10-751 Assignment 1

Due: Tuesday September 17

Problem 1. If the probability that student A will fail a certain computer science exam is 0.5, the probability that student B will fail the exam is 0.2, and the probability that both A and B will fail the exam is 0.1, what is the probability that at least one of these two students will fail the exam? (DeGroot 1.5.1.)

Problem 2. Prove that for every two events A and B, the expression

$$\Pr(A) + \Pr(B) - 2\Pr(AB)$$

is the probability that exactly one of the two events will occur. (DeGroot 1.5.9.)

Problem 3. Consider an experiment in which a fair coin is tossed once and a balanced die is rolled once.

1. Describe the sample space for this experiment.
2. What is a probability that a head will be obtained on the coin and an odd number will be obtained on the die? (DeGroot 1.6.8.)

Problem 4. Suppose you want to find someone with the same birthday as yours. What is the smallest number of people whose birthdays you need to ask about in order to have at least a 50% chance of finding someone with the same birthday? (This summer, I met identical twins having the same birthday as me!)

Problem 5. This problem will involve some computing using Matlab. (To get acquainted with the basics of Matlab, please read David Touretzky's tutorial, or one of the other tutorials on the Web.)

Write a Matlab program to generate n pairs of random integers, between 2 and 1000. Then, find the percentage that have greatest common divisor (gcd) $1, 2, 3, \ldots$, and make a histogram of the results for different values of n. Can you come up with any analytical explanation for the results you see? The Matlab functions `gcd`, `histc`, and `bar` might be useful for this problem.

Pass in plots of your results, your Matlab program, and your analysis of the probability that explains the curve.