Challenges in evaluating costs of known lattice attacks

Daniel J. Bernstein
Tanja Lange

Based on attack survey from 2019 Bernstein–Chuengsatiansup–Lange–van Vredendaal.

Why analysis is important:

- Guide attack optimization.
- Guide attack selection.
- Evaluate crypto parameters.
- Evaluate crypto designs.
- Advise users on security.

Three typical attack problems

Define $\mathcal{R} = \mathbf{Z}[x]/(x^{761} - x - 1)$; "small" $=$ all coeffs in $\{-1, 0, 1\}$; $w = 286$; $q = 4591$.

Attacker wants to find small weight-w secret $a \in \mathcal{R}$.

Problem 1: Public $G \in \mathcal{R}/q$ with $aG + e = 0$. Small secret $e \in \mathcal{R}$.

Problem 2: Public $G \in \mathcal{R}/q$ and $aG + e$. Small secret $e \in \mathcal{R}$.

Problem 3: Public $G_1, G_2 \in \mathcal{R}/q$. Public $aG_1 + e_1, aG_2 + e_2$. Small secrets $e_1, e_2 \in \mathcal{R}$.

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Example

Secret k

Public k
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Public k
$G = -e$

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Examples of target

Secret key: small

Public key reveals
and approximation

Public key for "N
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Examples of target cryptosys

Secret key: small a; small e.

Public key reveals multiplier

and approximation $A = aG$ -

Public key for "NTRU": $G = -e/a$, and $A = 0$.

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Systematization of naming,
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"NTRU" $\Rightarrow$ Quotient NTRU.
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Encryption for Qu

Input small b, sma

Ciphertext: $B = 3$

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Ciphertext: $B = 3Gb + d$.

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Encryption for Product NTRU:
Input encoded message M.
Randomly generate small b, small d, small c.
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Next slides: survey of G, a, e, c, M details and variants in NISTPQC submissions. Source: Bernstein, "Comparing proofs of security for lattice-based encryption".

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system	parame
frodo	
frodo	
frodo	
kyber	
kyber	
kyber	
lac	
lac	
lac	
newhope	
newhope	
ntru	hps20
ntru	hps20
ntru	hps40
ntru	hr
ntrulpr	
ntrulpr	
ntrulpr	
round5n1	
round5n1	
round5n1	
round5nd	
round5nd	
round5nd	
round5nd	
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round5nd	
saber	
saber	
saber	
sntrup	
sntrup	
sntrup	
threebears	
threebears	
threebears	

t cryptosystems

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n $A = aG + e$.

TRU":

$= 0$.

ng-LWE":

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system	parameter set	type
frodo	640	Product
frodo	976	Product
frodo	1344	Product
kyber	512	Product
kyber	768	Product
kyber	1024	Product
lac	128	Product
lac	192	Product
lac	256	Product
newhope	512	Product
newhope	1024	Product
ntru	hps2048509	Quotient
ntru	hps2048677	Quotient
ntru	hps4096821	Quotient
ntru	hrss701	Quotient
ntrulpr	653	Product
ntrulpr	761	Product
ntrulpr	857	Product
round5n1	1	Product
round5n1	3	Product
round5n1	5	Product
round5nd	1.0d	Product
round5nd	3.0d	Product
round5nd	5.0d	Product
round5nd	1.5d	Product
round5nd	3.5d	Product
round5nd	5.5d	Product
saber	light	Product
saber	main	Product
saber	fire	Product
sntrup	653	Quotient
sntrup	761	Quotient
sntrup	857	Quotient
threebears	baby	Product
threebears	mama	Product
threebears	papa	Product

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kyber	512	Product	$((\mathbf{Z}/3329)[x]/(x^{256}$
kyber	768	Product	$((\mathbf{Z}/3329)[x]/(x^{256}$
kyber	1024	Product	$((\mathbf{Z}/3329)[x]/(x^{256}$
lac	128	Product	$(\mathbf{Z}/251)[x]/(x^{512}+$
lac	192	Product	$(\mathbf{Z}/251)[x]/(x^{1024}$
lac	256	Product	$(\mathbf{Z}/251)[x]/(x^{1024}$
newhope	512	Product	$(\mathbf{Z}/12289)[x]/(x^{512}$
newhope	1024	Product	$(\mathbf{Z}/12289)[x]/(x^{102}$
ntru	hps2048509	Quotient	$(\mathbf{Z}/2048)[x]/(x^{509}$
ntru	hps2048677	Quotient	$(\mathbf{Z}/2048)[x]/(x^{677}$
ntru	hps4096821	Quotient	$(\mathbf{Z}/4096)[x]/(x^{821}$
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ntrulpr	653	Product	$(\mathbf{Z}/4621)[x]/(x^{653}$
ntrulpr	761	Product	$(\mathbf{Z}/4591)[x]/(x^{761}$
ntrulpr	857	Product	$(\mathbf{Z}/5167)[x]/(x^{857}$
round5n1	1	Product	$(\mathbf{Z}/4096)^{636\times 636}$
round5n1	3	Product	$(\mathbf{Z}/32768)^{876\times 876}$
round5n1	5	Product	$(\mathbf{Z}/32768)^{1217\times 1217}$
round5nd	1.0d	Product	$(\mathbf{Z}/8192)[x]/(x^{586}$
round5nd	3.0d	Product	$(\mathbf{Z}/4096)[x]/(x^{852}$
round5nd	5.0d	Product	$(\mathbf{Z}/8192)[x]/(x^{1170}$
round5nd	1.5d	Product	$(\mathbf{Z}/1024)[x]/(x^{509}$
round5nd	3.5d	Product	$(\mathbf{Z}/4096)[x]/(x^{757}$
round5nd	5.5d	Product	$(\mathbf{Z}/2048)[x]/(x^{947}$
saber	light	Product	$((\mathbf{Z}/8192)[x]/(x^{256}$
saber	main	Product	$((\mathbf{Z}/8192)[x]/(x^{256}$
saber	fire	Product	$((\mathbf{Z}/8192)[x]/(x^{256}$
sntrup	653	Quotient	$(\mathbf{Z}/4621)[x]/(x^{653}$
sntrup	761	Quotient	$(\mathbf{Z}/4591)[x]/(x^{761}$
sntrup	857	Quotient	$(\mathbf{Z}/5167)[x]/(x^{857}$
threebears	baby	Product	$(\mathbf{Z}/(2^{3120}-2^{1560}$
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kyber	1024	Product	$((\mathbf{Z}/3329)[x]/(x^{256}+1))^{4\times 4}$
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saber	light	Product	$((\mathbf{Z}/8192)[x]/(x^{256}+1))^{2\times2}$
saber	main	Product	$((\mathbf{Z}/8192)[x]/(x^{256}+1))^{3\times3}$
saber	fire	Product	$((\mathbf{Z}/8192)[x]/(x^{256}+1))^{4\times4}$
sntrup	653	Quotient	$(\mathbf{Z}/4621)[x]/(x^{653}-x-1)$
sntrup	761	Quotient	$(\mathbf{Z}/4591)[x]/(x^{761}-x-1)$
sntrup	857	Quotient	$(\mathbf{Z}/5167)[x]/(x^{857}-x-1)$
threebears	baby	Product	$(\mathbf{Z}/(2^{3120}-2^{1560}-1))^{2\times2}$
threebears	mama	Product	$(\mathbf{Z}/(2^{3120}-2^{1560}-1))^{3\times3}$
threebears	papa	Product	$(\mathbf{Z}/(2^{3120}-2^{1560}-1))^{4\times4}$

short element
$\mathbf{Z}^{640\times8}$; $\{-12, \ldots$
$\mathbf{Z}^{976\times8}$; $\{-10, \ldots$
$\mathbf{Z}^{1344\times8}$; $\{-6, \ldots$
$(\mathbf{Z}[x]/(x^{256}+1)$
$(\mathbf{Z}[x]/(x^{256}+1)$
$(\mathbf{Z}[x]/(x^{256}+1)$
$\mathbf{Z}[x]/(x^{512}+1)$;
$\mathbf{Z}[x]/(x^{1024}+1)$
$\mathbf{Z}[x]/(x^{1024}+1)$
$\mathbf{Z}[x]/(x^{512}+1)$;
$\mathbf{Z}[x]/(x^{1024}+1)$
$\mathbf{Z}[x]/(x^{509}-1)$;
$\mathbf{Z}[x]/(x^{677}-1)$;
$\mathbf{Z}[x]/(x^{821}-1)$;
$\mathbf{Z}[x]/(x^{701}-1)$;
$\mathbf{Z}[x]/(x^{653}-x-$
$\mathbf{Z}[x]/(x^{761}-x-$
$\mathbf{Z}[x]/(x^{857}-x-$
$\mathbf{Z}^{636\times8}$; $\{-1, 0,$
$\mathbf{Z}^{876\times8}$; $\{-1, 0,$
$\mathbf{Z}^{1217\times8}$; $\{-1, 0,$
$\mathbf{Z}[x]/(x^{586}+\ldots$
$\mathbf{Z}[x]/(x^{852}+\ldots$
$\mathbf{Z}[x]/(x^{1170}+\ldots$
$\mathbf{Z}[x]/(x^{509}-1)$;
$\mathbf{Z}[x]/(x^{757}-1)$;
$\mathbf{Z}[x]/(x^{947}-1)$;
$(\mathbf{Z}[x]/(x^{256}+1)$
$(\mathbf{Z}[x]/(x^{256}+1)$
$(\mathbf{Z}[x]/(x^{256}+1)$
$\mathbf{Z}[x]/(x^{653}-x-$
$\mathbf{Z}[x]/(x^{761}-x-$
$\mathbf{Z}[x]/(x^{857}-x-$
$\mathbf{Z}^2$; $\sum_{0\le i<312} 2$
$\mathbf{Z}^3$; $\sum_{0\le i<312} 2$
$\mathbf{Z}^4$; $\sum_{0\le i<312} 2$

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$+ c$.

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system	parameter set	type	set of multipliers
frodo	640	Product	$(\mathbf{Z}/32768)^{640\times 640}$
frodo	976	Product	$(\mathbf{Z}/65536)^{976\times 976}$
frodo	1344	Product	$(\mathbf{Z}/65536)^{1344\times 1344}$
kyber	512	Product	$((\mathbf{Z}/3329)[x]/(x^{256}+1))^{2\times 2}$
kyber	768	Product	$((\mathbf{Z}/3329)[x]/(x^{256}+1))^{3\times 3}$
kyber	1024	Product	$((\mathbf{Z}/3329)[x]/(x^{256}+1))^{4\times 4}$
lac	128	Product	$(\mathbf{Z}/251)[x]/(x^{512}+1)$
lac	192	Product	$(\mathbf{Z}/251)[x]/(x^{1024}+1)$
lac	256	Product	$(\mathbf{Z}/251)[x]/(x^{1024}+1)$
newhope	512	Product	$(\mathbf{Z}/12289)[x]/(x^{512}+1)$
newhope	1024	Product	$(\mathbf{Z}/12289)[x]/(x^{1024}+1)$
ntru	hps2048509	Quotient	$(\mathbf{Z}/2048)[x]/(x^{509}-1)$
ntru	hps2048677	Quotient	$(\mathbf{Z}/2048)[x]/(x^{677}-1)$
ntru	hps4096821	Quotient	$(\mathbf{Z}/4096)[x]/(x^{821}-1)$
ntru	hrss701	Quotient	$(\mathbf{Z}/8192)[x]/(x^{701}-1)$
ntrulpr	653	Product	$(\mathbf{Z}/4621)[x]/(x^{653}-x-1)$
ntrulpr	761	Product	$(\mathbf{Z}/4591)[x]/(x^{761}-x-1)$
ntrulpr	857	Product	$(\mathbf{Z}/5167)[x]/(x^{857}-x-1)$
round5n1	1	Product	$(\mathbf{Z}/4096)^{636\times 636}$
round5n1	3	Product	$(\mathbf{Z}/32768)^{876\times 876}$
round5n1	5	Product	$(\mathbf{Z}/32768)^{1217\times 1217}$
round5nd	1.0d	Product	$(\mathbf{Z}/8192)[x]/(x^{586}+\ldots+1)$
round5nd	3.0d	Product	$(\mathbf{Z}/4096)[x]/(x^{852}+\ldots+1)$
round5nd	5.0d	Product	$(\mathbf{Z}/8192)[x]/(x^{1170}+\ldots+1)$
round5nd	1.5d	Product	$(\mathbf{Z}/1024)[x]/(x^{509}-1)$
round5nd	3.5d	Product	$(\mathbf{Z}/4096)[x]/(x^{757}-1)$
round5nd	5.5d	Product	$(\mathbf{Z}/2048)[x]/(x^{947}-1)$
saber	light	Product	$((\mathbf{Z}/8192)[x]/(x^{256}+1))^{2\times 2}$
saber	main	Product	$((\mathbf{Z}/8192)[x]/(x^{256}+1))^{3\times 3}$
saber	fire	Product	$((\mathbf{Z}/8192)[x]/(x^{256}+1))^{4\times 4}$
sntrup	653	Quotient	$(\mathbf{Z}/4621)[x]/(x^{653}-x-1)$
sntrup	761	Quotient	$(\mathbf{Z}/4591)[x]/(x^{761}-x-1)$
sntrup	857	Quotient	$(\mathbf{Z}/5167)[x]/(x^{857}-x-1)$
threebears	baby	Product	$(\mathbf{Z}/(2^{3120}-2^{1560}-1))^{2\times 2}$
threebears	mama	Product	$(\mathbf{Z}/(2^{3120}-2^{1560}-1))^{3\times 3}$
threebears	papa	Product	$(\mathbf{Z}/(2^{3120}-2^{1560}-1))^{4\times 4}$

short element
$\mathbf{Z}^{640\times 8}$; $\{-12,\ldots,12\}$; Pr 1, 4, 17, .
$\mathbf{Z}^{976\times 8}$; $\{-10,\ldots,10\}$; Pr 1, 6, 29, .
$\mathbf{Z}^{1344\times 8}$; $\{-6,\ldots,6\}$; Pr 2, 40, 364,
$(\mathbf{Z}[x]/(x^{256}+1))^2$; $\sum_{0\le i<4}\{-0.5,$
$(\mathbf{Z}[x]/(x^{256}+1))^3$; $\sum_{0\le i<4}\{-0.5,$
$(\mathbf{Z}[x]/(x^{256}+1))^4$; $\sum_{0\le i<4}\{-0.5,$
$\mathbf{Z}[x]/(x^{512}+1)$; $\{-1,0,1\}$; Pr 1, 2,
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr 1, 6
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr 1, 2
$\mathbf{Z}[x]/(x^{512}+1)$; $\sum_{0\le i<16}\{-0.5, 0.5$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\sum_{0\le i<16}\{-0.5, 0.$
$\mathbf{Z}[x]/(x^{509}-1)$; $\{-1,0,1\}$
$\mathbf{Z}[x]/(x^{677}-1)$; $\{-1,0,1\}$
$\mathbf{Z}[x]/(x^{821}-1)$; $\{-1,0,1\}$
$\mathbf{Z}[x]/(x^{701}-1)$; $\{-1,0,1\}$; key corr
$\mathbf{Z}[x]/(x^{653}-x-1)$; $\{-1,0,1\}$; wei
$\mathbf{Z}[x]/(x^{761}-x-1)$; $\{-1,0,1\}$; wei
$\mathbf{Z}[x]/(x^{857}-x-1)$; $\{-1,0,1\}$; wei
$\mathbf{Z}^{636\times 8}$; $\{-1,0,1\}$; weight 57, 57
$\mathbf{Z}^{876\times 8}$; $\{-1,0,1\}$; weight 223, 223
$\mathbf{Z}^{1217\times 8}$; $\{-1,0,1\}$; weight 231, 231
$\mathbf{Z}[x]/(x^{586}+\ldots+1)$; $\{-1,0,1\}$; we
$\mathbf{Z}[x]/(x^{852}+\ldots+1)$; $\{-1,0,1\}$; we
$\mathbf{Z}[x]/(x^{1170}+\ldots+1)$; $\{-1,0,1\}$; w
$\mathbf{Z}[x]/(x^{509}-1)$; $\{-1,0,1\}$; weight 6
$\mathbf{Z}[x]/(x^{757}-1)$; $\{-1,0,1\}$; weight
$\mathbf{Z}[x]/(x^{947}-1)$; $\{-1,0,1\}$; weight
$(\mathbf{Z}[x]/(x^{256}+1))^2$; $\sum_{0\le i<10}\{-0.5,$
$(\mathbf{Z}[x]/(x^{256}+1))^3$; $\sum_{0\le i<8}\{-0.5,$
$(\mathbf{Z}[x]/(x^{256}+1))^4$; $\sum_{0\le i<6}\{-0.5,$
$\mathbf{Z}[x]/(x^{653}-x-1)$; $\{-1,0,1\}$; wei
$\mathbf{Z}[x]/(x^{761}-x-1)$; $\{-1,0,1\}$; wei
$\mathbf{Z}[x]/(x^{857}-x-1)$; $\{-1,0,1\}$; wei
$\mathbf{Z}^2$; $\sum_{0\le i<312} 2^{10i}\{-2,-1,0,1,2\}$;
$\mathbf{Z}^3$; $\sum_{0\le i<312} 2^{10i}\{-1,0,1\}$; Pr 13,
$\mathbf{Z}^4$; $\sum_{0\le i<312} 2^{10i}\{-1,0,1\}$; Pr 5, 2

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system	parameter set	type	set of multipliers	short element
frodo	640	Product	$(\mathbf{Z}/32768)^{640\times640}$	$\mathbf{Z}^{640\times8}$; $\{-12,\ldots,12\}$; Pr 1, 4, 17, ... (spec page 23)
frodo	976	Product	$(\mathbf{Z}/65536)^{976\times976}$	$\mathbf{Z}^{976\times8}$; $\{-10,\ldots,10\}$; Pr 1, 6, 29, ... (spec page 23)
frodo	1344	Product	$(\mathbf{Z}/65536)^{1344\times1344}$	$\mathbf{Z}^{1344\times8}$; $\{-6,\ldots,6\}$; Pr 2, 40, 364, ... (spec page 23)
kyber	512	Product	$((\mathbf{Z}/3329)[x]/(x^{256}+1))^{2\times2}$	$(\mathbf{Z}[x]/(x^{256}+1))^2$; $\sum_{0\le i<4}\{-0.5,0.5\}$
kyber	768	Product	$((\mathbf{Z}/3329)[x]/(x^{256}+1))^{3\times3}$	$(\mathbf{Z}[x]/(x^{256}+1))^3$; $\sum_{0\le i<4}\{-0.5,0.5\}$
kyber	1024	Product	$((\mathbf{Z}/3329)[x]/(x^{256}+1))^{4\times4}$	$(\mathbf{Z}[x]/(x^{256}+1))^4$; $\sum_{0\le i<4}\{-0.5,0.5\}$
lac	128	Product	$(\mathbf{Z}/251)[x]/(x^{512}+1)$	$\mathbf{Z}[x]/(x^{512}+1)$; $\{-1,0,1\}$; Pr 1, 2, 1; weight 128, 128
lac	192	Product	$(\mathbf{Z}/251)[x]/(x^{1024}+1)$	$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr 1, 6, 1; weight 128, 128
lac	256	Product	$(\mathbf{Z}/251)[x]/(x^{1024}+1)$	$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr 1, 2, 1; weight 256, 256
newhope	512	Product	$(\mathbf{Z}/12289)[x]/(x^{512}+1)$	$\mathbf{Z}[x]/(x^{512}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
newhope	1024	Product	$(\mathbf{Z}/12289)[x]/(x^{1024}+1)$	$\mathbf{Z}[x]/(x^{1024}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
ntru	hps2048509	Quotient	$(\mathbf{Z}/2048)[x]/(x^{509}-1)$	$\mathbf{Z}[x]/(x^{509}-1)$; $\{-1,0,1\}$
ntru	hps2048677	Quotient	$(\mathbf{Z}/2048)[x]/(x^{677}-1)$	$\mathbf{Z}[x]/(x^{677}-1)$; $\{-1,0,1\}$
ntru	hps4096821	Quotient	$(\mathbf{Z}/4096)[x]/(x^{821}-1)$	$\mathbf{Z}[x]/(x^{821}-1)$; $\{-1,0,1\}$
ntru	hrss701	Quotient	$(\mathbf{Z}/8192)[x]/(x^{701}-1)$	$\mathbf{Z}[x]/(x^{701}-1)$; $\{-1,0,1\}$; key correlation ≥ 0
ntrulpr	653	Product	$(\mathbf{Z}/4621)[x]/(x^{653}-x-1)$	$\mathbf{Z}[x]/(x^{653}-x-1)$; $\{-1,0,1\}$; weight 252
ntrulpr	761	Product	$(\mathbf{Z}/4591)[x]/(x^{761}-x-1)$	$\mathbf{Z}[x]/(x^{761}-x-1)$; $\{-1,0,1\}$; weight 250
ntrulpr	857	Product	$(\mathbf{Z}/5167)[x]/(x^{857}-x-1)$	$\mathbf{Z}[x]/(x^{857}-x-1)$; $\{-1,0,1\}$; weight 281
round5n1	1	Product	$(\mathbf{Z}/4096)^{636\times636}$	$\mathbf{Z}^{636\times8}$; $\{-1,0,1\}$; weight 57, 57
round5n1	3	Product	$(\mathbf{Z}/32768)^{876\times876}$	$\mathbf{Z}^{876\times8}$; $\{-1,0,1\}$; weight 223, 223
round5n1	5	Product	$(\mathbf{Z}/32768)^{1217\times1217}$	$\mathbf{Z}^{1217\times8}$; $\{-1,0,1\}$; weight 231, 231
round5nd	1.0d	Product	$(\mathbf{Z}/8192)[x]/(x^{586}+\ldots+1)$	$\mathbf{Z}[x]/(x^{586}+\ldots+1)$; $\{-1,0,1\}$; weight 91, 91
round5nd	3.0d	Product	$(\mathbf{Z}/4096)[x]/(x^{852}+\ldots+1)$	$\mathbf{Z}[x]/(x^{852}+\ldots+1)$; $\{-1,0,1\}$; weight 106, 106
round5nd	5.0d	Product	$(\mathbf{Z}/8192)[x]/(x^{1170}+\ldots+1)$	$\mathbf{Z}[x]/(x^{1170}+\ldots+1)$; $\{-1,0,1\}$; weight 111, 111
round5nd	1.5d	Product	$(\mathbf{Z}/1024)[x]/(x^{509}-1)$	$\mathbf{Z}[x]/(x^{509}-1)$; $\{-1,0,1\}$; weight 68, 68; ending 0
round5nd	3.5d	Product	$(\mathbf{Z}/4096)[x]/(x^{757}-1)$	$\mathbf{Z}[x]/(x^{757}-1)$; $\{-1,0,1\}$; weight 121, 121; ending 0
round5nd	5.5d	Product	$(\mathbf{Z}/2048)[x]/(x^{947}-1)$	$\mathbf{Z}[x]/(x^{947}-1)$; $\{-1,0,1\}$; weight 194, 194; ending 0
saber	light	Product	$((\mathbf{Z}/8192)[x]/(x^{256}+1))^{2\times2}$	$(\mathbf{Z}[x]/(x^{256}+1))^2$; $\sum_{0\le i<10}\{-0.5,0.5\}$
saber	main	Product	$((\mathbf{Z}/8192)[x]/(x^{256}+1))^{3\times3}$	$(\mathbf{Z}[x]/(x^{256}+1))^3$; $\sum_{0\le i<8}\{-0.5,0.5\}$
saber	fire	Product	$((\mathbf{Z}/8192)[x]/(x^{256}+1))^{4\times4}$	$(\mathbf{Z}[x]/(x^{256}+1))^4$; $\sum_{0\le i<6}\{-0.5,0.5\}$
sntrup	653	Quotient	$(\mathbf{Z}/4621)[x]/(x^{653}-x-1)$	$\mathbf{Z}[x]/(x^{653}-x-1)$; $\{-1,0,1\}$; weight 288
sntrup	761	Quotient	$(\mathbf{Z}/4591)[x]/(x^{761}-x-1)$	$\mathbf{Z}[x]/(x^{761}-x-1)$; $\{-1,0,1\}$; weight 286
sntrup	857	Quotient	$(\mathbf{Z}/5167)[x]/(x^{857}-x-1)$	$\mathbf{Z}[x]/(x^{857}-x-1)$; $\{-1,0,1\}$; weight 322
threebears	baby	Product	$(\mathbf{Z}/(2^{3120}-2^{1560}-1))^{2\times2}$	$\mathbf{Z}^2$; $\sum_{0\le i<312} 2^{10i}\{-2,-1,0,1,2\}$; Pr 1, 32, 62, 32, 1; *
threebears	mama	Product	$(\mathbf{Z}/(2^{3120}-2^{1560}-1))^{3\times3}$	$\mathbf{Z}^3$; $\sum_{0\le i<312} 2^{10i}\{-1,0,1\}$; Pr 13, 38, 13; *
threebears	papa	Product	$(\mathbf{Z}/(2^{3120}-2^{1560}-1))^{4\times4}$	$\mathbf{Z}^4$; $\sum_{0\le i<312} 2^{10i}\{-1,0,1\}$; Pr 5, 22, 5; *

system	parameter set	type	set of multipliers	short element
frodo	640	Product	$(\mathbf{Z}/32768)^{640\times640}$	$\mathbf{Z}^{640\times8}$; $\{-12,\ldots,12\}$; Pr 1, 4, 17, ... (spec page 23)
frodo	976	Product	$(\mathbf{Z}/65536)^{976\times976}$	$\mathbf{Z}^{976\times8}$; $\{-10,\ldots,10\}$; Pr 1, 6, 29, ... (spec page 23)
frodo	1344	Product	$(\mathbf{Z}/65536)^{1344\times1344}$	$\mathbf{Z}^{1344\times8}$; $\{-6,\ldots,6\}$; Pr 2, 40, 364, ... (spec page 23)
kyber	512	Product	$((\mathbf{Z}/3329)[x]/(x^{256}+1))^{2\times2}$	$(\mathbf{Z}[x]/(x^{256}+1))^2$; $\sum_{0\le i<4}\{-0.5,0.5\}$
kyber	768	Product	$((\mathbf{Z}/3329)[x]/(x^{256}+1))^{3\times3}$	$(\mathbf{Z}[x]/(x^{256}+1))^3$; $\sum_{0\le i<4}\{-0.5,0.5\}$
kyber	1024	Product	$((\mathbf{Z}/3329)[x]/(x^{256}+1))^{4\times4}$	$(\mathbf{Z}[x]/(x^{256}+1))^4$; $\sum_{0\le i<4}\{-0.5,0.5\}$
lac	128	Product	$(\mathbf{Z}/251)[x]/(x^{512}+1)$	$\mathbf{Z}[x]/(x^{512}+1)$; $\{-1,0,1\}$; Pr 1, 2, 1; weight 128, 128
lac	192	Product	$(\mathbf{Z}/251)[x]/(x^{1024}+1)$	$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr 1, 6, 1; weight 128, 128
lac	256	Product	$(\mathbf{Z}/251)[x]/(x^{1024}+1)$	$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr 1, 2, 1; weight 256, 256
newhope	512	Product	$(\mathbf{Z}/12289)[x]/(x^{512}+1)$	$\mathbf{Z}[x]/(x^{512}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
newhope	1024	Product	$(\mathbf{Z}/12289)[x]/(x^{1024}+1)$	$\mathbf{Z}[x]/(x^{1024}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
ntru	hps2048509	Quotient	$(\mathbf{Z}/2048)[x]/(x^{509}-1)$	$\mathbf{Z}[x]/(x^{509}-1)$; $\{-1,0,1\}$
ntru	hps2048677	Quotient	$(\mathbf{Z}/2048)[x]/(x^{677}-1)$	$\mathbf{Z}[x]/(x^{677}-1)$; $\{-1,0,1\}$
ntru	hps4096821	Quotient	$(\mathbf{Z}/4096)[x]/(x^{821}-1)$	$\mathbf{Z}[x]/(x^{821}-1)$; $\{-1,0,1\}$
ntru	hrss701	Quotient	$(\mathbf{Z}/8192)[x]/(x^{701}-1)$	$\mathbf{Z}[x]/(x^{701}-1)$; $\{-1,0,1\}$; key correlation ≥ 0
ntrulpr	653	Product	$(\mathbf{Z}/4621)[x]/(x^{653}-x-1)$	$\mathbf{Z}[x]/(x^{653}-x-1)$; $\{-1,0,1\}$; weight 252
ntrulpr	761	Product	$(\mathbf{Z}/4591)[x]/(x^{761}-x-1)$	$\mathbf{Z}[x]/(x^{761}-x-1)$; $\{-1,0,1\}$; weight 250
ntrulpr	857	Product	$(\mathbf{Z}/5167)[x]/(x^{857}-x-1)$	$\mathbf{Z}[x]/(x^{857}-x-1)$; $\{-1,0,1\}$; weight 281
round5n1	1	Product	$(\mathbf{Z}/4096)^{636\times636}$	$\mathbf{Z}^{636\times8}$; $\{-1,0,1\}$; weight 57, 57
round5n1	3	Product	$(\mathbf{Z}/32768)^{876\times876}$	$\mathbf{Z}^{876\times8}$; $\{-1,0,1\}$; weight 223, 223
round5n1	5	Product	$(\mathbf{Z}/32768)^{1217\times1217}$	$\mathbf{Z}^{1217\times8}$; $\{-1,0,1\}$; weight 231, 231
round5nd	1.0d	Product	$(\mathbf{Z}/8192)[x]/(x^{586}+\ldots+1)$	$\mathbf{Z}[x]/(x^{586}+\ldots+1)$; $\{-1,0,1\}$; weight 91, 91
round5nd	3.0d	Product	$(\mathbf{Z}/4096)[x]/(x^{852}+\ldots+1)$	$\mathbf{Z}[x]/(x^{852}+\ldots+1)$; $\{-1,0,1\}$; weight 106, 106
round5nd	5.0d	Product	$(\mathbf{Z}/8192)[x]/(x^{1170}+\ldots+1)$	$\mathbf{Z}[x]/(x^{1170}+\ldots+1)$; $\{-1,0,1\}$; weight 111, 111
round5nd	1.5d	Product	$(\mathbf{Z}/1024)[x]/(x^{509}-1)$	$\mathbf{Z}[x]/(x^{509}-1)$; $\{-1,0,1\}$; weight 68, 68; ending 0
round5nd	3.5d	Product	$(\mathbf{Z}/4096)[x]/(x^{757}-1)$	$\mathbf{Z}[x]/(x^{757}-1)$; $\{-1,0,1\}$; weight 121, 121; ending 0
round5nd	5.5d	Product	$(\mathbf{Z}/2048)[x]/(x^{947}-1)$	$\mathbf{Z}[x]/(x^{947}-1)$; $\{-1,0,1\}$; weight 194, 194; ending 0
saber	light	Product	$((\mathbf{Z}/8192)[x]/(x^{256}+1))^{2\times2}$	$(\mathbf{Z}[x]/(x^{256}+1))^2$; $\sum_{0\le i<10}\{-0.5,0.5\}$
saber	main	Product	$((\mathbf{Z}/8192)[x]/(x^{256}+1))^{3\times3}$	$(\mathbf{Z}[x]/(x^{256}+1))^3$; $\sum_{0\le i<8}\{-0.5,0.5\}$
saber	fire	Product	$((\mathbf{Z}/8192)[x]/(x^{256}+1))^{4\times4}$	$(\mathbf{Z}[x]/(x^{256}+1))^4$; $\sum_{0\le i<6}\{-0.5,0.5\}$
sntrup	653	Quotient	$(\mathbf{Z}/4621)[x]/(x^{653}-x-1)$	$\mathbf{Z}[x]/(x^{653}-x-1)$; $\{-1,0,1\}$; weight 288
sntrup	761	Quotient	$(\mathbf{Z}/4591)[x]/(x^{761}-x-1)$	$\mathbf{Z}[x]/(x^{761}-x-1)$; $\{-1,0,1\}$; weight 286
sntrup	857	Quotient	$(\mathbf{Z}/5167)[x]/(x^{857}-x-1)$	$\mathbf{Z}[x]/(x^{857}-x-1)$; $\{-1,0,1\}$; weight 322
threebears	baby	Product	$(\mathbf{Z}/(2^{3120}-2^{1560}-1))^{2\times2}$	$\mathbf{Z}^2$; $\sum_{0\le i<312}2^{10i}\{-2,-1,0,1,2\}$; Pr 1, 32, 62, 32, 1; *
threebears	mama	Product	$(\mathbf{Z}/(2^{3120}-2^{1560}-1))^{3\times3}$	$\mathbf{Z}^3$; $\sum_{0\le i<312}2^{10i}\{-1,0,1\}$; Pr 13, 38, 13; *
threebears	papa	Product	$(\mathbf{Z}/(2^{3120}-2^{1560}-1))^{4\times4}$	$\mathbf{Z}^4$; $\sum_{0\le i<312}2^{10i}\{-1,0,1\}$; Pr 5, 22, 5; *

eter set	type	set of multipliers
640	Product	$(\mathbf{Z}/32768)^{640\times 640}$
976	Product	$(\mathbf{Z}/65536)^{976\times 976}$
1344	Product	$(\mathbf{Z}/65536)^{1344\times 1344}$
512	Product	$((\mathbf{Z}/3329)[x]/(x^{256}+1))^{2\times 2}$
768	Product	$((\mathbf{Z}/3329)[x]/(x^{256}+1))^{3\times 3}$
1024	Product	$((\mathbf{Z}/3329)[x]/(x^{256}+1))^{4\times 4}$
128	Product	$(\mathbf{Z}/251)[x]/(x^{512}+1)$
192	Product	$(\mathbf{Z}/251)[x]/(x^{1024}+1)$
256	Product	$(\mathbf{Z}/251)[x]/(x^{1024}+1)$
512	Product	$(\mathbf{Z}/12289)[x]/(x^{512}+1)$
1024	Product	$(\mathbf{Z}/12289)[x]/(x^{1024}+1)$
048509	Quotient	$(\mathbf{Z}/2048)[x]/(x^{509}-1)$
048677	Quotient	$(\mathbf{Z}/2048)[x]/(x^{677}-1)$
096821	Quotient	$(\mathbf{Z}/4096)[x]/(x^{821}-1)$
rss701	Quotient	$(\mathbf{Z}/8192)[x]/(x^{701}-1)$
653	Product	$(\mathbf{Z}/4621)[x]/(x^{653}-x-1)$
761	Product	$(\mathbf{Z}/4591)[x]/(x^{761}-x-1)$
857	Product	$(\mathbf{Z}/5167)[x]/(x^{857}-x-1)$
1	Product	$(\mathbf{Z}/4096)^{636\times 636}$
3	Product	$(\mathbf{Z}/32768)^{876\times 876}$
5	Product	$(\mathbf{Z}/32768)^{1217\times 1217}$
1.0d	Product	$(\mathbf{Z}/8192)[x]/(x^{586}+\ldots+1)$
3.0d	Product	$(\mathbf{Z}/4096)[x]/(x^{852}+\ldots+1)$
5.0d	Product	$(\mathbf{Z}/8192)[x]/(x^{1170}+\ldots+1)$
1.5d	Product	$(\mathbf{Z}/1024)[x]/(x^{509}-1)$
3.5d	Product	$(\mathbf{Z}/4096)[x]/(x^{757}-1)$
5.5d	Product	$(\mathbf{Z}/2048)[x]/(x^{947}-1)$
light	Product	$((\mathbf{Z}/8192)[x]/(x^{256}+1))^{2\times 2}$
main	Product	$((\mathbf{Z}/8192)[x]/(x^{256}+1))^{3\times 3}$
fire	Product	$((\mathbf{Z}/8192)[x]/(x^{256}+1))^{4\times 4}$
653	Quotient	$(\mathbf{Z}/4621)[x]/(x^{653}-x-1)$
761	Quotient	$(\mathbf{Z}/4591)[x]/(x^{761}-x-1)$
857	Quotient	$(\mathbf{Z}/5167)[x]/(x^{857}-x-1)$
baby	Product	$(\mathbf{Z}/(2^{3120}-2^{1560}-1))^{2\times 2}$
mama	Product	$(\mathbf{Z}/(2^{3120}-2^{1560}-1))^{3\times 3}$
papa	Product	$(\mathbf{Z}/(2^{3120}-2^{1560}-1))^{4\times 4}$

short element
$\mathbf{Z}^{640\times 8}$; $\{-12,\ldots,12\}$; Pr $1,4,17,\ldots$ (spec page 23)
$\mathbf{Z}^{976\times 8}$; $\{-10,\ldots,10\}$; Pr $1,6,29,\ldots$ (spec page 23)
$\mathbf{Z}^{1344\times 8}$; $\{-6,\ldots,6\}$; Pr $2,40,364,\ldots$ (spec page 23)
$(\mathbf{Z}[x]/(x^{256}+1))^2$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^3$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^4$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{512}+1)$; $\{-1,0,1\}$; Pr $1,2,1$; weight $128,128$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr $1,6,1$; weight $128,128$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr $1,2,1$; weight $256,256$
$\mathbf{Z}[x]/(x^{512}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{509}-1)$; $\{-1,0,1\}$
$\mathbf{Z}[x]/(x^{677}-1)$; $\{-1,0,1\}$
$\mathbf{Z}[x]/(x^{821}-1)$; $\{-1,0,1\}$
$\mathbf{Z}[x]/(x^{701}-1)$; $\{-1,0,1\}$; key correlation ≥ 0
$\mathbf{Z}[x]/(x^{653}-x-1)$; $\{-1,0,1\}$; weight 252
$\mathbf{Z}[x]/(x^{761}-x-1)$; $\{-1,0,1\}$; weight 250
$\mathbf{Z}[x]/(x^{857}-x-1)$; $\{-1,0,1\}$; weight 281
$\mathbf{Z}^{636\times 8}$; $\{-1,0,1\}$; weight $57,57$
$\mathbf{Z}^{876\times 8}$; $\{-1,0,1\}$; weight $223,223$
$\mathbf{Z}^{1217\times 8}$; $\{-1,0,1\}$; weight $231,231$
$\mathbf{Z}[x]/(x^{586}+\ldots+1)$; $\{-1,0,1\}$; weight $91,91$
$\mathbf{Z}[x]/(x^{852}+\ldots+1)$; $\{-1,0,1\}$; weight $106,106$
$\mathbf{Z}[x]/(x^{1170}+\ldots+1)$; $\{-1,0,1\}$; weight $111,111$
$\mathbf{Z}[x]/(x^{509}-1)$; $\{-1,0,1\}$; weight $68,68$; ending 0
$\mathbf{Z}[x]/(x^{757}-1)$; $\{-1,0,1\}$; weight $121,121$; ending 0
$\mathbf{Z}[x]/(x^{947}-1)$; $\{-1,0,1\}$; weight $194,194$; ending 0
$(\mathbf{Z}[x]/(x^{256}+1))^2$; $\sum_{0\le i<10}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^3$; $\sum_{0\le i<8}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^4$; $\sum_{0\le i<6}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{653}-x-1)$; $\{-1,0,1\}$; weight 288
$\mathbf{Z}[x]/(x^{761}-x-1)$; $\{-1,0,1\}$; weight 286
$\mathbf{Z}[x]/(x^{857}-x-1)$; $\{-1,0,1\}$; weight 322
$\mathbf{Z}^2$; $\sum_{0\le i<312}2^{10i}\{-2,-1,0,1,2\}$; Pr $1,32,62,32,1$; *
$\mathbf{Z}^3$; $\sum_{0\le i<312}2^{10i}\{-1,0,1\}$; Pr $13,38,13$; *
$\mathbf{Z}^4$; $\sum_{0\le i<312}2^{10i}\{-1,0,1\}$; Pr $5,22,5$; *

key offset (nume
$\mathbf{Z}^{640\times 8}$; $\{-12,\ldots$
$\mathbf{Z}^{976\times 8}$; $\{-10,\ldots$
$\mathbf{Z}^{1344\times 8}$; $\{-6,\ldots$
$(\mathbf{Z}[x]/(x^{256}+1$
$(\mathbf{Z}[x]/(x^{256}+1$
$(\mathbf{Z}[x]/(x^{256}+1$
$\mathbf{Z}[x]/(x^{512}+1)$;
$\mathbf{Z}[x]/(x^{1024}+1$
$\mathbf{Z}[x]/(x^{1024}+1$
$\mathbf{Z}[x]/(x^{512}+1)$;
$\mathbf{Z}[x]/(x^{1024}+1$
$\mathbf{Z}[x]/(x^{509}-1)$;
$\mathbf{Z}[x]/(x^{677}-1)$;
$\mathbf{Z}[x]/(x^{821}-1)$;
$\mathbf{Z}[x]/(x^{701}-1)$;
round $\{-2310,\ldots$
round $\{-2295,\ldots$
round $\{-2583,\ldots$
round $\mathbf{Z}/4096$ to
round $\mathbf{Z}/32768$ t
round $\mathbf{Z}/32768$ t
round $\mathbf{Z}/8192$ to
round $\mathbf{Z}/4096$ to
round $\mathbf{Z}/8192$ to
reduce mod x^{508}
reduce mod x^{756}
reduce mod x^{946}
round $\mathbf{Z}/8192$ to
round $\mathbf{Z}/8192$ to
round $\mathbf{Z}/8192$ to
$\mathbf{Z}[x]/(x^{653}-x$
$\mathbf{Z}[x]/(x^{761}-x$
$\mathbf{Z}[x]/(x^{857}-x$
$\mathbf{Z}^2$; $\sum_{0\le i<312}2$
$\mathbf{Z}^3$; $\sum_{0\le i<312}2$
$\mathbf{Z}^4$; $\sum_{0\le i<312}2$

set of multipliers

$(\mathbf{Z}/32768)^{640\times 640}$
$(\mathbf{Z}/65536)^{976\times 976}$
$(\mathbf{Z}/65536)^{1344\times 1344}$
$((\mathbf{Z}/3329)[x]/(x^{256}+1))^{2\times 2}$
$((\mathbf{Z}/3329)[x]/(x^{256}+1))^{3\times 3}$
$((\mathbf{Z}/3329)[x]/(x^{256}+1))^{4\times 4}$
$(\mathbf{Z}/251)[x]/(x^{512}+1)$
$(\mathbf{Z}/251)[x]/(x^{1024}+1)$
$(\mathbf{Z}/251)[x]/(x^{1024}+1)$
$(\mathbf{Z}/12289)[x]/(x^{512}+1)$
$(\mathbf{Z}/12289)[x]/(x^{1024}+1)$
$(\mathbf{Z}/2048)[x]/(x^{509}-1)$
$(\mathbf{Z}/2048)[x]/(x^{677}-1)$
$(\mathbf{Z}/4096)[x]/(x^{821}-1)$
$(\mathbf{Z}/8192)[x]/(x^{701}-1)$
$(\mathbf{Z}/4621)[x]/(x^{653}-x-1)$
$(\mathbf{Z}/4591)[x]/(x^{761}-x-1)$
$(\mathbf{Z}/5167)[x]/(x^{857}-x-1)$
$(\mathbf{Z}/4096)^{636\times 636}$
$(\mathbf{Z}/32768)^{876\times 876}$
$(\mathbf{Z}/32768)^{1217\times 1217}$
$(\mathbf{Z}/8192)[x]/(x^{586}+\ldots+1)$
$(\mathbf{Z}/4096)[x]/(x^{852}+\ldots+1)$
$(\mathbf{Z}/8192)[x]/(x^{1170}+\ldots+1)$
$(\mathbf{Z}/1024)[x]/(x^{509}-1)$
$(\mathbf{Z}/4096)[x]/(x^{757}-1)$
$(\mathbf{Z}/2048)[x]/(x^{947}-1)$
$((\mathbf{Z}/8192)[x]/(x^{256}+1))^{2\times 2}$
$((\mathbf{Z}/8192)[x]/(x^{256}+1))^{3\times 3}$
$((\mathbf{Z}/8192)[x]/(x^{256}+1))^{4\times 4}$
$(\mathbf{Z}/4621)[x]/(x^{653}-x-1)$
$(\mathbf{Z}/4591)[x]/(x^{761}-x-1)$
$(\mathbf{Z}/5167)[x]/(x^{857}-x-1)$
$(\mathbf{Z}/(2^{3120}-2^{1560}-1))^{2\times 2}$
$(\mathbf{Z}/(2^{3120}-2^{1560}-1))^{3\times 3}$
$(\mathbf{Z}/(2^{3120}-2^{1560}-1))^{4\times 4}$

short element

$\mathbf{Z}^{640\times 8}$; $\{-12,\ldots,12\}$; Pr $1,4,17,\ldots$ (spec page 23)
$\mathbf{Z}^{976\times 8}$; $\{-10,\ldots,10\}$; Pr $1,6,29,\ldots$ (spec page 23)
$\mathbf{Z}^{1344\times 8}$; $\{-6,\ldots,6\}$; Pr $2,40,364,\ldots$ (spec page 23)
$(\mathbf{Z}[x]/(x^{256}+1))^2$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^3$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^4$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{512}+1)$; $\{-1,0,1\}$; Pr $1,2,1$; weight $128,128$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr $1,6,1$; weight $128,128$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr $1,2,1$; weight $256,256$
$\mathbf{Z}[x]/(x^{512}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{509}-1)$; $\{-1,0,1\}$
$\mathbf{Z}[x]/(x^{677}-1)$; $\{-1,0,1\}$
$\mathbf{Z}[x]/(x^{821}-1)$; $\{-1,0,1\}$
$\mathbf{Z}[x]/(x^{701}-1)$; $\{-1,0,1\}$; key correlation ≥ 0
$\mathbf{Z}[x]/(x^{653}-x-1)$; $\{-1,0,1\}$; weight 252
$\mathbf{Z}[x]/(x^{761}-x-1)$; $\{-1,0,1\}$; weight 250
$\mathbf{Z}[x]/(x^{857}-x-1)$; $\{-1,0,1\}$; weight 281
$\mathbf{Z}^{636\times 8}$; $\{-1,0,1\}$; weight $57,57$
$\mathbf{Z}^{876\times 8}$; $\{-1,0,1\}$; weight $223,223$
$\mathbf{Z}^{1217\times 8}$; $\{-1,0,1\}$; weight $231,231$
$\mathbf{Z}[x]/(x^{586}+\ldots+1)$; $\{-1,0,1\}$; weight $91,91$
$\mathbf{Z}[x]/(x^{852}+\ldots+1)$; $\{-1,0,1\}$; weight $106,106$
$\mathbf{Z}[x]/(x^{1170}+\ldots+1)$; $\{-1,0,1\}$; weight $111,111$
$\mathbf{Z}[x]/(x^{509}-1)$; $\{-1,0,1\}$; weight $68,68$; ending 0
$\mathbf{Z}[x]/(x^{757}-1)$; $\{-1,0,1\}$; weight $121,121$; ending 0
$\mathbf{Z}[x]/(x^{947}-1)$; $\{-1,0,1\}$; weight $194,194$; ending 0
$(\mathbf{Z}[x]/(x^{256}+1))^2$; $\sum_{0\le i<10}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^3$; $\sum_{0\le i<8}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^4$; $\sum_{0\le i<6}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{653}-x-1)$; $\{-1,0,1\}$; weight 288
$\mathbf{Z}[x]/(x^{761}-x-1)$; $\{-1,0,1\}$; weight 286
$\mathbf{Z}[x]/(x^{857}-x-1)$; $\{-1,0,1\}$; weight 322
$\mathbf{Z}^2$; $\sum_{0\le i<312}2^{10i}\{-2,-1,0,1,2\}$; Pr $1,32,62,32,1$; *
$\mathbf{Z}^3$; $\sum_{0\le i<312}2^{10i}\{-1,0,1\}$; Pr $13,38,13$; *
$\mathbf{Z}^4$; $\sum_{0\le i<312}2^{10i}\{-1,0,1\}$; Pr $5,22,5$; *

key offset (numerator or noise or rou

$\mathbf{Z}^{640\times 8}$; $\{-12,\ldots,12\}$; Pr $1,4,17,$.
$\mathbf{Z}^{976\times 8}$; $\{-10,\ldots,10\}$; Pr $1,6,29,$.
$\mathbf{Z}^{1344\times 8}$; $\{-6,\ldots,6\}$; Pr $2,40,364,$
$(\mathbf{Z}[x]/(x^{256}+1))^2$; $\sum_{0\le i<4}\{-0.5,$
$(\mathbf{Z}[x]/(x^{256}+1))^3$; $\sum_{0\le i<4}\{-0.5,$
$(\mathbf{Z}[x]/(x^{256}+1))^4$; $\sum_{0\le i<4}\{-0.5,$
$\mathbf{Z}[x]/(x^{512}+1)$; $\{-1,0,1\}$; Pr $1,2,$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr 1, 6
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr 1, 2
$\mathbf{Z}[x]/(x^{512}+1)$; $\sum_{0\le i<16}\{-0.5,0.5$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\sum_{0\le i<16}\{-0.5,0.$
$\mathbf{Z}[x]/(x^{509}-1)$; $\{-1,0,1\}$; weight
$\mathbf{Z}[x]/(x^{677}-1)$; $\{-1,0,1\}$; weight
$\mathbf{Z}[x]/(x^{821}-1)$; $\{-1,0,1\}$; weight
$\mathbf{Z}[x]/(x^{701}-1)$; $\{-1,0,1\}$; key corr
round $\{-2310,\ldots,2310\}$ to $3\mathbf{Z}$
round $\{-2295,\ldots,2295\}$ to $3\mathbf{Z}$
round $\{-2583,\ldots,2583\}$ to $3\mathbf{Z}$
round $\mathbf{Z}/4096$ to $8\mathbf{Z}$
round $\mathbf{Z}/32768$ to $16\mathbf{Z}$
round $\mathbf{Z}/32768$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $16\mathbf{Z}$
round $\mathbf{Z}/4096$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $16\mathbf{Z}$
reduce mod $x^{508}+\ldots+1$; round $\mathbf{Z}$
reduce mod $x^{756}+\ldots+1$; round $\mathbf{Z}$
reduce mod $x^{946}+\ldots+1$; round $\mathbf{Z}$
round $\mathbf{Z}/8192$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $8\mathbf{Z}$
$\mathbf{Z}[x]/(x^{653}-x-1)$; $\{-1,0,1\}$; inve
$\mathbf{Z}[x]/(x^{761}-x-1)$; $\{-1,0,1\}$; inve
$\mathbf{Z}[x]/(x^{857}-x-1)$; $\{-1,0,1\}$; inve
$\mathbf{Z}^2$; $\sum_{0\le i<312}2^{10i}\{-2,-1,0,1,2\}$;
$\mathbf{Z}^3$; $\sum_{0\le i<312}2^{10i}\{-1,0,1\}$; Pr 13,
$\mathbf{Z}^4$; $\sum_{0\le i<312}2^{10i}\{-1,0,1\}$; Pr 5, 2

$+1))^{2\times 2}$
$+1))^{3\times 3}$
$+1))^{4\times 4}$
$1)$
$+1)$
$+1)$
$+1)$
$^4+1)$
$-1)$
$-1)$
$-1)$
$-1)$
$-x-1)$
$-x-1)$
$-x-1)$
$+\ldots+1)$
$+\ldots+1)$
$+\ldots+1)$
$-1)$
$-1)$
$-1)$
$+1))^{2\times 2}$
$+1))^{3\times 3}$
$+1))^{4\times 4}$
$-x-1)$
$-x-1)$
$-x-1)$
$-1))^{2\times 2}$
$-1))^{3\times 3}$
$-1))^{4\times 4}$

short element
$\mathbf{Z}^{640\times 8}$; $\{-12,\ldots,12\}$; Pr $1,4,17,\ldots$ (spec page 23)
$\mathbf{Z}^{976\times 8}$; $\{-10,\ldots,10\}$; Pr $1,6,29,\ldots$ (spec page 23)
$\mathbf{Z}^{1344\times 8}$; $\{-6,\ldots,6\}$; Pr $2,40,364,\ldots$ (spec page 23)
$(\mathbf{Z}[x]/(x^{256}+1))^2$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^3$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^4$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{512}+1)$; $\{-1,0,1\}$; Pr $1,2,1$; weight $128,128$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr $1,6,1$; weight $128,128$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr $1,2,1$; weight $256,256$
$\mathbf{Z}[x]/(x^{512}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{509}-1)$; $\{-1,0,1\}$
$\mathbf{Z}[x]/(x^{677}-1)$; $\{-1,0,1\}$
$\mathbf{Z}[x]/(x^{821}-1)$; $\{-1,0,1\}$
$\mathbf{Z}[x]/(x^{701}-1)$; $\{-1,0,1\}$; key correlation ≥ 0
$\mathbf{Z}[x]/(x^{653}-x-1)$; $\{-1,0,1\}$; weight 252
$\mathbf{Z}[x]/(x^{761}-x-1)$; $\{-1,0,1\}$; weight 250
$\mathbf{Z}[x]/(x^{857}-x-1)$; $\{-1,0,1\}$; weight 281
$\mathbf{Z}^{636\times 8}$; $\{-1,0,1\}$; weight $57,57$
$\mathbf{Z}^{876\times 8}$; $\{-1,0,1\}$; weight $223,223$
$\mathbf{Z}^{1217\times 8}$; $\{-1,0,1\}$; weight $231,231$
$\mathbf{Z}[x]/(x^{586}+\ldots+1)$; $\{-1,0,1\}$; weight $91,91$
$\mathbf{Z}[x]/(x^{852}+\ldots+1)$; $\{-1,0,1\}$; weight $106,106$
$\mathbf{Z}[x]/(x^{1170}+\ldots+1)$; $\{-1,0,1\}$; weight $111,111$
$\mathbf{Z}[x]/(x^{509}-1)$; $\{-1,0,1\}$; weight $68,68$; ending 0
$\mathbf{Z}[x]/(x^{757}-1)$; $\{-1,0,1\}$; weight $121,121$; ending 0
$\mathbf{Z}[x]/(x^{947}-1)$; $\{-1,0,1\}$; weight $194,194$; ending 0
$(\mathbf{Z}[x]/(x^{256}+1))^2$; $\sum_{0\le i<10}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^3$; $\sum_{0\le i<8}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^4$; $\sum_{0\le i<6}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{653}-x-1)$; $\{-1,0,1\}$; weight 288
$\mathbf{Z}[x]/(x^{761}-x-1)$; $\{-1,0,1\}$; weight 286
$\mathbf{Z}[x]/(x^{857}-x-1)$; $\{-1,0,1\}$; weight 322
$\mathbf{Z}^2$; $\sum_{0\le i<312} 2^{10i}\{-2,-1,0,1,2\}$; Pr $1,32,62,32,1$; *
$\mathbf{Z}^3$; $\sum_{0\le i<312} 2^{10i}\{-1,0,1\}$; Pr $13,38,13$; *
$\mathbf{Z}^4$; $\sum_{0\le i<312} 2^{10i}\{-1,0,1\}$; Pr $5,22,5$; *

key offset (numerator or noise or rounding method)
$\mathbf{Z}^{640\times 8}$; $\{-12,\ldots,12\}$; Pr $1,4,17,\ldots$ (spec page 23)
$\mathbf{Z}^{976\times 8}$; $\{-10,\ldots,10\}$; Pr $1,6,29,\ldots$ (spec page 23)
$\mathbf{Z}^{1344\times 8}$; $\{-6,\ldots,6\}$; Pr $2,40,364,\ldots$ (spec page 23)
$(\mathbf{Z}[x]/(x^{256}+1))^2$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^3$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^4$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{512}+1)$; $\{-1,0,1\}$; Pr $1,2,1$; weight $128,128$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr $1,6,1$; weight $128,128$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr $1,2,1$; weight $256,256$
$\mathbf{Z}[x]/(x^{512}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{509}-1)$; $\{-1,0,1\}$; weight $127,127$
$\mathbf{Z}[x]/(x^{677}-1)$; $\{-1,0,1\}$; weight $127,127$
$\mathbf{Z}[x]/(x^{821}-1)$; $\{-1,0,1\}$; weight $255,255$
$\mathbf{Z}[x]/(x^{701}-1)$; $\{-1,0,1\}$; key correlation ≥ 0; $\cdot(x-1$
round $\{-2310,\ldots,2310\}$ to $3\mathbf{Z}$
round $\{-2295,\ldots,2295\}$ to $3\mathbf{Z}$
round $\{-2583,\ldots,2583\}$ to $3\mathbf{Z}$
round $\mathbf{Z}/4096$ to $8\mathbf{Z}$
round $\mathbf{Z}/32768$ to $16\mathbf{Z}$
round $\mathbf{Z}/32768$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $16\mathbf{Z}$
round $\mathbf{Z}/4096$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $16\mathbf{Z}$
reduce mod $x^{508}+\ldots+1$; round $\mathbf{Z}/1024$ to $8\mathbf{Z}$
reduce mod $x^{756}+\ldots+1$; round $\mathbf{Z}/4096$ to $16\mathbf{Z}$
reduce mod $x^{946}+\ldots+1$; round $\mathbf{Z}/2048$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $8\mathbf{Z}$
$\mathbf{Z}[x]/(x^{653}-x-1)$; $\{-1,0,1\}$; invertible mod 3
$\mathbf{Z}[x]/(x^{761}-x-1)$; $\{-1,0,1\}$; invertible mod 3
$\mathbf{Z}[x]/(x^{857}-x-1)$; $\{-1,0,1\}$; invertible mod 3
$\mathbf{Z}^2$; $\sum_{0\le i<312} 2^{10i}\{-2,-1,0,1,2\}$; Pr $1,32,62,32,1$; *
$\mathbf{Z}^3$; $\sum_{0\le i<312} 2^{10i}\{-1,0,1\}$; Pr $13,38,13$; *
$\mathbf{Z}^4$; $\sum_{0\le i<312} 2^{10i}\{-1,0,1\}$; Pr $5,22,5$; *

short element

$\mathbf{Z}^{640\times 8}$; $\{-12,\ldots,12\}$; Pr $1,4,17,\ldots$ (spec page 23)
$\mathbf{Z}^{976\times 8}$; $\{-10,\ldots,10\}$; Pr $1,6,29,\ldots$ (spec page 23)
$\mathbf{Z}^{1344\times 8}$; $\{-6,\ldots,6\}$; Pr $2,40,364,\ldots$ (spec page 23)
$(\mathbf{Z}[x]/(x^{256}+1))^2$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^3$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^4$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{512}+1)$; $\{-1,0,1\}$; Pr $1,2,1$; weight $128,128$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr $1,6,1$; weight $128,128$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr $1,2,1$; weight $256,256$
$\mathbf{Z}[x]/(x^{512}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{509}-1)$; $\{-1,0,1\}$
$\mathbf{Z}[x]/(x^{677}-1)$; $\{-1,0,1\}$
$\mathbf{Z}[x]/(x^{821}-1)$; $\{-1,0,1\}$
$\mathbf{Z}[x]/(x^{701}-1)$; $\{-1,0,1\}$; key correlation ≥ 0
$\mathbf{Z}[x]/(x^{653}-x-1)$; $\{-1,0,1\}$; weight 252
$\mathbf{Z}[x]/(x^{761}-x-1)$; $\{-1,0,1\}$; weight 250
$\mathbf{Z}[x]/(x^{857}-x-1)$; $\{-1,0,1\}$; weight 281
$\mathbf{Z}^{636\times 8}$; $\{-1,0,1\}$; weight $57,57$
$\mathbf{Z}^{876\times 8}$; $\{-1,0,1\}$; weight $223,223$
$\mathbf{Z}^{1217\times 8}$; $\{-1,0,1\}$; weight $231,231$
$\mathbf{Z}[x]/(x^{586}+\ldots+1)$; $\{-1,0,1\}$; weight $91,91$
$\mathbf{Z}[x]/(x^{852}+\ldots+1)$; $\{-1,0,1\}$; weight $106,106$
$\mathbf{Z}[x]/(x^{1170}+\ldots+1)$; $\{-1,0,1\}$; weight $111,111$
$\mathbf{Z}[x]/(x^{509}-1)$; $\{-1,0,1\}$; weight $68,68$; ending 0
$\mathbf{Z}[x]/(x^{757}-1)$; $\{-1,0,1\}$; weight $121,121$; ending 0
$\mathbf{Z}[x]/(x^{947}-1)$; $\{-1,0,1\}$; weight $194,194$; ending 0
$(\mathbf{Z}[x]/(x^{256}+1))^2$; $\sum_{0\le i<10}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^3$; $\sum_{0\le i<8}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^4$; $\sum_{0\le i<6}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{653}-x-1)$; $\{-1,0,1\}$; weight 288
$\mathbf{Z}[x]/(x^{761}-x-1)$; $\{-1,0,1\}$; weight 286
$\mathbf{Z}[x]/(x^{857}-x-1)$; $\{-1,0,1\}$; weight 322
$\mathbf{Z}^2$; $\sum_{0\le i<312}2^{10i}\{-2,-1,0,1,2\}$; Pr $1,32,62,32,1$; *
$\mathbf{Z}^3$; $\sum_{0\le i<312}2^{10i}\{-1,0,1\}$; Pr $13,38,13$; *
$\mathbf{Z}^4$; $\sum_{0\le i<312}2^{10i}\{-1,0,1\}$; Pr $5,22,5$; *

key offset (numerator or noise or rounding method)

$\mathbf{Z}^{640\times 8}$; $\{-12,\ldots,12\}$; Pr $1,4,17,\ldots$ (spec page 23)
$\mathbf{Z}^{976\times 8}$; $\{-10,\ldots,10\}$; Pr $1,6,29,\ldots$ (spec page 23)
$\mathbf{Z}^{1344\times 8}$; $\{-6,\ldots,6\}$; Pr $2,40,364,\ldots$ (spec page 23)
$(\mathbf{Z}[x]/(x^{256}+1))^2$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^3$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^4$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{512}+1)$; $\{-1,0,1\}$; Pr $1,2,1$; weight $128,128$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr $1,6,1$; weight $128,128$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr $1,2,1$; weight $256,256$
$\mathbf{Z}[x]/(x^{512}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{509}-1)$; $\{-1,0,1\}$; weight $127,127$
$\mathbf{Z}[x]/(x^{677}-1)$; $\{-1,0,1\}$; weight $127,127$
$\mathbf{Z}[x]/(x^{821}-1)$; $\{-1,0,1\}$; weight $255,255$
$\mathbf{Z}[x]/(x^{701}-1)$; $\{-1,0,1\}$; key correlation ≥ 0; $\cdot(x-1)$
round $\{-2310,\ldots,2310\}$ to $3\mathbf{Z}$
round $\{-2295,\ldots,2295\}$ to $3\mathbf{Z}$
round $\{-2583,\ldots,2583\}$ to $3\mathbf{Z}$
round $\mathbf{Z}/4096$ to $8\mathbf{Z}$
round $\mathbf{Z}/32768$ to $16\mathbf{Z}$
round $\mathbf{Z}/32768$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $16\mathbf{Z}$
round $\mathbf{Z}/4096$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $16\mathbf{Z}$
reduce mod $x^{508}+\ldots+1$; round $\mathbf{Z}/1024$ to $8\mathbf{Z}$
reduce mod $x^{756}+\ldots+1$; round $\mathbf{Z}/4096$ to $16\mathbf{Z}$
reduce mod $x^{946}+\ldots+1$; round $\mathbf{Z}/2048$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $8\mathbf{Z}$
$\mathbf{Z}[x]/(x^{653}-x-1)$; $\{-1,0,1\}$; invertible mod 3
$\mathbf{Z}[x]/(x^{761}-x-1)$; $\{-1,0,1\}$; invertible mod 3
$\mathbf{Z}[x]/(x^{857}-x-1)$; $\{-1,0,1\}$; invertible mod 3
$\mathbf{Z}^2$; $\sum_{0\le i<312}2^{10i}\{-2,-1,0,1,2\}$; Pr $1,32,62,32,1$; *
$\mathbf{Z}^3$; $\sum_{0\le i<312}2^{10i}\{-1,0,1\}$; Pr $13,38,13$; *
$\mathbf{Z}^4$; $\sum_{0\le i<312}2^{10i}\{-1,0,1\}$; Pr $5,22,5$; *

$\ldots, 12\}$; Pr $1, 4, 17, \ldots$ (spec page 23)
$\ldots, 10\}$; Pr $1, 6, 29, \ldots$ (spec page 23)
$\ldots, 6\}$; Pr $2, 40, 364, \ldots$ (spec page 23)
$))^2$; $\sum_{0\le i<4}\{-0.5, 0.5\}$
$))^3$; $\sum_{0\le i<4}\{-0.5, 0.5\}$
$))^4$; $\sum_{0\le i<4}\{-0.5, 0.5\}$
$\{-1, 0, 1\}$; Pr $1, 2, 1$; weight $128, 128$
$)$; $\{-1, 0, 1\}$; Pr $1, 6, 1$; weight $128, 128$
$)$; $\{-1, 0, 1\}$; Pr $1, 2, 1$; weight $256, 256$
$\sum_{0\le i<16}\{-0.5, 0.5\}$
$)$; $\sum_{0\le i<16}\{-0.5, 0.5\}$
$\{-1, 0, 1\}$
$\{-1, 0, 1\}$
$\{-1, 0, 1\}$
$\{-1, 0, 1\}$; key correlation ≥ 0
$-1)$; $\{-1, 0, 1\}$; weight 252
$-1)$; $\{-1, 0, 1\}$; weight 250
$-1)$; $\{-1, 0, 1\}$; weight 281
$1\}$; weight $57, 57$
$1\}$; weight $223, 223$
$1\}$; weight $231, 231$
$+1)$; $\{-1, 0, 1\}$; weight $91, 91$
$+1)$; $\{-1, 0, 1\}$; weight $106, 106$
$.+1)$; $\{-1, 0, 1\}$; weight $111, 111$
$\{-1, 0, 1\}$; weight $68, 68$; ending 0
$\{-1, 0, 1\}$; weight $121, 121$; ending 0
$\{-1, 0, 1\}$; weight $194, 194$; ending 0
$))^2$; $\sum_{0\le i<10}\{-0.5, 0.5\}$
$))^3$; $\sum_{0\le i<8}\{-0.5, 0.5\}$
$))^4$; $\sum_{0\le i<6}\{-0.5, 0.5\}$
$-1)$; $\{-1, 0, 1\}$; weight 288
$-1)$; $\{-1, 0, 1\}$; weight 286
$-1)$; $\{-1, 0, 1\}$; weight 322
${}^{0i}\{-2, -1, 0, 1, 2\}$; Pr $1, 32, 62, 32, 1$; *
${}^{0i}\{-1, 0, 1\}$; Pr $13, 38, 13$; *
${}^{0i}\{-1, 0, 1\}$; Pr $5, 22, 5$; *

key offset (numerator or noise or rounding method)

$\mathbf{Z}^{640\times 8}$; $\{-12, \ldots, 12\}$; Pr $1, 4, 17, \ldots$ (spec page 23)
$\mathbf{Z}^{976\times 8}$; $\{-10, \ldots, 10\}$; Pr $1, 6, 29, \ldots$ (spec page 23)
$\mathbf{Z}^{1344\times 8}$; $\{-6, \ldots, 6\}$; Pr $2, 40, 364, \ldots$ (spec page 23)
$(\mathbf{Z}[x]/(x^{256}+1))^2$; $\sum_{0\le i<4}\{-0.5, 0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^3$; $\sum_{0\le i<4}\{-0.5, 0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^4$; $\sum_{0\le i<4}\{-0.5, 0.5\}$
$\mathbf{Z}[x]/(x^{512}+1)$; $\{-1, 0, 1\}$; Pr $1, 2, 1$; weight $128, 128$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1, 0, 1\}$; Pr $1, 6, 1$; weight $128, 128$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1, 0, 1\}$; Pr $1, 2, 1$; weight $256, 256$
$\mathbf{Z}[x]/(x^{512}+1)$; $\sum_{0\le i<16}\{-0.5, 0.5\}$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\sum_{0\le i<16}\{-0.5, 0.5\}$
$\mathbf{Z}[x]/(x^{509}-1)$; $\{-1, 0, 1\}$; weight $127, 127$
$\mathbf{Z}[x]/(x^{677}-1)$; $\{-1, 0, 1\}$; weight $127, 127$
$\mathbf{Z}[x]/(x^{821}-1)$; $\{-1, 0, 1\}$; weight $255, 255$
$\mathbf{Z}[x]/(x^{701}-1)$; $\{-1, 0, 1\}$; key correlation ≥ 0; $\cdot(x-1)$
round $\{-2310, \ldots, 2310\}$ to $3\mathbf{Z}$
round $\{-2295, \ldots, 2295\}$ to $3\mathbf{Z}$
round $\{-2583, \ldots, 2583\}$ to $3\mathbf{Z}$
round $\mathbf{Z}/4096$ to $8\mathbf{Z}$
round $\mathbf{Z}/32768$ to $16\mathbf{Z}$
round $\mathbf{Z}/32768$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $16\mathbf{Z}$
round $\mathbf{Z}/4096$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $16\mathbf{Z}$
reduce mod $x^{508}+\ldots+1$; round $\mathbf{Z}/1024$ to $8\mathbf{Z}$
reduce mod $x^{756}+\ldots+1$; round $\mathbf{Z}/4096$ to $16\mathbf{Z}$
reduce mod $x^{946}+\ldots+1$; round $\mathbf{Z}/2048$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $8\mathbf{Z}$
$\mathbf{Z}[x]/(x^{653}-x-1)$; $\{-1, 0, 1\}$; invertible mod 3
$\mathbf{Z}[x]/(x^{761}-x-1)$; $\{-1, 0, 1\}$; invertible mod 3
$\mathbf{Z}[x]/(x^{857}-x-1)$; $\{-1, 0, 1\}$; invertible mod 3
$\mathbf{Z}^2$; $\sum_{0\le i<312} 2^{10i}\{-2, -1, 0, 1, 2\}$; Pr $1, 32, 62, 32, 1$; *
$\mathbf{Z}^3$; $\sum_{0\le i<312} 2^{10i}\{-1, 0, 1\}$; Pr $13, 38, 13$; *
$\mathbf{Z}^4$; $\sum_{0\le i<312} 2^{10i}\{-1, 0, 1\}$; Pr $5, 22, 5$; *

ciphertext offset

$\mathbf{Z}^{8\times 8}$; $\{-12, \ldots$
$\mathbf{Z}^{8\times 8}$; $\{-10, \ldots$
$\mathbf{Z}^{8\times 8}$; $\{-6, \ldots,$
$\mathbf{Z}[x]/(x^{256}+1)$;
$\mathbf{Z}[x]/(x^{256}+1)$;
$\mathbf{Z}[x]/(x^{256}+1)$;
$\mathbf{Z}[x]/(x^{512}+1)$;
$\mathbf{Z}[x]/(x^{1024}+1$
$\mathbf{Z}[x]/(x^{1024}+1$
$\mathbf{Z}[x]/(x^{512}+1)$;
$\mathbf{Z}[x]/(x^{1024}+1$
not applicable
not applicable
not applicable
not applicable
bottom 256 coef
bottom 256 coef
bottom 256 coef
round $\mathbf{Z}/4096$ to
round $\mathbf{Z}/32768$ t
round $\mathbf{Z}/32768$ t
bottom 128 coef
bottom 192 coef
bottom 256 coef
bottom 318 coef
bottom 410 coef
bottom 490 coef
round $\mathbf{Z}/8192$ to
round $\mathbf{Z}/8192$ to
round $\mathbf{Z}/8192$ to
not applicable
not applicable
not applicable
$\mathbf{Z}$; $\sum_{0\le i<312} 2^{10}$
$\mathbf{Z}$; $\sum_{0\le i<312} 2^{10}$
$\mathbf{Z}$; $\sum_{0\le i<312} 2^{10}$

. . (spec page 23)
. . (spec page 23)
. . . (spec page 23)
$0.5\}$
$0.5\}$
$0.5\}$
1; weight 128, 128
, 1; weight 128, 128
, 1; weight 256, 256
$5\}$
$.5\}$

relation ≥ 0
ght 252
ght 250
ght 281

eight 91, 91
eight 106, 106
veight 111, 111
68, 68; ending 0
121, 121; ending 0
194, 194; ending 0
$0.5\}$
$0.5\}$
$0.5\}$
ght 288
ght 286
ght 322
Pr 1, 32, 62, 32, 1; *
38, 13; *
22, 5; *

key offset (numerator or noise or rounding method)

$\mathbf{Z}^{640\times 8}$; $\{-12,\ldots,12\}$; Pr $1,4,17,\ldots$ (spec page 23)
$\mathbf{Z}^{976\times 8}$; $\{-10,\ldots,10\}$; Pr $1,6,29,\ldots$ (spec page 23)
$\mathbf{Z}^{1344\times 8}$; $\{-6,\ldots,6\}$; Pr $2,40,364,\ldots$ (spec page 23)
$(\mathbf{Z}[x]/(x^{256}+1))^2$; $\sum_{0\leq i<4}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^3$; $\sum_{0\leq i<4}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^4$; $\sum_{0\leq i<4}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{512}+1)$; $\{-1,0,1\}$; Pr 1, 2, 1; weight 128, 128
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr 1, 6, 1; weight 128, 128
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr 1, 2, 1; weight 256, 256
$\mathbf{Z}[x]/(x^{512}+1)$; $\sum_{0\leq i<16}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\sum_{0\leq i<16}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{509}-1)$; $\{-1,0,1\}$; weight 127, 127
$\mathbf{Z}[x]/(x^{677}-1)$; $\{-1,0,1\}$; weight 127, 127
$\mathbf{Z}[x]/(x^{821}-1)$; $\{-1,0,1\}$; weight 255, 255
$\mathbf{Z}[x]/(x^{701}-1)$; $\{-1,0,1\}$; key correlation ≥ 0; $\cdot(x-1)$
round $\{-2310,\ldots,2310\}$ to $3\mathbf{Z}$
round $\{-2295,\ldots,2295\}$ to $3\mathbf{Z}$
round $\{-2583,\ldots,2583\}$ to $3\mathbf{Z}$
round $\mathbf{Z}/4096$ to $8\mathbf{Z}$
round $\mathbf{Z}/32768$ to $16\mathbf{Z}$
round $\mathbf{Z}/32768$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $16\mathbf{Z}$
round $\mathbf{Z}/4096$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $16\mathbf{Z}$
reduce mod $x^{508}+\ldots+1$; round $\mathbf{Z}/1024$ to $8\mathbf{Z}$
reduce mod $x^{756}+\ldots+1$; round $\mathbf{Z}/4096$ to $16\mathbf{Z}$
reduce mod $x^{946}+\ldots+1$; round $\mathbf{Z}/2048$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $8\mathbf{Z}$
$\mathbf{Z}[x]/(x^{653}-x-1)$; $\{-1,0,1\}$; invertible mod 3
$\mathbf{Z}[x]/(x^{761}-x-1)$; $\{-1,0,1\}$; invertible mod 3
$\mathbf{Z}[x]/(x^{857}-x-1)$; $\{-1,0,1\}$; invertible mod 3
$\mathbf{Z}^2$; $\sum_{0\leq i<312} 2^{10i}\{-2,-1,0,1,2\}$; Pr 1, 32, 62, 32, 1; *
$\mathbf{Z}^3$; $\sum_{0\leq i<312} 2^{10i}\{-1,0,1\}$; Pr 13, 38, 13; *
$\mathbf{Z}^4$; $\sum_{0\leq i<312} 2^{10i}\{-1,0,1\}$; Pr 5, 22, 5; *

ciphertext offset (noise or rounding r

$\mathbf{Z}^{8\times 8}$; $\{-12,\ldots,12\}$; Pr $1,4,17,\ldots$
$\mathbf{Z}^{8\times 8}$; $\{-10,\ldots,10\}$; Pr $1,6,29,\ldots$
$\mathbf{Z}^{8\times 8}$; $\{-6,\ldots,6\}$; Pr $2,40,364,\ldots$
$\mathbf{Z}[x]/(x^{256}+1)$; $\sum_{0\leq i<4}\{-0.5,0.5$
$\mathbf{Z}[x]/(x^{256}+1)$; $\sum_{0\leq i<4}\{-0.5,0.5$
$\mathbf{Z}[x]/(x^{256}+1)$; $\sum_{0\leq i<4}\{-0.5,0.5$
$\mathbf{Z}[x]/(x^{512}+1)$; $\{-1,0,1\}$; Pr 1, 2,
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr 1, 6
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr 1, 2
$\mathbf{Z}[x]/(x^{512}+1)$; $\sum_{0\leq i<16}\{-0.5,0.5$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\sum_{0\leq i<16}\{-0.5,0.$
not applicable
not applicable
not applicable
not applicable
bottom 256 coeffs; $z\mapsto\lfloor(114(z+2$
bottom 256 coeffs; $z\mapsto\lfloor(113(z+2$
bottom 256 coeffs; $z\mapsto\lfloor(101(z+2$
round $\mathbf{Z}/4096$ to $64\mathbf{Z}$
round $\mathbf{Z}/32768$ to $512\mathbf{Z}$
round $\mathbf{Z}/32768$ to $64\mathbf{Z}$
bottom 128 coeffs; round $\mathbf{Z}/8192$ to
bottom 192 coeffs; round $\mathbf{Z}/4096$ to
bottom 256 coeffs; round $\mathbf{Z}/8192$ to
bottom 318 coeffs; round $\mathbf{Z}/1024$ to
bottom 410 coeffs; round $\mathbf{Z}/4096$ to
bottom 490 coeffs; round $\mathbf{Z}/2048$ to
round $\mathbf{Z}/8192$ to $1024\mathbf{Z}$
round $\mathbf{Z}/8192$ to $512\mathbf{Z}$
round $\mathbf{Z}/8192$ to $128\mathbf{Z}$
not applicable
not applicable
not applicable
$\mathbf{Z}$; $\sum_{0\leq i<312} 2^{10i}\{-2,-1,0,1,2\}$; P
$\mathbf{Z}$; $\sum_{0\leq i<312} 2^{10i}\{-1,0,1\}$; Pr 13, 3
$\mathbf{Z}$; $\sum_{0\leq i<312} 2^{10i}\{-1,0,1\}$; Pr 5, 22

key offset (numerator or noise or rounding method)

$\mathbf{Z}^{640\times 8}$; $\{-12,\ldots,12\}$; Pr $1,4,17,\ldots$ (spec page 23)
$\mathbf{Z}^{976\times 8}$; $\{-10,\ldots,10\}$; Pr $1,6,29,\ldots$ (spec page 23)
$\mathbf{Z}^{1344\times 8}$; $\{-6,\ldots,6\}$; Pr $2,40,364,\ldots$ (spec page 23)
$(\mathbf{Z}[x]/(x^{256}+1))^2$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^3$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^4$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{512}+1)$; $\{-1,0,1\}$; Pr $1,2,1$; weight $128,128$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr $1,6,1$; weight $128,128$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr $1,2,1$; weight $256,256$
$\mathbf{Z}[x]/(x^{512}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{509}-1)$; $\{-1,0,1\}$; weight $127,127$
$\mathbf{Z}[x]/(x^{677}-1)$; $\{-1,0,1\}$; weight $127,127$
$\mathbf{Z}[x]/(x^{821}-1)$; $\{-1,0,1\}$; weight $255,255$
$\mathbf{Z}[x]/(x^{701}-1)$; $\{-1,0,1\}$; key correlation ≥ 0; $\cdot(x-1)$
round $\{-2310,\ldots,2310\}$ to $3\mathbf{Z}$
round $\{-2295,\ldots,2295\}$ to $3\mathbf{Z}$
round $\{-2583,\ldots,2583\}$ to $3\mathbf{Z}$
round $\mathbf{Z}/4096$ to $8\mathbf{Z}$
round $\mathbf{Z}/32768$ to $16\mathbf{Z}$
round $\mathbf{Z}/32768$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $16\mathbf{Z}$
round $\mathbf{Z}/4096$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $16\mathbf{Z}$
reduce mod $x^{508}+\ldots+1$; round $\mathbf{Z}/1024$ to $8\mathbf{Z}$
reduce mod $x^{756}+\ldots+1$; round $\mathbf{Z}/4096$ to $16\mathbf{Z}$
reduce mod $x^{946}+\ldots+1$; round $\mathbf{Z}/2048$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $8\mathbf{Z}$
$\mathbf{Z}[x]/(x^{653}-x-1)$; $\{-1,0,1\}$; invertible mod 3
$\mathbf{Z}[x]/(x^{761}-x-1)$; $\{-1,0,1\}$; invertible mod 3
$\mathbf{Z}[x]/(x^{857}-x-1)$; $\{-1,0,1\}$; invertible mod 3
$\mathbf{Z}^2$; $\sum_{0\le i<312}2^{10i}\{-2,-1,0,1,2\}$; Pr $1,32,62,32,1$; *
$\mathbf{Z}^3$; $\sum_{0\le i<312}2^{10i}\{-1,0,1\}$; Pr $13,38,13$; *
$\mathbf{Z}^4$; $\sum_{0\le i<312}2^{10i}\{-1,0,1\}$; Pr $5,22,5$; *

ciphertext offset (noise or rounding method)

$\mathbf{Z}^{8\times 8}$; $\{-12,\ldots,12\}$; Pr $1,4,17,\ldots$ (spec page 23)
$\mathbf{Z}^{8\times 8}$; $\{-10,\ldots,10\}$; Pr $1,6,29,\ldots$ (spec page 23)
$\mathbf{Z}^{8\times 8}$; $\{-6,\ldots,6\}$; Pr $2,40,364,\ldots$ (spec page 23)
$\mathbf{Z}[x]/(x^{256}+1)$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{256}+1)$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{256}+1)$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{512}+1)$; $\{-1,0,1\}$; Pr $1,2,1$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr $1,6,1$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr $1,2,1$
$\mathbf{Z}[x]/(x^{512}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
not applicable
not applicable
not applicable
not applicable
bottom 256 coeffs; $z\mapsto\lfloor(114(z+2156)+16384)/3276$
bottom 256 coeffs; $z\mapsto\lfloor(113(z+2175)+16384)/3276$
bottom 256 coeffs; $z\mapsto\lfloor(101(z+2433)+16384)/3276$
round $\mathbf{Z}/4096$ to $64\mathbf{Z}$
round $\mathbf{Z}/32768$ to $512\mathbf{Z}$
round $\mathbf{Z}/32768$ to $64\mathbf{Z}$
bottom 128 coeffs; round $\mathbf{Z}/8192$ to $512\mathbf{Z}$
bottom 192 coeffs; round $\mathbf{Z}/4096$ to $128\mathbf{Z}$
bottom 256 coeffs; round $\mathbf{Z}/8192$ to $256\mathbf{Z}$
bottom 318 coeffs; round $\mathbf{Z}/1024$ to $64\mathbf{Z}$
bottom 410 coeffs; round $\mathbf{Z}/4096$ to $512\mathbf{Z}$
bottom 490 coeffs; round $\mathbf{Z}/2048$ to $64\mathbf{Z}$
round $\mathbf{Z}/8192$ to $1024\mathbf{Z}$
round $\mathbf{Z}/8192$ to $512\mathbf{Z}$
round $\mathbf{Z}/8192$ to $128\mathbf{Z}$
not applicable
not applicable
not applicable
$\mathbf{Z}$; $\sum_{0\le i<312}2^{10i}\{-2,-1,0,1,2\}$; Pr $1,32,62,32,1$; *
$\mathbf{Z}$; $\sum_{0\le i<312}2^{10i}\{-1,0,1\}$; Pr $13,38,13$; *
$\mathbf{Z}$; $\sum_{0\le i<312}2^{10i}\{-1,0,1\}$; Pr $5,22,5$; *

key offset (numerator or noise or rounding method)

$\mathbf{Z}^{640\times 8}$; $\{-12,\dots,12\}$; Pr $1,4,17,\dots$ (spec page 23)
$\mathbf{Z}^{976\times 8}$; $\{-10,\dots,10\}$; Pr $1,6,29,\dots$ (spec page 23)
$\mathbf{Z}^{1344\times 8}$; $\{-6,\dots,6\}$; Pr $2,40,364,\dots$ (spec page 23)
$(\mathbf{Z}[x]/(x^{256}+1))^2$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^3$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$(\mathbf{Z}[x]/(x^{256}+1))^4$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{512}+1)$; $\{-1,0,1\}$; Pr $1,2,1$; weight $128,128$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr $1,6,1$; weight $128,128$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr $1,2,1$; weight $256,256$
$\mathbf{Z}[x]/(x^{512}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{509}-1)$; $\{-1,0,1\}$; weight $127,127$
$\mathbf{Z}[x]/(x^{677}-1)$; $\{-1,0,1\}$; weight $127,127$
$\mathbf{Z}[x]/(x^{821}-1)$; $\{-1,0,1\}$; weight $255,255$
$\mathbf{Z}[x]/(x^{701}-1)$; $\{-1,0,1\}$; key correlation ≥ 0; $\cdot(x-1)$
round $\{-2310,\dots,2310\}$ to $3\mathbf{Z}$
round $\{-2295,\dots,2295\}$ to $3\mathbf{Z}$
round $\{-2583,\dots,2583\}$ to $3\mathbf{Z}$
round $\mathbf{Z}/4096$ to $8\mathbf{Z}$
round $\mathbf{Z}/32768$ to $16\mathbf{Z}$
round $\mathbf{Z}/32768$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $16\mathbf{Z}$
round $\mathbf{Z}/4096$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $16\mathbf{Z}$
reduce mod $x^{508}+\dots+1$; round $\mathbf{Z}/1024$ to $8\mathbf{Z}$
reduce mod $x^{756}+\dots+1$; round $\mathbf{Z}/4096$ to $16\mathbf{Z}$
reduce mod $x^{946}+\dots+1$; round $\mathbf{Z}/2048$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $8\mathbf{Z}$
round $\mathbf{Z}/8192$ to $8\mathbf{Z}$
$\mathbf{Z}[x]/(x^{653}-x-1)$; $\{-1,0,1\}$; invertible mod 3
$\mathbf{Z}[x]/(x^{761}-x-1)$; $\{-1,0,1\}$; invertible mod 3
$\mathbf{Z}[x]/(x^{857}-x-1)$; $\{-1,0,1\}$; invertible mod 3
$\mathbf{Z}^2$; $\sum_{0\le i<312} 2^{10i}\{-2,-1,0,1,2\}$; Pr $1,32,62,32,1$; *
$\mathbf{Z}^3$; $\sum_{0\le i<312} 2^{10i}\{-1,0,1\}$; Pr $13,38,13$; *
$\mathbf{Z}^4$; $\sum_{0\le i<312} 2^{10i}\{-1,0,1\}$; Pr $5,22,5$; *

ciphertext offset (noise or rounding method)

$\mathbf{Z}^{8\times 8}$; $\{-12,\dots,12\}$; Pr $1,4,17,\dots$ (spec page 23)
$\mathbf{Z}^{8\times 8}$; $\{-10,\dots,10\}$; Pr $1,6,29,\dots$ (spec page 23)
$\mathbf{Z}^{8\times 8}$; $\{-6,\dots,6\}$; Pr $2,40,364,\dots$ (spec page 23)
$\mathbf{Z}[x]/(x^{256}+1)$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{256}+1)$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{256}+1)$; $\sum_{0\le i<4}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{512}+1)$; $\{-1,0,1\}$; Pr $1,2,1$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr $1,6,1$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr $1,2,1$
$\mathbf{Z}[x]/(x^{512}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$
not applicable
not applicable
not applicable
not applicable
bottom 256 coeffs; $z\mapsto\lfloor(114(z+2156)+16384)/32768\rfloor$
bottom 256 coeffs; $z\mapsto\lfloor(113(z+2175)+16384)/32768\rfloor$
bottom 256 coeffs; $z\mapsto\lfloor(101(z+2433)+16384)/32768\rfloor$
round $\mathbf{Z}/4096$ to $64\mathbf{Z}$
round $\mathbf{Z}/32768$ to $512\mathbf{Z}$
round $\mathbf{Z}/32768$ to $64\mathbf{Z}$
bottom 128 coeffs; round $\mathbf{Z}/8192$ to $512\mathbf{Z}$
bottom 192 coeffs; round $\mathbf{Z}/4096$ to $128\mathbf{Z}$
bottom 256 coeffs; round $\mathbf{Z}/8192$ to $256\mathbf{Z}$
bottom 318 coeffs; round $\mathbf{Z}/1024$ to $64\mathbf{Z}$
bottom 410 coeffs; round $\mathbf{Z}/4096$ to $512\mathbf{Z}$
bottom 490 coeffs; round $\mathbf{Z}/2048$ to $64\mathbf{Z}$
round $\mathbf{Z}/8192$ to $1024\mathbf{Z}$
round $\mathbf{Z}/8192$ to $512\mathbf{Z}$
round $\mathbf{Z}/8192$ to $128\mathbf{Z}$
not applicable
not applicable
not applicable
$\mathbf{Z}$; $\sum_{0\le i<312} 2^{10i}\{-2,-1,0,1,2\}$; Pr $1,32,62,32,1$; *
$\mathbf{Z}$; $\sum_{0\le i<312} 2^{10i}\{-1,0,1\}$; Pr $13,38,13$; *
$\mathbf{Z}$; $\sum_{0\le i<312} 2^{10i}\{-1,0,1\}$; Pr $5,22,5$; *

rator or noise or rounding method)

. . . , 12}; Pr $1, 4, 17, \ldots$ (spec page 23)
. . . , 10}; Pr $1, 6, 29, \ldots$ (spec page 23)
. . , 6}; Pr $2, 40, 364, \ldots$ (spec page 23)
$))^2$; $\sum_{0 \le i < 4}\{-0.5, 0.5\}$
$))^3$; $\sum_{0 \le i < 4}\{-0.5, 0.5\}$
$))^4$; $\sum_{0 \le i < 4}\{-0.5, 0.5\}$
$\{-1, 0, 1\}$; Pr $1, 2, 1$; weight $128, 128$
); $\{-1, 0, 1\}$; Pr $1, 6, 1$; weight $128, 128$
); $\{-1, 0, 1\}$; Pr $1, 2, 1$; weight $256, 256$
$\sum_{0 \le i < 16}\{-0.5, 0.5\}$
); $\sum_{0 \le i < 16}\{-0.5, 0.5\}$
$\{-1, 0, 1\}$; weight $127, 127$
$\{-1, 0, 1\}$; weight $127, 127$
$\{-1, 0, 1\}$; weight $255, 255$
$\{-1, 0, 1\}$; key correlation ≥ 0; $\cdot(x-1)$
. . . , 2310} to $3\mathbf{Z}$
. . . , 2295} to $3\mathbf{Z}$
. . . , 2583} to $3\mathbf{Z}$
o $8\mathbf{Z}$
to $16\mathbf{Z}$
to $8\mathbf{Z}$
o $16\mathbf{Z}$
o $8\mathbf{Z}$
o $16\mathbf{Z}$
$+ \ldots + 1$; round $\mathbf{Z}/1024$ to $8\mathbf{Z}$
$+ \ldots + 1$; round $\mathbf{Z}/4096$ to $16\mathbf{Z}$
$+ \ldots + 1$; round $\mathbf{Z}/2048$ to $8\mathbf{Z}$
o $8\mathbf{Z}$
o $8\mathbf{Z}$
o $8\mathbf{Z}$
$-1)$; $\{-1, 0, 1\}$; invertible mod 3
$-1)$; $\{-1, 0, 1\}$; invertible mod 3
$-1)$; $\{-1, 0, 1\}$; invertible mod 3
$^{10i}\{-2, -1, 0, 1, 2\}$; Pr $1, 32, 62, 32, 1$; *
$^{10i}\{-1, 0, 1\}$; Pr $13, 38, 13$; *
$^{10i}\{-1, 0, 1\}$; Pr $5, 22, 5$; *

ciphertext offset (noise or rounding method)

$\mathbf{Z}^{8\times 8}$; $\{-12, \ldots, 12\}$; Pr $1, 4, 17, \ldots$ (spec page 23)
$\mathbf{Z}^{8\times 8}$; $\{-10, \ldots, 10\}$; Pr $1, 6, 29, \ldots$ (spec page 23)
$\mathbf{Z}^{8\times 8}$; $\{-6, \ldots, 6\}$; Pr $2, 40, 364, \ldots$ (spec page 23)
$\mathbf{Z}[x]/(x^{256}+1)$; $\sum_{0 \le i < 4}\{-0.5, 0.5\}$
$\mathbf{Z}[x]/(x^{256}+1)$; $\sum_{0 \le i < 4}\{-0.5, 0.5\}$
$\mathbf{Z}[x]/(x^{256}+1)$; $\sum_{0 \le i < 4}\{-0.5, 0.5\}$
$\mathbf{Z}[x]/(x^{512}+1)$; $\{-1, 0, 1\}$; Pr $1, 2, 1$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1, 0, 1\}$; Pr $1, 6, 1$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1, 0, 1\}$; Pr $1, 2, 1$
$\mathbf{Z}[x]/(x^{512}+1)$; $\sum_{0 \le i < 16}\{-0.5, 0.5\}$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\sum_{0 \le i < 16}\{-0.5, 0.5\}$
not applicable
not applicable
not applicable
not applicable
bottom 256 coeffs; $z \mapsto \lfloor(114(z+2156)+16384)/32768\rfloor$
bottom 256 coeffs; $z \mapsto \lfloor(113(z+2175)+16384)/32768\rfloor$
bottom 256 coeffs; $z \mapsto \lfloor(101(z+2433)+16384)/32768\rfloor$
round $\mathbf{Z}/4096$ to $64\mathbf{Z}$
round $\mathbf{Z}/32768$ to $512\mathbf{Z}$
round $\mathbf{Z}/32768$ to $64\mathbf{Z}$
bottom 128 coeffs; round $\mathbf{Z}/8192$ to $512\mathbf{Z}$
bottom 192 coeffs; round $\mathbf{Z}/4096$ to $128\mathbf{Z}$
bottom 256 coeffs; round $\mathbf{Z}/8192$ to $256\mathbf{Z}$
bottom 318 coeffs; round $\mathbf{Z}/1024$ to $64\mathbf{Z}$
bottom 410 coeffs; round $\mathbf{Z}/4096$ to $512\mathbf{Z}$
bottom 490 coeffs; round $\mathbf{Z}/2048$ to $64\mathbf{Z}$
round $\mathbf{Z}/8192$ to $1024\mathbf{Z}$
round $\mathbf{Z}/8192$ to $512\mathbf{Z}$
round $\mathbf{Z}/8192$ to $128\mathbf{Z}$
not applicable
not applicable
not applicable
$\mathbf{Z}$; $\sum_{0 \le i < 312} 2^{10i}\{-2, -1, 0, 1, 2\}$; Pr $1, 32, 62, 32, 1$; *
$\mathbf{Z}$; $\sum_{0 \le i < 312} 2^{10i}\{-1, 0, 1\}$; Pr $13, 38, 13$; *
$\mathbf{Z}$; $\sum_{0 \le i < 312} 2^{10i}\{-1, 0, 1\}$; Pr $5, 22, 5$; *

set of encoded m

8×8 matrix ove
8×8 matrix ove
8×8 matrix ove
$\sum_{0 \le i < 256}\{0, 16$
$\sum_{0 \le i < 256}\{0, 16$
$\sum_{0 \le i < 256}\{0, 16$
256-dim subcode
256-dim subcode
256-dim subcode
$\sum_{0 \le i < 256}\{0, 61$
$\sum_{0 \le i < 256}\{0, 61$
not applicable
not applicable
not applicable
not applicable
$\sum_{0 \le i < 256}\{0, 23$
$\sum_{0 \le i < 256}\{0, 22$
$\sum_{0 \le i < 256}\{0, 25$
8×8 matrix ove
8×8 matrix ove
8×8 matrix ove
$\sum_{0 \le i < 128}\{0, 40$
$\sum_{0 \le i < 192}\{0, 20$
$\sum_{0 \le i < 256}\{0, 40$
128-dim subcode
192-dim subcode
256-dim subcode
$\sum_{0 \le i < 256}\{0, 40$
$\sum_{0 \le i < 256}\{0, 40$
$\sum_{0 \le i < 256}\{0, 40$
not applicable
not applicable
not applicable
256-dim subcode
256-dim subcode
256-dim subcode

nding method)
. . (spec page 23)
. . (spec page 23)
. . . (spec page 23)
0.5}
0.5}
0.5}
1; weight 128, 128
, 1; weight 128, 128
, 1; weight 256, 256
5}
.5}
127, 127
127, 127
255, 255
relation ≥ 0; $\cdot(x-1)$
/1024 to 8**Z**
/4096 to 16**Z**
/2048 to 8**Z**
ertible mod 3
ertible mod 3
ertible mod 3
Pr 1, 32, 62, 32, 1; *
38, 13; *
22, 5; *

ciphertext offset (noise or rounding method)

$\mathbf{Z}^{8\times8}$; $\{-12,\ldots,12\}$; Pr $1,4,17,\ldots$ (spec page 23)
$\mathbf{Z}^{8\times8}$; $\{-10,\ldots,10\}$; Pr $1,6,29,\ldots$ (spec page 23)
$\mathbf{Z}^{8\times8}$; $\{-6,\ldots,6\}$; Pr $2,40,364,\ldots$ (spec page 23)
$\mathbf{Z}[x]/(x^{256}+1)$; $\sum_{0\leq i<4}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{256}+1)$; $\sum_{0\leq i<4}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{256}+1)$; $\sum_{0\leq i<4}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{512}+1)$; $\{-1,0,1\}$; Pr 1, 2, 1
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr 1, 6, 1
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr 1, 2, 1
$\mathbf{Z}[x]/(x^{512}+1)$; $\sum_{0\leq i<16}\{-0.5,0.5\}$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\sum_{0\leq i<16}\{-0.5,0.5\}$
not applicable
not applicable
not applicable
not applicable
bottom 256 coeffs; $z\mapsto\lfloor(114(z+2156)+16384)/32768\rfloor$
bottom 256 coeffs; $z\mapsto\lfloor(113(z+2175)+16384)/32768\rfloor$
bottom 256 coeffs; $z\mapsto\lfloor(101(z+2433)+16384)/32768\rfloor$
round $\mathbf{Z}/4096$ to $64\mathbf{Z}$
round $\mathbf{Z}/32768$ to $512\mathbf{Z}$
round $\mathbf{Z}/32768$ to $64\mathbf{Z}$
bottom 128 coeffs; round $\mathbf{Z}/8192$ to $512\mathbf{Z}$
bottom 192 coeffs; round $\mathbf{Z}/4096$ to $128\mathbf{Z}$
bottom 256 coeffs; round $\mathbf{Z}/8192$ to $256\mathbf{Z}$
bottom 318 coeffs; round $\mathbf{Z}/1024$ to $64\mathbf{Z}$
bottom 410 coeffs; round $\mathbf{Z}/4096$ to $512\mathbf{Z}$
bottom 490 coeffs; round $\mathbf{Z}/2048$ to $64\mathbf{Z}$
round $\mathbf{Z}/8192$ to $1024\mathbf{Z}$
round $\mathbf{Z}/8192$ to $512\mathbf{Z}$
round $\mathbf{Z}/8192$ to $128\mathbf{Z}$
not applicable
not applicable
not applicable
$\mathbf{Z}$; $\sum_{0\leq i<312}2^{10i}\{-2,-1,0,1,2\}$; Pr 1, 32, 62, 32, 1; *
$\mathbf{Z}$; $\sum_{0\leq i<312}2^{10i}\{-1,0,1\}$; Pr 13, 38, 13; *
$\mathbf{Z}$; $\sum_{0\leq i<312}2^{10i}\{-1,0,1\}$; Pr 5, 22, 5; *

set of encoded messages

8×8 matrix over $\{0,8192,16384,24$
8×8 matrix over $\{0,8192,\ldots,5734$
8×8 matrix over $\{0,4096,\ldots,6144$
$\sum_{0\leq i<256}\{0,1665\}x^i$
$\sum_{0\leq i<256}\{0,1665\}x^i$
$\sum_{0\leq i<256}\{0,1665\}x^i$
256-dim subcode (see spec) of $\sum_{0\leq}$
256-dim subcode (see spec) of $\sum_{0\leq}$
256-dim subcode (see spec) of $\sum_{0\leq}$
$\sum_{0\leq i<256}\{0,6145\}x^i(1+x^{256})$
$\sum_{0\leq i<256}\{0,6145\}x^i(1+x^{256}+x^5$
not applicable
not applicable
not applicable
not applicable
$\sum_{0\leq i<256}\{0,2310\}x^i$
$\sum_{0\leq i<256}\{0,2295\}x^i$
$\sum_{0\leq i<256}\{0,2583\}x^i$
8×8 matrix over $\{0,1024,2048,307$
8×8 matrix over $\{0,4096,\ldots,2867$
8×8 matrix over $\{0,2048,\ldots,3072$
$\sum_{0\leq i<128}\{0,4096\}x^i$
$\sum_{0\leq i<192}\{0,2048\}x^i$
$\sum_{0\leq i<256}\{0,4096\}x^i$
128-dim subcode (see spec) of $\sum_{0\leq}$
192-dim subcode (see spec) of $\sum_{0\leq}$
256-dim subcode (see spec) of $\sum_{0\leq}$
$\sum_{0\leq i<256}\{0,4096\}x^i$
$\sum_{0\leq i<256}\{0,4096\}x^i$
$\sum_{0\leq i<256}\{0,4096\}x^i$
not applicable
not applicable
not applicable
256-dim subcode (see spec) of $\sum_{0\leq}$
256-dim subcode (see spec) of $\sum_{0\leq}$
256-dim subcode (see spec) of $\sum_{0\leq}$

7	8
ciphertext offset (noise or rounding method)	set of encoded messages
$\mathbf{Z}^{8\times 8}$; $\{-12,\dots,12\}$; Pr $1,4,17,\dots$ (spec page 23)	8×8 matrix over $\{0,8192,16384,24576\}$
$\mathbf{Z}^{8\times 8}$; $\{-10,\dots,10\}$; Pr $1,6,29,\dots$ (spec page 23)	8×8 matrix over $\{0,8192,\dots,57344\}$
$\mathbf{Z}^{8\times 8}$; $\{-6,\dots,6\}$; Pr $2,40,364,\dots$ (spec page 23)	8×8 matrix over $\{0,4096,\dots,61440\}$
$\mathbf{Z}[x]/(x^{256}+1)$; $\sum_{0\le i<4}\{-0.5,0.5\}$	$\sum_{0\le i<256}\{0,1665\}x^i$
$\mathbf{Z}[x]/(x^{256}+1)$; $\sum_{0\le i<4}\{-0.5,0.5\}$	$\sum_{0\le i<256}\{0,1665\}x^i$
$\mathbf{Z}[x]/(x^{256}+1)$; $\sum_{0\le i<4}\{-0.5,0.5\}$	$\sum_{0\le i<256}\{0,1665\}x^i$
$\mathbf{Z}[x]/(x^{512}+1)$; $\{-1,0,1\}$; Pr $1,2,1$	256-dim subcode (see spec) of $\sum_{0\le i<512}\{0,126\}x^i$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr $1,6,1$	256-dim subcode (see spec) of $\sum_{0\le i<1024}\{0,126\}x^i$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1,0,1\}$; Pr $1,2,1$	256-dim subcode (see spec) of $\sum_{0\le i<1024}\{0,126\}x^i$
$\mathbf{Z}[x]/(x^{512}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$	$\sum_{0\le i<256}\{0,6145\}x^i(1+x^{256})$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\sum_{0\le i<16}\{-0.5,0.5\}$	$\sum_{0\le i<256}\{0,6145\}x^i(1+x^{256}+x^{512}+x^{768})$
not applicable	not applicable
not applicable	not applicable
not applicable	not applicable
not applicable	not applicable
bottom 256 coeffs; $z\mapsto\lfloor(114(z+2156)+16384)/32768\rfloor$	$\sum_{0\le i<256}\{0,2310\}x^i$
bottom 256 coeffs; $z\mapsto\lfloor(113(z+2175)+16384)/32768\rfloor$	$\sum_{0\le i<256}\{0,2295\}x^i$
bottom 256 coeffs; $z\mapsto\lfloor(101(z+2433)+16384)/32768\rfloor$	$\sum_{0\le i<256}\{0,2583\}x^i$
round $\mathbf{Z}/4096$ to $64\mathbf{Z}$	8×8 matrix over $\{0,1024,2048,3072\}$
round $\mathbf{Z}/32768$ to $512\mathbf{Z}$	8×8 matrix over $\{0,4096,\dots,28672\}$
round $\mathbf{Z}/32768$ to $64\mathbf{Z}$	8×8 matrix over $\{0,2048,\dots,30720\}$
bottom 128 coeffs; round $\mathbf{Z}/8192$ to $512\mathbf{Z}$	$\sum_{0\le i<128}\{0,4096\}x^i$
bottom 192 coeffs; round $\mathbf{Z}/4096$ to $128\mathbf{Z}$	$\sum_{0\le i<192}\{0,2048\}x^i$
bottom 256 coeffs; round $\mathbf{Z}/8192$ to $256\mathbf{Z}$	$\sum_{0\le i<256}\{0,4096\}x^i$
bottom 318 coeffs; round $\mathbf{Z}/1024$ to $64\mathbf{Z}$	128-dim subcode (see spec) of $\sum_{0\le i<318}\{0,512\}x^i$
bottom 410 coeffs; round $\mathbf{Z}/4096$ to $512\mathbf{Z}$	192-dim subcode (see spec) of $\sum_{0\le i<410}\{0,2048\}x^i$
bottom 490 coeffs; round $\mathbf{Z}/2048$ to $64\mathbf{Z}$	256-dim subcode (see spec) of $\sum_{0\le i<490}\{0,1024\}x^i$
round $\mathbf{Z}/8192$ to $1024\mathbf{Z}$	$\sum_{0\le i<256}\{0,4096\}x^i$
round $\mathbf{Z}/8192$ to $512\mathbf{Z}$	$\sum_{0\le i<256}\{0,4096\}x^i$
round $\mathbf{Z}/8192$ to $128\mathbf{Z}$	$\sum_{0\le i<256}\{0,4096\}x^i$
not applicable	not applicable
not applicable	not applicable
not applicable	not applicable
$\mathbf{Z}$; $\sum_{0\le i<312}2^{10i}\{-2,-1,0,1,2\}$; Pr $1,32,62,32,1$; *	256-dim subcode (see spec) of $\sum_{0\le i<274}\{0,512\}2^{10i}$
$\mathbf{Z}$; $\sum_{0\le i<312}2^{10i}\{-1,0,1\}$; Pr $13,38,13$; *	256-dim subcode (see spec) of $\sum_{0\le i<274}\{0,512\}2^{10i}$
$\mathbf{Z}$; $\sum_{0\le i<312}2^{10i}\{-1,0,1\}$; Pr $5,22,5$; *	256-dim subcode (see spec) of $\sum_{0\le i<274}\{0,512\}2^{10i}$

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ciphertext offset (noise or rounding method)
$\mathbf{Z}^{8\times 8}$; $\{-12,\dots,12\}$; Pr $1, 4, 17, \dots$ (spec page 23)
$\mathbf{Z}^{8\times 8}$; $\{-10,\dots,10\}$; Pr $1, 6, 29, \dots$ (spec page 23)
$\mathbf{Z}^{8\times 8}$; $\{-6,\dots,6\}$; Pr $2, 40, 364, \dots$ (spec page 23)
$\mathbf{Z}[x]/(x^{256}+1)$; $\sum_{0\le i<4}\{-0.5, 0.5\}$
$\mathbf{Z}[x]/(x^{256}+1)$; $\sum_{0\le i<4}\{-0.5, 0.5\}$
$\mathbf{Z}[x]/(x^{256}+1)$; $\sum_{0\le i<4}\{-0.5, 0.5\}$
$\mathbf{Z}[x]/(x^{512}+1)$; $\{-1, 0, 1\}$; Pr $1, 2, 1$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1, 0, 1\}$; Pr $1, 6, 1$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\{-1, 0, 1\}$; Pr $1, 2, 1$
$\mathbf{Z}[x]/(x^{512}+1)$; $\sum_{0\le i<16}\{-0.5, 0.5\}$
$\mathbf{Z}[x]/(x^{1024}+1)$; $\sum_{0\le i<16}\{-0.5, 0.5\}$
not applicable
not applicable
not applicable
not applicable
bottom 256 coeffs; $z \mapsto \lfloor (114(z+2156)+16384)/32768 \rfloor$
bottom 256 coeffs; $z \mapsto \lfloor (113(z+2175)+16384)/32768 \rfloor$
bottom 256 coeffs; $z \mapsto \lfloor (101(z+2433)+16384)/32768 \rfloor$
round $\mathbf{Z}/4096$ to $64\mathbf{Z}$
round $\mathbf{Z}/32768$ to $512\mathbf{Z}$
round $\mathbf{Z}/32768$ to $64\mathbf{Z}$
bottom 128 coeffs; round $\mathbf{Z}/8192$ to $512\mathbf{Z}$
bottom 192 coeffs; round $\mathbf{Z}/4096$ to $128\mathbf{Z}$
bottom 256 coeffs; round $\mathbf{Z}/8192$ to $256\mathbf{Z}$
bottom 318 coeffs; round $\mathbf{Z}/1024$ to $64\mathbf{Z}$
bottom 410 coeffs; round $\mathbf{Z}/4096$ to $512\mathbf{Z}$
bottom 490 coeffs; round $\mathbf{Z}/2048$ to $64\mathbf{Z}$
round $\mathbf{Z}/8192$ to $1024\mathbf{Z}$
round $\mathbf{Z}/8192$ to $512\mathbf{Z}$
round $\mathbf{Z}/8192$ to $128\mathbf{Z}$
not applicable
not applicable
not applicable
$\mathbf{Z}$; $\sum_{0\le i<312} 2^{10i}\{-2,-1,0,1,2\}$; Pr $1, 32, 62, 32, 1$; *
$\mathbf{Z}$; $\sum_{0\le i<312} 2^{10i}\{-1,0,1\}$; Pr $13, 38, 13$; *
$\mathbf{Z}$; $\sum_{0\le i<312} 2^{10i}\{-1,0,1\}$; Pr $5, 22, 5$; *

set of encoded messages
8×8 matrix over $\{0, 8192, 16384, 24576\}$
8×8 matrix over $\{0, 8192, \dots, 57344\}$
8×8 matrix over $\{0, 4096, \dots, 61440\}$
$\sum_{0\le i<256}\{0, 1665\}x^i$
$\sum_{0\le i<256}\{0, 1665\}x^i$
$\sum_{0\le i<256}\{0, 1665\}x^i$
256-dim subcode (see spec) of $\sum_{0\le i<512}\{0, 126\}x^i$
256-dim subcode (see spec) of $\sum_{0\le i<1024}\{0, 126\}x^i$
256-dim subcode (see spec) of $\sum_{0\le i<1024}\{0, 126\}x^i$
$\sum_{0\le i<256}\{0, 6145\}x^i(1+x^{256})$
$\sum_{0\le i<256}\{0, 6145\}x^i(1+x^{256}+x^{512}+x^{768})$
not applicable
not applicable
not applicable
not applicable
$\sum_{0\le i<256}\{0, 2310\}x^i$
$\sum_{0\le i<256}\{0, 2295\}x^i$
$\sum_{0\le i<256}\{0, 2583\}x^i$
8×8 matrix over $\{0, 1024, 2048, 3072\}$
8×8 matrix over $\{0, 4096, \dots, 28672\}$
8×8 matrix over $\{0, 2048, \dots, 30720\}$
$\sum_{0\le i<128}\{0, 4096\}x^i$
$\sum_{0\le i<192}\{0, 2048\}x^i$
$\sum_{0\le i<256}\{0, 4096\}x^i$
128-dim subcode (see spec) of $\sum_{0\le i<318}\{0, 512\}x^i$
192-dim subcode (see spec) of $\sum_{0\le i<410}\{0, 2048\}x^i$
256-dim subcode (see spec) of $\sum_{0\le i<490}\{0, 1024\}x^i$
$\sum_{0\le i<256}\{0, 4096\}x^i$
$\sum_{0\le i<256}\{0, 4096\}x^i$
$\sum_{0\le i<256}\{0, 4096\}x^i$
not applicable
not applicable
not applicable
256-dim subcode (see spec) of $\sum_{0\le i<274}\{0, 512\}2^{10i}$
256-dim subcode (see spec) of $\sum_{0\le i<274}\{0, 512\}2^{10i}$
256-dim subcode (see spec) of $\sum_{0\le i<274}\{0, 512\}2^{10i}$

(noise or rounding method)	set of encoded messages
, 12}; Pr 1, 4, 17, . . . (spec page 23)	8×8 matrix over $\{0, 8192, 16384, 24576\}$
, 10}; Pr 1, 6, 29, . . . (spec page 23)	8×8 matrix over $\{0, 8192, \ldots, 57344\}$
6}; Pr 2, 40, 364, . . . (spec page 23)	8×8 matrix over $\{0, 4096, \ldots, 61440\}$
$\sum_{0 \le i < 4}\{-0.5, 0.5\}$	$\sum_{0 \le i < 256}\{0, 1665\}x^i$
$\sum_{0 \le i < 4}\{-0.5, 0.5\}$	$\sum_{0 \le i < 256}\{0, 1665\}x^i$
$\sum_{0 \le i < 4}\{-0.5, 0.5\}$	$\sum_{0 \le i < 256}\{0, 1665\}x^i$
$\{-1, 0, 1\}$; Pr 1, 2, 1	256-dim subcode (see spec) of $\sum_{0 \le i < 512}\{0, 126\}x^i$
); $\{-1, 0, 1\}$; Pr 1, 6, 1	256-dim subcode (see spec) of $\sum_{0 \le i < 1024}\{0, 126\}x^i$
); $\{-1, 0, 1\}$; Pr 1, 2, 1	256-dim subcode (see spec) of $\sum_{0 \le i < 1024}\{0, 126\}x^i$
$\sum_{0 \le i < 16}\{-0.5, 0.5\}$	$\sum_{0 \le i < 256}\{0, 6145\}x^i(1 + x^{256})$
); $\sum_{0 \le i < 16}\{-0.5, 0.5\}$	$\sum_{0 \le i < 256}\{0, 6145\}x^i(1 + x^{256} + x^{512} + x^{768})$
	not applicable
	not applicable
	not applicable
	not applicable
fs; $z \mapsto \lfloor(114(z + 2156) + 16384)/32768\rfloor$	$\sum_{0 \le i < 256}\{0, 2310\}x^i$
fs; $z \mapsto \lfloor(113(z + 2175) + 16384)/32768\rfloor$	$\sum_{0 \le i < 256}\{0, 2295\}x^i$
fs; $z \mapsto \lfloor(101(z + 2433) + 16384)/32768\rfloor$	$\sum_{0 \le i < 256}\{0, 2583\}x^i$
o 64**Z**	8×8 matrix over $\{0, 1024, 2048, 3072\}$
to 512**Z**	8×8 matrix over $\{0, 4096, \ldots, 28672\}$
to 64**Z**	8×8 matrix over $\{0, 2048, \ldots, 30720\}$
fs; round **Z**/8192 to 512**Z**	$\sum_{0 \le i < 128}\{0, 4096\}x^i$
fs; round **Z**/4096 to 128**Z**	$\sum_{0 \le i < 192}\{0, 2048\}x^i$
fs; round **Z**/8192 to 256**Z**	$\sum_{0 \le i < 256}\{0, 4096\}x^i$
fs; round **Z**/1024 to 64**Z**	128-dim subcode (see spec) of $\sum_{0 \le i < 318}\{0, 512\}x^i$
fs; round **Z**/4096 to 512**Z**	192-dim subcode (see spec) of $\sum_{0 \le i < 410}\{0, 2048\}x^i$
fs; round **Z**/2048 to 64**Z**	256-dim subcode (see spec) of $\sum_{0 \le i < 490}\{0, 1024\}x^i$
o 1024**Z**	$\sum_{0 \le i < 256}\{0, 4096\}x^i$
o 512**Z**	$\sum_{0 \le i < 256}\{0, 4096\}x^i$
o 128**Z**	$\sum_{0 \le i < 256}\{0, 4096\}x^i$
	not applicable
	not applicable
	not applicable
$^{i}\{-2, -1, 0, 1, 2\}$; Pr 1, 32, 62, 32, 1; *	256-dim subcode (see spec) of $\sum_{0 \le i < 274}\{0, 512\}2^{10i}$
$^{i}\{-1, 0, 1\}$; Pr 13, 38, 13; *	256-dim subcode (see spec) of $\sum_{0 \le i < 274}\{0, 512\}2^{10i}$
$^{i}\{-1, 0, 1\}$; Pr 5, 22, 5; *	256-dim subcode (see spec) of $\sum_{0 \le i < 274}\{0, 512\}2^{10i}$

Attackin

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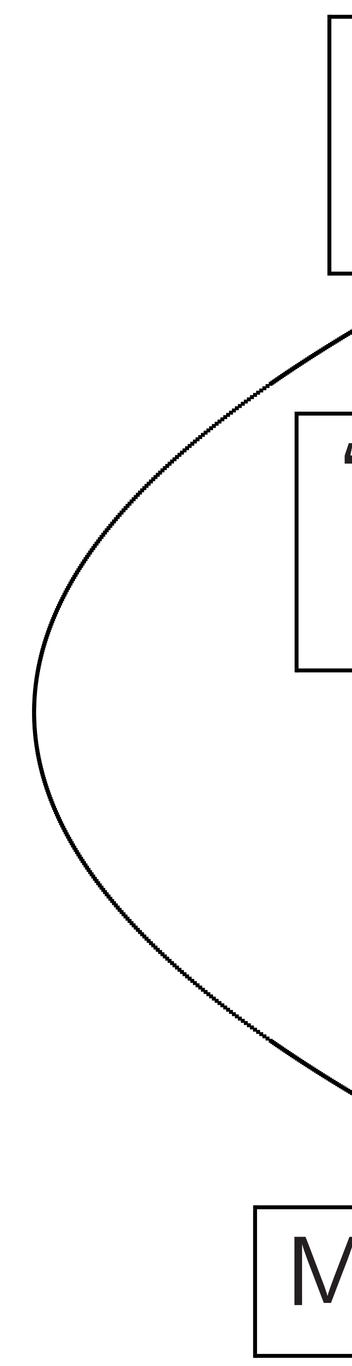
strategy.
Normal



method)
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}
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156) + 16384)/32768⌋
175) + 16384)/32768⌋
433) + 16384)/32768⌋

512**Z**
128**Z**
256**Z**
64**Z**
512**Z**
64**Z**

Pr 1, 32, 62, 32, 1; *
38, 13; *
2, 5; *

set of encoded messages

8×8 matrix over $\{0, 8192, 16384, 24576\}$
8×8 matrix over $\{0, 8192, \ldots, 57344\}$
8×8 matrix over $\{0, 4096, \ldots, 61440\}$
$\sum_{0 \le i < 256} \{0, 1665\} x^i$
$\sum_{0 \le i < 256} \{0, 1665\} x^i$
$\sum_{0 \le i < 256} \{0, 1665\} x^i$
256-dim subcode (see spec) of $\sum_{0 \le i < 512} \{0, 126\} x^i$
256-dim subcode (see spec) of $\sum_{0 \le i < 1024} \{0, 126\} x^i$
256-dim subcode (see spec) of $\sum_{0 \le i < 1024} \{0, 126\} x^i$
$\sum_{0 \le i < 256} \{0, 6145\} x^i (1 + x^{256})$
$\sum_{0 \le i < 256} \{0, 6145\} x^i (1 + x^{256} + x^{512} + x^{768})$
not applicable
not applicable
not applicable
not applicable
$\sum_{0 \le i < 256} \{0, 2310\} x^i$
$\sum_{0 \le i < 256} \{0, 2295\} x^i$
$\sum_{0 \le i < 256} \{0, 2583\} x^i$
8×8 matrix over $\{0, 1024, 2048, 3072\}$
8×8 matrix over $\{0, 4096, \ldots, 28672\}$
8×8 matrix over $\{0, 2048, \ldots, 30720\}$
$\sum_{0 \le i < 128} \{0, 4096\} x^i$
$\sum_{0 \le i < 192} \{0, 2048\} x^i$
$\sum_{0 \le i < 256} \{0, 4096\} x^i$
128-dim subcode (see spec) of $\sum_{0 \le i < 318} \{0, 512\} x^i$
192-dim subcode (see spec) of $\sum_{0 \le i < 410} \{0, 2048\} x^i$
256-dim subcode (see spec) of $\sum_{0 \le i < 490} \{0, 1024\} x^i$
$\sum_{0 \le i < 256} \{0, 4096\} x^i$
$\sum_{0 \le i < 256} \{0, 4096\} x^i$
$\sum_{0 \le i < 256} \{0, 4096\} x^i$
not applicable
not applicable
not applicable
256-dim subcode (see spec) of $\sum_{0 \le i < 274} \{0, 512\} 2^{10i}$
256-dim subcode (see spec) of $\sum_{0 \le i < 274} \{0, 512\} 2^{10i}$
256-dim subcode (see spec) of $\sum_{0 \le i < 274} \{0, 512\} 2^{10i}$

Attacking these pr

Attack strategy wi
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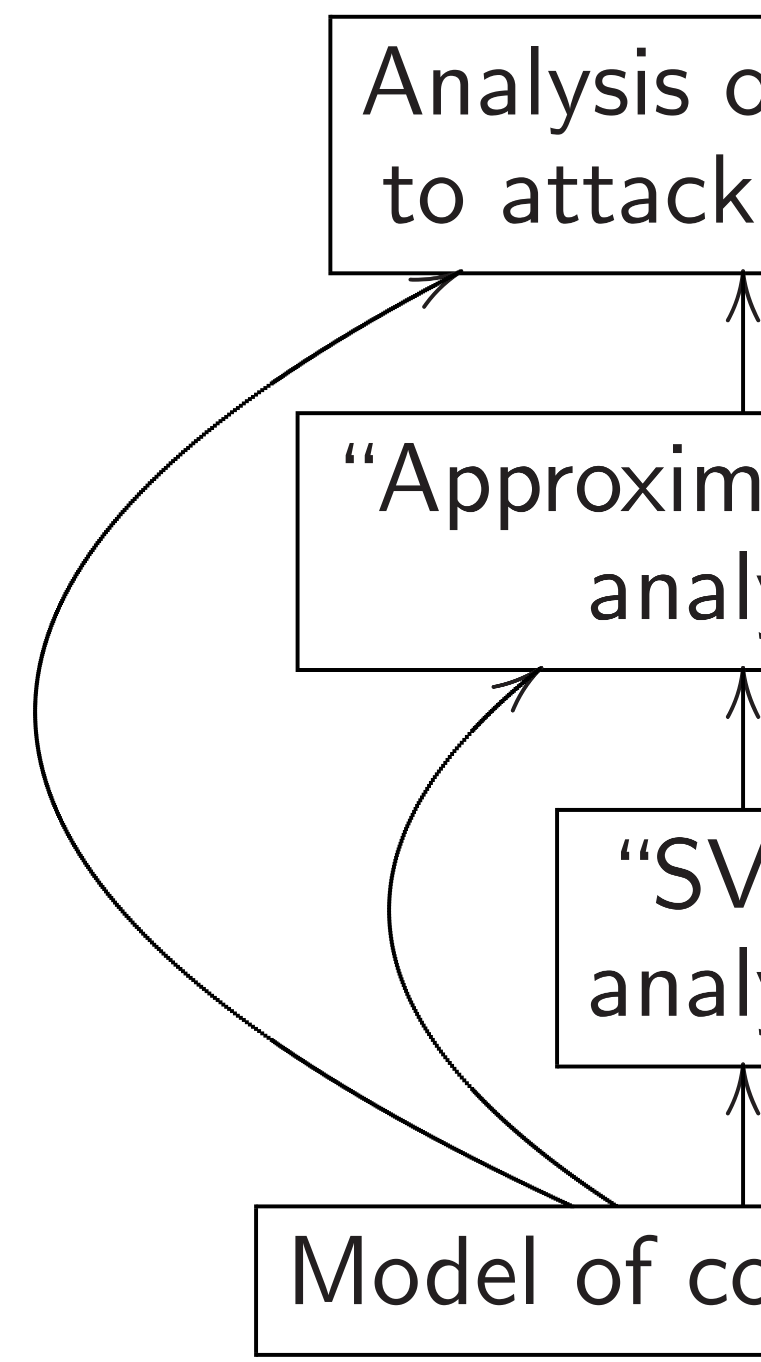
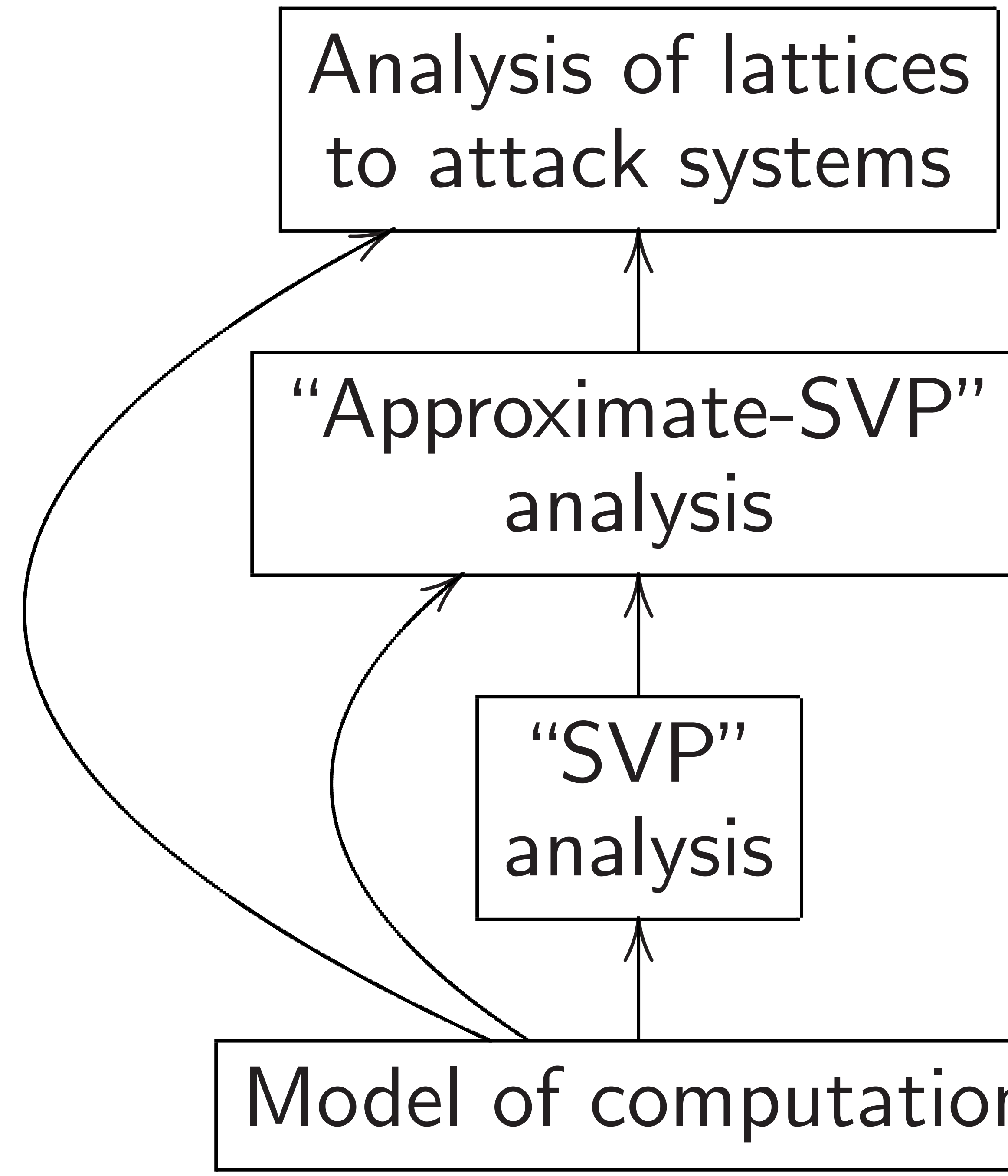
strategy. Focus of
Normal layers in a



set of encoded messages
8×8 matrix over $\{0, 8192, 16384, 24576\}$
8×8 matrix over $\{0, 8192, \ldots, 57344\}$
8×8 matrix over $\{0, 4096, \ldots, 61440\}$
$\sum_{0 \le i < 256} \{0, 1665\} x^i$
$\sum_{0 \le i < 256} \{0, 1665\} x^i$
$\sum_{0 \le i < 256} \{0, 1665\} x^i$
256-dim subcode (see spec) of $\sum_{0 \le i < 512} \{0, 126\} x^i$
256-dim subcode (see spec) of $\sum_{0 \le i < 1024} \{0, 126\} x^i$
256-dim subcode (see spec) of $\sum_{0 \le i < 1024} \{0, 126\} x^i$
$\sum_{0 \le i < 256} \{0, 6145\} x^i (1 + x^{256})$
$\sum_{0 \le i < 256} \{0, 6145\} x^i (1 + x^{256} + x^{512} + x^{768})$
not applicable
not applicable
not applicable
not applicable
$\sum_{0 \le i < 256} \{0, 2310\} x^i$
$\sum_{0 \le i < 256} \{0, 2295\} x^i$
$\sum_{0 \le i < 256} \{0, 2583\} x^i$
8×8 matrix over $\{0, 1024, 2048, 3072\}$
8×8 matrix over $\{0, 4096, \ldots, 28672\}$
8×8 matrix over $\{0, 2048, \ldots, 30720\}$
$\sum_{0 \le i < 128} \{0, 4096\} x^i$
$\sum_{0 \le i < 192} \{0, 2048\} x^i$
$\sum_{0 \le i < 256} \{0, 4096\} x^i$
128-dim subcode (see spec) of $\sum_{0 \le i < 318} \{0, 512\} x^i$
192-dim subcode (see spec) of $\sum_{0 \le i < 410} \{0, 2048\} x^i$
256-dim subcode (see spec) of $\sum_{0 \le i < 490} \{0, 1024\} x^i$
$\sum_{0 \le i < 256} \{0, 4096\} x^i$
$\sum_{0 \le i < 256} \{0, 4096\} x^i$
$\sum_{0 \le i < 256} \{0, 4096\} x^i$
not applicable
not applicable
not applicable
256-dim subcode (see spec) of $\sum_{0 \le i < 274} \{0, 512\} 2^{10i}$
256-dim subcode (see spec) of $\sum_{0 \le i < 274} \{0, 512\} 2^{10i}$
256-dim subcode (see spec) of $\sum_{0 \le i < 274} \{0, 512\} 2^{10i}$

Attacking these problems

Attack strategy with reputat
of usually being best: "prim
strategy. Focus of this talk.
Normal layers in analysis:

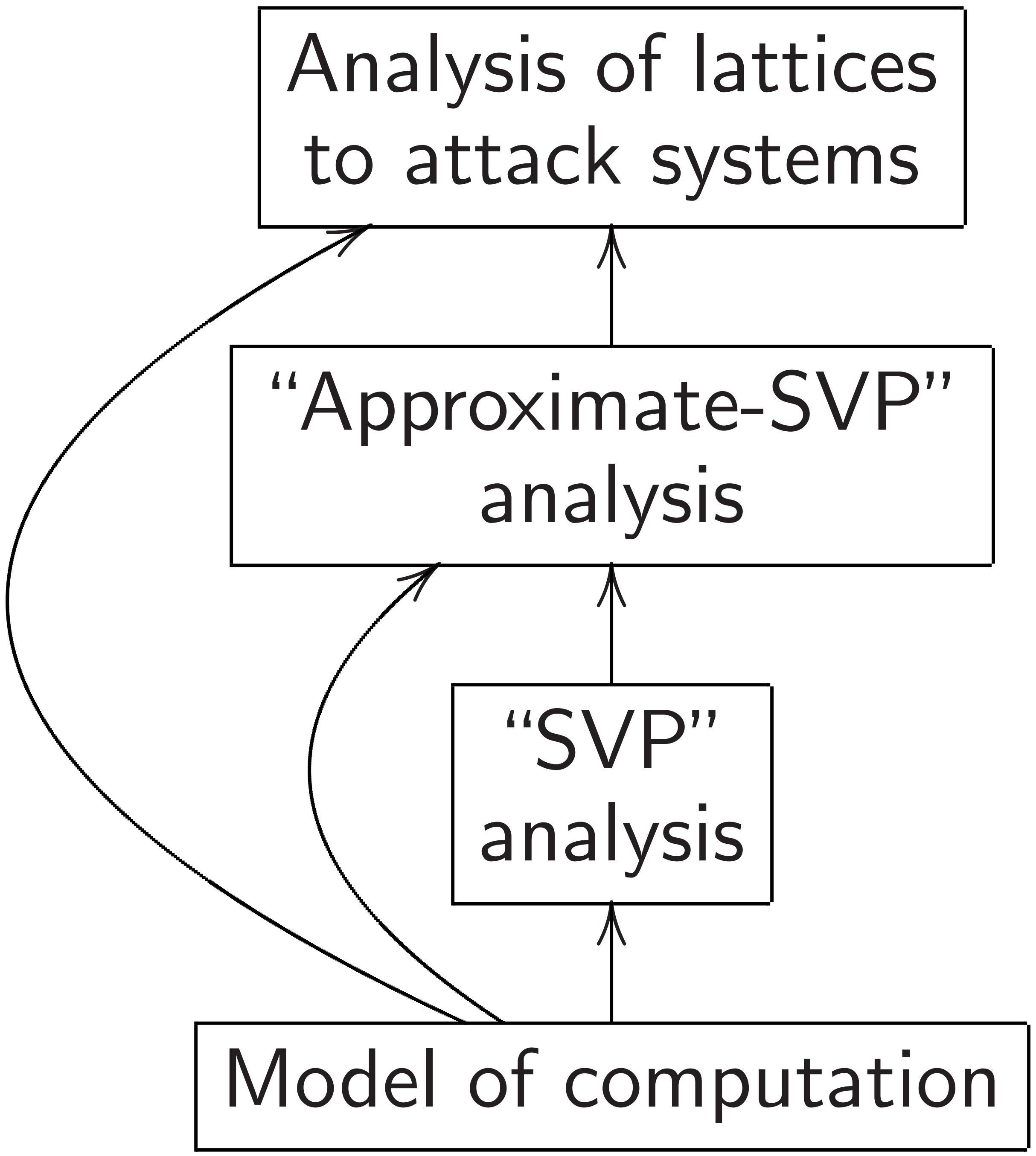


set of encoded messages
8×8 matrix over $\{0, 8192, 16384, 24576\}$
8×8 matrix over $\{0, 8192, \ldots, 57344\}$
8×8 matrix over $\{0, 4096, \ldots, 61440\}$
$\sum_{0 \le i < 256} \{0, 1665\} x^i$
$\sum_{0 \le i < 256} \{0, 1665\} x^i$
$\sum_{0 \le i < 256} \{0, 1665\} x^i$
256-dim subcode (see spec) of $\sum_{0 \le i < 512} \{0, 126\} x^i$
256-dim subcode (see spec) of $\sum_{0 \le i < 1024} \{0, 126\} x^i$
256-dim subcode (see spec) of $\sum_{0 \le i < 1024} \{0, 126\} x^i$
$\sum_{0 \le i < 256} \{0, 6145\} x^i (1 + x^{256})$
$\sum_{0 \le i < 256} \{0, 6145\} x^i (1 + x^{256} + x^{512} + x^{768})$
not applicable
not applicable
not applicable
not applicable
$\sum_{0 \le i < 256} \{0, 2310\} x^i$
$\sum_{0 \le i < 256} \{0, 2295\} x^i$
$\sum_{0 \le i < 256} \{0, 2583\} x^i$
8×8 matrix over $\{0, 1024, 2048, 3072\}$
8×8 matrix over $\{0, 4096, \ldots, 28672\}$
8×8 matrix over $\{0, 2048, \ldots, 30720\}$
$\sum_{0 \le i < 128} \{0, 4096\} x^i$
$\sum_{0 \le i < 192} \{0, 2048\} x^i$
$\sum_{0 \le i < 256} \{0, 4096\} x^i$
128-dim subcode (see spec) of $\sum_{0 \le i < 318} \{0, 512\} x^i$
192-dim subcode (see spec) of $\sum_{0 \le i < 410} \{0, 2048\} x^i$
256-dim subcode (see spec) of $\sum_{0 \le i < 490} \{0, 1024\} x^i$
$\sum_{0 \le i < 256} \{0, 4096\} x^i$
$\sum_{0 \le i < 256} \{0, 4096\} x^i$
$\sum_{0 \le i < 256} \{0, 4096\} x^i$
not applicable
not applicable
not applicable
256-dim subcode (see spec) of $\sum_{0 \le i < 274} \{0, 512\} 2^{10i}$
256-dim subcode (see spec) of $\sum_{0 \le i < 274} \{0, 512\} 2^{10i}$
256-dim subcode (see spec) of $\sum_{0 \le i < 274} \{0, 512\} 2^{10i}$

Attacking these problems

Attack strategy with reputation of usually being best: "primal" strategy. Focus of this talk.

Normal layers in analysis:



nessages

r $\{0, 8192, 16384, 24576\}$

r $\{0, 8192, \ldots, 57344\}$

r $\{0, 4096, \ldots, 61440\}$

$55\}x^i$

$55\}x^i$

$55\}x^i$

e (see spec) of $\sum_{0 \le i < 512} \{0, 126\} x^i$

e (see spec) of $\sum_{0 \le i < 1024} \{0, 126\} x^i$

e (see spec) of $\sum_{0 \le i < 1024} \{0, 126\} x^i$

$45\}x^i(1 + x^{256})$

$45\}x^i(1 + x^{256} + x^{512} + x^{768})$

$10\}x^i$

$95\}x^i$

$33\}x^i$

r $\{0, 1024, 2048, 3072\}$

r $\{0, 4096, \ldots, 28672\}$

r $\{0, 2048, \ldots, 30720\}$

$96\}x^i$

$48\}x^i$

$96\}x^i$

e (see spec) of $\sum_{0 \le i < 318} \{0, 512\} x^i$

e (see spec) of $\sum_{0 \le i < 410} \{0, 2048\} x^i$

e (see spec) of $\sum_{0 \le i < 490} \{0, 1024\} x^i$

$96\}x^i$

$96\}x^i$

$96\}x^i$

e (see spec) of $\sum_{0 \le i < 274} \{0, 512\} 2^{10i}$

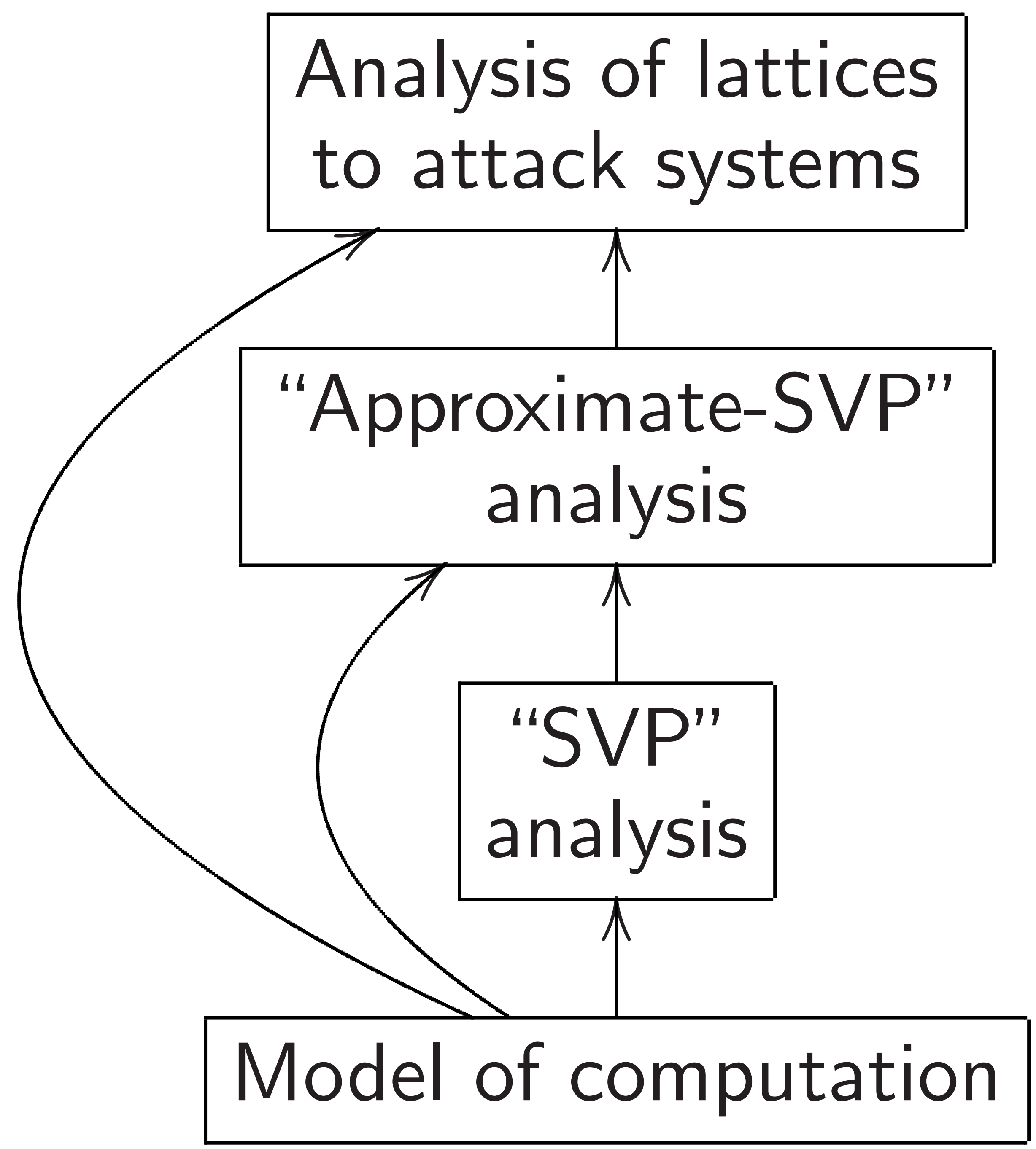
e (see spec) of $\sum_{0 \le i < 274} \{0, 512\} 2^{10i}$

e (see spec) of $\sum_{0 \le i < 274} \{0, 512\} 2^{10i}$

Attacking these problems

Attack strategy with reputation of usually being best: “primal” strategy. Focus of this talk.

Normal layers in analysis:



Models o

Multitap

sort N in

time N^{1}

4576}
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$_{i<512}\{0,126\}x^i$
$_{i<1024}\{0,126\}x^i$
$_{i<1024}\{0,126\}x^i$

$^{512}+x^{768})$

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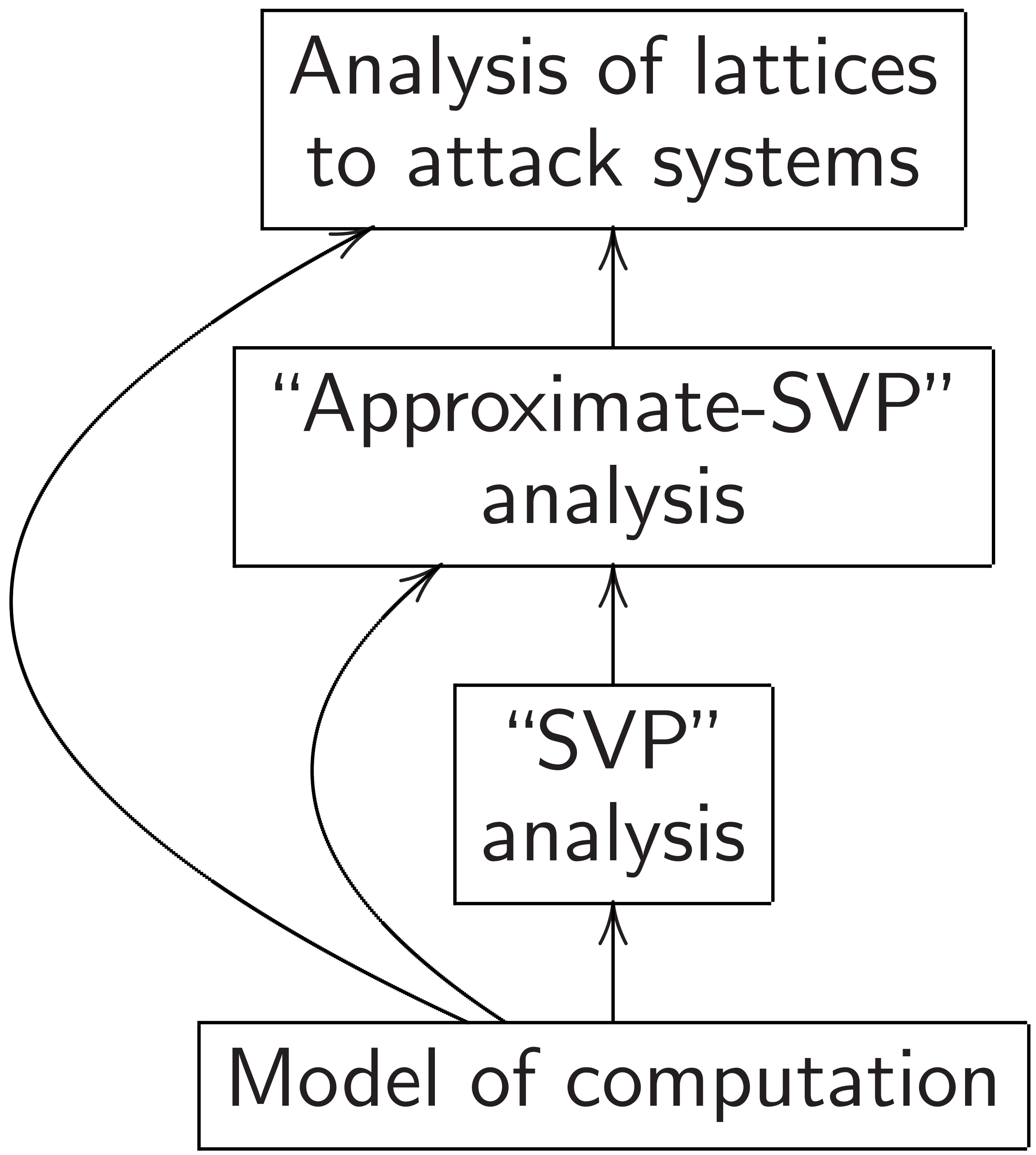
$_{i<318}\{0,512\}x^i$
$_{i<410}\{0,2048\}x^i$
$_{i<490}\{0,1024\}x^i$

$_{i<274}\{0,512\}2^{10i}$
$_{i<274}\{0,512\}2^{10i}$
$_{i<274}\{0,512\}2^{10i}$

Attacking these problems

Attack strategy with reputation of usually being best: "primal" strategy. Focus of this talk.

Normal layers in analysis:



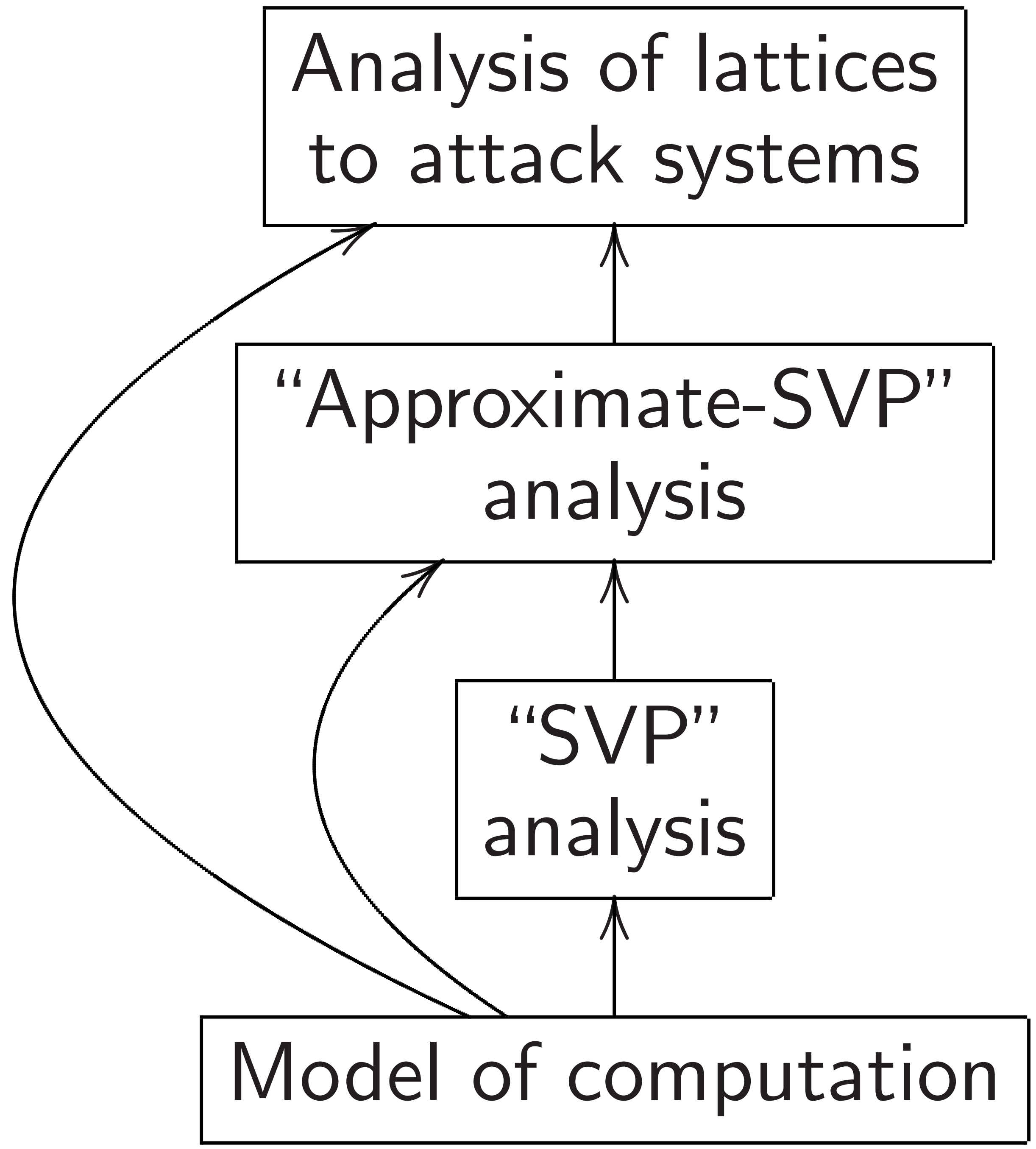
Models of comput

Multitape Turing
sort N ints, each
time $N^{1+o(1)}$, spa

Attacking these problems

Attack strategy with reputation of usually being best: "primal" strategy. Focus of this talk.

Normal layers in analysis:



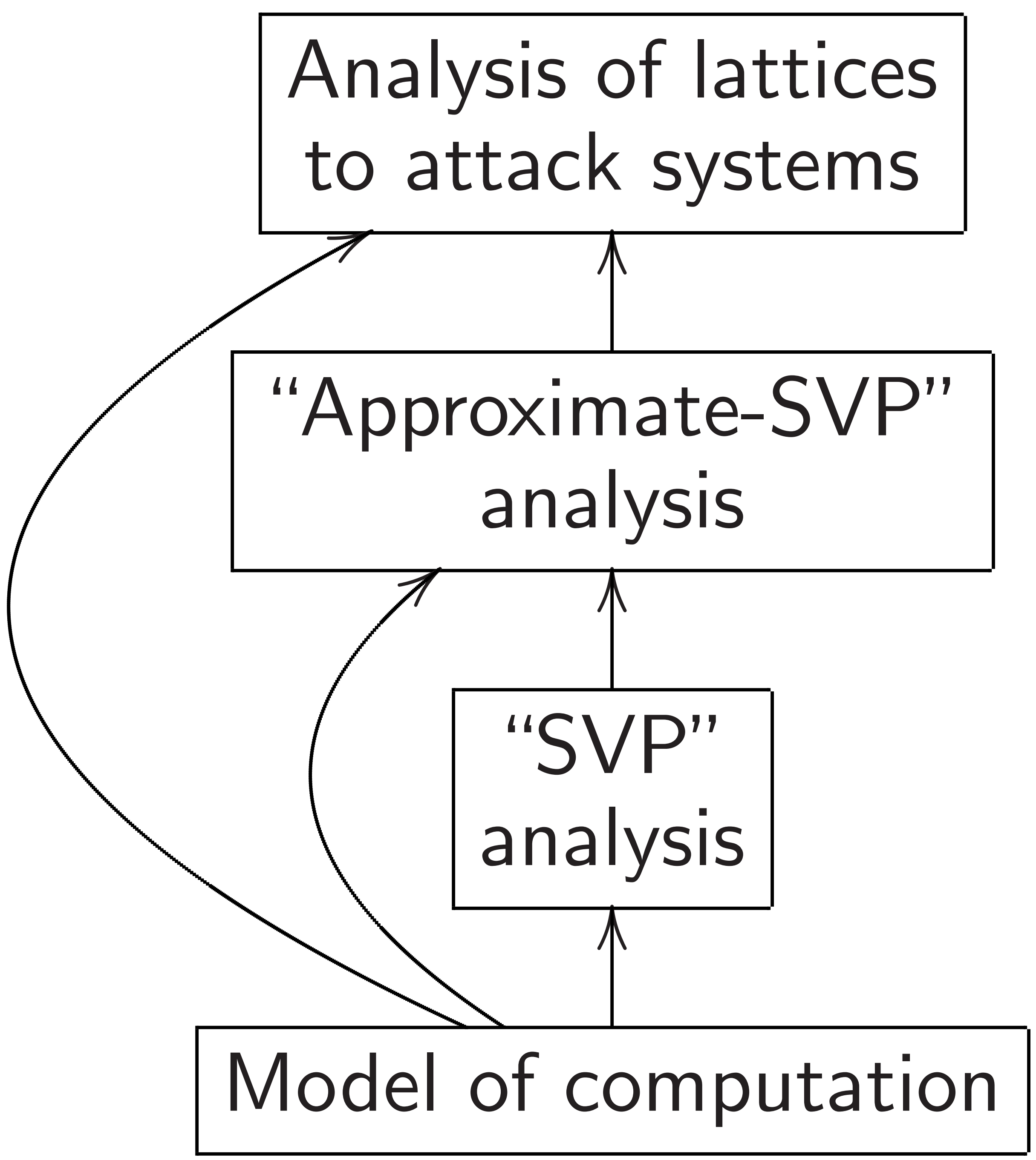
Models of computation

Multitape Turing machine: c
sort N ints, each $N^{o(1)}$ bits,
time $N^{1+o(1)}$, space $N^{1+o(1}$

Attacking these problems

Attack strategy with reputation of usually being best: "primal" strategy. Focus of this talk.

Normal layers in analysis:



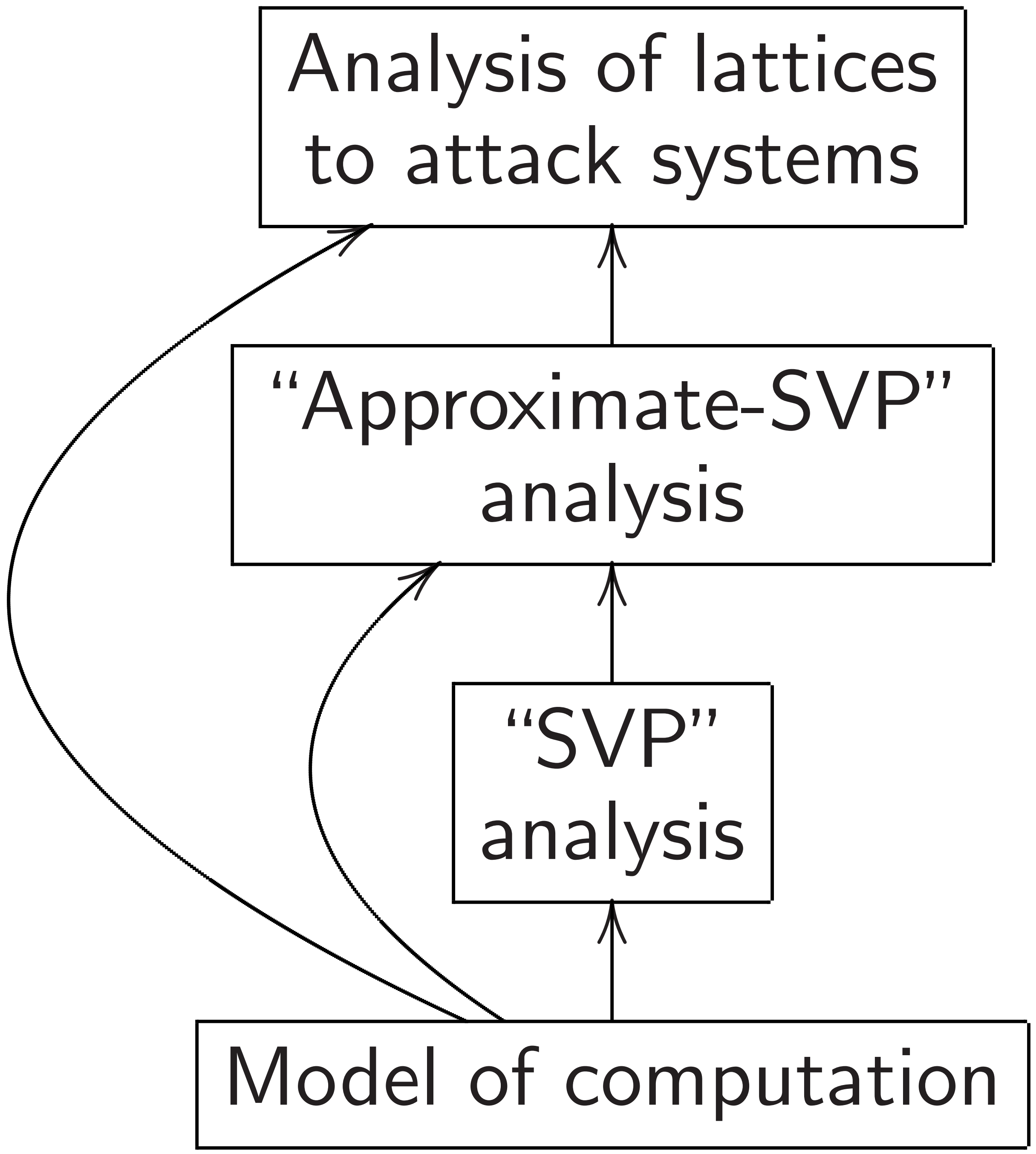
Models of computation

Multitape Turing machine: e.g., sort N ints, each $N^{o(1)}$ bits, in time $N^{1+o(1)}$, space $N^{1+o(1)}$.

Attacking these problems

Attack strategy with reputation of usually being best: "primal" strategy. Focus of this talk.

Normal layers in analysis:



Models of computation

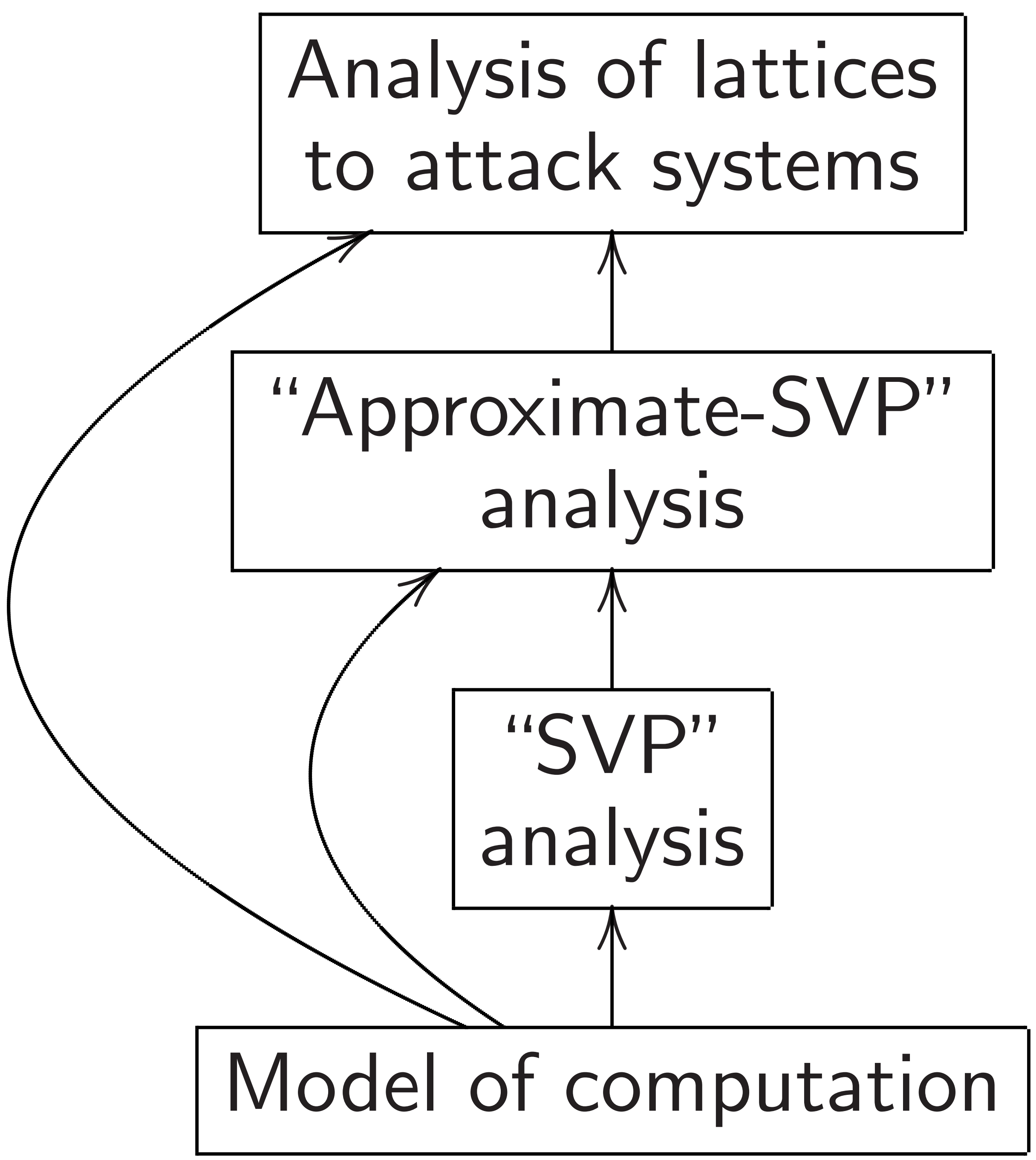
Multitape Turing machine: e.g., sort N ints, each $N^{o(1)}$ bits, in time $N^{1+o(1)}$, space $N^{1+o(1)}$.

Brent–Kung 2D circuit model allows parallelism—e.g., sort in time $N^{0.5+o(1)}$, space $N^{1+o(1)}$.

Attacking these problems

Attack strategy with reputation of usually being best: "primal" strategy. Focus of this talk.

Normal layers in analysis:



Models of computation

Multitape Turing machine: e.g., sort N ints, each $N^{o(1)}$ bits, in time $N^{1+o(1)}$, space $N^{1+o(1)}$.

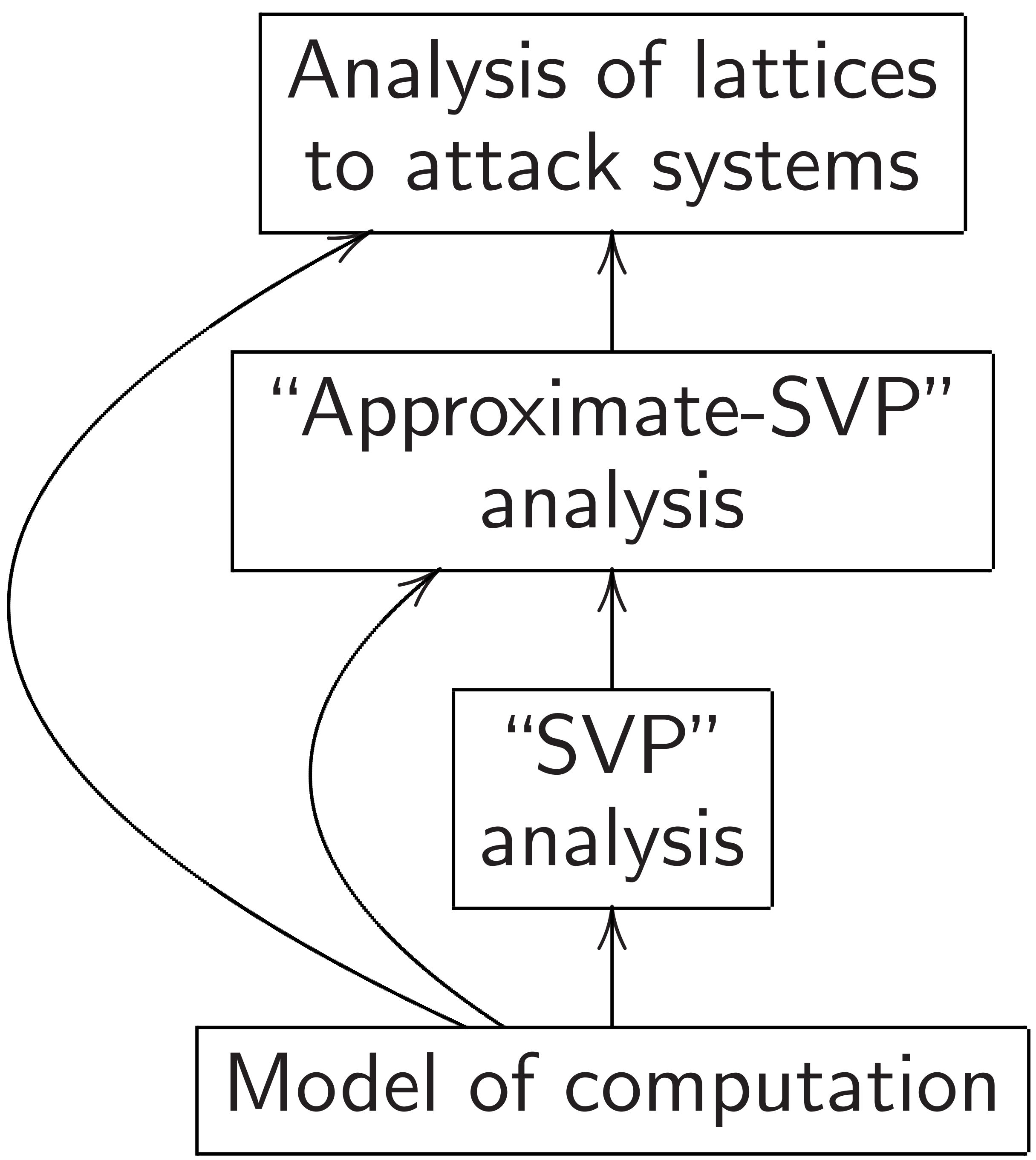
Brent–Kung 2D circuit model allows parallelism—e.g., sort in time $N^{0.5+o(1)}$, space $N^{1+o(1)}$.

PRAM: multiple inequivalent definitions, untethered to physical explanations. Sort in time $N^{o(1)}$.

Attacking these problems

Attack strategy with reputation of usually being best: "primal" strategy. Focus of this talk.

Normal layers in analysis:



Models of computation

Multitape Turing machine: e.g., sort N ints, each $N^{o(1)}$ bits, in time $N^{1+o(1)}$, space $N^{1+o(1)}$.

Brent–Kung 2D circuit model allows parallelism—e.g., sort in time $N^{0.5+o(1)}$, space $N^{1+o(1)}$.

PRAM: multiple inequivalent definitions, untethered to physical explanations. Sort in time $N^{o(1)}$.

Quantum computing: similar divergence of models.

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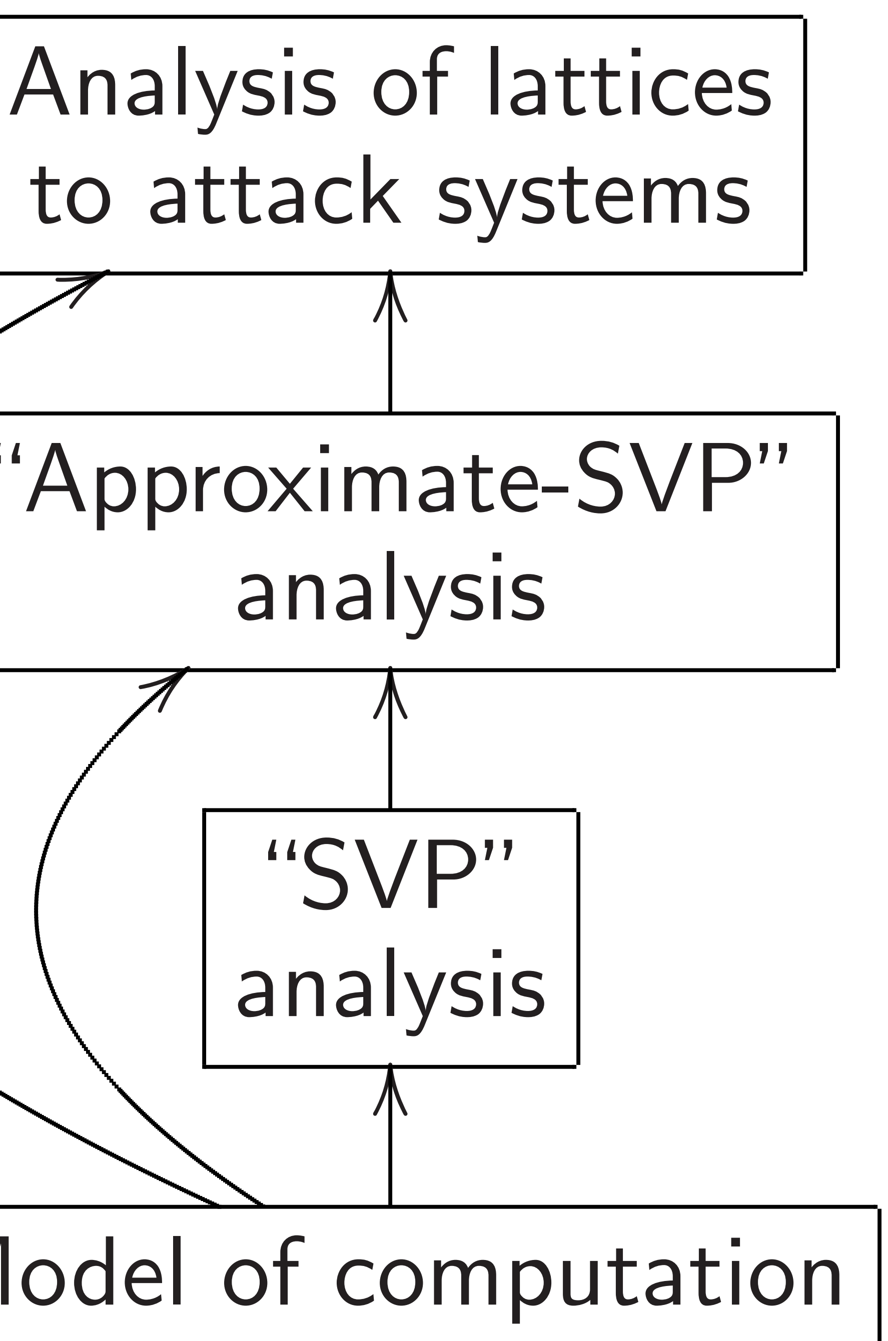
layers in analysis:



Models of computation

Multitape Turing machine: e.g., sort N ints, each $N^{o(1)}$ bits, in time $N^{1+o(1)}$, space $N^{1+o(1)}$.

Brent–Kung 2D circuit model allows parallelism—e.g., sort in time $N^{0.5+o(1)}$, space $N^{1+o(1)}$.

PRAM: multiple inequivalent definitions, untethered to physical explanations. Sort in time $N^{o(1)}$.

Quantum computing: similar divergence of models.

Lattices

Rewrite
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Problem
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Models of computation

Multitape Turing machine: e.g., sort N ints, each $N^{o(1)}$ bits, in time $N^{1+o(1)}$, space $N^{1+o(1)}$.

Brent–Kung 2D circuit model allows parallelism—e.g., sort in time $N^{0.5+o(1)}$, space $N^{1+o(1)}$.

PRAM: multiple inequivalent definitions, untethered to physical explanations. Sort in time $N^{o(1)}$.

Quantum computing:
similar divergence of models.

Lattices

Rewrite each prob
short nonzero solu
of homogeneous $\mathcal{R}$

Problem 1: Find (
with $aG + e = 0$,

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Models of computation

Multitape Turing machine: e.g., sort N ints, each $N^{o(1)}$ bits, in time $N^{1+o(1)}$, space $N^{1+o(1)}$.

Brent–Kung 2D circuit model allows parallelism—e.g., sort in time $N^{0.5+o(1)}$, space $N^{1+o(1)}$.

PRAM: multiple inequivalent definitions, untethered to physical explanations. Sort in time $N^{o(1)}$.

Quantum computing: similar divergence of models.

Lattices

Rewrite each problem as fin

short nonzero solution to sy

of homogeneous $\mathcal{R}/q$ equat

Problem 1: Find $(a, e) \in \mathcal{R}^2$

with $aG + e = 0$, given $G \in$

Models of computation

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PRAM: multiple inequivalent definitions, untethered to physical explanations. Sort in time $N^{o(1)}$.

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Lattice has rank $2 \cdot 761 = 1$

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Attack parameter: $k = 13$. Force k positions in a to be 0: restrict to sublattice of rank 1509.

$\Pr[a \text{ is in sublattice}] \approx 0.2\%$.

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Attack parameter: $m = 600$.

Ignore $761 - m = 161$ equations: i.e., project e onto 600 positions.

Projected sublattice rank $d = 1509 - 161 = 1348$; det q^{600}.

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How long does BKZ-β take?

Standard answer: $2^{0.292\beta} = 2^{153.3}$ operations by "sieving".

(Plugging $o(1) = 0$ into the $2^{(0.292+o(1))\beta}$ *asymptotic* does not match experiments. What's the actual performance? And what exactly is an "operation"?)

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Common claim: This saves time only for sufficiently narrow $\{a\}$. (Is this true, or a calculation error in existing algorithm analyses?)