

Model Checking Assignment 2

Exercise 1. The “weak until” operator **W** has similar semantics to the “strong until” operator **U** except that $g_1 \mathbf{W} g_2$ doesn’t require g_2 to ever hold true if g_1 is globally true:

$$\pi \models g_1 \mathbf{W} g_2 \quad \Leftrightarrow \quad \text{for all } j \geq 0, \text{ if } \pi^j \models \neg g_1 \text{ then there exists a } k \leq j \text{ such } \pi^k \models g_2$$

Show that **W** can be expressed in terms of the existing temporal operators:

- a. Express $\pi \models g_1 \mathbf{W} g_2$ in terms of the existing temporal operators (**F**, **G**, **U**). You may check your answer by consulting the Wikipedia page on Linear Temporal Logic.
- b. Prove your expression in part (a) means the same as $\pi \models g_1 \mathbf{W} g_2$ provided $\pi \models \mathbf{F} g_2$.
- c. Prove your expression in part (a) means the same as $\pi \models g_1 \mathbf{W} g_2$ provided $\pi \models \neg \mathbf{F} g_2$.

Exercise 2. Write an expression equivalent to $\mathbf{AG} \mathbf{EF} p$ without using any of the operators $\{\mathbf{A}, \mathbf{F}, \mathbf{G}\}$. (Hint: use the identities on Slide 13 of lecture2.pdf.)

Exercise 3. Prove that $\mathbf{A} \mathbf{G} \mathbf{F} p$ is equivalent to $\mathbf{AG} \mathbf{AF} p$.

Exercise 4. Show that $\mathbf{A}[f \mathbf{U} g] \equiv \neg \mathbf{E}[\neg g \mathbf{U} \neg f \wedge \neg g] \wedge \neg \mathbf{EG} \neg g$

Hints:

- a. Note that $\neg \mathbf{E} x = \mathbf{A} \neg x$ and $\mathbf{A}(x \wedge y) = (\mathbf{A} x) \wedge (\mathbf{A} y)$.
- b. Rewrite $[f \mathbf{U} g]$ as $[(f \mathbf{W} g) \wedge \mathbf{F} g]$.
- c. Show that $(\pi \models \neg(f \mathbf{W} g)) \equiv (\pi \models \neg g \mathbf{U} \neg f \wedge \neg g)$.