15110 PRINCIPLES OF COMPUTING – FINAL EXAM – FALL 2012

Name Answer Key Section ________ Andrew ID ____________________

Directions: Answer each question neatly in the space provided. Please read each question carefully. You have 180 minutes for this exam. No electronic devices or notes are allowed. Good luck!

	POINTS	GRADER
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
BONUS	2	
TOTAL	102	

1. (10 pts)

1a. Compute the value of each of the following Ruby expressions.

`10 * 8 + 5 - 6 / 3` 83

`(73 / 10) + (73 % 10)` 10

1b. Suppose you have a Ruby function `student_count(class, section)` that returns the number of students in a given class `class` and section `section`. Assume that the parameters `class` and `section` have the type integer. Write an assignment expression that uses the function `student_count` and gives the sum of the number of students in the class 15110 in sections 1 and 2.

total_student_count = student_count(15110, 1) + student-count(15110, 2)

1c. The following Ruby function `print_elems_1(mylist)`, which includes a `while` loop, can be written using a `for` loop instead. Fill in the blanks in the function `print_elems_2(mylist)` such that it is equivalent to `print_elems_1(mylist)`, which means that for a given argument `mylist` it would print the same numbers as `print_elems_1(mylist)`.

```
def print_elems_1(mylist)
  i = 0
  len = mylist.length
  while  i <  len do
    print mylist[i], "\n"
    i = i + 1
  end
end
```

```
def print_elems_2(mylist)
  for item in mylist do
    print item, "\n"
  end
end
```

1d. Let `colors = ["Red", "Blue", "Green", "Purple", "White", "Black"]`

What is the output in irb for each of the following Ruby code fragments?

```
colors.each { |item| print item }
```

Red Blue Green Purple White Black

```
colors.each { |item| print item if item > "Blue" }
```

Red Green Purple White

1e. Write a Ruby function `create_table()` to create and return a two-dimensional array named `table` with 100 rows and 100 columns such that all the elements in the `table` are initialized to 0. Recall that you can create a one-dimensional array of size `n` using the method `Array.new(n)`.

```
def create_table()
  table = Array.new(100)
  for row in 0..99 do
      table[row]= Array.new(100, 0)
  end
  return table
end
```

2. (10 pts)

2a. Recall the algorithm merge sort for sorting elements in a list, which works by first dividing the input array into several chunks, and then combining them into bigger and bigger groups until the final merged group includes the full array. The function `merge_groups(a, gs)` below is a function that can be used to merge all adjacent groups of size `gs` to form groups of size `2*gs`.

```
def merge_groups(a, gs)
  i = 0                              # start of first group
  while i < a.length
    j = i + (2 * gs) - 1             # end of second group
    a[i..j] = merge_pair(a, i, gs)
    i = i + (2*gs)                   # start of next group
 end
```

Assume that the function `merge_pair(a, i, gs)` does the actual merge operation by merging the two groups at `a[i]` and `a[i+gs]`, and returning a new array with all the items in the merged group sorted in ascending order. Note that `a` stands for the array to be sorted, and `gs` for the group size. That is, an assignment statement such as `a[i..j] = merge_pair(a, i, gs)` tells Ruby to replace the current items in locations `i` through `j` of the array `a` with the array returned by `merge_pair(a, i, gs)`.

(i) If `a = [3, 8, 6, 7, 1, 9, 2, 10]`, what are the contents of `a[0..3]` right after the assignment statement `a[0..3] = merge_pair(a, 0, 2)` is executed?

`a[0..3]=` [3,6,7,8] or a= [3,6,7,8,1,9,2,10]

(ii) If `a = [3, 6, 7, 8, 1, 9, 2, 10]`, what are the contents of `a[4..7]` right after the assignment statement `a[4..7] = merge_pair(a, 4, 2)` is executed?

`a[4..7] =` [1,2,9,10] or a=[3,6,7,8,1,2,9,10]

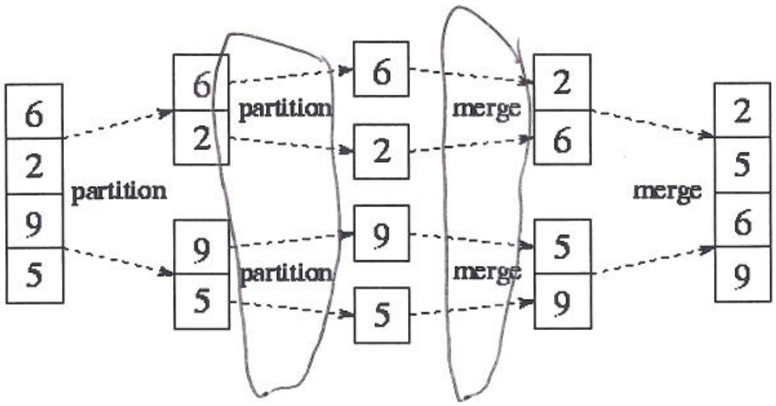
2b. Below is a Ruby function for a non-recursive implementation of merge sort that calls the `merge_groups(a, gs)` function from part (a). Complete the missing parts in the code.

```
def msort(array)
  a = array.clone   #creates an identical copy of array and stores it in a
  size = 1
  while size < a.length
    merge_groups(a , size)
    size = size * 2
  end
  return a
end
```

2c. Given the function `msort(array)` in part 2b, how many times would the function `merge_groups(a, gs)` be called to sort an array of 8 elements? ____3____ times.

2d. What is the worst-case order of complexity of this merge sort for `n` elements using the big O notation? You can assume that the complexity of `merge_groups(a, gs)` is linear in the number of elements in `a`.
O(n log n).

2e. Merge sort is an algorithm that lends itself to parallel execution of certain steps, where each step is executed on a different processor. Which steps of the algorithm can be executed in parallel (simultaneously)? Give your answer using the following figure as a reference. Draw a circle around each pair of steps that can be run in parallel.



2f. Part (2e) assumes that multiple processors exist to perform certain steps of the algorithm in parallel. In general, multiple processes share a single processor giving rise to the need to coordinate the actions of individual processes. For example, a condition to be avoided is deadlock, which is the condition when two or more processes are all waiting for some shared resource that other processes of the group hold, causing all processes to wait forever without proceeding.

3. (10 pts)

3a. Suppose we want to perform binary search on a sorted array of elements, returning either a position in the array, or nil if the requested item does not appear in the array. Complete the following code:

```
def search(array,item)
  low = -1
  high = a.length
  while high > low + 1 do
    mid = (low +high) /2
    if array[mid] == item then
      return mid
    elsif array[mid] < item then
      low = mid
    else
      high = mid
    end
  end
  return nil
end
```

Or:

```
low = 0
high = a.length - 1
while low ≤ high do

low = mid + 1

high = mid - 1

return nil
```

3b. Trace the binary search that results from the following function call (using the search function from problem 3a) by filling in the table below. You might not need to fill in all the rows.

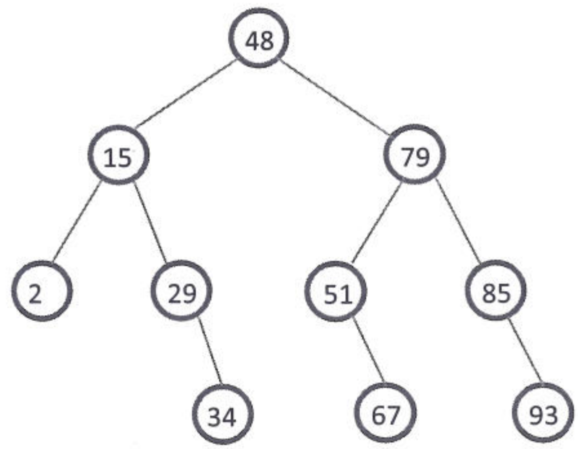
search([2,15,29,34,48,51,67,79,85,93], 67)

low	high	mid	array[mid]
-1	10	4	48
4	10	7	79
4	7	5	51
5	7	6	67

Answer is different if they use the alternative algorithm where low starts at 0 and high at 9.

(3c) Now consider doing binary search using a binary tree such as the one below. Note that each node is a list of the form `[item, left_child, right_child]`, where `item` is an integer and `left_child` and `right_child` are either nodes or nil (if that child is missing).

```
[48, [15, [2, nil nil],
          [29, nil,
               [34, nil, nil]]],
     [79, [51, nil,
               [67, nil, nil]],
          [85, nil,
               [93, nil, nil]]]]
```



Fill in the code for the binary search algorithm that operates on trees of this form:

```
def search(tree, item)
  if tree == __nil__ then
    return false
  elsif tree[0] == __item__ then
    return true
  elsif tree[0] > item then
    return __search(tree[1], item)__
  else
    return search(tree[2], item)
  end
end
```

3d. Given a balanced binary tree with *n* terminal nodes, what is the largest number of recursive calls that could be made by the search function of problem 3c when the item does not appear in the tree? __log n__

3e. Fill in the blank by circling one of the choices below. For maximum efficiency, a hash function should distribute items __B__ across the buckets.

A. randomly
B. uniformly (circled)
C. sequentially
D. logarithmically

4. (10 pts)

4a. Complete the recursive function `doubler(list)` that doubles every element of a list and returns the result. Example: `doubler([2,5,30])` returns `[4,10,60]`. Note: the "+" in the code below denotes array concatenation, not integer addition.

```
def doubler(list)
  if list.empty?
    return [ ]
  else
    return [list[0]*2] + doubler(list.drop(1))
  end
end
```

4b. Consider the recursive function twos(x) shown below, that factors all the 2's out of an integer. Find the result of twos(80) by tracing each recursive call, by filling in the table below. Hint: begin by filling in the call column until you hit the base case, then work your way back up to fill in the result column.

```
def twos(x)
  if x%2 == 0 then
    return [2] + twos(x/2)
  else
    return [x]
  end
end
```

#	Call to twos	Result
1	twos(80)	[2,2,2,5]
2	twos(20)	[2,2,5]
3	twos(10)	[2,5]
4	twos(5)	[5]
5		

4c. What does the function `f(x,y)` shown below do? Hint: consider `f(3,4)`.

```
def f(x,y)
  if x == 0 then
    return y
  else
    return f(x-1, y+1)
  end
end
```

Computes x + y.

4d. A palindrome is a sequence that reads the same forward and backward, such as "racecar" or [1,4,7,4,1]. An empty sequence is obviously also a palindrome, as is a sequence of length one. In Ruby we can test for palindromes with `x==x.reverse`, but in this problem we're going to write a recursive test, dropping both the first and last element of the sequence with each recursive call. Fill in the missing code. Note that this solution works for both lists and strings.

```
def palindrome?(x)
  if x.length <= 1 then
    return true
  elsif x[0] != x[x.length-1] then
    return false
  else
    return palindrome?(x[1.. x.length -2])
  end
end
```

5. (10 pts)

5a. In a reliable message delivery system messages are received by the recipient in the order they are sent by the sender. If we wanted to implement a message buffer that stores the messages in transmission which data structure would be the most appropriate: *graph*, *queue*, *stack* or *tree*? queue

5b. If you have an array of 5000 elements in computer memory, and you want to insert an element at the beginning of the array, how many elements of the array need to be moved? 5000

5c. If you have a linked list of 5000 elements in computer memory, and you want to insert an element at the beginning of the linked list, how many nodes of the list need to be moved? 0

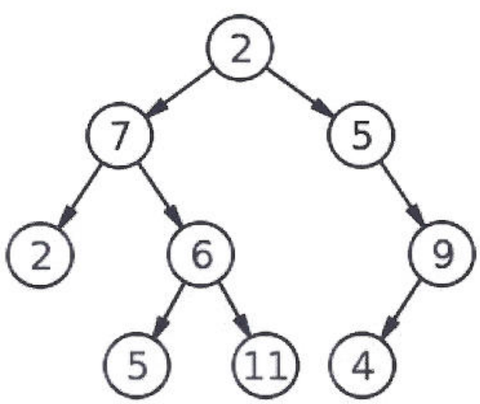
5d. Consider the function `balanced?(list)` that returns *true* if its input is a properly balanced parenthetical expression, and *false* otherwise. An expression is properly balanced if every left parenthesis is matched by a closing right parenthesis, and every right parenthesis closes some left parenthesis. We can use a stack to keep track of all the parentheses waiting to be closed. In a balanced expression, a right parenthesis will never be encountered when the stack is empty, and the stack will always be empty after we've processed the last element of the input. Examples: `balanced?(["(", "(", ")", ")"]` should return *true*, and `balanced?(["(", ")", ")", "("])` should return *false*. Complete the function.

```
def balanced?(list)
  stack = []
  for x in list do
    if x == "(" then
      stack.push(x)
    else  # x must be ")"
      if stack.empty? then
        return false      # no left paren to close!
      else
        stack.pop
    end
  end
  return stack.empty?
end
```

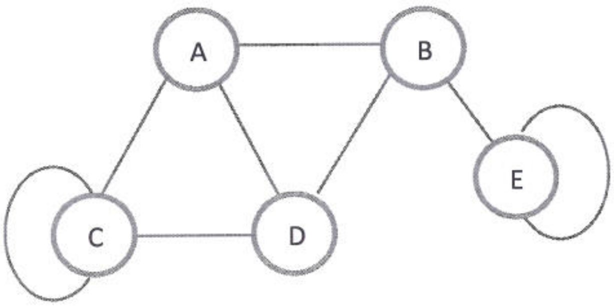
5e. Answer the following questions based on the tree given below.

(i) Is the given tree a binary tree? Why or why not? Yes, because it's a tree and every parent has at most two children

(ii) Is the given tree a binary search tree? Why or why not? No, because the left and right children of the root both contain values > 2.



5f. (i) Recall that an undirected graph can be represented using an adjacency matrix `M` such that for all `i` and `j`, `M[i][j]` = 1 and `M[j][i]` = 1 if there is a link between `i` and `j`, and `M[i][j]` = 0otherwise. Given the example graph below, write down its 5-by-5 adjacency matrix.



	A	B	C	D	E
A	0	1	1	1	0
B	1	0	0	1	1
C	1	0	1	1	0
D	1	1	1	0	0
E	0	1	0	0	1

(ii) If a given adjacency matrix `M` is such that `M[i][i] = 0` for all `i`, can the graph have any self-links?

Yes or No? No

6. (10 pts)

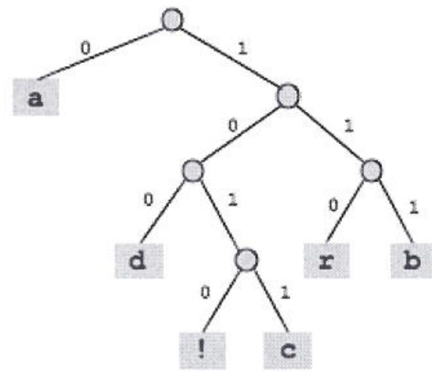
6a. Write the integer +73 as an 8-bit signed integer. 01001001
(The powers of 2 are: 1, 2, 4, 8, 16, 32, 64, 128, 256, 1024, …)

6b. Based on your answer from part (a), write the integer -73 as an 8-bit signed integer. 10110111

6c. How many distinct colors can be represented using the Red-Green-Blue (RGB) encoding for images? Note that in this encoding the intensity of each color component is represented using 8 bits. You may express your answer as a power of 2.

2^{24}

6d. A Huffman tree is designed for a collection of 6 characters as shown below.



(i) Based on this Huffman tree, we can conclude that the letter a is more commonly used than the letter c. Fill in the blank with the answer "less" or "more".

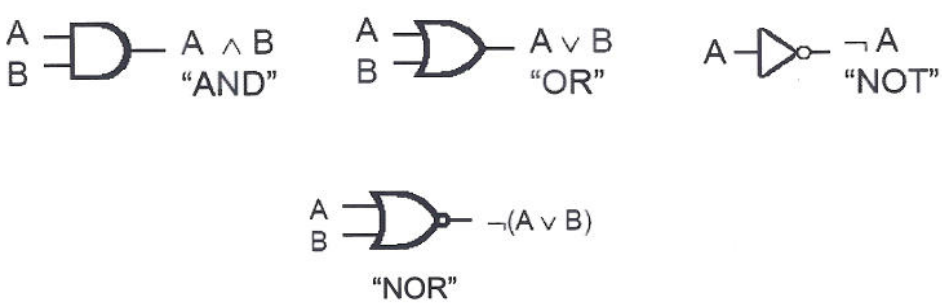
(ii) Decode the following message that was encoded using the given Huffman tree:

0111110010110100011111001010 abracadabra!

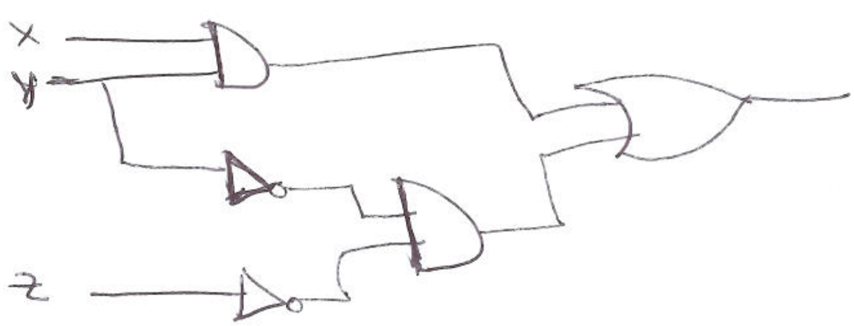
6e. Complete the following truth table:

X	Y	Z	(X ∧ Y)	(¬Y ∧ ¬Z)	(X ∧ Y) ∨ (¬Y ∧ ¬Z)
0	0	0	0	1	1
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	1	1
1	0	1	0	0	0
1	1	0	1	0	1
1	1	1	1	0	1

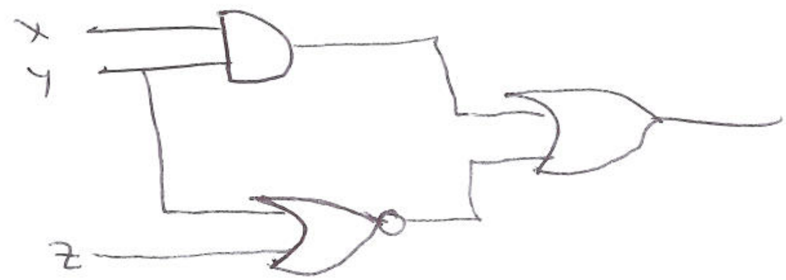
6f. A gate is an electronic device that operates on a collection of binary inputs to produce a binary output. Below are diagrammatic representations for a set of logic gates.



Draw a diagram for the formula (X ∧ Y) ∨ (¬Y ∧ ¬Z) using AND, OR, and NOT gates.



6g. Draw a diagram for the same formula from part (6e) this time using AND, OR, and NOR gates. Note that you will need to make a simple transformation using one of DeMorgan's laws on part of the expression.



6h. Put the following computer components in order from lowest to highest level of abstraction. Hint: If a component M is used to build a component N then M is at a lower abstraction level than N.

A. Full adder
B. Transistors
C. Central Processing Unit
D. Gates
E. Arithmetic Logic Unit

B, D, A, E, C

7. (10 pts) Recall that Ruby has a `rand(n)` function that returns a random integer between 0 and n-1, inclusive.

7a. Write a Ruby expression to generate a random integer value between -5 and 5, inclusive. Hint: how many possible values are there?

rand(11) - 5

7b. Show how to use rand to generate a random string from the list below.

```
list = ["rock", "paper", "scissors"].
```

list[rand(3)]

8b. Given rows A, G, O, and T of the Vigenere table:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	Z	Y	Z
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S

2pts −0.25

Decode the following message using the keyword GOAT:

M F A I K Z E T B S S   GRAPELEAVES

✓ no partial credit

8c. What idea did Jacquard's loom contribute to computing? punched cards   1pt

8d. Fill in the blanks using the terms below; some may be used more than once, and others not at all. Alice wants to publish a document and allow any recipient to verify that she is the author and that the document is unmodified. To do this, she computes a __e__ of the document and encrypts this with __b__; we call this a __g__. She then publishes the document along with her signature. When Bob reads this, he can verify the document's authenticity by decrypting the __g__ using __a__, then verify that it matches the __e__ of the document.

3pts. −0.5

a. Alice's public key.
b. Alice's private key
c. Bob's public key
d. Bob's private key
e. MD5 hash
f. prime factorization
g. digital signature
h. ciphertext version
i. plaintext version
j. key exchange protocol

8e. In a quantum computer with *n* independent qubits, the state space can be described by how many amplitude values? __2n__   −1

When the qubits are fully entangled, then the state space has __2^n__ amplitudes.

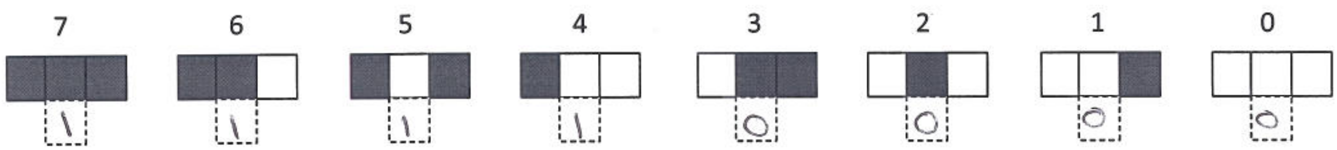
2pts

7c. In a continuous-time simulation, such as the oscillatory motion of a weight attached to a spring, time can't really be continuous. How do we deal with this?

divide time into tiny discrete intervals

7d. Consider a 1D cellular automaton where with each new generation, a cell takes on the value of its left neighbor. Thus, whatever pattern the automaton was initialized to would shift one place to the right with each new generation, and gradually be replaced by all zeros (white squares) shifting in from the left. Here is an example initial state and the next three generations. Note that the first and last cells are fixed at 0 (white). And if a square does not exist and is outside of the picture, then it is white.

Write down the rule for this automaton by putting 0's and 1's in the dotted boxes below.



7e. Write down the number of the rule you created in problem 7d. 240

8. (10 pts)

8a. Match the following: 2pts − 0.5 pts for wrong

5	IP address	1. http, https
3	domain name	2. RSA
1	protocol	3. www.cmu.edu
7	URL	4. Packet switching
2	encryption	5. 128.2.1.101
6	TCP/IP reference model	6. Link layer
4	router	7. http://www.amazon.com

9. (10 pts)

9a. In the game of Capture, there is a board with 16 numbers in a 4 X 4 grid as shown below. Players alternate turns placing a stick on an empty horizontal or vertical line on the board. The player that places the fourth stick around a number captures that number. The player with the highest total captured at the end of the game wins.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Original game board

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Player 1 plays between 9 and 10.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Player 2 plays between 3 and 7.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Player 1 plays below 16. .... etc...

If we build a game tree to analyze all of the moves of this game starting from an initial game with an empty board:

(i) How many nodes would be at the first level below the root? 40

(ii) How many nodes would be at the second level below the root? 40 × 39

(iii) How many terminal nodes (nodes without any children) would there be? 40!

9b. Suppose that you are given a pattern expressed as the regular expression `/.a.*/` in Ruby. Write 2 strings that would yield a successful match against this pattern.

(i) __________ Any string of length ≥ 2 with "a" as

(ii) __________ the second character.

9c. An engineer is designing a digital circuit with 10 separate input lines that are interconnected through some logic gates to output a single bit. What is the maximum number of input settings he would have to test in order to find out if the circuit would ever output the bit 1? $2^{10} = 1024$

9d. If an algorithm has $O(n^4)$ running time do we classify it as tractable or intractable? tractable

9e. P is the class of problems that can be decided in polynomial time and NP is the class of problems for which a candidate solution can be verified in polynomial time. Which one of the statements below is correct about P and NP. Circle your answer.

i. P = NP   ii. P ≠ NP   iii. Not proven whether i or ii. (circled)

9f. A fundamental result in computability theory, proved by Alan Turing in 1936, establishes that it is not possible to write a single universal program that can determine if any program __halts__ on any given input.

10. (10 pts – 5 pts each)

10a. Write a Ruby function `first_even(list)` that takes as argument a list consisting of integers and returns the position of the first element that is an even number if such an element exists, and returns nil if all elements are odd. You can assume that 0 is an even number.

```
def first-even (list)
    for i in 0.. list.length-1 do
        if list [i].even? then
            return i
        end
    end
    return nil
end
```

10b. Write a Ruby function `max_rand(c, n)` that uses the `rand` function to print `c` random numbers in the range 0 to `n`, one per line, and return the maximum number that is generated. You can use a for loop for this; you will have to keep track of the maximum value seen so far as you iterate.

```
def max_rand (c, n)
  max = 0
  for i in 0..c-1 do
    r = rand (n+1)
    puts r
    if r > max then
      max = r
    end
  end
  return max
end
```

BONUS QUESTIONS (1 pt each) – Your answer must be exactly correct to receive the bonus point.

1. What is the name of the procedure that has been famously proposed to determine whether a computer has achieved "artificial intelligence"?

Turing test or Imitation Game

2. What is the value of this Ruby expression? Remember that Ruby evaluates expressions left to right, and pay careful attention to the nesting of delimiters; there is no typo here. `[[[[2]]][ [].length ].length]`

[1]