-FFT Recap (or, what am I expected to know?)
- Learning Finite State Environments

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FFT Recap

The basic result: Given vectors
- $A = (a_0, a_1, a_2, \ldots, a_{n-1})$, and
- $B = (b_0, b_1, \ldots, b_{n-1})$,

the FFT allows us in O(n log n) time to compute the convolution
- $C = (c_0, c_1, \ldots, c_{2n-2})$

where $c_j = a_0b_j + a_1b_{j-1} + \ldots + a_jb_0$.

I.e., this is polynomial multiplication, where A,B,C are vectors of coefficients.

How does it work?

Compute $F^{-1}(F(A)\cdot F(B))$, where "F" is FFT.

- F(A) is evaluation of A at $1,\omega,\omega^2,\ldots,\omega^{m-1}$
  - ω is principal m^{th} root of unity. m = 2n-1. E.g., $\omega = e^{2\pi i/m}$. Or use modular arithmetic.
  - Able to do this quickly with divide-and-conquer.
- F(A)·F(B) gives C(x) at these points.
- We then saw that $F^{-1} = (1/m)F'$, where F' is same as F but using ω^{-1}.

Applications (not on test)

- signal analysis, lots more
- Pattern matching with don't cares:
  - Given text string $X = x_1,x_2,\ldots,x_n$. $x_i \in \{0..25\}$
  - Given pattern $Y = y_1,y_2,\ldots,y_k$. $y_i \in \{0..25\} \cup \{*\}$.
  - Want to find instances of Y inside X.
- Idea [Adam Kalai based on Karp-Rabin]:
  - Pick random R: $r_1,r_2,\ldots,r_k$, $r_i \in 1..N$. E.g, $N=n^2$
  - Set $r_i = 0$ if $y_{k-i+1} = *$.
  - Let $T = r_1y_k + \ldots + r_ky_1$. (can do mod $p > N$)
  - Now do convolution of R and X. See if any entries match T. Each entry at most 1/N chance of false positive.

OK, on to machine learning...