McEliece verification

Daniel J. Bernstein

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- 1 key + 10^6 ciphertexts for McEliece is several times less network traffic than 1 key + 10^6 ciphertexts for lattices.
- McEliece deployment is underway: e.g., McEliece is already used in some end-to-end secure-messaging systems and the Mullvad and Rosenpass VPNs.

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New: CryptAttackTester includes full attack circuits + analyses passing systematic tests.

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Observed speeds match algorithm analyses. Security levels are remarkably stable.

Classic McEliece implementations

Official software for Classic McEliece is distributed via SUPERCOP benchmarking framework. Four implementations for each parameter set, all passing TIMECOP:

- `ref`: portable, prioritizing simplicity.
- `vec`: portable, 64-bit vectorization.
- `sse`: Intel/AMD, 128-bit vectorization.
- `avx`: Intel/AMD, 256-bit vectorization.

Unofficial implementations: M4, FPGAs, McTiny, McOutsourcing, Bouncy Castle (Java and C#), Rust. Integrations: PQClean, liboqs, Node.js. New: Easy-to-use libmceliece.

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Plus: Put everything together into "keygen, enc, dec always work". Automate the entire process to handle many implementations.

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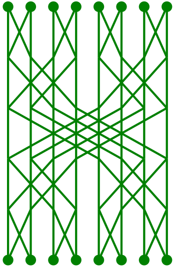
`sorting.cr.yp.to` includes fast constant-time *N*-input sorting built from min/max ("sorting networks") for `int32`; automated verif with `angr` + DAG analysis.
Classic McEliece also uses `int16`, `int64`.

Verified formulas for control bits

Can permute 8192 items in constant time via sorting. Simpler, faster: “Control bits” specify



- swap 0 with 1? swap 2 with 3? etc.;
- swap 0 with 2? swap 1 with 3? etc.;
- swap 0 with 4? swap 1 with 5? etc.;
- and so on: $1, 2, 4, 8, \ldots, 8, 4, 2, 1$.

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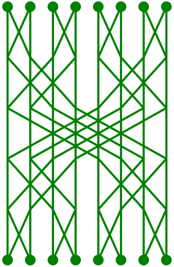

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cr.yp.to/papers.html#controlbits
presents a proof of fast formulas mapping any given permutation *to* control bits.
Proof is computer-verified using HOL Light.

Verified formulas for decoding

`mceliece8192128` secrets: deg-128 irred poly $g \in \mathbb{F}_{8192}[x]$; distinct $s_0, \ldots, s_{8191} \in \mathbb{F}_{8192}$.

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`cr.yp.to/papers.html#goppadecoding`: minicourse on decoding formulas used in the Classic McEliece software. New: Proofs are computer-verified in HOL Light and Lean.

The end is in sight

What I'm working on: More code-analysis tools, automatically matching up stages in the Classic McEliece keygen/enc/dec specification to segments of machine code.

HOL Light already includes a model of basic machine instructions; `angr` already includes a model of instructions through AVX2.

Binary-field mult is challenging to optimize, but the optimized code is easy to verify: simply trace bilinear operations on bits.