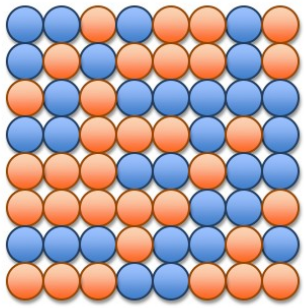


UNIT 7B

Data Representation: Compression

Hash Tables in Ruby

- Last week we looked at hash tables as a means of determining whether a key is in a list in O(1) time.
- We can generalize this idea to associate a key with a value.
- Examples:
  – Employee name => Employee number
  – Product code => Price
  – Name in contacts list => Email address

Hash Tables in Ruby

```
>> h = Hash.new
⇒{}

>> h["Mercedes"] = 50000
50000

>> h["Bentley"] = 120000
120000
```


Hash Tables in Ruby

```
>> h
{"Mercedes" => 50000,
 "Bentley" => 120000}

>> h["Mercedes"]
⇒50000
```

Hash Tables in Ruby

```
>> h2 = {:apple => :red,
         :banana => :yellow,
         :cherry => :red}

>> h2[:banana]
⇒:yellow
>> h2.invert
⇒{:red => :cherry,
  :yellow => :banana}
```


Alternative Constructor Syntax

```
>> h3 = {1, 2, 3, 4, 5, 6}
=> {5=>6, 1=>2, 3=>4}

>> h3.size
⇒3

>> h3[:woof]
⇒nil
```

Fixed-Width Encoding

- In a fixed-width encoding scheme, each character is given a binary code with the same number of bits.
  - Example:
    Standard ASCII is a fixed width encoding scheme, where each character is encoded with 7 bits.
    This gives us 2^7 = 128 different codes for characters.

Fixed-Width Encoding

- Given a character set with n characters, what is the minimum number of bits needed for a fixed-width encoding of these characters?
  - Since a fixed width of k bits gives us n unique codes to use for characters, where $n = 2^k$.
  - So given n characters, the number of bits needed is given by $k = \lceil \log_2 n \rceil$. (We use the ceiling function since $\log_2 n$ may not be an integer.)
  - Example: To encode just the alphabet A-Z using a fixed-width encoding, we would need $\lceil \log_2 26 \rceil = 5$ bits:
    e.g. A => 00000, B => 00001, C => 00010, ..., Z => 11001.

Using Fixed-Width Encoding

- If we have a fixed-width encoding scheme using *n* bits for a character set and we want to transmit or store a file with *m* characters, we would need *mn* bits to store the entire file.
- Can we do better?
  - If we assign fewer bits to more frequent characters, and more bits to less frequent characters, then the overall length of the message might be shorter.

Huffman Coding

- We can use an encoding scheme named after David A. Huffman to compress our text without losing any information.

- Based on the idea that some characters occur more frequently than others.
- Huffman codes are not fixed-width.

The Hawaiian Alphabet

- The Hawaiian alphabet consists of 13 characters.
  - **'** is the okina which sometimes occurs between vowels (e.g. **KAMA'AINA**)
- The table to the right shows each character along with its relative frequency in Hawaiian words.

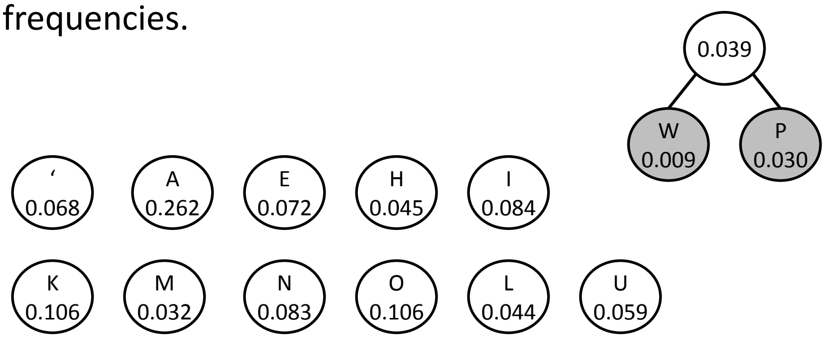
'	0.068
A	0.262
E	0.072
H	0.045
I	0.084
K	0.106
L	0.044
M	0.032
N	0.083
O	0.106
P	0.030
U	0.059
W	0.009

The Huffman Tree

- We use a tree structure to develop the unique binary code for each letter.
- Start with each letter/frequency as its own node:

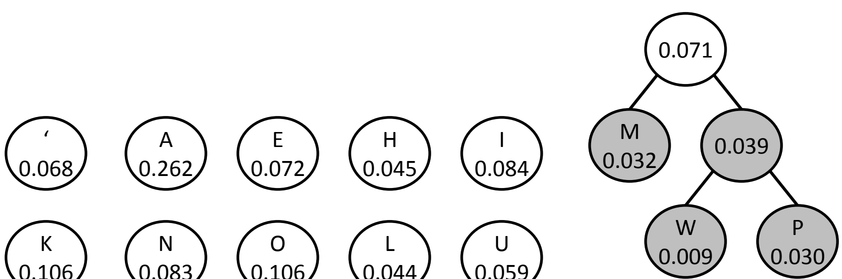
The Huffman Tree

- Combine lowest two frequency nodes into a tree with a new parent with the sum of their frequencies.



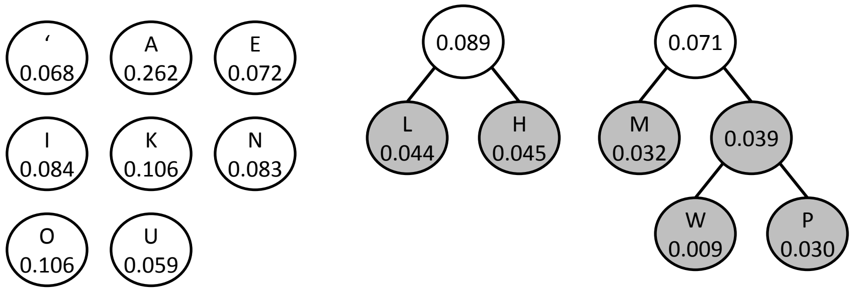
The Huffman Tree

- Combine lowest two frequency nodes (including the new node we just created) into a tree with a new parent with the sum of their frequencies.



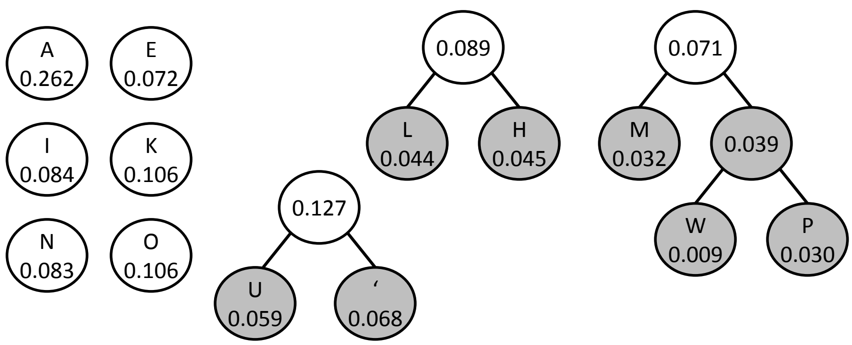
The Huffman Tree

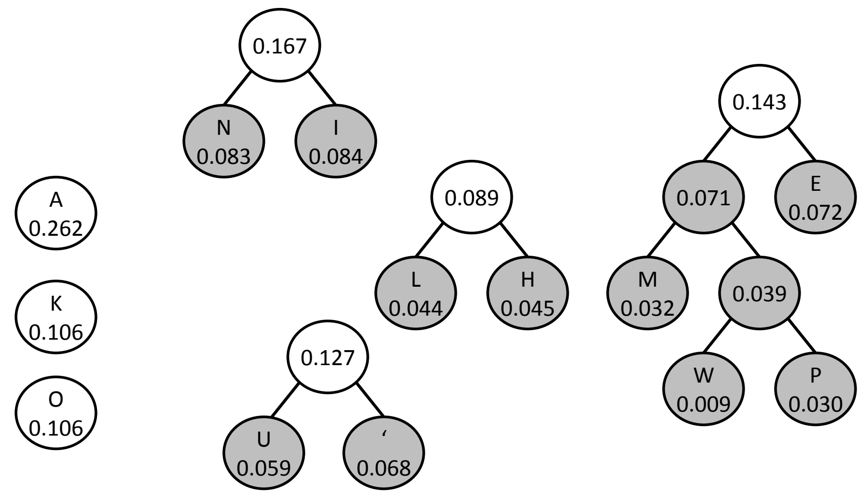
- Combine lowest two frequency nodes (including the new node we just created) into a tree with a new parent with the sum of their frequencies.



The Huffman Tree

- Combine lowest two frequency nodes (including the new node we just created) into a tree with a new parent with the sum of their frequencies...





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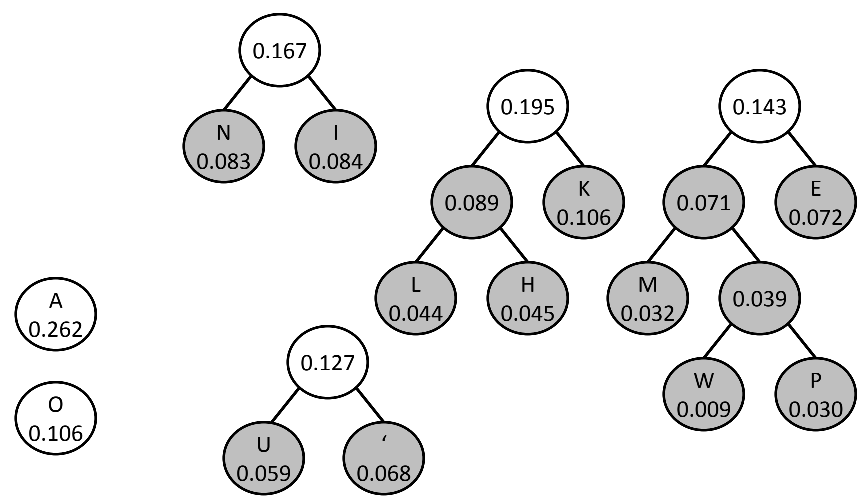
17



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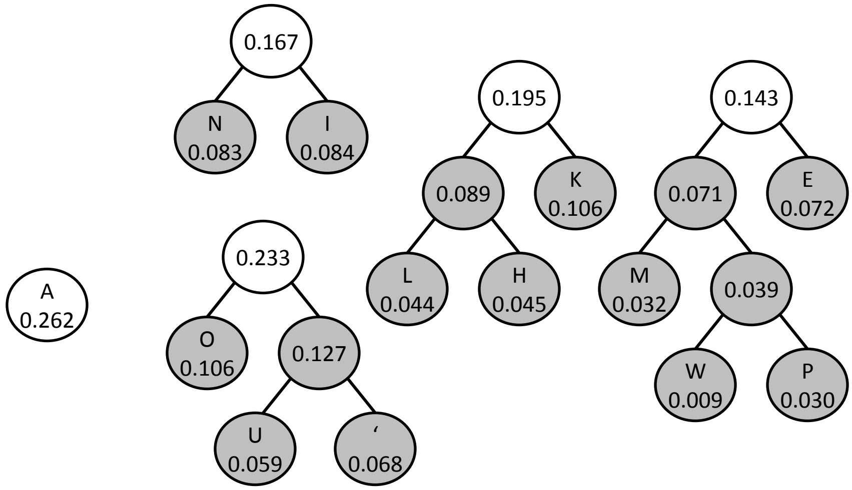
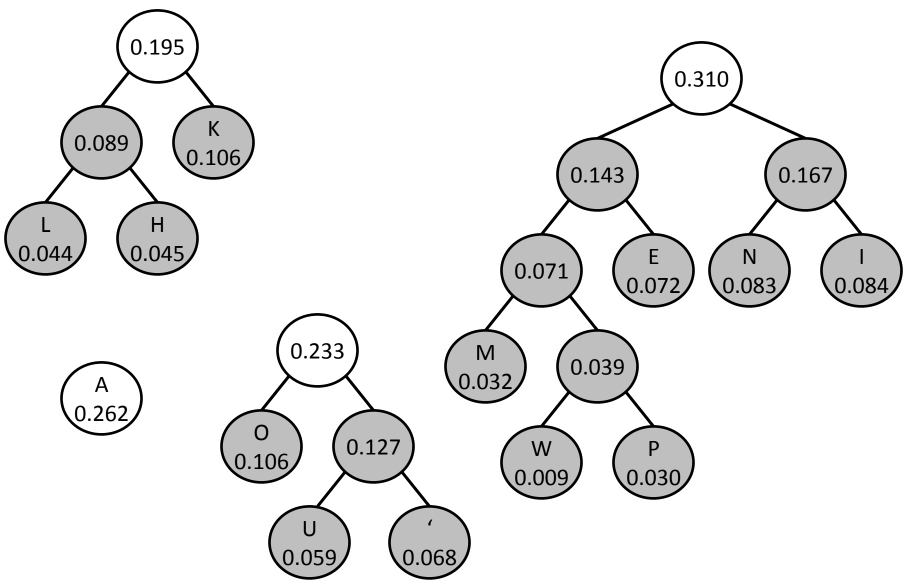
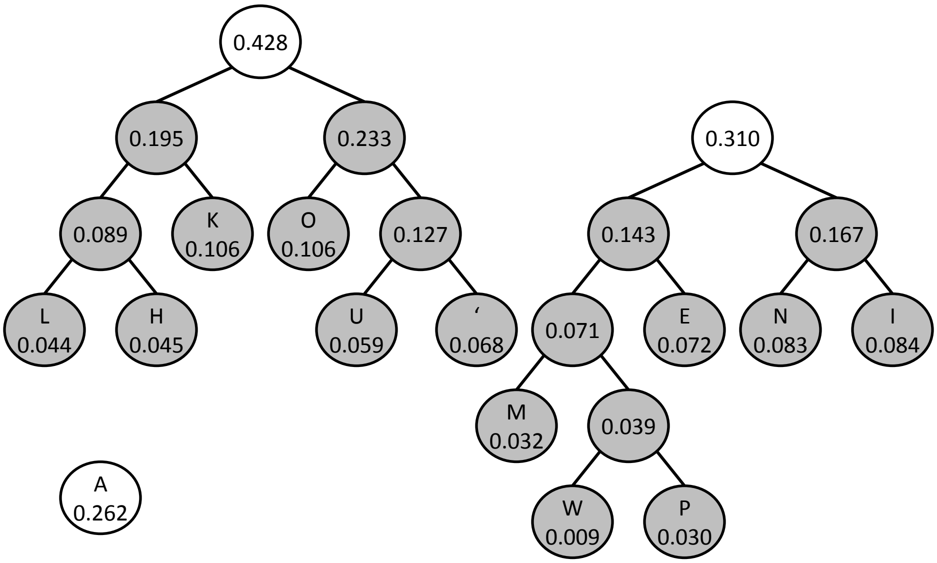
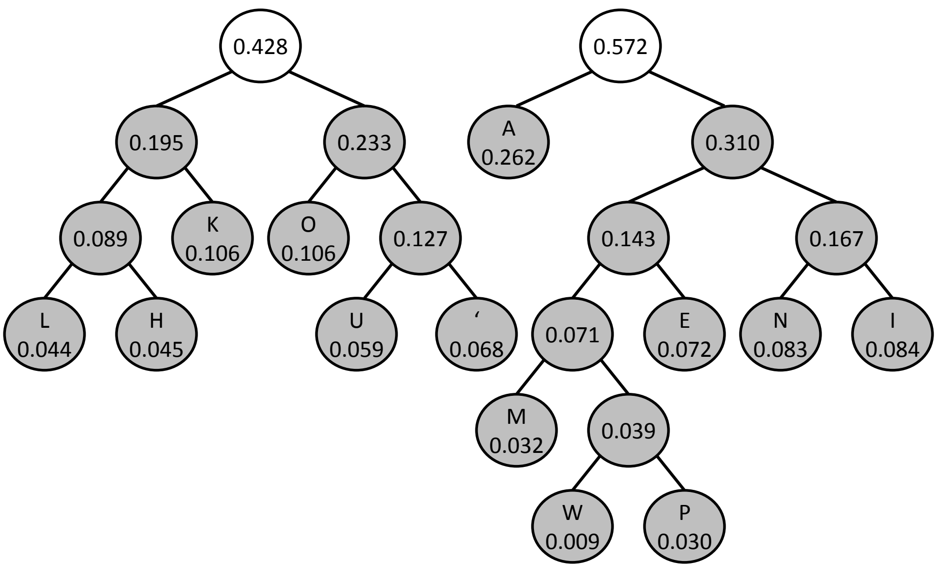
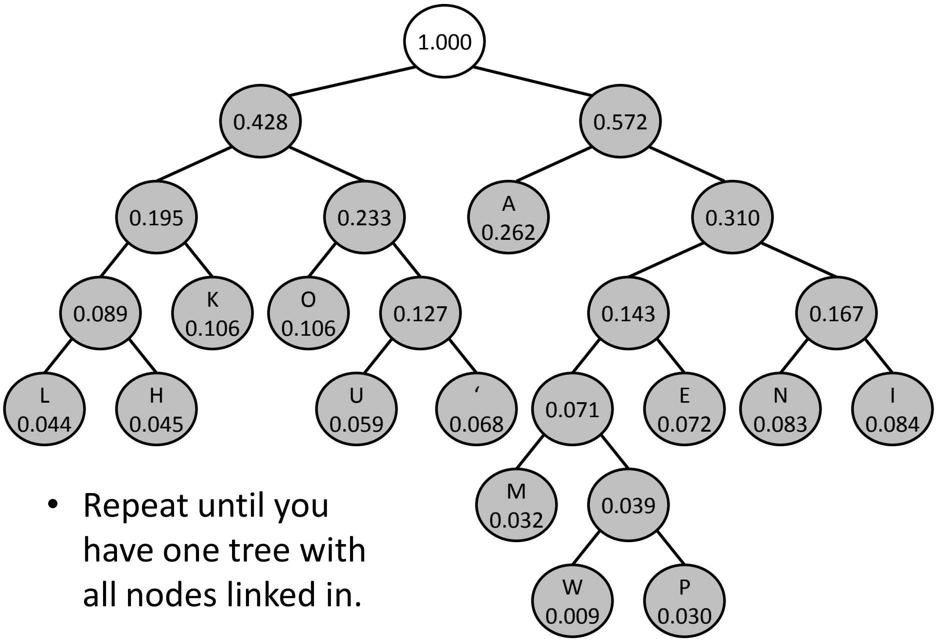
18


0.167
N 0.083
I 0.084
0.195
0.089
K 0.106
L 0.044
H 0.045
0.143
0.071
E 0.072
M 0.032
0.039
W 0.009
P 0.030
A 0.262
0.233
O 0.106
0.127
U 0.059
‘ 0.068

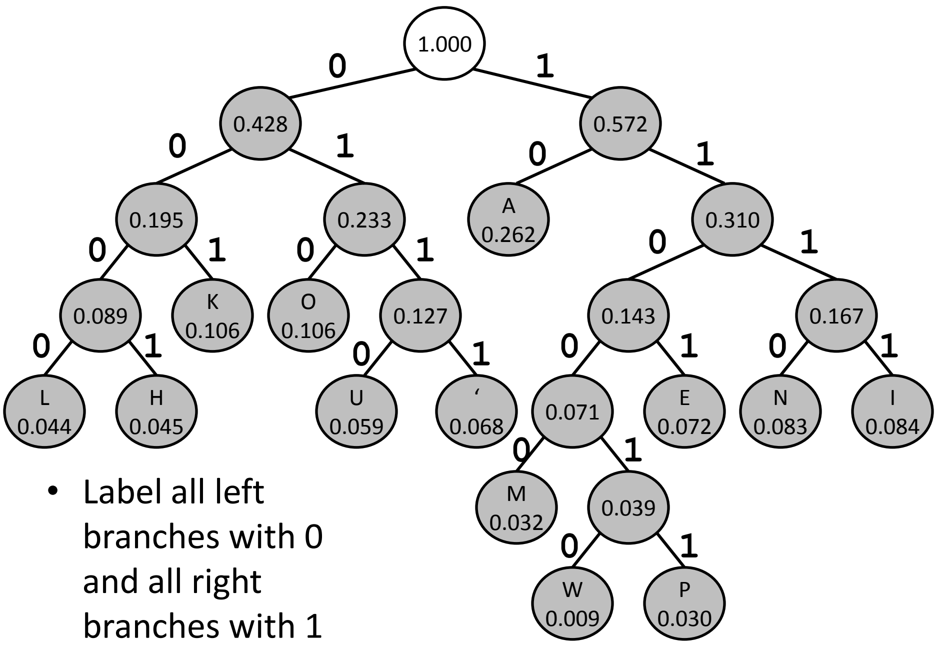

0.195
0.089
K 0.106
L 0.044
H 0.045
0.310
0.143
0.167
0.071
E 0.072
N 0.083
I 0.084
M 0.032
0.039
W 0.009
P 0.030
A 0.262
0.233
O 0.106
0.127
U 0.059
‘ 0.068


0.428
0.195
0.233
0.310
0.089
K 0.106
O 0.106
0.127
0.143
0.167
L 0.044
H 0.045
U 0.059
‘ 0.068
0.071
E 0.072
N 0.083
I 0.084
M 0.032
0.039
A 0.262
W 0.009
P 0.030

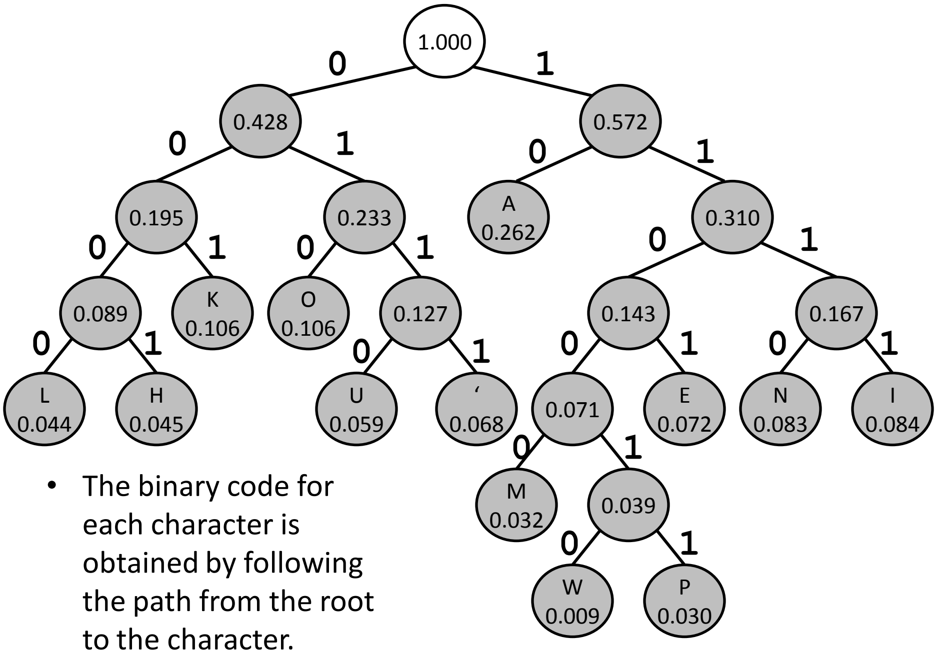

0.428
0.572
0.195
0.233
A 0.262
0.310
0.089
K 0.106
O 0.106
0.127
0.143
0.167
L 0.044
H 0.045
U 0.059
‘ 0.068
0.071
E 0.072
N 0.083
I 0.084
M 0.032
0.039
W 0.009
P 0.030



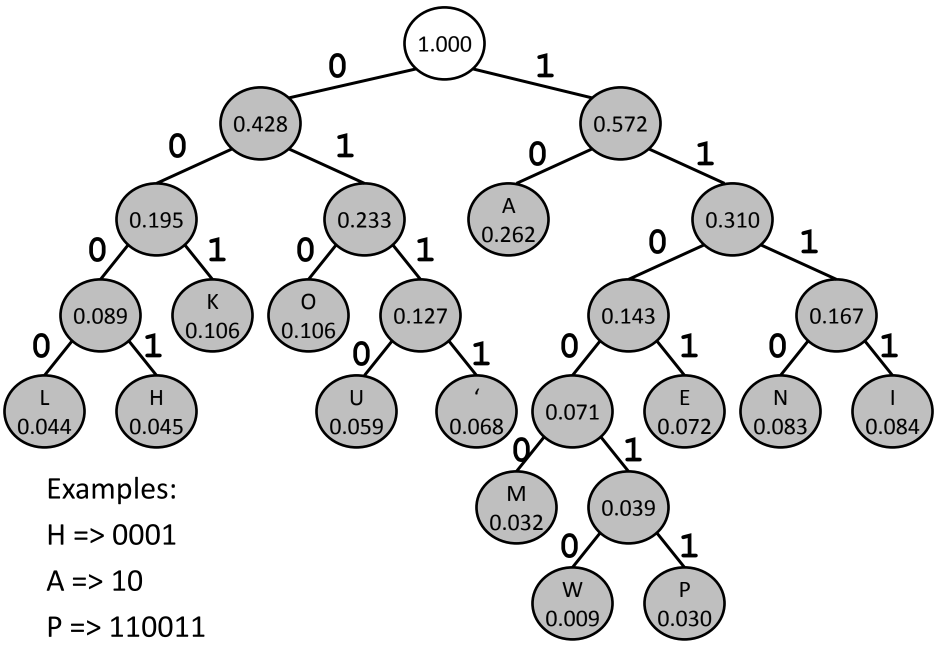
- Repeat until you have one tree with all nodes linked in.



- Label all left branches with 0 and all right branches with 1



- The binary code for each character is obtained by following the path from the root to the character.



Examples:

H => 0001

A => 10

P => 110011

Fixed Width vs. Huffman Coding

'	0000	'	0111
A	0001	A	10
E	0010	E	1101
H	0011	H	0001
I	0100	I	1111
K	0101	K	001
L	0110	L	0000
M	0111	M	11000
N	1000	N	1110
O	1001	O	010
P	1010	P	110011
U	1011	U	0110
W	1100	W	110010

ALOHA

Fixed Width:
0001 0110 1001 0011 0001
20 bits

Huffman Code:
10 0000 010 0001 10
15 bits

Variable Length Codes

- In a fixed-width code, the boundaries between letters are fixed in advance:
  0001 0110 1001 0011 0001
- With a variable-length code, the boundaries are determined by the letters themselves.
  - No letter's code can be a prefix of another letter.
  - Example: since A is "10", no other letter's code can begin with "10". All the remaining codes begin with "00", "01", or "11".

Programming the Huffman Tree

- Let's write Ruby code to produce a Huffman encoding of an alphabet.
- At each step we need to find the two nodes with the lowest frequency scores.
- This will be easy if nodes are kept in a list that is sorted by score value.
- Solution: use a **priority queue**.

Priority Queues

NOTE: For this unit, you will need RubyLabs set up and you will need to include BitLab (see p. 167)

- A priority queue (PQ) is like an array that is sorted.

```
pq = PriorityQueue.new
=> []
```

- To add element into the priority queue in its correct position, we use the **<<** operator:

```
pq << "peach"
pq << "apple"
pq << "banana"
=> ["apple", "banana", "peach"]
```

Priority Queues (cont'd)

- To remove the first element from the priority queue, we will use the **`shift`** method:

```
fruit1 = pq.shift
=> "apple"
pq
=> ["banana", "peach"]
fruit2 = pq.shift
=> "banana"
pq
=> ["peach"]
```


Tree Nodes

- We can store all of the node data into a 2-dimensional array:

```
table = [ ["'", 0.068], ["A", 0.262],
 ["E", 0.072], ["H", 0.045], ["I", 0.084],
 ["K", 0.106], ["L", 0.044], ["M", 0.032],
 ["N", 0.083], ["O", 0.106], ["P", 0.030],
 ["U", 0.059], ["W", 0.009] ]
```

- A tree node consists of two values, the character and its frequency. Making one of the tree nodes:

```
char = table[2].first     # "E"
freq = table[2].last      # 0.072
node = Node.new(char, freq)
```

Building a PQ of Single Nodes

```
def make_pq(table)
    pq = PriorityQueue.new
    for item in table do
        char = item.first
        freq = item.last
        node = Node.new(char, freq)
        pq << node
    end
    return pq
end
```

Remember: each item in the table is a 2-element array with a character and a frequency.

Building our Priority Queue

```
pq = make_pq(table)
=> [( W: 0.009 ), ( P: 0.030 ),
   ( M: 0.032 ), ( L: 0.044 ),
   ( H: 0.045 ), ( U: 0.059 ),
   ( ': 0.068 ), ( E: 0.072 ),
   ( N: 0.083 ), ( I: 0.084 ),
   ( K: 0.106 ), ( O: 0.106 ),
   ( A: 0.262 )]
```

This is our priority queue showing the 13 nodes in sorted order based on frequency.

Building a Huffman Tree

(Slightly different than book version fig 7.9)

```
def build_tree(pq)
   while pq.length > 1
        node1 = pq.shift
        node2 = pq.shift
        pq << Node.combine(node1, node2)
   end
   return pq.first
end
```

Creates a new node with node1 as its left child and node2 as its right child

Building our Huffman Tree

```
tree = build_tree(pq)
=> ( 1.000 ( 0.428 ( 0.195 ( 0.089
  ( L: 0.044 ) ( H: 0.045 ) ) ( K: 0.106 ) )
  ( 0.233 ( O: 0.106 ) ( 0.127 ( U: 0.059 )
  ( ': 0.068 ) ) ) ) ( 0.572 ( A: 0.262 )
  ( 0.310 ( 0.143 ( 0.071 ( M: 0.032 )
  ( 0.039 ( W: 0.009 ) ( P: 0.030 ) ) )
  ( E: 0.072 ) ) ( 0.167 ( N: 0.083 )
  ( I: 0.084 ) ) ) ) )
```

This is just our Huffman tree expressed using recursively nested parenthetical components:
`( root ( left ) ( right ) )`

Assigning Codes, Encoding & Decoding

```
ht = assign_codes(tree)
```

from BitLab
takes a Huffman tree and returns a hash table that maps each letter to its binary code

```
ht["W"]
=> 110010
ht["A"]
=> 10
```

Note the [ ] syntax.
This returns the code associated with the character from the hash table.

```
msg = encode("ALOHA", tree)
=> 100000010000110
decode(msg, tree)
=> "ALOHA"
```

from BitLab
encode and decode functions