

Sorting integer arrays:
security, speed, and verification

D. J. Bernstein

University of Illinois at Chicago,
Ruhr-University Bochum

Bob's laptop screen:

From: Alice

Thank you for your
submission. We received
many interesting papers,
and unfortunately your

Bob assumes this message is
something Alice actually sent.

But today's "security" systems
fail to guarantee this property.
Attacker could have modified
or forged the message.

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Any hope of fixing this?

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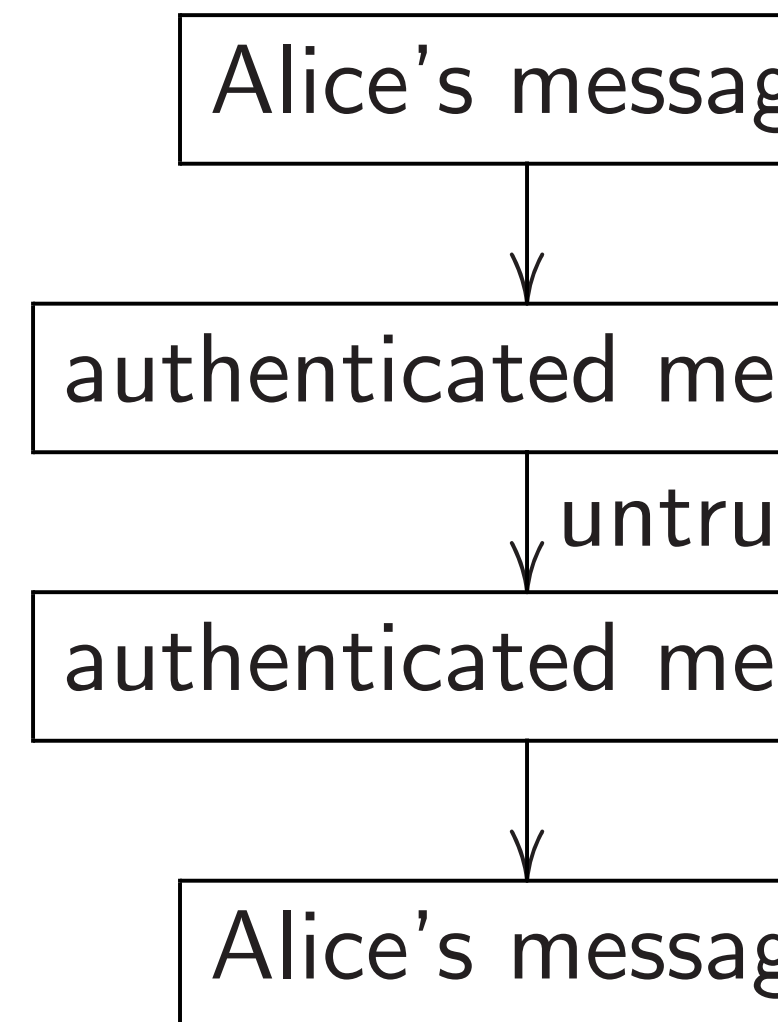
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Cryptographic solution

Message-authentication



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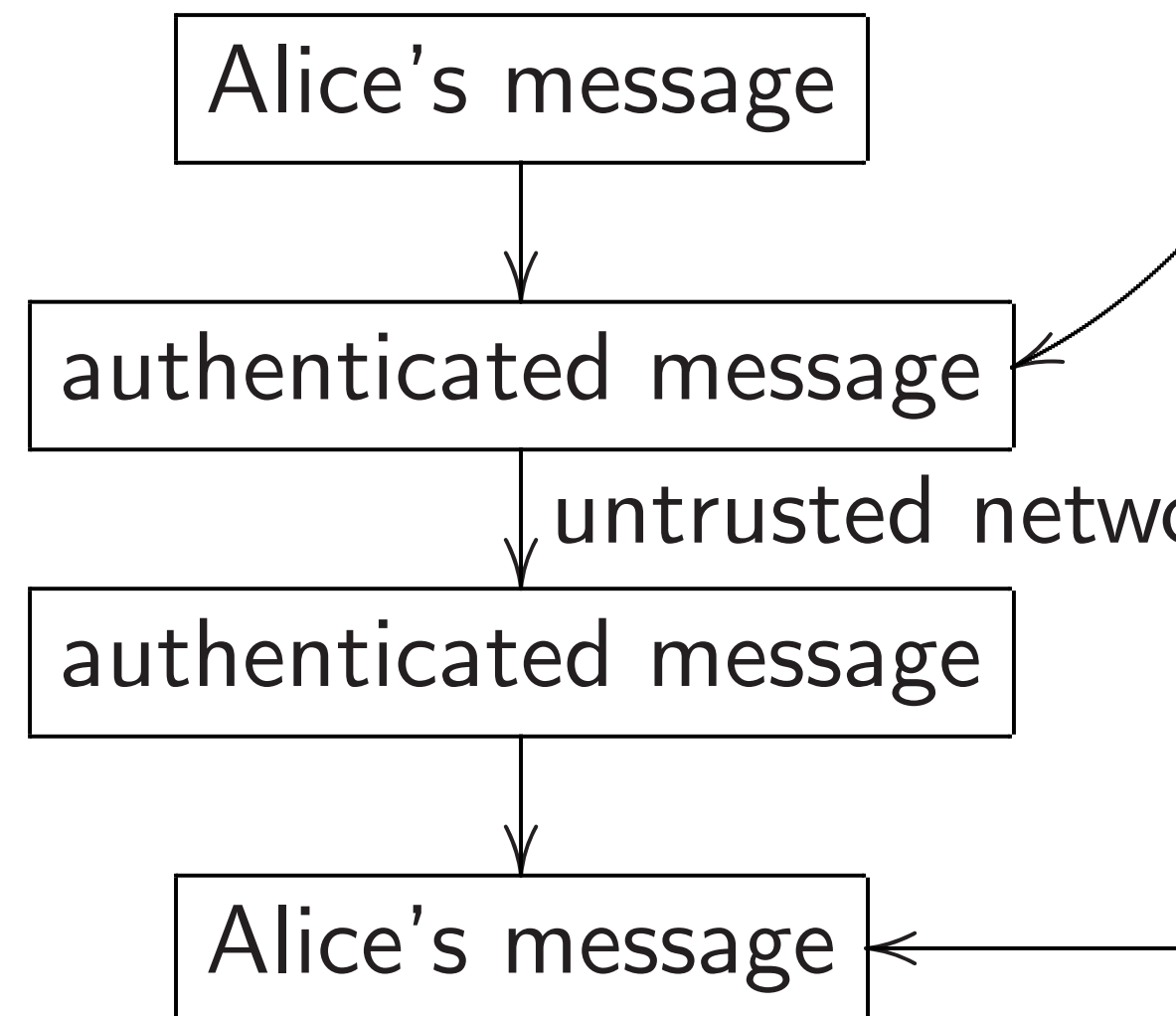
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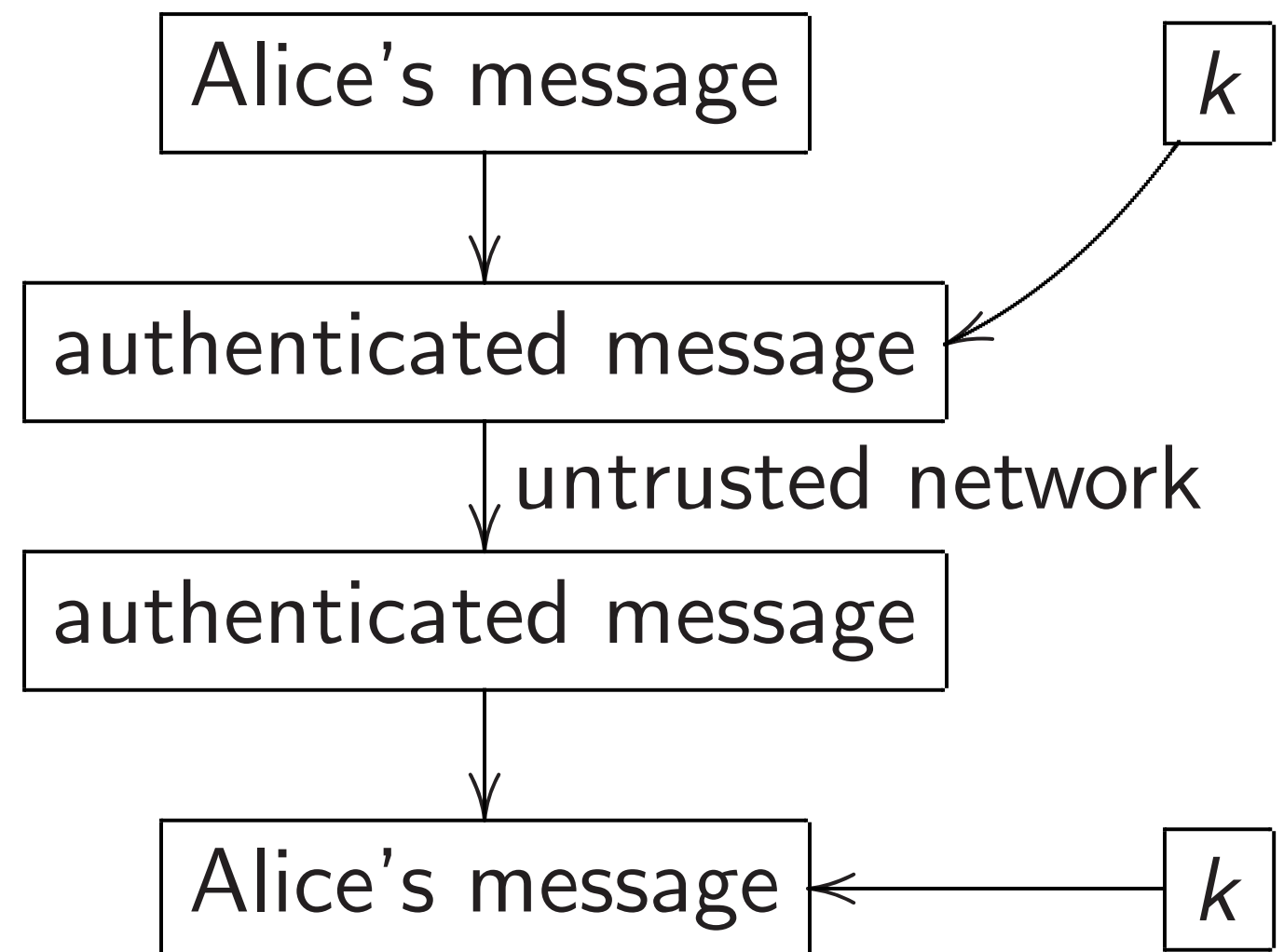
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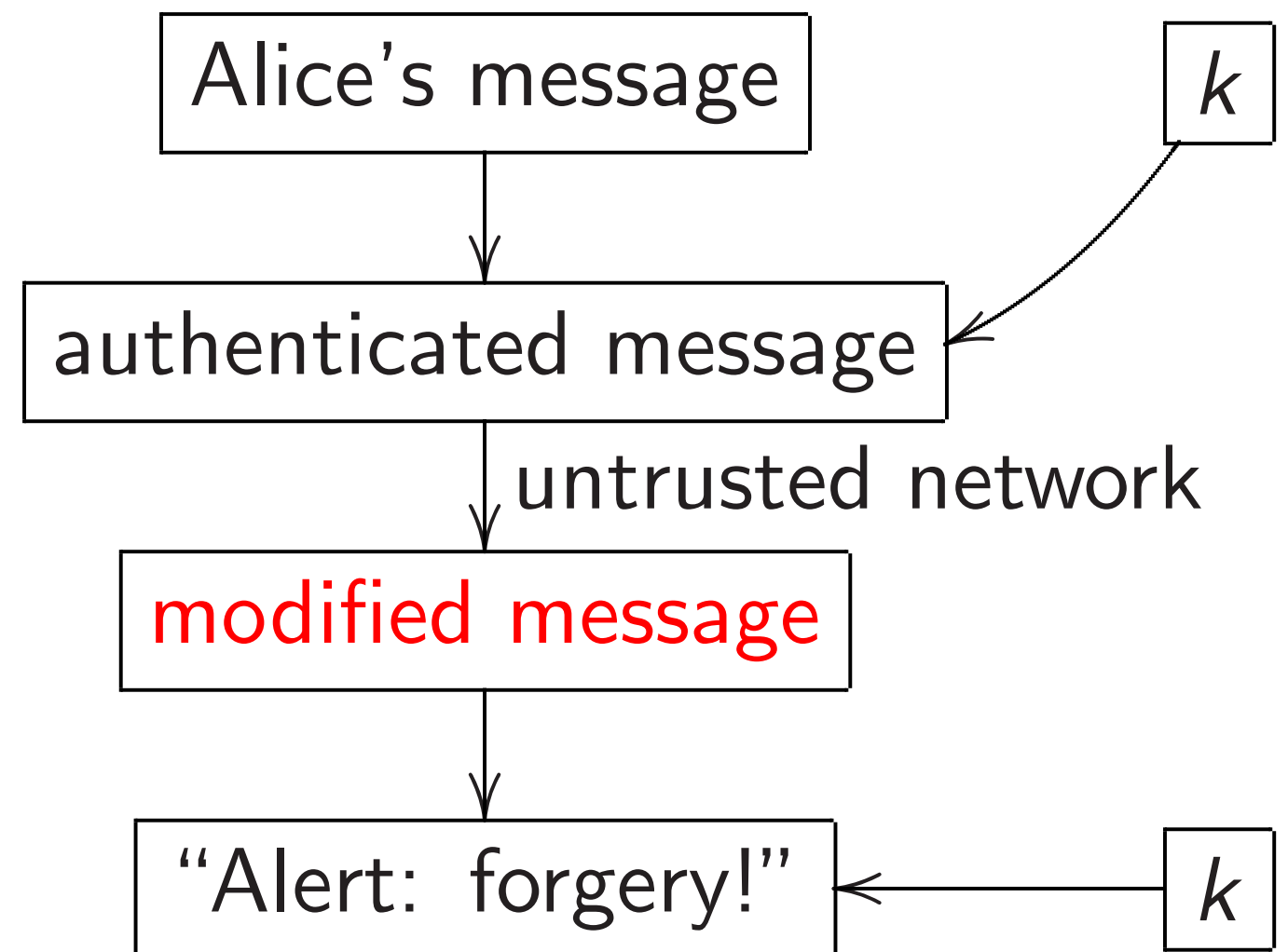
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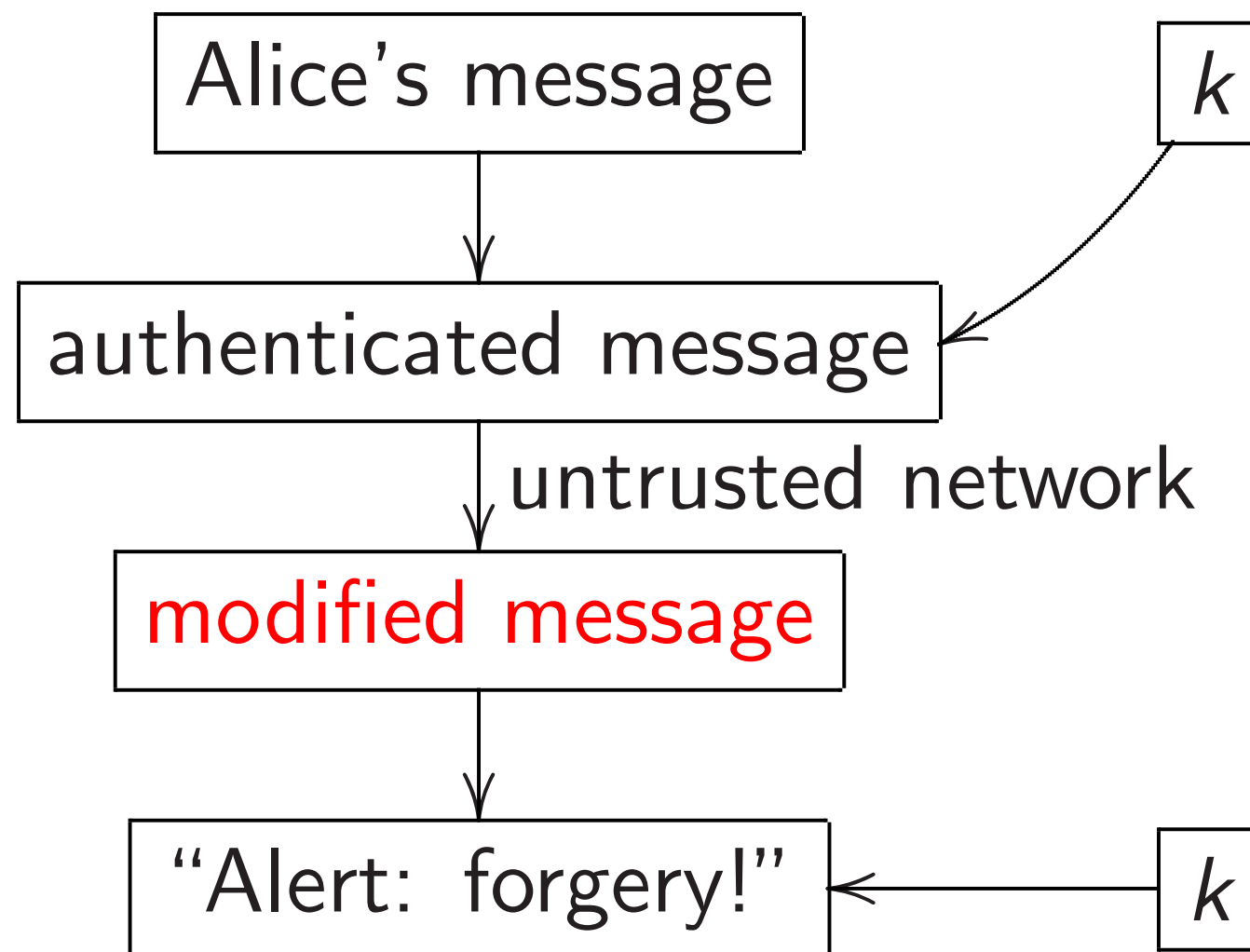
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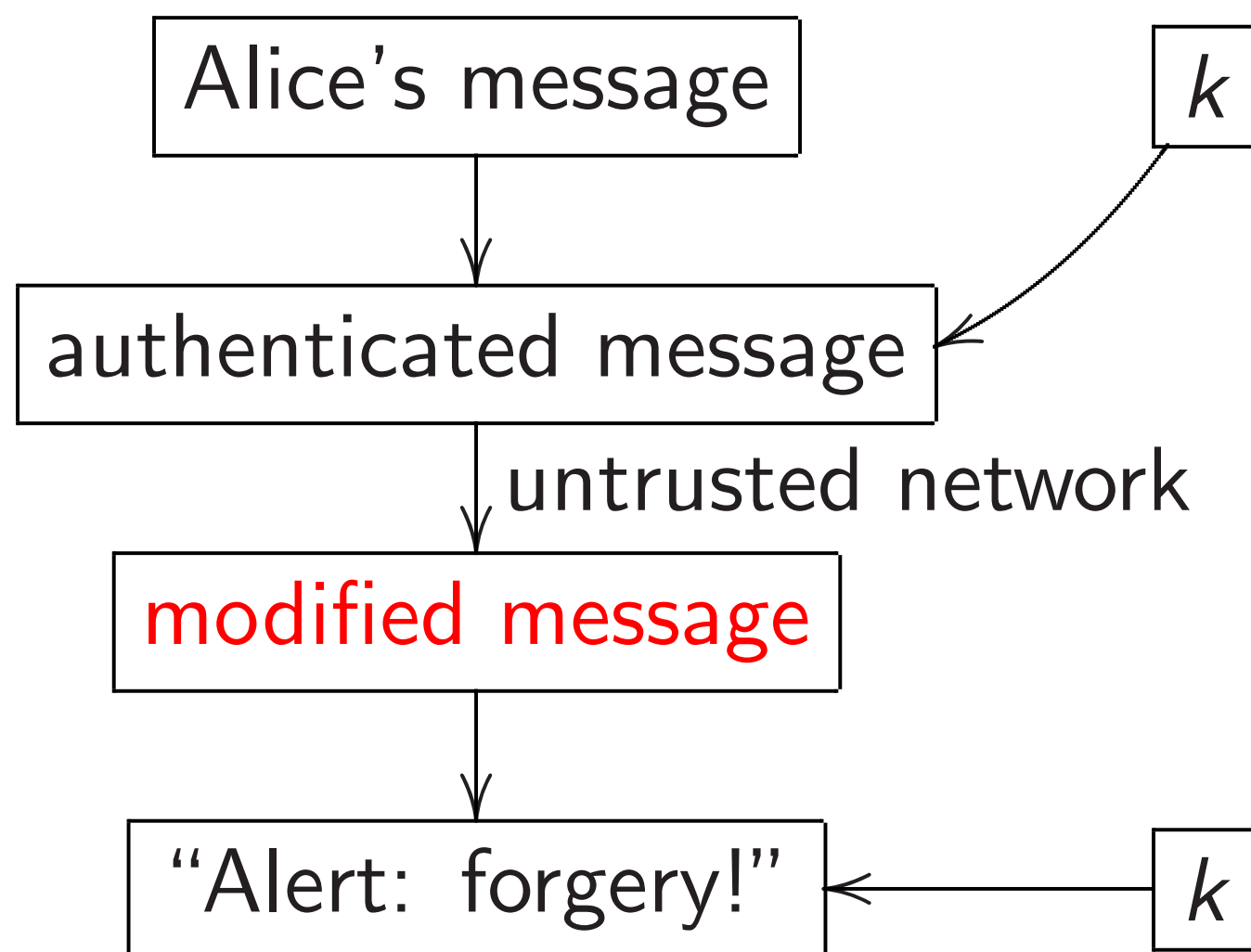
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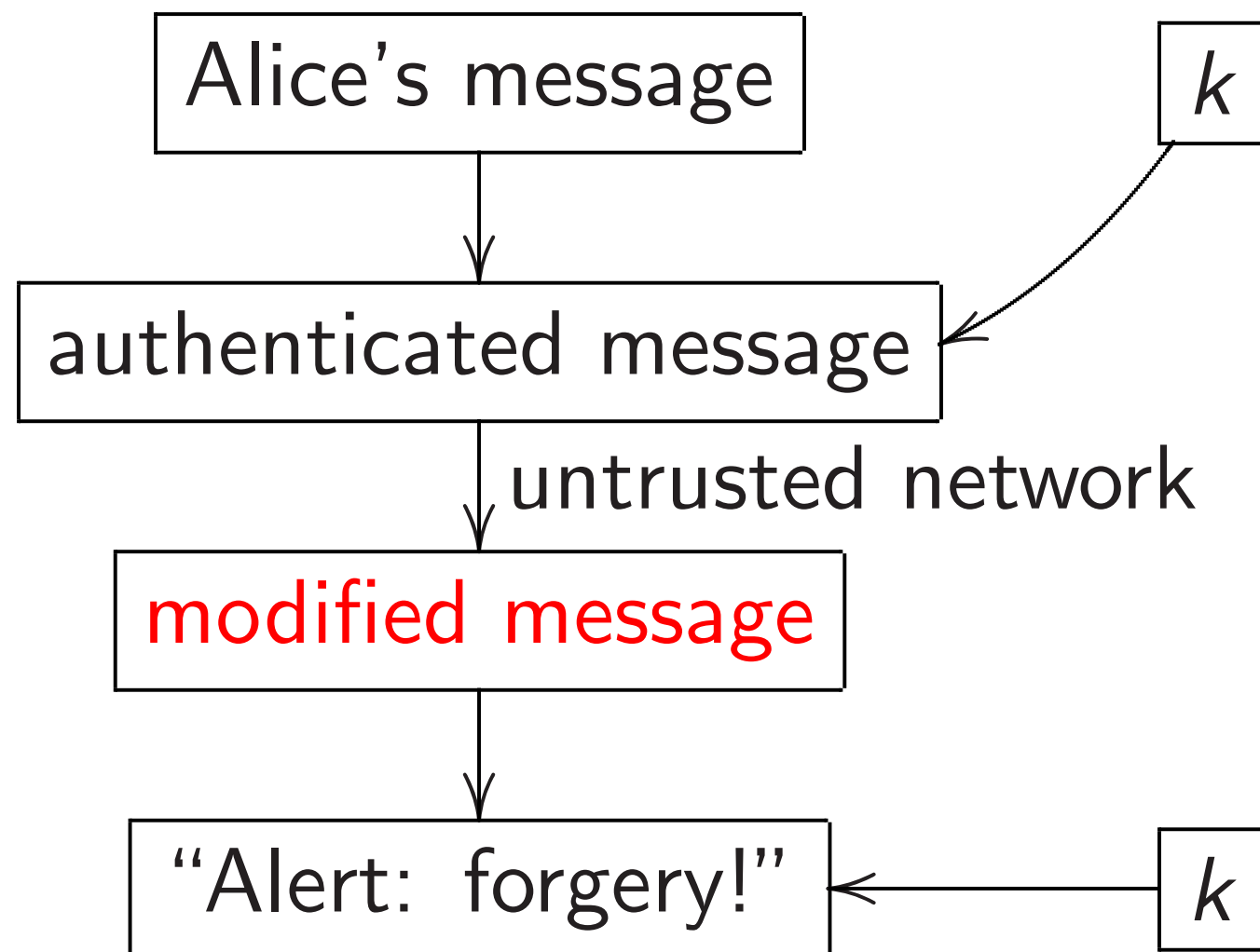
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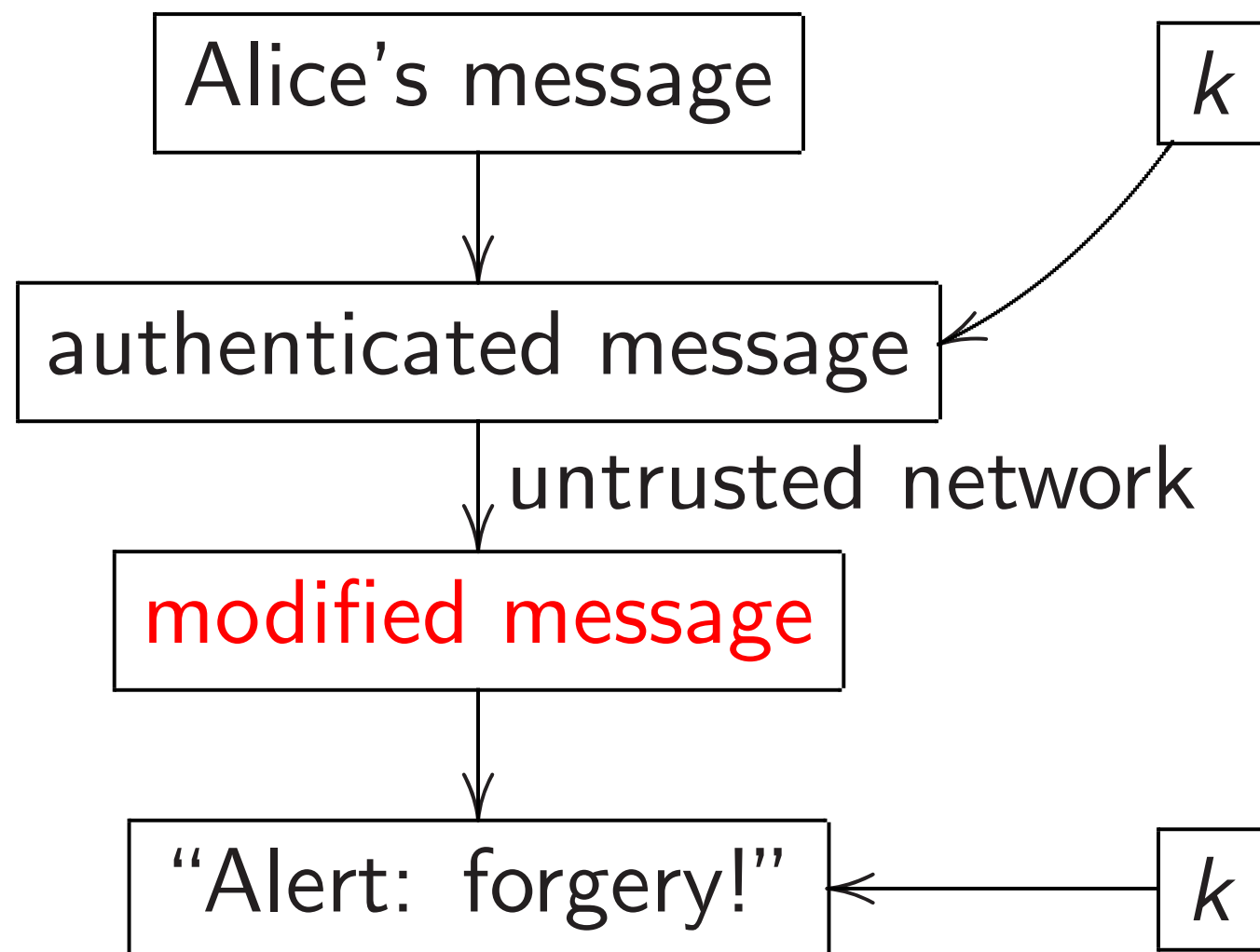
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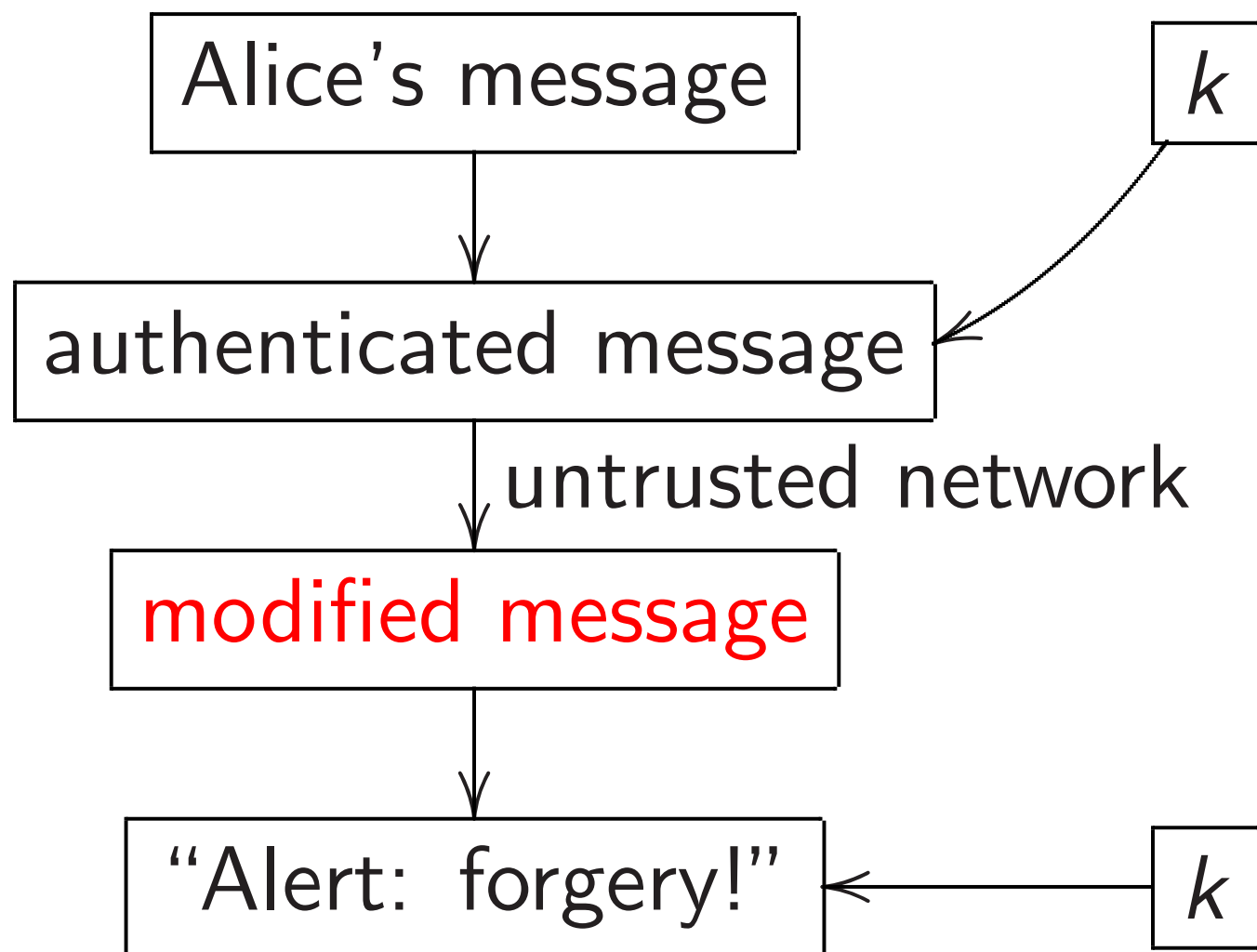
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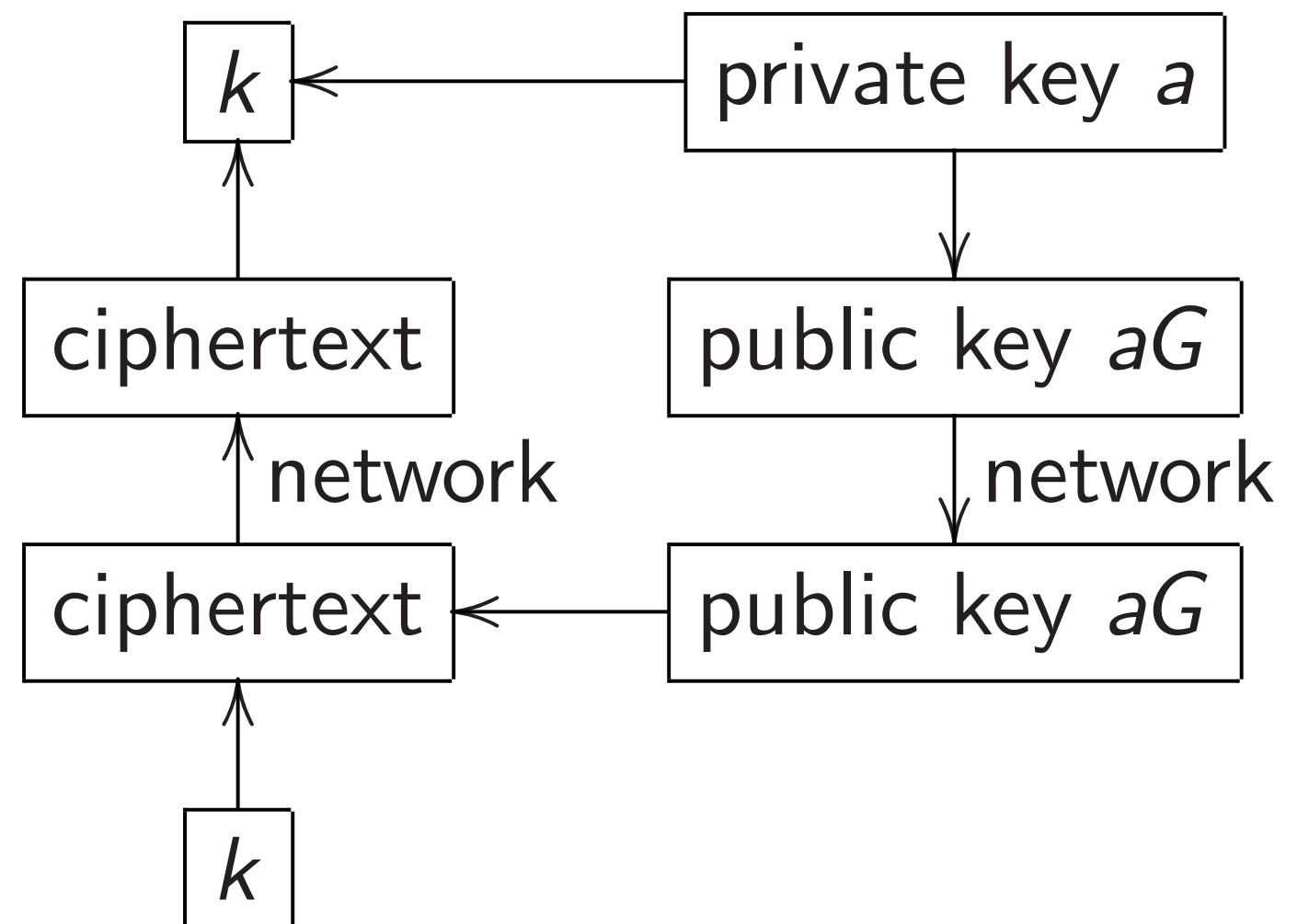


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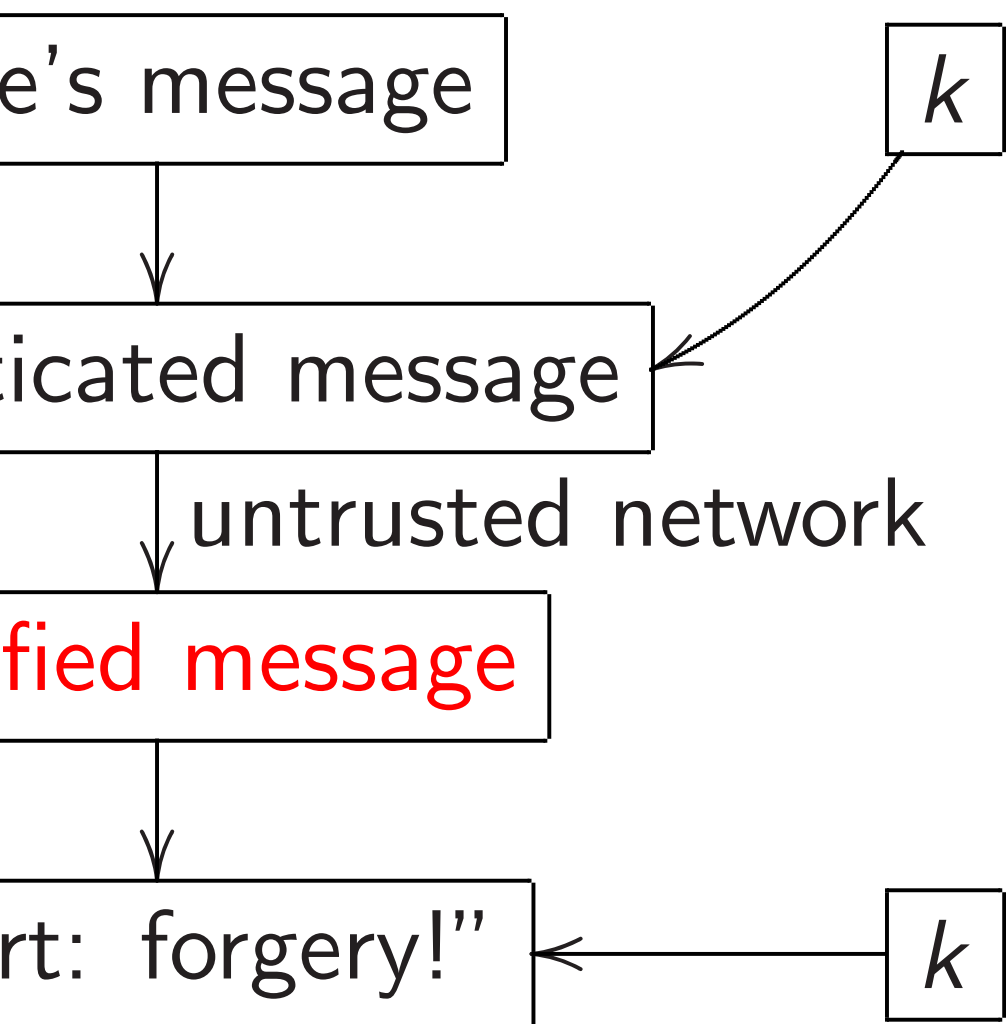
Public-key encryption.



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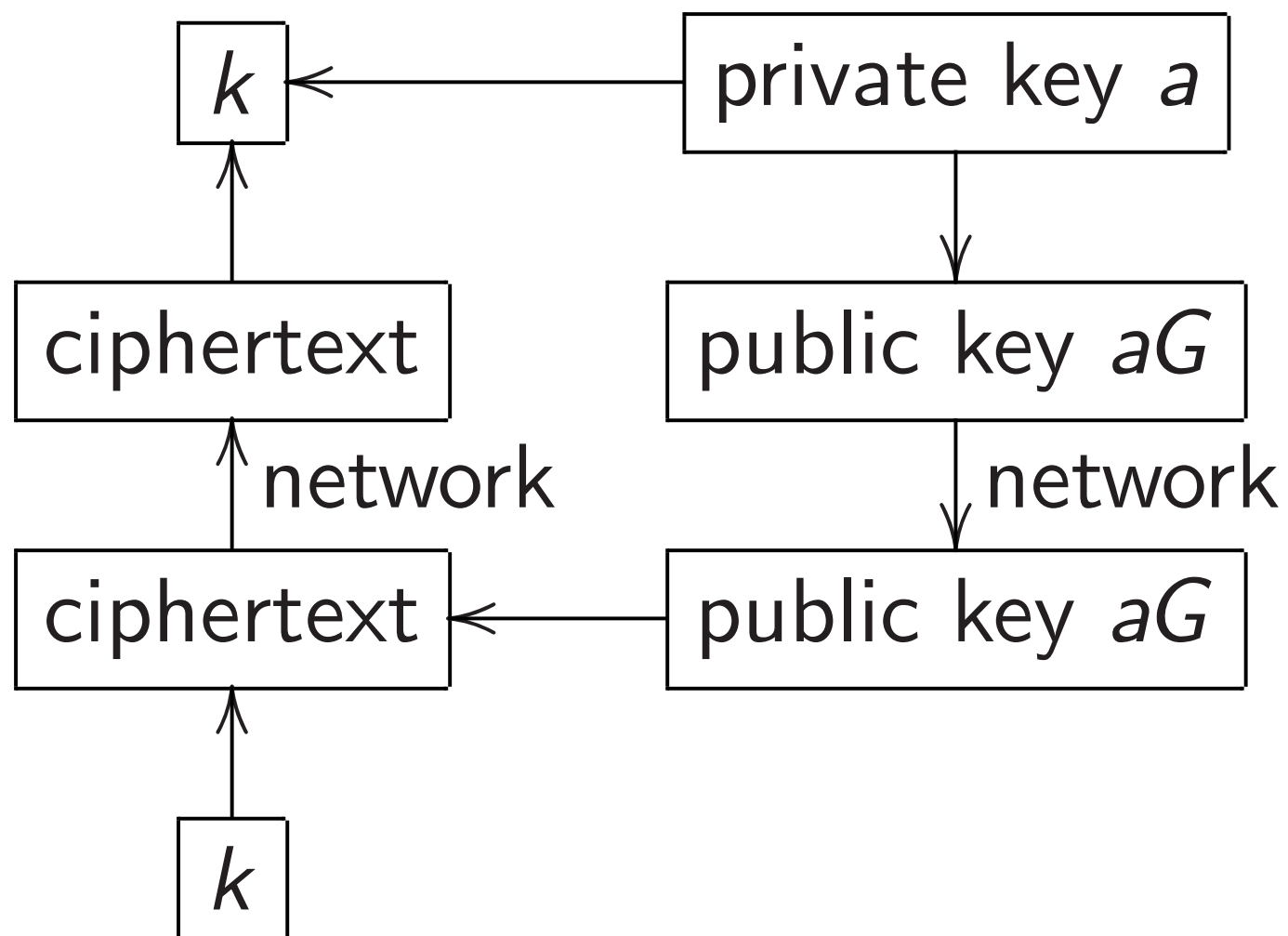
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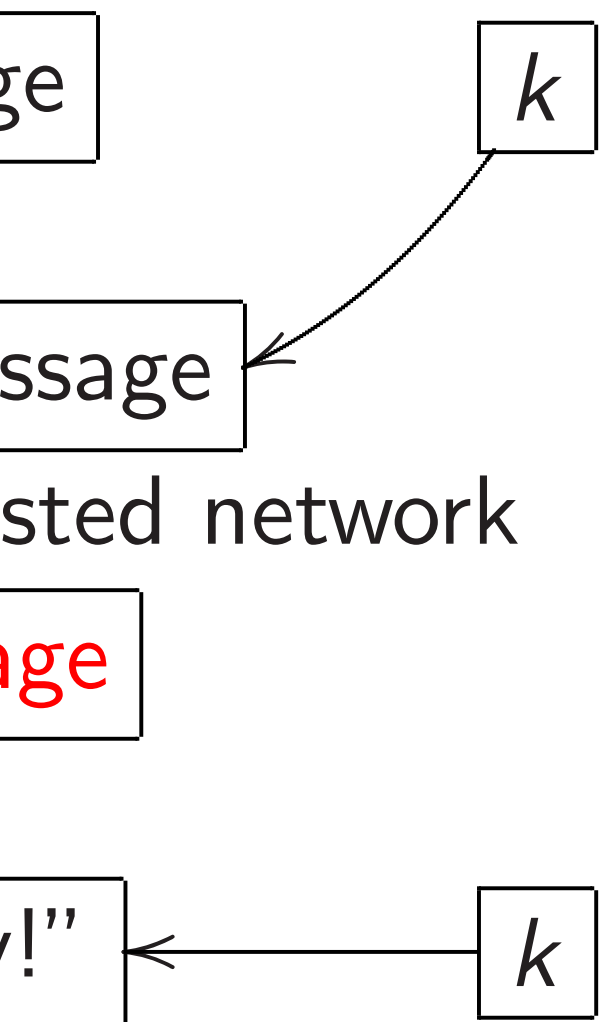
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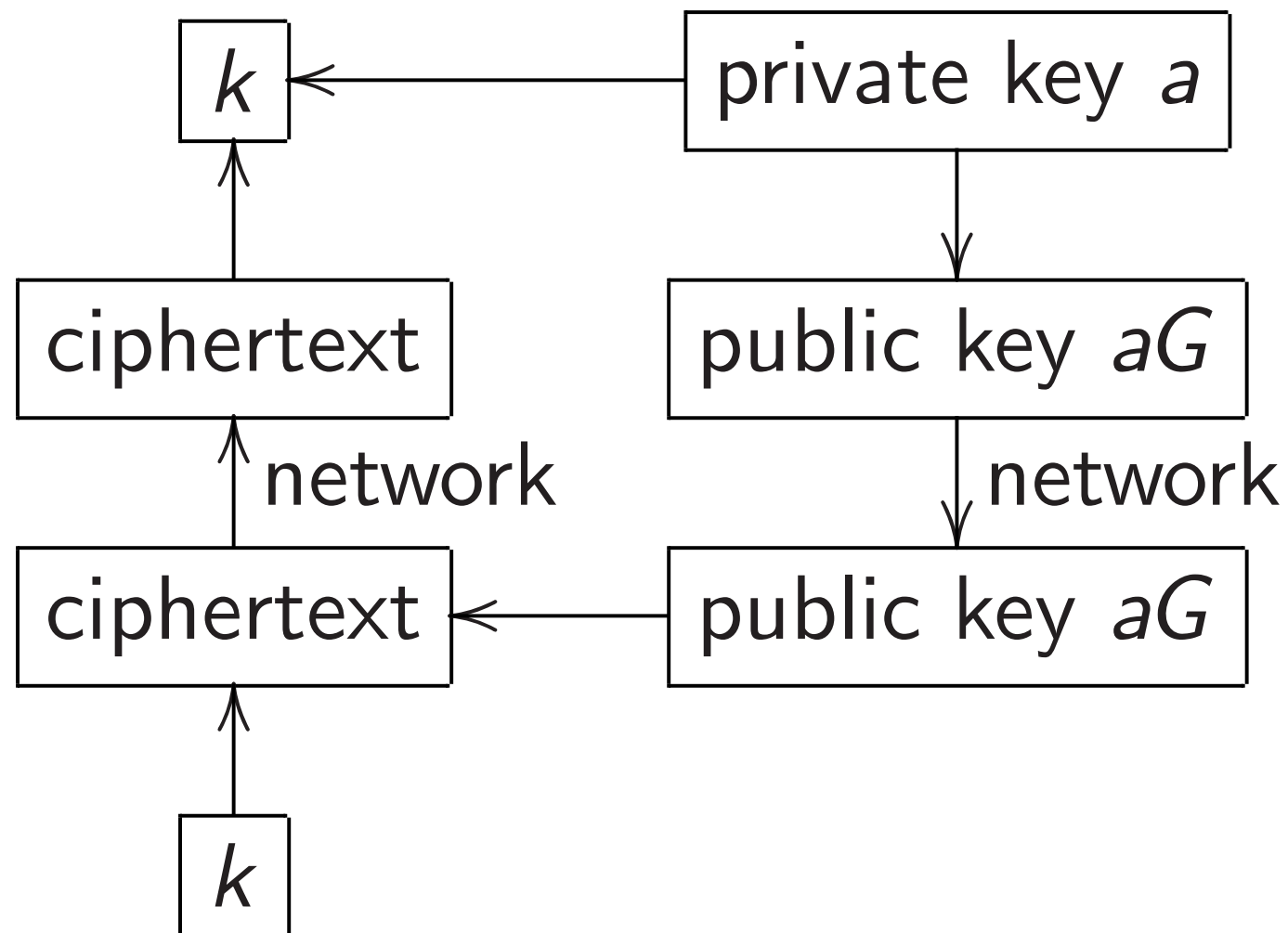
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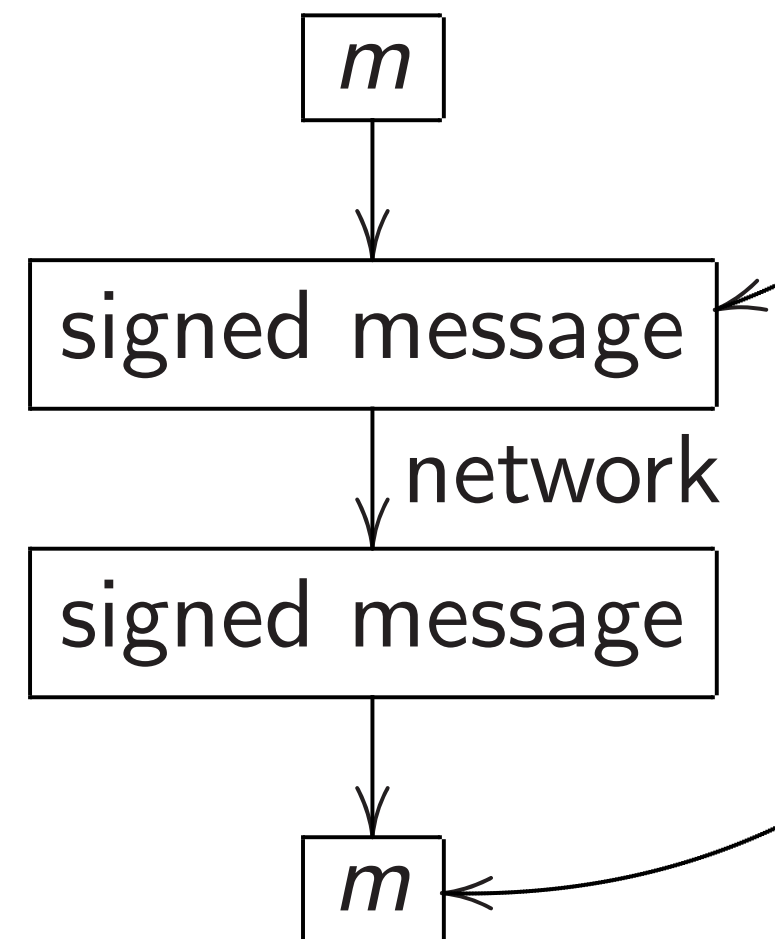
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Solution 2:
Public-key signatu



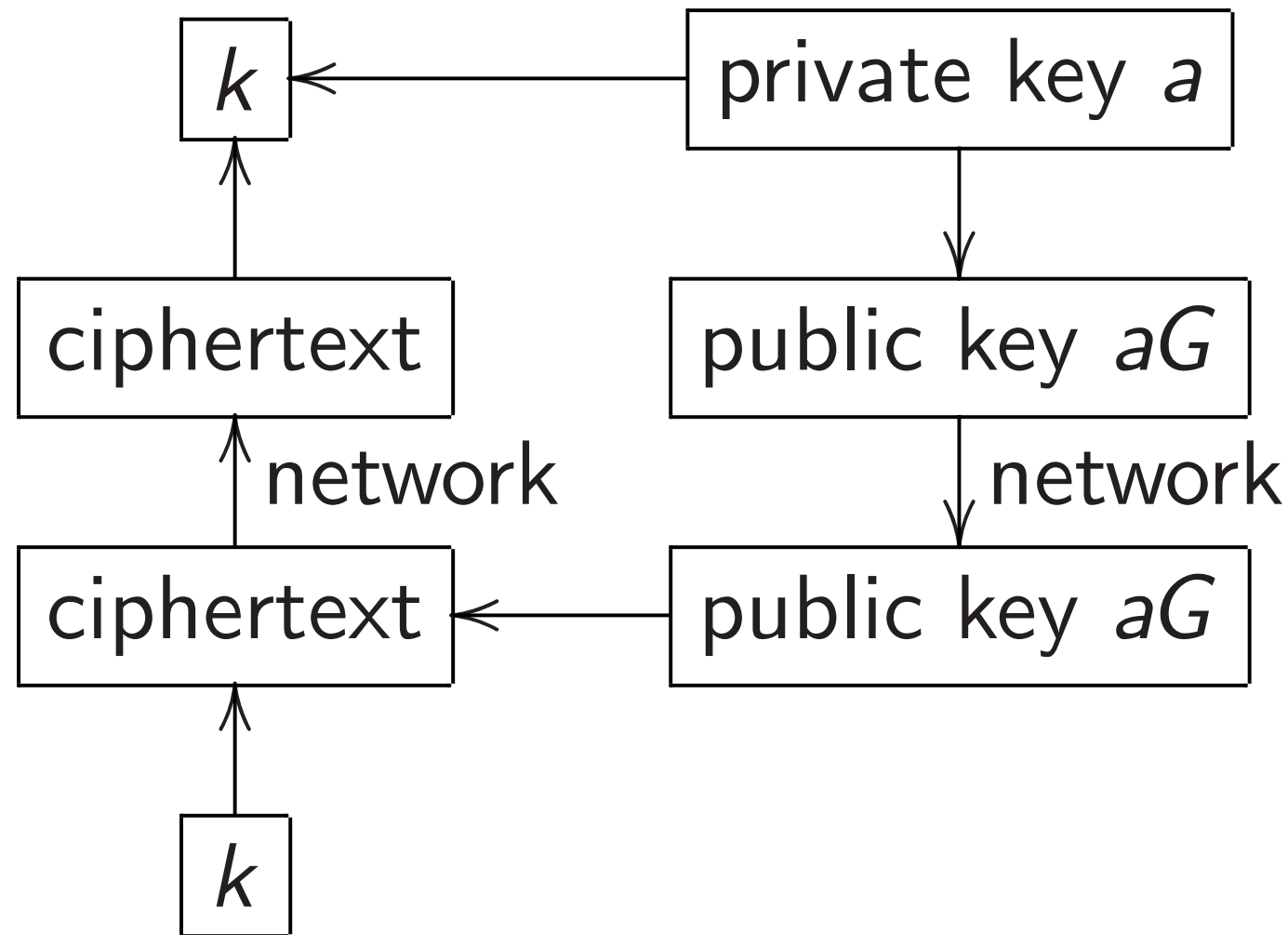
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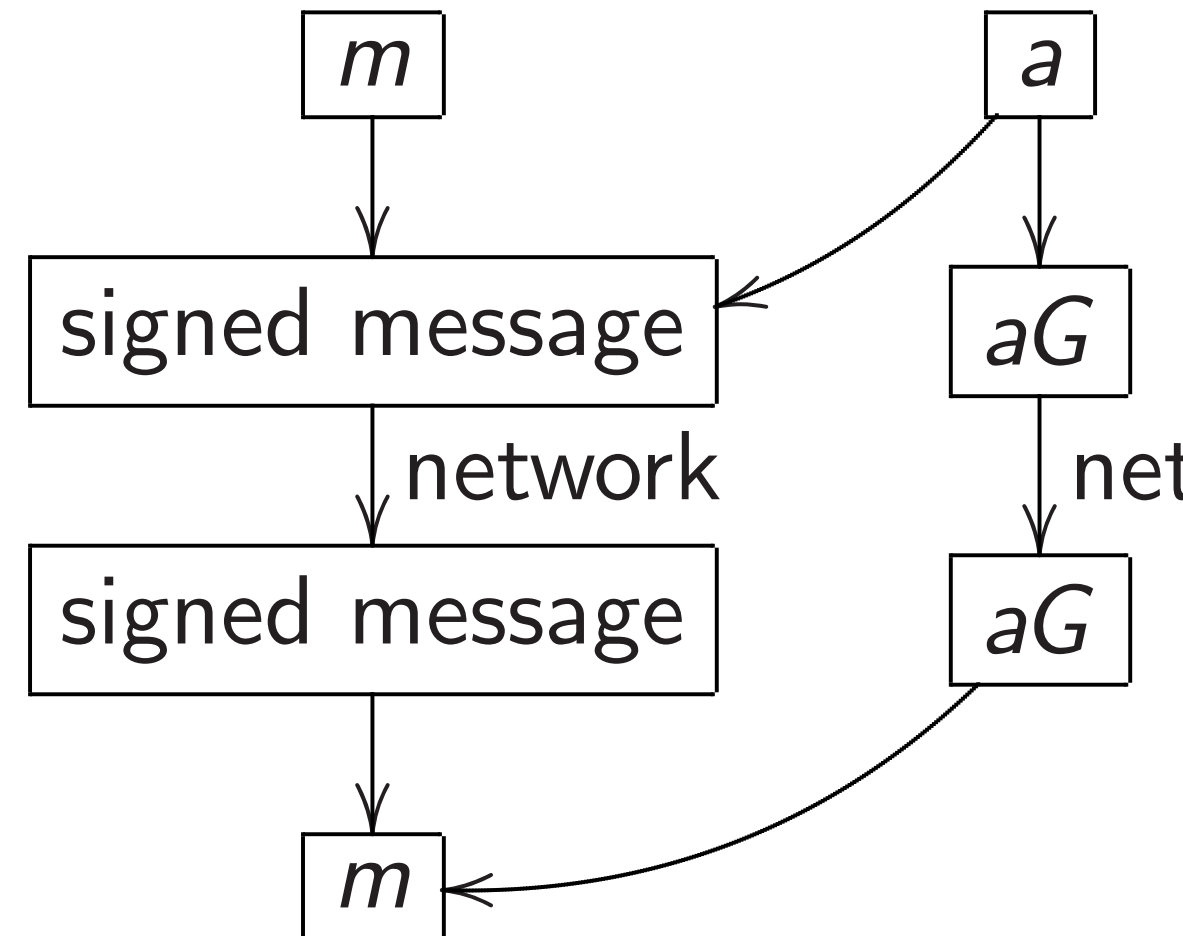
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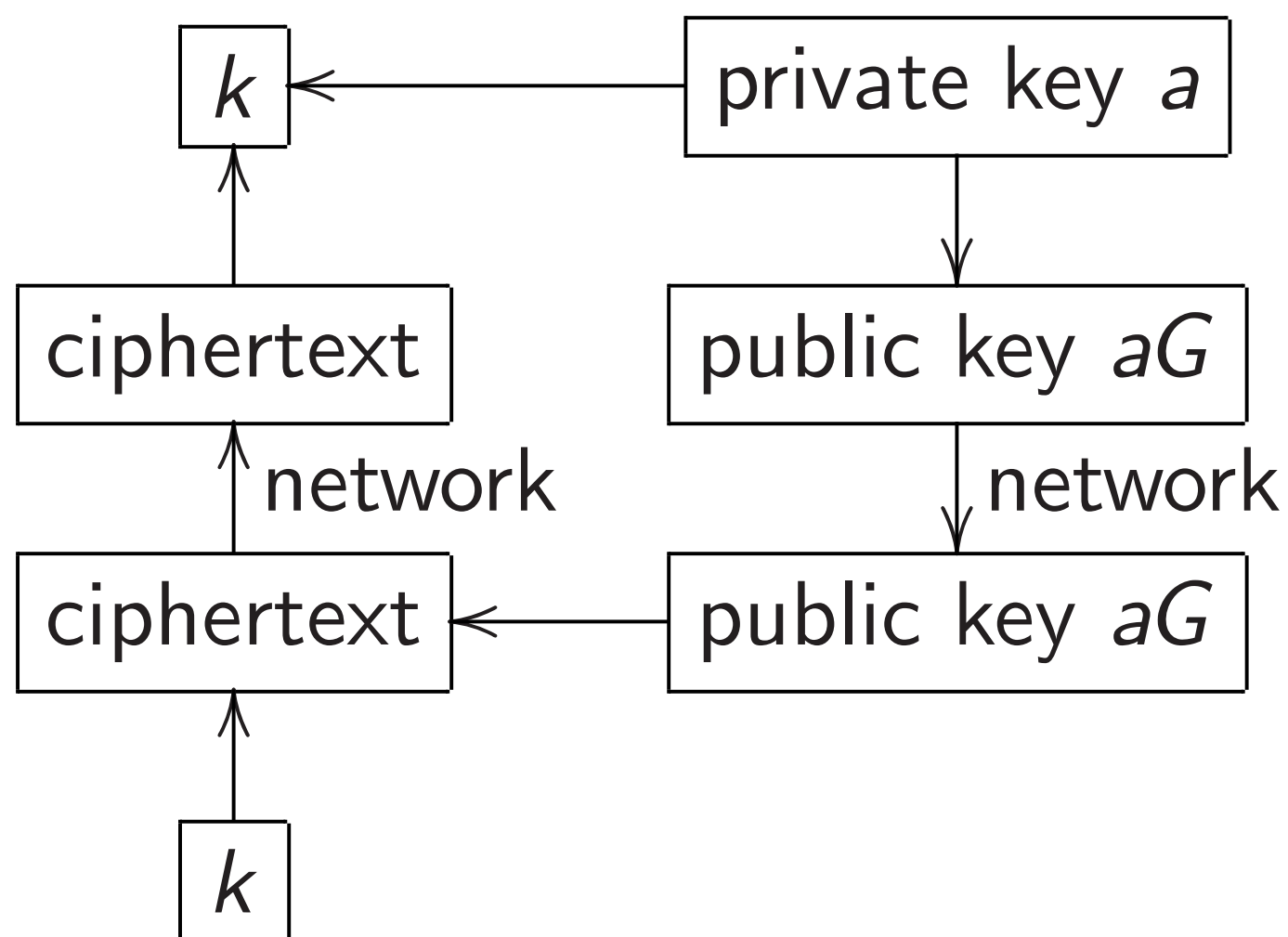


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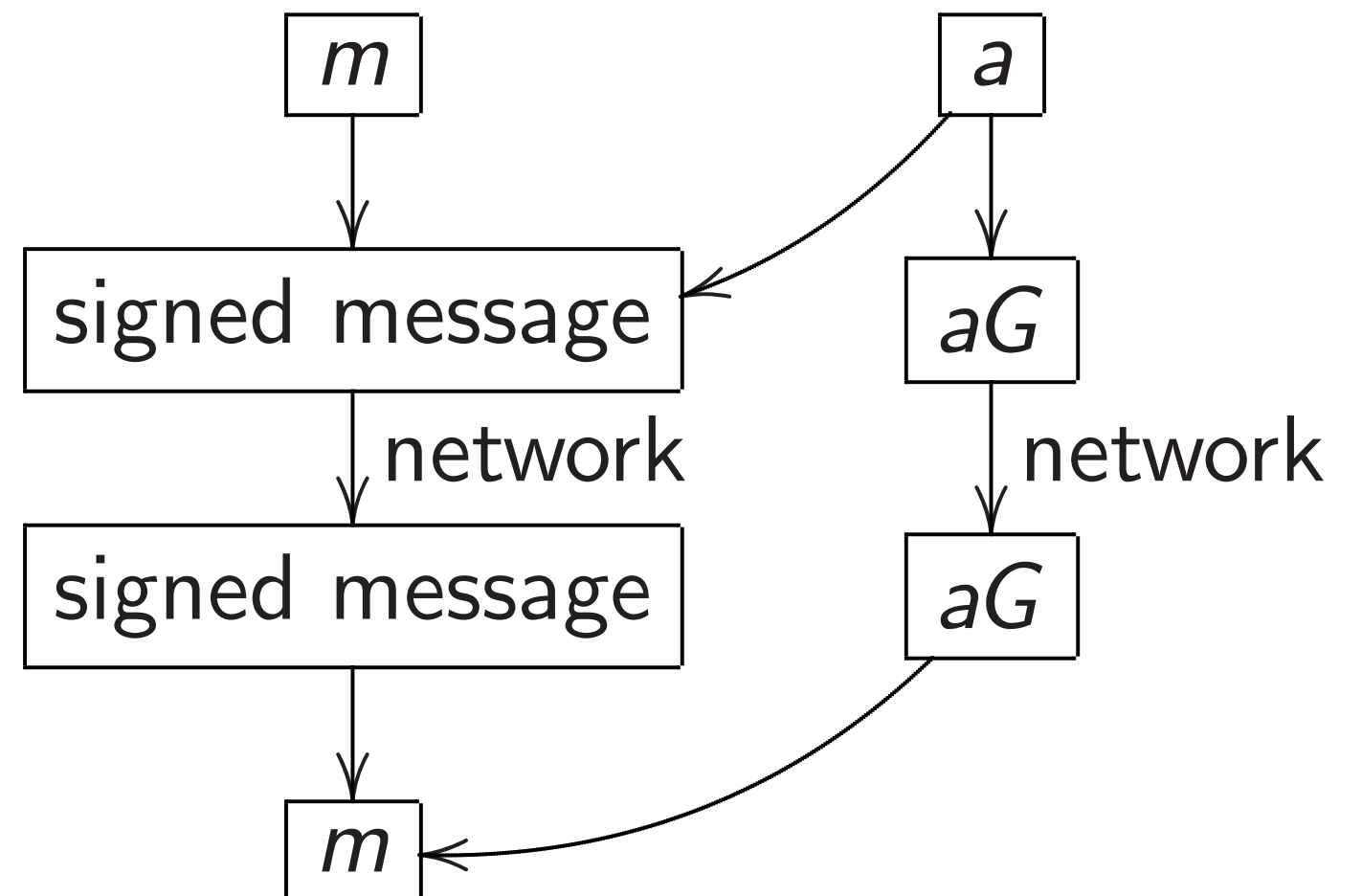
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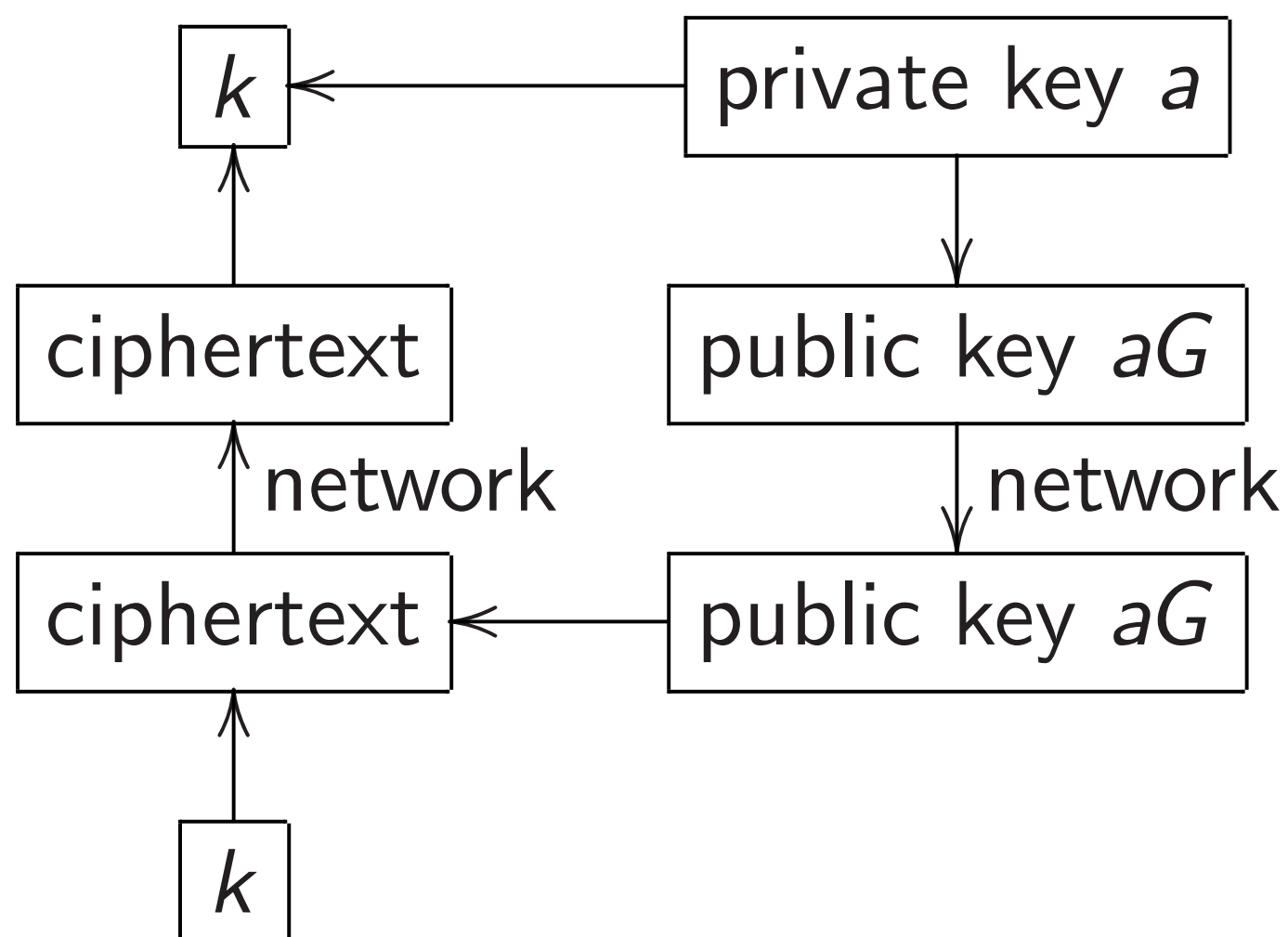


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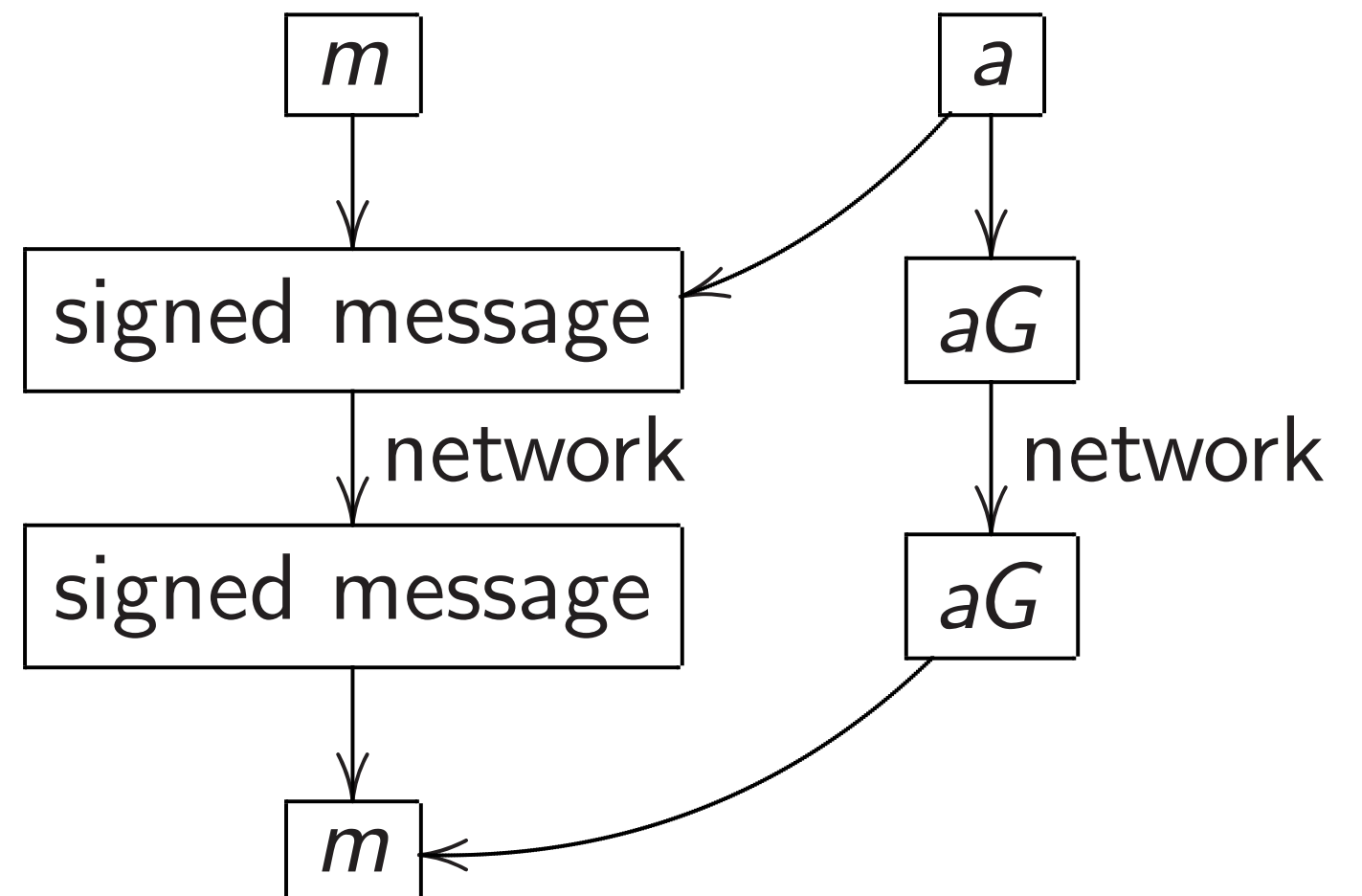
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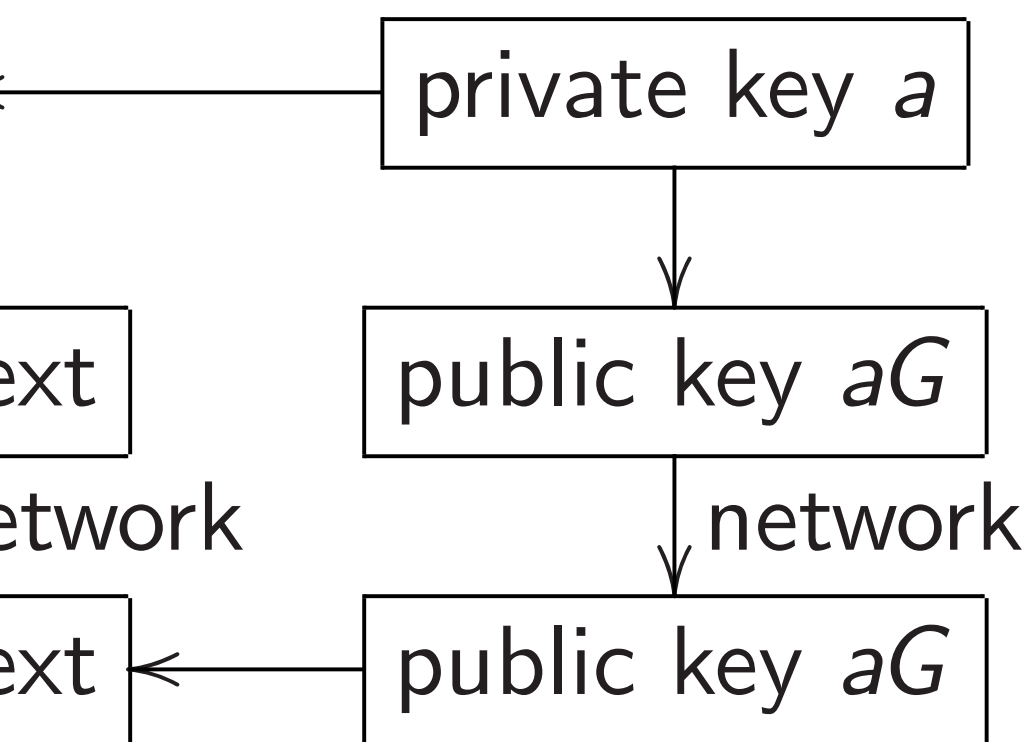
No more shared secret k but Alice still has secret a .

Cryptography requires TCB to protect secrecy of keys, even if user has no other secrets.

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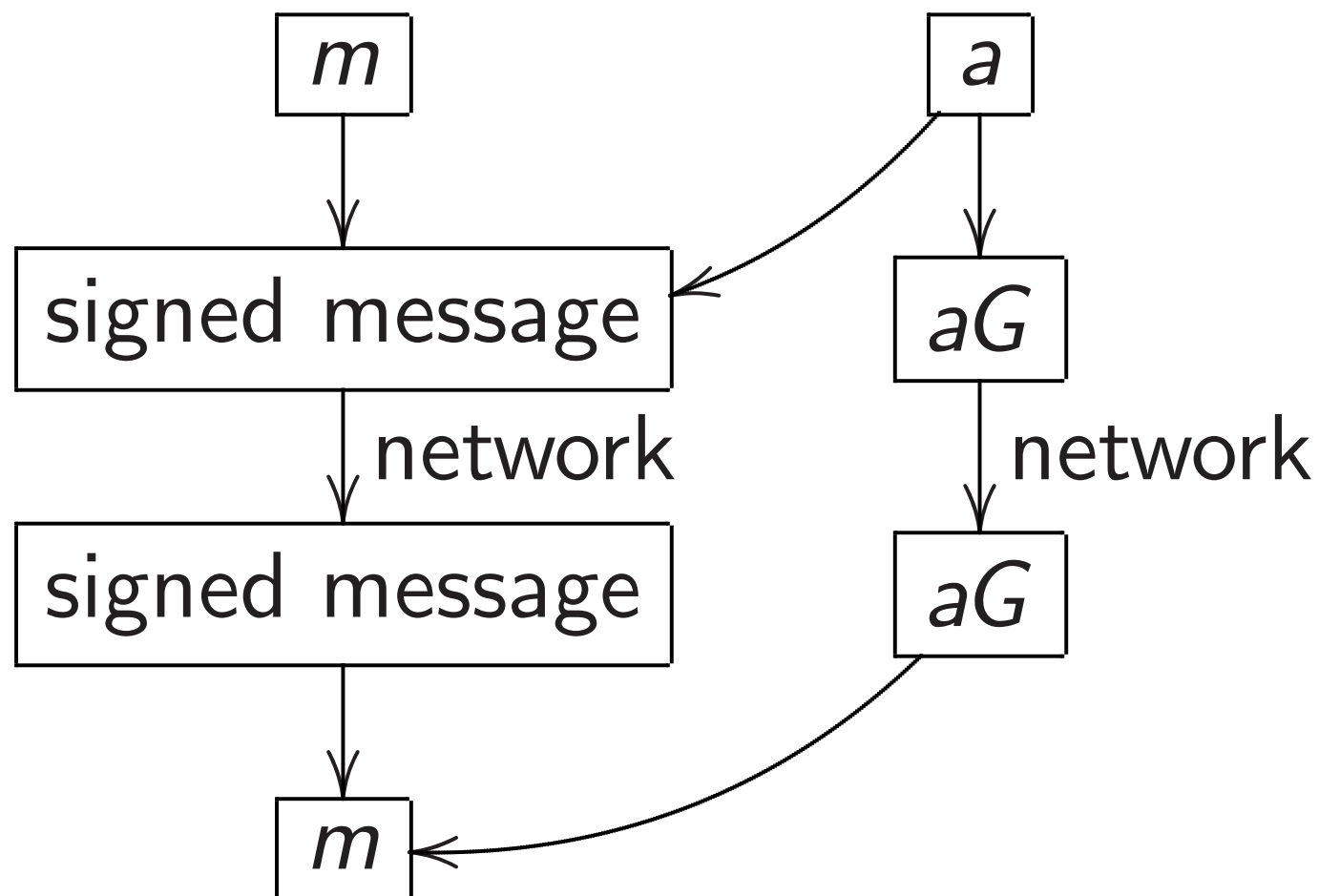
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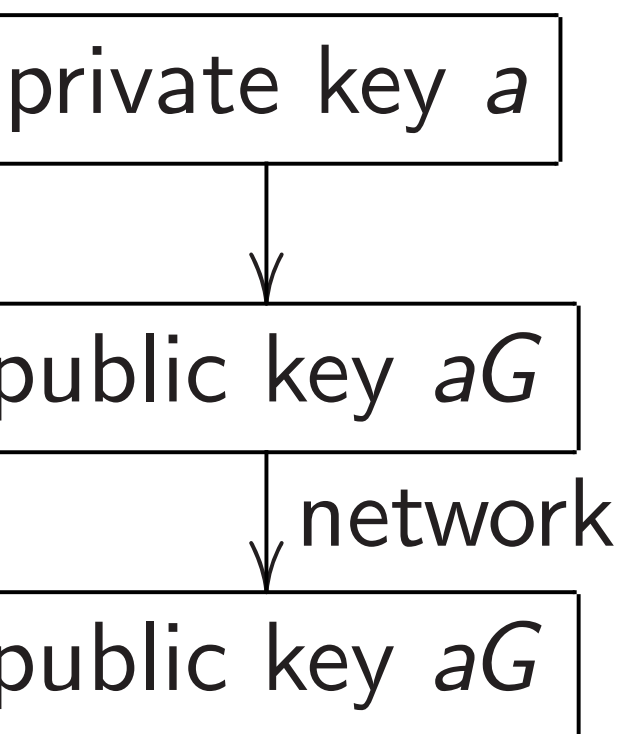
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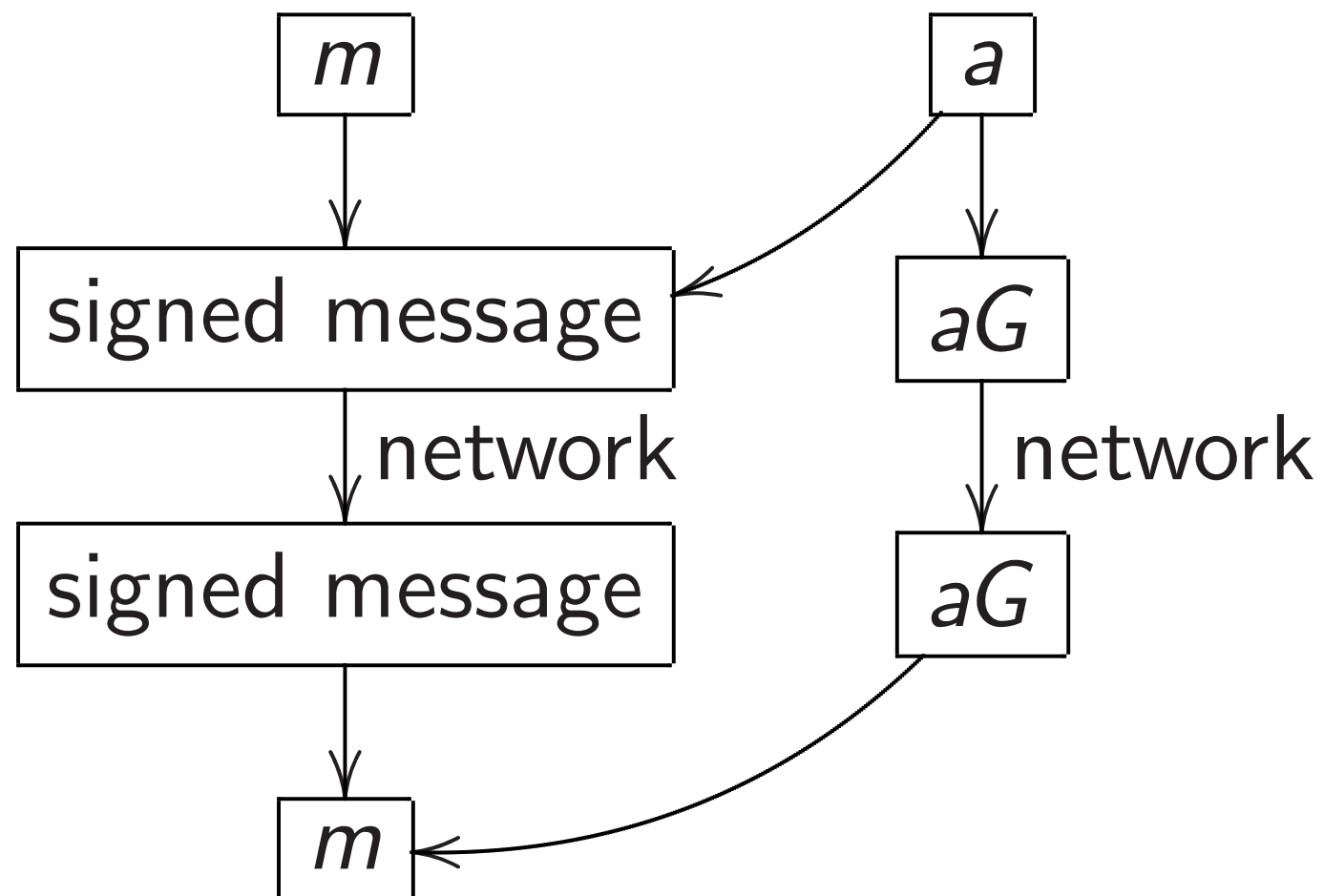
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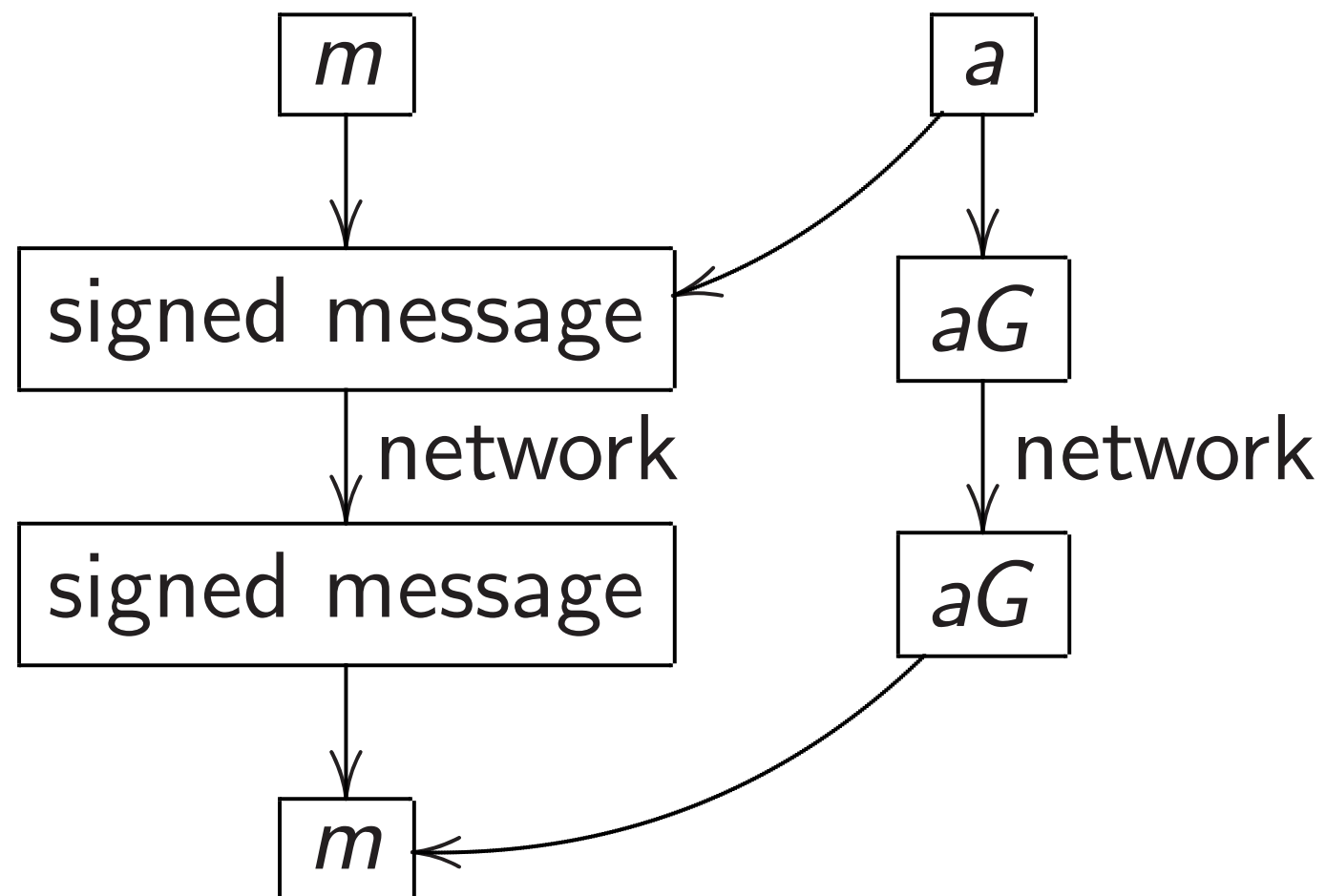
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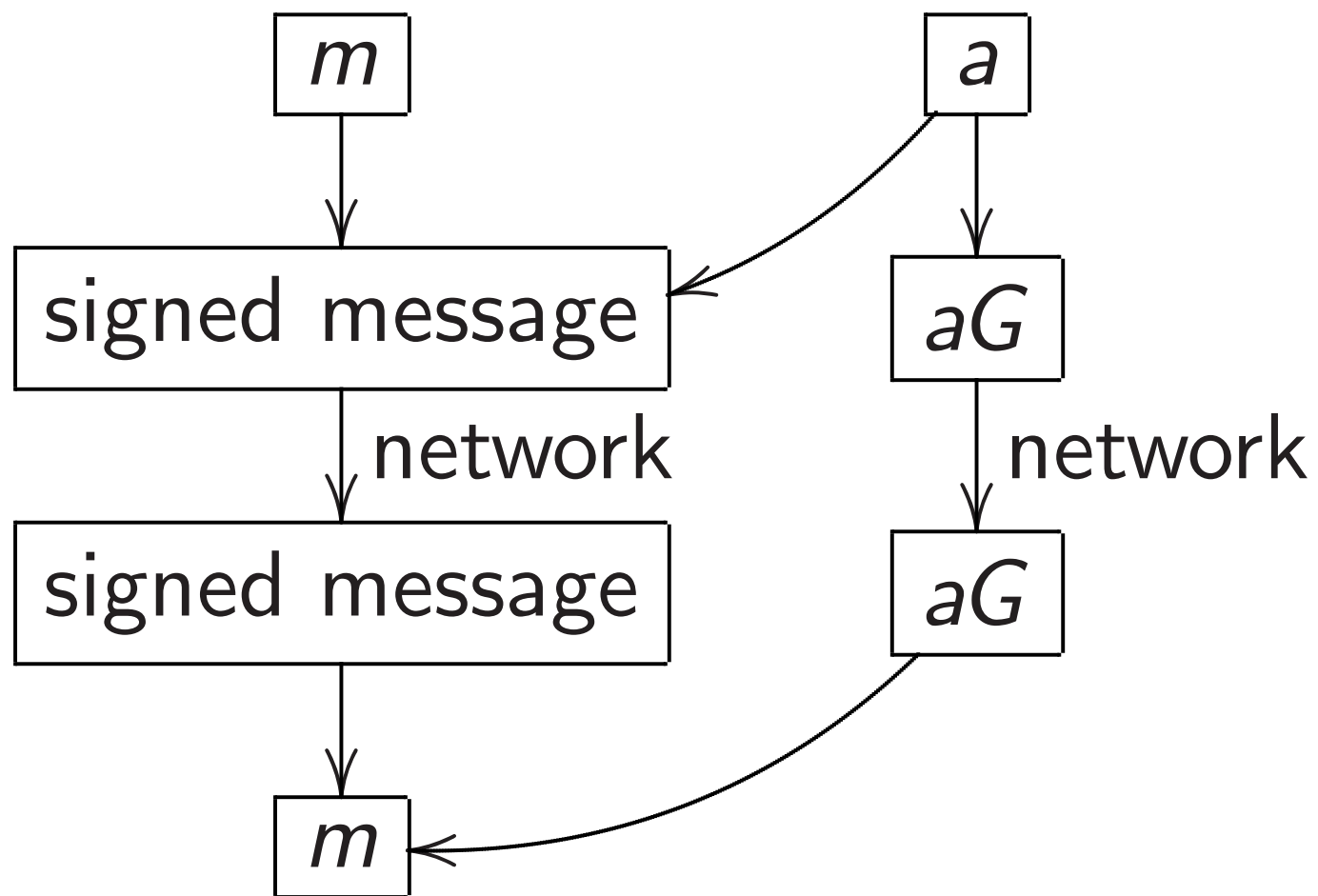
Constant-time software

Large portion of CPU hardware
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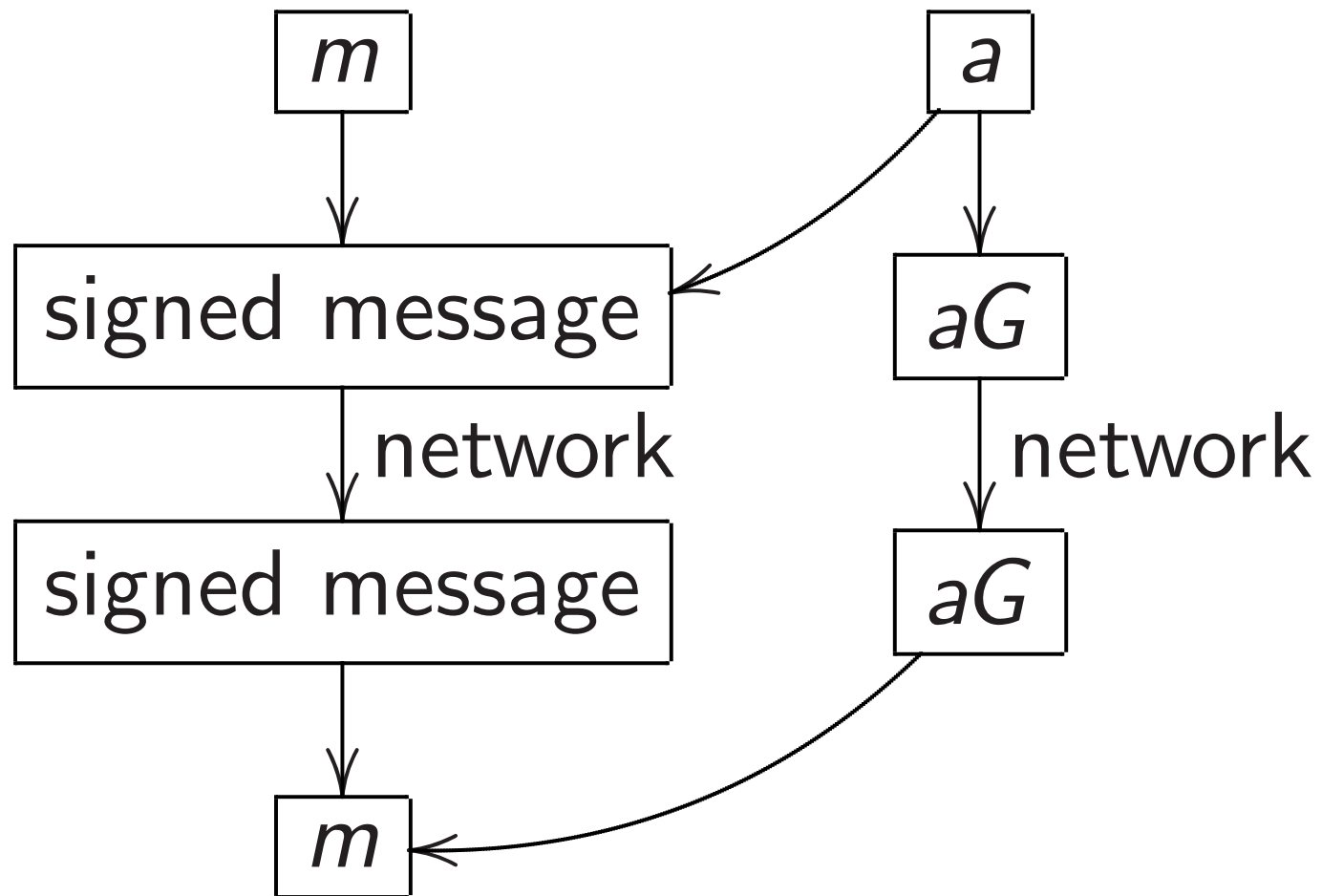
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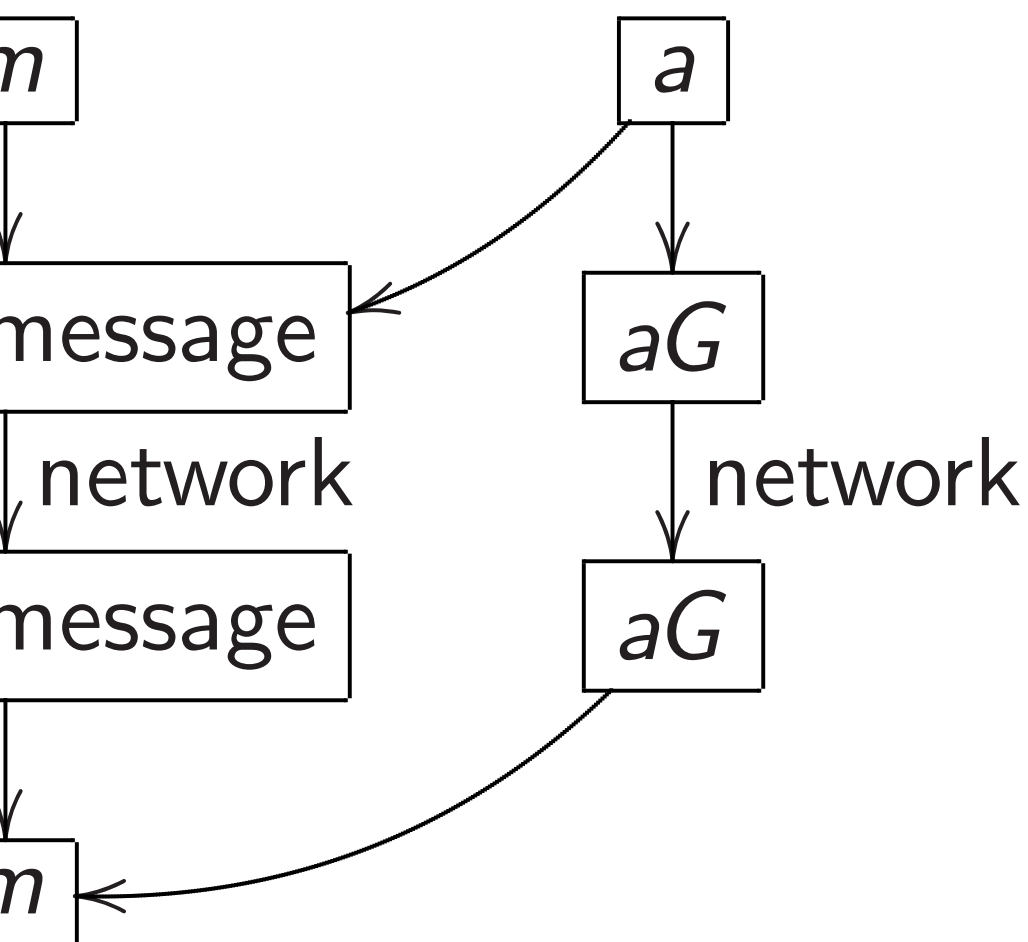
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Schaik–Milburn–Österlund–Frigo–
Maisuradze–Razavi–Bos–Giuffrida.

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secret secrecy of keys,

user has no other secrets.

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Constant-time software

Large portion of CPU hardware:
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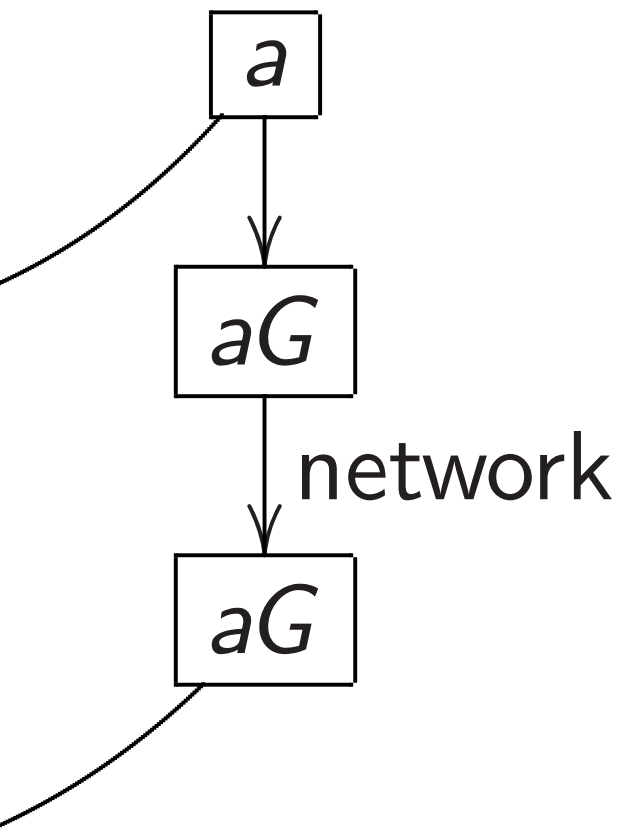
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Try to push attacks
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For auditors: This is a nightmare.

Many years of security failures.

No confidence in future security.

Real-time software

Portion of CPU hardware:
variations depending on
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Makes auditing much easier.

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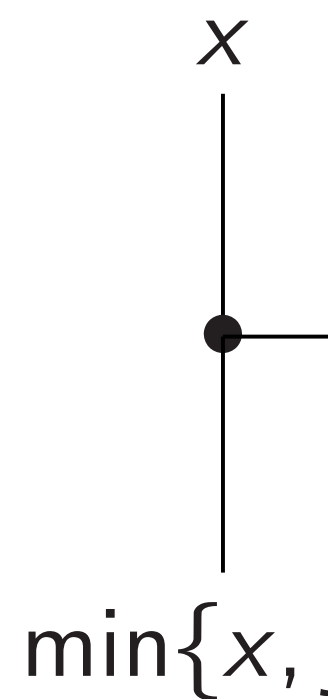
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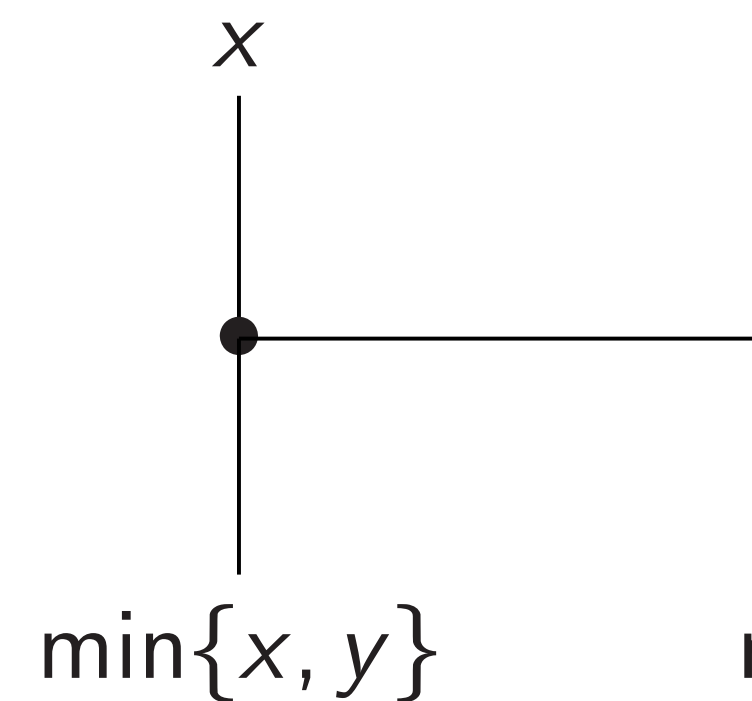
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Easy constant-time

Warning: C standard

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Even easier exercise

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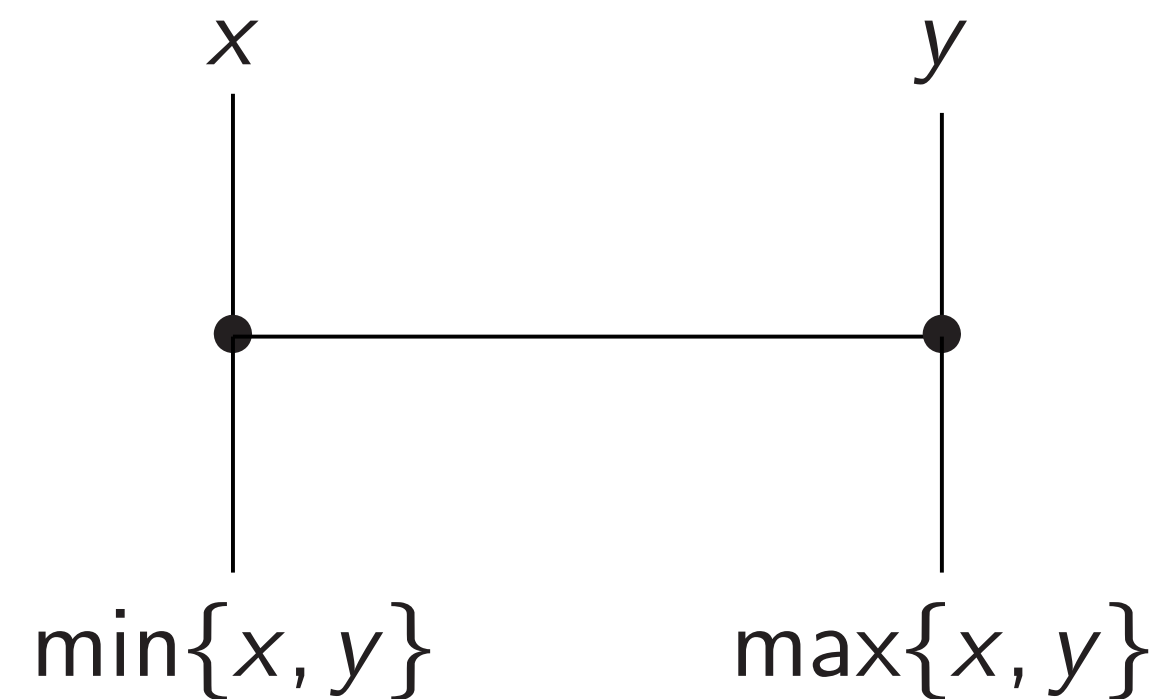
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Easy constant-time exercise
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Even easier exercise in asm.

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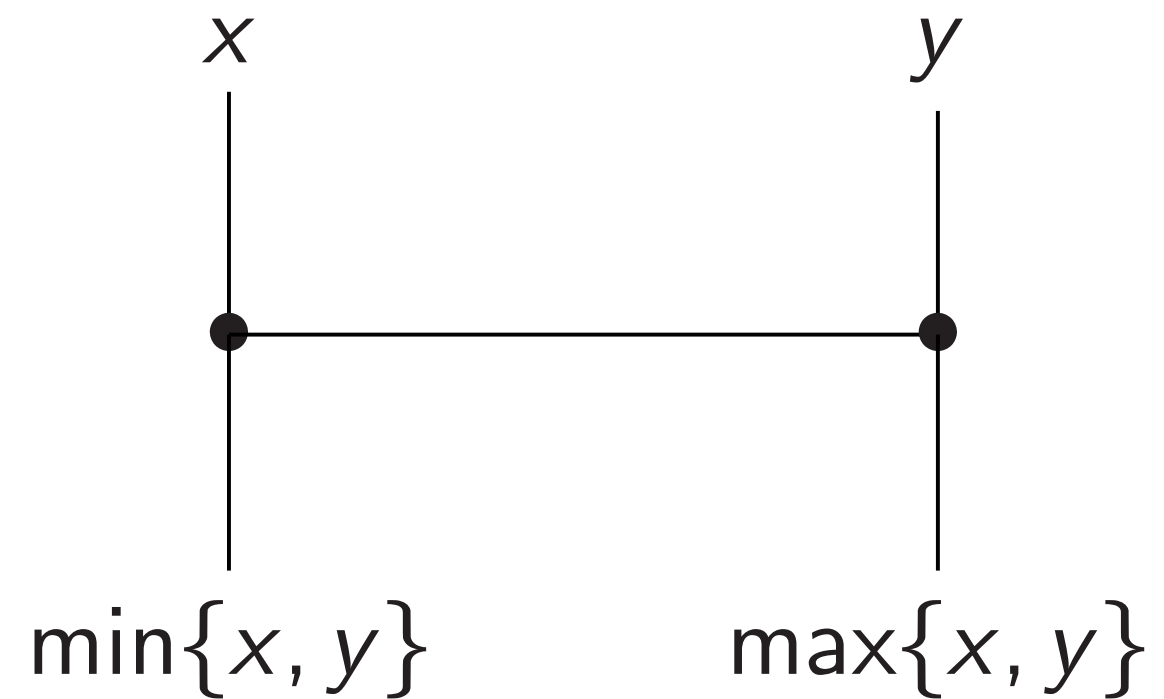
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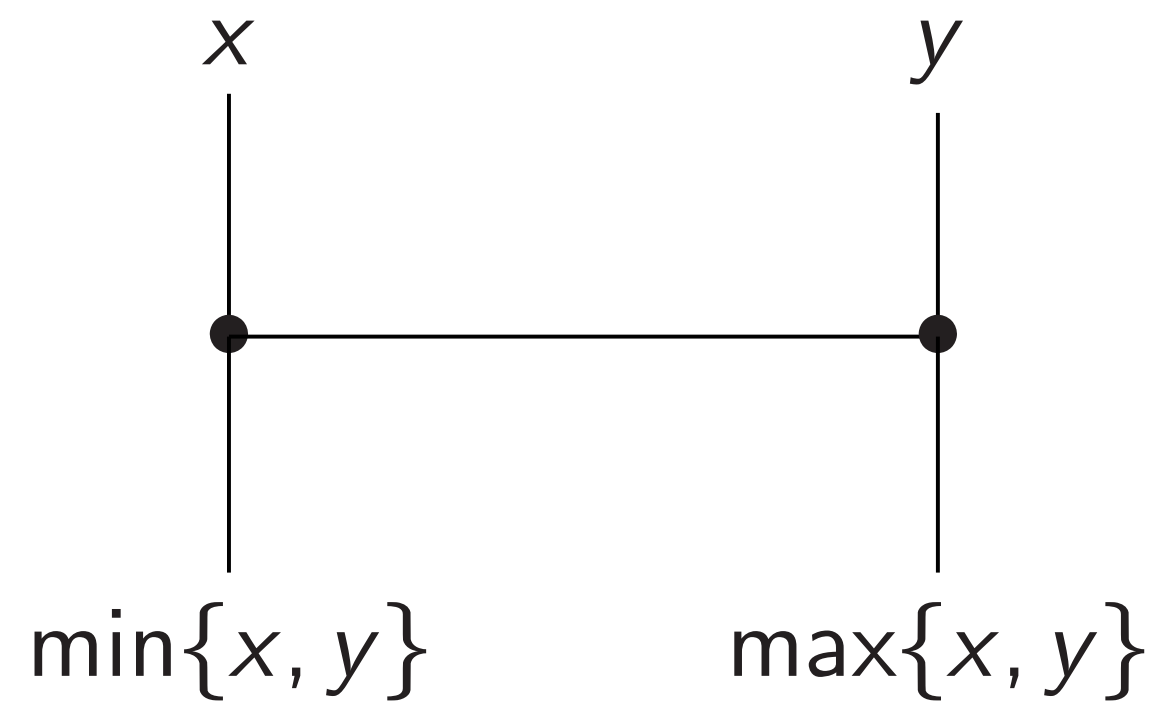
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Combined

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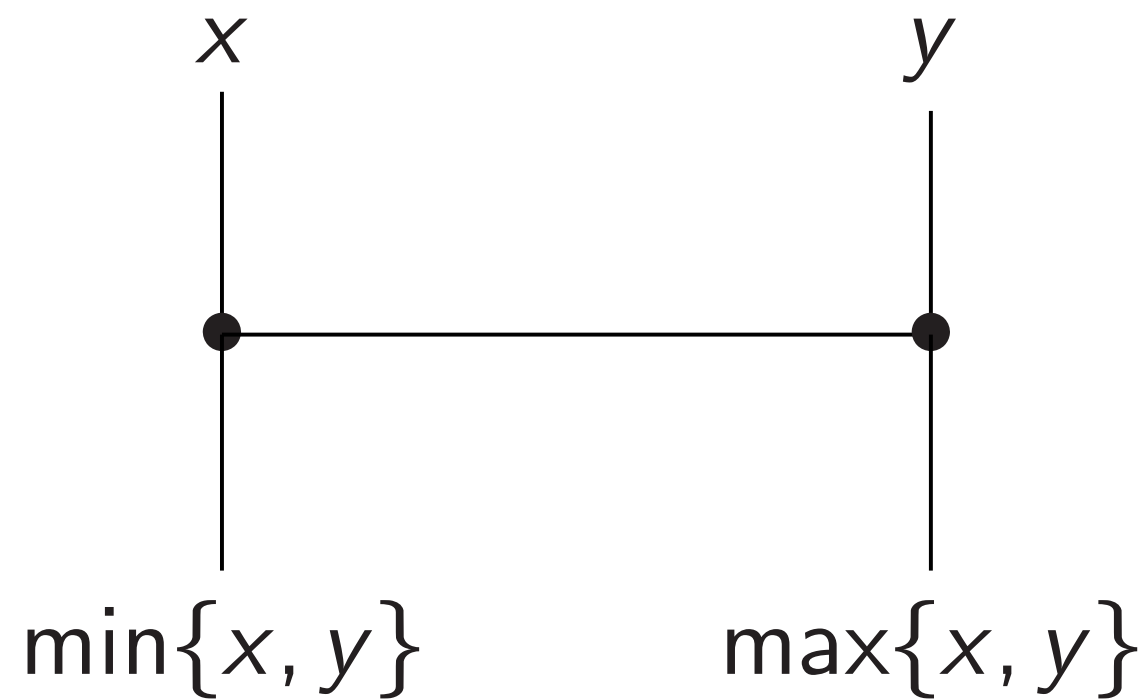
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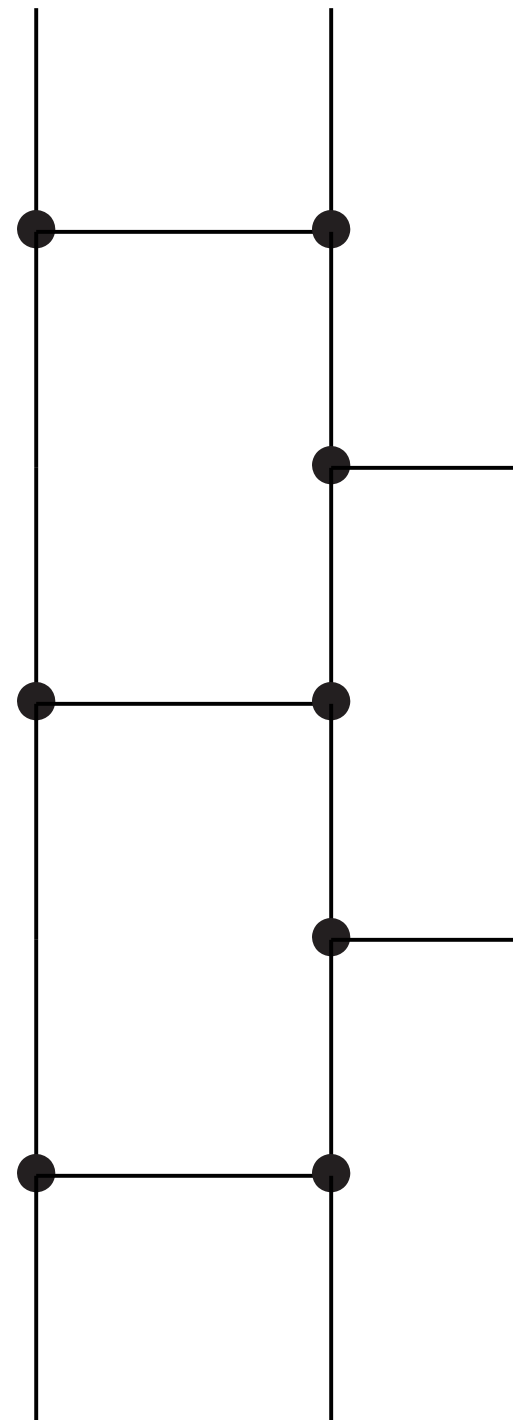


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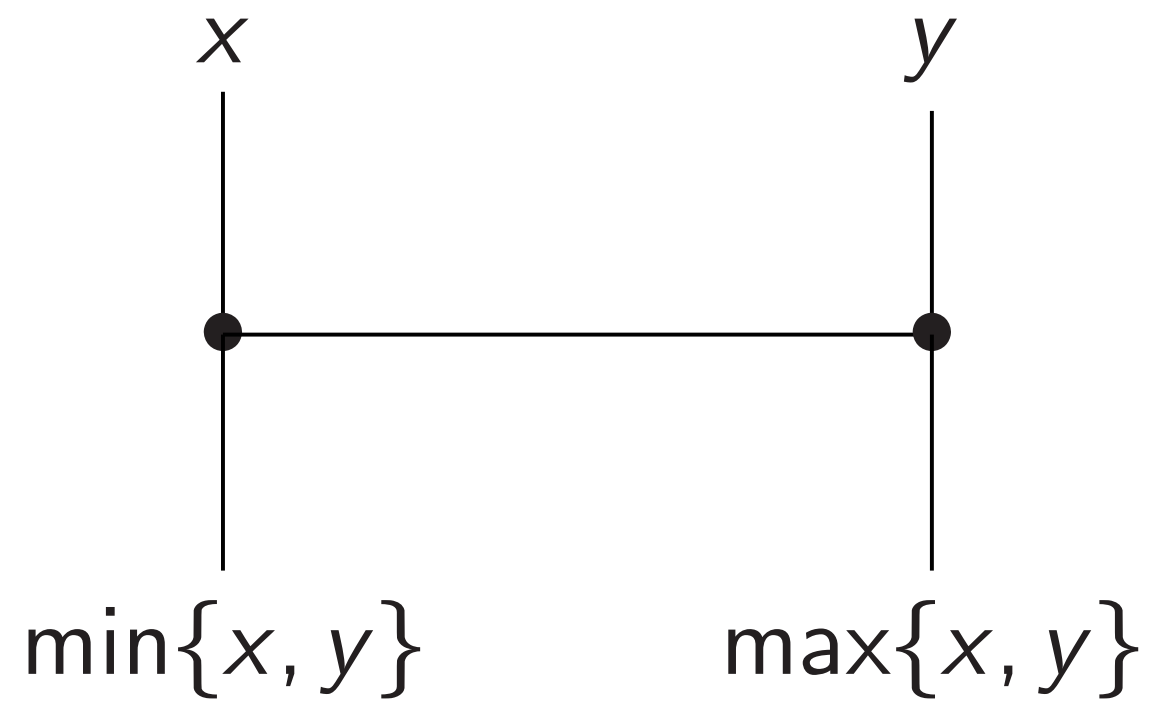
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Combine comparators
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Example of a sorting



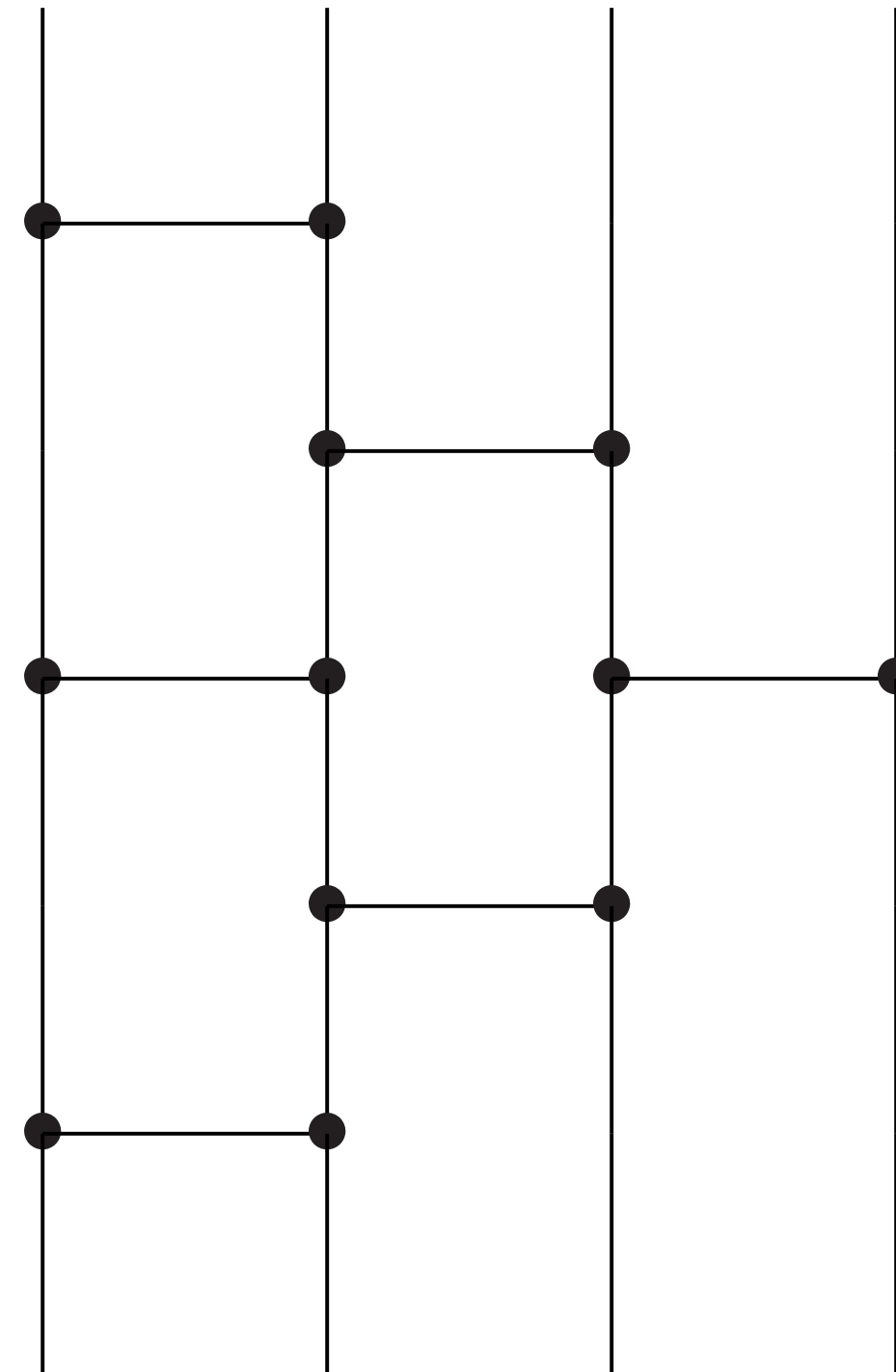
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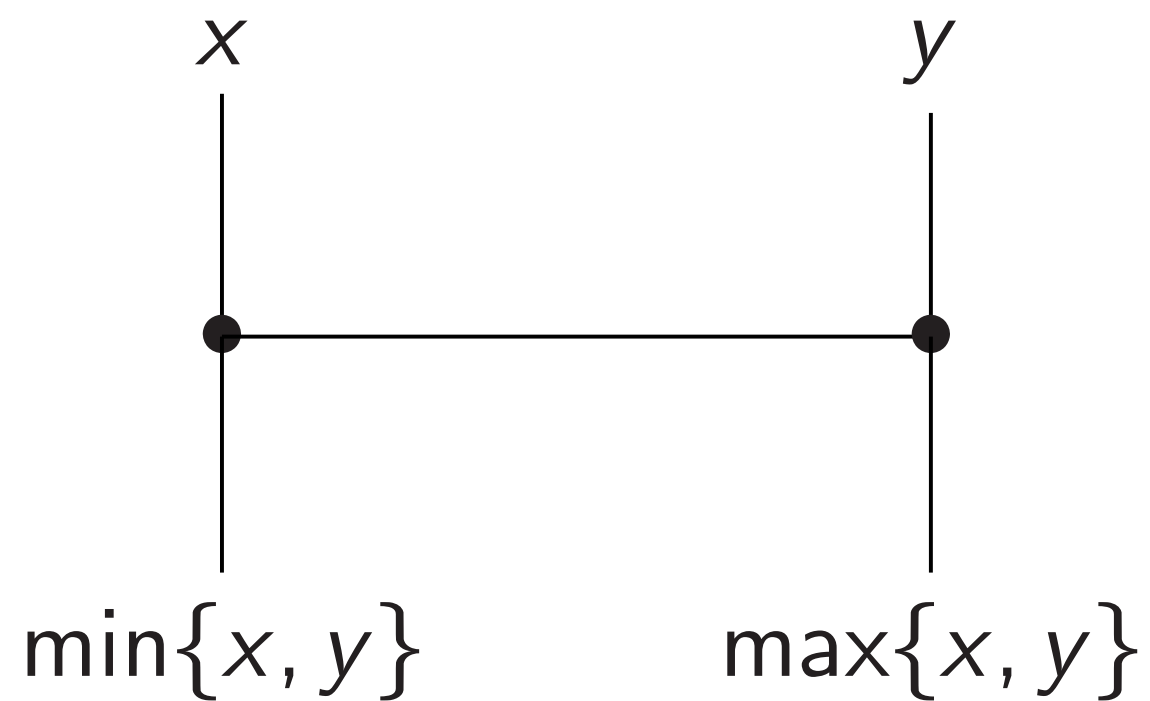
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Combine comparators into a
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Example of a sorting network



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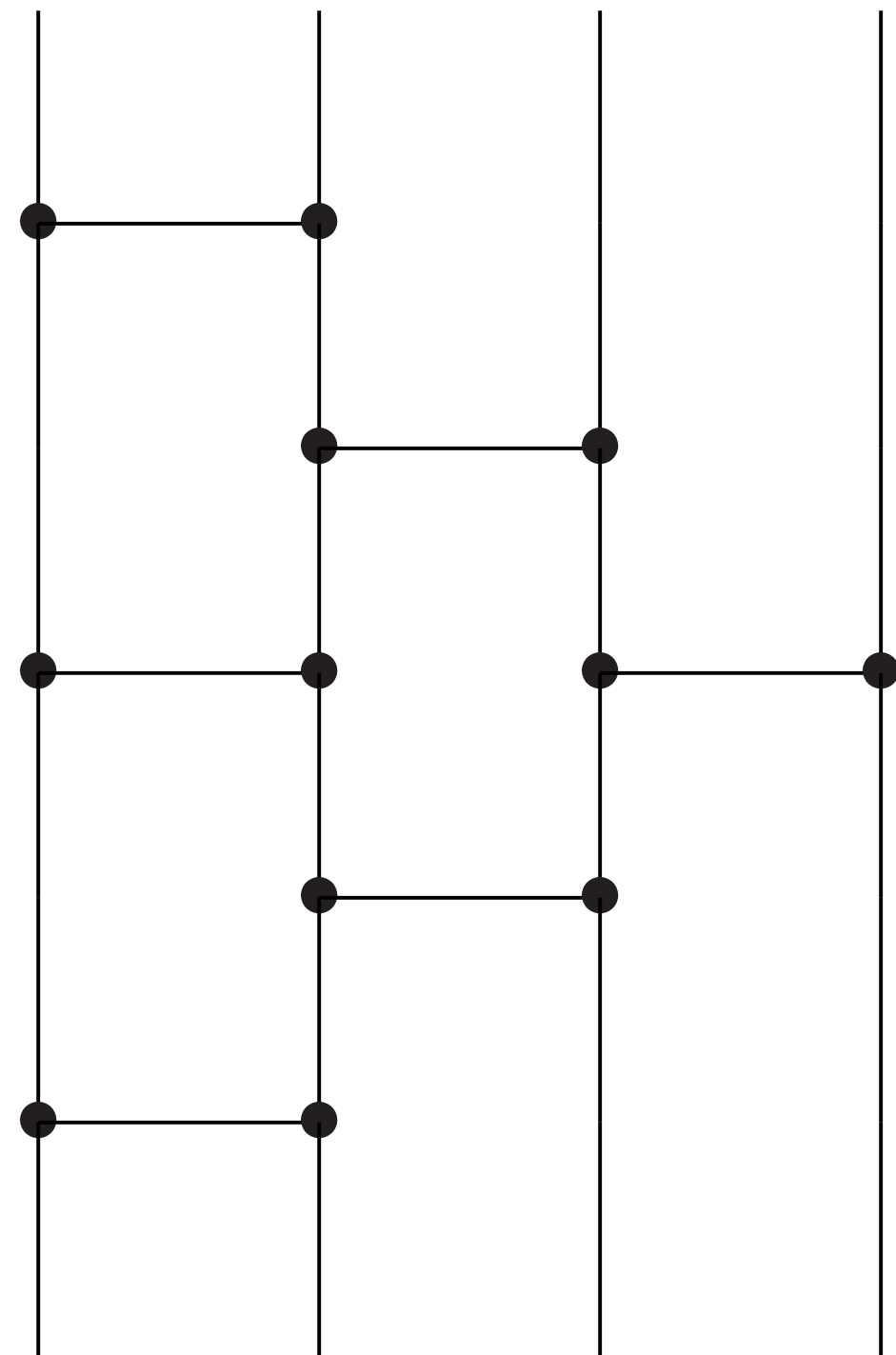
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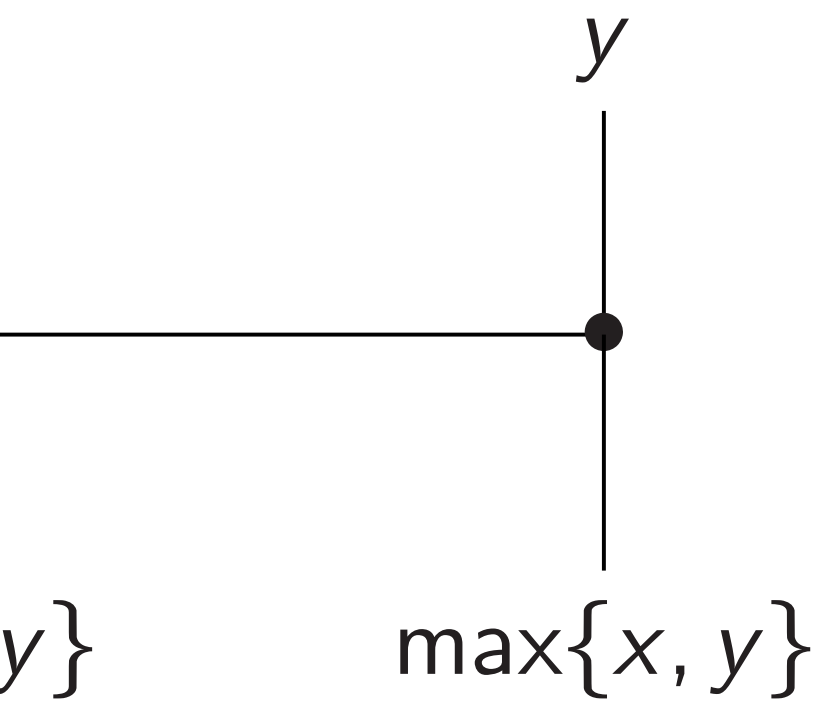
Combine comparators into a
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Example of a sorting network:



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Comparator sorting 2 integers.



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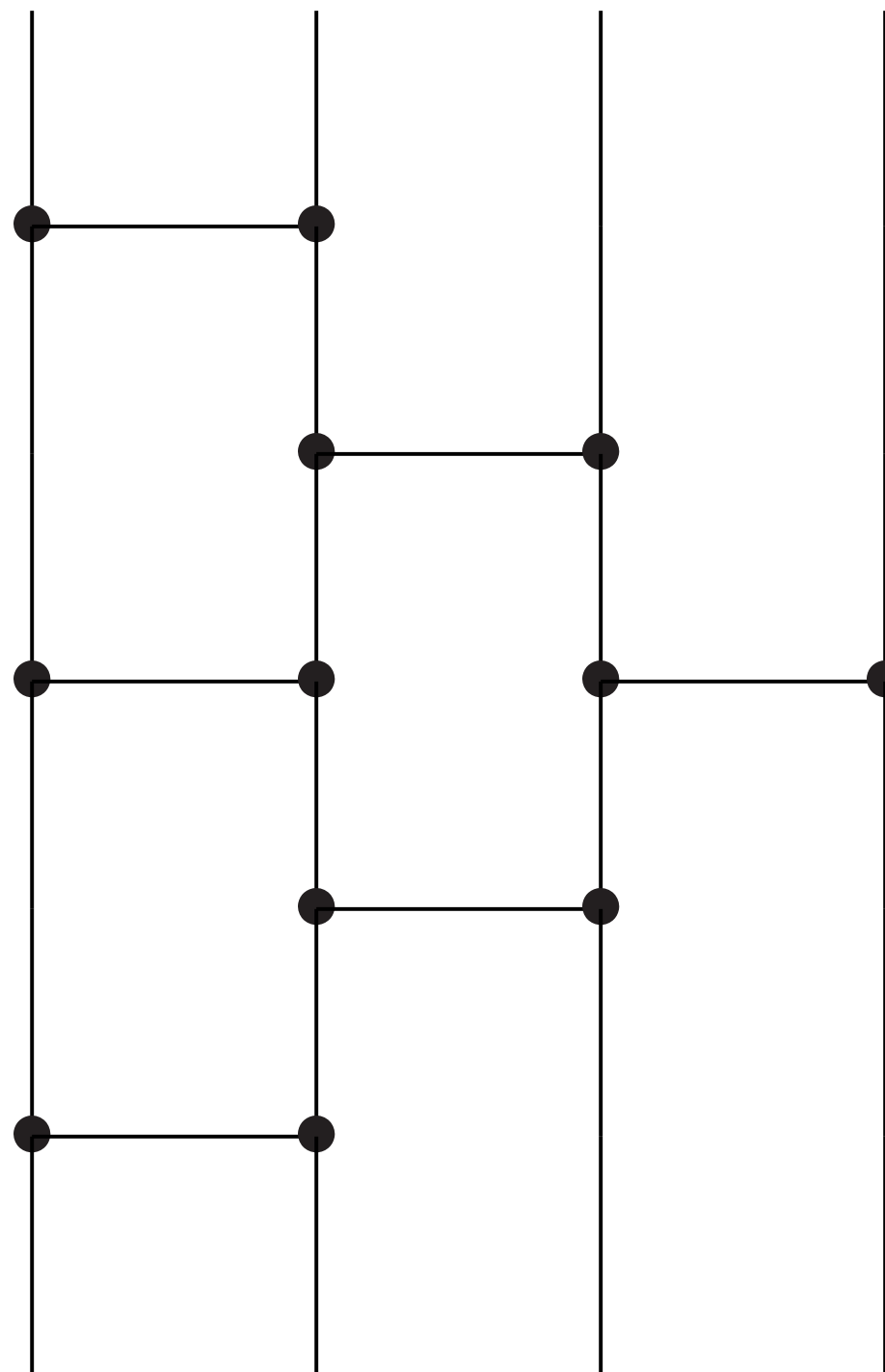
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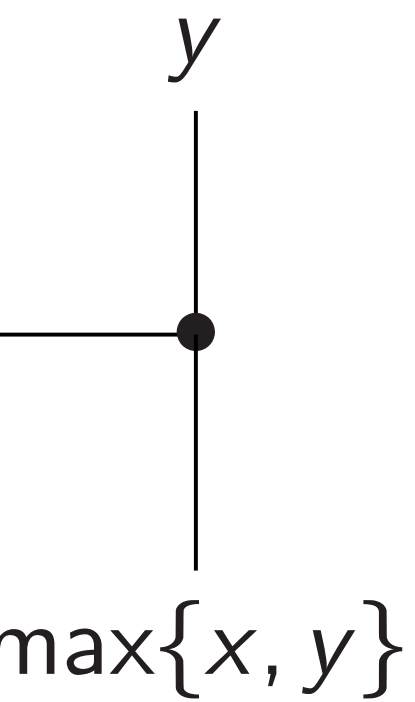
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Example of a sorting network:



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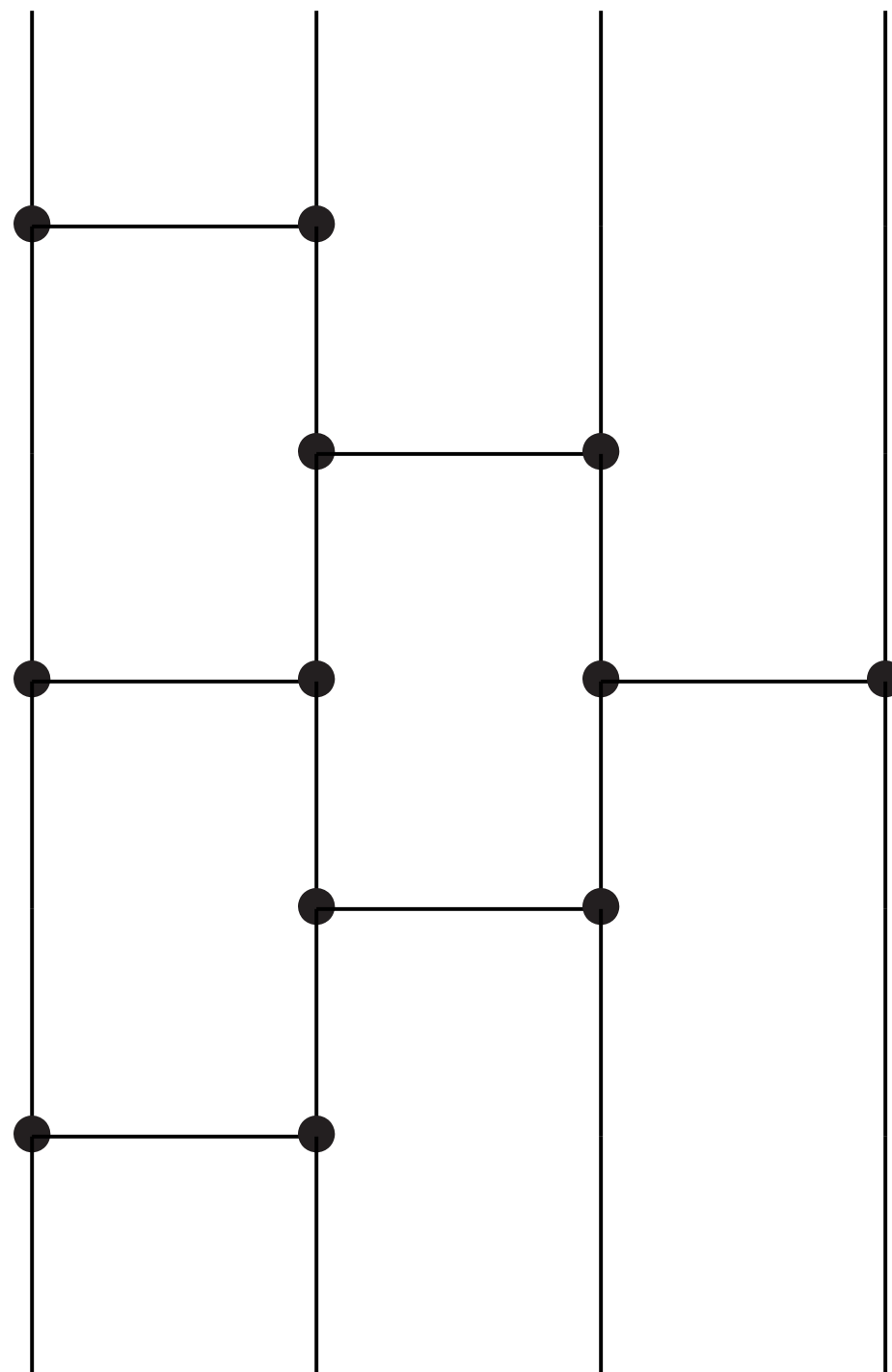
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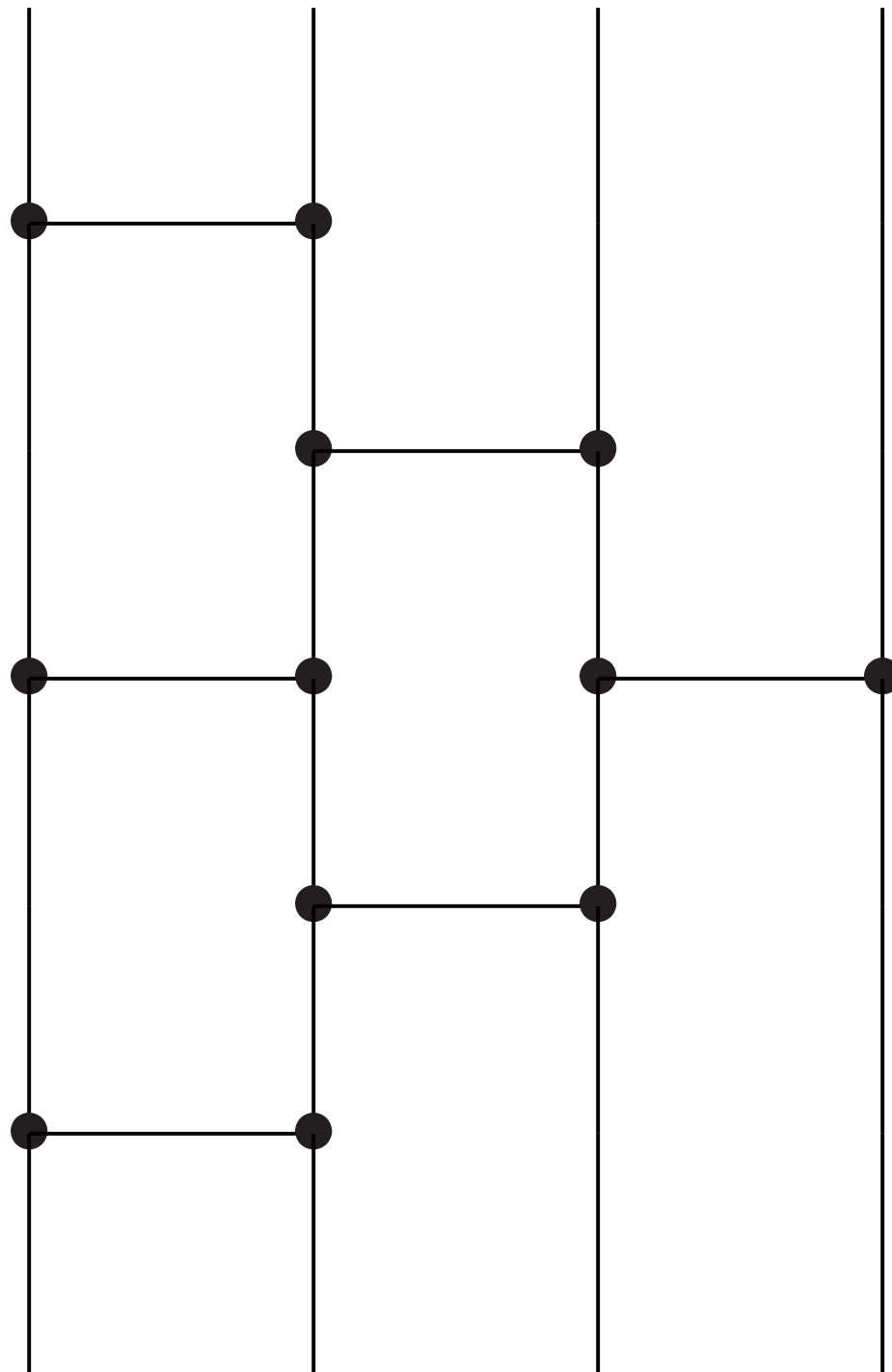


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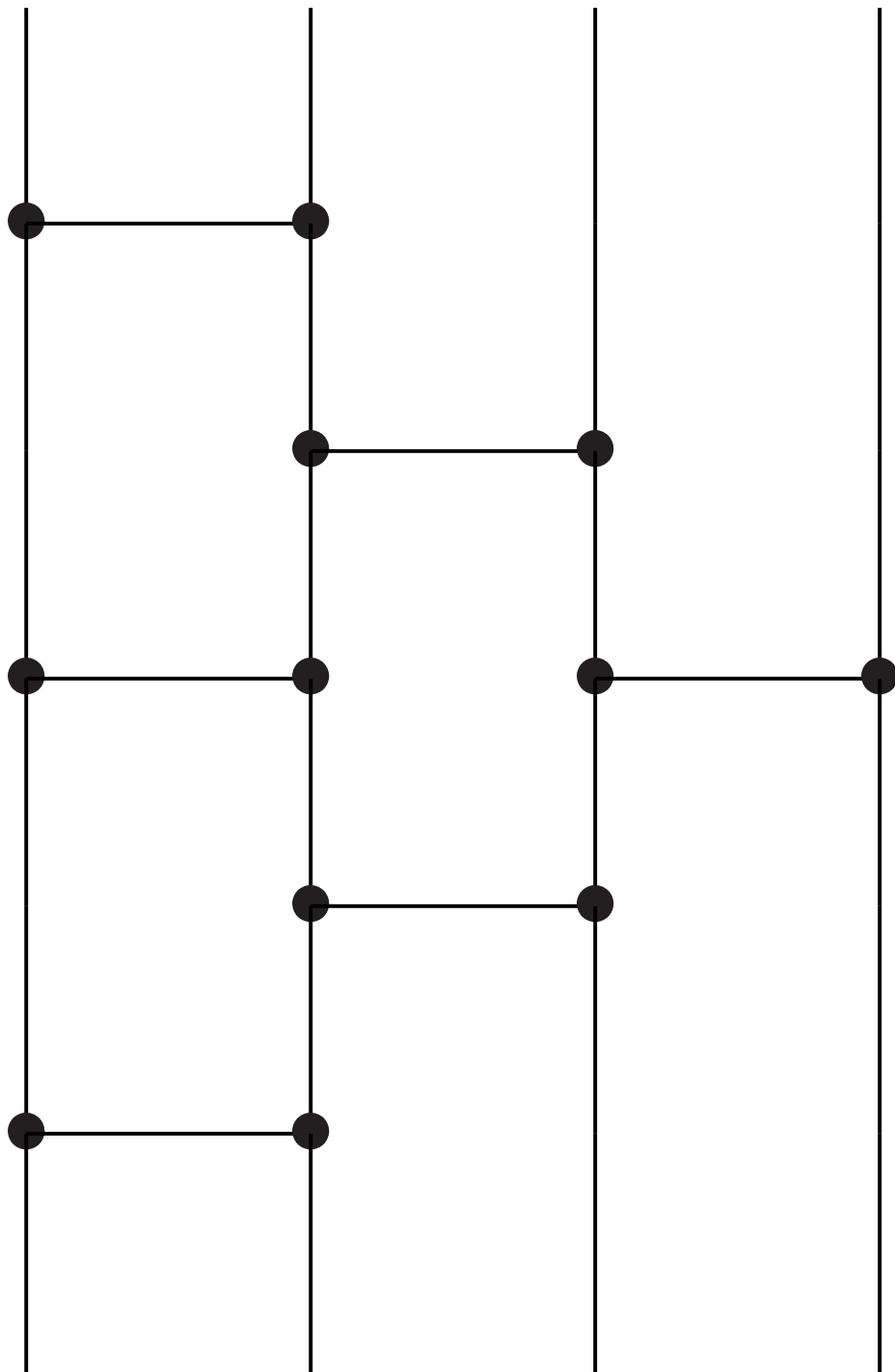


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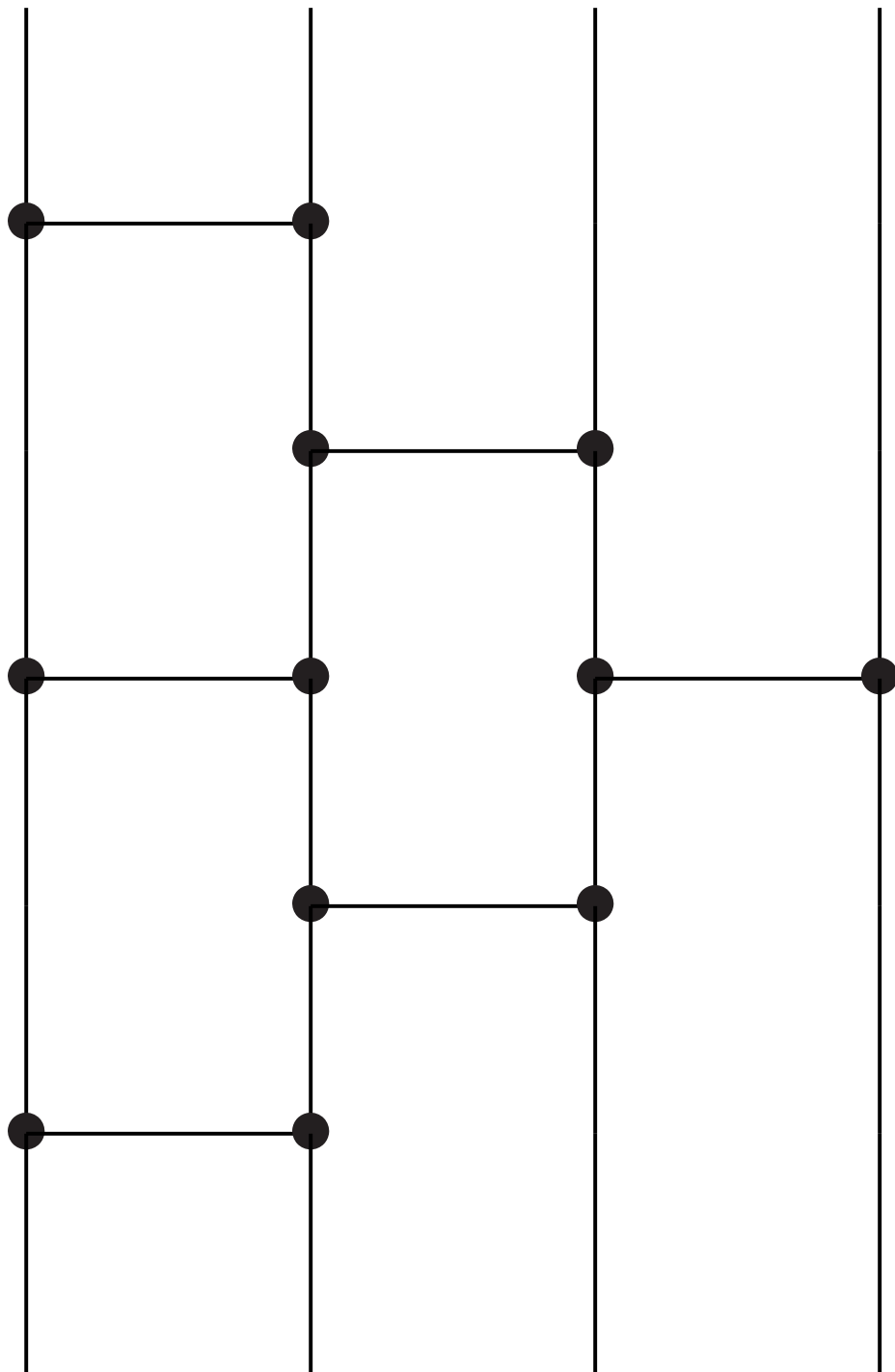
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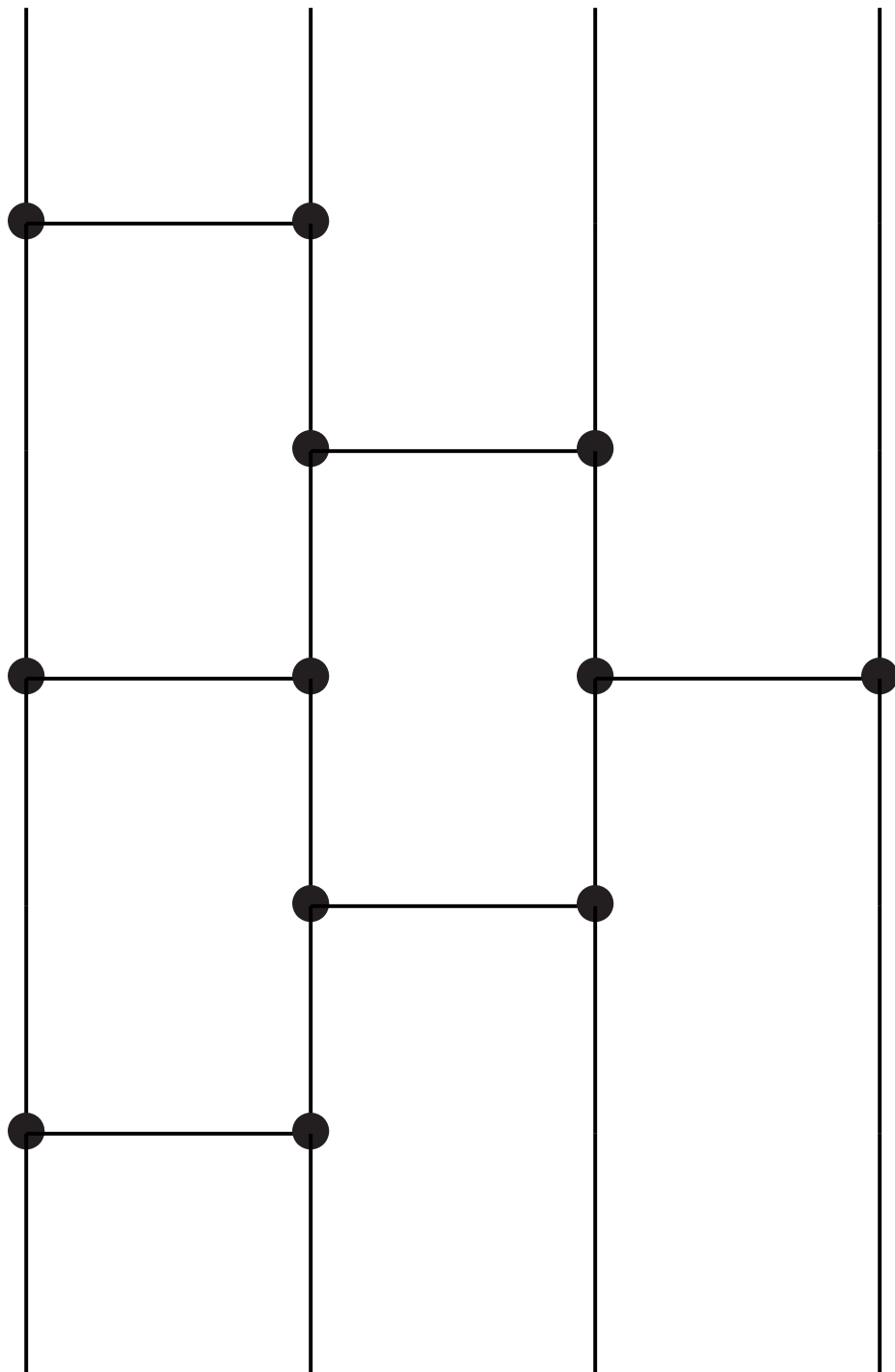


Positions of comparators in a sorting network are independent of the input. Naturally constant-time.

But $(n^2 - n)/2$ comparators produce complaints about performance as n increases.

Combine comparators into a **sorting network** for more inputs.

Example of a sorting network:



Positions of comparators in a sorting network are independent of the input. Naturally constant-time.

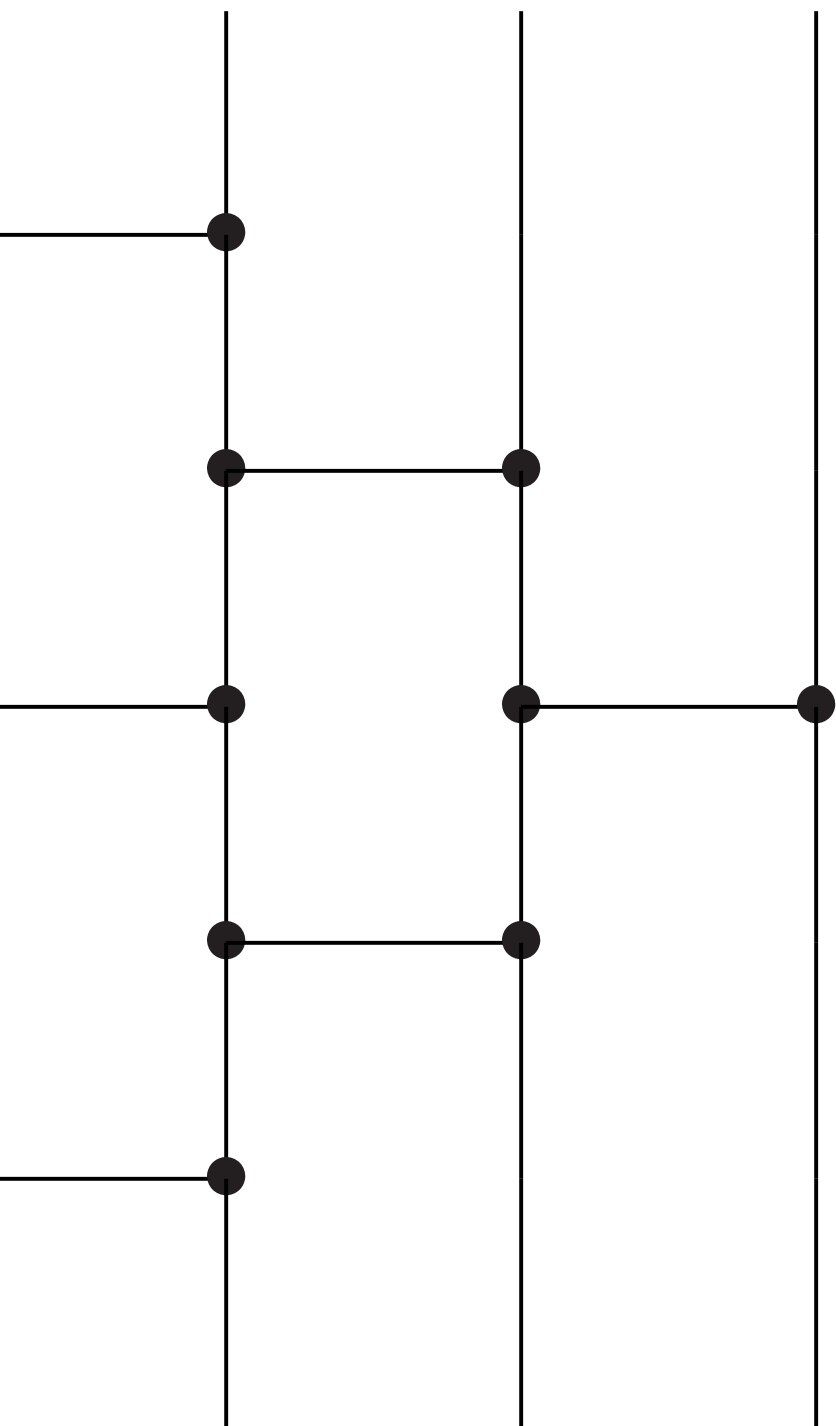
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Speed is a serious issue in the post-quantum competition.

“Cost” is evaluation criterion; “we’d like to stress this once again on the forum that we’d really like to see more platform-optimized implementations”; etc.

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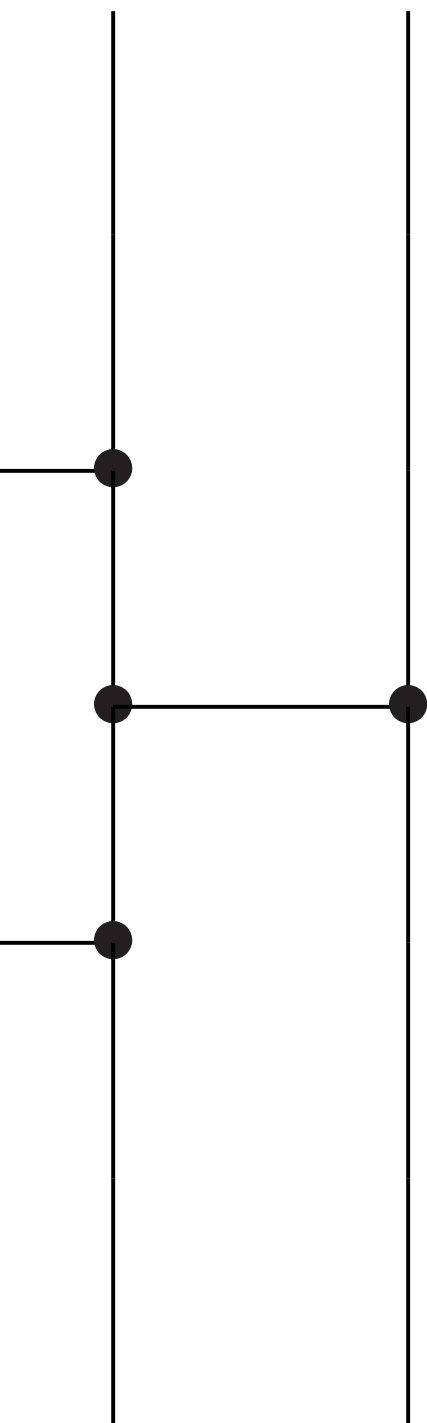
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void int  
{ int64  
  if (n  
    t = 1  
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```
void int32_sort(
{ int64 t,p,q,i;
  if (n < 2) ret
  t = 1;
  while (t < n -
  for (p = t;p >
    for (i = 0;i
      if (!(i &
        minmax(x
  for (q = t;q
    for (i = 0
      if (!(i
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    }
  }
}
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void int32_sort(int32 *x,
{ int64 t,p,q,i;
  if (n < 2) return;
  t = 1;
  while (t < n - t) t +=
  for (p = t;p > 0;p >>=
    for (i = 0;i < n - p;
      if (!(i & p))
        minmax(x+i,x+i+p)
    for (q = t;q > p;q >>
      for (i = 0;i < n -
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  }
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      if (!(i & p))
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      for (i = 0; i < n - q; ++i)
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Conditional branch: much slower.

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Sorting software is in the TCB.

Does it work correctly?

Test the sorting software on many random inputs, increasing inputs, decreasing inputs. Seems to work.

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Sorting software is in the TCB.
Does it work correctly?

Test the sorting software on many random inputs, increasing inputs, decreasing inputs. Seems to work.

But are there *occasional* inputs where this sorting software fails to sort correctly?

History: Many security problems involve occasional inputs where TCB works incorrectly.

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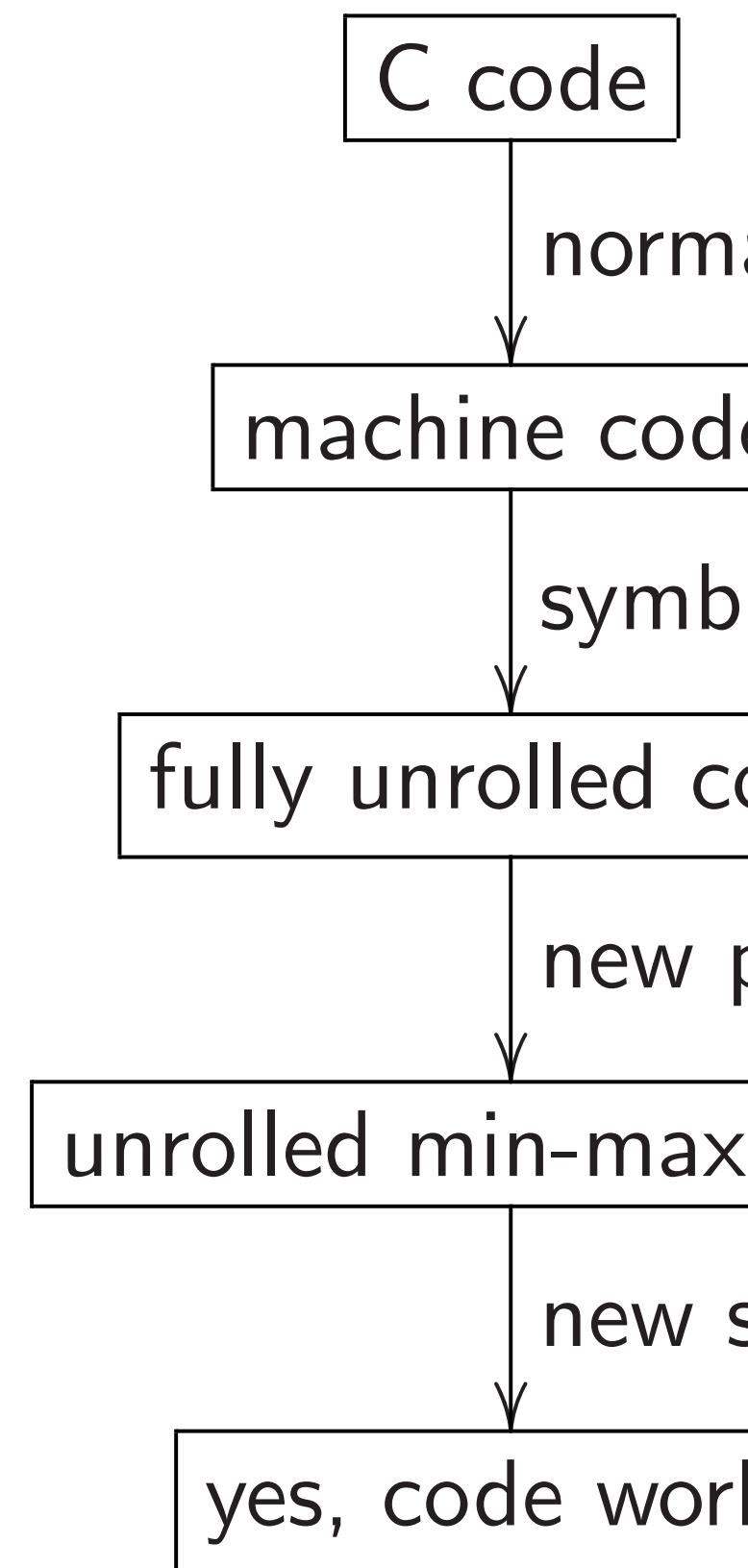
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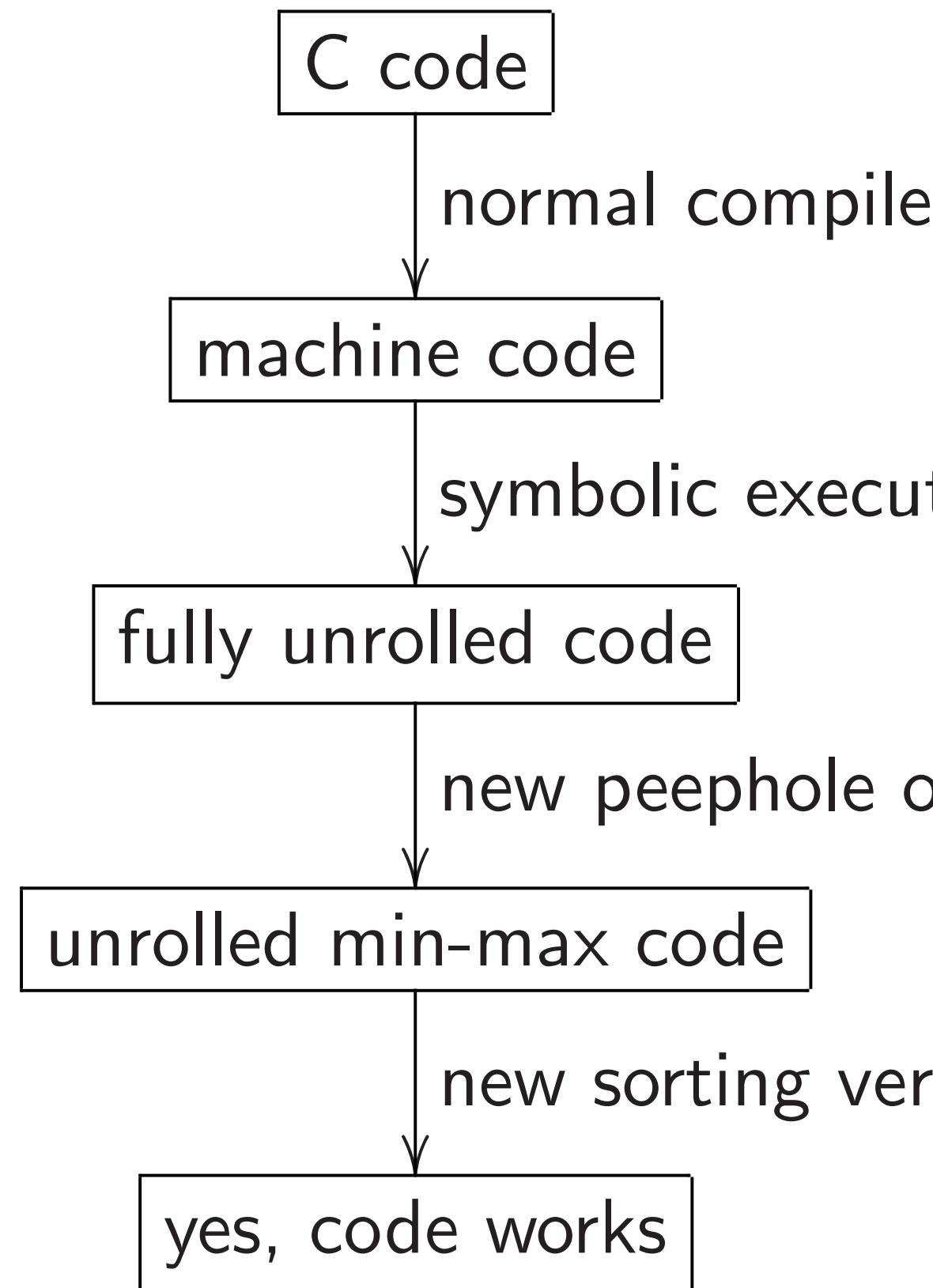
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Verification

Sorting software is in the TCB.

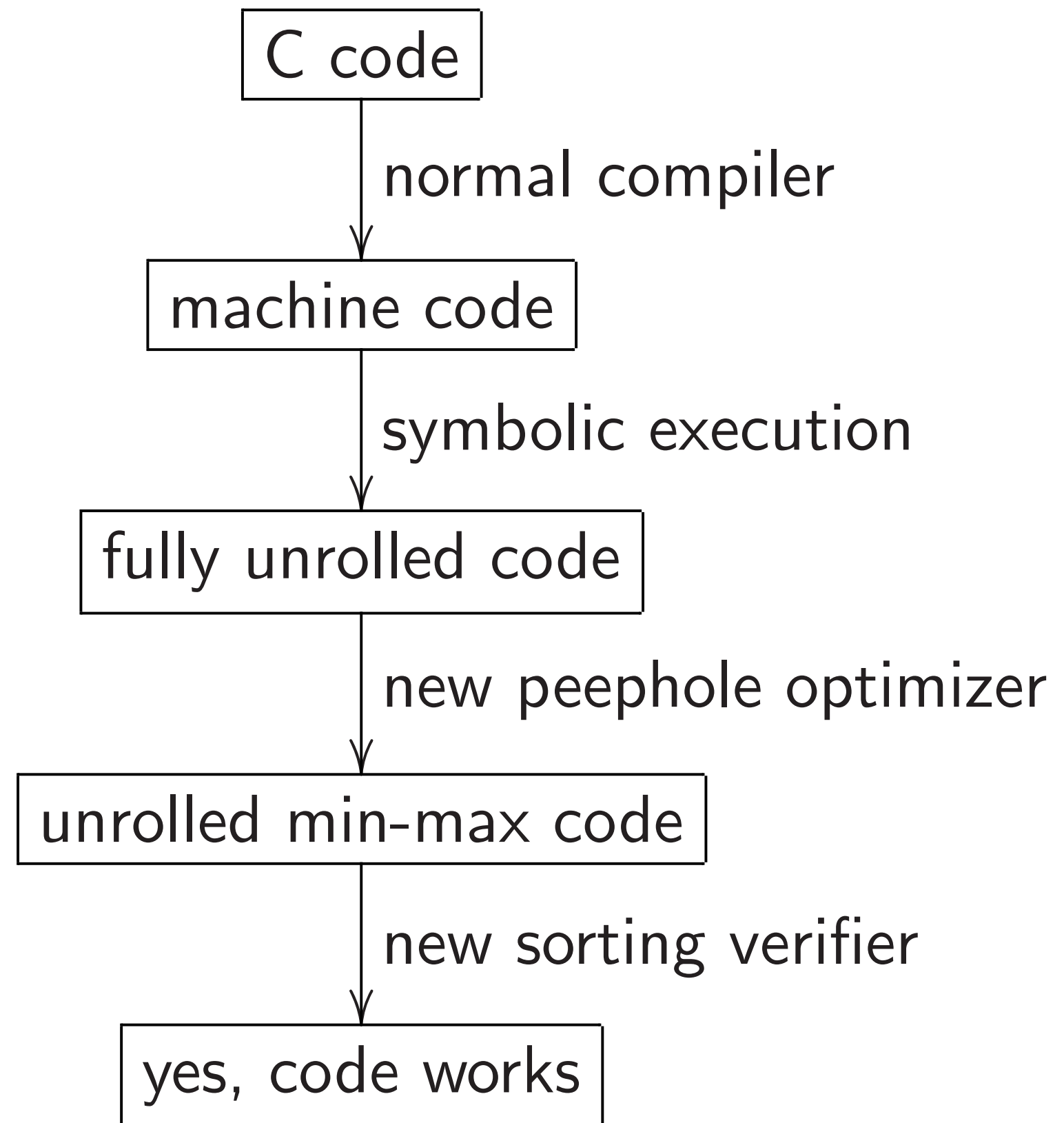
Does it work correctly?

Test the sorting software on many random inputs, increasing inputs, decreasing inputs. Seems to work.

But are there *occasional* inputs where this sorting software fails to sort correctly?

History: Many security problems involve occasional inputs where TCB works incorrectly.

For each used n (e.g., 768):



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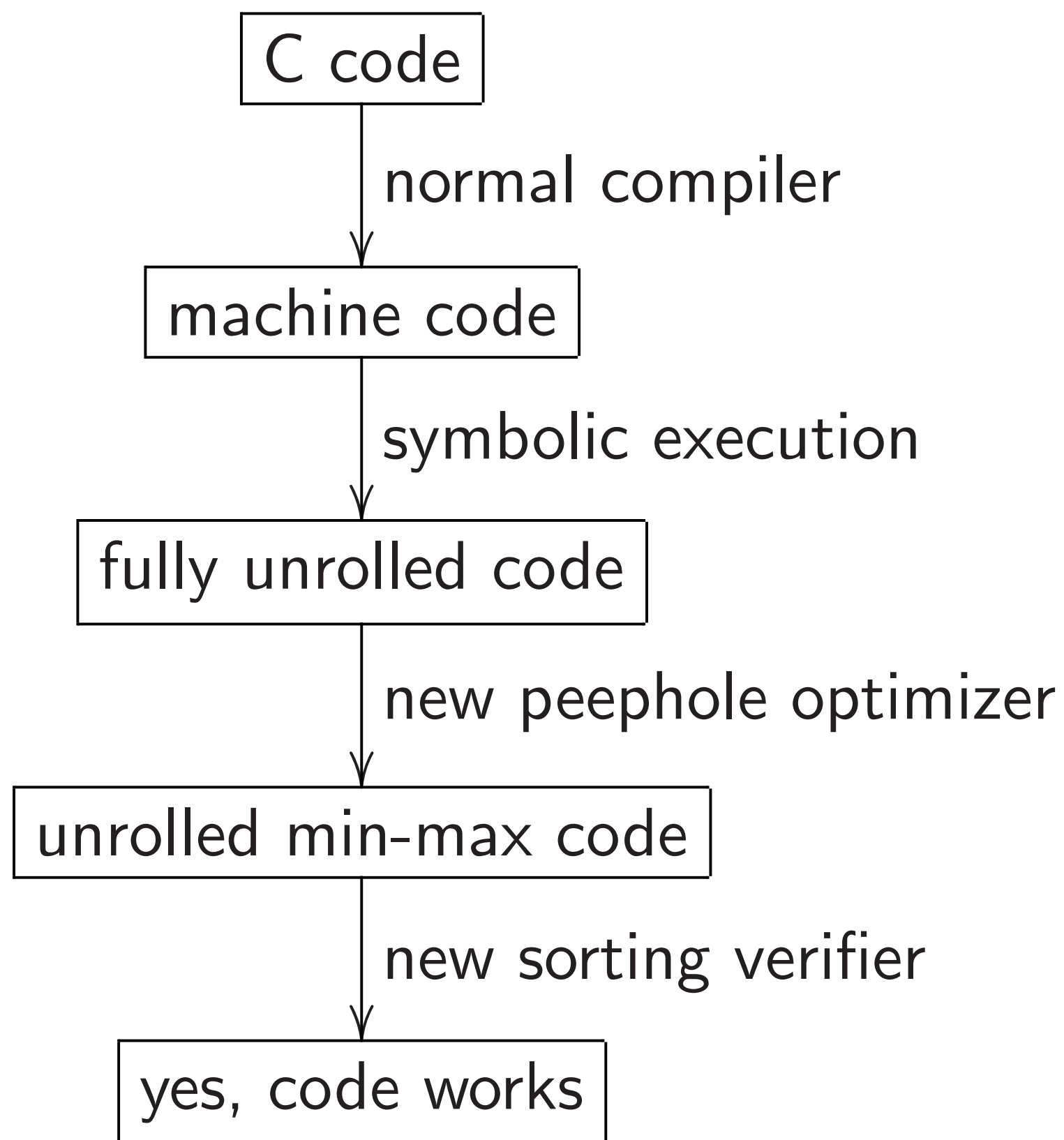
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Symbolic

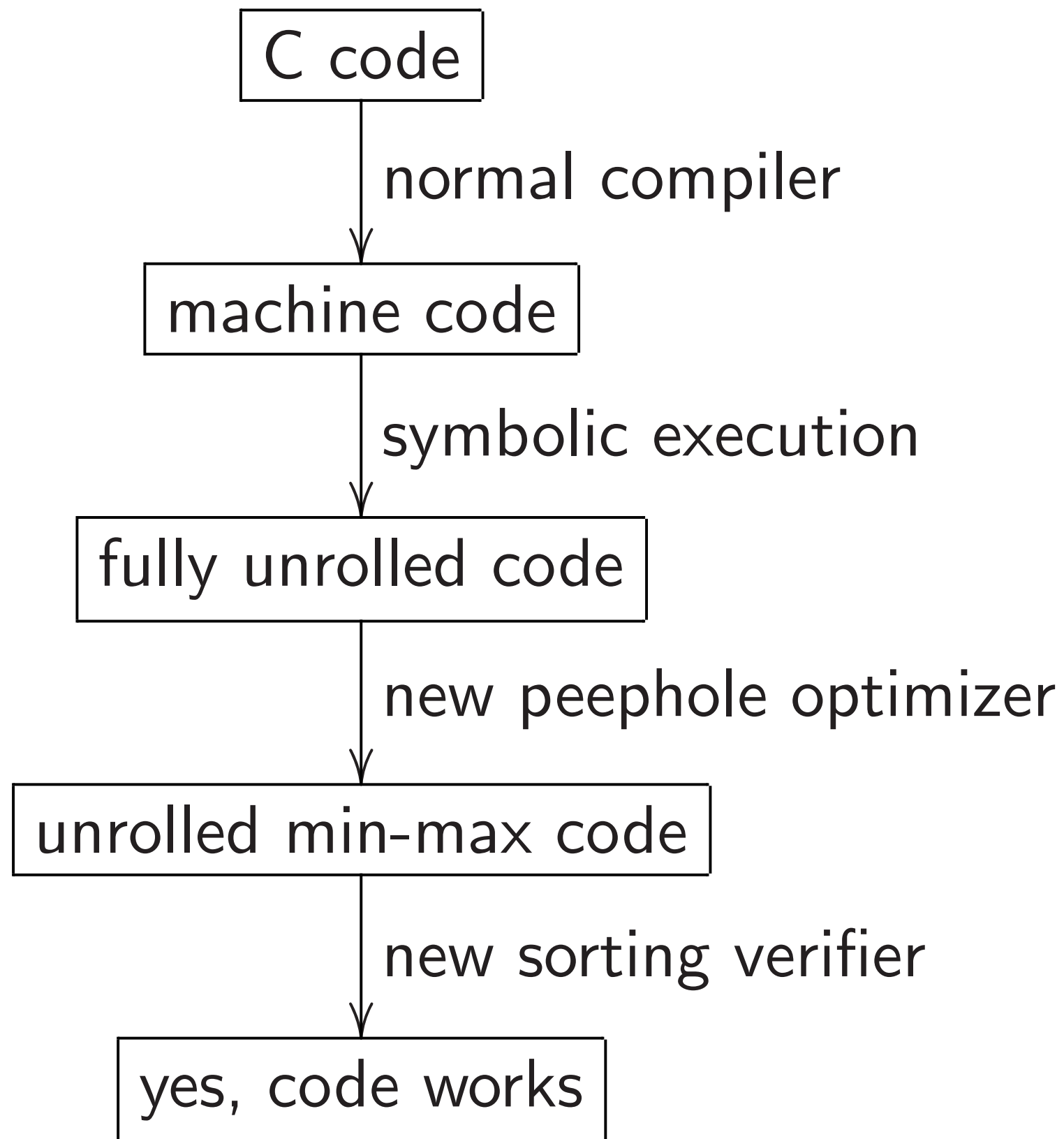
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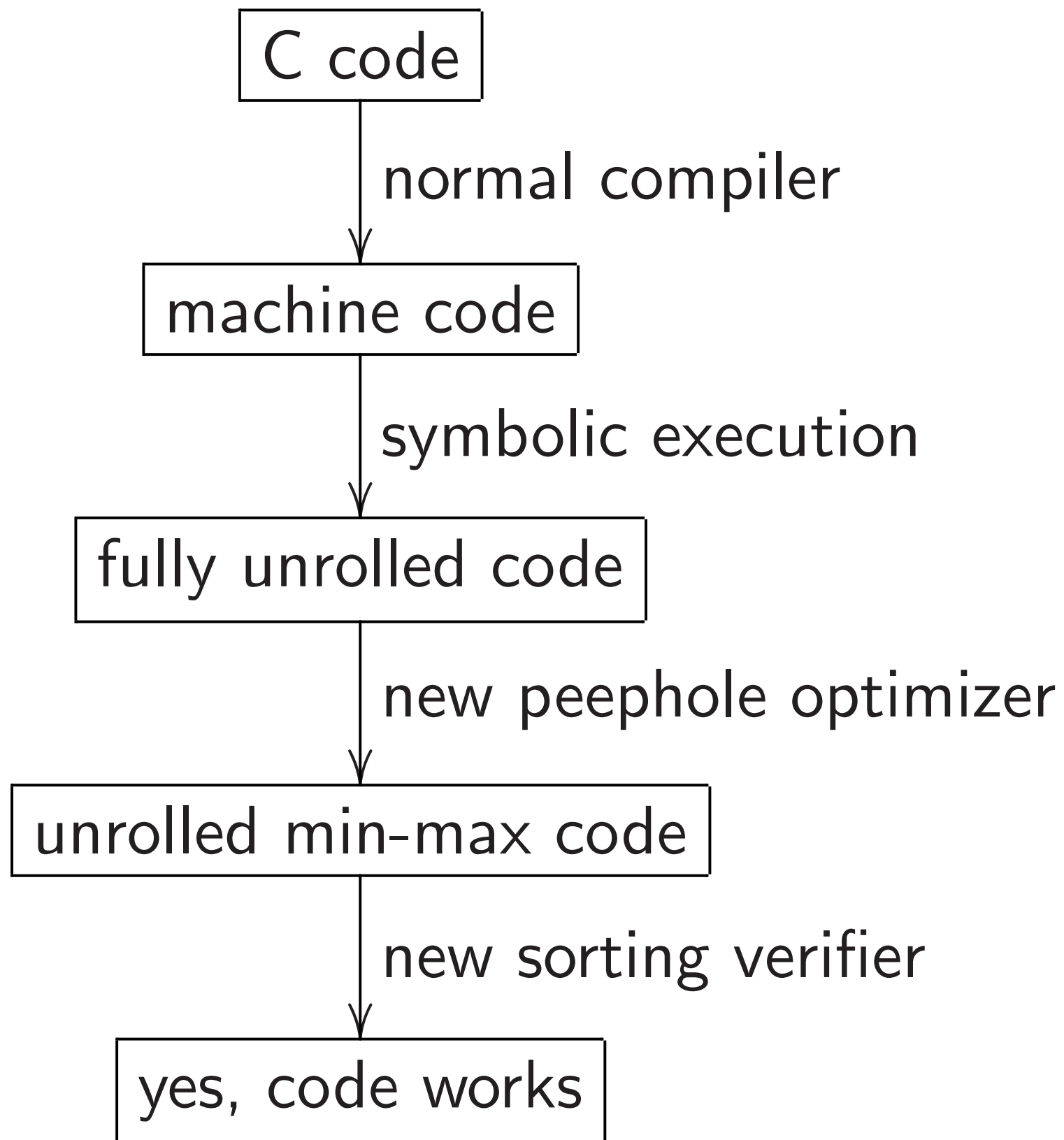
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Symbolic execution
 use existing [angr](#).
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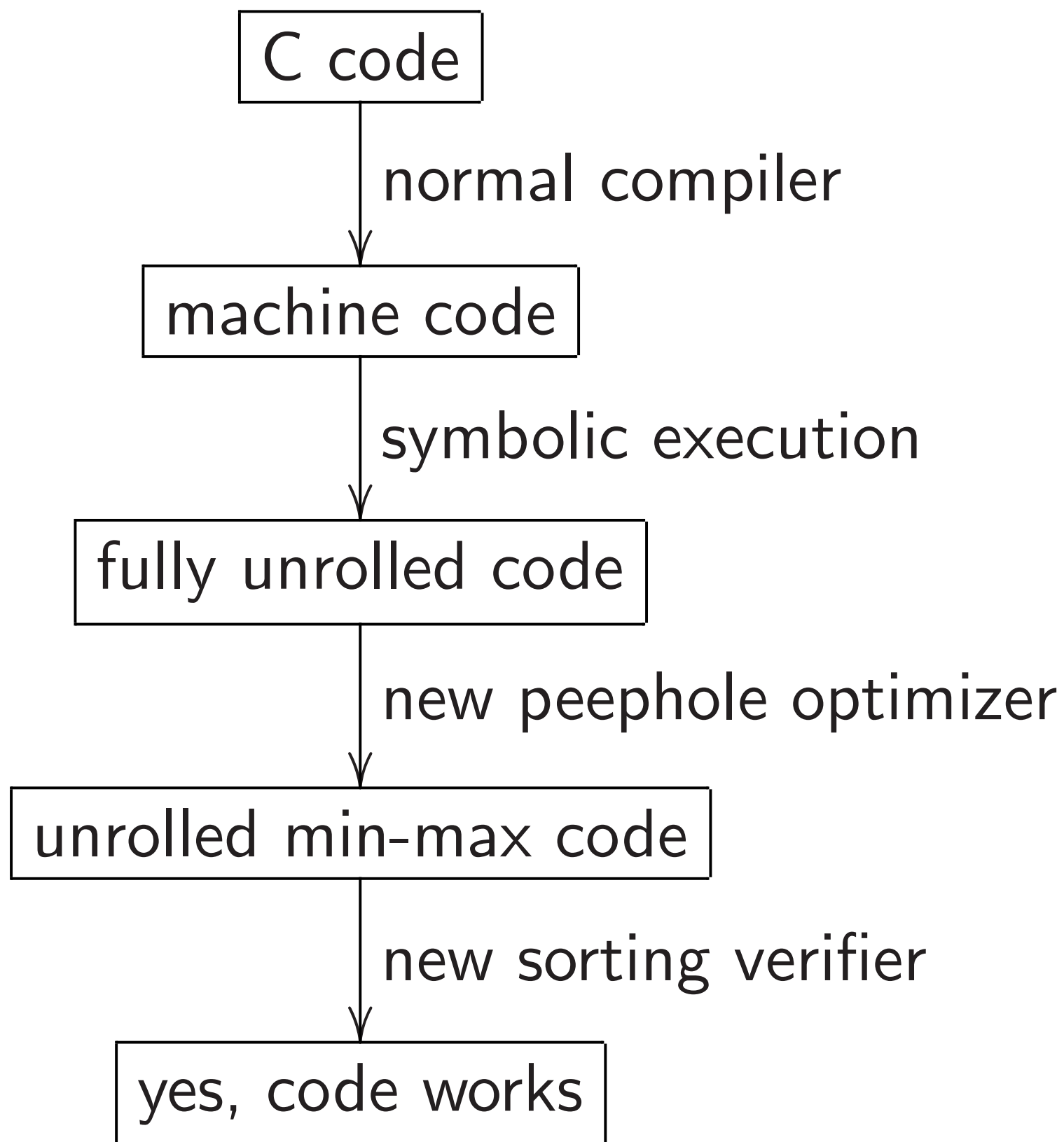
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Symbolic execution:

use existing angr.io toolkit
with several tiny new patches
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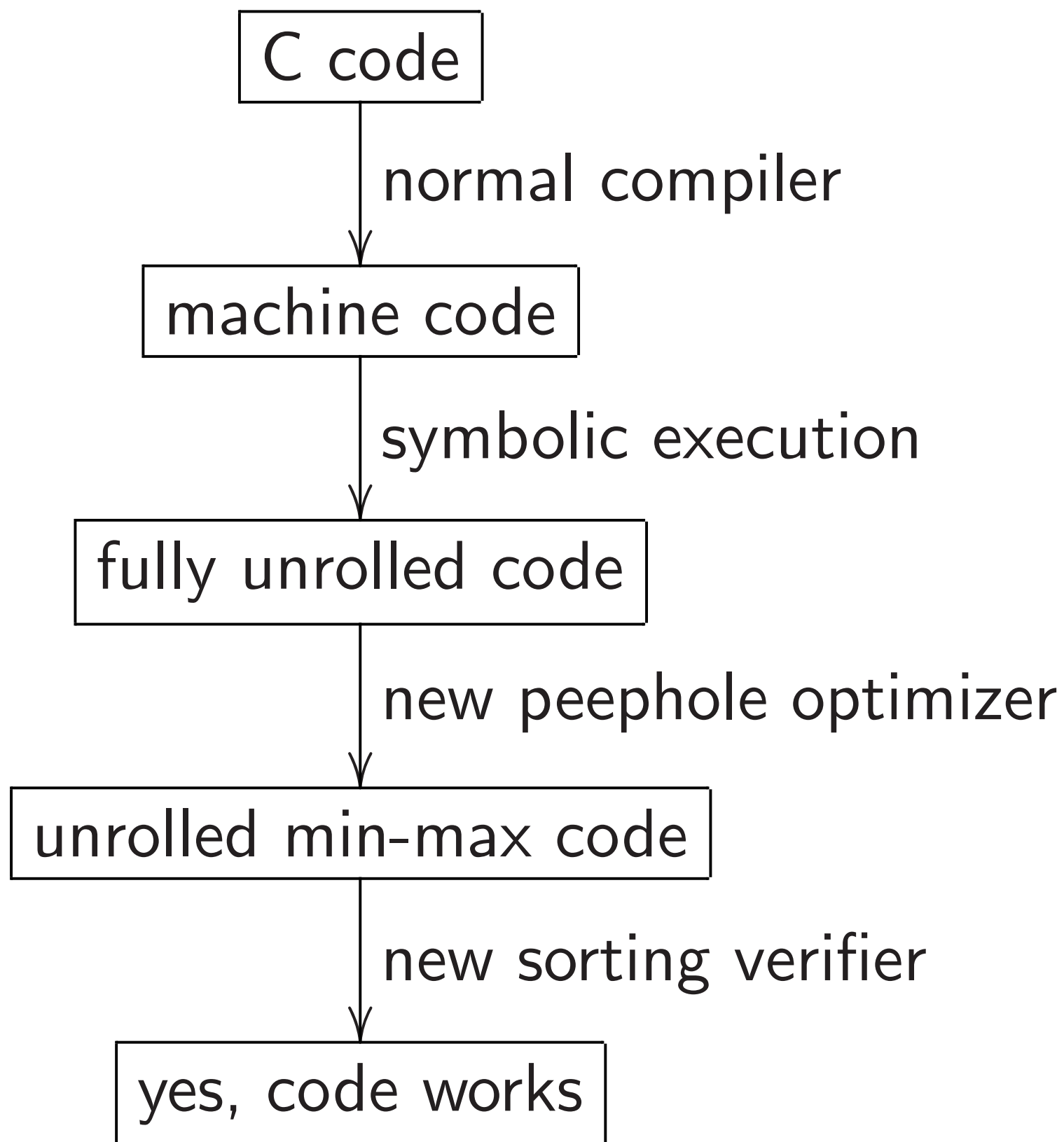
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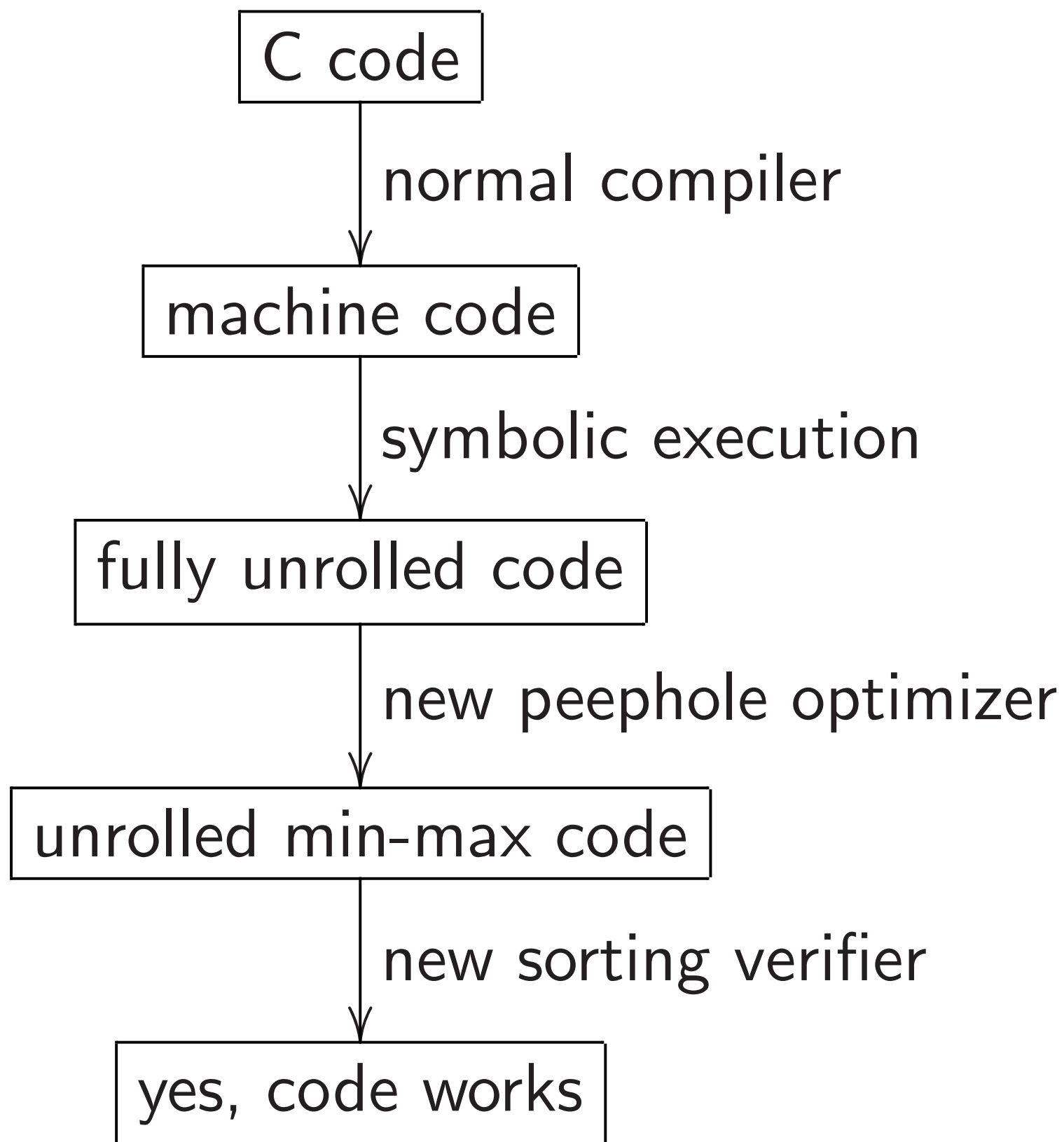
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Peephole optimizer:

recognize instruction patterns equivalent to min, max.

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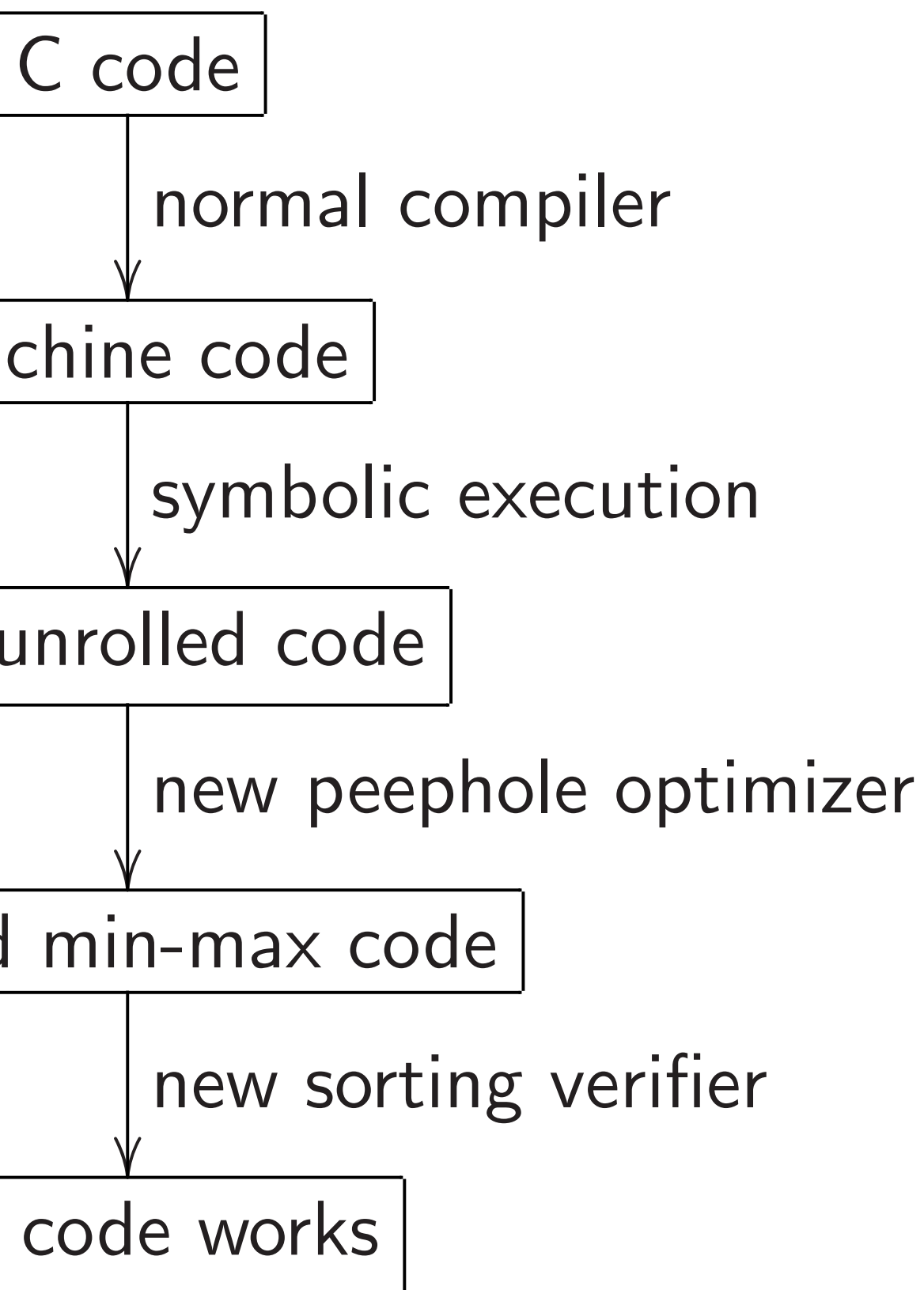
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Sorting verifier: decompose DAG into merging networks.

Verify each merging network using generalization of 2007

Even–Levi–Litman, correction of 1990 Chung–Ravikumar.

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Current djbsort release
(verified fast int32 on AVX,
verified portable int32,
fast uint32, fast float32):

sorting.cr.yp.to

Includes the sorting code;
automatic build-time tests;
simple benchmarking program
verification tools.

Web site shows how to
use the verification tools.

Next release planned:
verified ARM NEON code.

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