10-751 Assignment 2

Due: Thursday September 26

Problem 1.

1. Show that conditional independence of A and B given C is neither implied nor implied by the independence of A and B.
2. For which events C *is* it the case that for all A and B, the events A and B are independent if and only if they are conditionally independent given C?

Problem 2. Starting from a well-shuffled deck of 52 playing cards containing four aces, cards are turned up one-by-one off the top until the first ace appears. On average, how many cards are required to be taken to produce the first ace?

Problem 3. A random number N of dice is thrown. Let A_i be the event that $N = i$ and assume that $\Pr(A_i) = 2^{-i},\ i \geq 1$. Let S be the sum of the number of dots that appear on each of the N throws. Find the probability that

1. $N = 2$ given that $S = 4$;
2. $S = 4$ given that N is even;
3. $N = 2$, given that $S = 4$ and the first die showed 1;
4. the largest number shown by any die is r, where S is unknown

Problem 4. A bowl contains twenty cherries, exactly fifteen of which have had their pits removed. A man eats five cherries, picked at random, without noticing whether or not they contain pits. Subsequently, a cherry is picked randomly from the remaining fifteen.

1. What is the probability that this cherry contains a pit?
2. Given that this cherry contains a pit, what is the probability that the man consumed at least one pit?

Problem 5. Using Matlab, implement Adam Kalai's algorithm to generate factored random numbers (easily): `http://www.mit.edu/~akalai`. Using this implementation, generate a bunch of random factored numbers $1 \leq r \leq n$ (for a sufficiently large n of your choice), and make a histogram showing the empirical probability of each of the small primes (say, $p \leq 31$) dividing a random integer at least once.