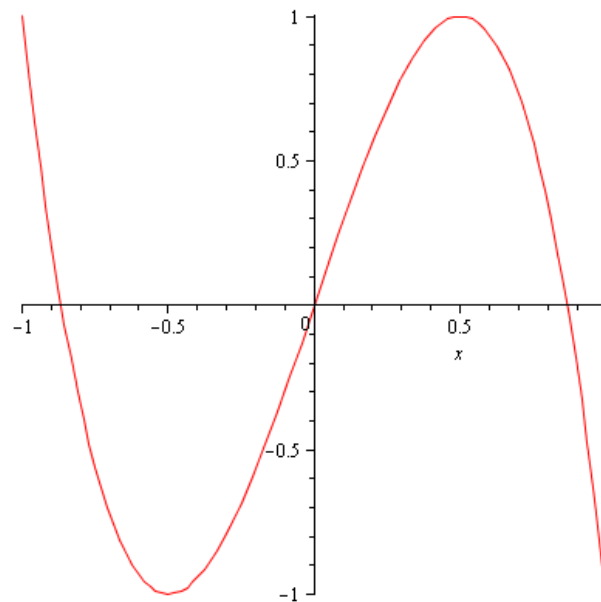
(2)

```
> BestPoly := subs(op(Soln[2]), P(x));
```

BestPoly := 0.0000121482101641112857 + 3.00028125652306343 x - 0.000326369013471309970 x² - 4.00112504781797629 x³ + 0.00111128812486230256 x⁴

(3)

```
> plot(BestPoly, x = -1..1);
```



deg = 4: $p(x) = \mathbf{3x - 4x^3}$ again (!)

deg := 5

(1)

```
> P := x -> add(c[i]*x^i, i = 0..deg):  
> ConstrPts := [seq(2*Generate(float(range = 0..1, method = uniform))-1, i=1..numPts)]:  
> Constrs := map(y -> (P(y) >= -1, P(y) <= 1), ConstrPts):  
> Soln := LPSolve(c[1], Constrs, maximize):  
> BestDeriv := Soln[1];
```

BestDeriv := 5.00120532339474

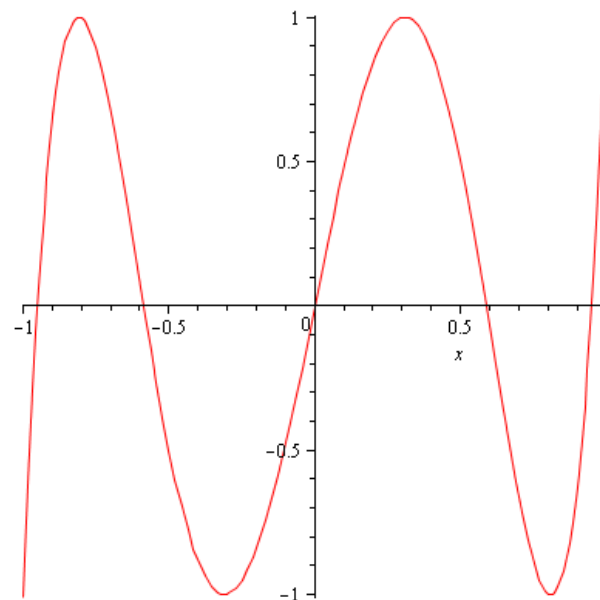
(2)

```
> BestPoly := subs(op(Soln[2]), P(x));
```

BestPoly := -0.00106623559767311328 + 5.00120532339473822 x + 0.0127965087812789788 x² - 20.0144708379618130 x³ - 0.0170841289591253838 x⁴ + 16.0192856795011309 x⁵

(3)

```
> plot(BestPoly, x = -1..1);
```



deg = 5: $p(x) = 5x - 20x^3 + 16x^5$

deg := 6

(1)

```
> P := x -> add(c[i]*x^i, i = 0..deg):  
> ConstrPts := [seq(2*Generate(float(range = 0..1, method = uniform))-1, i=1..numPts)]:  
> Constrs := map(y -> (P(y) >= -1, P(y) <= 1), ConstrPts):  
> Soln := LPSolve(c[1], Constrs, maximize):  
> BestDeriv := Soln[1];
```

BestDeriv = 5.00176146095726

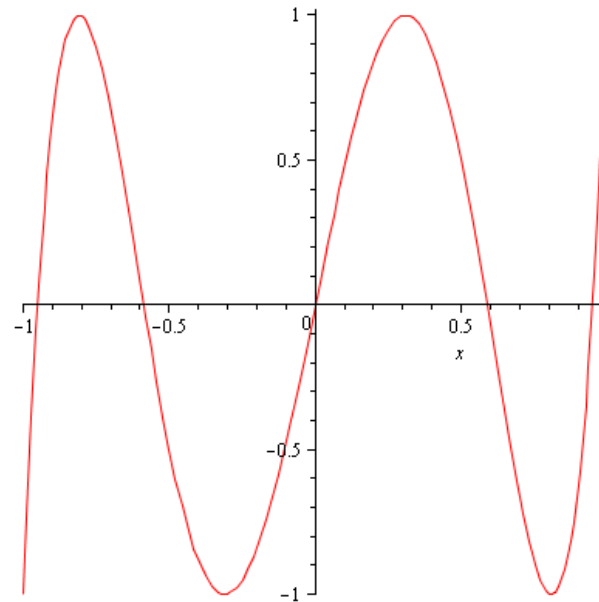
(2)

```
> BestPoly := subs(op(Soln[2]), P(x));
```

BestPoly := 0.000368034916638261978 + 5.00176146095725738 x - 0.00321856542933066266 x² - 20.0210030986764060 x³ - 0.00827062048655928528 x⁴ + 16.0279861868147115 x⁵
+ 0.0188502914009799974 x⁶

(3)

```
> plot(BestPoly, x = -1..1);
```



deg = 6: $p(x) = 5x - 20x^3 + 16x^5$ again

Summary:

Except for weird anomaly at degree 2,
looks like degree $2k$ optimizer is the same
as the degree $2k-1$ optimizer.

(In fact, that's true; can you see why?)

So let's focus on odd degree.

$$p_1(x) = \mathbf{1}x$$

$$p_3(x) = \mathbf{3}x - 4x^3$$

$$p_5(x) = \mathbf{5}x - 20x^3 + 16x^5$$

Largest $p'(0)$ seems to equal degree,
but now what?

Summary:

Except for weird anomalies, p_k looks like degree $2k$ optimal as the degree $2k-1$ optimal.

(In fact

Try typing these coefficients into oeis.org

So let's focus on odd degree.

$$p_1(x) = \mathbf{1}x$$

$$p_3(x) = \mathbf{3}x - 4x^3$$

$$p_5(x) = \mathbf{5}x - 20x^3 + 16x^5$$

Largest $p'(0)$ seems to equal degree, but now what?

Search: seq:1,3,-4,5,-20,16

Displaying 1-1 of 1 result found.

page 1

Sort: relevance | [references](#) | [number](#) | [modified](#) | [created](#) | [Format: long](#) | [short](#) | [data](#)[A084930](#) Triangle of coefficients of Chebyshev polynomials $T_{2n+1}(x)$.+10
3

1, -3, 4, 5, -20, 16, -7, 56, -112, 64, 0, -120, 192, -576, 256, -11, 220, -1232, 2816, -2816, 1024, 13, -364, 2912, -9984, 16640, -13312, 4096, -15, 560, -6048, 28800, -70400, 92160, -61440, 16384, 17, -816, 11424, -71808, 239360, -452608, 487424, -278528, 65536, -19, 1140, -20064, 160512, -695552 ([list](#); [table](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

OFFSET 0,2

COMMENTS Comment from Herb Conn, HCR 83, Box 93, Custer, SD 57730, Jan 28 2005:
 "Letting $x = 2 \cos 2A$, we have the following trigonometric identities:
 $\sin 3A = 3\sin A - 4\sin^3 A$
 $\sin 5A = 5\sin A - 20\sin^3 A + 16\sin^5 A$
 $\sin 7A = 7\sin A - 56\sin^3 A + 112\sin^5 A - 64\sin^7 A$
 $\sin 9A = 9\sin A - 120\sin^3 A + 432\sin^5 A - 576\sin^7 A + 256\sin^9 A$,
 etc."
 Cayley (1876) states "Write $\sin u = x$, then we have $\sin u = x$, [...] $\sin 3u = 3x - 4x^3$, [...] $\sin 5u = 5x - 20x^3 + 16x^5$, [...]". Since $T_n(\cos(u)) = \cos(nu)$ for all integer n , $\sin(u) = \cos(u - \pi/2)$, and $\sin(u + k\pi) = (-1)^k \sin(u)$ it follows that $T_n(\sin(u)) = (-1)^{(n-1)/2} \sin(nu)$ for all odd integer n . - Michael Somos, Jun 19 2012

REFERENCES A. Cayley, On an Expression for $1 \pm \sin(2p+1)u$ in Terms of $\sin u$, Messenger of Mathematics, 5 (1876), pp. 7-8 = Mathematical Papers Vol. 10, n. 630, pp. 1-2.
 Theodore J. Rivlin, Chebyshev polynomials: from approximation theory to algebra and number theory, 2. ed., Wiley, New York, 1990. p. 37, eq. (1.96) and p. 4, eq. (1.10).

LINKS [Table of \$n\$, \$a\(n\)\$ for \$n=0..49\$.](#)

M. Abramowitz and I. A. Stegun, eds., [Handbook of Mathematical Functions](#), National Bureau of Standards, Applied Math. Series 55, Tenth Printing, 1972 [alternative scanned copy].

M. Abramowitz and I. A. Stegun, eds., [Handbook of Mathematical Functions](#), National Bureau of Standards Applied Math. Series 55, Tenth Printing, 1972, p. 795.

[Index entries for sequences related to Chebyshev polynomials.](#)

FORMULA

Alternate rows of [A008310](#).



WIKIPEDIA
The Free Encyclopedia

Main page
Contents
Featured content
Current events
Random article
Donate to Wikipedia
Wikimedia Shop

Interaction
Help
About Wikipedia
Community portal
Recent changes
Contact page

Toolbox
Print/export

Languages
العربية
Български

Create account Log in

Article Talk

Read

Edit source

Edit ^{beta}

View history

Search



Chebyshev polynomials

From Wikipedia, the free encyclopedia

Not to be confused with [discrete Chebyshev polynomials](#).



It has been suggested that *[Dickson polynomial](#)* be merged into this article. ([Discuss](#)) *Proposed since September 2011.*

In mathematics the **Chebyshev polynomials**, named after [Pafnuty Chebyshev](#),^[1] are a [sequence](#) of [orthogonal polynomials](#) which are related to [de Moivre's formula](#) and which can be defined [recursively](#). One usually distinguishes between **Chebyshev polynomials of the first kind** which are denoted T_n and **Chebyshev polynomials of the second kind** which are denoted U_n . The letter T is used because of the alternative [transliterations](#) of the name *Chebyshev* as *Tchebycheff*, *Tchebyshev* (French) or *Tschebyschow* (German).

The Chebyshev polynomials T_n or U_n are polynomials of degree n and the [sequence](#) of Chebyshev polynomials of either kind composes a [polynomial sequence](#).




Chebyshev polynomials are polynomials with the largest possible leading coefficient, but subject to the condition that their absolute value is bounded on the interval $[-1, 1]$. They are also the extremal polynomials for many other properties.^[2]

Chebyshev polynomials are important in [approximation theory](#) because the roots of the Chebyshev polynomials of the first kind, which are also called [Chebyshev nodes](#), are used as nodes in [polynomial interpolation](#). The resulting interpolation polynomial minimizes the problem of [Runge's phenomenon](#) and provides an approximation that is close to the polynomial of best approximation to a [continuous function](#) under the [maximum norm](#). This approximation leads directly to the method of [Clenshaw–Curtis quadrature](#).









In the study of [differential equations](#) they arise as the solution to the [Chebyshev differential equations](#)

$$(1 - x^2) y'' - x y' + n^2 y = 0$$





Notes [\[edit source \]](#) [\[edit beta \]](#)

1. [^] Chebyshev polynomials were first presented in: P. L. Chebyshev (1854) "Théorie des mécanismes connus sous le nom de parallélogrammes," *Mémoires des Savants étrangers présentés à l'Académie de Saint-Petersbourg*, vol. 7, pages 539–586.
2. [^] Rivlin, Theodore J. **The Chebyshev polynomials**. Pure and Applied Mathematics. Wiley-Interscience [John Wiley & Sons], New York-London-Sydney, 1974. Chapter 2, "Extremal Properties", pp. 56--123.
3. [^] Jeroen Demeyer Diophantine Sets over Polynomial Rings and Hilbert's Tenth Problem for Function Fields , Ph.D. theses (2007), p.70.
4. [^] ^a ^b ^c Boyd, John P. (2001). *Chebyshev and Fourier Spectral Methods*  (second ed.). Dover. ISBN 0-486-41183-4.
5. [^] Chebyshev Interpolation: An Interactive Tour 

References [\[edit source \]](#) [\[edit beta \]](#)

- Abramowitz, Milton; Stegun, Irene A., eds. (1965), "Chapter 22" , *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, New York: Dover, p. 773, ISBN 978-0486612720, MR 0167642 .
- Dette, Holger (1995), A Note on Some Peculiar Nonlinear Extremal Phenomena of the Chebyshev Polynomials , *Proceedings of the Edinburgh Mathematical Society* **38**, 343-355
- Eremenko, A.; Lempert, L. (1994), An Extremal Problem For Polynomials , *Proceedings of the American Mathematical Society*, Volume 122, Number 1, 191-193
- Koornwinder, Tom H.; Wong, Roderick S. C.; Koekoek, Roelof; Swarttouw, René F. (2010), "Orthogonal Polynomials" , in Olver, Frank W. J.; Lozier, Daniel M.; Boisvert, Ronald F.; Clark, Charles W., *NIST Handbook of Mathematical Functions*, Cambridge University Press, ISBN 978-0521192255, MR2723248 
- Remes, Eugene, On an Extremal Property of Chebyshev Polynomials 
- Suetin, P.K. (2001), "C/c021940" , in Hazewinkel, Michiel, *Encyclopedia of Mathematics*, Springer, ISBN 978-1-55608-010-4

External links [\[edit source \]](#) [\[edit beta \]](#)

- Weisstein, Eric W., "Chebyshev Polynomial of the First Kind" , *MathWorld*.
- Module for Chebyshev Polynomials by John H. Mathews 
- Chebyshev Interpolation: An Interactive Tour , includes illustrative Java applet.
- Numerical Computing with Functions: The Chebfun Project 

Books

Add to my library

Write review

GET PRINT BOOK

No eBook available

[Amazon.com](#)[Barnes&Noble.com](#)[Books-A-Million](#)[IndieBound](#)[Find in a library](#)[All sellers »](#)**New! Get Textbooks on Google Play**

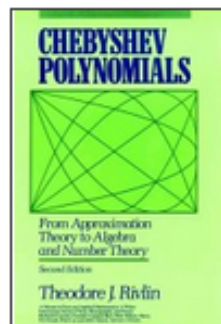
Rent and save from the world's largest eBookstore. Read, highlight, and take notes, across web, tablet, and phone.

[Go to Google Play Now »](#)

► My library

► My History

Books on Google Play

Chebyshev polynomials: from approximation theory to algebra and number theory

Theodore J. Rivlin

★★★★★

0 Reviews

J. Wiley, Jul 4, 1990 - [Mathematics](#) - 249 pages

This Second Edition continues the fine tradition of its predecessor by surveying the most important properties. New to this edition are approximately 80 exercises and a chapter which introduces some elementary algebra. Additional coverage focuses on extremal and iterative properties.

From inside the book

Rivlin Chebyshev Polynor

18 pages matching Rivlin Chebyshev Polynomials Theorem 2.20 in this book

Page 107

CHEBYSHEV POLYNOMIALS ARE EXTREMAL

107

Page 108

Theorem 4.40. Let T be a linear functional on \mathcal{P}_n such that (i) neither ± 1 is an extremal for T , (ii) $p \in \mathcal{P}_n$, $p \neq 0$, having n distinct zeros in I , implies $TP \neq 0$. Then

$$|Tp| \leq |Tt_n|, \quad p \in C_n, \quad (2.36)$$

with equality holding if and only if $p = \pm T$

Page 114

3 Bounding the approximate degree of the majority function

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

We will require the following well-known fact, which follows from, e.g., [Riv90, Theorem 2.20].

Proposition 3.1. *Let $p(x)$ be a polynomial of degree at most k which satisfies $|p(x)| \leq 1$ whenever $|x| \leq 1$. Then $|p'(0)| \leq 2\lceil \frac{k}{2} \rceil - 1 \leq k$.*

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

References

[Riv90] Theodore J. Rivlin. *Chebyshev Polynomials*. John Wiley & Sons, New York, second edition, 1990.

Do Today

Set up your LaTeX workflow (editor, etc.)
Make stub .tex, .sty, lifetime .bib files
Get reference manager software
Get version control software
Get presentation software + LaTeX addin
Practice drawing figures in it,
& importing them into LaTeX documents
Start a giant folder of saved papers
Get access to Maple/Mathematica/Sage
Get access to Matlab
Start Homework #1

www.youtube.com/channel/UCWFp4UWNiOv71j0sPbdNiqw
sites.google.com/site/plustcs/
video.ias.edu/csdm
simons.berkeley.edu/videos
www.birs.ca/videos/2016
www.youtube.com/user/MicrosoftResearch
theory.cs.princeton.edu/videos.html

Bookmark Today

feedworld.net/toc
scholar.google.com
dblp.org
wikipedia.org
arxiv.org
eccc.hpi-web.edu
scirate.com
ams.org/mrlookup
oeis.org
isc.carma.newcastle.edu.au
wolframalpha.com
books.google.com
illiad.library.cmu.edu/illiad/illiad.dll
theory.cs.cmu.edu
mathoverflow.net
[\[tex,cstheory,cs,math,mathematica\].stackexchange.com](http://[tex,cstheory,cs,math,mathematica].stackexchange.com)
mailman.srv.cs.cmu.edu/mailman/listinfo/theory-announce
www.cs.cmu.edu/~kayvonf/misc/cleartalktips.pdf