Data Representation

15-110 – Friday 09/03

Announcements

- Eberly Study Information
- No class or office hours next Monday
  - Enjoy Labor Day!
- Check1 due next Wednesday at **noon**
  - Don't forget about Piazza and Office Hours!
  - **Demo:** how to submit files on Gradescope

Muddiest Points

- Hard to keep up with new terminology
  - Try having the slides open during lecture! Download from https://www.cs.cmu.edu/~110/schedule.html before lecture starts.
  - If we don't define a term in the slides, let us know so we can add it

- How to tell apart different error types
  - We'll learn more about errors in Week 3
  - For now, just try using the debugging approach we introduced

Learning Objectives

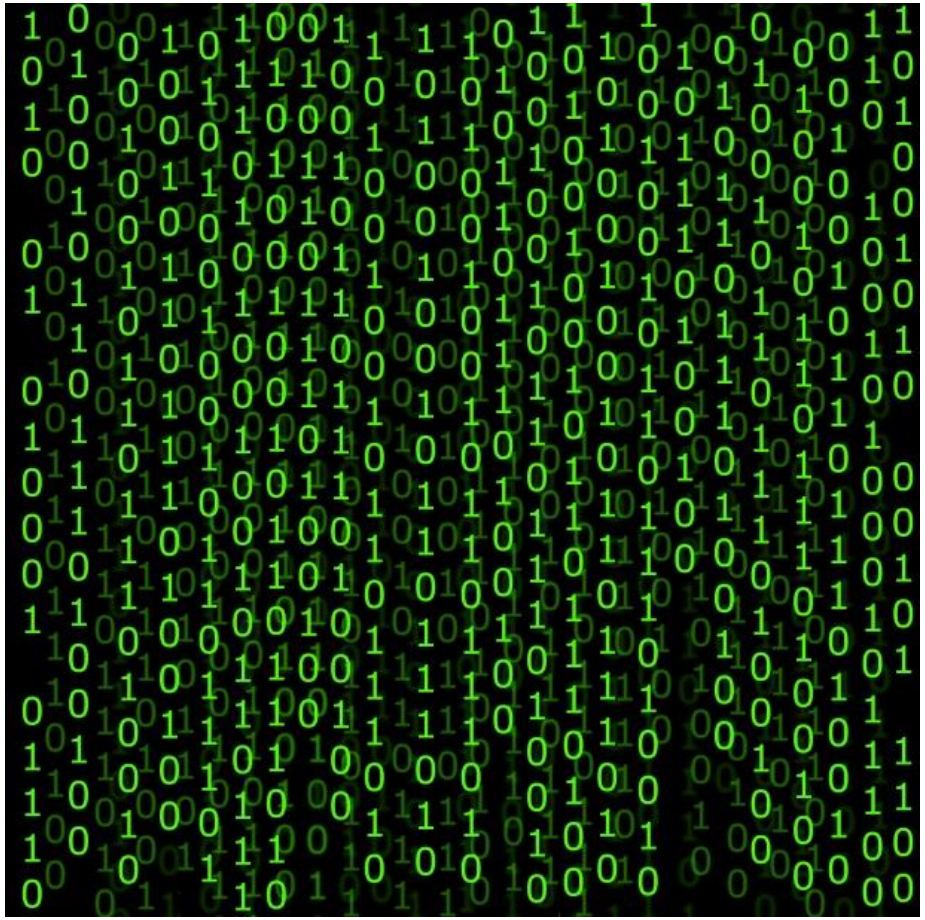
- Understand how different **number systems** can represent the same information
- Translate **binary numbers** to decimal, and vice versa
- Interpret binary numbers as abstracted types, including **colors** and **text**

Number Systems

Computers Run on 0s and 1s

Computers represent everything by using 0s and 1s. You've likely seen references to this before.

How can we represent text, or images, or sound with 0s and 1s? This brings us back to **abstraction**.



Abstraction is About Representation

Recall our definition of abstraction from the first lecture:

Abstraction is a technique used to make complex systems manageable by changing the amount of detail used to **represent** or interact with the system.

We'll use abstraction to translate 0s and 1s to decimal numbers, then translate those numbers to other types.

Number Systems – Coins

A **number system** is a way of representing numbers using symbols.

One example of a number system is currency. In the US currency system, how much is each of the following symbols worth?

Penny
1 cent

Nickel
5 cents

Dime
10 cents

Quarter
25 cents

Number Systems – Dollars

Alternatively, we can represent money using **dollars and cents**, in decimal form.

For example, a medium coffee at Tazza is **$2.65**.

Converting Coins to Dollars

We can **convert between number systems** by translating a value from one system to the other.

For example, the coins on the left represent the same value as $0.87

Using pictures is clunky. Let's make a new representation system for coins.

Coin Number Representation

To represent coins, we'll make a number with four digits.

The first represents quarters, the second dimes, the third nickels, and the fourth pennies.

c.3.1.0.2 =

3*$0.25 + 1*$0.10 + 0*$0.05 + 2*$0.01 =

$0.87

	Q	D	N	P
c	3	1	0	2

Converting Dollars to Coins

In recitation, you created an algorithm to convert money from dollars to coins, minimizing the number of coins used.

How did your algorithm work?

Conversion Example

What is $0.59 in coin representation?

$0.59 = 2*$0.25 + 0*$0.10 + 1*$0.05 + 4*$0.01 = c.2.0.1.4

Activity: Coin Conversion

You do: Now try the following calculations:

What is c.1.1.1.2 in dollars?

What is $0.61 in coin representation?

Number Systems - Decimal

When we work with ordinary numbers outside of any specific context, we usually use the **decimal** number system.

Moving from the right, the first digit is the number of 1s, the second is 10s, the third is 100s, etc. Each digit represents a **power of 10**. For example, 1980 in decimal is 1 * 1000 + 9 * 100 + 8 * 10 + 0 * 1

But this isn't the only abstract number system we can use!

10^3 **1000**	10^2 **100**	10^1 **10**	10^0 **1**
1	9	8	0

Number Systems – Binary

We can represent numbers using only 0s and 1s with the **binary number system**.

Instead of counting the number of 1s, 5s, 10s, and 25s in coins, or 1s, 10s, 100s, and 1000s in abstract amounts, count the number of 1s, 2s, 4s, and 8s. For example, 1101 in binary is 1 * 8 + 1 * 4 + 0 * 2 + 1 * 1.

Why these numbers? They're **powers of 2**. This is a number in **base 2,** which only needs the digits 0 and 1.

2^3 **8**	2^2 **4**	2^1 **2**	2^0 **1**
1	1	0	1

Bits and Bytes

When working with binary and computers, we often refer to a set of binary values used together to represent a number.

A single binary value is called a **bit**.

A set of 8 bits is called a **byte**.

We commonly use some number of **bytes** to represent data values.

Counting in Binary

0 =

128	64	32	16	8	4	2	1
0	0	0	0	0	0	0	0

1 =

128	64	32	16	8	4	2	1
0	0	0	0	0	0	0	1

2 =

128	64	32	16	8	4	2	1
0	0	0	0	0	0	1	0

3 =

128	64	32	16	8	4	2	1
0	0	0	0	0	0	1	1

4 =

128	64	32	16	8	4	2	1
0	0	0	0	0	1	0	0

5 =

128	64	32	16	8	4	2	1
0	0	0	0	0	1	0	1

6 =

128	64	32	16	8	4	2	1
0	0	0	0	0	1	1	0

7 =

128	64	32	16	8	4	2	1
0	0	0	0	0	1	1	1

Converting Binary to Decimal

To convert a binary number to decimal, just add each power of 2 that is represented by a 1.

For example, 00011000 = 16 + 8 = 24

128	64	32	16	8	4	2	1
0	0	0	1	1	0	0	0

Another example:

10010001 = 128 + 16 + 1 = 145

128	64	32	16	8	4	2	1
1	0	0	1	0	0	0	1

Converting Decimal to Binary

Converting decimal to binary uses the **same process** as converting dollars to coins.

Look for the largest power of 2 that can fit in the number and subtract it from the number. Repeat with the next power of 2, etc., until you reach 0.

For example, 36 = 32 + 4 = 00100100

128	64	32	16	8	4	2	1
0	0	1	0	0	1	0	0

Another example:

103 = 64 + 32 + 4 + 2 + 1

128	64	32	16	8	4	2	1
0	1	1	0	0	1	1	1

Activity: Converting Binary

You do: Now try converting numbers on your own.

First: what is **01011011** in decimal?

Second: what is **75** in binary?

Abstracted Types

Binary and Abstraction

Now that we can represent numbers using binary, we can represent **everything** computers store using binary.

We just need to use **abstraction** to interpret bits or numbers in particular ways.

Let's consider numbers, images, and text.

Discussion: Representing Negative Numbers

It can be helpful to think logically about how to represent a value before learning how it's done in practice. Let's do that now.

Discuss: We can convert binary directly into positive numbers, but how do we represent negative numbers?

Answer: Representing Negative Numbers

Simple Approach: reserve one bit to represent whether the number is positive or negative. Convert the rest normally.

Actual Approach: mathematically, X + (-X) = 0. Set up the negative binary so that when it is added to the binary of the positive version of the number, the result is 0. Do this by **restricting the number of bits** that will be considered to a preset amount (perhaps 8).

The value 11111111 is 255, but it is also -1 because 11111111 + 1 = 100000000, which becomes 00000000 if we only have 8 bits. 11111110 is -2 (or 254), and so on.

How do we decide whether 11111111 is 255 or -1? It all depends on our **interpretation**. Is the integer signed (possibly negative) or unsigned (definitely positive)?

Size of Integers

Your machine is either classified as 32-bit or 64-bit. This refers to the **size of integers** used by your computer's operating system.

The largest signed integer that can be represented with N bits is 2^N-1 (why?). This means that...

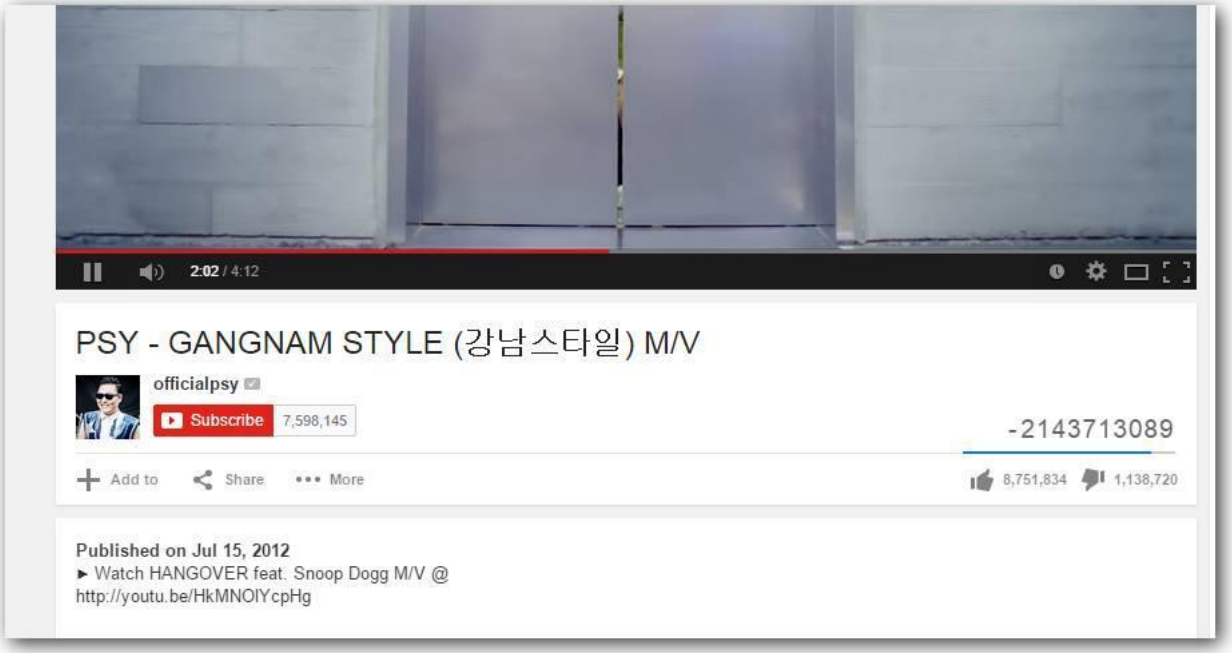
Largest int for 32 bits: 4,294,967,295 (or 2,147,483,647 with negative numbers)

Largest int for 64 bits: 18,446,744,073,709,551,615 (18.4 quintillion)

Integer Overflow

Why does this matter?

By late 2014, the music video Gangnam Style received more than **2 billion** views. When it passed the largest positive number that could be represented with 32 bits, YouTube showed the number of views as **negative** instead!



Now YouTube uses a 64-bit counter instead.

Represent Images as Grids of Colors

What if we want to represent an image? How can we convert that to numbers?

First, break the image down into a grid of colors, where each square of color has a distinct hue. A square of color in this context is called a **pixel**.

If we can represent a pixel in binary, we can interpret a series of pixels as an image.

Representing Colors in Binary

We need to represent a single color (a pixel) as a number.

There are a few ways to do this, but we'll focus on **RGB**. Any color can be represented as a combination of Red, Green, and Blue.

Red, green, and blue intensity can be represented using one **byte** each, where 00000000 (0) is none and 11111111 (255) is very intense. Each pixel will therefore require 3 bytes to encode.

Try it out here: w3schools.com/colors/colors_rgb.asp

Represent Text as Individual Characters

Next, how do we represent text?

First, we break it down into smaller parts, like with images. In this case, we can break text down into individual **characters**.

For example, the text "Hello World" becomes

H, e, l, l, o, space, W, o, r, l, d

Use a Lookup Table to Convert Characters

Unlike colors, characters don't have a natural connection to numbers.

Instead, we can use a **lookup table** that maps each possible character to an integer.

As long as every computer uses the same lookup table, computers can always translate a set of numbers into the same set of characters.

ASCII is a Simple Lookup Table

For basic characters, we can use the encoding system called ASCII. This maps the numbers 0 to 255 to characters. Therefore, one character is represented by one byte.

Check it out here: www.asciitable.com

Dec	Hex	Oct	Chr	Dec	Hex	Oct	HTML	Chr	Dec	Hex	Oct	HTML	Chr	Dec	Hex	Oct	HTML	Chr
0	0	000	NULL	32	20	040	&#032;	Space	64	40	100	&#064;	@	96	60	140	&#096;	`
1	1	001	Start of Header	33	21	041	&#033;	!	65	41	101	&#065;	A	97	61	141	&#097;	a
2	2	002	Start of Text	34	22	042	&#034;	"	66	42	102	&#066;	B	98	62	142	&#098;	b
3	3	003	End of Text	35	23	043	&#035;	#	67	43	103	&#067;	C	99	63	143	&#099;	c
4	4	004	End of Transmission	36	24	044	&#036;	$	68	44	104	&#068;	D	100	64	144	&#100;	d
5	5	005	Enquiry	37	25	045	&#037;	%	69	45	105	&#069;	E	101	65	145	&#101;	e
6	6	006	Acknowledgment	38	26	046	&#038;	&	70	46	106	&#070;	F	102	66	146	&#102;	f
7	7	007	Bell	39	27	047	&#039;	'	71	47	107	&#071;	G	103	67	147	&#103;	g
8	8	010	Backspace	40	28	050	&#040;	(	72	48	110	&#072;	H	104	68	150	&#104;	h
9	9	011	Horizontal Tab	41	29	051	&#041;	)	73	49	111	&#073;	I	105	69	151	&#105;	i
10	A	012	Line feed	42	2A	052	&#042;	*	74	4A	112	&#074;	J	106	6A	152	&#106;	j
11	B	013	Vertical Tab	43	2B	053	&#043;	+	75	4B	113	&#075;	K	107	6B	153	&#107;	k
12	C	014	Form feed	44	2C	054	&#044;	,	76	4C	114	&#076;	L	108	6C	154	&#108;	l
13	D	015	Carriage return	45	2D	055	&#045;	-	77	4D	115	&#077;	M	109	6D	155	&#109;	m
14	E	016	Shift Out	46	2E	056	&#046;	.	78	4E	116	&#078;	N	110	6E	156	&#110;	n
15	F	017	Shift In	47	2F	057	&#047;	/	79	4F	117	&#079;	O	111	6F	157	&#111;	o
16	10	020	Data Link Escape	48	30	060	&#048;	0	80	50	120	&#080;	P	112	70	160	&#112;	p
17	11	021	Device Control 1	49	31	061	&#049;	1	81	51	121	&#081;	Q	113	71	161	&#113;	q
18	12	022	Device Control 2	50	32	062	&#050;	2	82	52	122	&#082;	R	114	72	162	&#114;	r
19	13	023	Device Control 3	51	33	063	&#051;	3	83	53	123	&#083;	S	115	73	163	&#115;	s
20	14	024	Device Control 4	52	34	064	&#052;	4	84	54	124	&#084;	T	116	74	164	&#116;	t
21	15	025	Negative Ack.	53	35	065	&#053;	5	85	55	125	&#085;	U	117	75	165	&#117;	u
22	16	026	Synchronous idle	54	36	066	&#054;	6	86	56	126	&#086;	V	118	76	166	&#118;	v
23	17	027	End of Trans. Block	55	37	067	&#055;	7	87	57	127	&#087;	W	119	77	167	&#119;	w
24	18	030	Cancel	56	38	070	&#056;	8	88	58	130	&#088;	X	120	78	170	&#120;	x
25	19	031	End of Medium	57	39	071	&#057;	9	89	59	131	&#089;	Y	121	79	171	&#121;	y
26	1A	032	Substitute	58	3A	072	&#058;	:	90	5A	132	&#090;	Z	122	7A	172	&#122;	z
27	1B	033	Escape	59	3B	073	&#059;	;	91	5B	133	&#091;	[	123	7B	173	&#123;	{
28	1C	034	File Separator	60	3C	074	&#060;	<	92	5C	134	&#092;	\	124	7C	174	&#124;	|
29	1D	035	Group Separator	61	3D	075	&#061;	=	93	5D	135	&#093;	]	125	7D	175	&#125;	}
30	1E	036	Record Separator	62	3E	076	&#062;	>	94	5E	136	&#094;	^	126	7E	176	&#126;	~
31	1F	037	Unit Separator	63	3F	077	&#063;	?	95	5F	137	&#095;	_	127	7F	177	&#127;	Del

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Translating Text to Numbers

"Yay" =

01011001 -> 89

01100001 -> 97

01111001 -> 121

Dec	Hex	Oct	Chr	Dec	Hex	Oct	HTML	Chr	Dec	Hex	Oct	HTML	Chr	Dec	Hex	Oct	HTML	Chr
0	0	000	NULL	32	20	040	&#032;	Space	64	40	100	&#064;	@	96	60	140	&#096;	`
1	1	001	Start of Header	33	21	041	&#033;	!	65	41	101	&#065;	A	97	61	141	&#097;	a
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3	3	003	End of Text	35	23	043	&#035;	#	67	43	103	&#067;	C	99	63	143	&#099;	c
4	4	004	End of Transmission	36	24	044	&#036;	$	68	44	104	&#068;	D	100	64	144	&#100;	d
5	5	005	Enquiry	37	25	045	&#037;	%	69	45	105	&#069;	E	101	65	145	&#101;	e
6	6	006	Acknowledgment	38	26	046	&#038;	&	70	46	106	&#070;	F	102	66	146	&#102;	f
7	7	007	Bell	39	27	047	&#039;	'	71	47	107	&#071;	G	103	67	147	&#103;	g
8	8	010	Backspace	40	28	050	&#040;	(	72	48	110	&#072;	H	104	68	150	&#104;	h
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10	A	012	Line feed	42	2A	052	&#042;	*	74	4A	112	&#074;	J	106	6A	152	&#106;	j
11	B	013	Vertical Tab	43	2B	053	&#043;	+	75	4B	113	&#075;	K	107	6B	153	&#107;	k
12	C	014	Form feed	44	2C	054	&#044;	,	76	4C	114	&#076;	L	108	6C	154	&#108;	l
13	D	015	Carriage return	45	2D	055	&#045;	-	77	4D	115	&#077;	M	109	6D	155	&#109;	m
14	E	016	Shift Out	46	2E	056	&#046;	.	78	4E	116	&#078;	N	110	6E	156	&#110;	n
15	F	017	Shift In	47	2F	057	&#047;	/	79	4F	117	&#079;	O	111	6F	157	&#111;	o
16	10	020	Data Link Escape	48	30	060	&#048;	0	80	50	120	&#080;	P	112	70	160	&#112;	p
17	11	021	Device Control 1	49	31	061	&#049;	1	81	51	121	&#081;	Q	113	71	161	&#113;	q
18	12	022	Device Control 2	50	32	062	&#050;	2	82	52	122	&#082;	R	114	72	162	&#114;	r
19	13	023	Device Control 3	51	33	063	&#051;	3	83	53	123	&#083;	S	115	73	163	&#115;	s
20	14	024	Device Control 4	52	34	064	&#052;	4	84	54	124	&#084;	T	116	74	164	&#116;	t
21	15	025	Negative Ack.	53	35	065	&#053;	5	85	55	125	&#085;	U	117	75	165	&#117;	u
22	16	026	Synchronous idle	54	36	066	&#054;	6	86	56	126	&#086;	V	118	76	166	&#118;	v
23	17	027	End of Trans. Block	55	37	067	&#055;	7	87	57	127	&#087;	W	119	77	167	&#119;	w
24	18	030	Cancel	56	38	070	&#056;	8	88	58	130	&#088;	X	120	78	170	&#120;	x
25	19	031	End of Medium	57	39	071	&#057;	9	89	59	131	&#089;	Y	121	79	171	&#121;	y
26	1A	032	Substitute	58	3A	072	&#058;	:	90	5A	132	&#090;	Z	122	7A	172	&#122;	z
27	1B	033	Escape	59	3B	073	&#059;	;	91	5B	133	&#091;	[	123	7B	173	&#123;	{
28	1C	034	File Separator	60	3C	074	&#060;	<	92	5C	134	&#092;	\	124	7C	174	&#124;	|
29	1D	035	Group Separator	61	3D	075	&#061;	=	93	5D	135	&#093;	]	125	7D	175	&#125;	}
30	1E	036	Record Separator	62	3E	076	&#062;	>	94	5E	136	&#094;	^	126	7E	176	&#126;	~
31	1F	037	Unit Separator	63	3F	077	&#063;	?	95	5F	137	&#095;	_	127	7F	177	&#127;	Del

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For More Characters, Use Unicode

There are plenty of characters that aren't available in ASCII (characters from non-English languages, advanced symbols, emoji...) due to the limited size.

The Unicode system represents every character that can be typed into a computer. It uses up to 5 bytes, which can represent up to 1 trillion characters! Find all the Unicode characters here: www.unicode-table.com

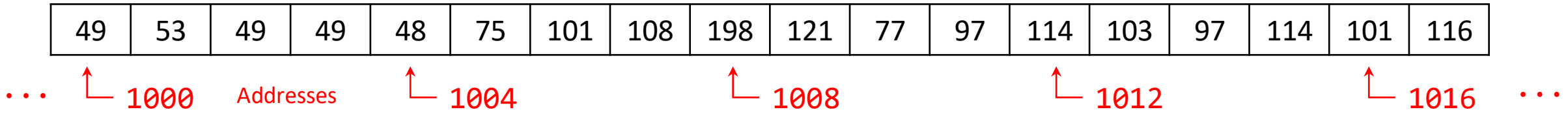
The Unicode system is also **actively under development**. The Unicode Consortium regularly updates the standard to add new types of characters and emoji.

Discuss: what are the potential repercussions of using a single standard for all text on computers?

Computer Memory is Stored as Binary

Your computer keeps track of saved data and all the information it needs to run in its **memory**, which is represented as binary. You can think about your computer's memory as a really long list of bits, where each bit can be set to 0 or 1. But usually we think in terms of bytes, groups of 8 bits.

Every byte in your computer has an **address**, which the computer uses to look up its value.



Binary Values Depend on Interpretation

When you open a file on your computer, the application goes to the appropriate address, reads the associated binary, and **interprets** the binary values based on the file encoding it expects. That interpretation depends on the **application** you use when opening the file, and the **filetype**.

You can attempt to open **any file** using **any program**, if you convince your computer to let you try. Some programs may crash, and others will show nonsense because the binary isn't being interpreted correctly.

Example: try changing a .docx filetype to .txt, then open it in a plain text editor. .docx files have extra encoding, whereas .txt files use plain ASCII.

We Use Lots of Bytes!

In modern computing, we use a **lot** of bytes to represent information.

Smartphone Memory: 64 gigabytes = 64 **billion** bytes

Google databases: Over 100 million gigabytes = 100 **quadrillion** bytes!

CMU Wifi: 15 **million** bytes per second

Learning Objectives

- Understand how different **number systems** can represent the same information
- Translate **binary numbers** to decimal, and vice versa
- Interpret binary numbers as abstracted types, including **colors** and **text**

Feedback form: https://bit.ly/110-f21-feedback