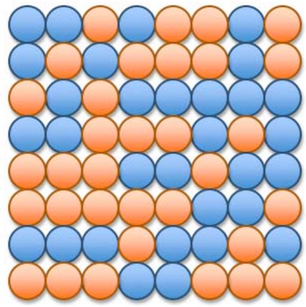


UNIT 4B
Iteration: Sorting

Exponential Notation

Floating point number syntax:

5.716e+4 means $5.716 \times 10^4 = 57160$

exponent marker

decimal point

```
>> 5.716e+4
=> 57160.0
>> 5.716 * 10**15
=> 5.716e+15
```

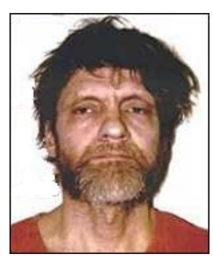
What Does Your Code Say About You?

```
def linsearch  (list ,key)
   len = list.length
   index=0
   while index < len do
      if list[index] == key then
         return index
      end
      index = index + 1
   end
   return nil
end
```

What Does Your Code Say About You?

```
 def linsearch  (list   ,key)
len =list.length
   index=0
          while index<len do
if list[index  ]==key then

    return index
      end
   index= index+1
          end  return nil
end
```



Grading on Code Formatting

- From now on, you will be graded on the appearance of your code.
- Proper indentation, no gratuitous blank lines. (But in long functions, blank lines can be a good way to group code into sections.)
- Why are we doing this?
  – Because we're mean.
  – Because you cannot find the bugs in your code if you cannot read it properly.

Indenting a FOR Loop

for *var* **in** *values* **do**
    loop body stuff
    more loop body stuff
    even mode loop body stuff
end

Indenting a WHILE Loop

while *test* **do**
  loop body stuff
  more loop body stuff
  even mode loop body stuff
end

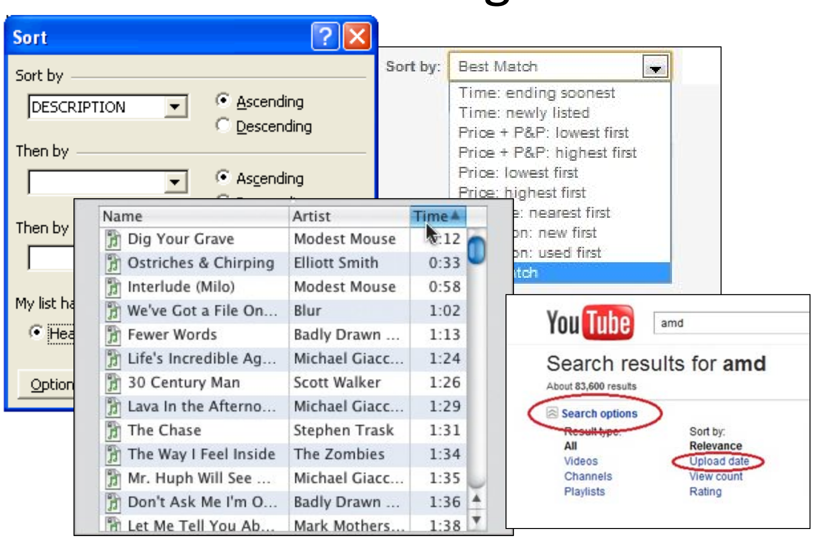
Indenting an IF

if test **then**
  some then stuff
  more then stuff
else
  some else stuff
  more else stuff
end

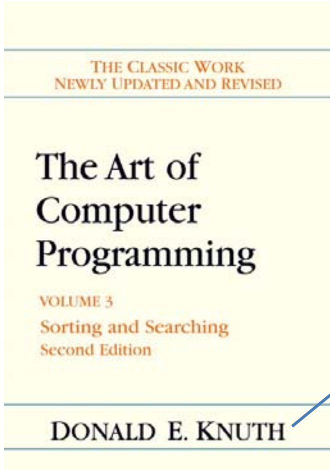
Nesting

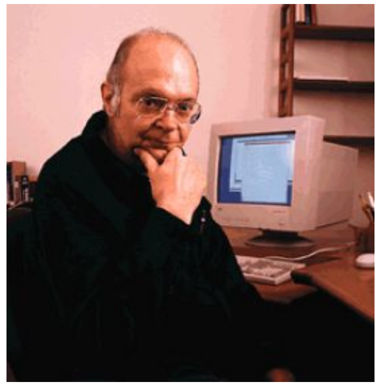
```
x = [3, 13, 5, 25, 4, 64]
for v in x do
  if v < 10 then
    print " ", v
  else
    print v
  end
  print "\n"
end
```


Sorting



The Art of Computer Programming Volume 3: Sorting and Searching





Insertion Sort

Insertion Sort Outline

```
def isort (list)
    result = [ ]
    for val in list do
        # insert val in its proper place in result
    end
    return result
end
```


insert

- list.insert(position, value)

```
>> a = [10, 20, 30]
=> [10, 20, 30]
>> a.insert(0,”foo”)
=> [“foo”, 10, 20, 30]
>> a.insert(2, “bar”)
=> [“foo”, 10, “bar”, 20, 30]
>> a.insert(5, “baz”)
=> [“foo”, 10, “bar”, 20, 30, “baz”]
```

Insertion Sort, Refined

```
def isort (list)
   result = [ ]
   for val in list do
      result.insert(place, val)
   end
   return result
end
```


gindex

```
# index of first element greater than item
def gindex (list, item)
   index = 0
   while index < list.length and
               list[index] < item do
      index = index + 1
   end
   return index
end
```

Testing gindex

```
>> a = [10, 20, 30, 40, 50]
=> [10, 20, 30, 40, 50]
>> gindex(a,3)
=> 0
>> gindex(a,14)
=> 1
>> gindex(a,37)
=> 3
>> gindex(a,99)
=> 5
```


Insertion Sort, Complete

```
def isort (list)
  result = [ ]
  for val in list do
    result.insert(gindex(result,val), val)
  end
  return result
end
```

Instrumenting Insertion Sort

```
def isort (list)
   result = [ ]
   p result     # for debugging
   for val in list do
      result.insert(gindex(result,val), val)
      p result     # for debugging
   end
   return result
end
```


Testing isort

```
>> isort([3, 1, 4, 1, 5, 9, 2, 6])
[]
```

Testing isort

```
>> isort([3, 1, 4, 1, 5, 9, 2, 6])
[]
[3]
```


Testing isort

```
>> isort([3, 1, 4, 1, 5, 9, 2, 6])
[]
[3]
[1, 3]
```

Testing isort

```
>> isort([3, 1, 4, 1, 5, 9, 2, 6])
[]
[3]
[1, 3]
[1, 3, 4]
```


Testing isort

```
>> isort([3, 1, 4, 1, 5, 9, 2, 6])
[]
[3]
[1, 3]
[1, 3, 4]
[1, 1, 3, 4]
```

Testing isort

```
>> isort([3, 1, 4, 1, 5, 9, 2, 6])
[]
[3]
[1, 3]
[1, 3, 4]
[1, 1, 3, 4]
[1, 1, 3, 4, 5]
```


Testing isort

```
>> isort([3, 1, 4, 1, 5, 9, 2, 6])
[]
[3]
[1, 3]
[1, 3, 4]
[1, 1, 3, 4]
[1, 1, 3, 4, 5]
[1, 1, 3, 4, 5, 9]
```

Testing isort

```
>> isort([3, 1, 4, 1, 5, 9, 2, 6])
[]
[3]
[1, 3]
[1, 3, 4]
[1, 1, 3, 4]
[1, 1, 3, 4, 5]
[1, 1, 3, 4, 5, 9]
[1, 1, 2, 3, 4, 5, 9]
```


Testing isort

```
>> isort([3, 1, 4, 1, 5, 9, 2, 6])
[]
[3]
[1, 3]
[1, 3, 4]
[1, 1, 3, 4]
[1, 1, 3, 4, 5]
[1, 1, 3, 4, 5, 9]
[1, 1, 3, 4, 5, 6, 9]
[1, 1, 2, 3, 4, 5, 6, 9]
=> [1, 1, 2, 3, 4, 5, 6, 9]
```

Can We Do Better?

- isort doesn't change its input list.
- Instead it makes a new list, called result.
- This takes twice as much memory.

- Can we write a destructive version of the algorithm that doesn't use extra memory?
- That is the version shown in the book (see chapter 4).

Destructive Insertion Sort

Given an array a of length n, n > 0.

1. Set i = 1.
2. While i is not equal to n, do the following:

   a. Insert a[i] into its correct position in a[0..i], shifting the other elements as necessary.

   b. Add 1 to i.
3. Return the array a which will now be sorted.

Example

```
a = [53, 26, 76, 30, 14, 91, 68, 42]
i =       1
```

Insert a[1] into its correct position in a[0..1]
and then add 1 to i:

53 moves to the right,

26 is inserted back into the array

```
a = [26, 53, 76, 30, 14, 91, 68, 42]
i =           2
```


Example

```
a = [26, 53, 76, 30, 14, 91, 68, 42]
i =           2
```

Insert a[2] into its correct position in a[0..2]
and then add 1 to i:

76 is already in the correct place in a[0..2]

```
a = [26, 53, 76, 30, 14, 91, 68, 42]
i =               3
```

Example

```
a = [26, 53, 76, 30, 14, 91, 68, 42]
i =           3
```

Insert a[3] into its correct position in a[0..3]
and then add 1 to i:

76 moves to the right, then 53 moves to the right,

now 30 is inserted back into the array

```
a = [26, 30, 53, 76, 14, 91, 68, 42]
i =               4
```


Look Closer at Insertion Sort

Given an array a of length n, n > 0.

1. Set i = 1.
2. While i is not equal to n, do the following:

Precondition for each iteration: a[0..i-1] is sorted

a. Insert a[i] into its correct position in a[0..i].
Now a[0..i] is sorted.

b. Add 1 to i.

Postcondition for each iteration: a[0..i-1] is sorted

3. Return the array a which will now be sorted.

Look Closer at Insertion Sort

Given an array a of length n, n > 0.

1. Set i = 1.
2. While i is not equal to n, do the following:

   Loop invariant: a[0..i-1] is sorted

   a. Insert a[i] into its correct position in a[0..i].

   b. Add 1 to i.
3. Return the array a which will now be sorted.

A <u>loop invariant</u> is a condition that is true at the start and end of each iteration of a loop.

Example (cont'd)

```
a = [26, 30, 53, 76, 14, 91, 68, 42]
i =                  4
```

Insert a[4] into its correct position in a[0..4] and then add 1 to i:

76 moves to the right, then 53 moves to the right, then 30 moves to the right, then 26 moves to the right, now 14 is inserted back into the array

```
a = [14, 26, 30, 53, 76, 91, 68, 42]
i =                      5
```

Example

```
a = [14, 26, 30, 53, 76, 91, 68, 42]
i =                      5
```

Insert a[5] into its correct position in a[0..5]
and then add 1 to i:

91 is already in its correct position

```
a = [14, 26, 30, 53, 76, 91, 68, 42]
i =                          6
```


Example

```
a = [14, 26, 30, 53, 76, 91, 68, 42]
i =                          6
```

Insert a[6] into its correct position in a[0..6]
and then add 1 to i:

91 moves to the right,

76 moves to the right,

now 68 is inserted back into the array

```
a = [14, 26, 30, 53, 68, 76, 91, 42]
i =                              7
```

Example

a = [14, 26, 30, 53, 68, 76, 91, 42]
i = 7

Insert a[7] into its correct position in a[0..7] and then add 1 to i:

91 moves to the right, then 76 moves to the right,

then 68 moves to the right, then 53 moves to the right,

then 42 is inserted back into the array

a = [14, 26, 30, 42, 53, 68, 76, 91]
i = 8

Example

a = [14, 26, 30, 42, 53, 68, 76, 91]
i = 8

The array is sorted.

But how do we know that the algorithm always sorts correctly?

Reasoning with the Loop Invariant

The loop invariant is true at the end of each iteration, including the last iteration. After the last iteration, when we go to step 3:

a[0..i-1] is sorted AND i is equal to n

These 2 conditions imply that a[0..n-1] is sorted, but this range covers the entire array, so the array must always be sorted when we return our final answer!

Insertion Sort in Ruby

```
def isort!(list)
     i = 1
     while i != list.length do
          move_left(list, i)
          i = i + 1          <----
     end
     return list
end
```

insert a[i] into a[0..i] in its correct sorted position

Moving left

To move the element x at index i "left" to its correct position, start at position i-1, and search left until we find the first element that is less than x.

Then insert x back into the array to the right of the first element that is less than x when you searched from right to left in the sorted part of the array.

(The insert operation does not overwrite. Think of it as "squeezing into the array".)

Can you think of a special case for the step above?

Moving left: examples

Insert 68:

a = [14, 26, 30, 53, 76, 91, 68, 42]

Searching from right to left starting with 91, the first element less than 68 is 53.

Insert 68 to the right of 53.

Insert 76:

a = [26, 53, 76, 30, 14, 91, 68, 42]

Searching from right to left starting with 53, the first element less than 76 is 53. Insert 76 to the right of 53 (where it was before).

Insert 14: SPECIAL CASE

a = [26, 30, 53, 76, 14, 91, 68, 42]

Searching from right to left starting with 76, all elements left of 14 are greater than 14. Insert 14 into the position 0.

The **move_left** algorithm

Given an array a of length n, n > 0 and a value at index i to be "moved left" in the array.

1. Remove a[i] from the array and store in x.
2. Set j = i-1.
3. While j >= 0 and a[j] > x, do the following:
   a. Subtract 1 from j.
4. Reinsert x into position a[j+1].

How is the special case handled here?

move_left in Ruby

```
def move_left(a, i)
    x = a.slice!(i)
    j = i-1
    while j >= 0 and a[j] > x do
        j = j - 1
    end
    a.insert(j+1, x)
end
```

remove the item at position i in array a and store it in x

logical operator AND: both conditions must be true for the loop to continue

insert x at position j+1 of array a, shifting all elements from j+1 and beyond over one position