Does cryptographic software work correctly?

1. The scale of the problem

Daniel J. Bernstein

University of Illinois at Chicago; Ruhr University Bochum

CVE-2018-0733, an OpenSSL bug

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— Yes, 2^{16} is “lower than” 2^{128}.

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Don't care about PA-RISC? How about Intel?

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“Attacks against DH1024 are considered just feasible”
— How much time? How much hardware?

CVE-2017-3738, continued

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2019.12: Similar OpenSSL advisory for CVE-2019-1551.

Part of the CVE-2017-3738 patch

```
@@ -1093,7 +1093,9 @@
  vmovdqu  -8+32*2-128($ap),$TEMP2

  mov      $r1, %rax
+ vpblendd \$0xfc, $ZERO, $ACC9, $ACC9 # correct $ACC3
  imull    $n0, %eax
+ vpaddq   $ACC9,$ACC4,$ACC4           # correct $ACC3
  and      \$0x1fffffff, %eax

  imulq    16-128($ap),%rbx
@@ -1329,15 +1331,12 @@
```

2019.09: bug announced in Falcon software

“The consequences of these bugs are the following:

- Produced signatures were valid but **leaked information on the private key**. [emphasis added]
- Performance was artificially inflated: . . .

The fact that these bugs existed in the first place shows that the traditional development methodology (i.e. ‘being super careful’) has failed.”

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2020.07: NIST post-quantum competition announces Dilithium and Falcon as the two lattice-based signature-system finalists.

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Cryptography is applied to large volumes of data.
Often individual cryptographic computations are time-consuming.
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cryptographic code optimized in many ways for particular CPUs.

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e.g. Keccak Code Package: >20 implementations of SHA-3.
e.g. Google added hand-written Cortex-A7 asm to Linux kernel for Speck128/128-XTS, then switched to (faster) Adiantum-XChaCha.

Is open-source software bug-free?

Eric S. Raymond, 1999: “Given a large enough beta-tester and co-developer base, almost every problem will be characterized quickly and the fix obvious to someone. Or, less formally, ‘Given enough eyeballs, all bugs are shallow.’ ”

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— “Beta-tester”: Ultimately, the unhappy user?

— “Almost every problem”: That’s not “all bugs”!
Don’t we care about the exceptions, the bugs not found quickly?
Rare bugs can be devastating, especially for security!

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— ESR advocates a development methodology
that releases a constant flood of new bugs.
Doesn't this make his "law" automatically true?
Is this the correctness metric that users want?

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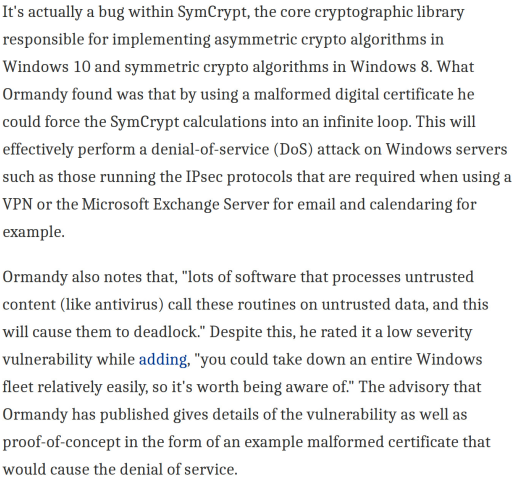
“Closed source scares away some lazy academics, so we have fewer public bug announcements to deal with.”
— Sounds plausible, but is the delay worthwhile?
e.g. Infineon deployed RSALib very widely before its keygen was broken by 2017 Nemec–Sys–Svenda–Klinec–Matyas “ROCA”.

Forbes Billionaires Innovation Leadership Money Business Small Business Lifes

210,878 views | Jun 12, 2019, 08:10am

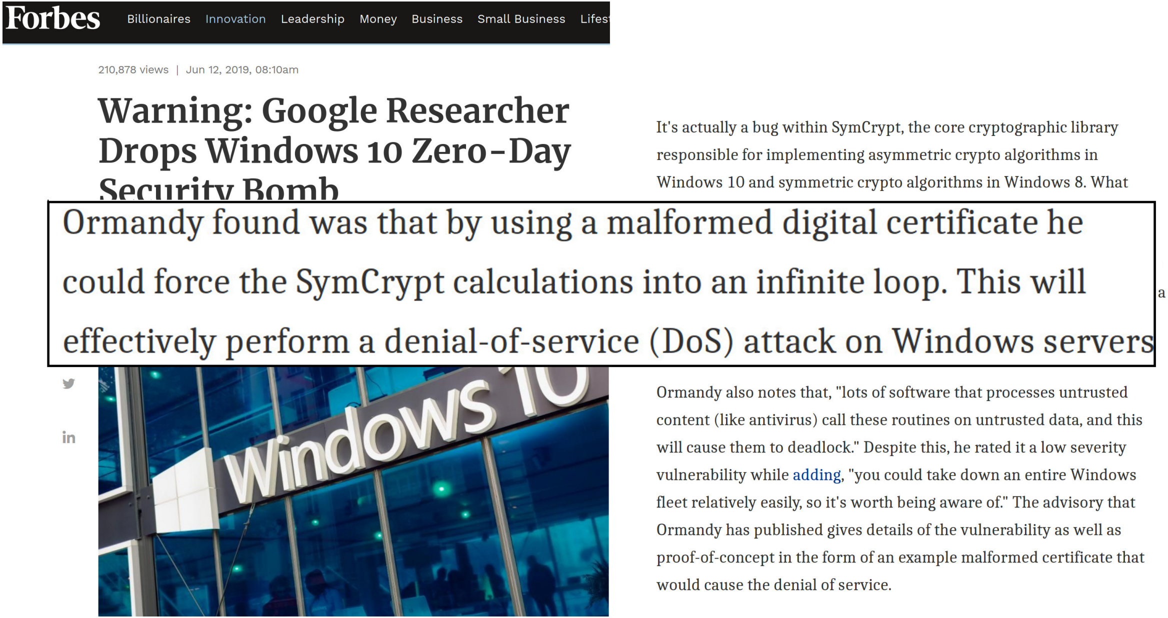
Warning: Google Researcher Drops Windows 10 Zero-Day Security Bomb

Davey Winder Senior Contributor
Cybersecurity
I report and analyse breaking cybersecurity and privacy stories



It's actually a bug within SymCrypt, the core cryptographic library responsible for implementing asymmetric crypto algorithms in Windows 10 and symmetric crypto algorithms in Windows 8. What Ormandy found was that by using a malformed digital certificate he could force the SymCrypt calculations into an infinite loop. This will effectively perform a denial-of-service (DoS) attack on Windows servers such as those running the IPsec protocols that are required when using a VPN or the Microsoft Exchange Server for email and calendaring for example.

Ormandy also notes that, "lots of software that processes untrusted content (like antivirus) call these routines on untrusted data, and this will cause them to deadlock." Despite this, he rated it a low severity vulnerability while adding, "you could take down an entire Windows fleet relatively easily, so it's worth being aware of." The advisory that Ormandy has published gives details of the vulnerability as well as proof-of-concept in the form of an example malformed certificate that would cause the denial of service.



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2. Computer-verified proofs

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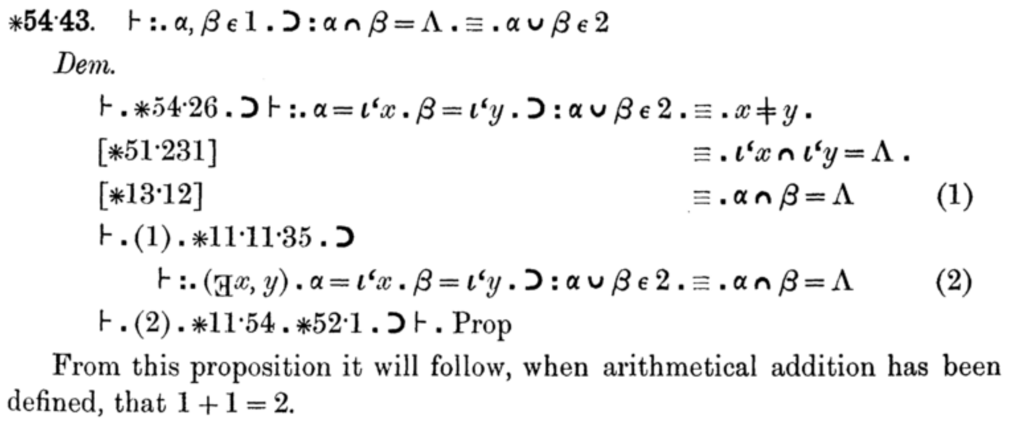
Formal logic to the rescue?

Whitehead and Russell, *Principia Mathematica*, volume 1,
1st edition (1910), page 379:



∗54·43. $\vdash :. \alpha, \beta \in 1 . \supset : \alpha \cap \beta = \Lambda . \equiv . \alpha \cup \beta \in 2$

Dem.

$$\vdash . {*}54{\cdot}26 . \supset \vdash :. \alpha = \iota\text{‘}x . \beta = \iota\text{‘}y . \supset : \alpha \cup \beta \in 2 . \equiv . x \neq y .$$

$$[{*}51{\cdot}231] \qquad \equiv . \iota\text{‘}x \cap \iota\text{‘}y = \Lambda .$$

$$[{*}13{\cdot}12] \qquad \equiv . \alpha \cap \beta = \Lambda \qquad (1)$$

$$\vdash . (1) . {*}11{\cdot}11{\cdot}35 . \supset$$

$$\vdash :. (\exists x, y) . \alpha = \iota\text{‘}x . \beta = \iota\text{‘}y . \supset : \alpha \cup \beta \in 2 . \equiv . \alpha \cap \beta = \Lambda \qquad (2)$$

$$\vdash . (2) . {*}11{\cdot}54 . {*}52{\cdot}1 . \supset \vdash . \text{Prop}$$

From this proposition it will follow, when arithmetical addition has been defined, that $1 + 1 = 2$.

Formal verification today

Require code reviewer to *prove* correctness.

Require proofs to pass a proof-checking computer program.

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Latest EverCrypt release: verified software for Curve25519, Ed25519, ChaCha20, Poly1305, AES-CTR (if CPU has AES-NI), AES-GCM (same), MD5, SHA-1, SHA-2, SHA-3, BLAKE2.

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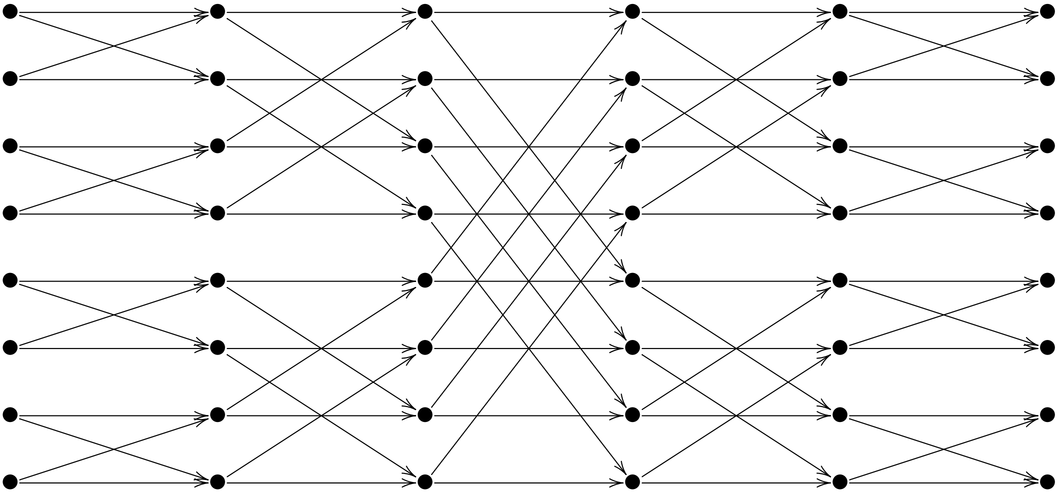
Bad: Tons of effort for each implementation.
e.g. EverCrypt doesn't have fast software for smartphone CPUs.

Case study: Beneš networks



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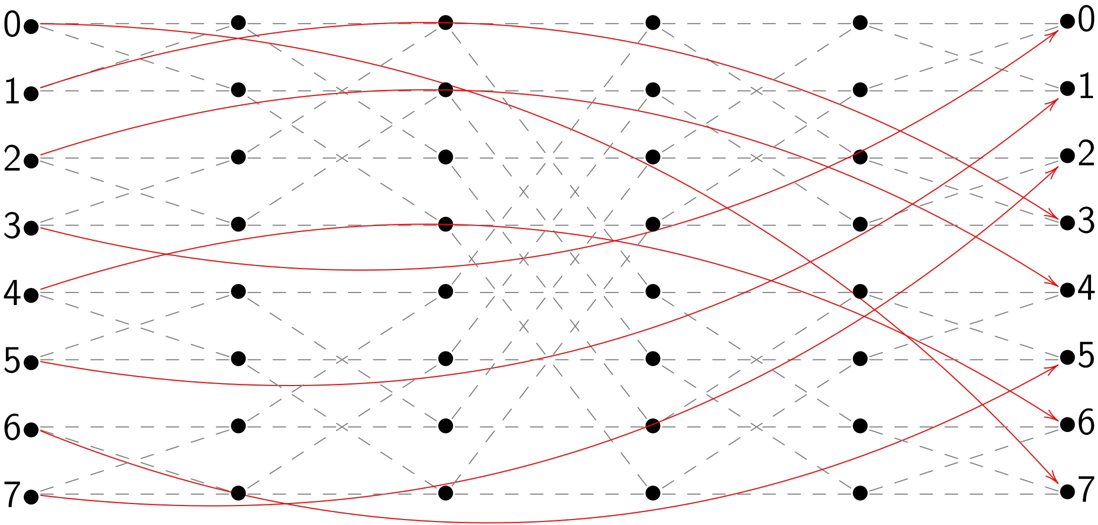
Post-quantum crypto (e.g., Classic McEliece) uses fast constant-time software to compute and apply control bits.
Is this software always computing the right control bits?

Stone's algorithm



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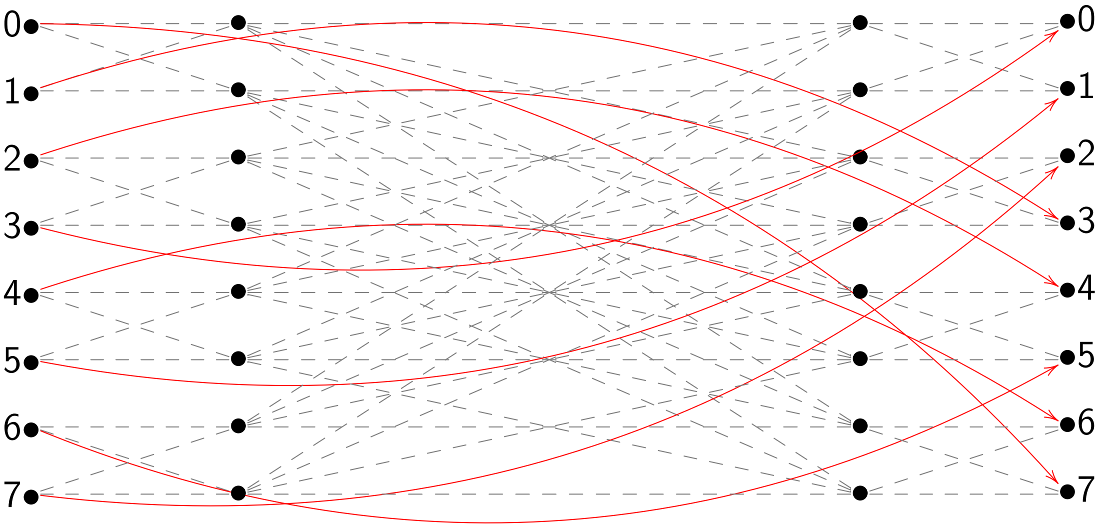

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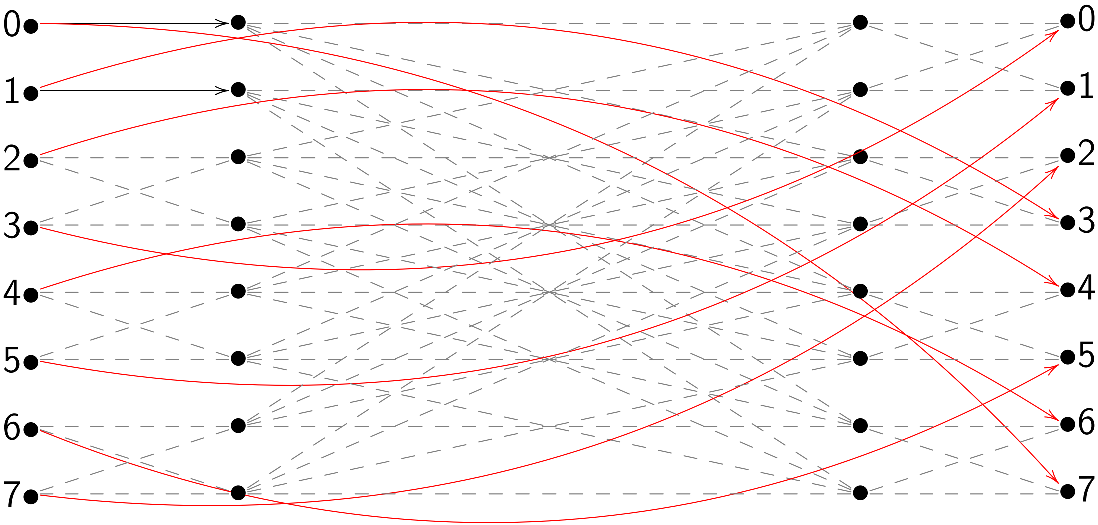

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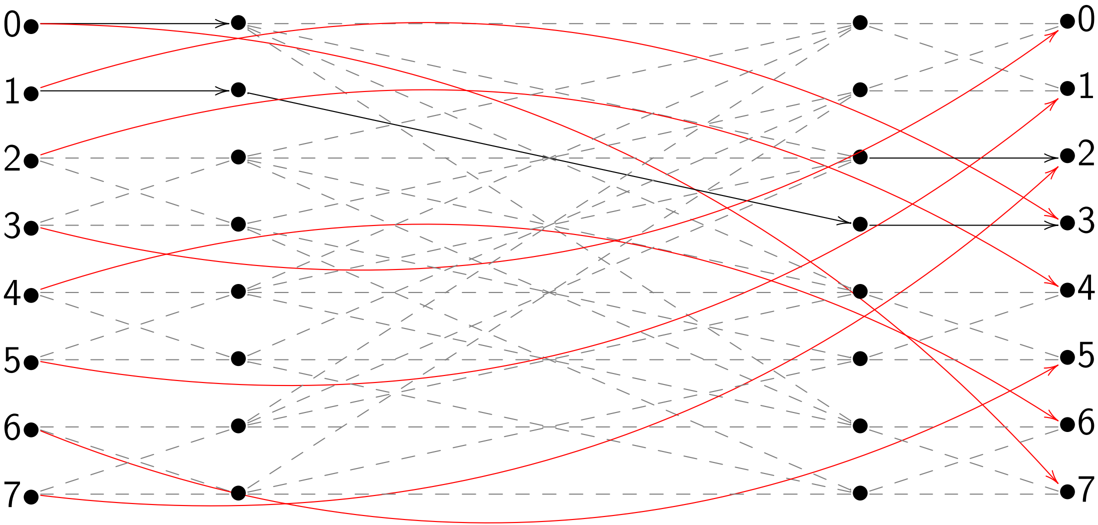

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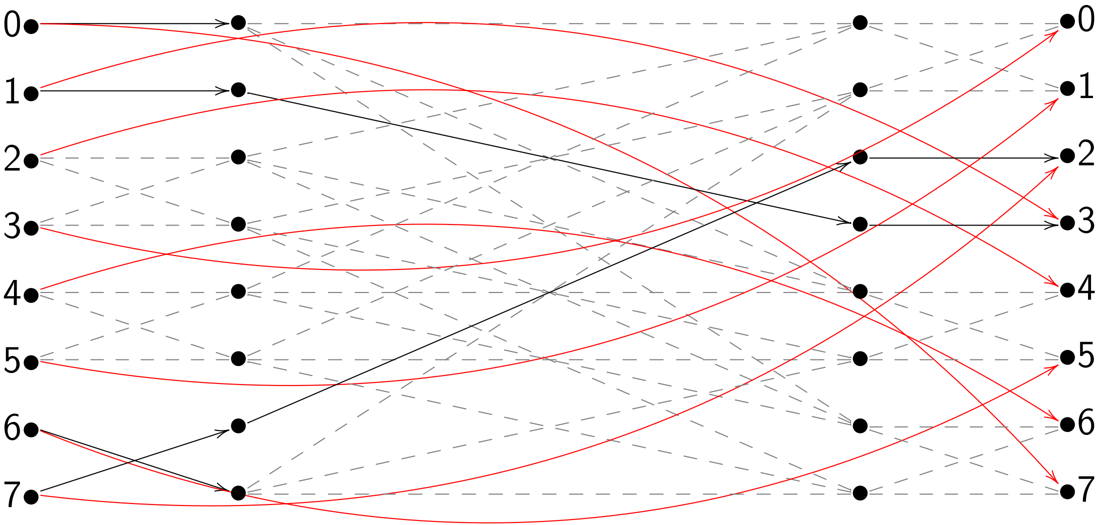

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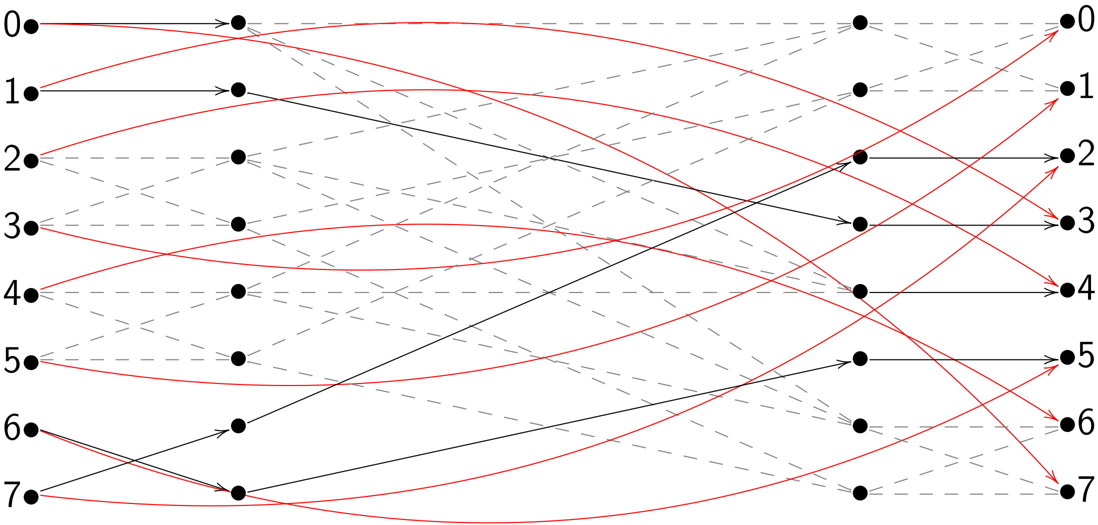

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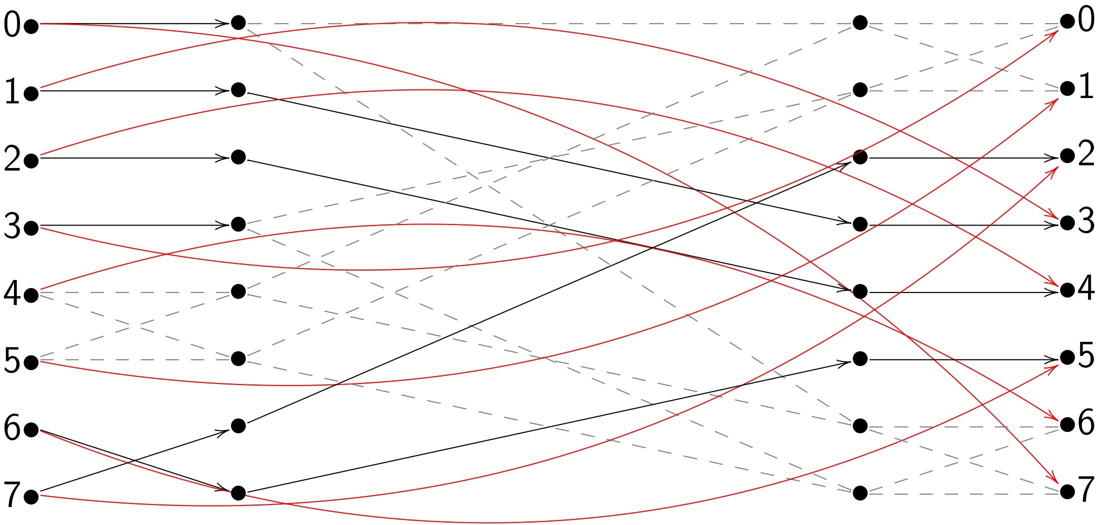

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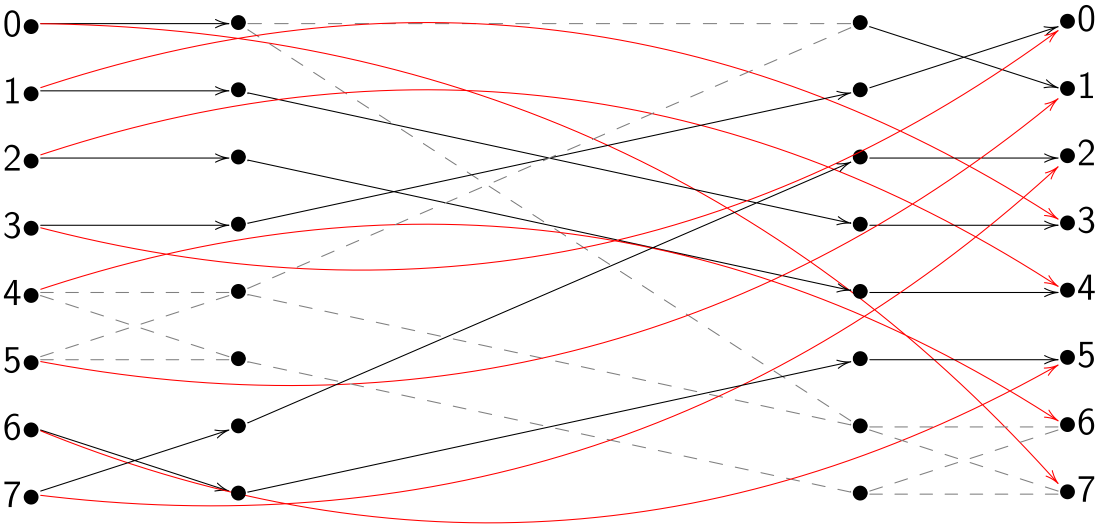

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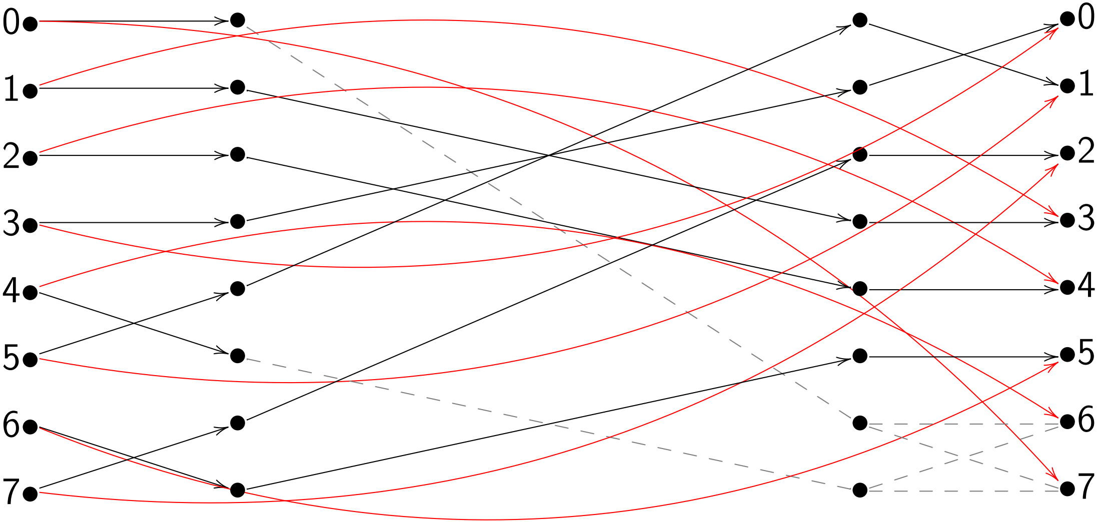

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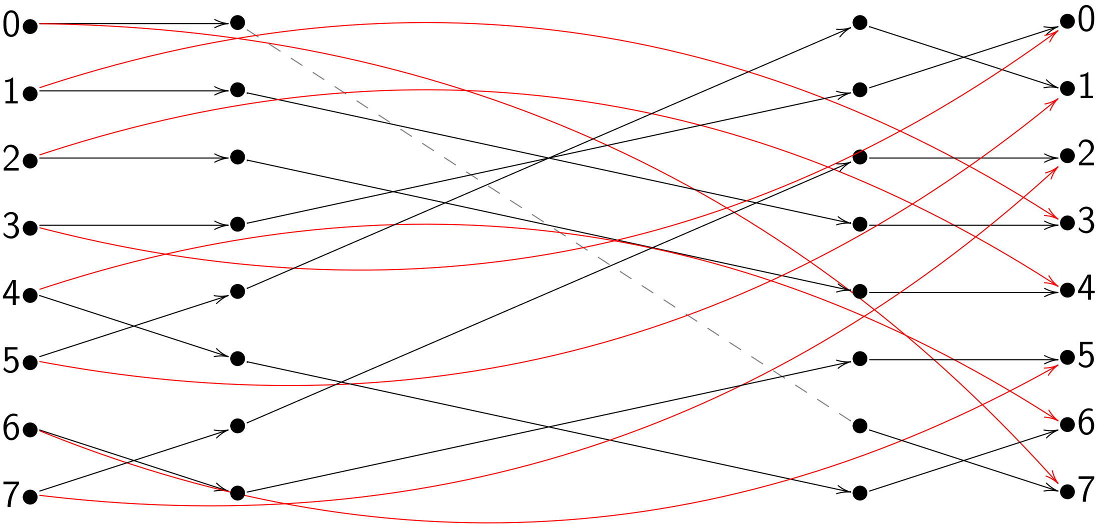

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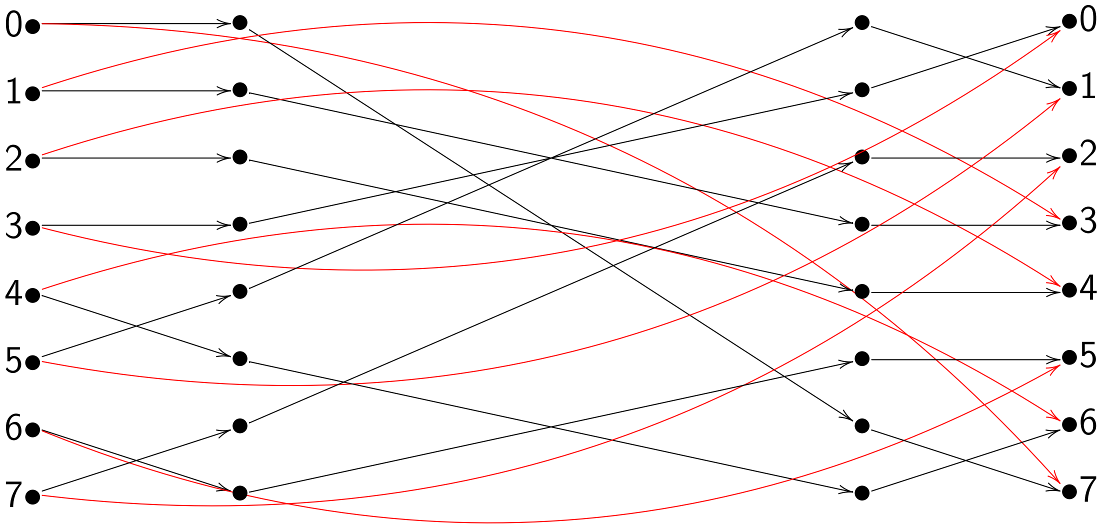

Control-bit formulas

"Verified fast formulas for control bits for permutation networks", `https://cr.yp.to/papers.html#controlbits`:

Start with any permutation π of $\{0, 1, \ldots, 2b-1\}$.

Compute first control bits $f_0, f_1, \ldots, f_{b-1}$ and last control bits $\ell_0, \ell_1, \ldots, \ell_{b-1}$ according to particular formulas in terms of π.

Define $F(x) = x \oplus f_{\lfloor x/2 \rfloor}$; $L(x) = x \oplus \ell_{\lfloor x/2 \rfloor}$; $M(x) = F(\pi(L(x)))$.

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Pages 21–66 of paper: Proof verified by HOL Light.

Verifying claimed theorems in HOL Light

In a new Debian Stretch VM: `# apt install git make camlp5`

As a new user, download and compile HOL Light:

```
$ git clone https://github.com/jrh13/hol-light.git
$ cd hol-light; make
```

Download someone's claimed HOL Light theorems: e.g.,

```
$ wget https://cr.yp.to/2020/controlbits-20200923.ml
```

Start HOL Light (takes a few minutes to verify built-in theorems):

```
$ ocaml
# #use "hol.ml";;
```

Ask HOL Light to verify the claimed theorems:

```
# #use "controlbits-20200923.ml";;
```

Defining a mathematical function in HOL Light

```
let xor1 = new_definition
  `xor1 (n:num) = if EVEN n then n+1 else n-1`;;
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i.e. `xor1(0)` is 1; `xor1(1)` is 0; `xor1(2)` is 3; `xor1(3)` is 2; etc.

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Warning: `n-1` doesn't mean exactly what you think it means.
If `n` is `0:num` then `n-1` is `0`. Error-prone definition of `-`. Yikes!
Analogy: `+` on `int` in C isn't math $+$ on integers; can overflow.

Quantifiers in HOL Light

"f is an involution" means: every x has $f(f(x)) = x$.

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Can even say `involution f = ...`; type-checker will invent an `A`.

Verified theorems in HOL Light: `thm`

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val it : thm = |- involution xor1
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Proving theorems in HOL Light

Somewhere inside `controlbits-20200923.ml`:

```
let xor1_involution = prove(
  `involution xor1`,
  MESON_TAC[xor1xor1;involution]);;
```

`MESON_TAC`: “model elimination subgoal oriented” theorem-proving tactic . . . meaning: this follows trivially.

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# xor1xor1;;
val it : thm = |- !n. xor1 (xor1 n) = n
```

Proving theorems in HOL Light, continued

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let xor1xor1 = prove(
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```

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# xor1xor1_ifodd;;
val it : thm = |- !n. ODD n ==> xor1 (xor1 n) = n
```

Sometimes proofs feel a bit more complicated

```
let pow_num_bijection = prove(
  `!p:A->A. bijection p ==> !n. bijection (p pow_num n)`,
  GEN_TAC THEN DISCH_TAC THEN
  INDUCT_TAC THENL
  [ REWRITE_TAC[pow_num_0;bijection_I]
  ; REWRITE_TAC[suc_isadd1] THEN
    ASM_MESON_TAC[pow_num_plus1;bijection_composes]
  ]);;
```

So we're done?

```
# middleperm_parity;;
val it : thm = |- !p x. bijection p ==>
  (ODD (middleperm p x) <=> ODD x)
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So we know $M(x) \equiv x \pmod 2$.

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What we actually want to know: this **software** is computing the same control bits, and this **software** is then applying the same π.

"Software" includes Python script in paper; reference C code; gcc output from the C code; optimized assembly language; etc.

Solution: More proofs?

CompCert is a compiler with

- a formal definition of a C-like input language;
- a formal definition of (e.g.) an “ARM assembly language” (at least some instructions), maybe perfectly matching ARM;
- a formally verified proof that, for each input program, the output program is equivalent to the input program.

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So: write C-like code, prove it applies π. Compile with CompCert.

Oops: the output is too slow, and have to pay to use CompCert.

So: write assembly, prove it applies π. Feasible? Yes. Tedious? Yes.

Does cryptographic software work correctly?

3. Symbolic testing

Daniel J. Bernstein

University of Illinois at Chicago; Ruhr University Bochum

Testing

Testing is great. Test everything. Design for tests.

Why wasn't the PA-RISC `CRYPTO_memcmp` software in OpenSSL run through millions of tests on random inputs?
And tests on inputs differing in just a few positions?
SUPERCOP crypto test framework has always done this.

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Good reaction to a bug:
"How can I build fast automated tests to catch this kind of bug?"
Even better to ask question before bug happens.

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Testing can miss attacker-triggerable bugs for rare inputs.

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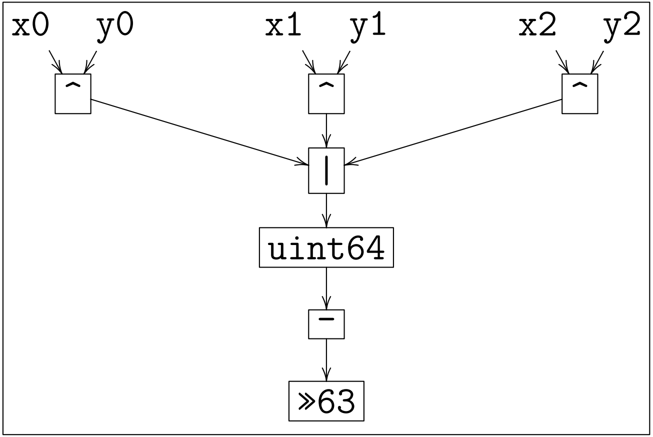
"On certain kinds of inputs, the code will lead to overflow conditions and hence to incorrect results. This, however, is a very low probability event and cannot be captured using some randomly generated known answer tests (KATs). . . . We believe that it is important to have proofs of correctness of the reduction algorithms to ensure that the algorithms works correctly for all possible inputs."

Symbolic testing: beyond testing particular inputs

```
.globl CRYPTO_memcmp
CRYPTO_memcmp:
xor    %rax,%rax
xor    %r10,%r10
cmp    $0x0,%rdx
je     no_data
cmp    $0x10,%rdx
jne    loop
mov    (%rdi),%r10
mov    0x8(%rdi),%r11
mov    $0x1,%rdx
xor    (%rsi),%r10
xor    0x8(%rsi),%r11
or     %r11,%r10
cmovne %rdx,%rax
repz retq
loop:
mov    (%rdi),%r10b
lea    0x1(%rdi),%rdi
xor    (%rsi),%r10b
lea    0x1(%rsi),%rsi
or     %r10b,%al
dec    %rdx
jne    loop
neg    %rax
shr    $0x3f,%rax
no_data:
repz retq
```

Arithmetic DAG for all 3-byte inputs:



The power of modern reverse-engineering tools

Easy to use `angr.io` for automatic **symbolic execution**:
machine-language software ➜ arithmetic DAG.
Simplifies analysis: simpler instructions, no memory, no jumps.

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`angr` (via Z3 SMT solver) often sees equivalence of small DAGs.
e.g. sees that OpenSSL `x86_64` `CRYPTO_memcmp` on 3-byte inputs outputs 0 if `x0==y0` and `x1==y1` and `x2==y2`,
and outputs 1 otherwise. Similarly for other input lengths.

```c
#include <openssl/crypto.h>

unsigned char x[N];
unsigned char y[N];
int z;

int main()
{
  z = CRYPTO_memcmp(x,y,N);
  return 0;
}
```

```
#!/usr/bin/env python3

import sys
import angr

N = int(sys.argv[1]) if len(sys.argv) > 1 else 16

proj = angr.Project(’cmp%d’%N)
state = proj.factory.full_init_state()

state.options |= {
  angr.options.ZERO_FILL_UNCONSTRAINED_MEMORY
}
```

```
x = {}
xaddr = proj.loader.find_symbol('x').rebased_addr
for i in range(N):
  x[i] = state.solver.BVS('x%d'%i,8)
  state.mem[xaddr+i].char = x[i]

y = {}
yaddr = proj.loader.find_symbol('y').rebased_addr
for i in range(N):
  y[i] = state.solver.BVS('y%d'%i,8)
  state.mem[yaddr+i].char = y[i]

simgr = proj.factory.simgr(state)
simgr.run()
```

```
assert len(simgr.errored) == 0
print(’%d universes’ % len(simgr.deadended))
for exit in simgr.deadended:
  zaddr = proj.loader.find_symbol(’z’).rebased_addr
  z = exit.mem[zaddr].int.resolved
  print(’out = %s’ % z)

  xeqy = True
  for i in range(N):
    xeqy = state.solver.And(xeqy,x[i]==y[i])
  xney = state.solver.Not(xeqy)
  for bugs in ((z!=0,z!=1),(z!=0,xeqy),(z!=1,xney)):
    assert not exit.satisfiable(extra_constraints=bugs)
```

Symbolic execution with better equivalence testing

What if the DAG is too complicated for the SMT solver?
Answer: **Build smarter tools to recognize DAG equivalence.**

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Case study, software library from `sorting.cr.yp.to`:

- New speed records for sorting of in-memory integer arrays. This is a subroutine in some post-quantum cryptosystems.
- Side-channel countermeasures: no secret branch conditions; no secret array indices.
- New tool verifies correct sorting of all size-N inputs. No need for manual review of per-CPU optimized code.