What do quantum computers do?

Daniel J. Bernstein

"Quantum algorithm" means an algorithm that a quantum computer can run.

i.e. a sequence of instructions, where each instruction is in a quantum computer's supported instruction set.

How do we know which instructions a quantum computer will support?

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Data stored in 4 qubits: a list of 16 numbers, not all zero. e.g.:
$(3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3)$.

The state of a computer

Data ("state") stored in 3 bits:
a list of 3 elements of $\{0, 1\}$.
e.g.: $(0, 0, 0)$.
e.g.: $(1, 1, 1)$.
e.g.: $(0, 1, 1)$.

Data stored in 64 bits:
a list of 64 elements of $\{0, 1\}$.
e.g.: $(1, 1, 1, 1, 1, 0, 0, 0, 1,$
$0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0,$
$0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1,$
$1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1,$
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The state of a quantum computer

Data stored in 3 qubits:
a list of 8 numbers, not all zero.
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"Quantum RNG."

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Measurement produces

$000 = 0$ with probability 0;

$001 = 1$ with probability 0;

$010 = 2$ with probability 0;

$011 = 3$ with probability 0;

$100 = 4$ with probability 0;

$101 = 5$ with probability 1;

$110 = 6$ with probability 0;

$111 = 7$ with probability 0.

5 is guaranteed outcome.

NOT gates

NOT_0 gate on 3 q

$(3, 1, 4, 1, 5, 9, 2, 6$

$(1, 3, 1, 4, 9, 5, 6, 2$

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$(3,1,4,1,5,9,2,6,5,3,5,8,9,7,9,3) \mapsto (1,3,1,4,9,5,6,2,3,5,8,5,7,9,3,9)$.

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NOT_1 gate on 3 qubits:
$(3,1,4,1,5,9,2,6) \mapsto (4,1,3,1,2,6,5,9)$.

e.g.: Say 3 qubits have state $(0,0,0,0,0,1,0,0)$.

Measurement produces
$000 = 0$ with probability 0;
$001 = 1$ with probability 0;
$010 = 2$ with probability 0;
$011 = 3$ with probability 0;
$100 = 4$ with probability 0;
$101 = 5$ with probability 1;
$110 = 6$ with probability 0;
$111 = 7$ with probability 0.

5 is guaranteed outcome.

NOT gates

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$(3,1,4,1,5,9,2,6,5,3,5,8,9,7,9,3) \mapsto$
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NOT_1 gate on 3 qubits:
$(3,1,4,1,5,9,2,6) \mapsto$
$(4,1,3,1,2,6,5,9)$.

NOT_2 gate on 3 qubits:
$(3,1,4,1,5,9,2,6) \mapsto$
$(5,9,2,6,3,1,4,1)$.

y 3 qubits have state

$0, 0, 1, 0, 0)$.

ement produces

with probability 0;

with probability 0;

with probability 0;

with probability 0;

with probability 0;

with probability 1;

with probability 0;

with probability 0.

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NOT gates

NOT_0 gate on 3 qubits:

$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$

$(1, 3, 1, 4, 9, 5, 6, 2)$.

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NOT_1 gate on 3 qubits:

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NOT_2 gate on 3 qubits:

$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$

$(5, 9, 2, 6, 3, 1, 4, 1)$.

$(1, 0, 0,$

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NOT gates

NOT_0 gate on 3 qubits:

$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$

$(1, 3, 1, 4, 9, 5, 6, 2)$.

NOT_0 gate on 4 qubits:

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NOT_2 gate on 3 qubits:

$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$

$(5, 9, 2, 6, 3, 1, 4, 1)$.

state

(1, 0, 0, 0, 0, 0, 0, 0

(0, 1, 0, 0, 0, 0, 0, 0

(0, 0, 1, 0, 0, 0, 0, 0

(0, 0, 0, 1, 0, 0, 0, 0

(0, 0, 0, 0, 1, 0, 0, 0

(0, 0, 0, 0, 0, 1, 0, 0

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NOT gates

NOT_0 gate on 3 qubits:
$(3,1,4,1,5,9,2,6) \mapsto$
$(1,3,1,4,9,5,6,2)$.

NOT_0 gate on 4 qubits:
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$(5,9,2,6,3,1,4,1)$.

state	measure
$(1,0,0,0,0,0,0,0)$	000
$(0,1,0,0,0,0,0,0)$	001
$(0,0,1,0,0,0,0,0)$	010
$(0,0,0,1,0,0,0,0)$	011
$(0,0,0,0,1,0,0,0)$	100
$(0,0,0,0,0,1,0,0)$	101
$(0,0,0,0,0,0,1,0)$	110
$(0,0,0,0,0,0,0,1)$	111

Operation on quantum state
NOT_0, swapping pairs.
Operation after measuremen
flipping bit 0 of result.
Flip: output is not input.

NOT gates

NOT_0 gate on 3 qubits:

$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(1, 3, 1, 4, 9, 5, 6, 2)$.

NOT_0 gate on 4 qubits:

$(3,1,4,1,5,9,2,6,5,3,5,8,9,7,9,3) \mapsto$
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NOT_1 gate on 3 qubits:

$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(4, 1, 3, 1, 2, 6, 5, 9)$.

NOT_2 gate on 3 qubits:

$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(5, 9, 2, 6, 3, 1, 4, 1)$.

state	measurement
$(1, 0, 0, 0, 0, 0, 0, 0)$	000
$(0, 1, 0, 0, 0, 0, 0, 0)$	001
$(0, 0, 1, 0, 0, 0, 0, 0)$	010
$(0, 0, 0, 1, 0, 0, 0, 0)$	011
$(0, 0, 0, 0, 1, 0, 0, 0)$	100
$(0, 0, 0, 0, 0, 1, 0, 0)$	101
$(0, 0, 0, 0, 0, 0, 1, 0)$	110
$(0, 0, 0, 0, 0, 0, 0, 1)$	111

Operation on quantum state:
NOT_0, swapping pairs.
Operation after measurement:
flipping bit 0 of result.
Flip: output is not input.

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ate on 3 qubits:

$1, 5, 9, 2, 6) \mapsto$

$4, 9, 5, 6, 2).$

ate on 4 qubits:

$5,9,2,6,5,3,5,8,9,7,9,3) \mapsto$

$9,5,6,2,3,5,8,5,7,9,3,9).$

ate on 3 qubits:

$1, 5, 9, 2, 6) \mapsto$

$1, 2, 6, 5, 9).$

ate on 3 qubits:

$1, 5, 9, 2, 6) \mapsto$

$6, 3, 1, 4, 1).$

state	measurement
$(1, 0, 0, 0, 0, 0, 0, 0)$	000
$(0, 1, 0, 0, 0, 0, 0, 0)$	001
$(0, 0, 1, 0, 0, 0, 0, 0)$	010
$(0, 0, 0, 1, 0, 0, 0, 0)$	011
$(0, 0, 0, 0, 1, 0, 0, 0)$	100
$(0, 0, 0, 0, 0, 1, 0, 0)$	101
$(0, 0, 0, 0, 0, 0, 1, 0)$	110
$(0, 0, 0, 0, 0, 0, 0, 1)$	111

Operation on quantum state:
NOT_0, swapping pairs.
Operation after measurement:
flipping bit 0 of result.
Flip: output is not input.

Controll

e.g. C_1N

$(3, 1, 4,$

$(3, 1, 1,$

ubits:

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ubits:

3,5,8,9,7,9,3) $\mapsto$

5,8,5,7,9,3,9).

ubits:

) $\mapsto$

).

ubits:

) $\mapsto$

).

state	measurement
$(1,0,0,0,0,0,0,0)$	000
$(0,1,0,0,0,0,0,0)$	001
$(0,0,1,0,0,0,0,0)$	010
$(0,0,0,1,0,0,0,0)$	011
$(0,0,0,0,1,0,0,0)$	100
$(0,0,0,0,0,1,0,0)$	101
$(0,0,0,0,0,0,1,0)$	110
$(0,0,0,0,0,0,0,1)$	111

Operation on quantum state:
NOT_0, swapping pairs.
Operation after measurement:
flipping bit 0 of result.
Flip: output is not input.

Controlled-NOT (

e.g. C_1NOT_0:
$(3,1,4,1,5,9,2,6$
$(3,1,1,4,5,9,6,2$

,3) $\mapsto$

,9).

state	measurement
$(1,0,0,0,0,0,0,0)$	000
$(0,1,0,0,0,0,0,0)$	001
$(0,0,1,0,0,0,0,0)$	010
$(0,0,0,1,0,0,0,0)$	011
$(0,0,0,0,1,0,0,0)$	100
$(0,0,0,0,0,1,0,0)$	101
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Operation on quantum state:
NOT_0, swapping pairs.
Operation after measurement:
flipping bit 0 of result.
Flip: output is not input.

Controlled-NOT (CNOT) ga

e.g. C_1NOT_0:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,1,4,5,9,6,2)$.

state	measurement
$(1,0,0,0,0,0,0,0)$	000
$(0,1,0,0,0,0,0,0)$	001
$(0,0,1,0,0,0,0,0)$	010
$(0,0,0,1,0,0,0,0)$	011
$(0,0,0,0,1,0,0,0)$	100
$(0,0,0,0,0,1,0,0)$	101
$(0,0,0,0,0,0,1,0)$	110
$(0,0,0,0,0,0,0,1)$	111

Operation on quantum state:
NOT_0, swapping pairs.

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Flip: output is not input.

Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,1,4,5,9,6,2)$.

state	measurement
$(1,0,0,0,0,0,0,0)$	000
$(0,1,0,0,0,0,0,0)$	001
$(0,0,1,0,0,0,0,0)$	010
$(0,0,0,1,0,0,0,0)$	011
$(0,0,0,0,1,0,0,0)$	100
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Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

state	measurement
$(1,0,0,0,0,0,0,0)$	000
$(0,1,0,0,0,0,0,0)$	001
$(0,0,1,0,0,0,0,0)$	010
$(0,0,0,1,0,0,0,0)$	011
$(0,0,0,0,1,0,0,0)$	100
$(0,0,0,0,0,1,0,0)$	101
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$(0,0,0,0,0,0,0,1)$	111

Operation on quantum state:
NOT_0, swapping pairs.
Operation after measurement:
flipping bit 0 of result.
Flip: output is not input.

Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,1,4,5,9,6,2)$.

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flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. C_2NOT_0:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,4,1,9,5,6,2)$.

state	measurement
$(1,0,0,0,0,0,0,0)$	000
$(0,1,0,0,0,0,0,0)$	001
$(0,0,1,0,0,0,0,0)$	010
$(0,0,0,1,0,0,0,0)$	011
$(0,0,0,0,1,0,0,0)$	100
$(0,0,0,0,0,1,0,0)$	101
$(0,0,0,0,0,0,1,0)$	110
$(0,0,0,0,0,0,0,1)$	111

Operation on quantum state:
NOT_0, swapping pairs.
Operation after measurement:
flipping bit 0 of result.
Flip: output is not input.

Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,1,4,5,9,6,2)$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. C_2NOT_0:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,4,1,9,5,6,2)$.

e.g. C_0NOT_2:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,9,4,6,5,1,2,1)$.

state	measurement
0, 0, 0, 0, 0)	000
0, 0, 0, 0, 0)	001
0, 0, 0, 0, 0)	010
1, 0, 0, 0, 0)	011
0, 1, 0, 0, 0)	100
0, 0, 1, 0, 0)	101
0, 0, 0, 1, 0)	110
0, 0, 0, 0, 1)	111

on on quantum state:
swapping pairs.
on after measurement:
bit 0 of result.
tput is not input.

Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 1, 4, 5, 9, 6, 2)$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. C_2NOT_0:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 9, 5, 6, 2)$.

e.g. C_0NOT_2:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 9, 4, 6, 5, 1, 2, 1)$.

Toffoli g

Also kno
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e.g. C_2C
$(3, 1, 4, 1$
$(3, 1, 4, 1$

measurement

0⟩	000
0⟩	001
0⟩	010
0⟩	011
0⟩	100
0⟩	101
0⟩	110
1⟩	111

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Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:

$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$

$(3, 1, 1, 4, 5, 9, 6, 2)$.

Operation after measurement:

flipping bit 0 *if* bit 1 is set; i.e.,

$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. C_2NOT_0:

$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$

$(3, 1, 4, 1, 9, 5, 6, 2)$.

e.g. C_0NOT_2:

$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$

$(3, 9, 4, 6, 5, 1, 2, 1)$.

Toffoli gates

Also known as CC

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e.g. $C_2C_1NOT_0$:

$(3, 1, 4, 1, 5, 9, 2, 6$

$(3, 1, 4, 1, 5, 9, 6, 2$

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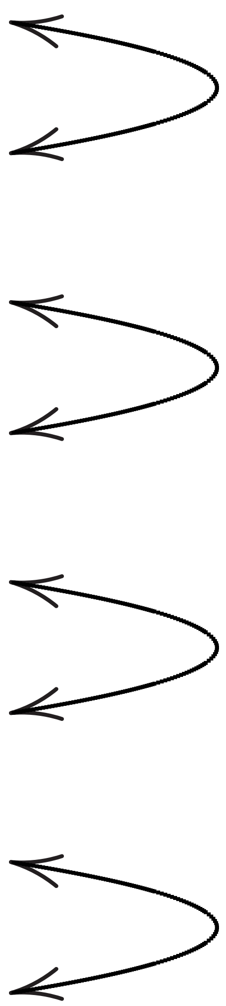

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Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 1, 4, 5, 9, 6, 2)$.

Operation after measurement: flipping bit 0 *if* bit 1 is set; i.e., $(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. C_2NOT_0:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 9, 5, 6, 2)$.

e.g. C_0NOT_2:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 9, 4, 6, 5, 1, 2, 1)$.

Toffoli gates

Also known as CCNOT gate
controlled-controlled-NOT g

e.g. $C_2C_1NOT_0$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 5, 9, 6, 2)$.

Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 1, 4, 5, 9, 6, 2)$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. C_2NOT_0:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 9, 5, 6, 2)$.

e.g. C_0NOT_2:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 9, 4, 6, 5, 1, 2, 1)$.

Toffoli gates

Also known as CCNOT gates:
controlled-controlled-NOT gates.

e.g. $C_2C_1NOT_0$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 5, 9, 6, 2)$.

Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 1, 4, 5, 9, 6, 2)$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. C_2NOT_0:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 9, 5, 6, 2)$.

e.g. C_0NOT_2:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 9, 4, 6, 5, 1, 2, 1)$.

Toffoli gates

Also known as CCNOT gates:
controlled-controlled-NOT gates.

e.g. $C_2C_1NOT_0$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 5, 9, 6, 2)$.

Operation after measurement:
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1 q_2)$.

Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,1,4,5,9,6,2)$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. C_2NOT_0:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,4,1,9,5,6,2)$.

e.g. C_0NOT_2:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,9,4,6,5,1,2,1)$.

Toffoli gates

Also known as CCNOT gates:
controlled-controlled-NOT gates.

e.g. $C_2C_1NOT_0$:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,4,1,5,9,6,2)$.

Operation after measurement:
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1 q_2)$.

e.g. $C_0C_1NOT_2$:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,4,6,5,9,2,1)$.

ed-NOT (CNOT) gates

IOT$_0$:

$1, 5, 9, 2, 6) \mapsto$

$4, 5, 9, 6, 2)$.

on after measurement:

bit 0 *if* bit 1 is set; i.e.,

$q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

IOT$_0$:

$1, 5, 9, 2, 6) \mapsto$

$1, 9, 5, 6, 2)$.

IOT$_2$:

$1, 5, 9, 2, 6) \mapsto$

$6, 5, 1, 2, 1)$.

Toffoli gates

Also known as CCNOT gates: controlled-controlled-NOT gates.

e.g. $C_2C_1NOT_0$:

$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$

$(3, 1, 4, 1, 5, 9, 6, 2)$.

Operation after measurement:

$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1 q_2)$.

e.g. $C_0C_1NOT_2$:

$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$

$(3, 1, 4, 6, 5, 9, 2, 1)$.

More sh

Combine

to build

CNOT) gates

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easurement:

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More shuffling

Combine NOT, CN

to build other perr

ates

nt:

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$q_1)$.

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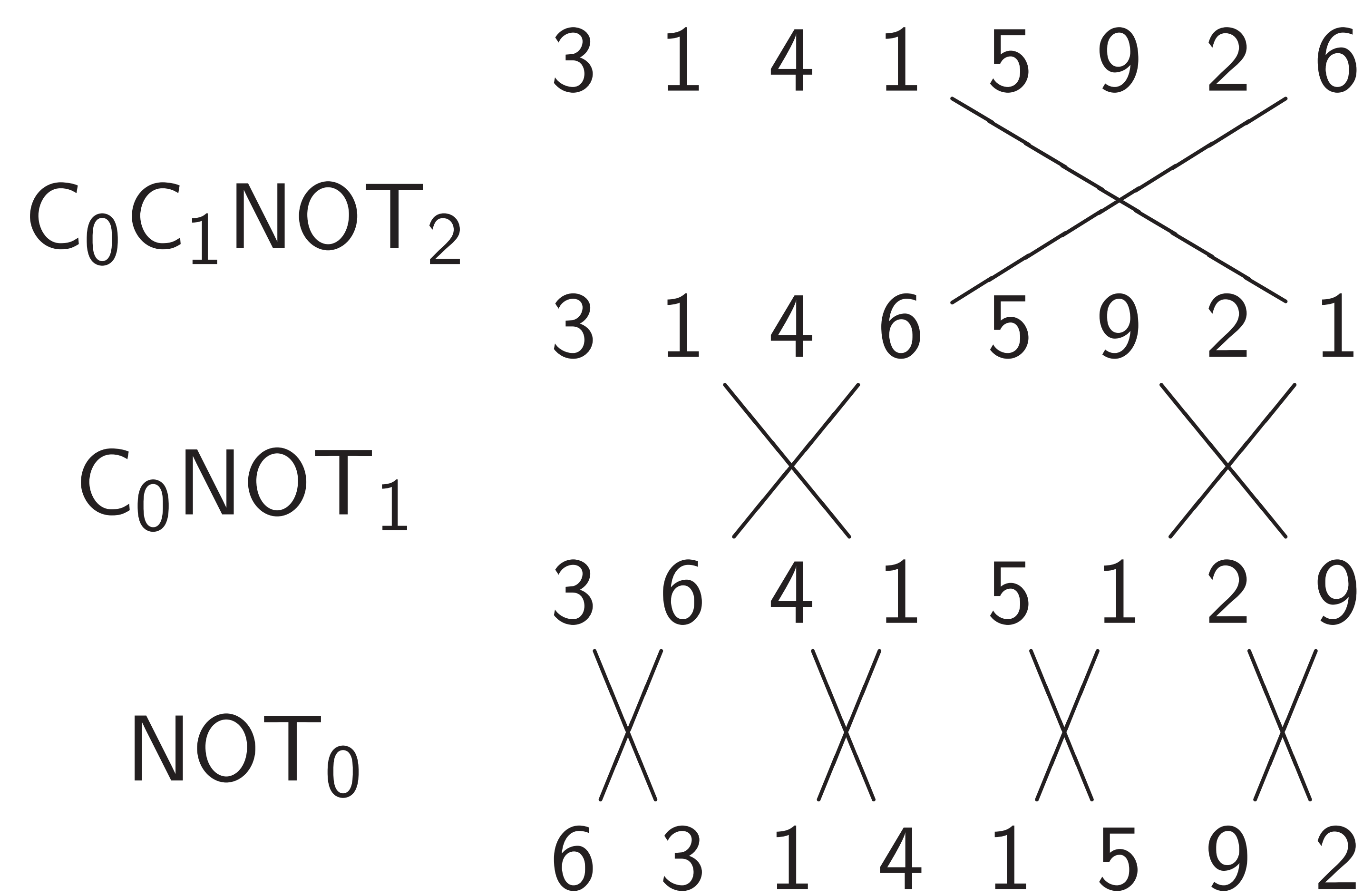
e.g. series of gates to rotate 8 positions by distance 1:



gates

own as CCNOT gates:
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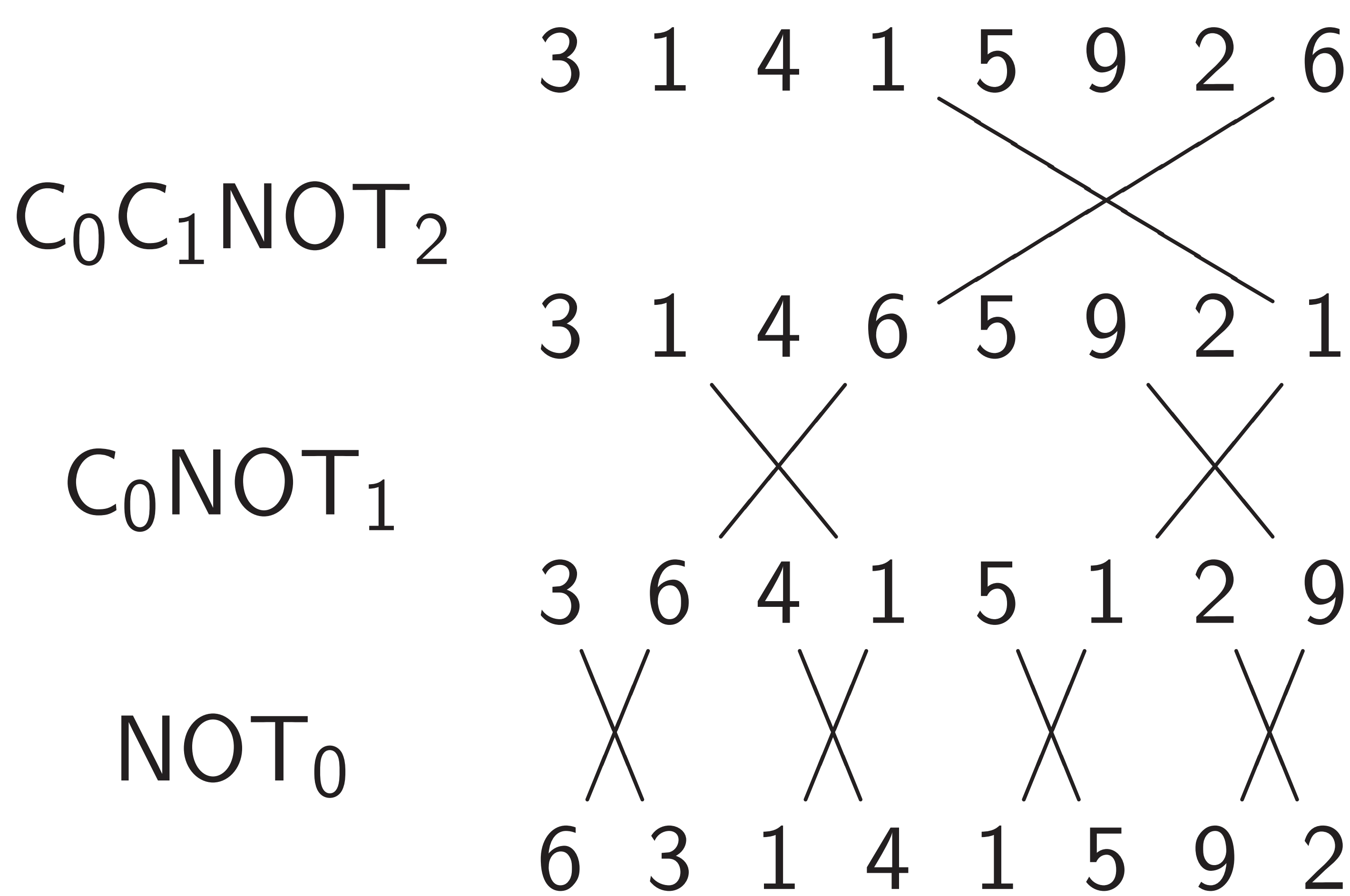
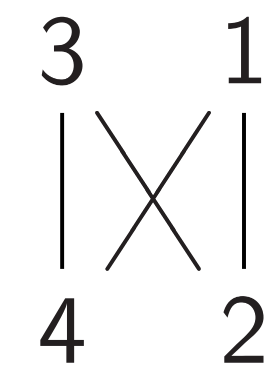


Hadama

Hadama

$(a, b) \mapsto$



NOT gates:
ed-NOT gates.

) $\mapsto$
).

easurement:
$, q_1, q_0 \oplus q_1 q_2)$.

) $\mapsto$
).

More shuffling

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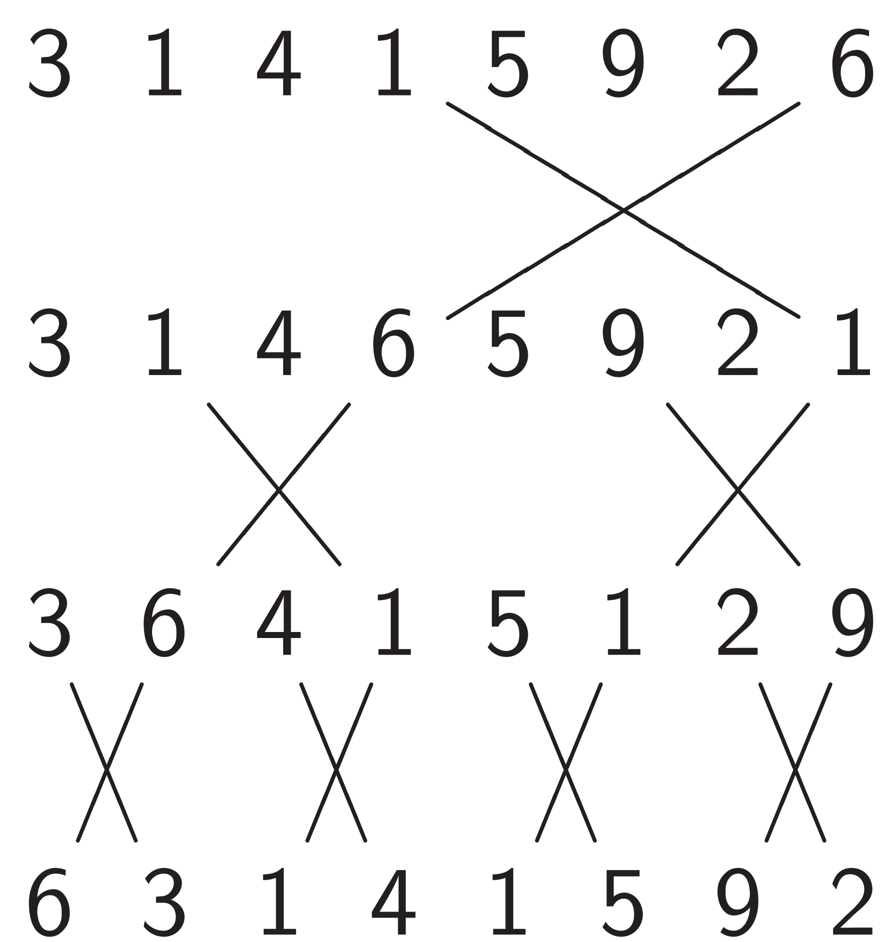
e.g. series of gates to
rotate 8 positions by distance 1:

$C_0C_1NOT_2$

C_0NOT_1

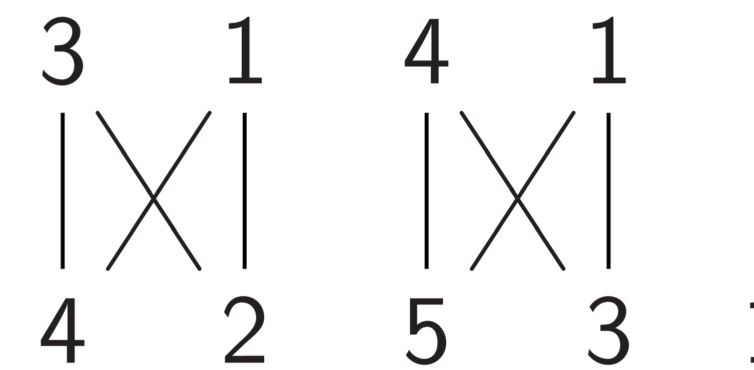
NOT_0



Hadamard gates

Hadamard_0:

$(a, b) \mapsto (a + b, a$



es:

ates.

nt:

$q_1 q_2)$.

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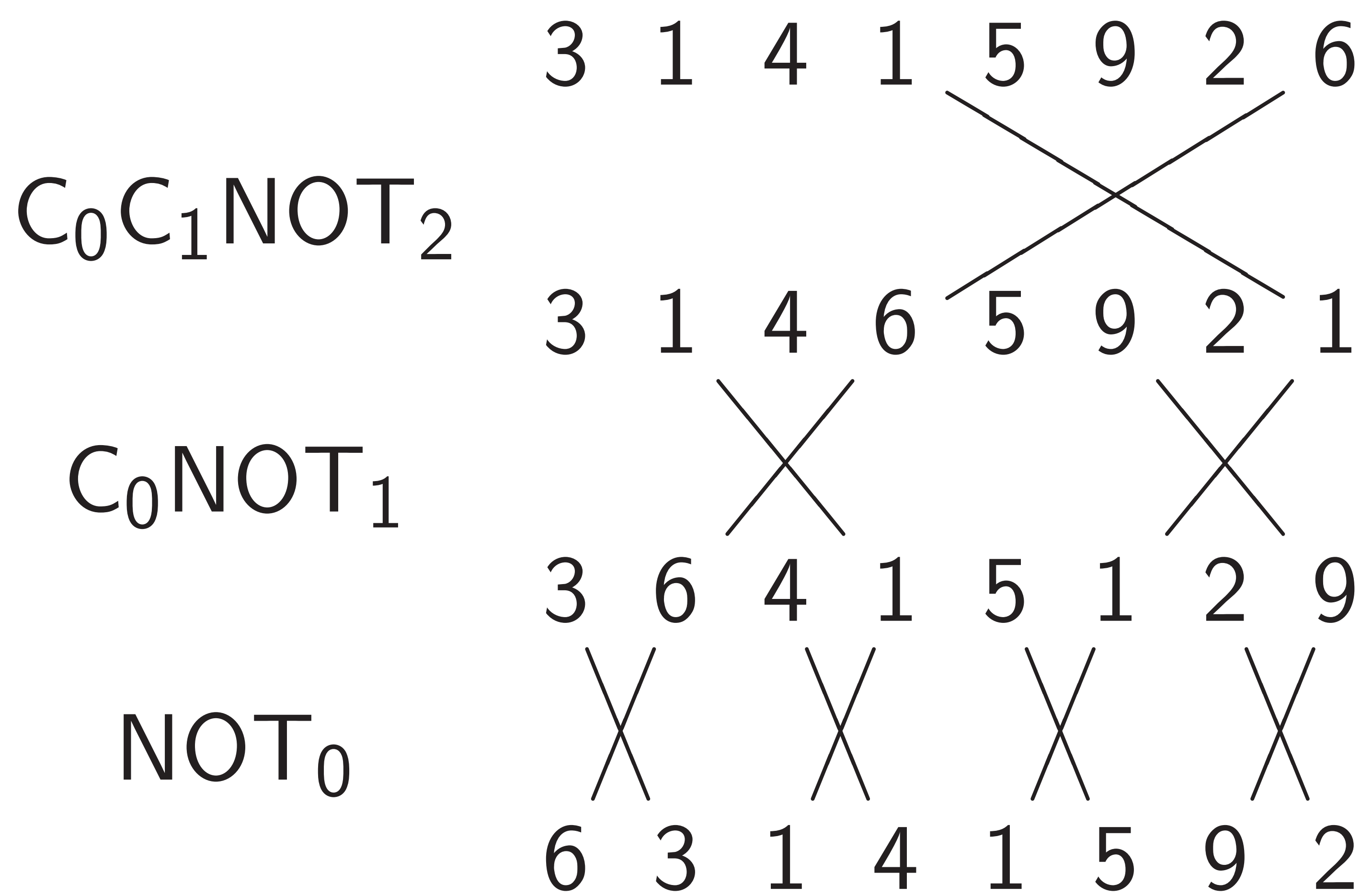

Hadamard gates

Hadamard_0:

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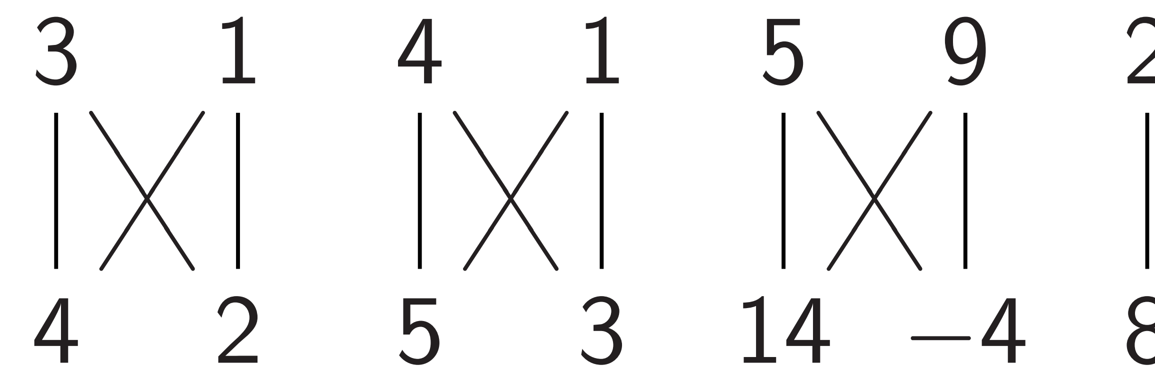

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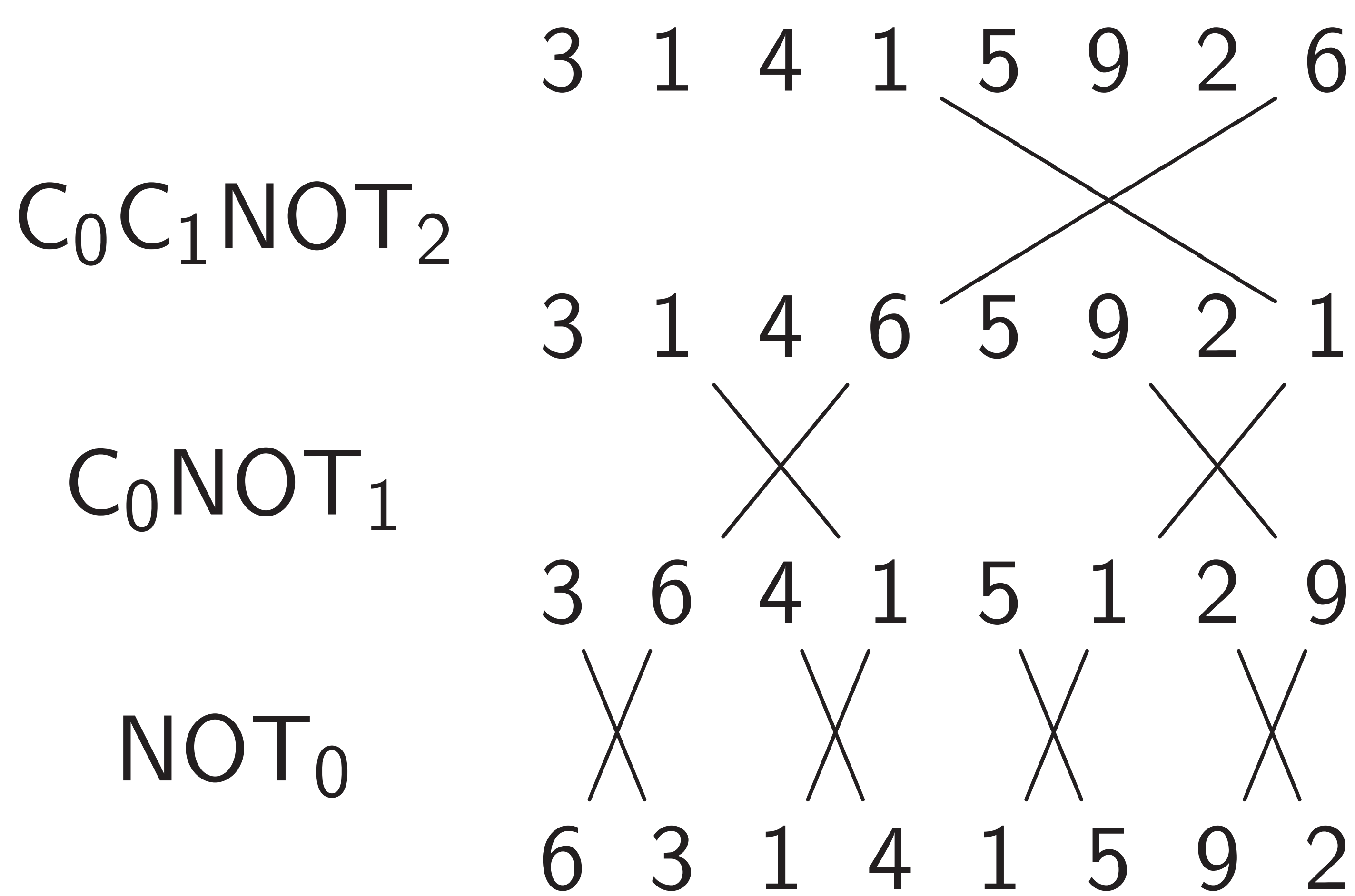

Hadamard gates

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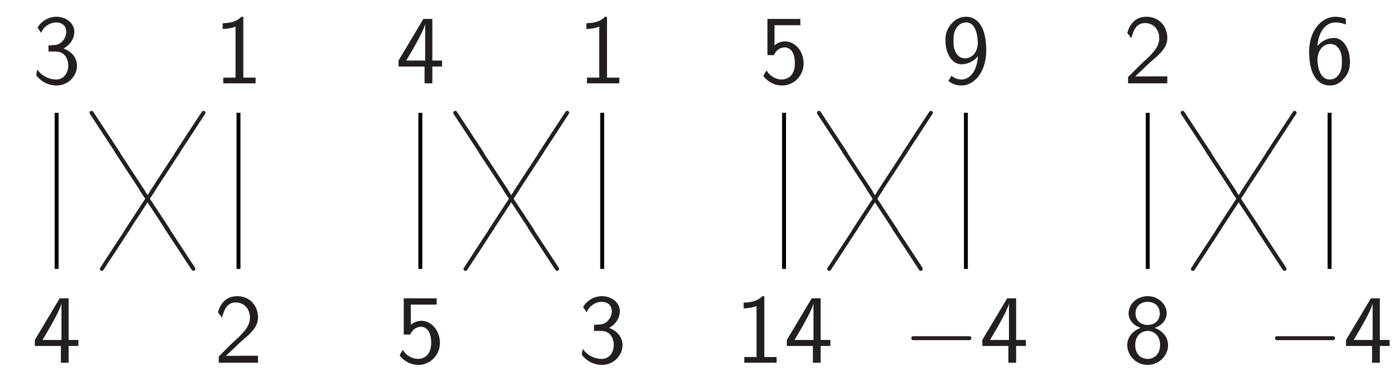

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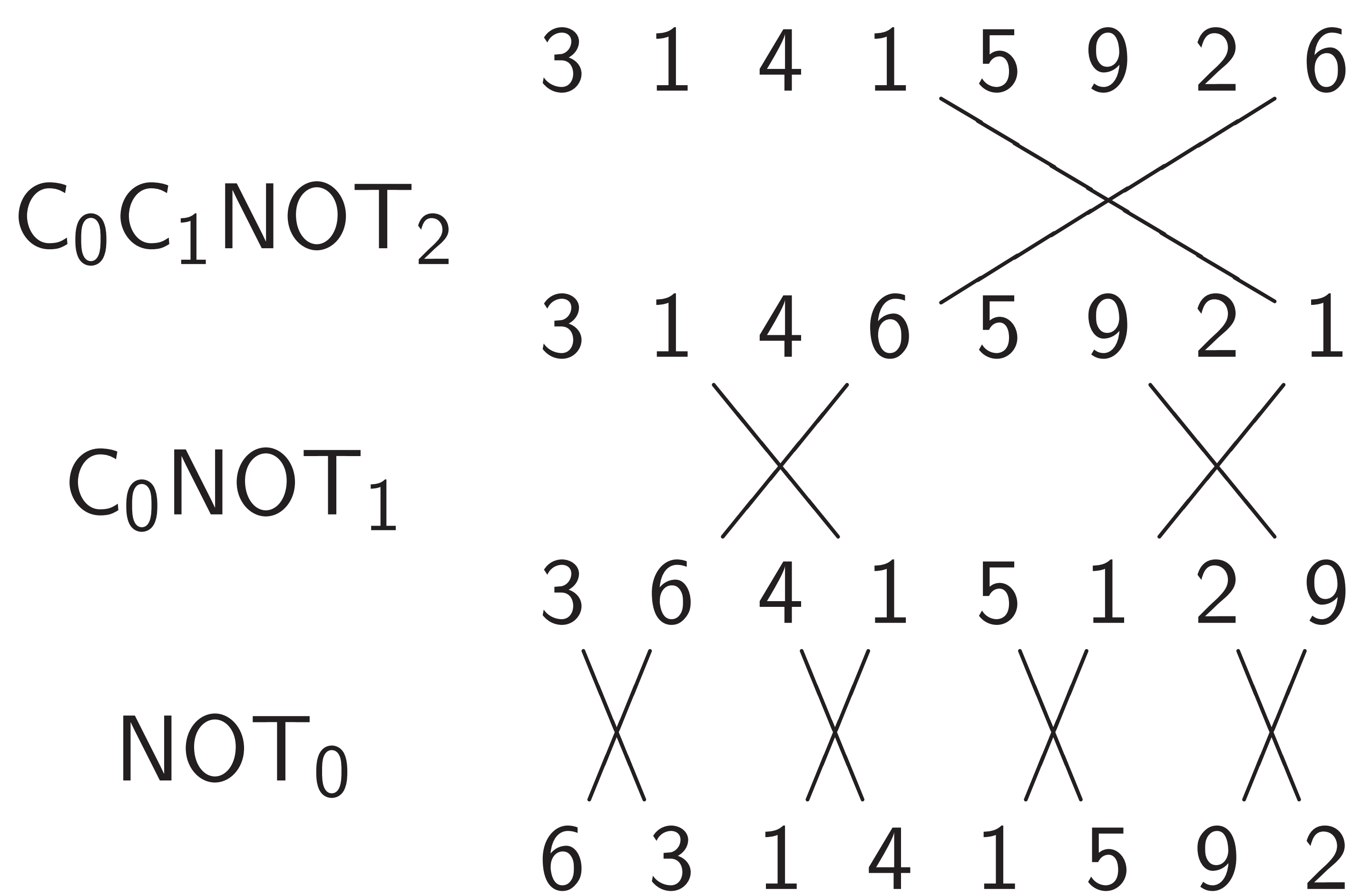

Hadamard gates

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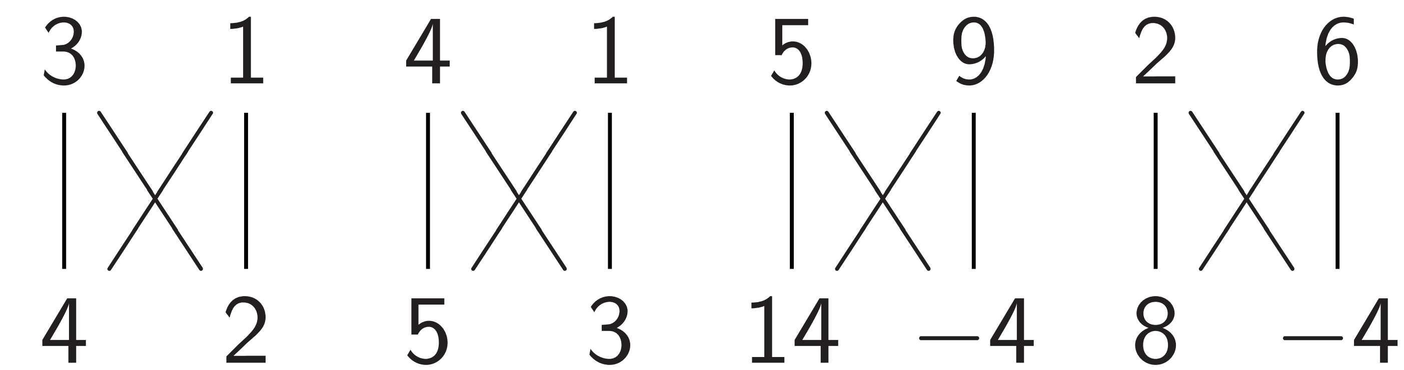
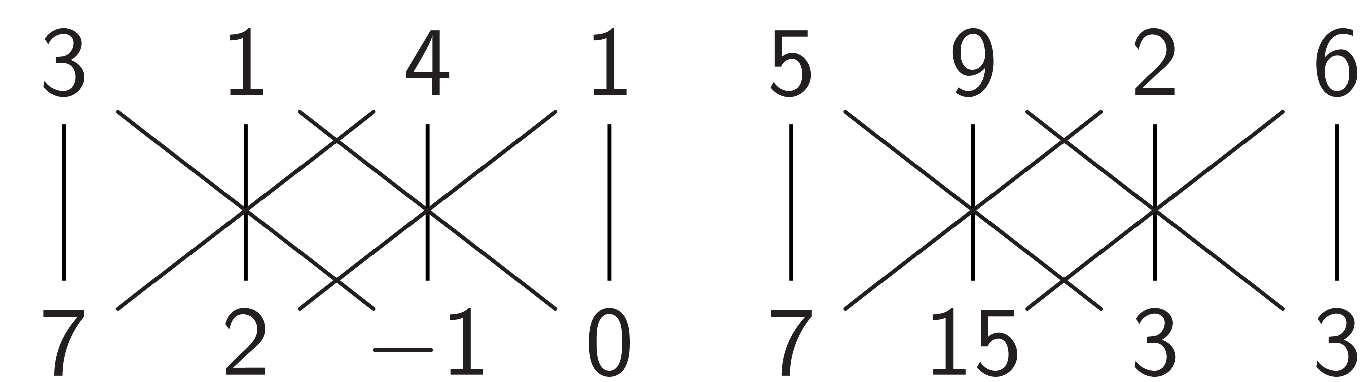


Hadamard_1:

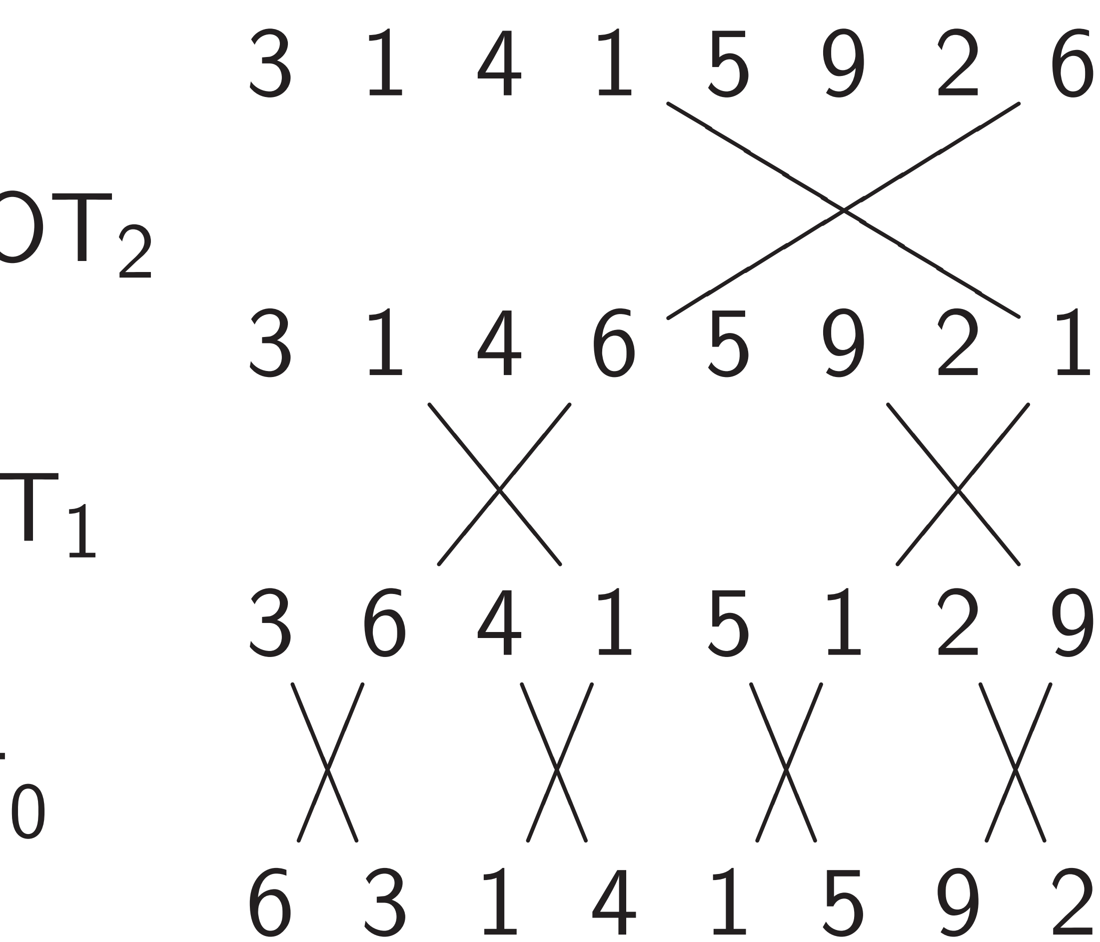
$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



uffling

e NOT, CNOT, Toffoli
other permutations.

es of gates to
positions by distance 1:



Hadamard gates

Hadamard$_0$:

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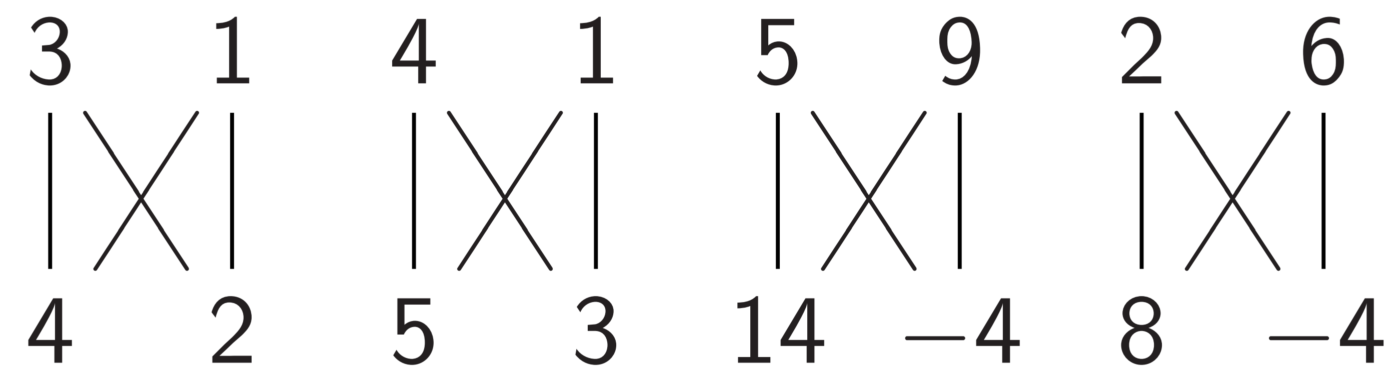

Hadamard$_1$:

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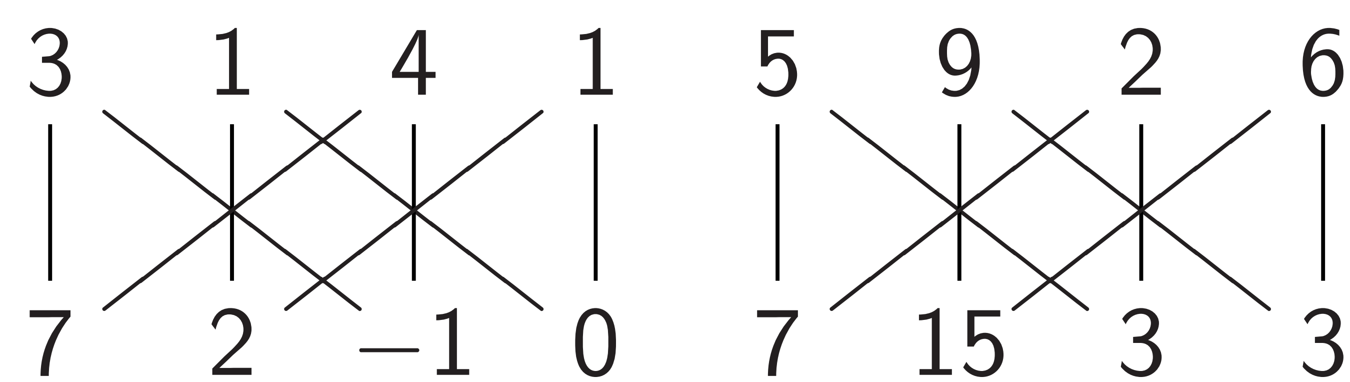

Some Ha

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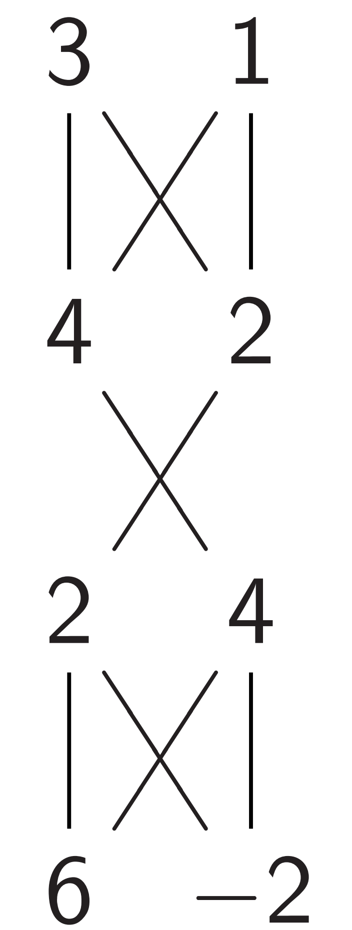
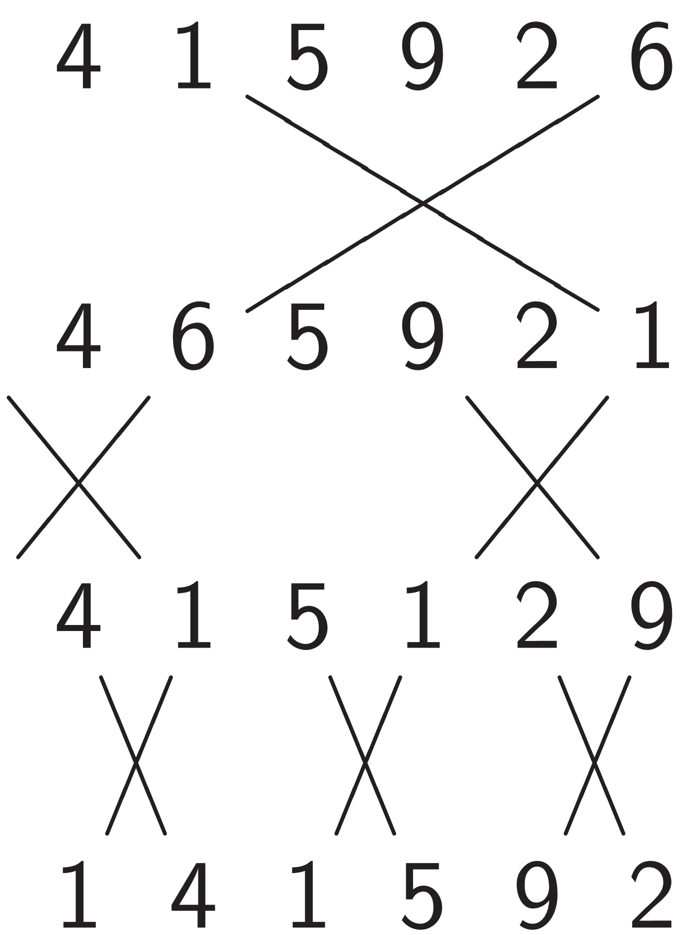


NOT, Toffoli
mutations.

s to
by distance 1:



Hadamard gates

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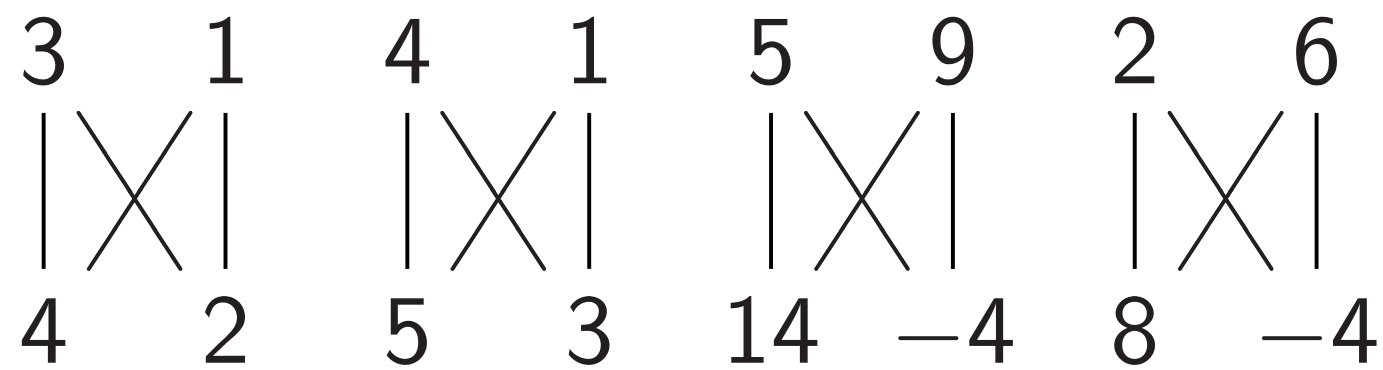

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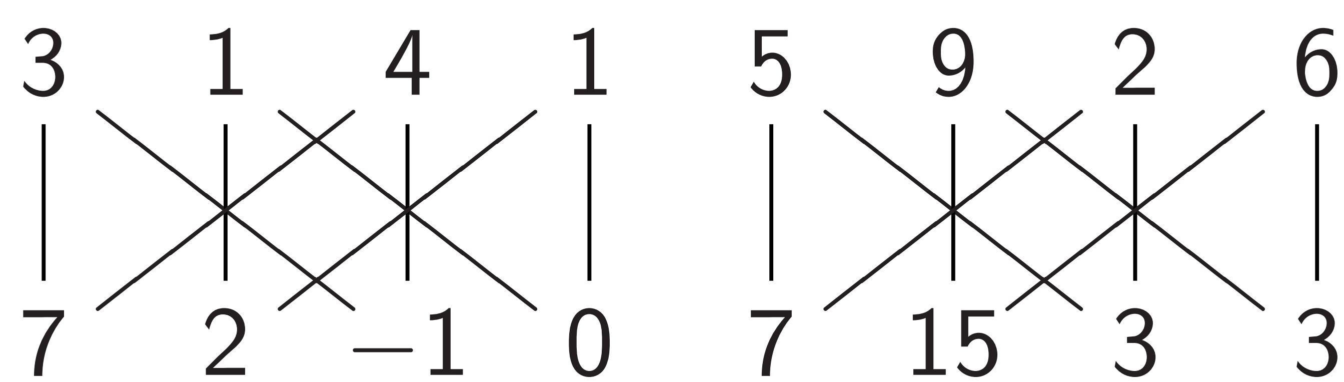
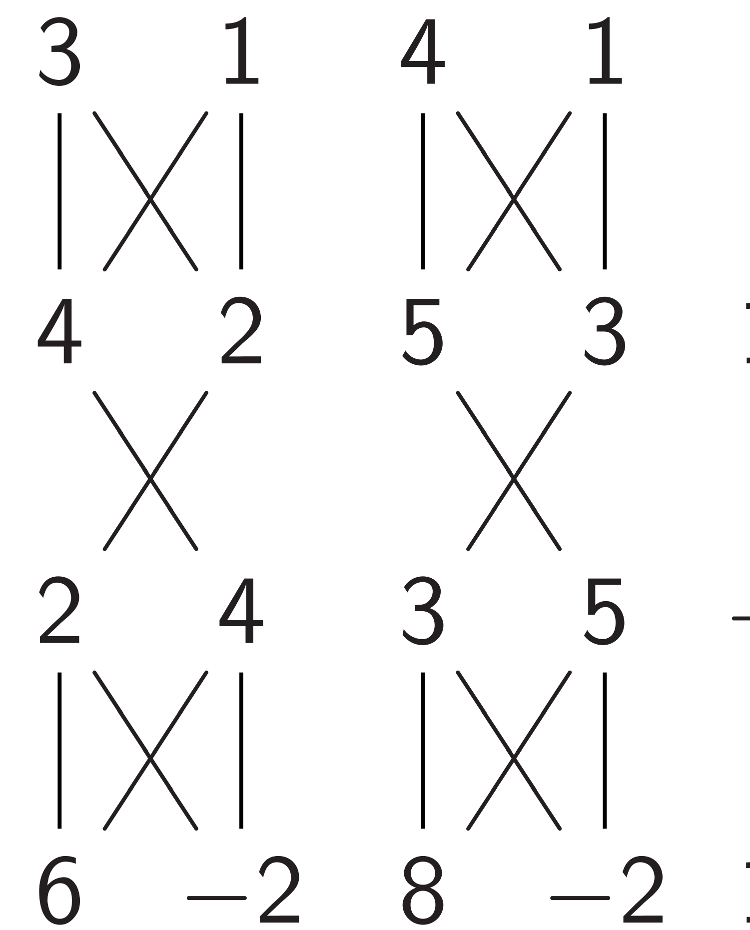


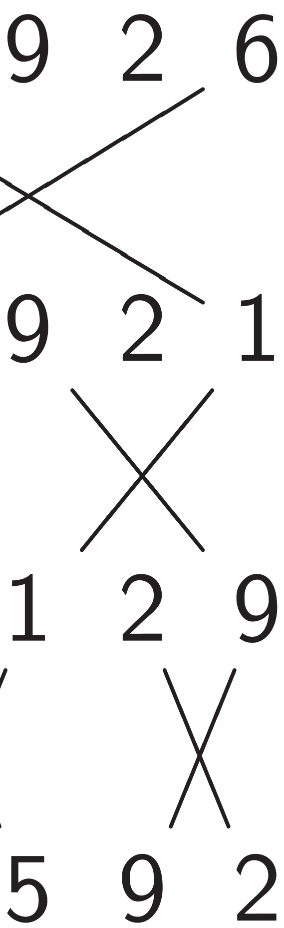
Some Hadamard a

Hadamard$_0$, NOT



foli

ce 1:



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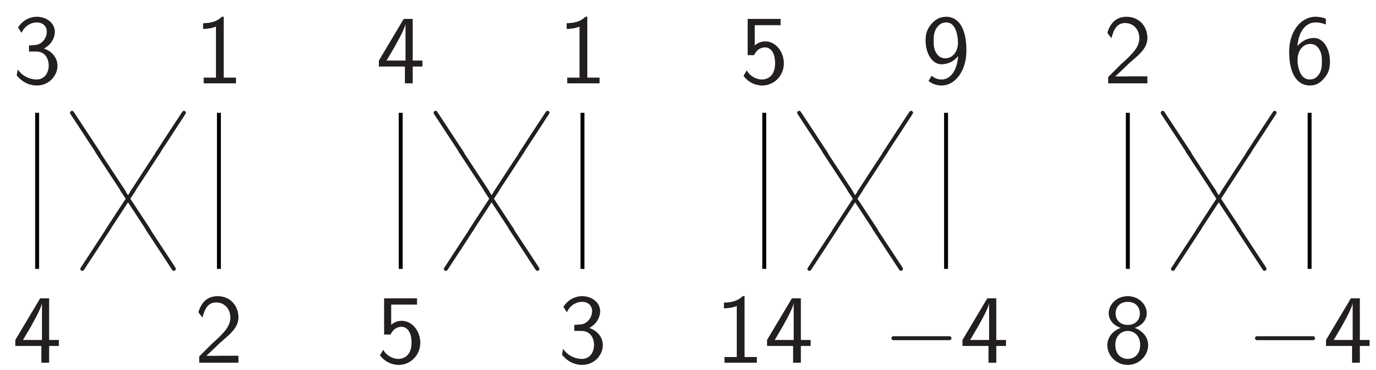

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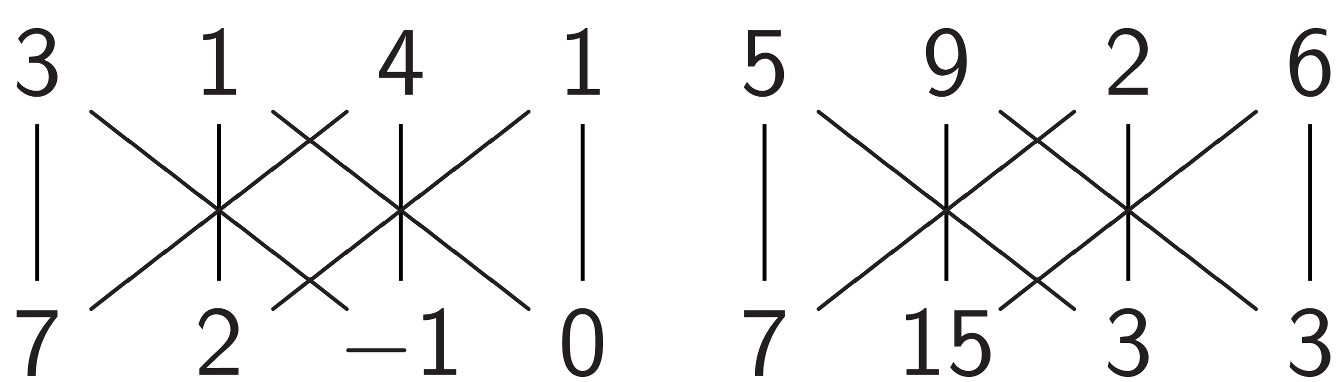
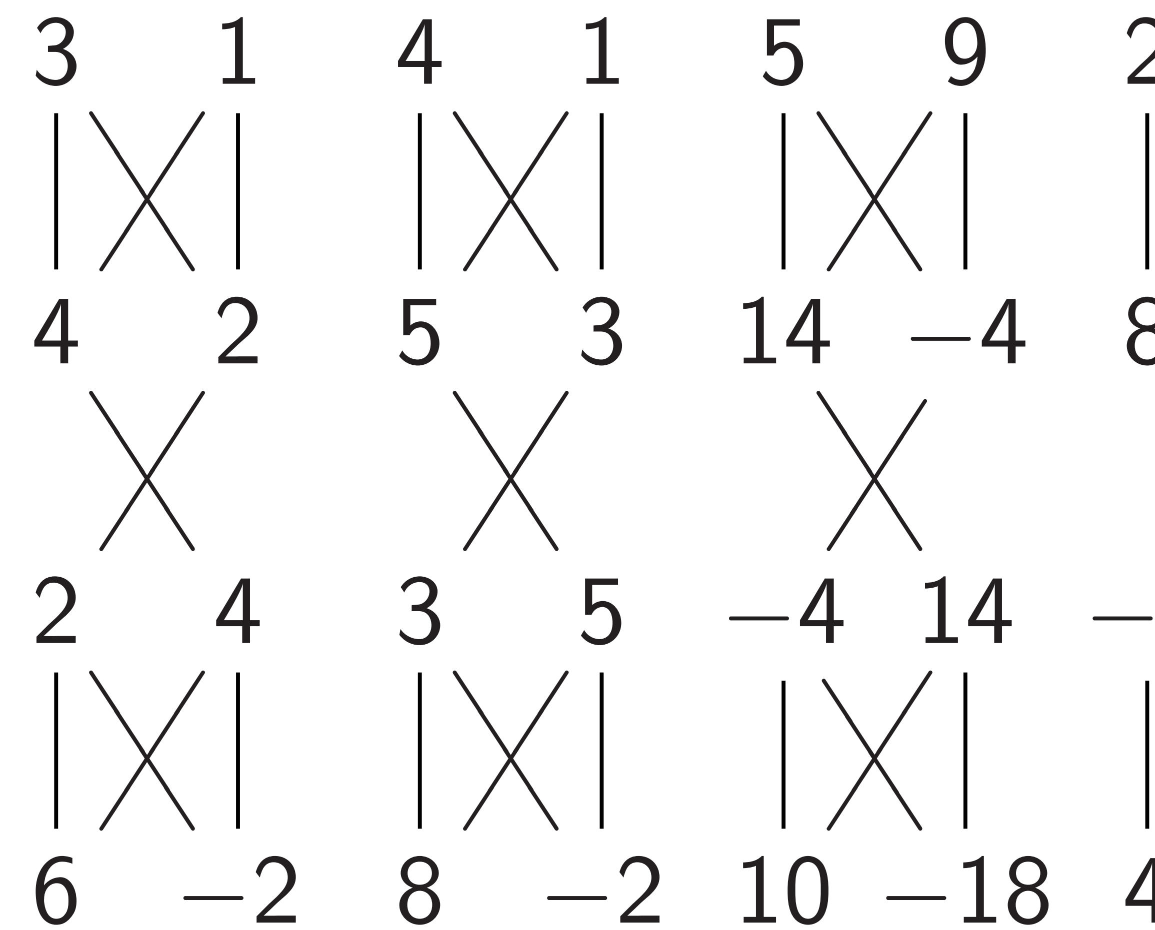


Some Hadamard application

Hadamard$_0$, NOT$_0$, Hadama



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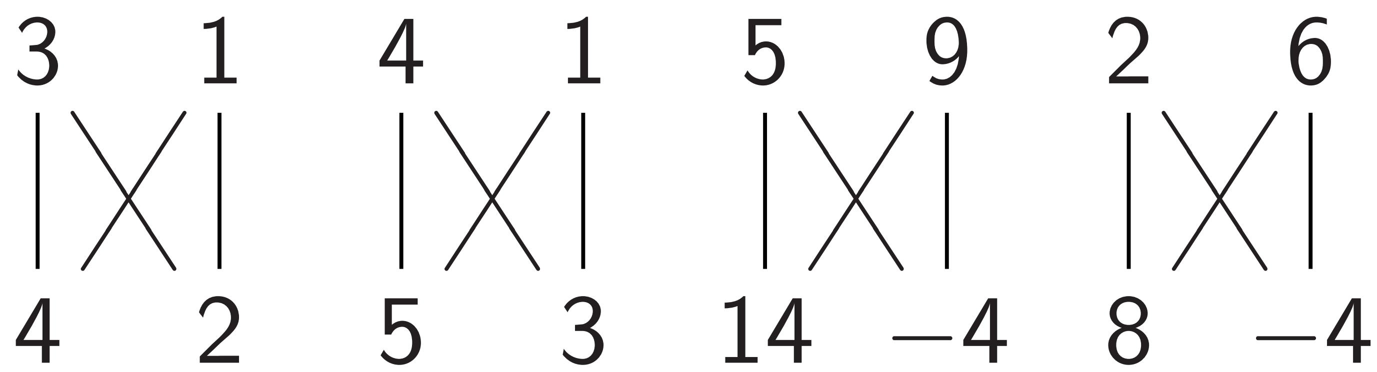

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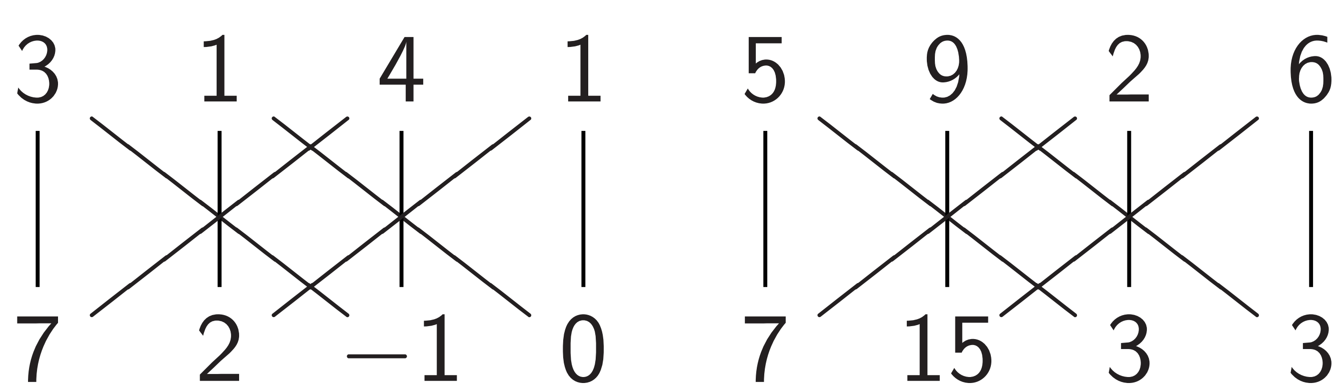
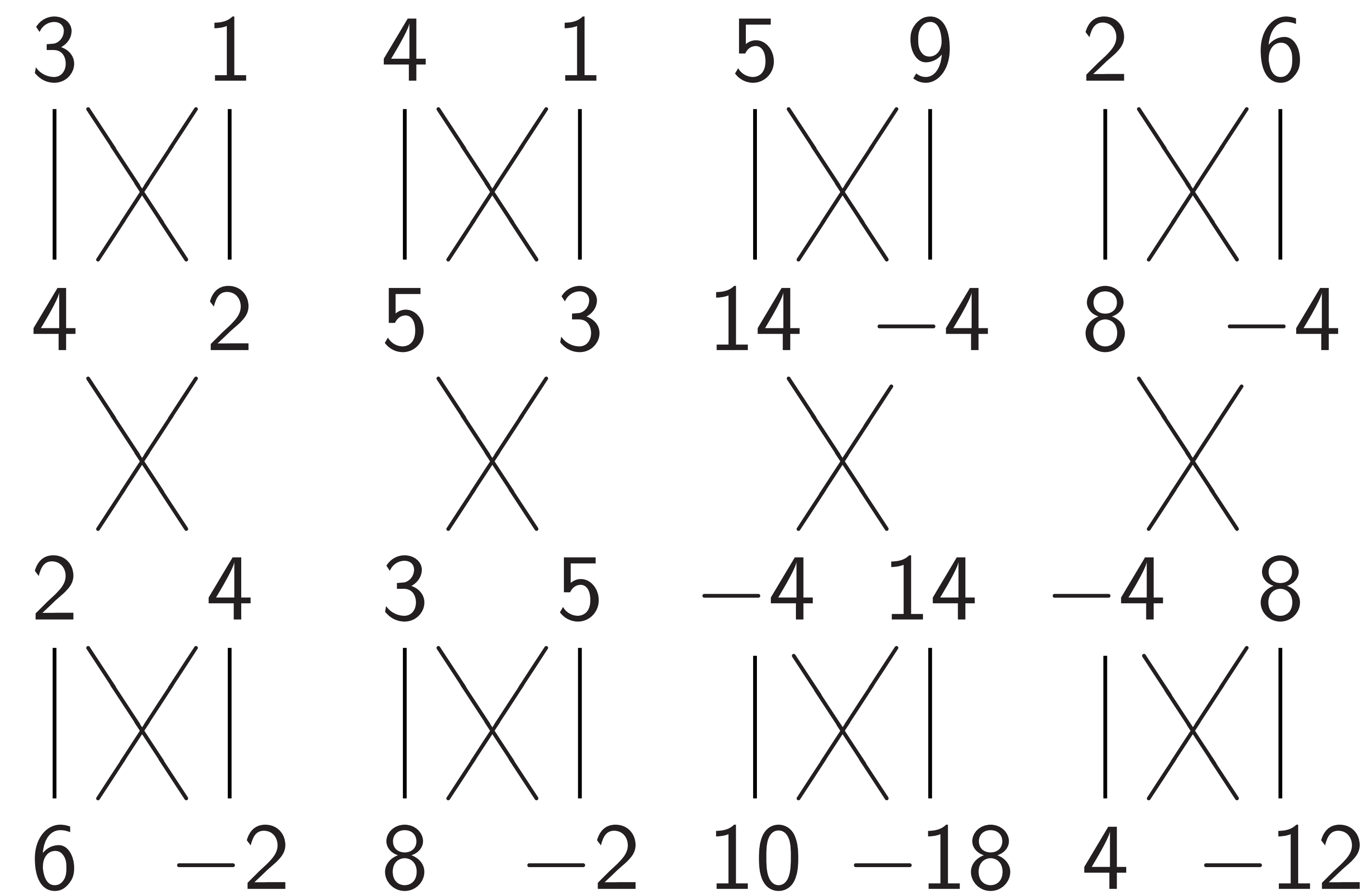


Some Hadamard applications

Hadamard$_0$, NOT$_0$, Hadamard$_0$:



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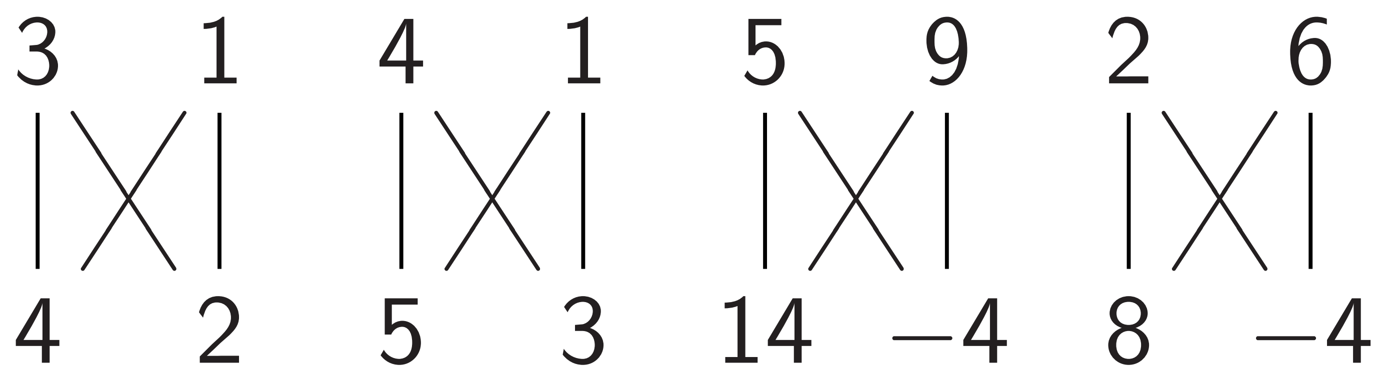

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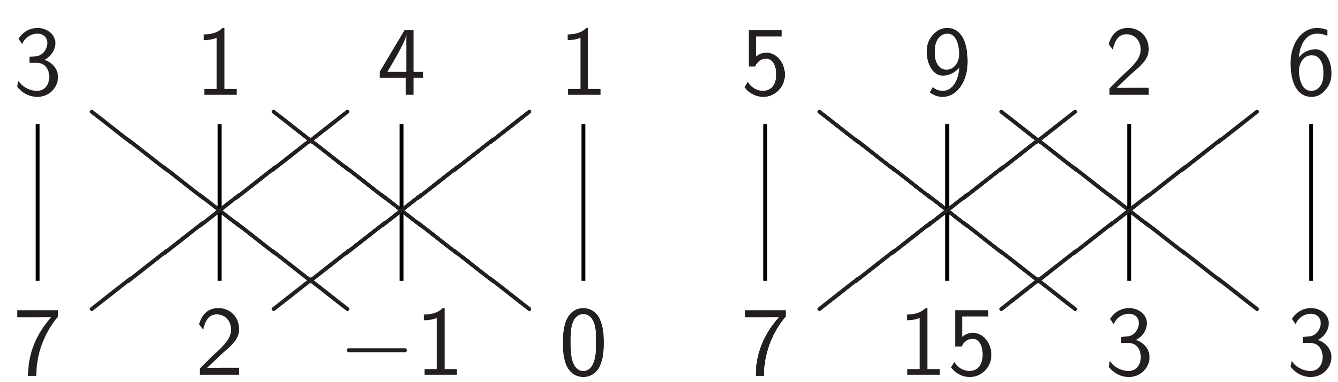

Some Hadamard applications

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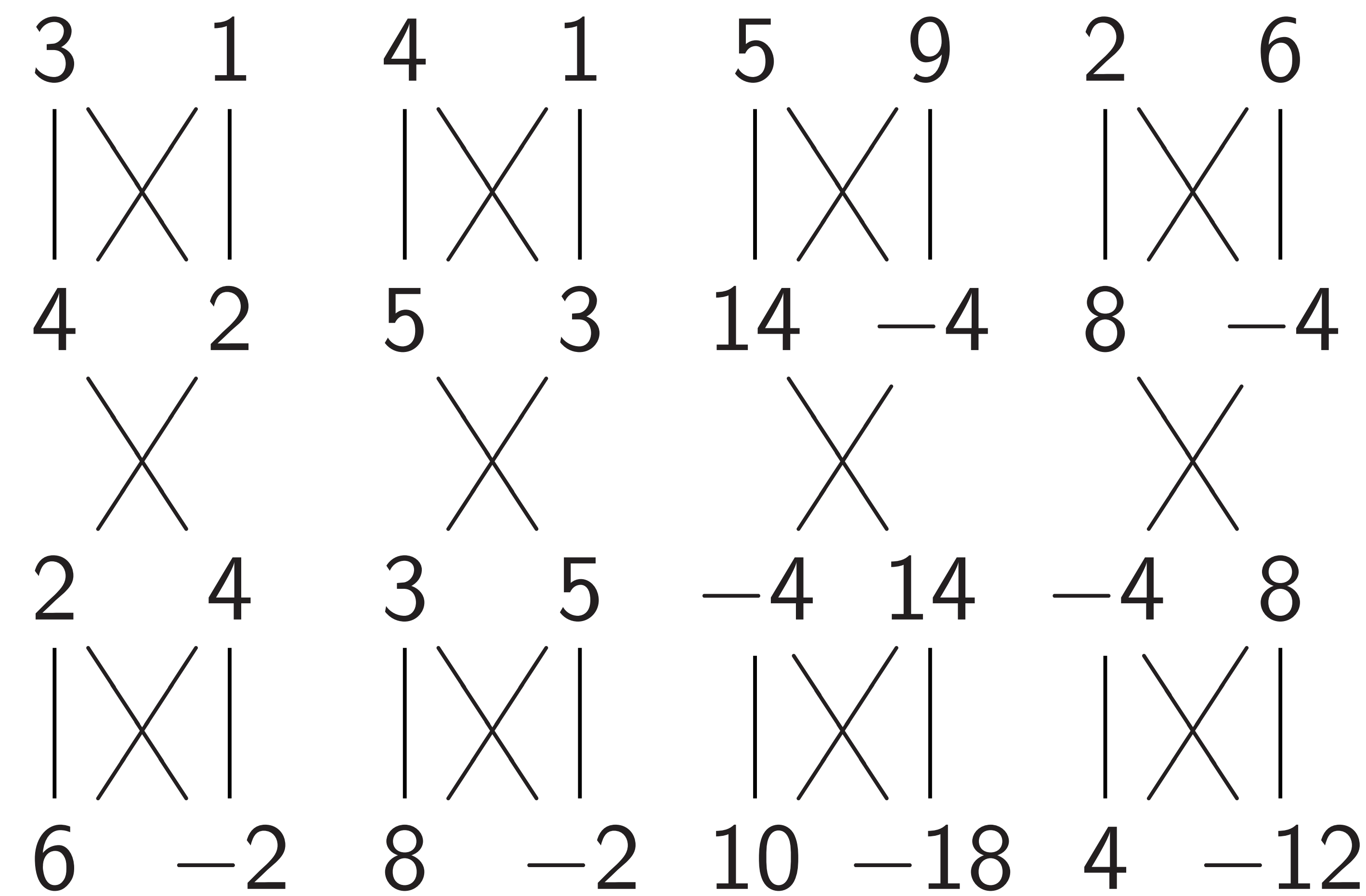

"Multiply each amplitude by 2."

This is not physically observable.

Hadamard gates

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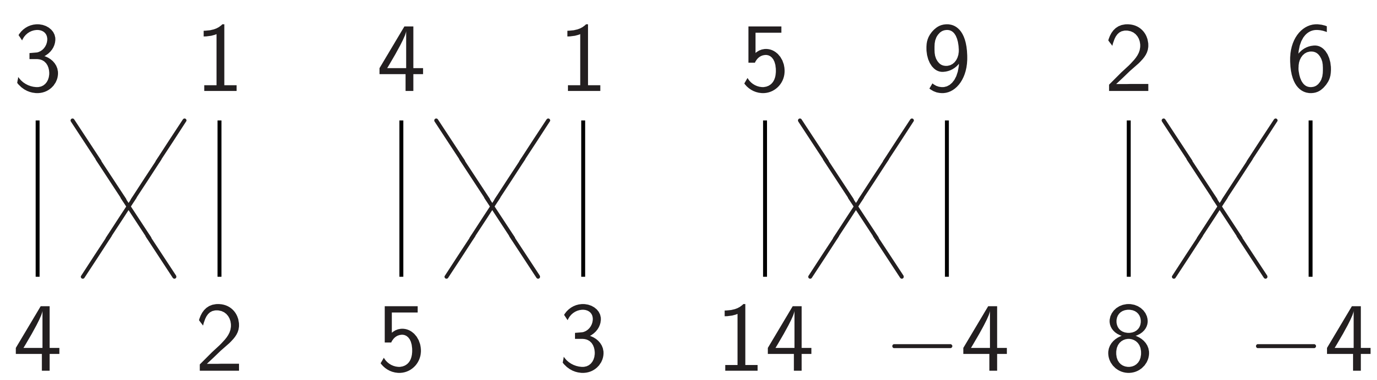

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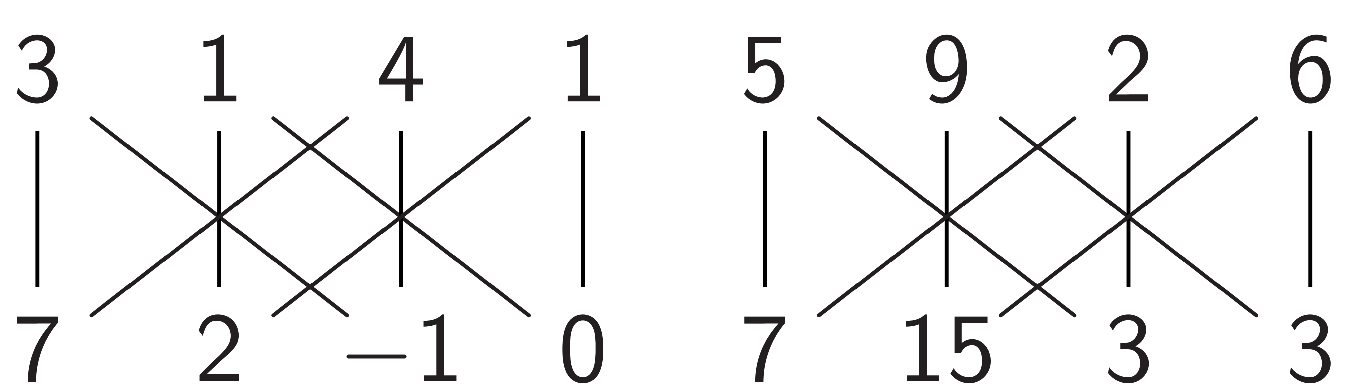

Some Hadamard applications

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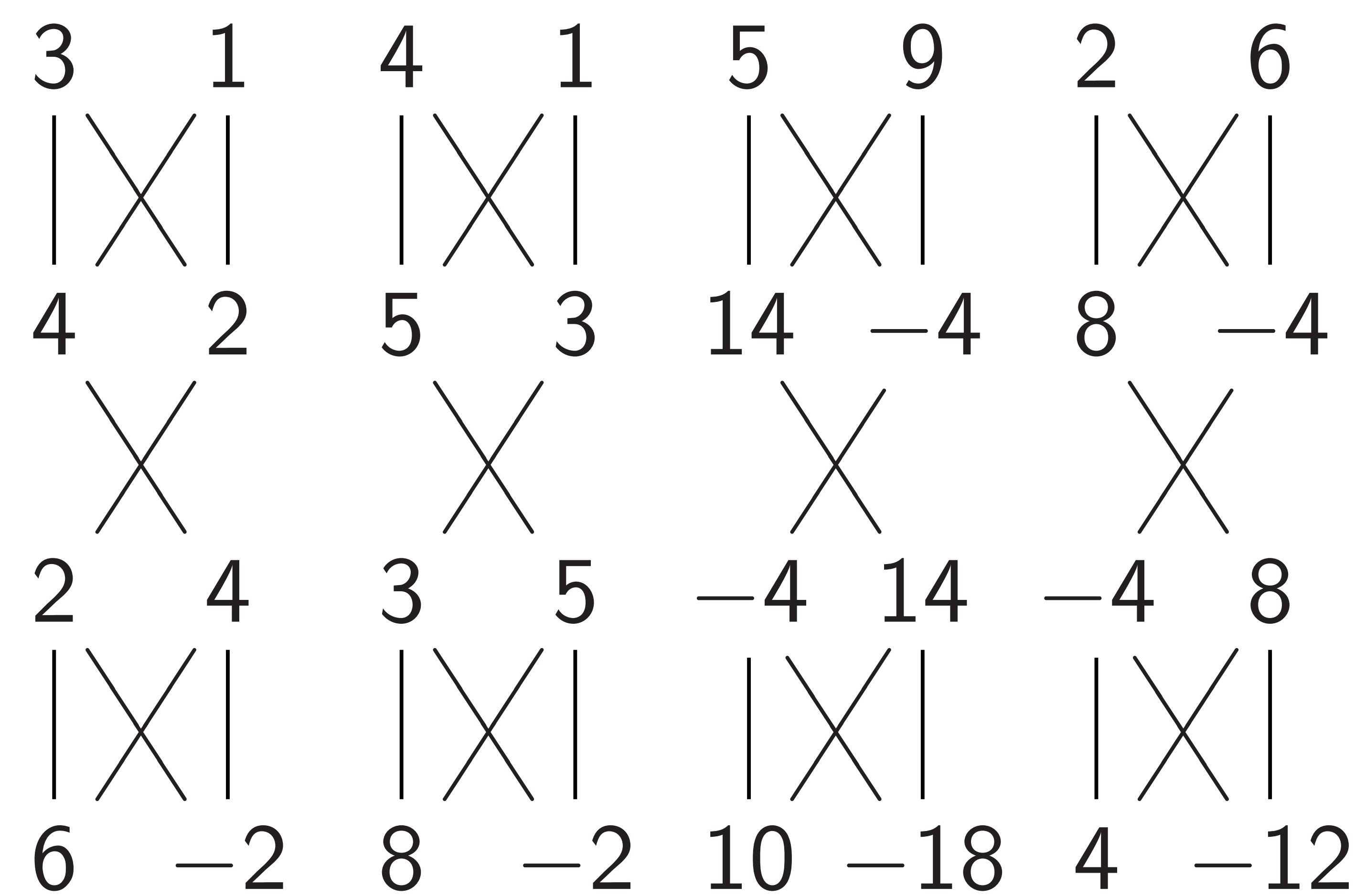


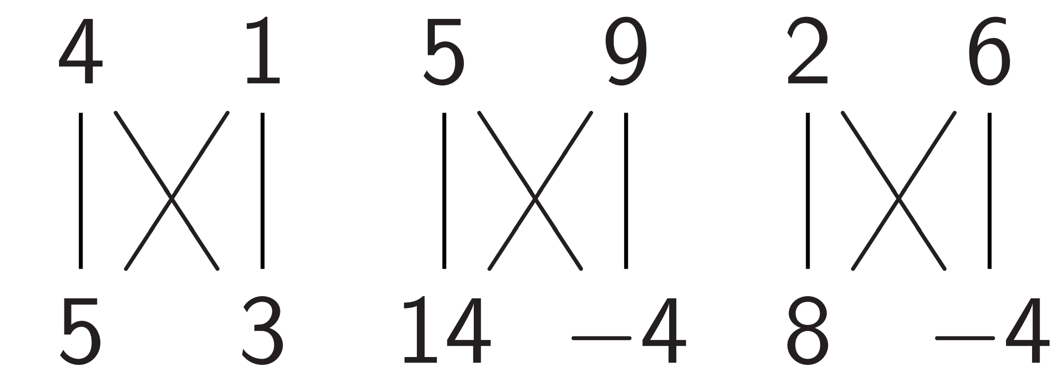
"Multiply each amplitude by 2."
This is not physically observable.

"Negate amplitude if q_0 is set."
No effect on measuring *now*.

rd gates

rd$_0$:

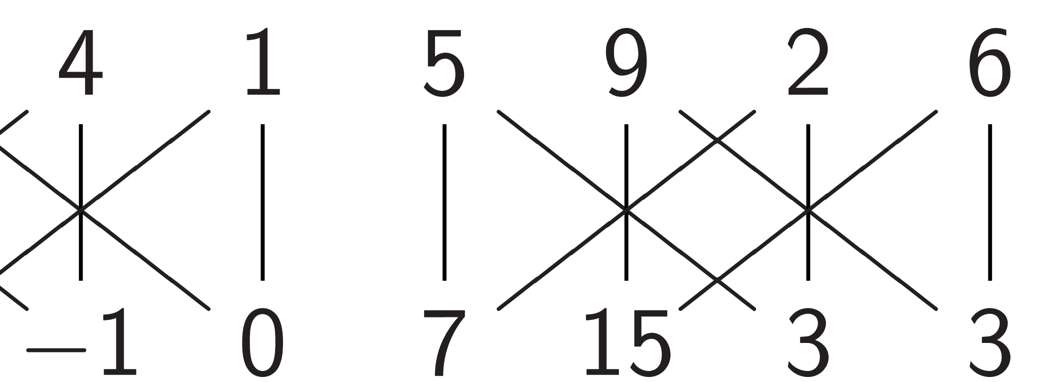
$\mapsto (a + b, a - b)$.



rd$_1$:

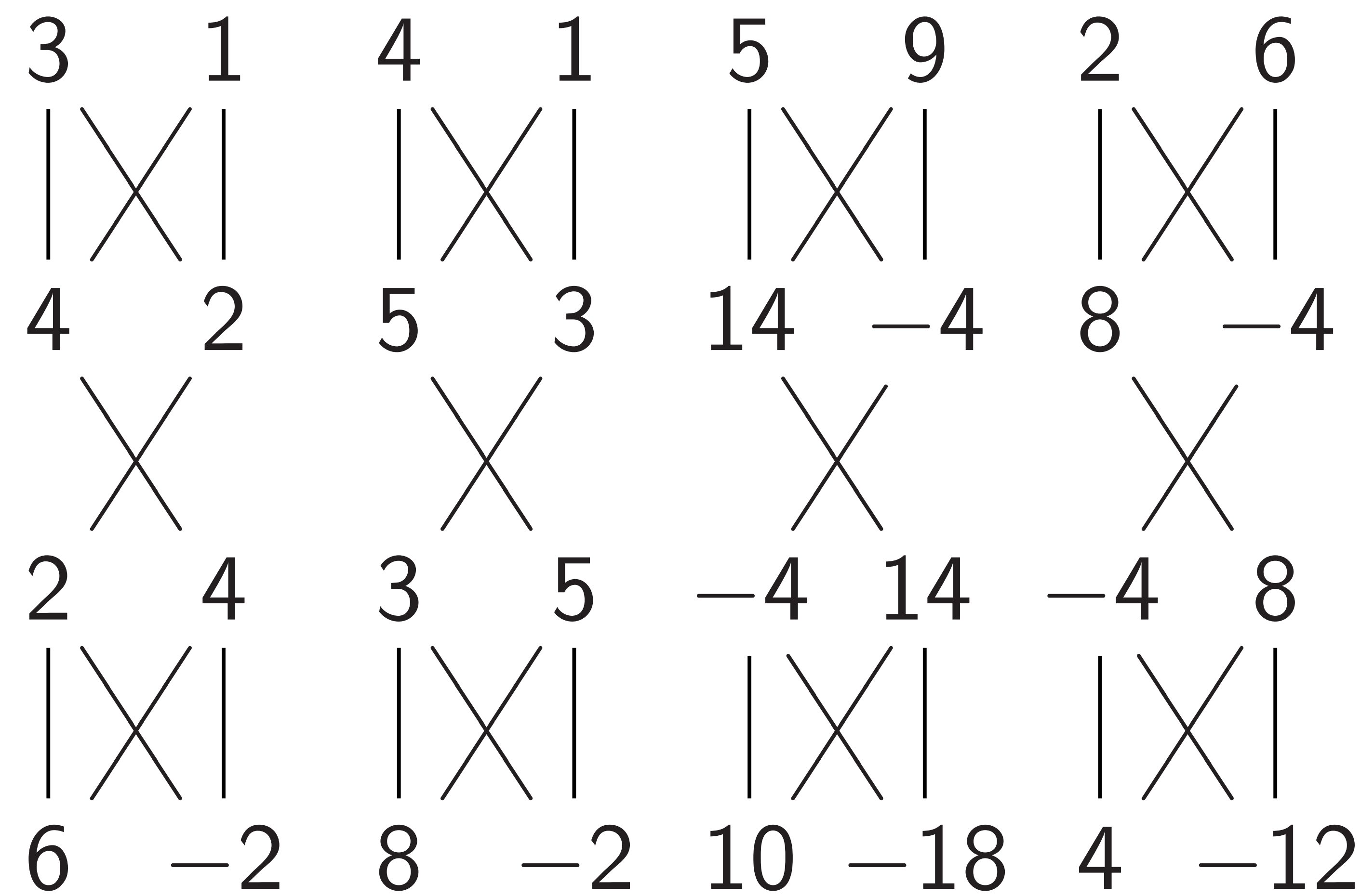
$d) \mapsto$

$b + d, a - c, b - d)$.



Some Hadamard applications

Hadamard$_0$, NOT$_0$, Hadamard$_0$:



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Fancier

"Negate

Assumes

C_0C_1NO

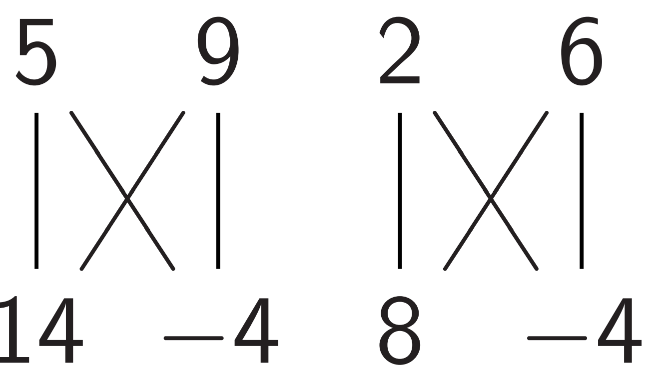
Hadama

NOT

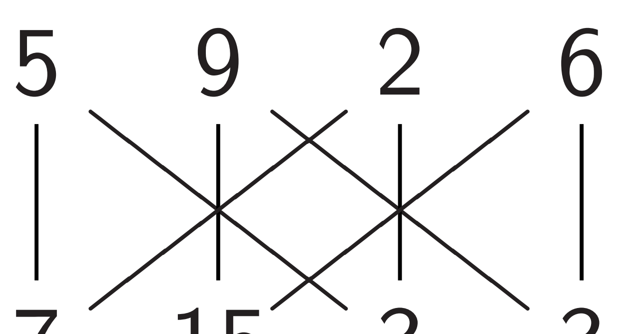
Hadama

C_0C_1NO

$- b)$.

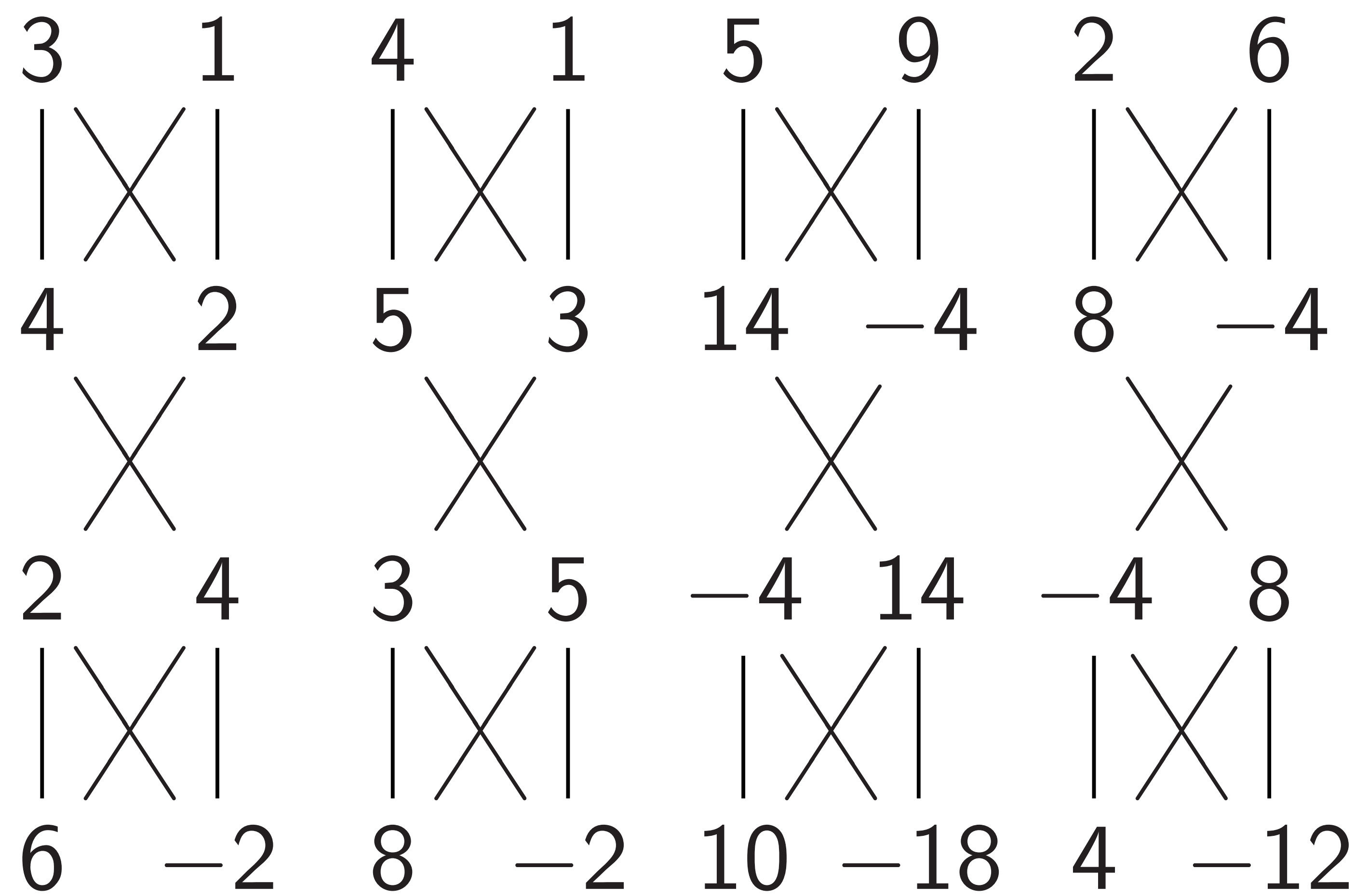


$c, b - d)$.



Some Hadamard applications

Hadamard_0, NOT_0, Hadamard_0:



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Fancier example:

"Negate amplitud

Assumes $q_2 = 0$:

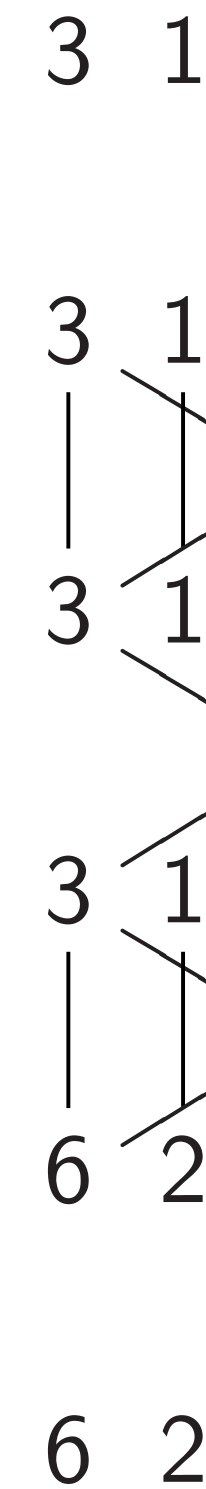
$\text{C}_0\text{C}_1\text{NOT}_2$

Hadamard_2

NOT_2

Hadamard_2

$\text{C}_0\text{C}_1\text{NOT}_2$



Some Hadamard applications

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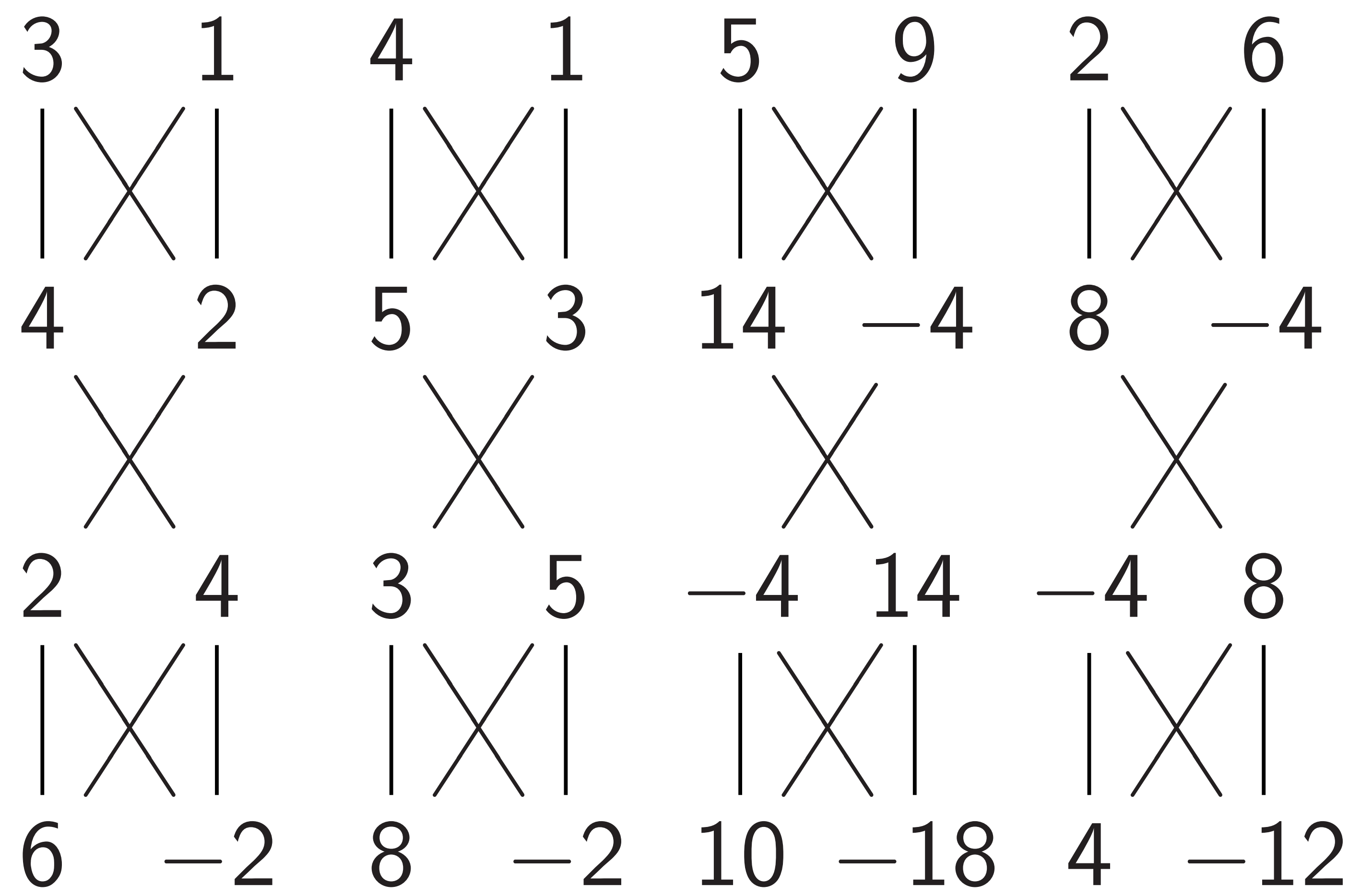

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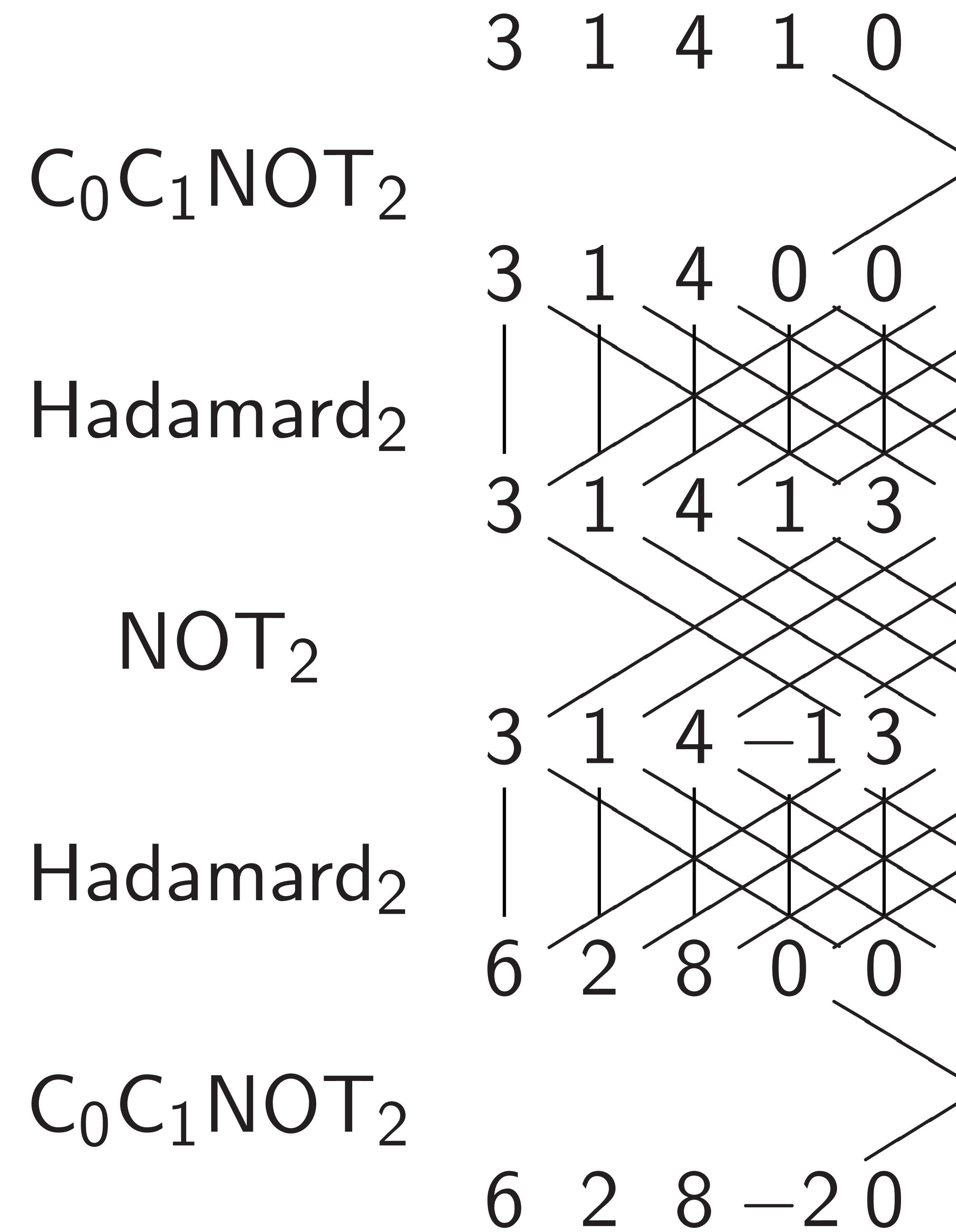
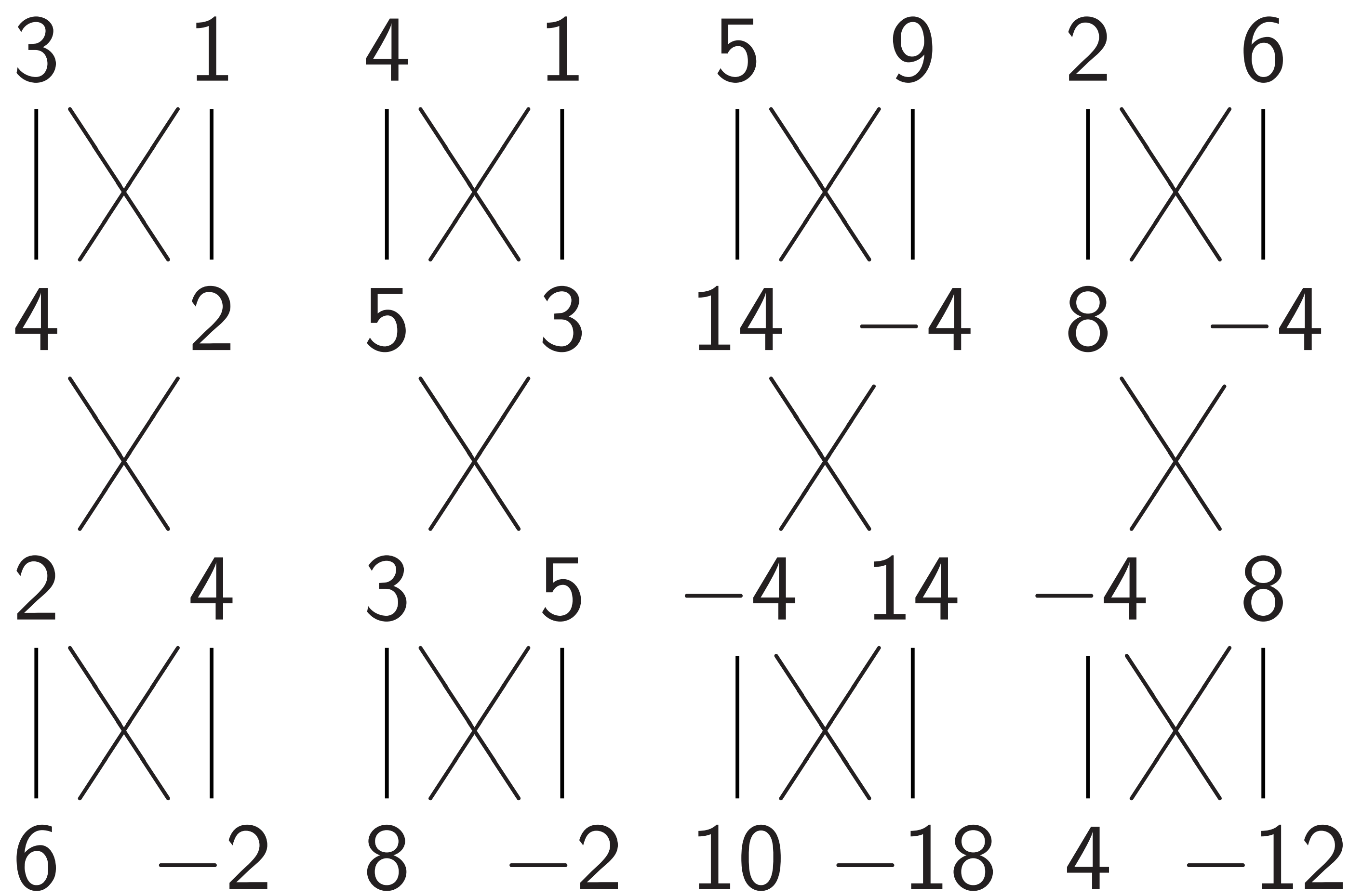


Some Hadamard applications

Hadamard_0, NOT_0, Hadamard_0:



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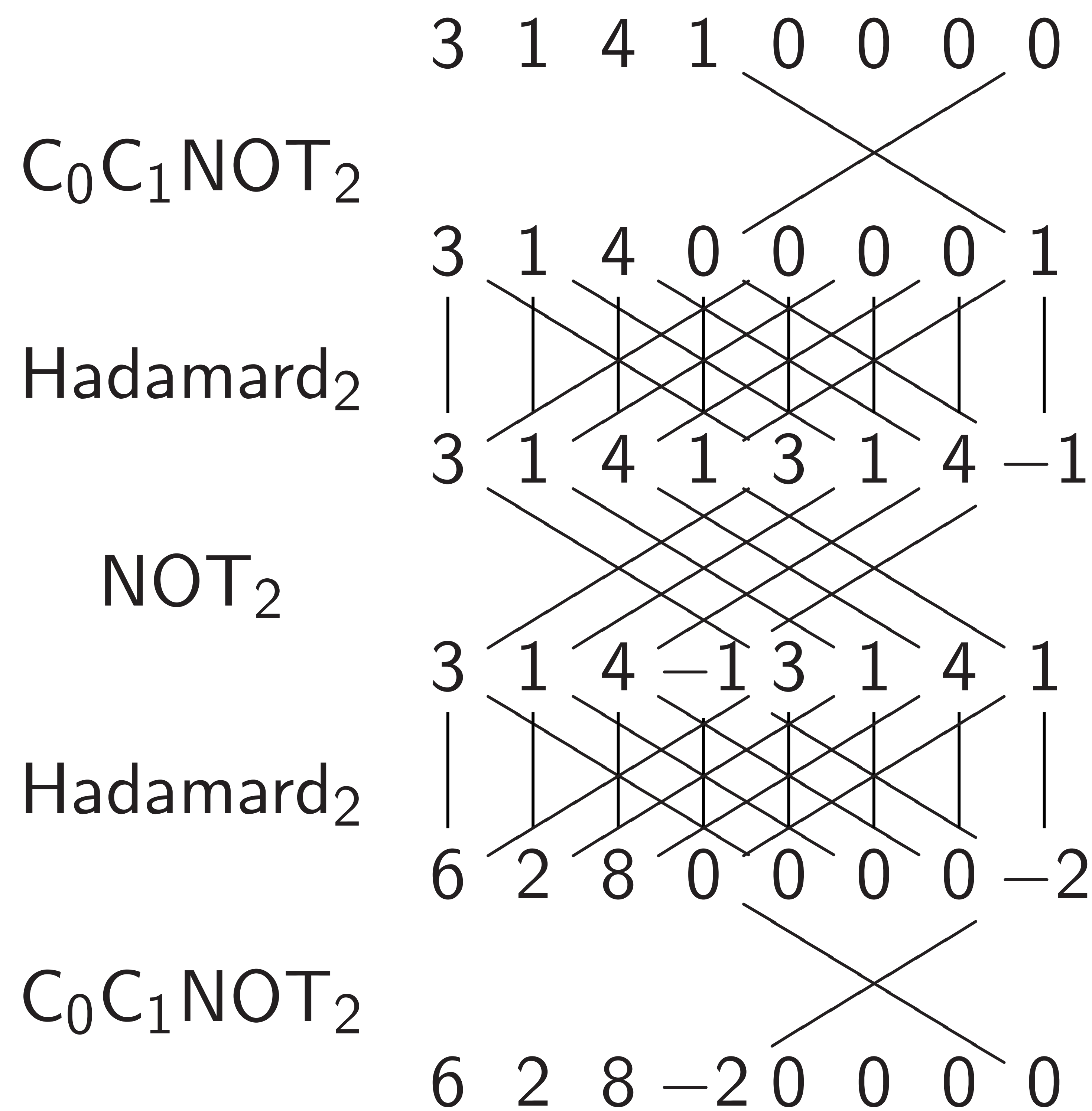
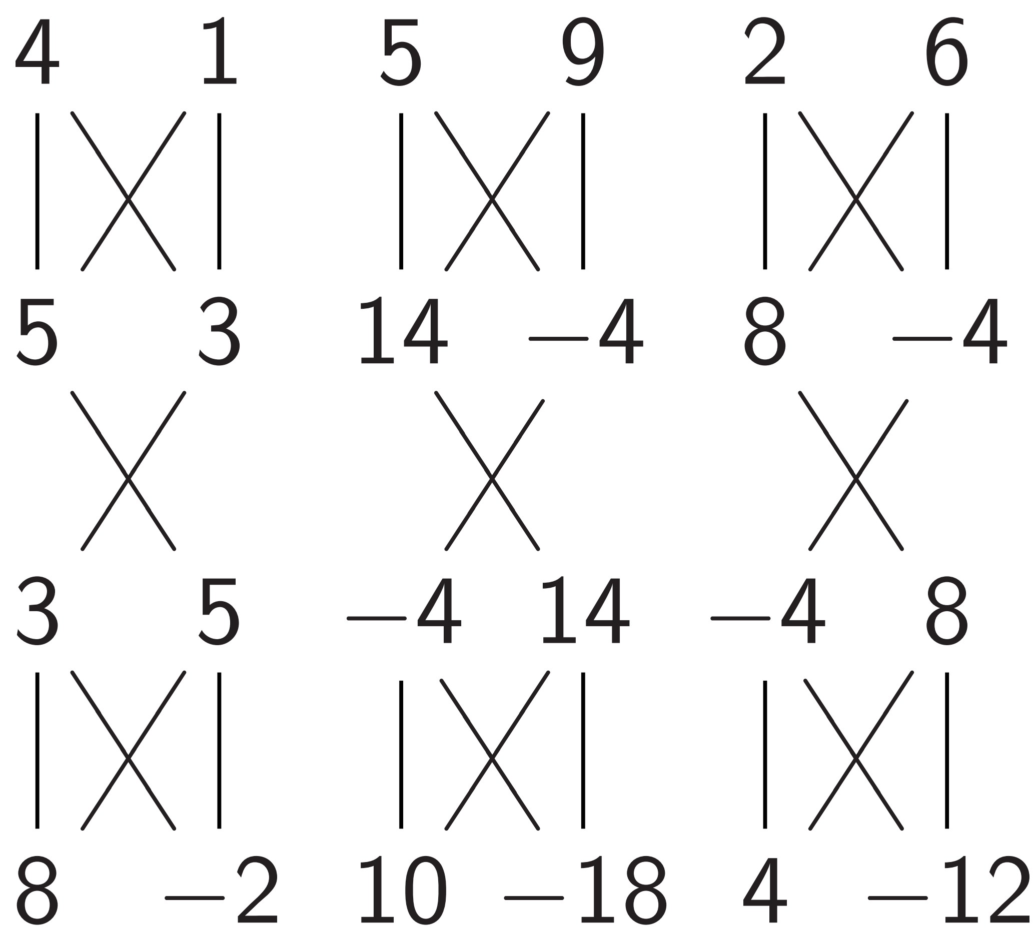


adamard applications

rd$_0$, NOT$_0$, Hadamard$_0$:



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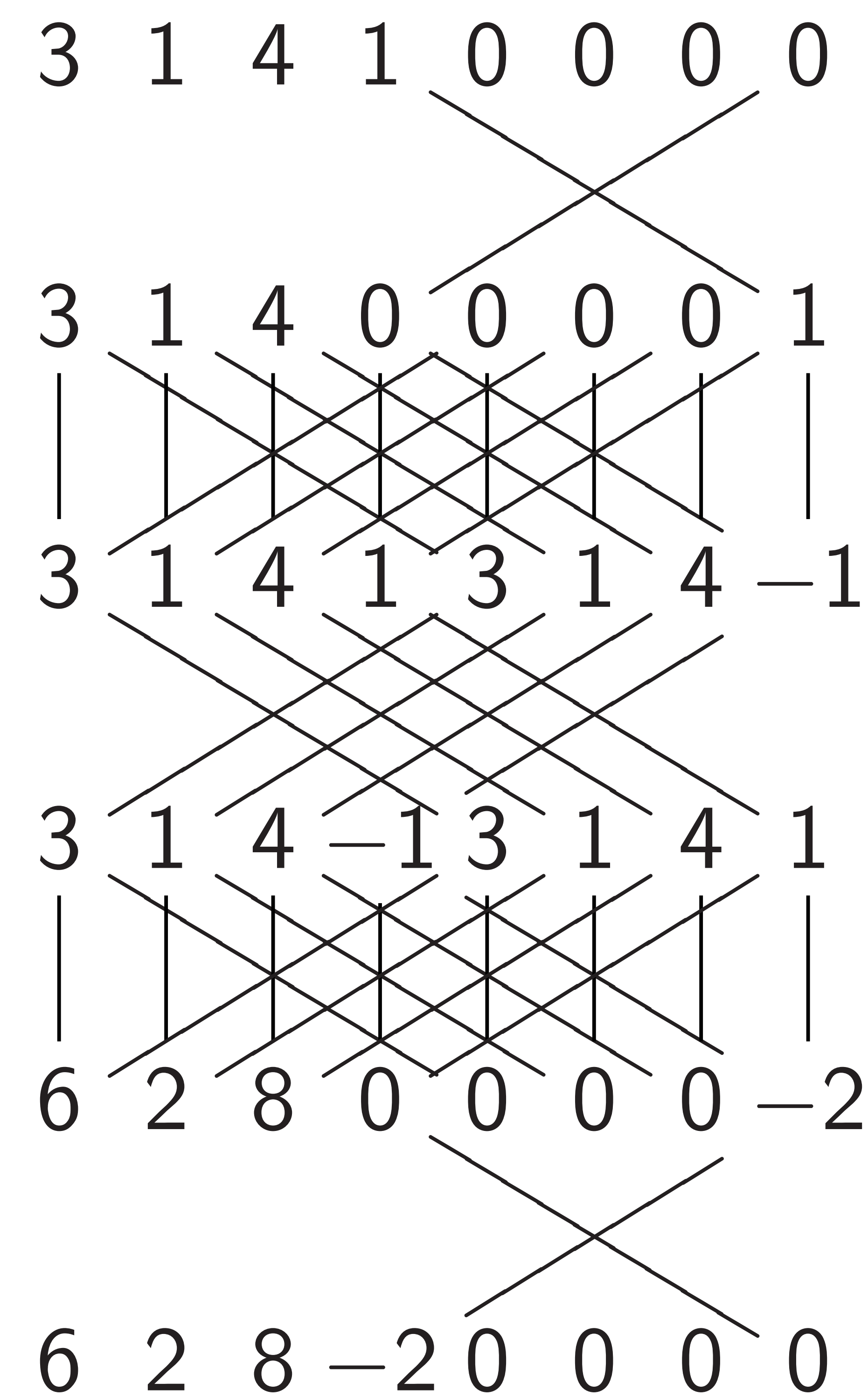

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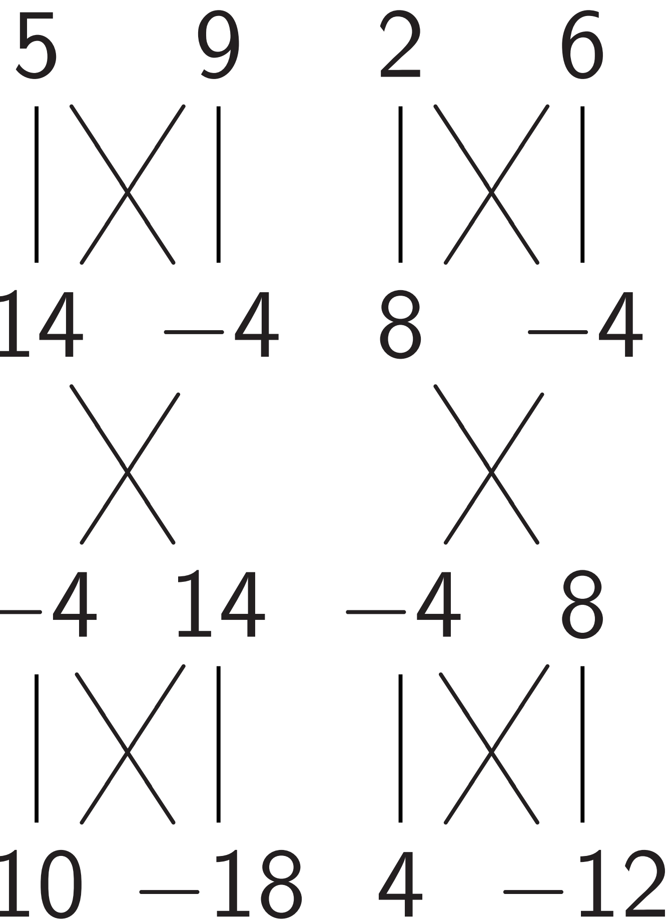
applications

$_0$, Hadamard$_0$:

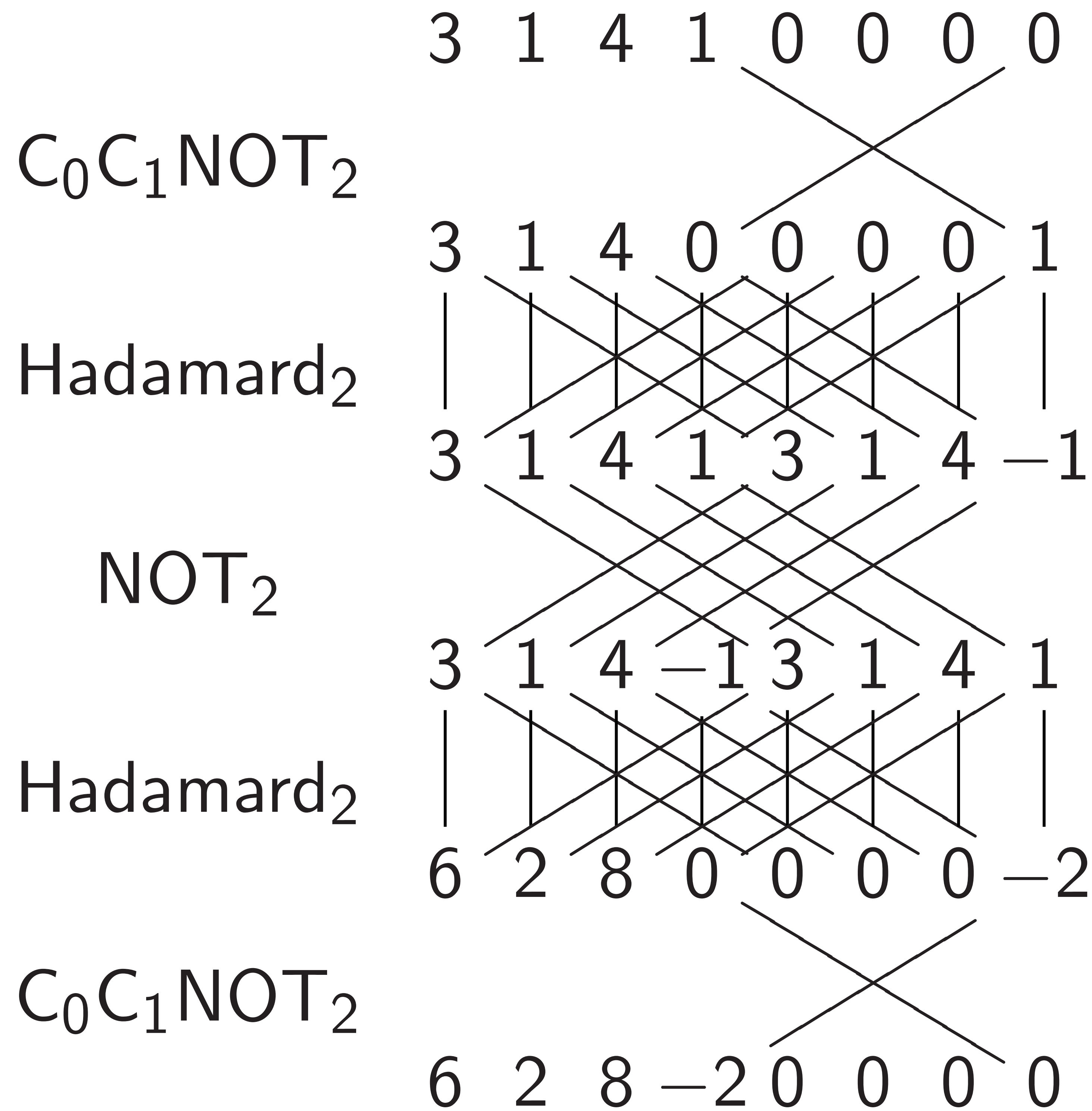


nplitude by 2.”
ally observable.

e if q_0 is set.”
uring *now*.

Fancier example:

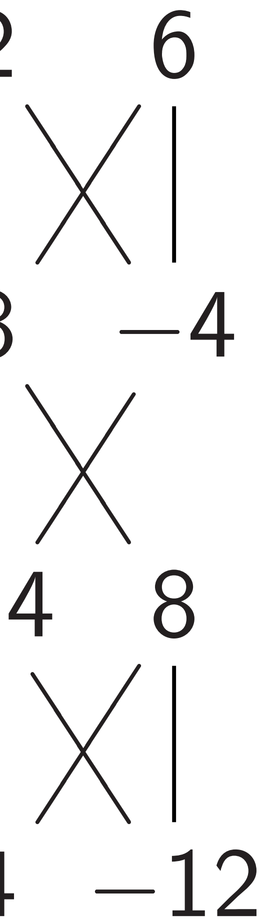
“Negate amplitude if $q_0 q_1$ is set.”
Assumes $q_2 = 0$: “ancilla” qubit.



Affects measureme
amplitude around
$(3, 1, 4, 1) \mapsto (1.5,$

s

ard$_0$:



2.”

able.

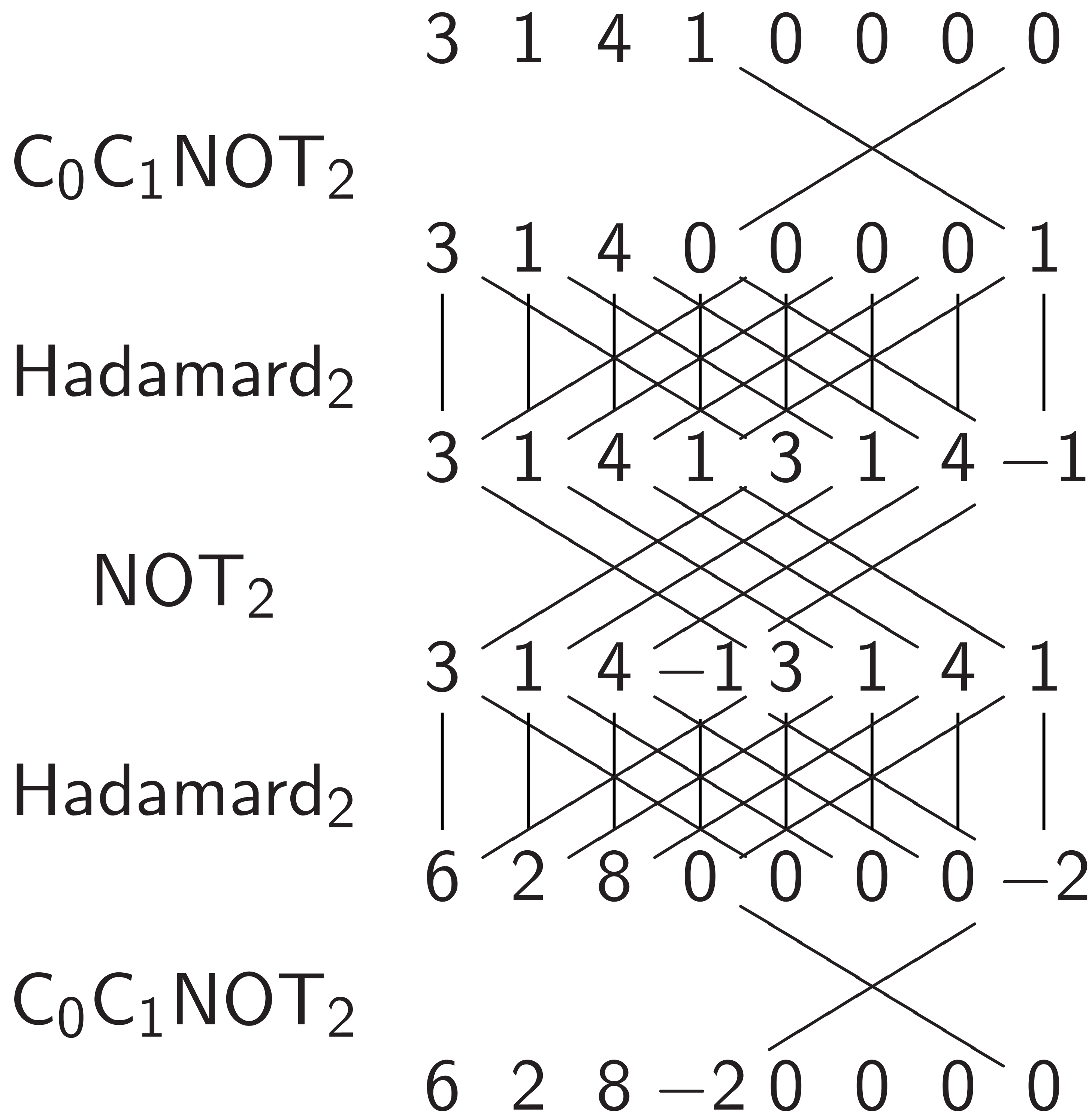
et.”

.

Fancier example:

“Negate amplitude if $q_0 q_1$ is set.”

Assumes $q_2 = 0$: “ancilla” qubit.



Affects measurements: “Neg

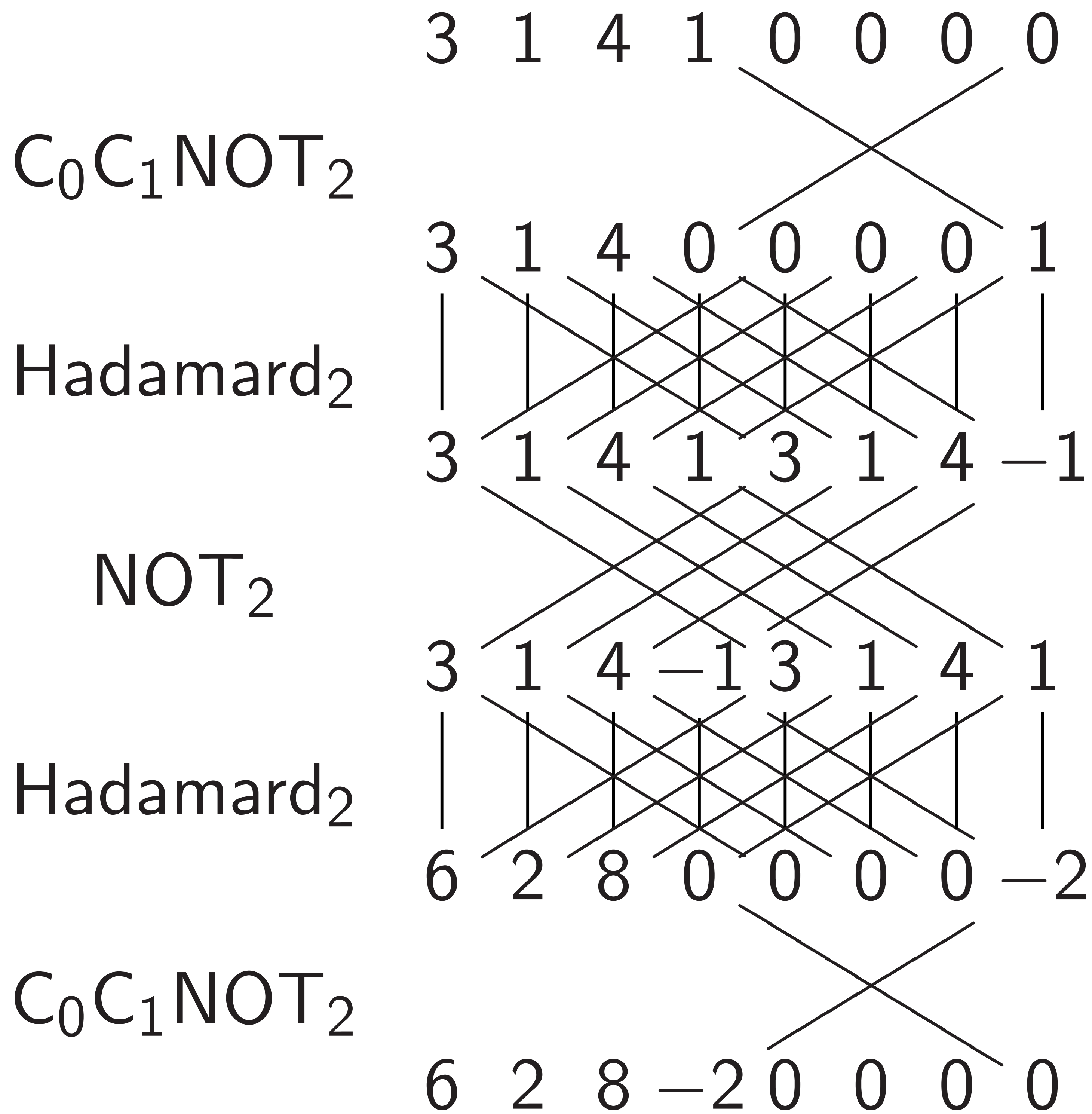
amplitude around its averag

$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3$

Fancier example:

"Negate amplitude if $q_0 q_1$ is set."

Assumes $q_2 = 0$: "ancilla" qubit.



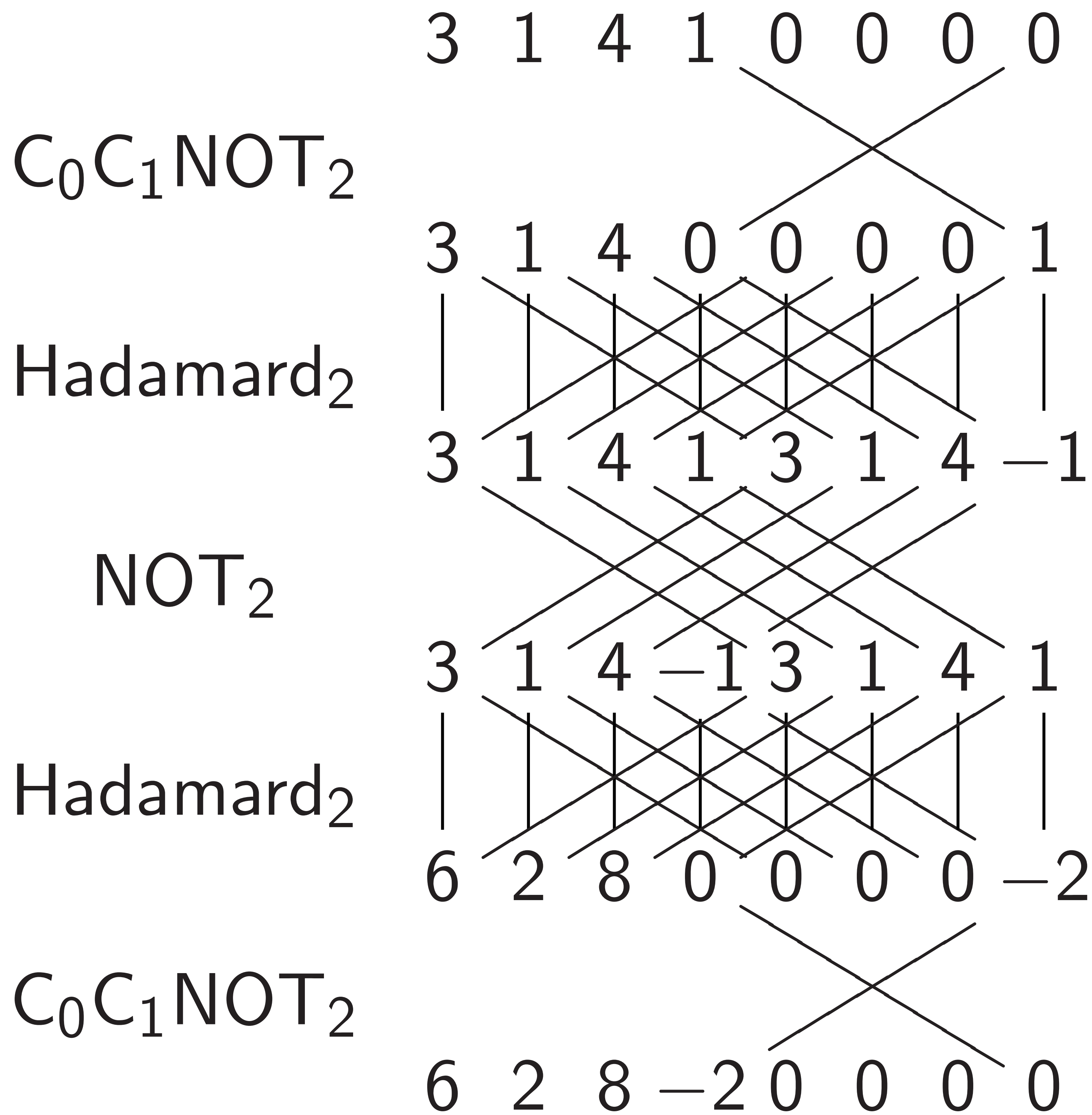
Affects measurements: "Negate amplitude around its average."

$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.

Fancier example:

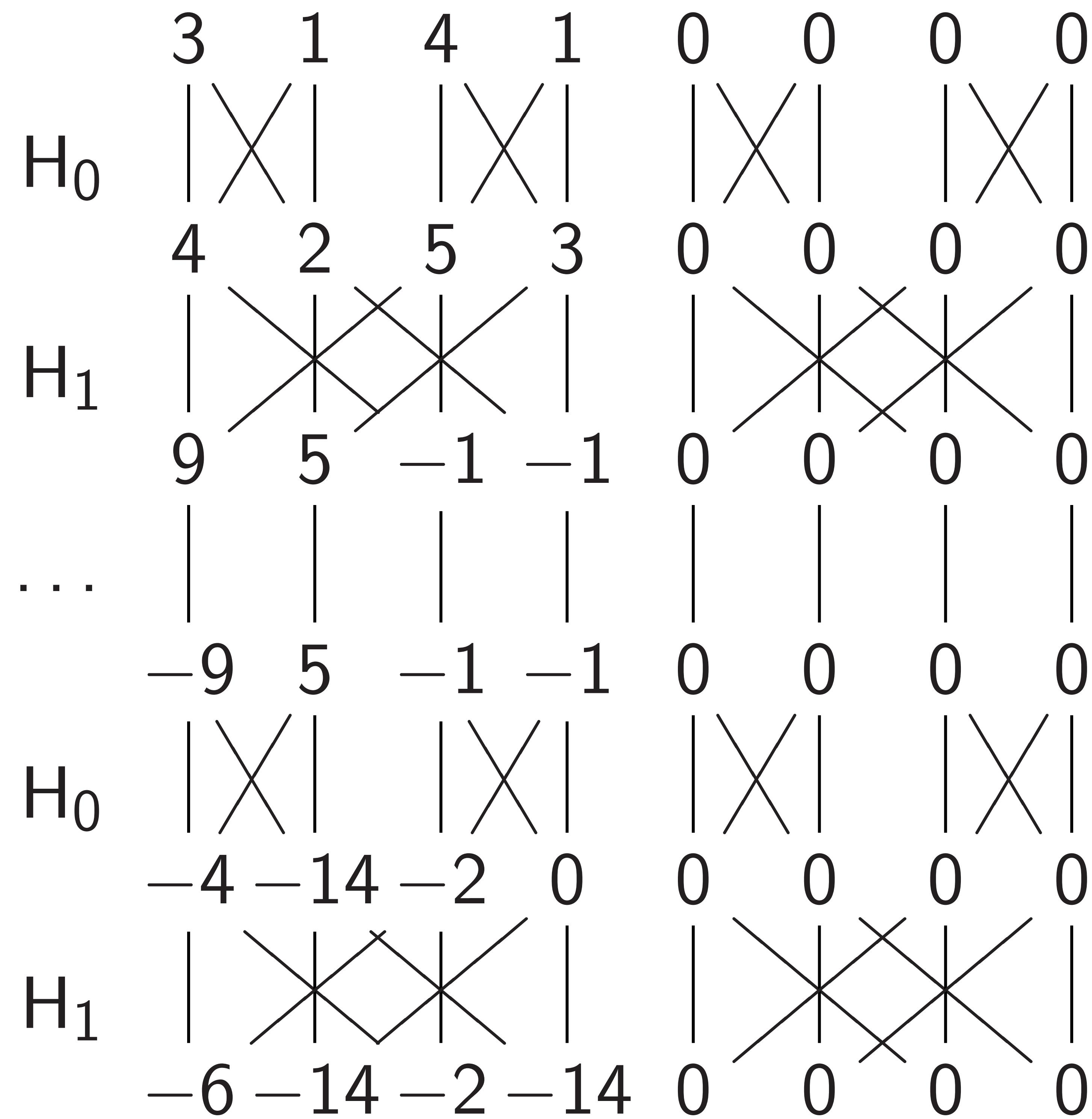
"Negate amplitude if $q_0 q_1$ is set."

Assumes $q_2 = 0$: "ancilla" qubit.



Affects measurements: "Negate amplitude around its average."

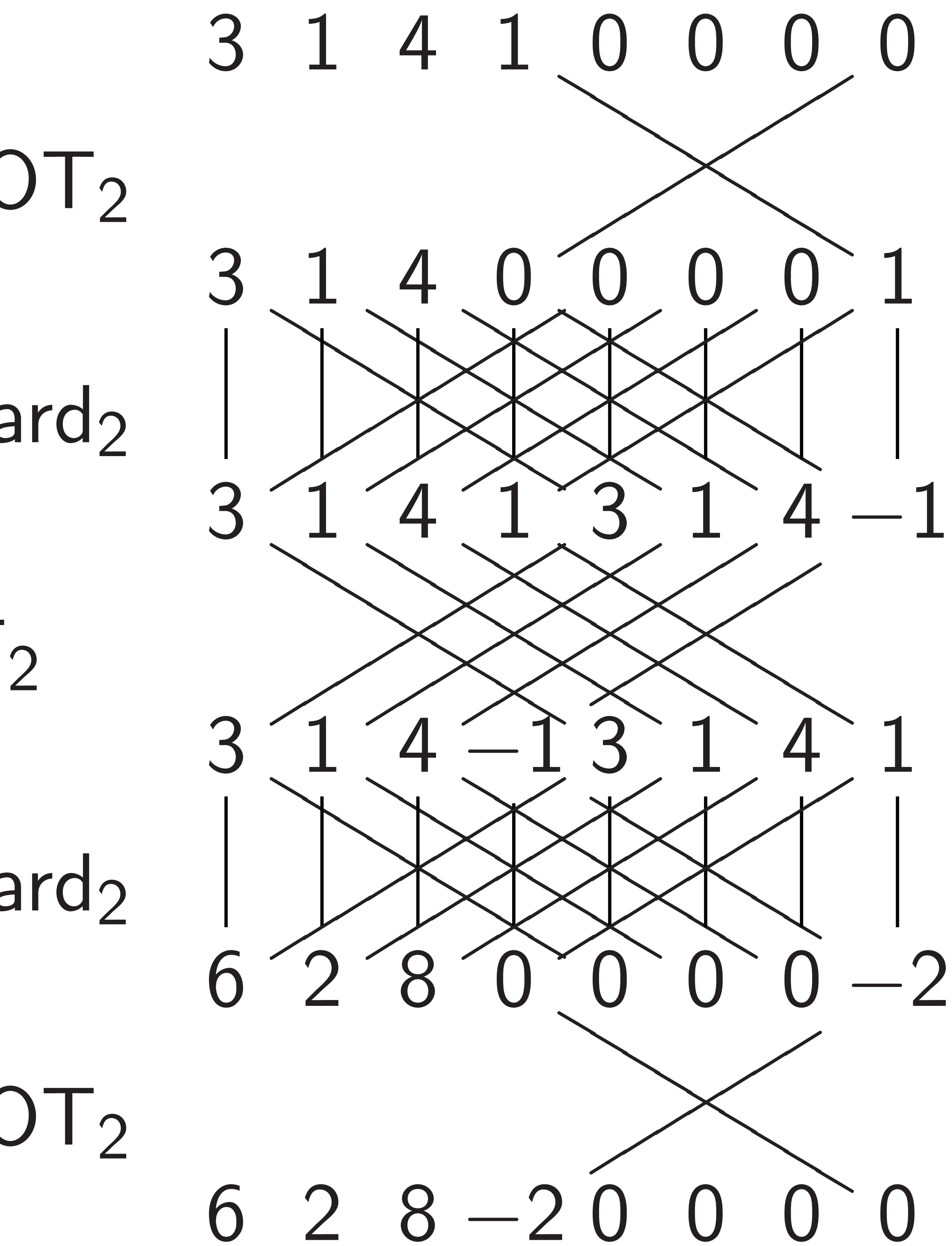
$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.



example:

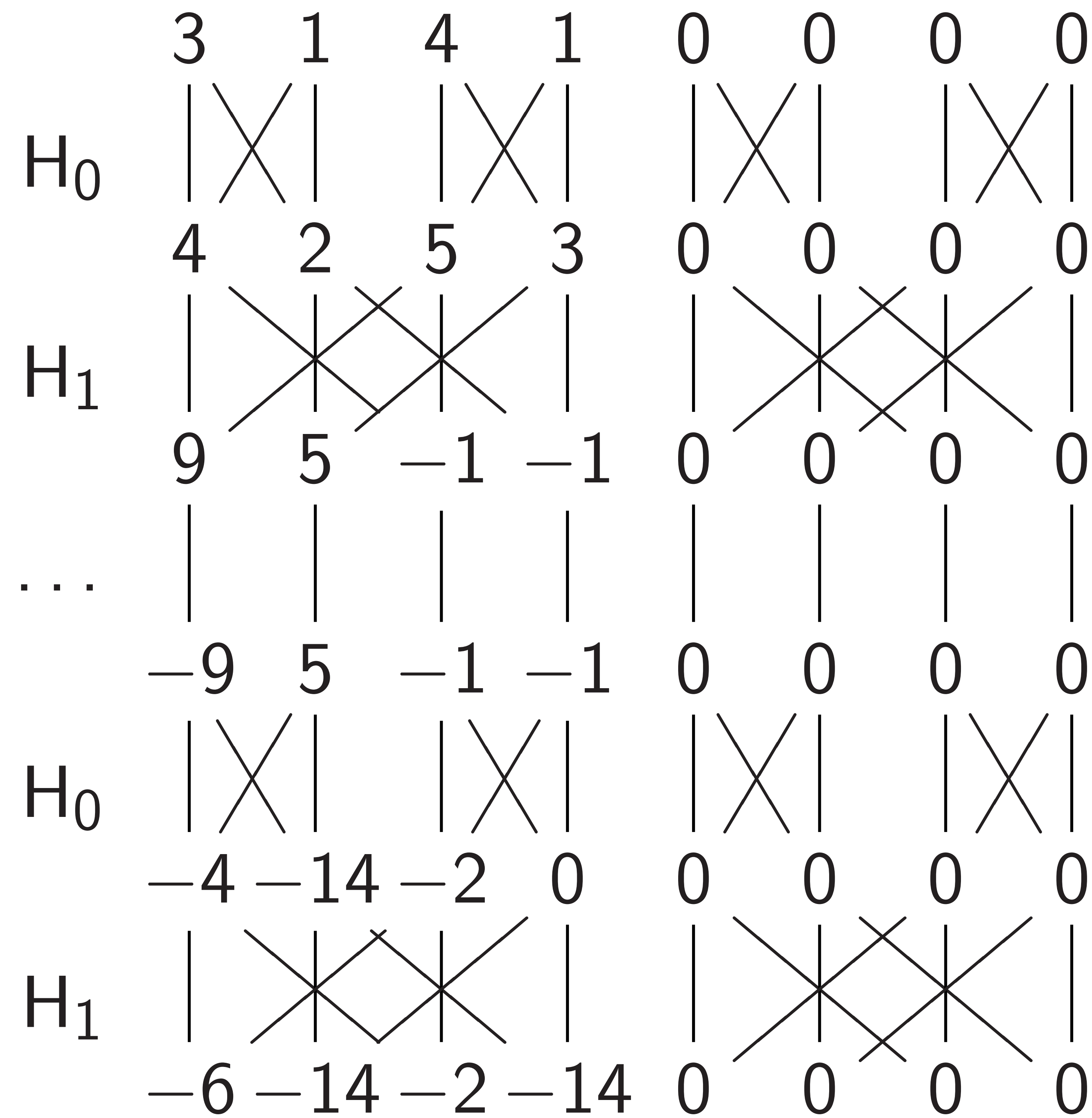
amplitude if $q_0 q_1$ is set."

s $q_2 = 0$: "ancilla" qubit.



Affects measurements: "Negate amplitude around its average."

$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.

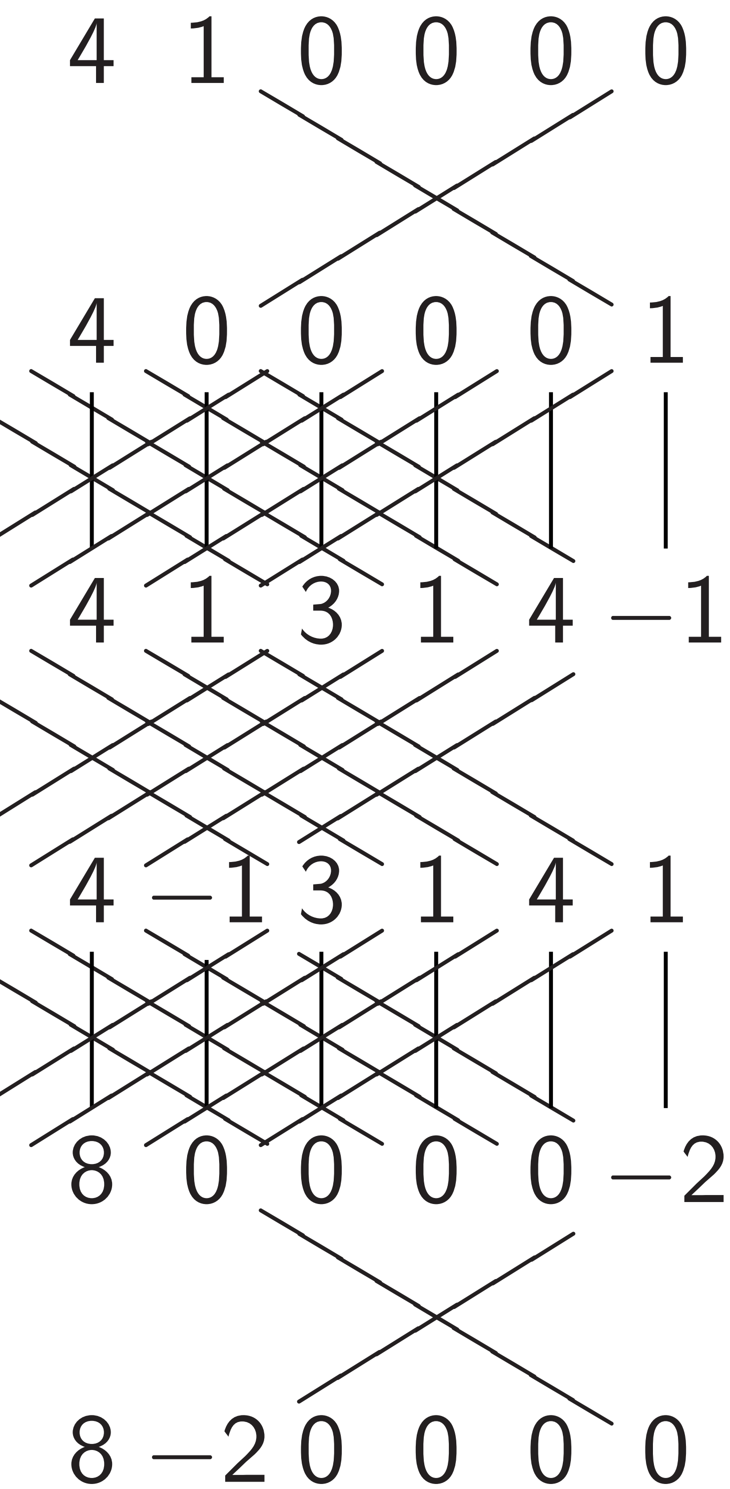


Simon's

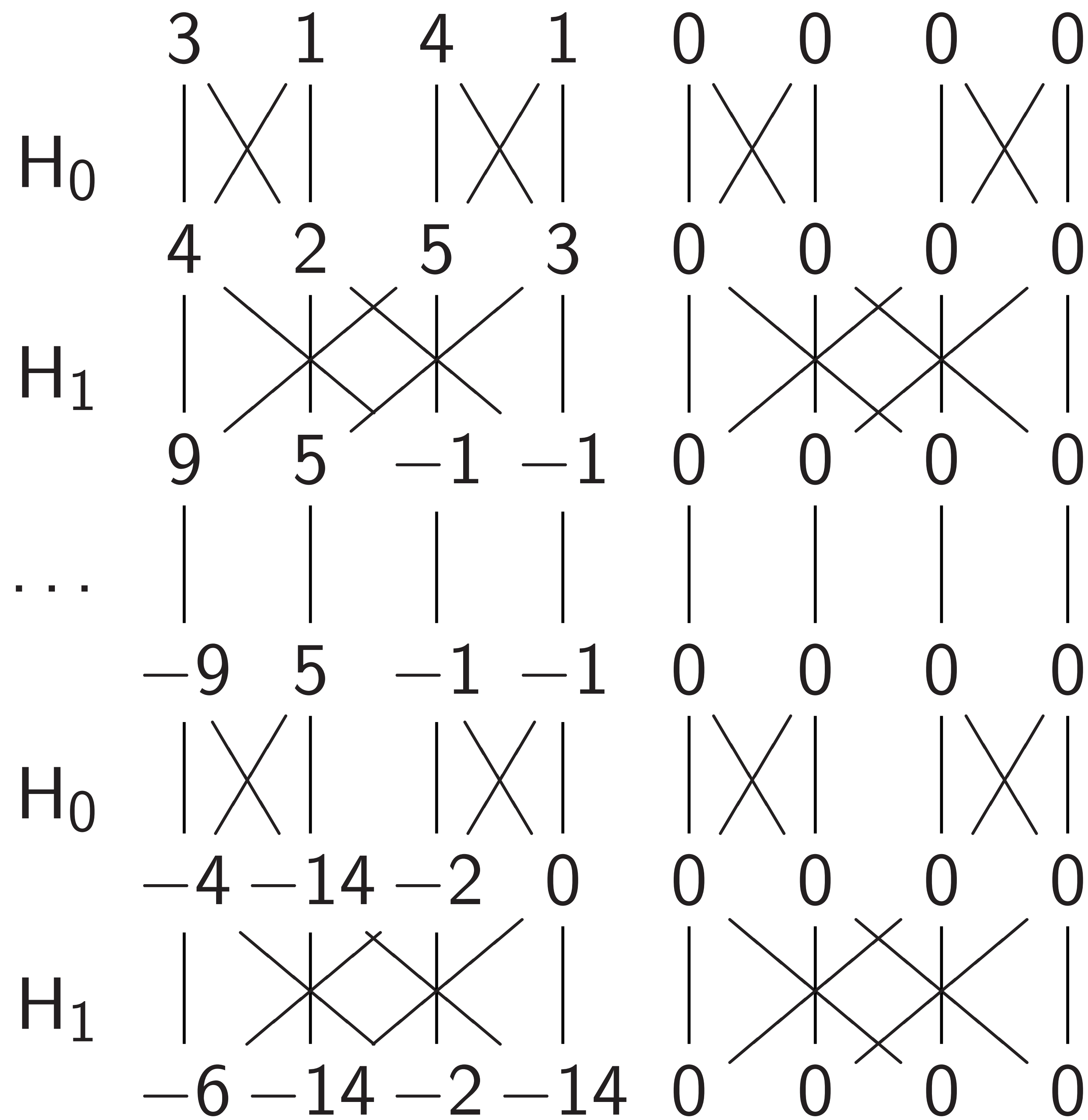
Step 1.

1, 0, 0,
0, 0, 0,
0, 0, 0,
0, 0, 0,
0, 0, 0,
0, 0, 0,
0, 0, 0,
0, 0, 0,

e if $q_0 q_1$ is set."
"ancilla" qubit.



Affects measurements: "Negate amplitude around its average."
$(3,1,4,1) \mapsto (1.5,3.5,0.5,3.5)$.



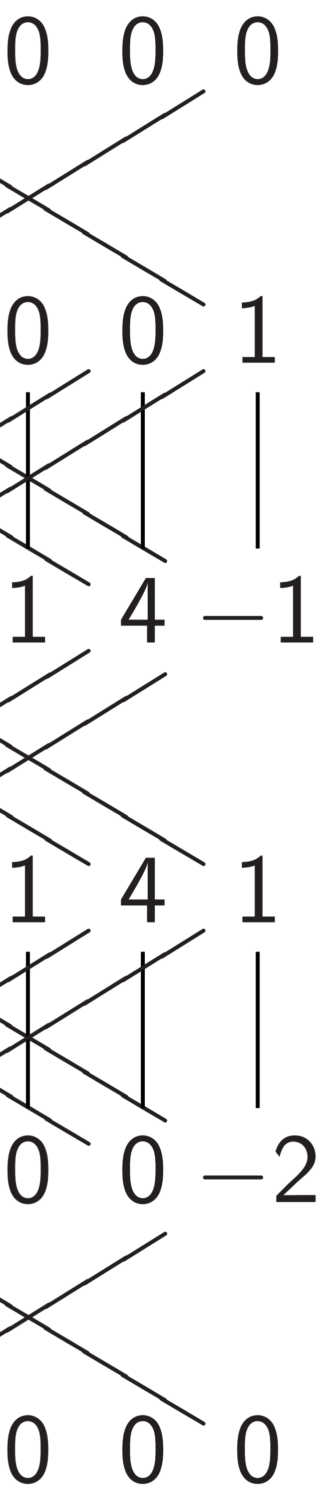
Simon's algorithm

Step 1. Set up pu

1, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0

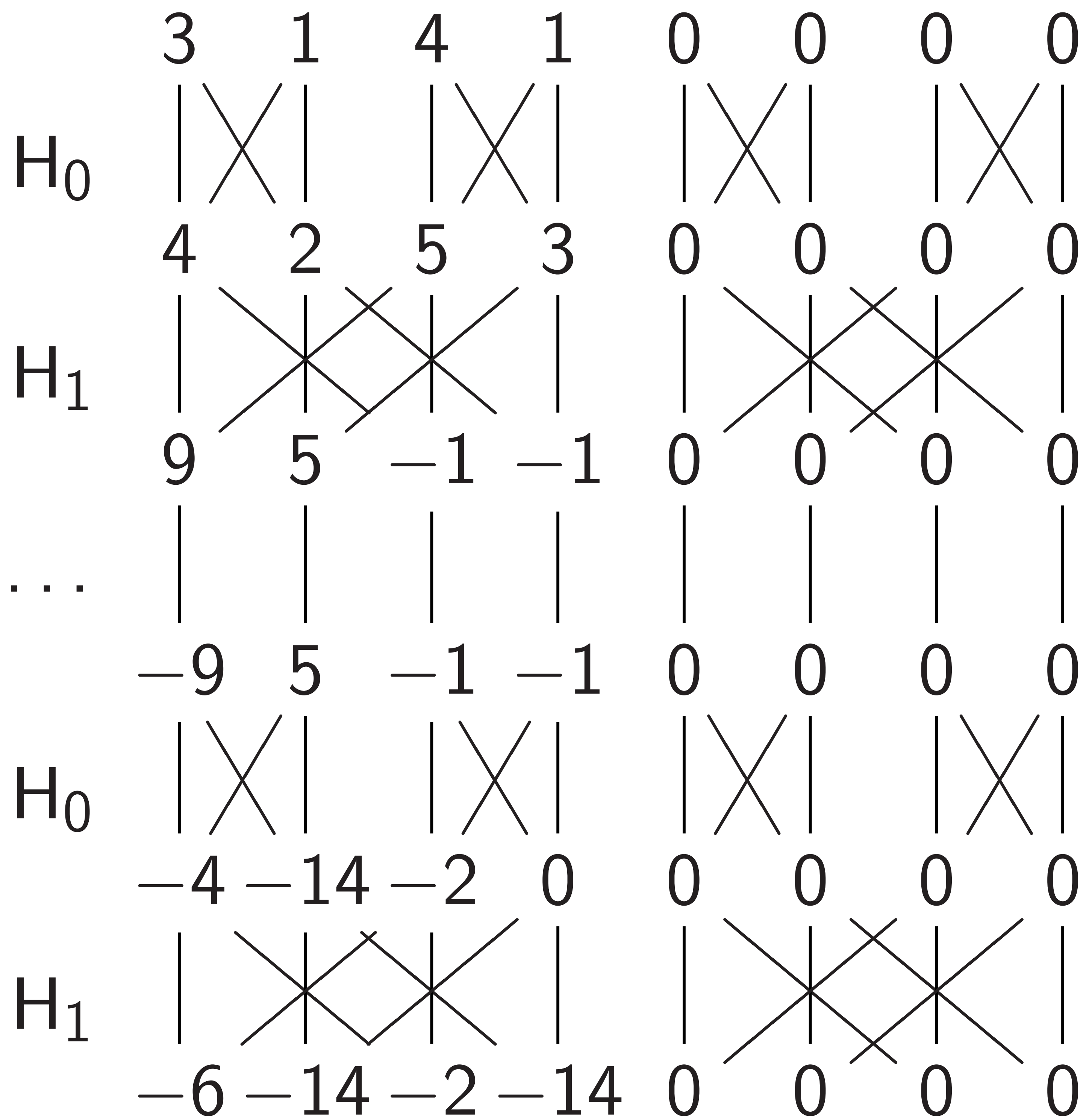
s set.”

qubit.



Affects measurements: “Negate amplitude around its average.”

$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.

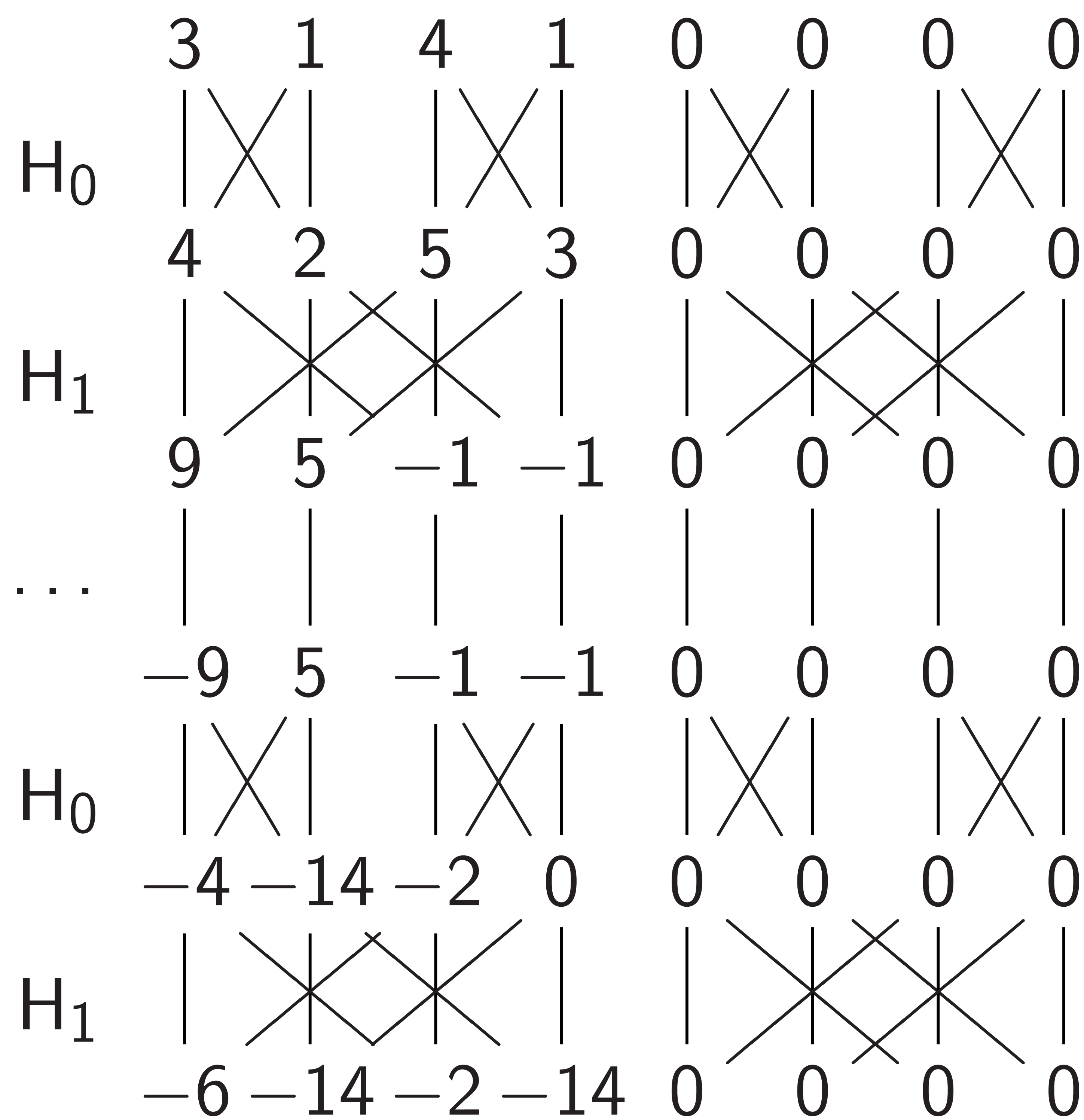


Simon’s algorithm

Step 1. Set up pure zero sta

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Affects measurements: "Negate amplitude around its average."
$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.

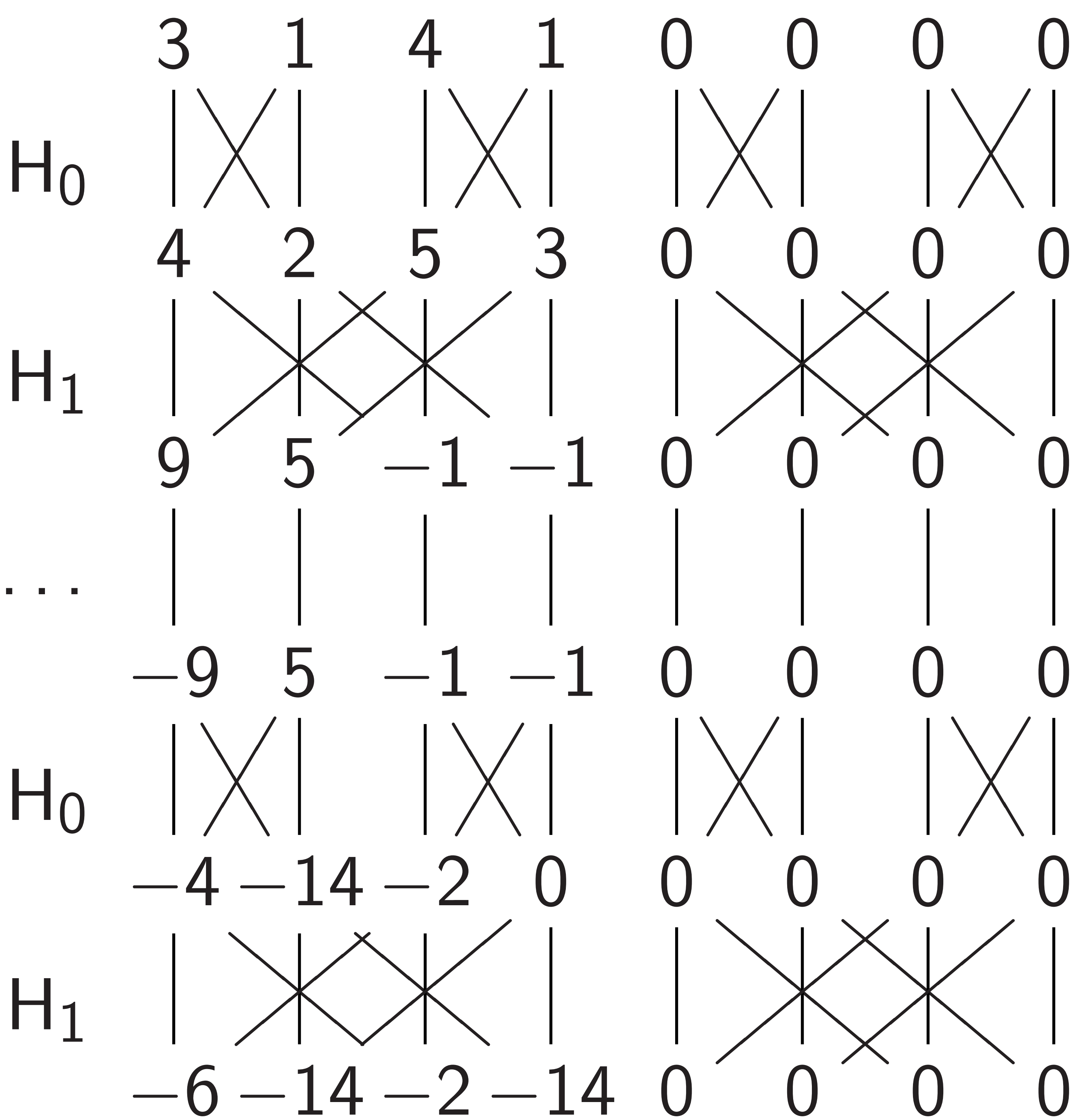


Simon's algorithm

Step 1. Set up pure zero state:

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Affects measurements: “Negate amplitude around its average.”
$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.

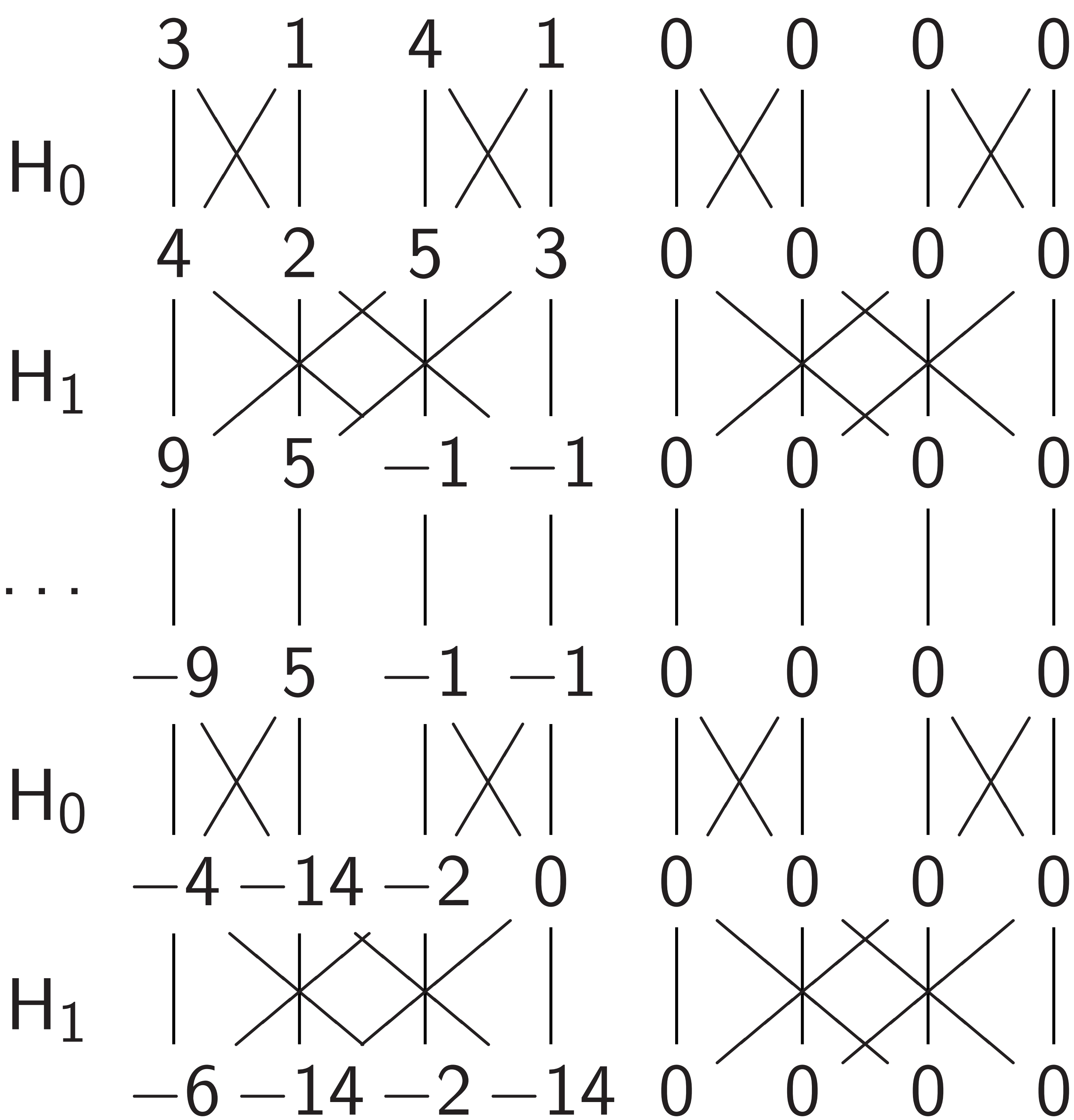


Simon’s algorithm

Step 2. Hadamard_0:

1, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Affects measurements: "Negate amplitude around its average."
$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.

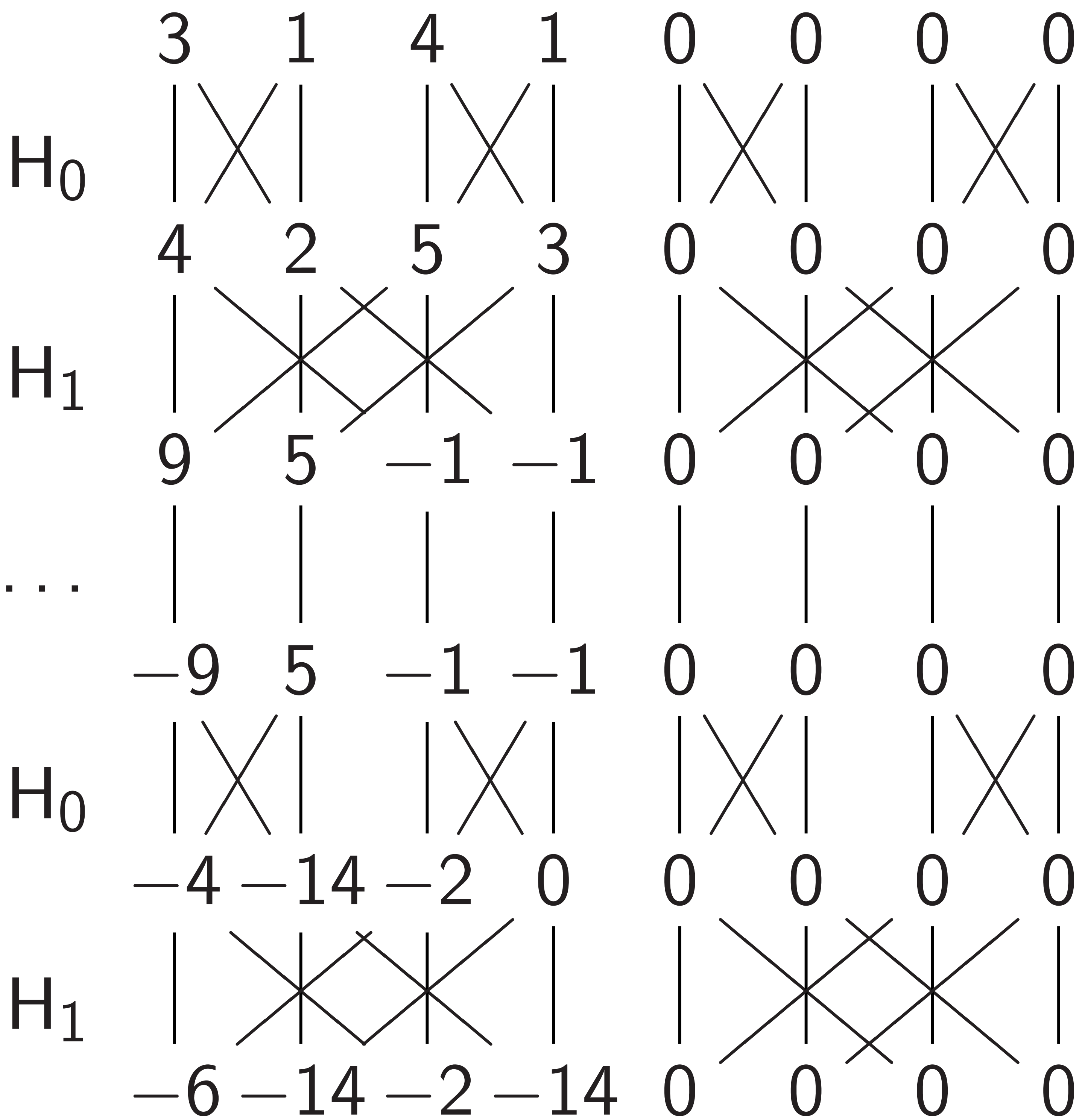


Simon's algorithm

Step 3. Hadamard$_1$:

1, 1, 1, 1, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Affects measurements: "Negate amplitude around its average."
$(3,1,4,1) \mapsto (1.5,3.5,0.5,3.5)$.



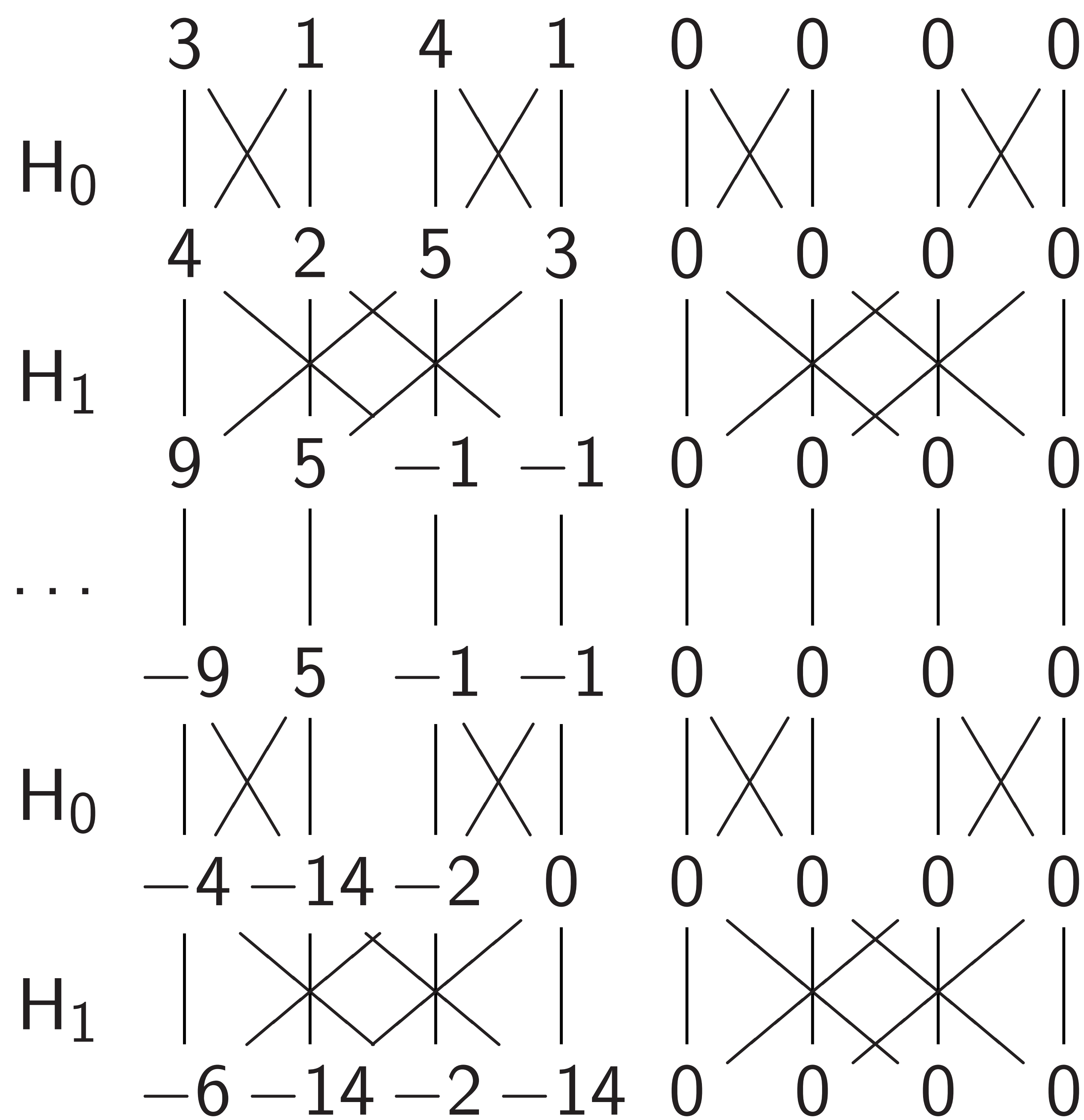
Simon's algorithm

Step 4. Hadamard$_2$:

1, 1, 1, 1, 1, 1, 1, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe.

Affects measurements: “Negate amplitude around its average.”
$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.



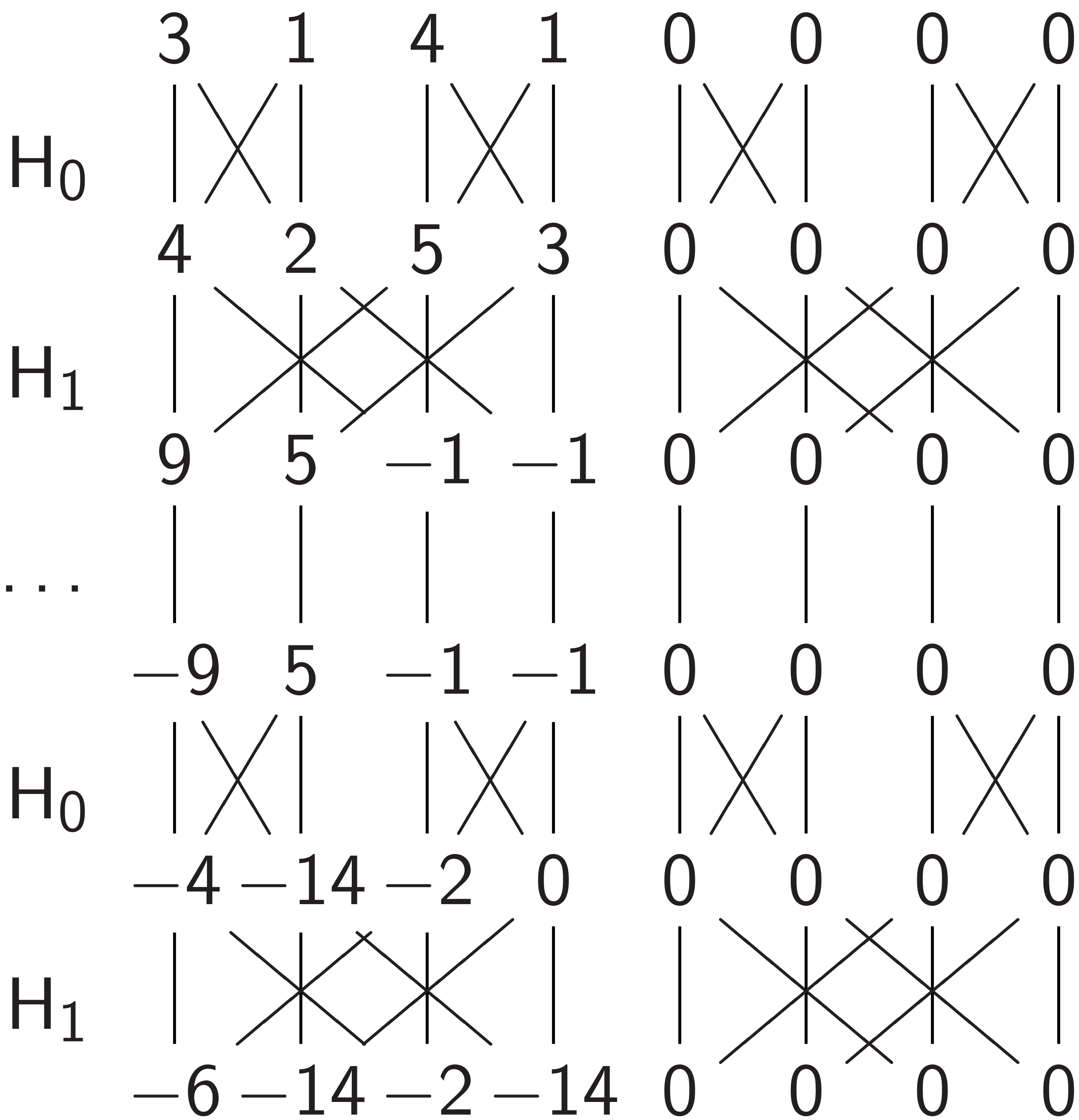
Simon's algorithm

Step 5. C_0NOT_3:

1, 0, 1, 0, 1, 0, 1, 0,
0, 1, 0, 1, 0, 1, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Affects measurements: “Negate amplitude around its average.” $(3,1,4,1) \mapsto (1.5,3.5,0.5,3.5)$.



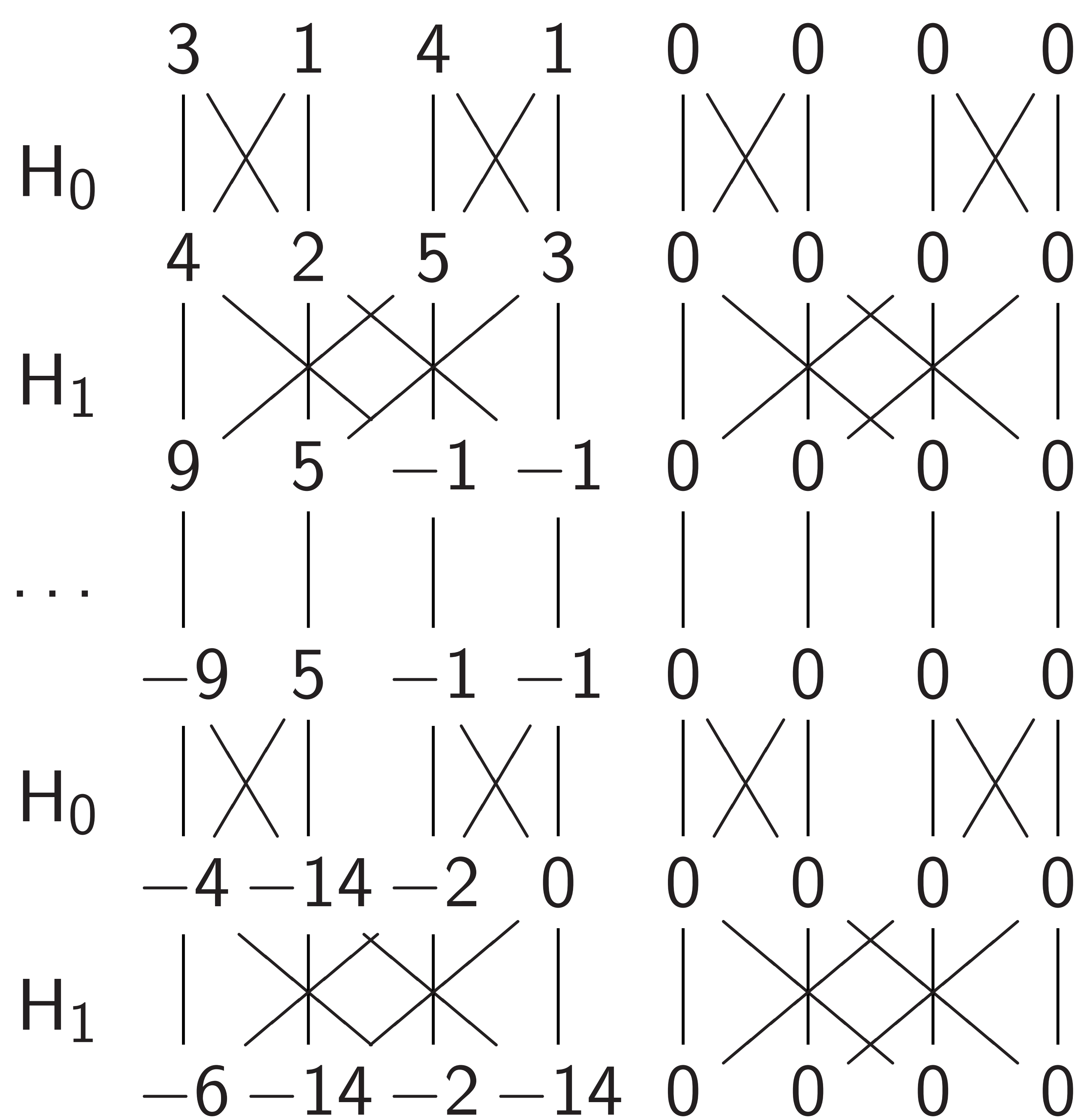
Simon’s algorithm

Step 5b. More shuffling:

1, 0, 0, 0, 1, 0, 0, 0,
0, 1, 0, 0, 0, 1, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 1, 0,
0, 0, 0, 1, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Affects measurements: "Negate amplitude around its average."
$(3,1,4,1) \mapsto (1.5,3.5,0.5,3.5)$.



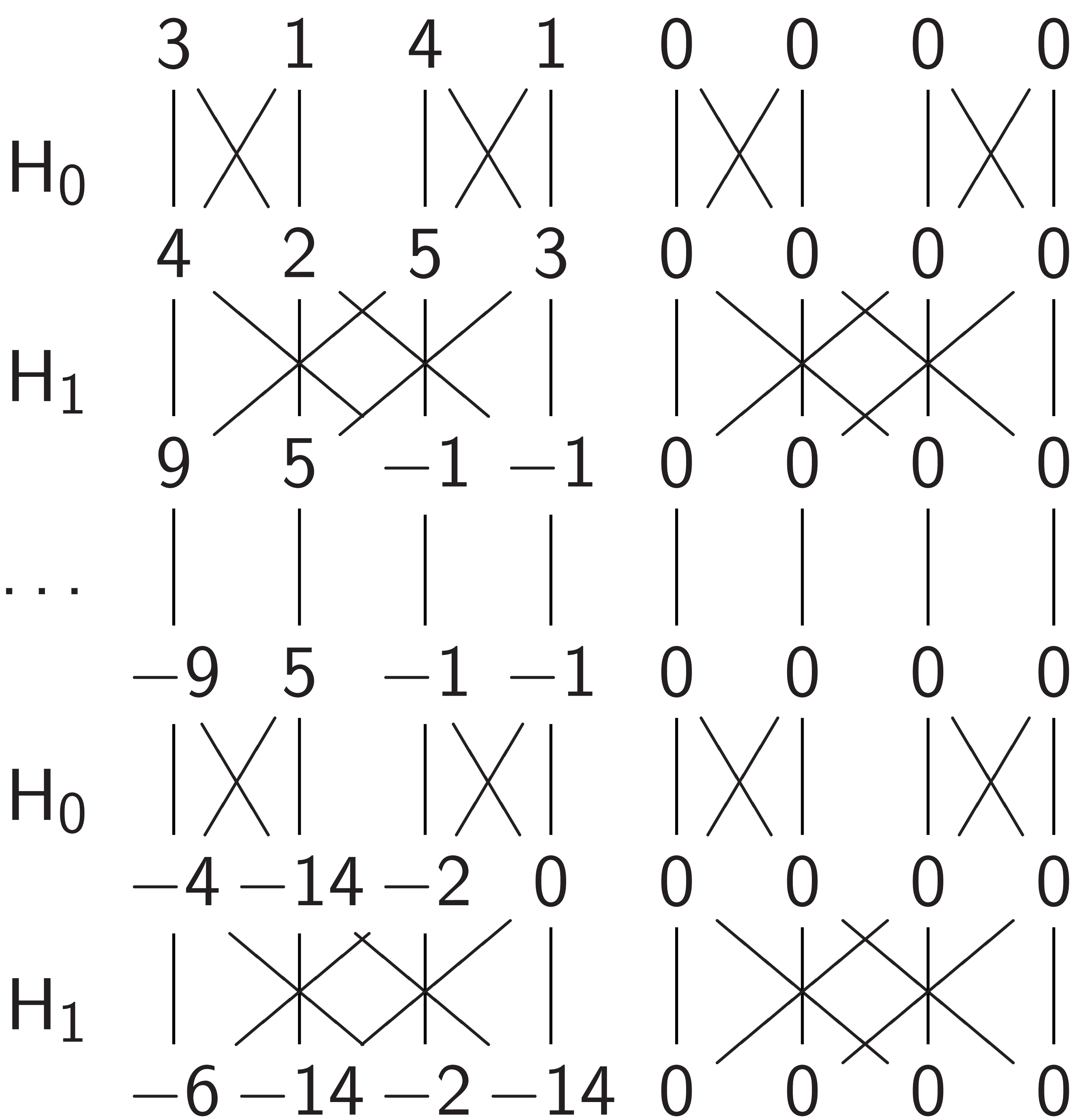
Simon's algorithm

Step 5c. More shuffling:

1, 0, 0, 0, 0, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
0, 0, 1, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 1.

Each column is a parallel universe performing its own computations.

Affects measurements: "Negate amplitude around its average." $(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.



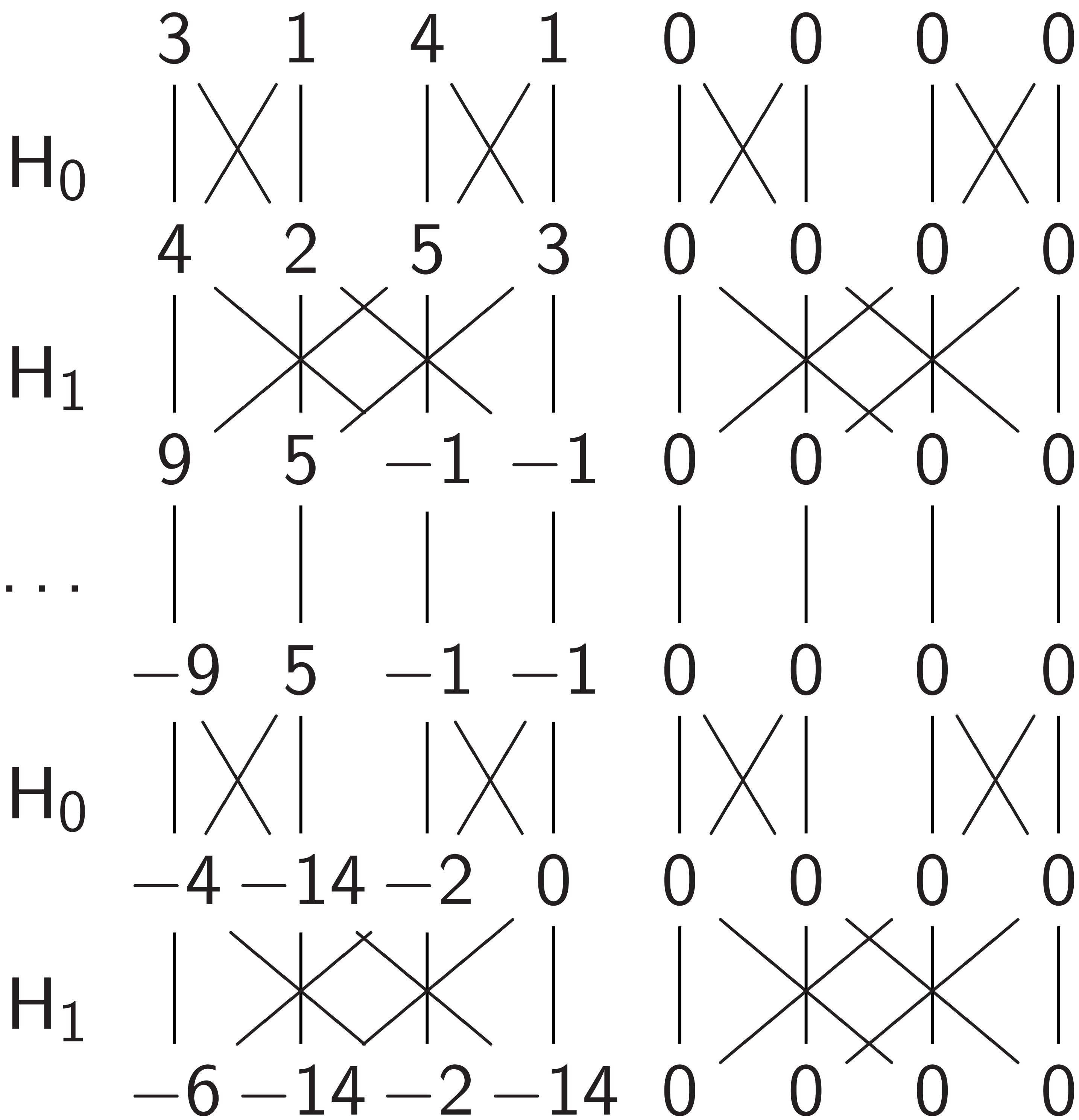
Simon's algorithm

Step 5d. More shuffling:

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 0, 1, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Affects measurements: "Negate amplitude around its average." $(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.



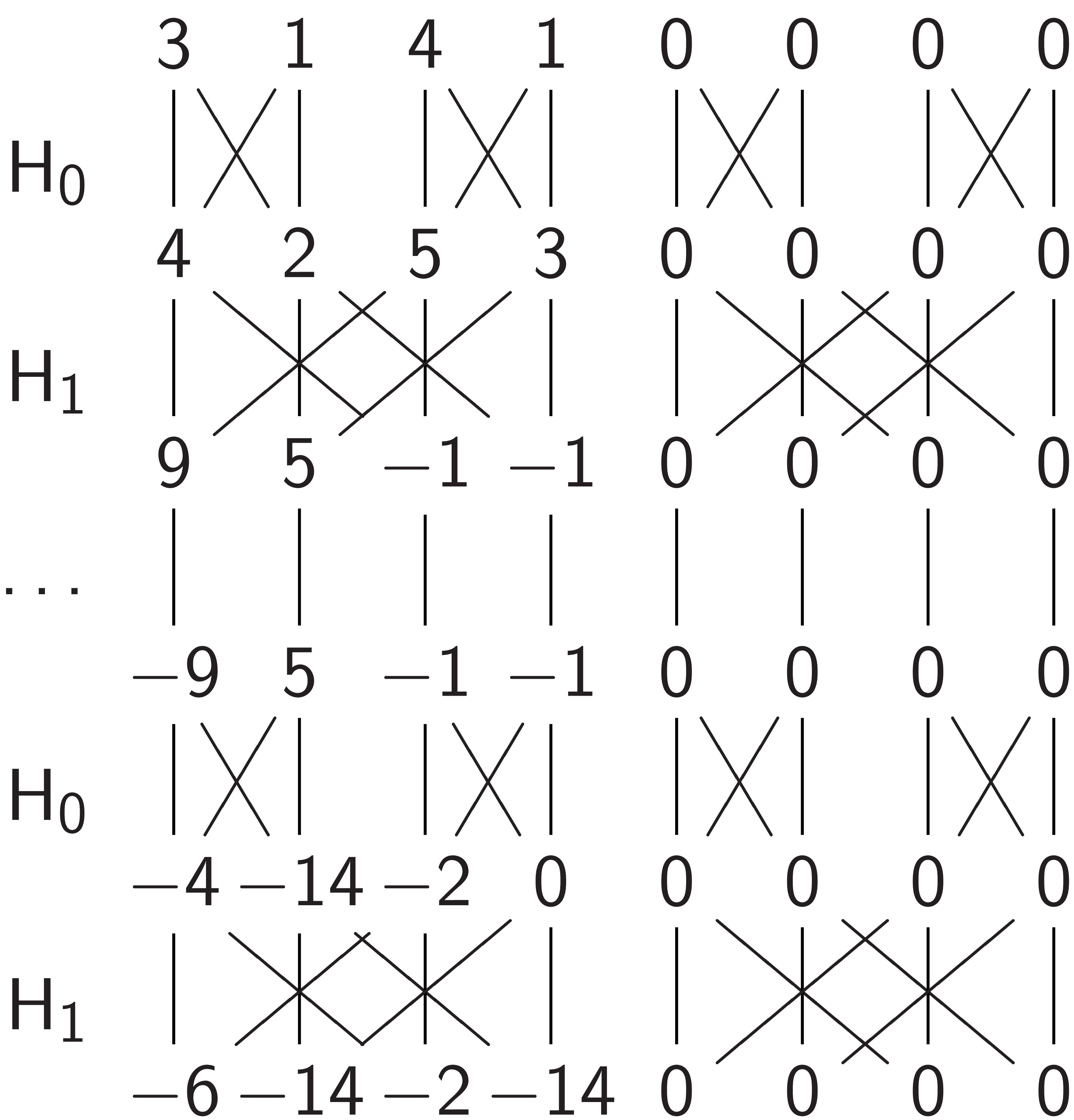
Simon's algorithm

Step 5e. More shuffling:

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Affects measurements: "Negate amplitude around its average."
$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.



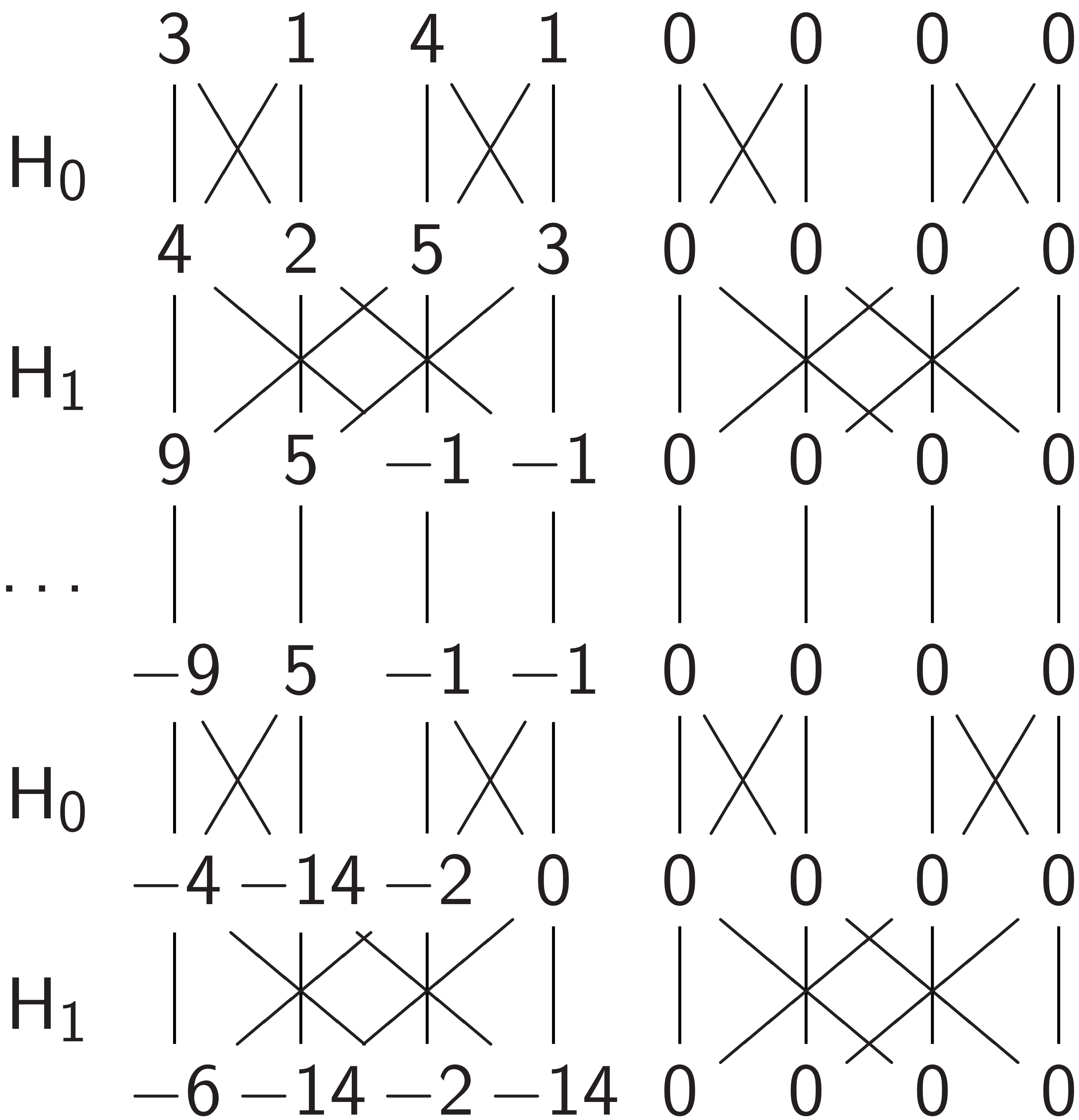
Simon's algorithm

Step 5f. More shuffling:

0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0.

Each column is a parallel universe performing its own computations.

Affects measurements: “Negate amplitude around its average.”
$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.



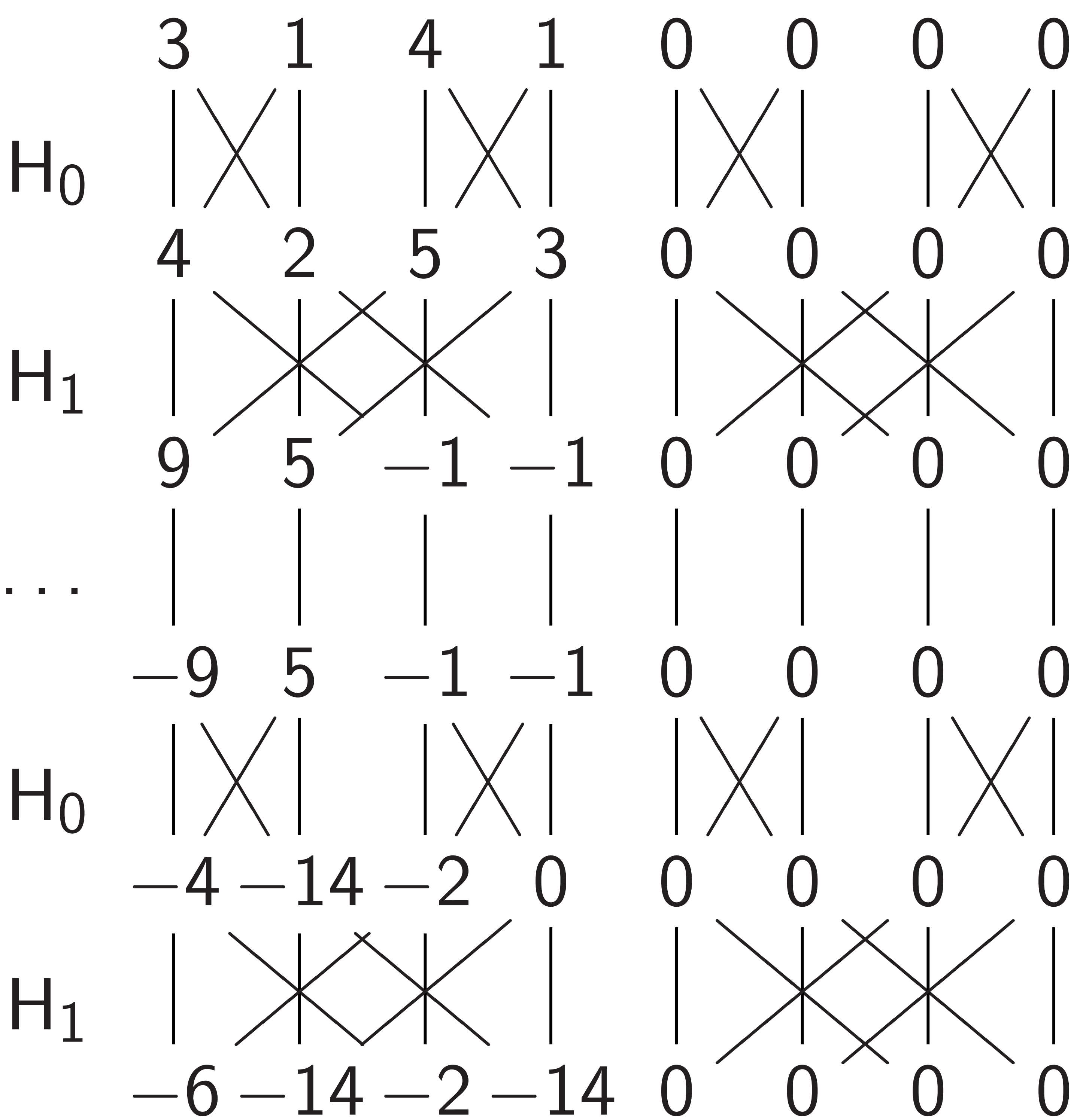
Simon’s algorithm

Step 5g. More shuffling:

0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1.

Each column is a parallel universe performing its own computations.

Affects measurements: "Negate amplitude around its average."
$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.



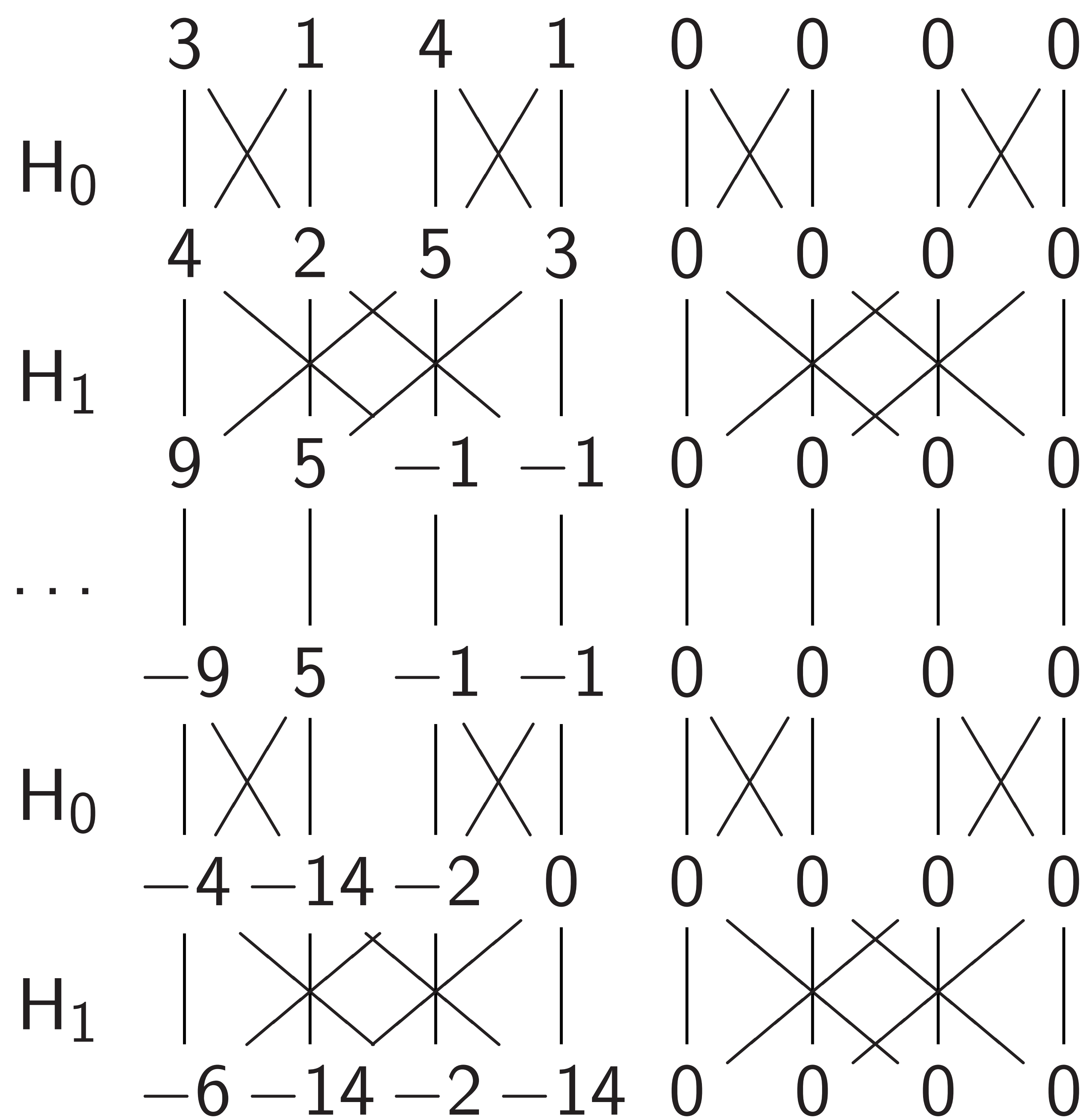
Simon's algorithm

Step 5h. More shuffling:

0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Affects measurements: “Negate amplitude around its average.” $(3,1,4,1) \mapsto (1.5,3.5,0.5,3.5)$.



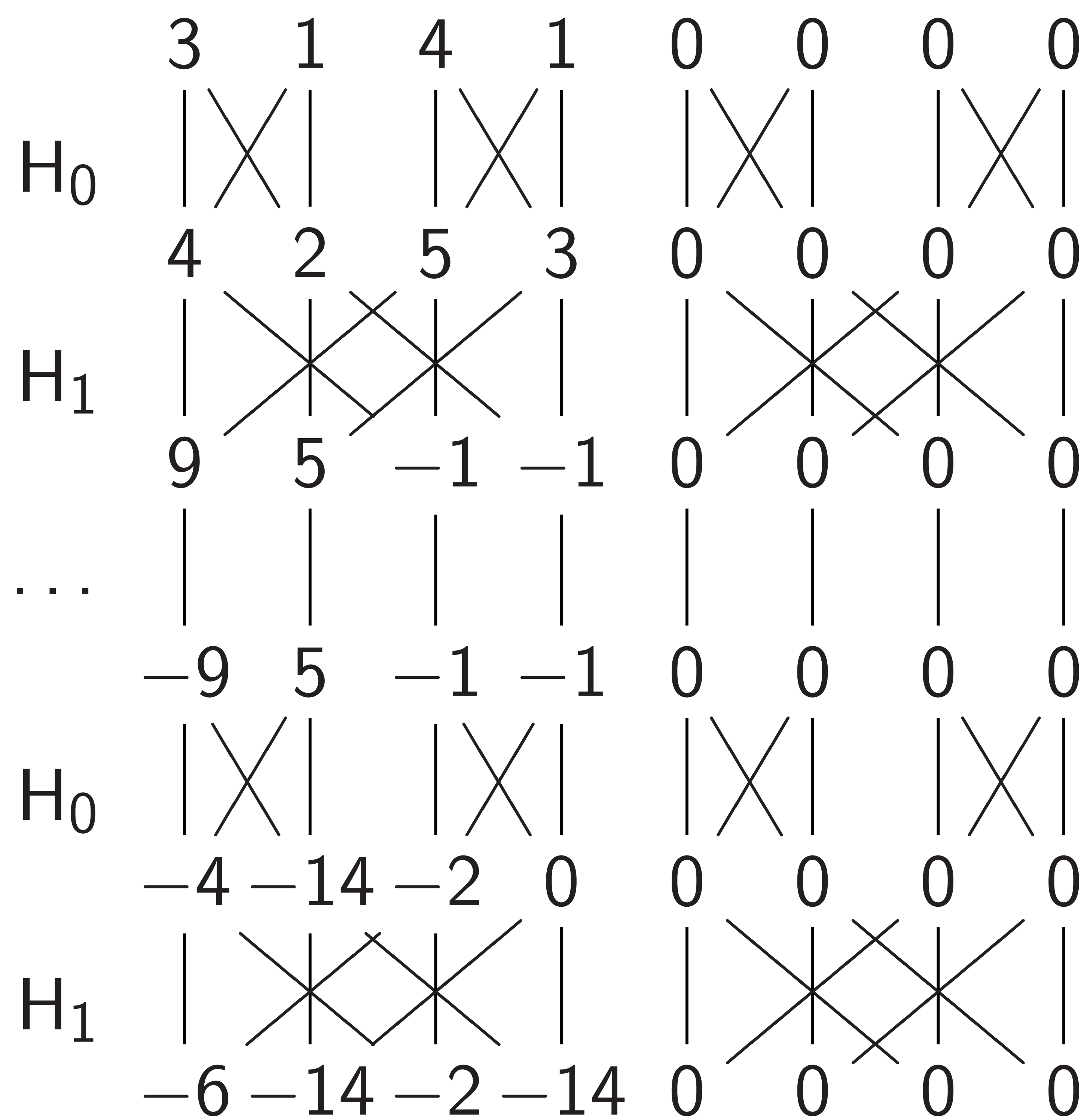
Simon’s algorithm

Step 5i. More shuffling:

0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 0, 1, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 1,
0, 0, 1, 0, 0, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Affects measurements: “Negate amplitude around its average.”
$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.



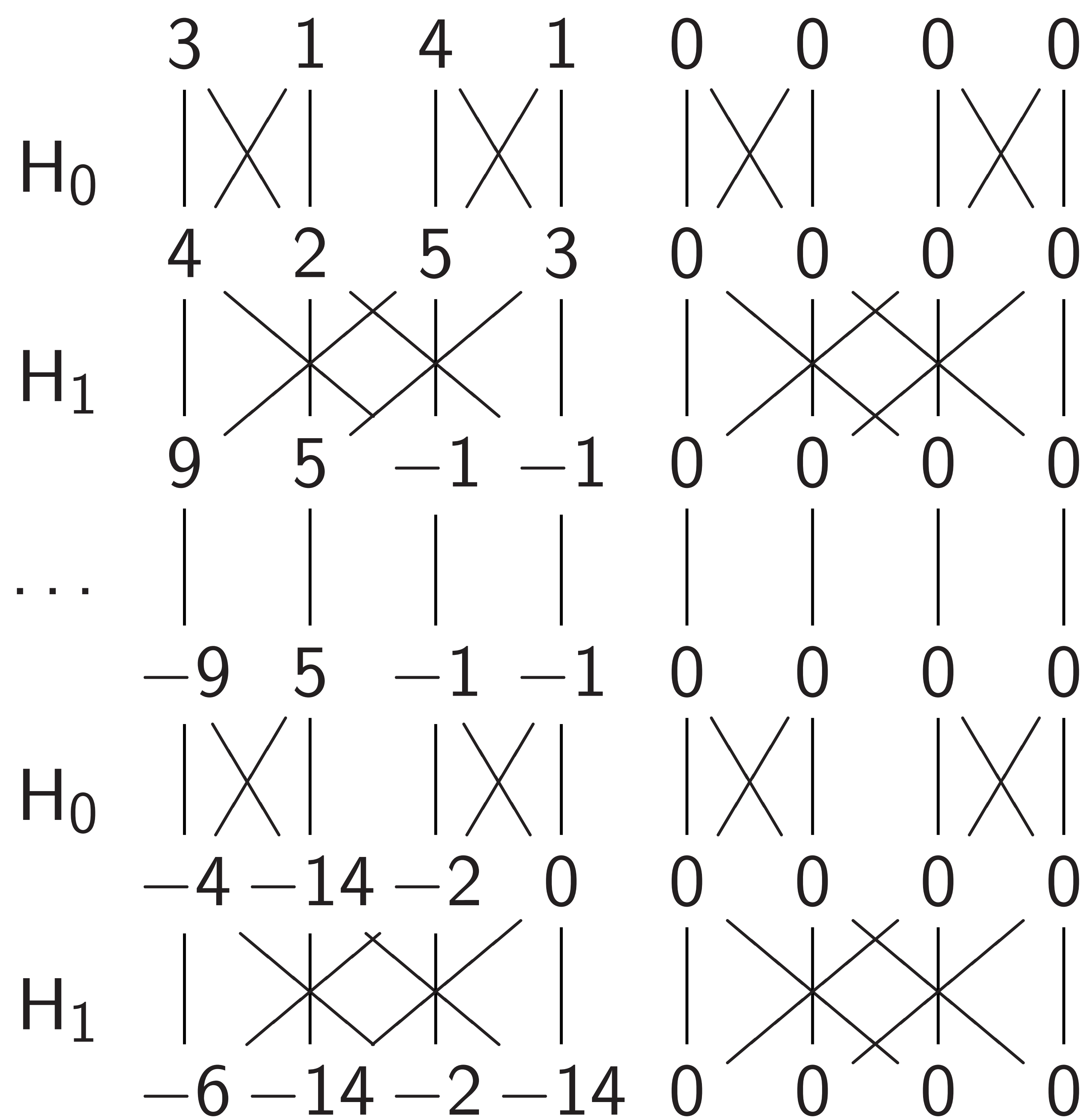
Simon’s algorithm

Step 5j. Final shuffling:

0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 1, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
1, 0, 0, 0, 0, 1, 0, 0.

Each column is a parallel universe performing its own computations.

Affects measurements: "Negate amplitude around its average."
$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.



Simon's algorithm

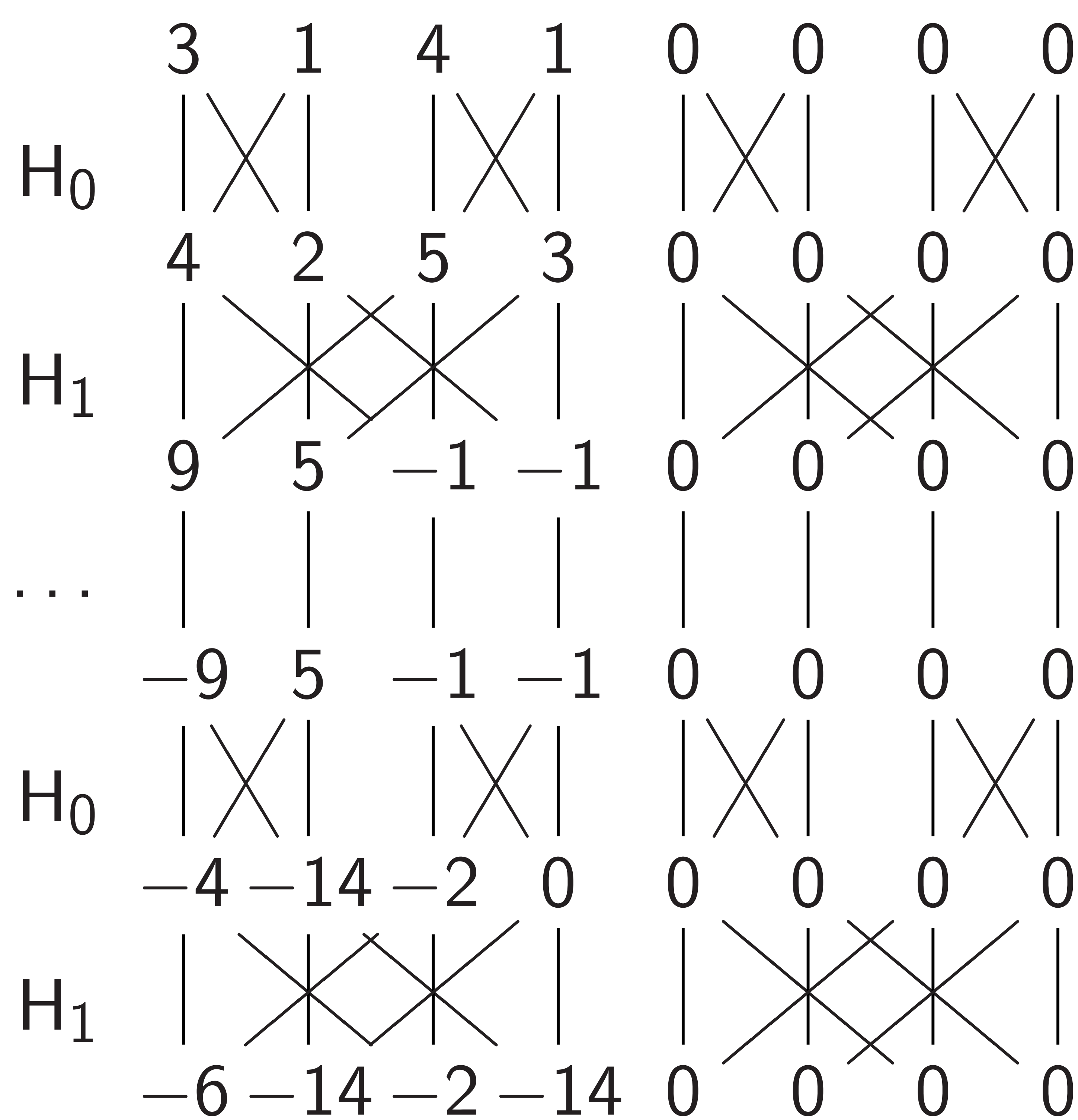
Step 5j. Final shuffling:

0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 1, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
1, 0, 0, 0, 0, 1, 0, 0.

Each column is a parallel universe performing its own computations.
Surprise: u and $u \oplus 101$ match.

Affects measurements: "Negate amplitude around its average."

$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.



Simon's algorithm

Step 6. Hadamard_0:

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 1, \overline{1}, 0, 0, 1, 1,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 1, 1, 0, 0, 1, \overline{1},$

$1, \overline{1}, 0, 0, 1, 1, 0, 0,$

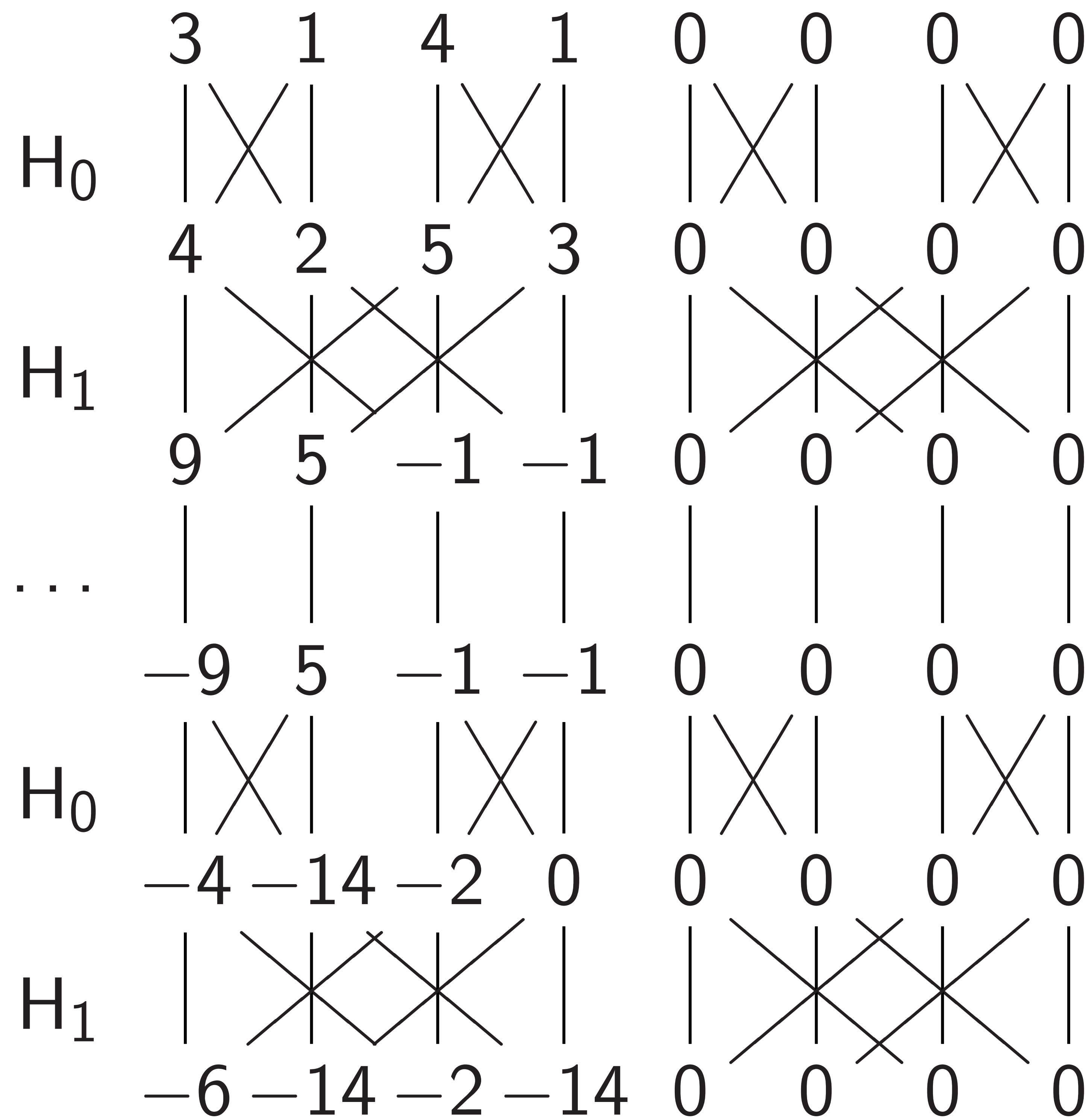
$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$1, 1, 0, 0, 1, \overline{1}, 0, 0.$

Notation: $\overline{1}$ means -1.

Affects measurements: "Negate amplitude around its average."
$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.

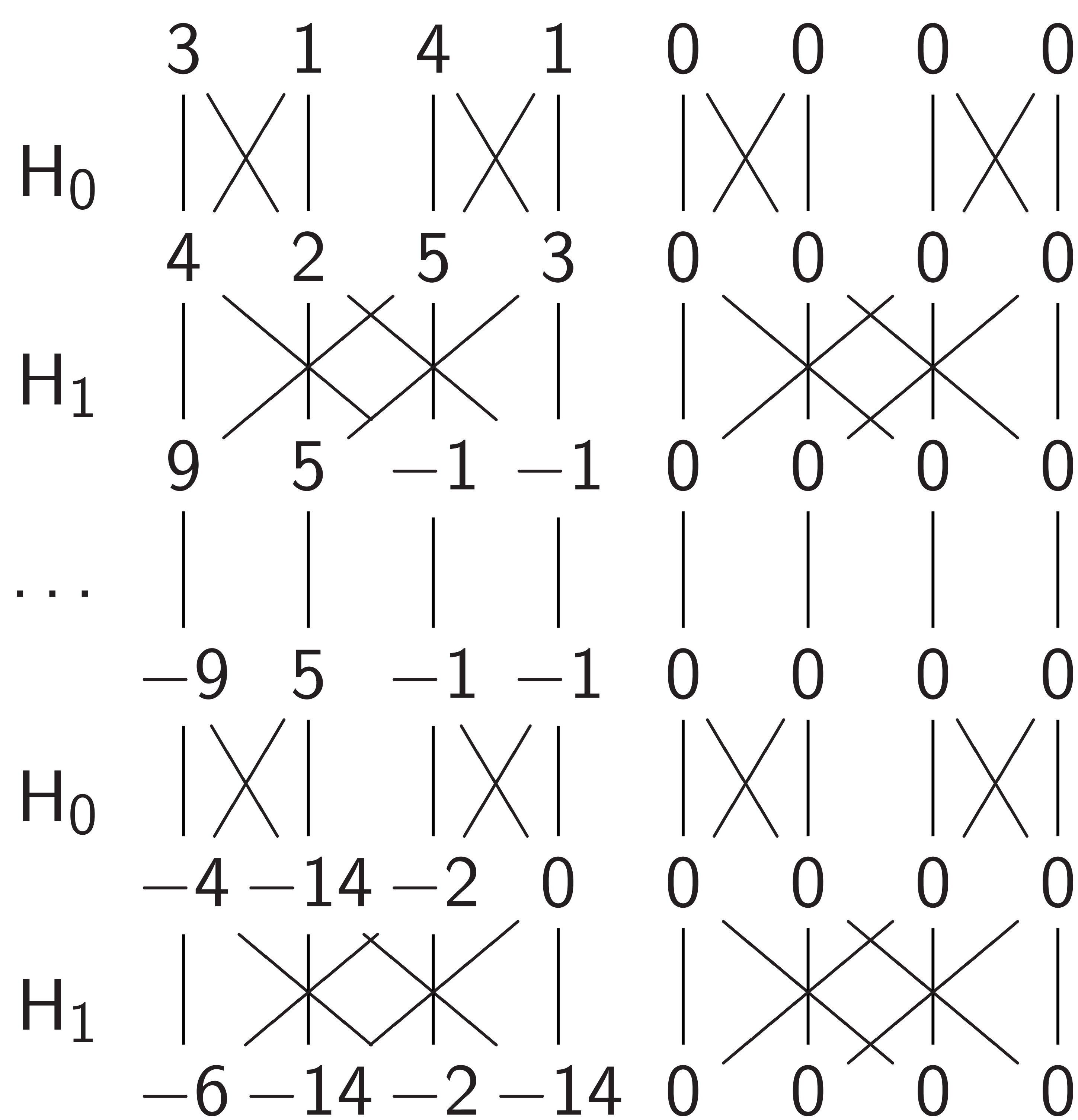


Simon's algorithm

Step 7. Hadamard$_1$:

$0, 0, 0, 0, 0, 0, 0, 0,$
$1, \bar{1}, \bar{1}, 1, 1, 1, \bar{1}, \bar{1},$
$0, 0, 0, 0, 0, 0, 0, 0,$
$1, 1, \bar{1}, \bar{1}, 1, \bar{1}, \bar{1}, 1,$
$1, \bar{1}, 1, \bar{1}, 1, 1, 1, 1,$
$0, 0, 0, 0, 0, 0, 0, 0,$
$0, 0, 0, 0, 0, 0, 0, 0,$
$1, 1, 1, 1, 1, \bar{1}, 1, \bar{1}.$

Affects measurements: "Negate amplitude around its average."
$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.

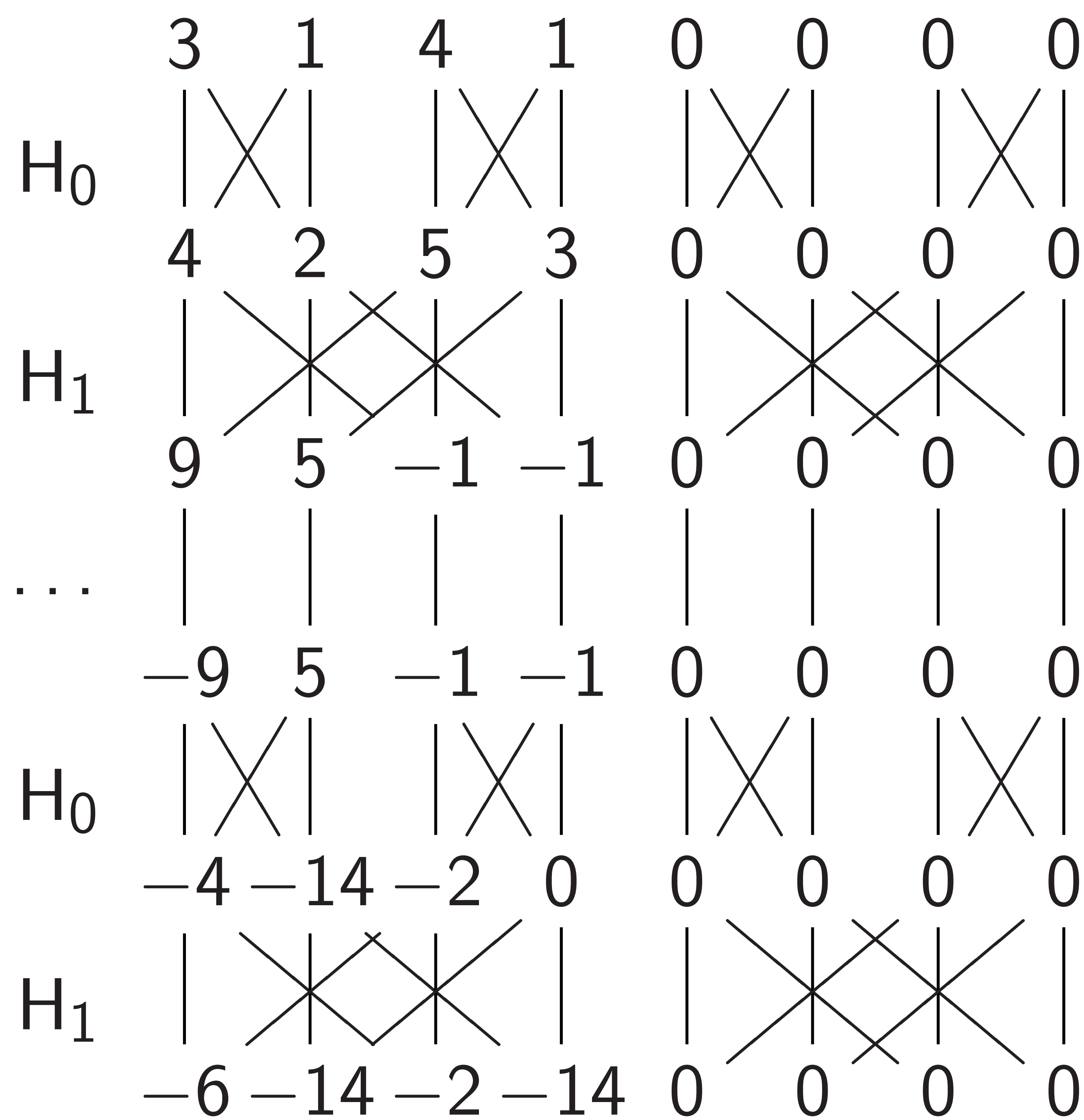


Simon's algorithm

Step 8. Hadamard_2:

$0, 0, 0, 0, 0, 0, 0, 0,$
$2, 0, \overline{2}, 0, 0, \overline{2}, 0, 2,$
$0, 0, 0, 0, 0, 0, 0, 0,$
$2, 0, \overline{2}, 0, 0, 2, 0, \overline{2},$
$2, 0, 2, 0, 0, \overline{2}, 0, \overline{2},$
$0, 0, 0, 0, 0, 0, 0, 0,$
$0, 0, 0, 0, 0, 0, 0, 0,$
$2, 0, 2, 0, 0, 2, 0, 2.$

Affects measurements: "Negate amplitude around its average."
$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.



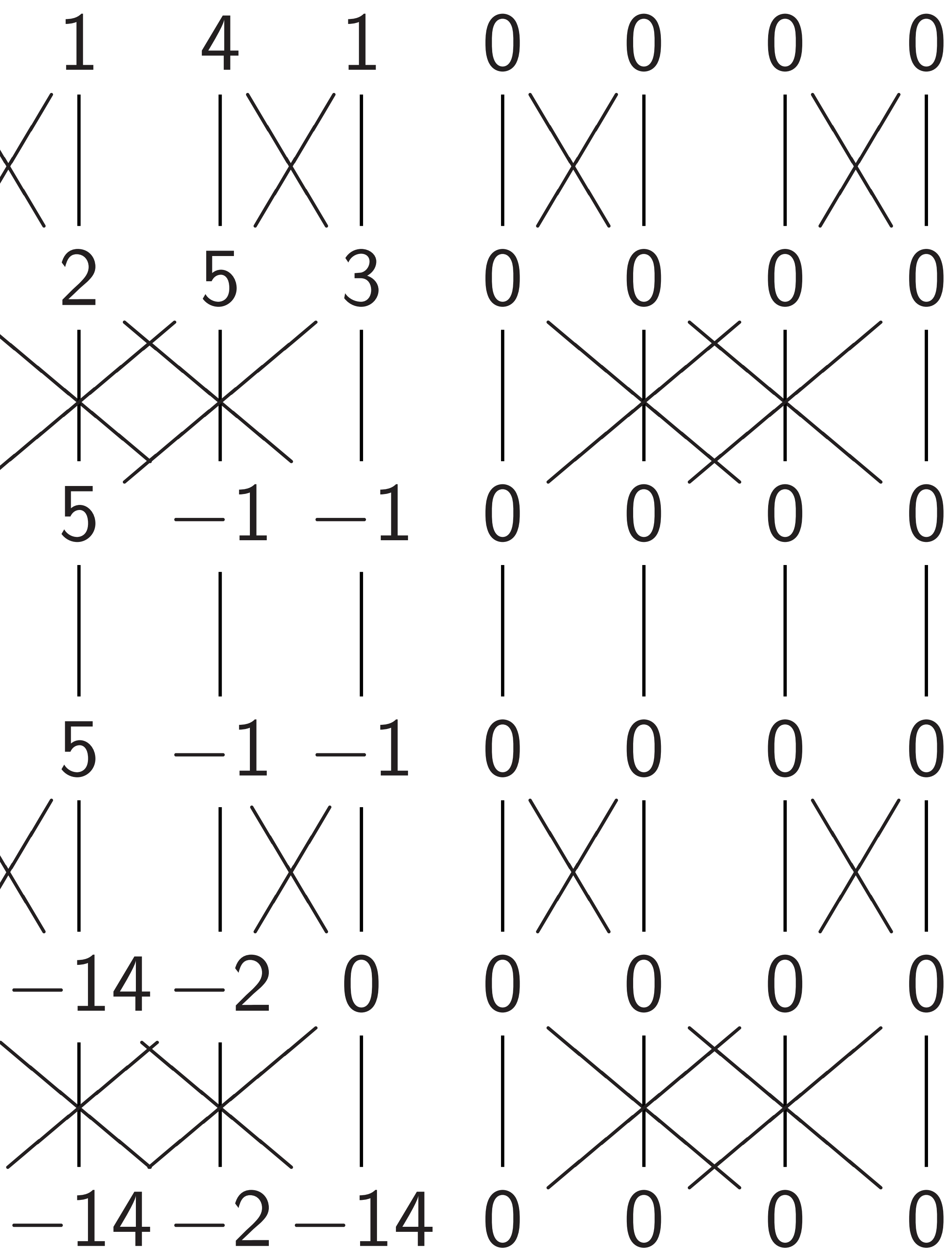
Simon's algorithm

Step 8. Hadamard_2:

$0, 0, 0, 0, 0, 0, 0, 0,$
$2, 0, \bar{2}, 0, 0, \bar{2}, 0, 2,$
$0, 0, 0, 0, 0, 0, 0, 0,$
$2, 0, \bar{2}, 0, 0, 2, 0, \bar{2},$
$2, 0, 2, 0, 0, \bar{2}, 0, \bar{2},$
$0, 0, 0, 0, 0, 0, 0, 0,$
$0, 0, 0, 0, 0, 0, 0, 0,$
$2, 0, 2, 0, 0, 2, 0, 2.$

Step 9: Measure. Obtain some information about the surprise: a random vector orthogonal to 101.

measurements: “Negate
de around its average.”
1) $\mapsto (1.5, 3.5, 0.5, 3.5)$.



Simon’s algorithm

Step 8. Hadamard$_2$:

$0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$
$2,\ 0,\ \overline{2},\ 0,\ 0,\ \overline{2},\ 0,\ 2,$
$0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$
$2,\ 0,\ \overline{2},\ 0,\ 0,\ 2,\ 0,\ \overline{2},$
$2,\ 0,\ 2,\ 0,\ 0,\ \overline{2},\ 0,\ \overline{2},$
$0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$
$0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$
$2,\ 0,\ 2,\ 0,\ 0,\ 2,\ 0,\ 2.$

Step 9: Measure. Obtain some information about the surprise: a random vector orthogonal to 101.

Repeat t

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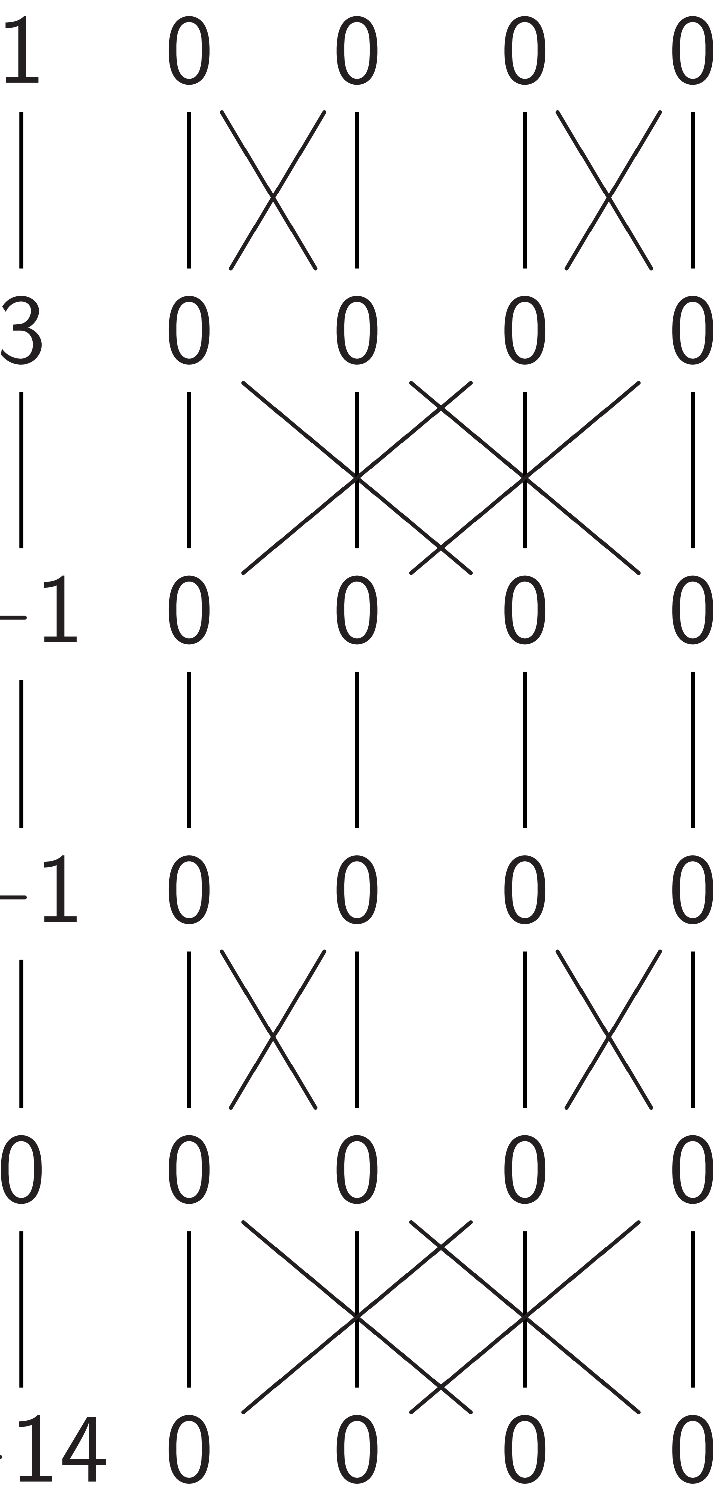
3.5, 0.5, 3.5).



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2, 0, $\overline{2}$, 0, 0, $\overline{2}$, 0, 2,
0, 0, 0, 0, 0, 0, 0, 0,
2, 0, $\overline{2}$, 0, 0, 2, 0, $\overline{2}$,
2, 0, 2, 0, 0, $\overline{2}$, 0, $\overline{2}$,
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Repeat to figure o

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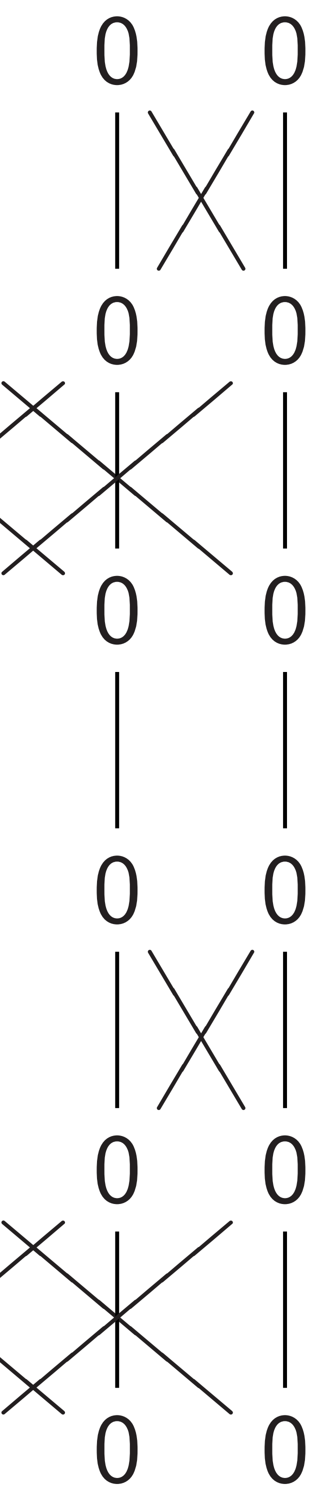

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2, 0, $\overline{2}$, 0, 0, 2, 0, $\overline{2}$,
2, 0, 2, 0, 0, $\overline{2}$, 0, $\overline{2}$,
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0, 0, 0, 0, 0, 0, 0, 0,
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Repeat to figure out 101.

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Repeat to figure out 101.

Generalize Step 5 to any function $u \mapsto f(u)$ with $f(u) = f(u \oplus s)$. "Usually" algorithm figures out s.

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2, 0, 2, 0, 0, $\overline{2}$, 0, $\overline{2}$,
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$0,\ 0,\ 0,\ 0,\ 0,$
$0,\ 0,\ \overline{2},\ 0,\ 2,$
$0,\ 0,\ 0,\ 0,\ 0,$
$0,\ 0,\ 2,\ 0,\ \overline{2},$
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Assume:
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Assume: unique s
has $f(s) = 0$.

Traditional algorit
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Success probability
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Assume: unique $s \in \{0,1\}^n$
has $f(s) = 0$.

Traditional algorithm to find
compute f for many inputs,
hope to find output 0.
Success probability is very lo
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Grover's algorithm takes only $2^{n/2}$ reversible computations of f. Typically: reversibility overhead is small enough that this easily beats traditional algorithm.

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Start from uniform superposition
over all n-bit strings q.

Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

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Negate a around its average.
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Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

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Measure the n qubits.
With high probability this finds s.

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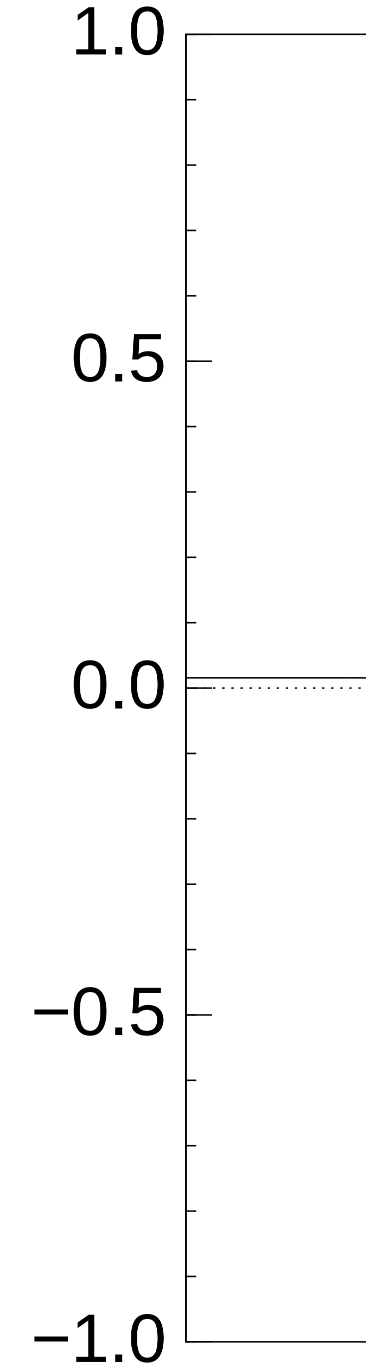

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Normalized graph
for an example wit
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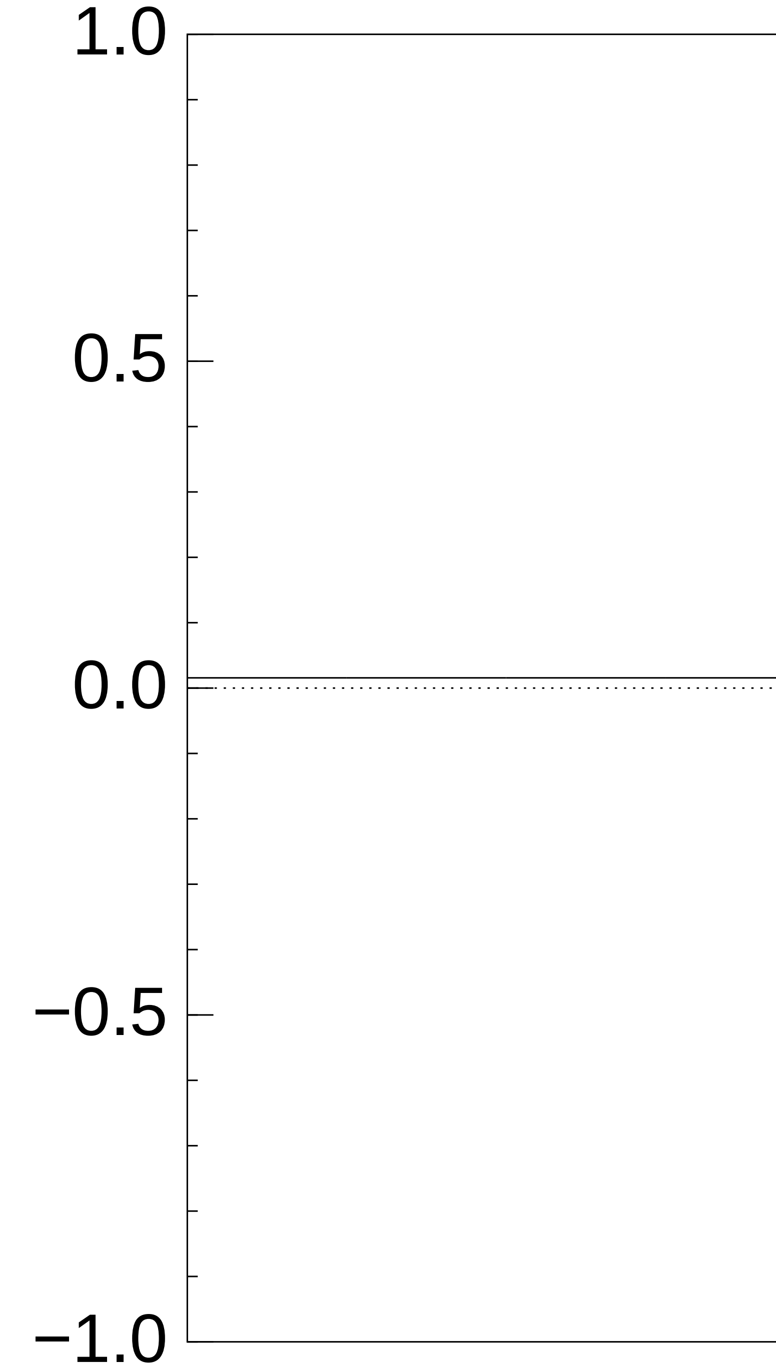

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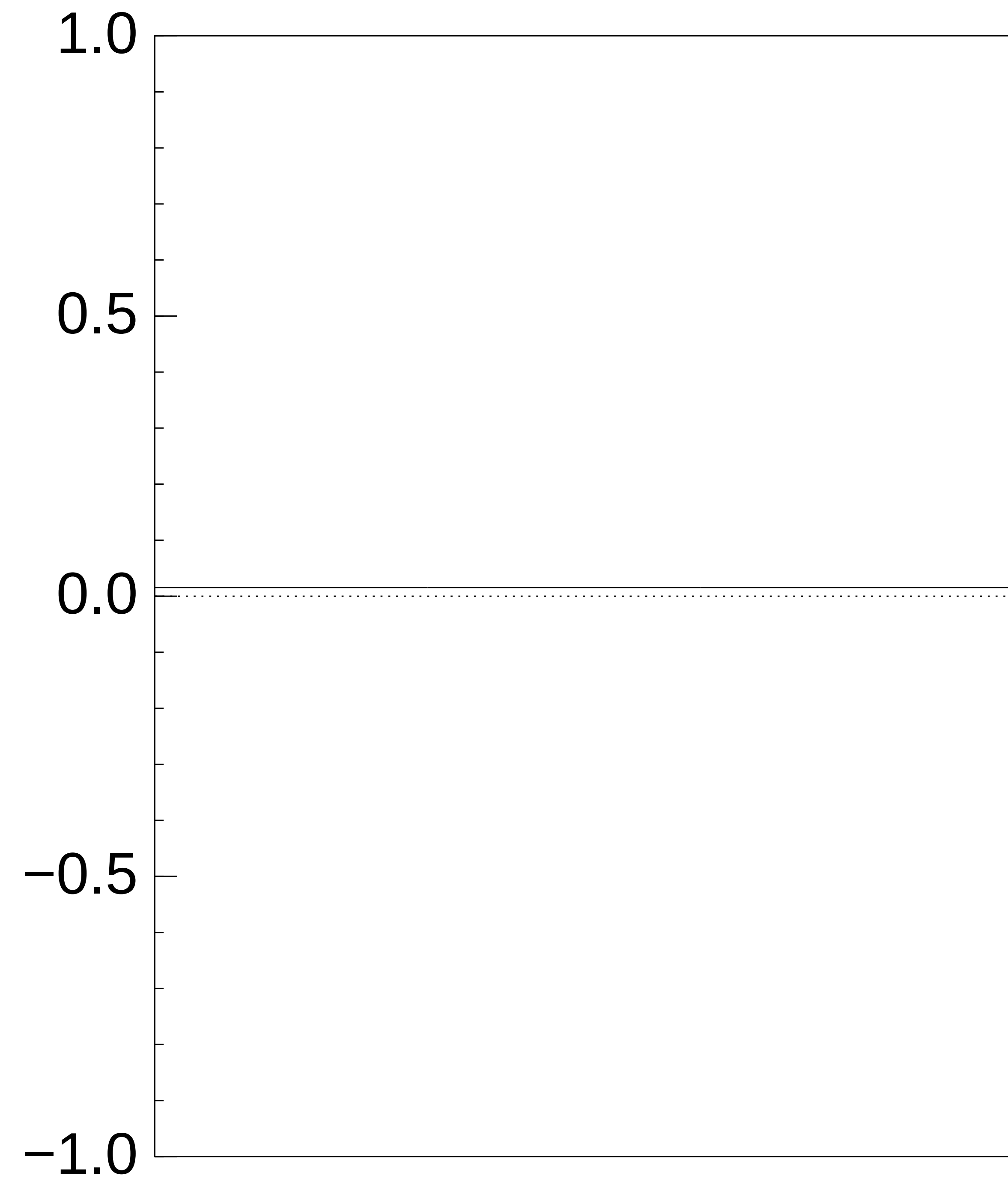
Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after 0 steps:



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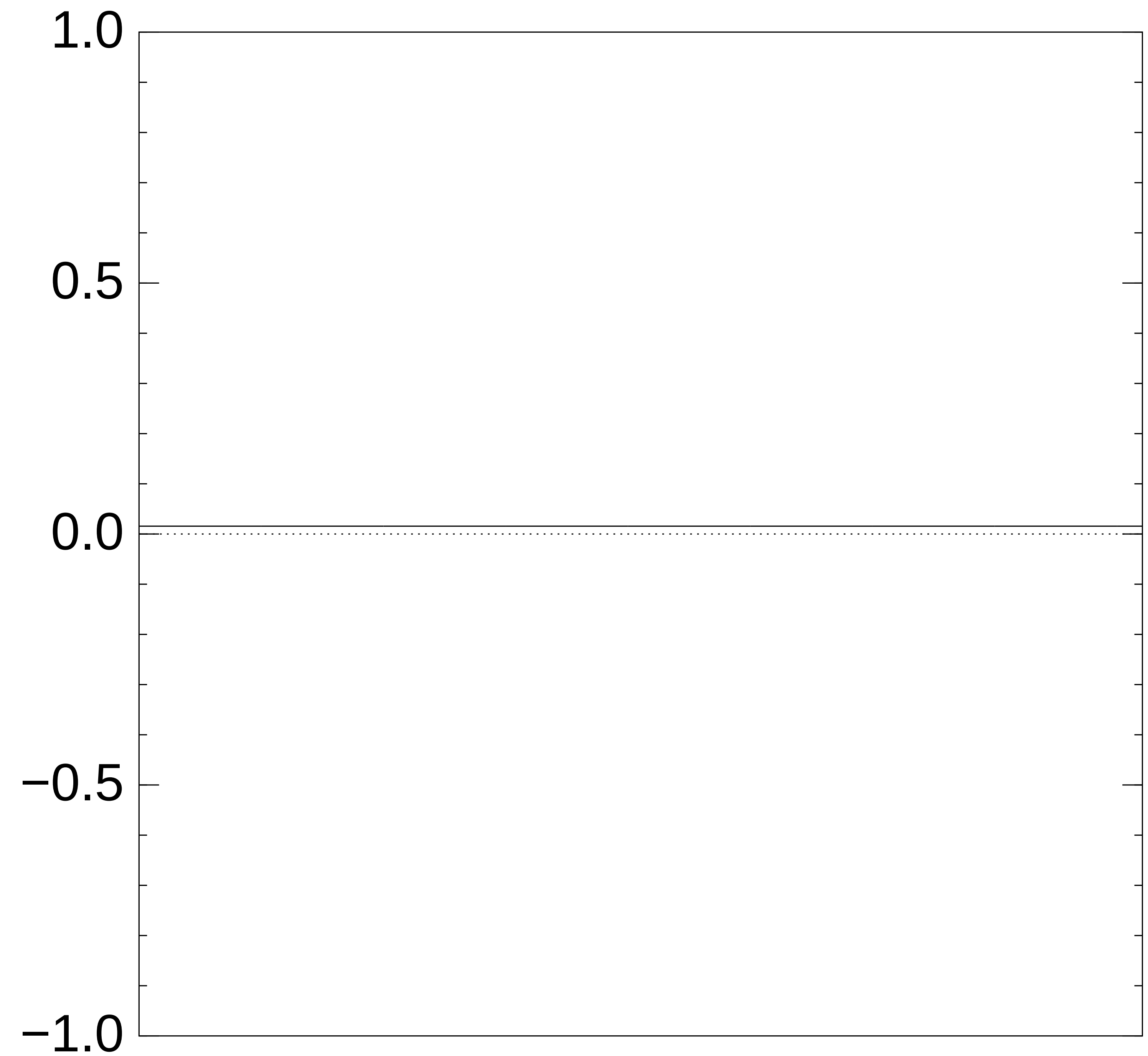

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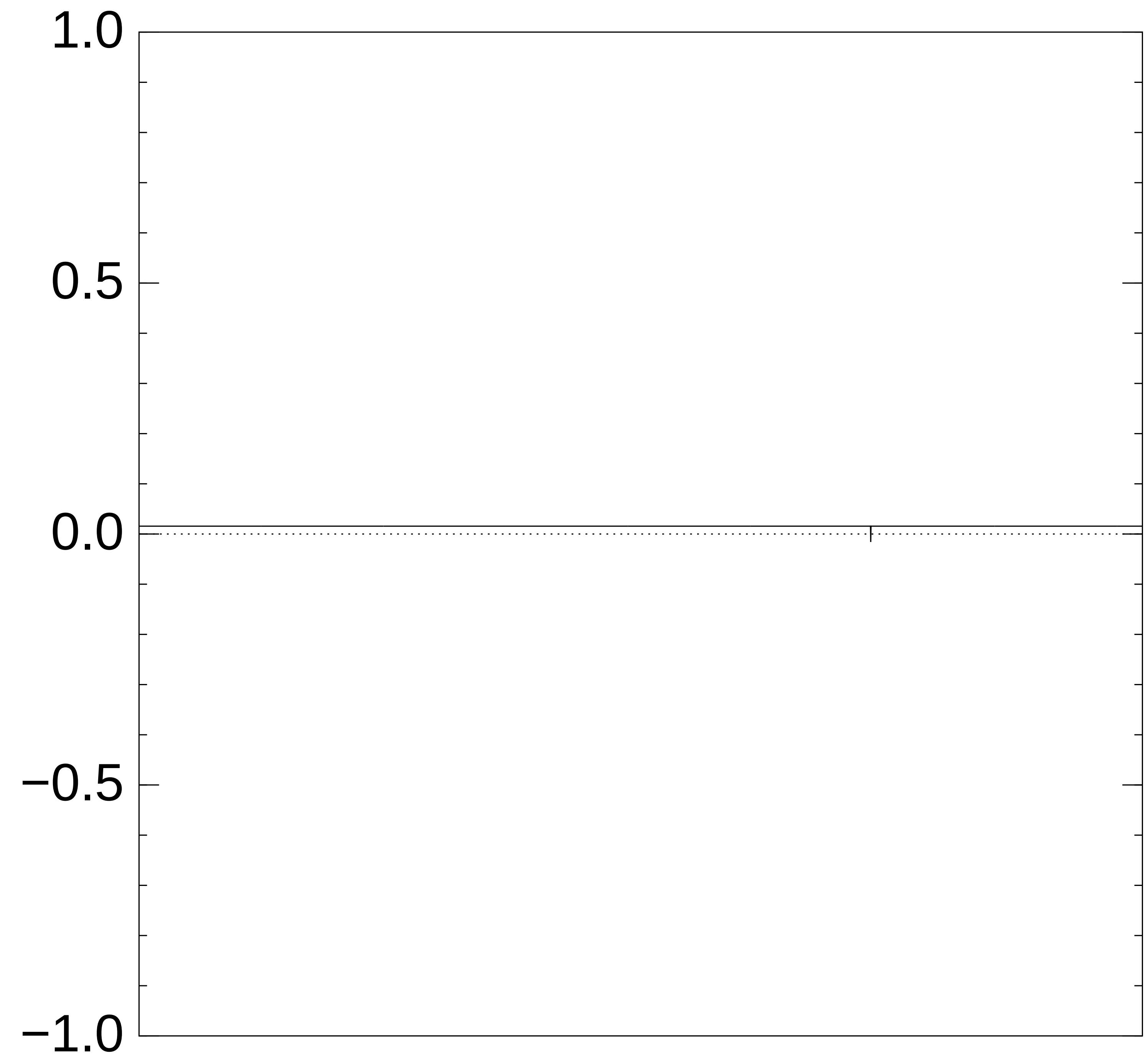
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after Step 1:



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Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
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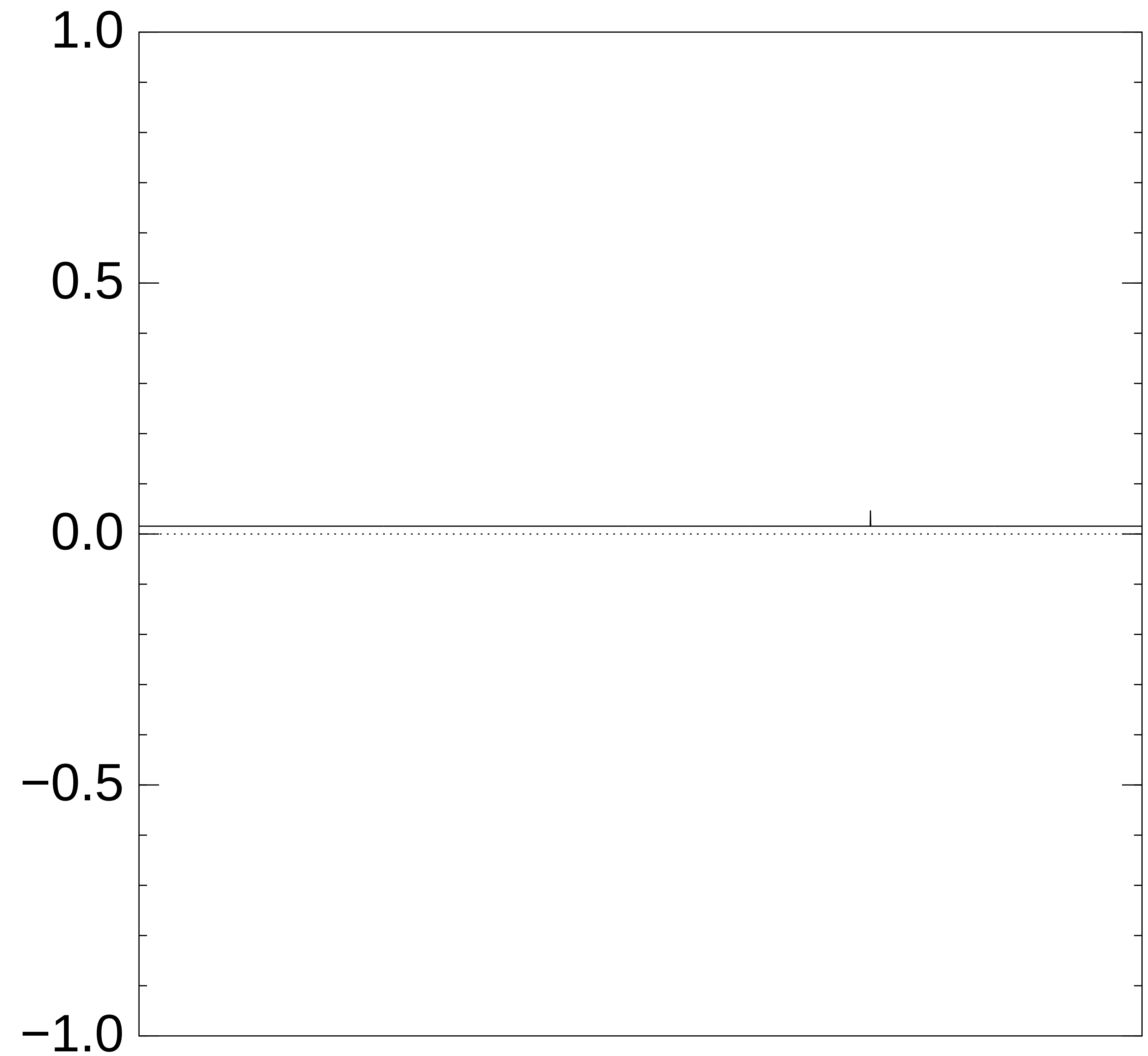
Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after Step 1 + Step 2:



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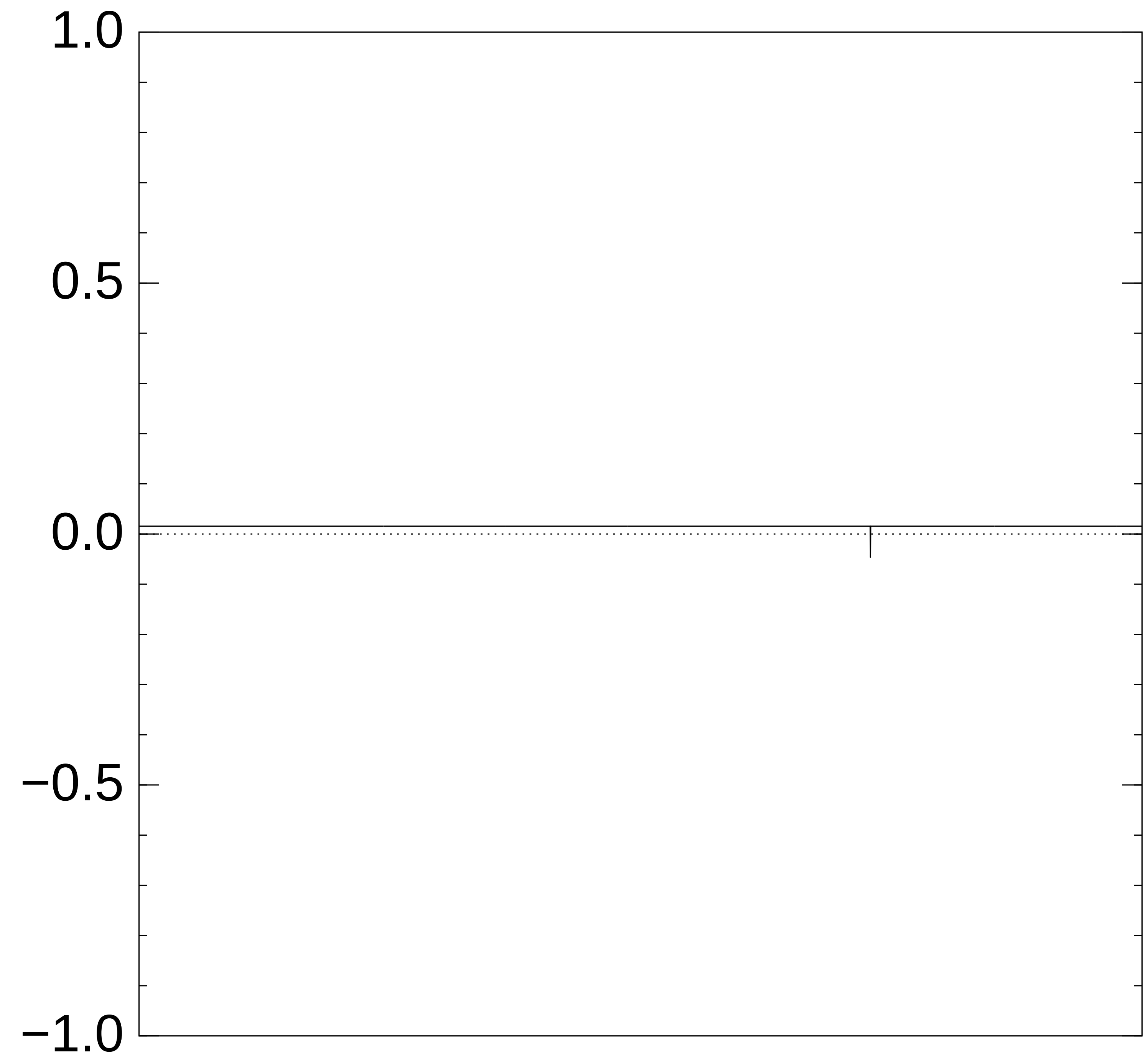
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after Step 1 + Step 2 + Step 1:



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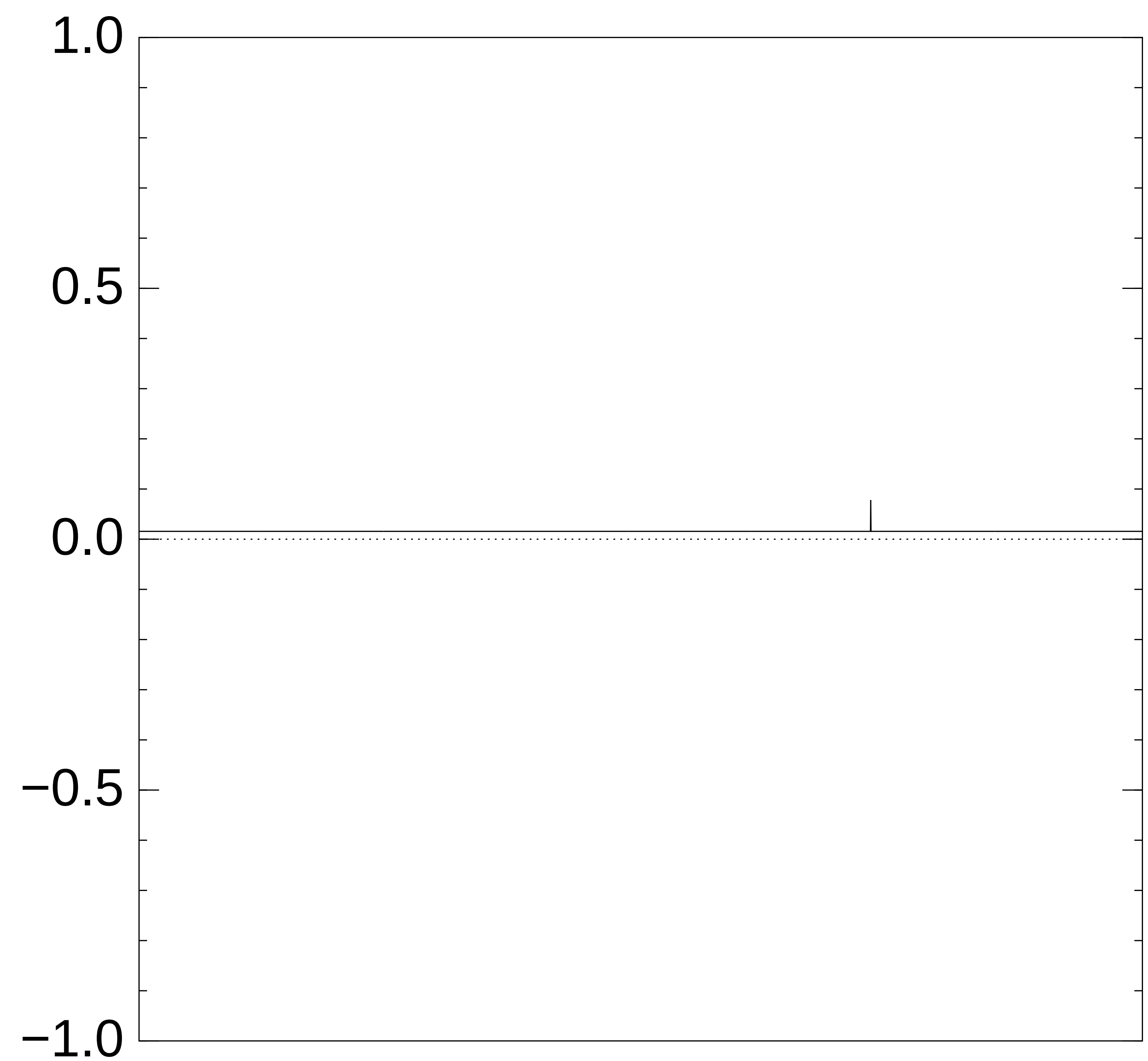
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $2 \times$ (Step 1 + Step 2):



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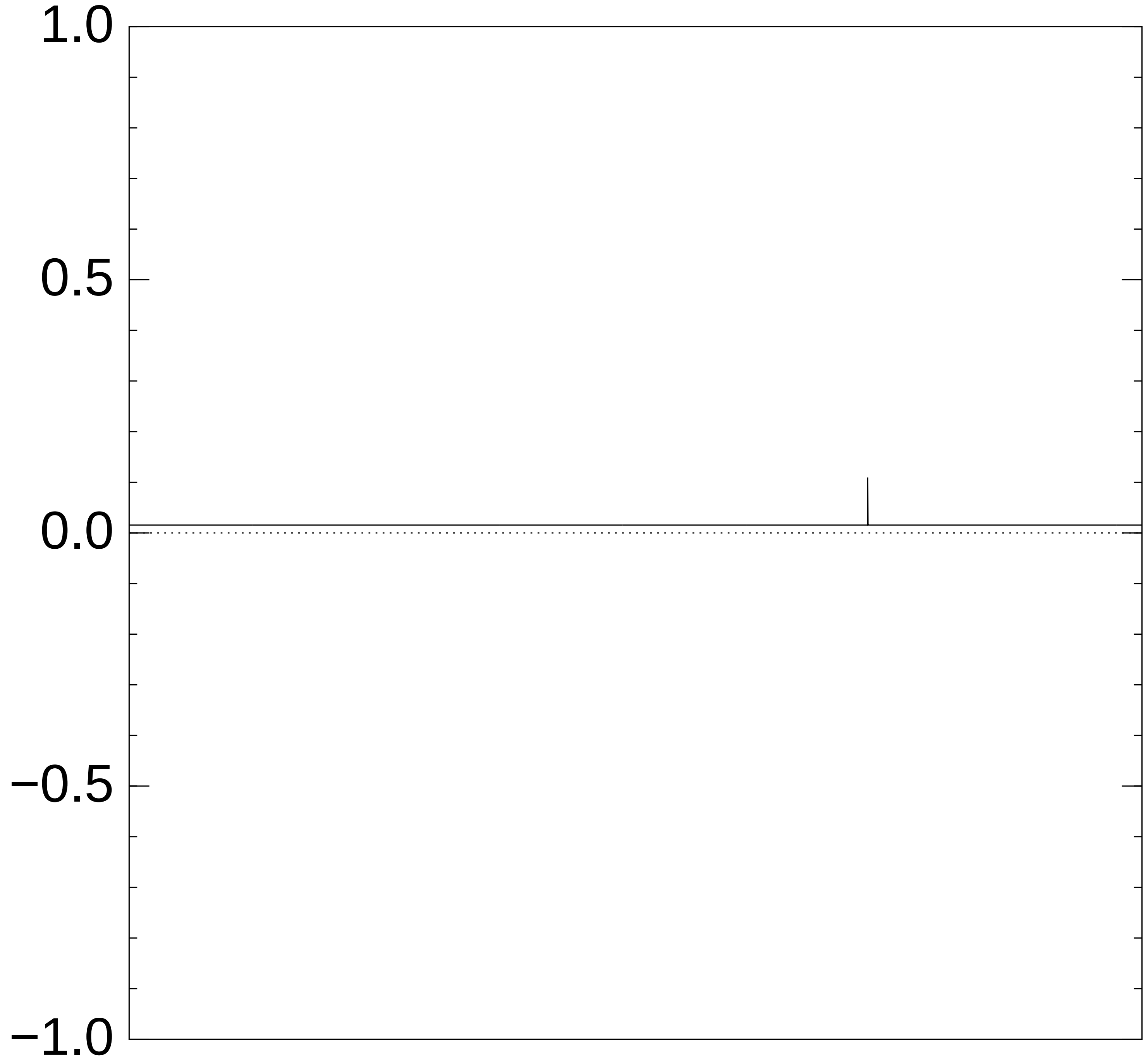
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $3 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

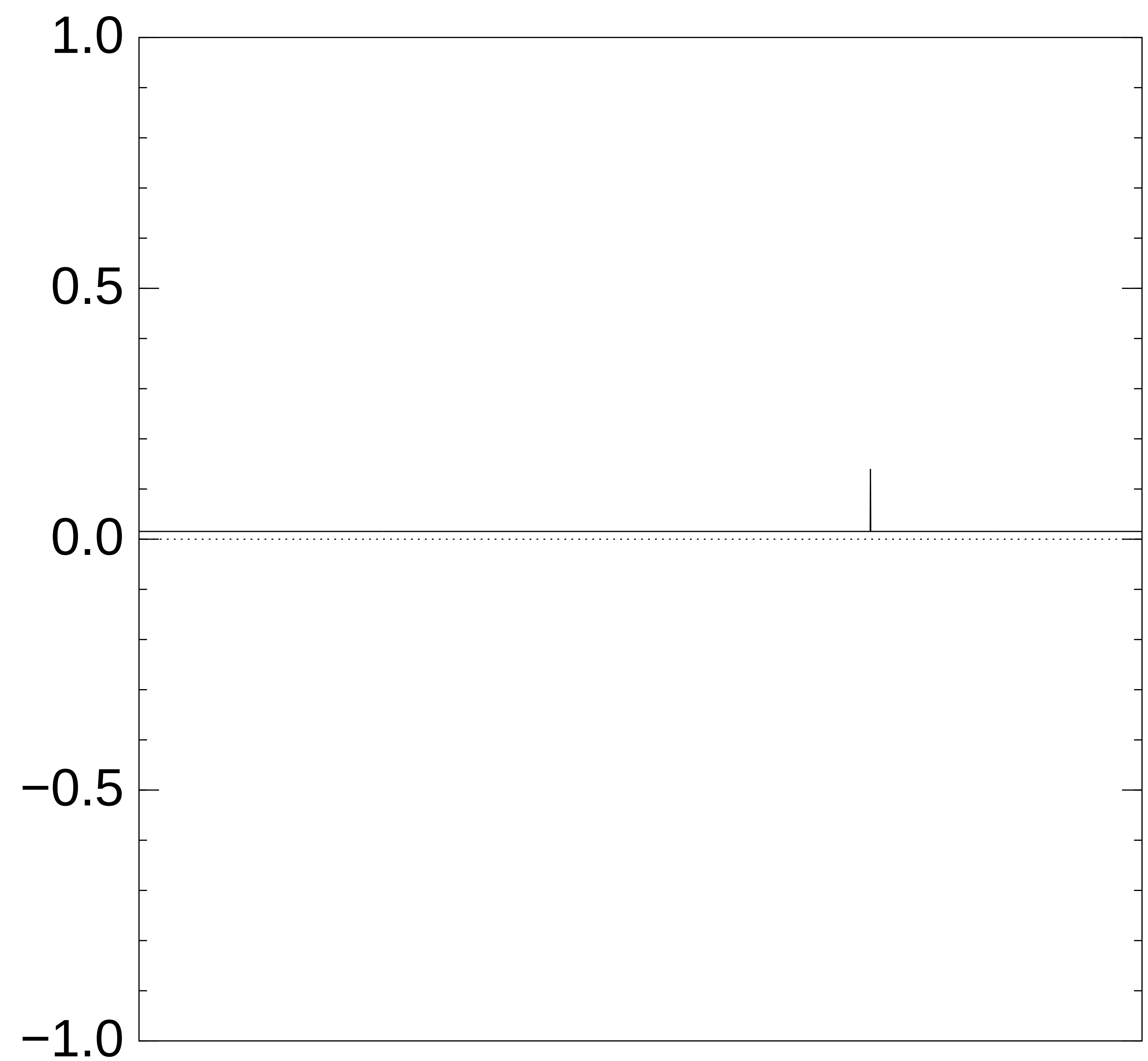
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $4 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

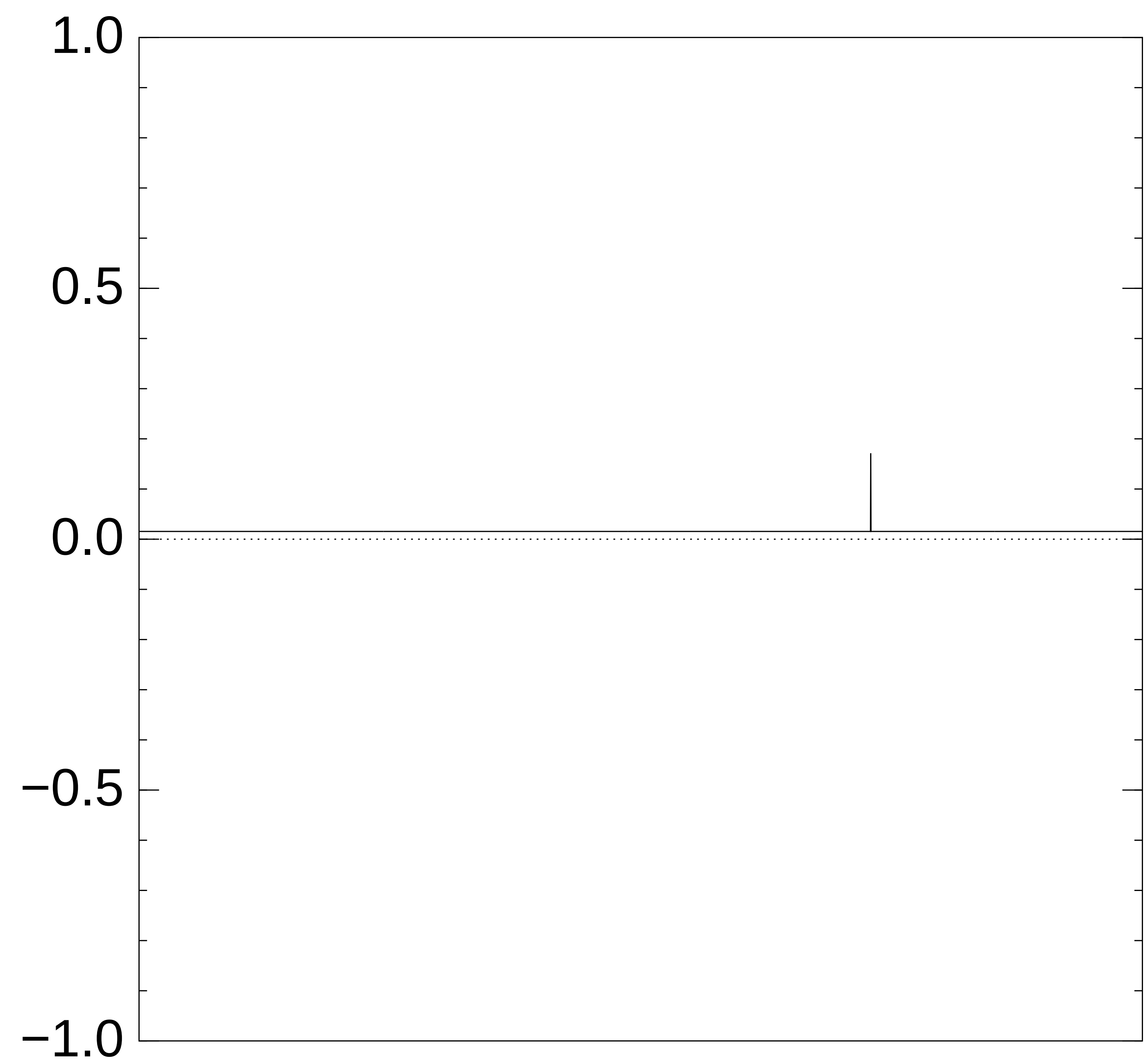
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $5 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

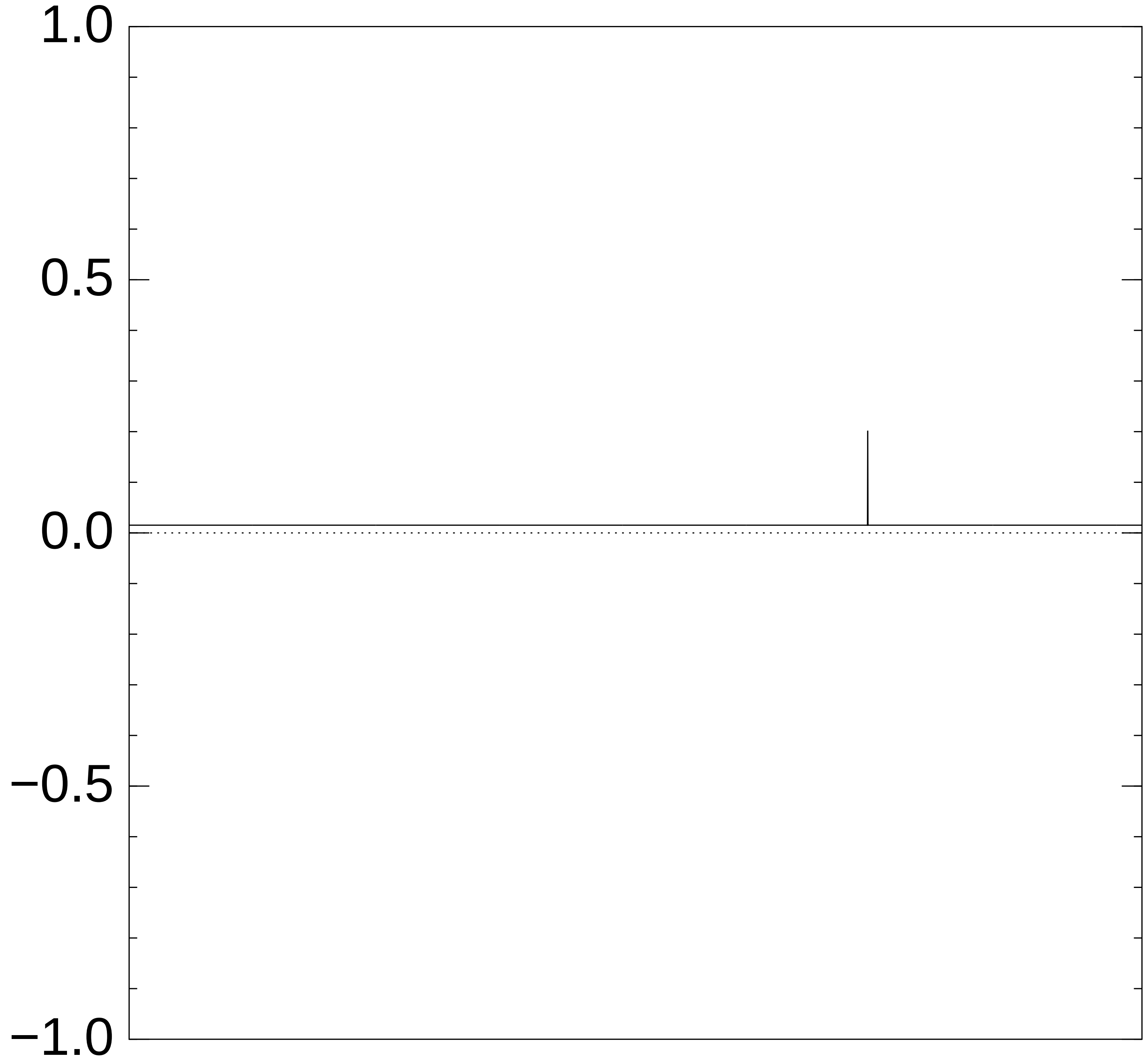
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $6 \times (\text{Step 1} + \text{Step 2})$:



Start from uniform superposition over all n-bit strings q.

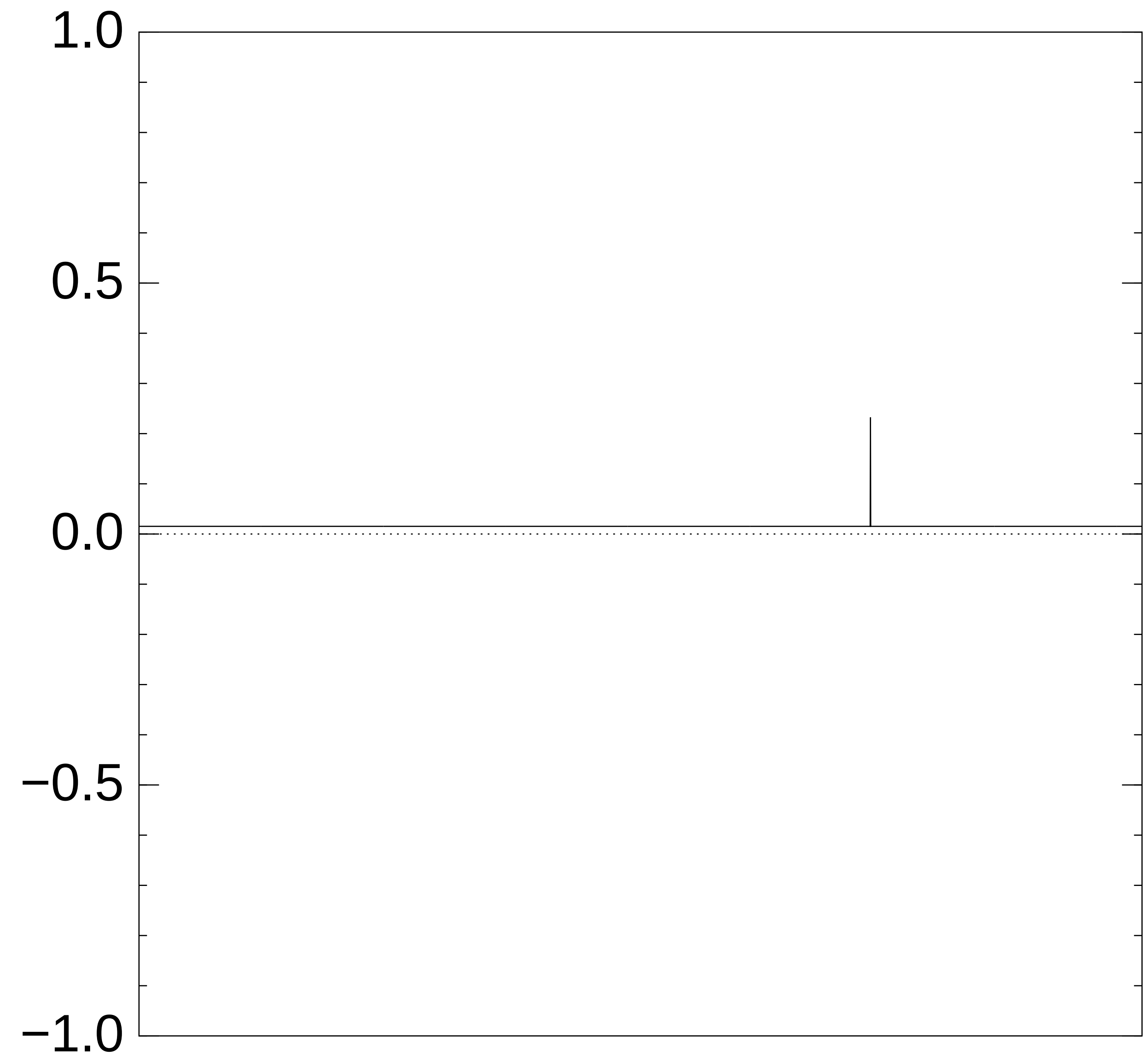
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $7 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

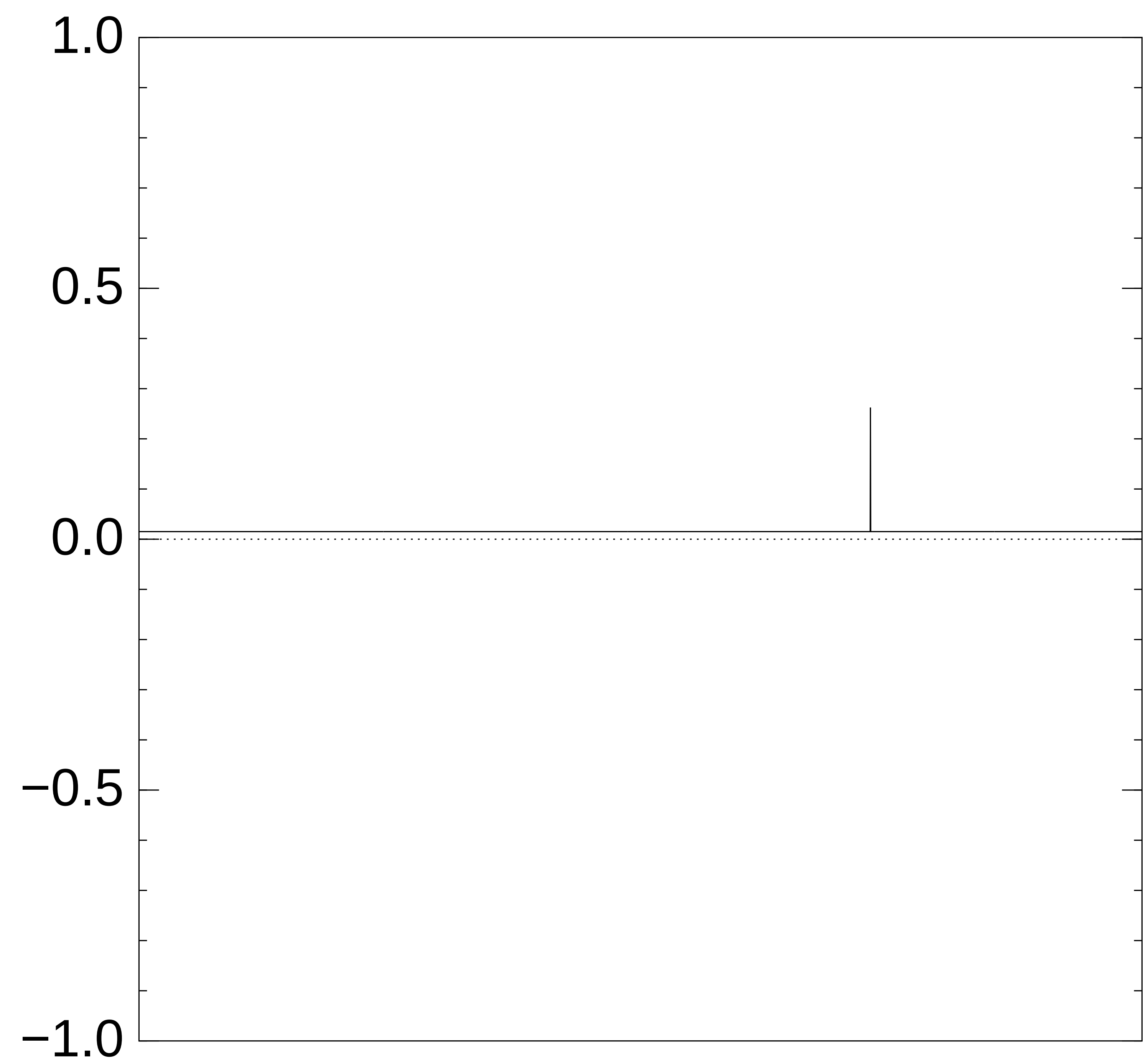
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $8 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

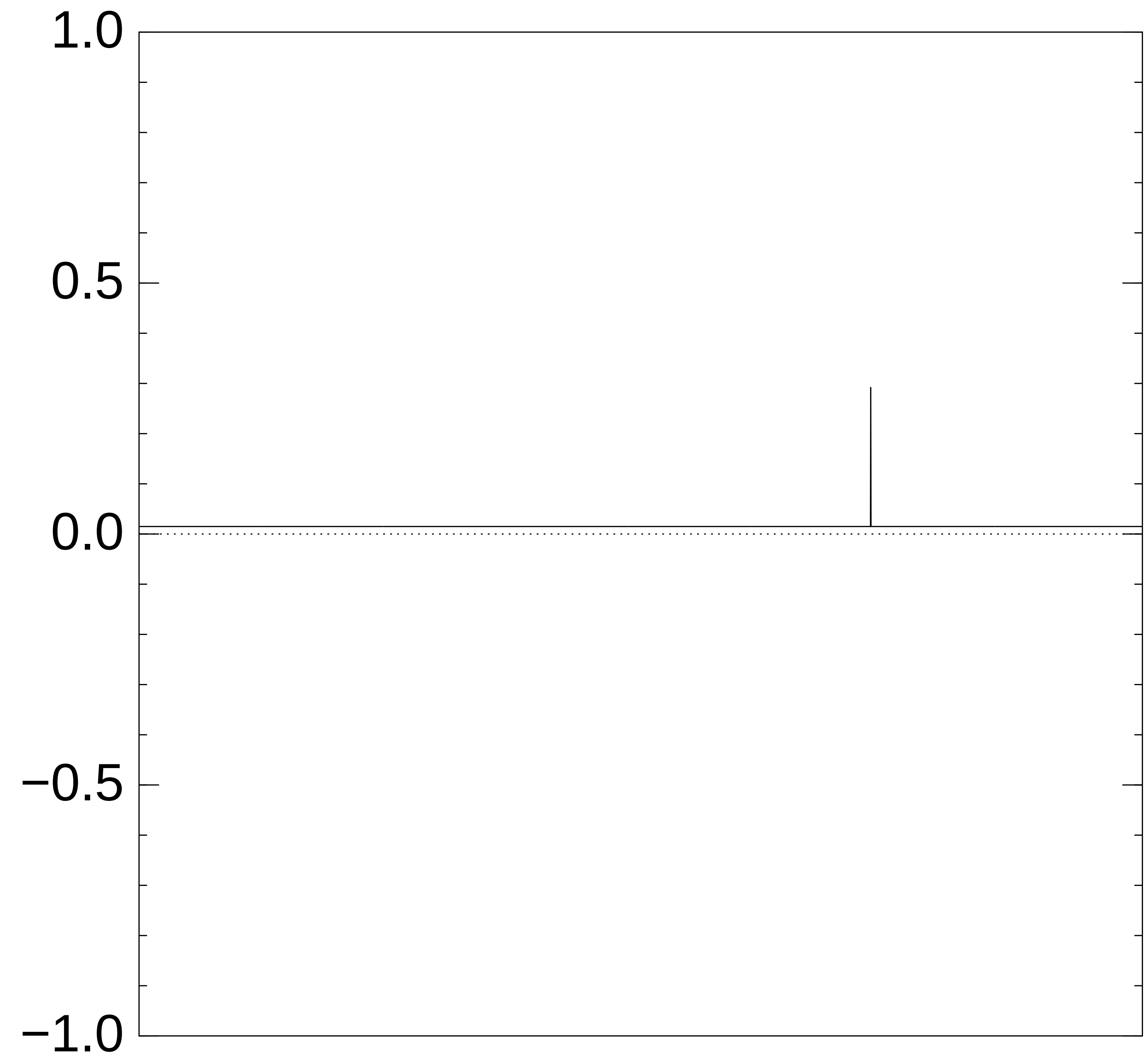
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $9 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

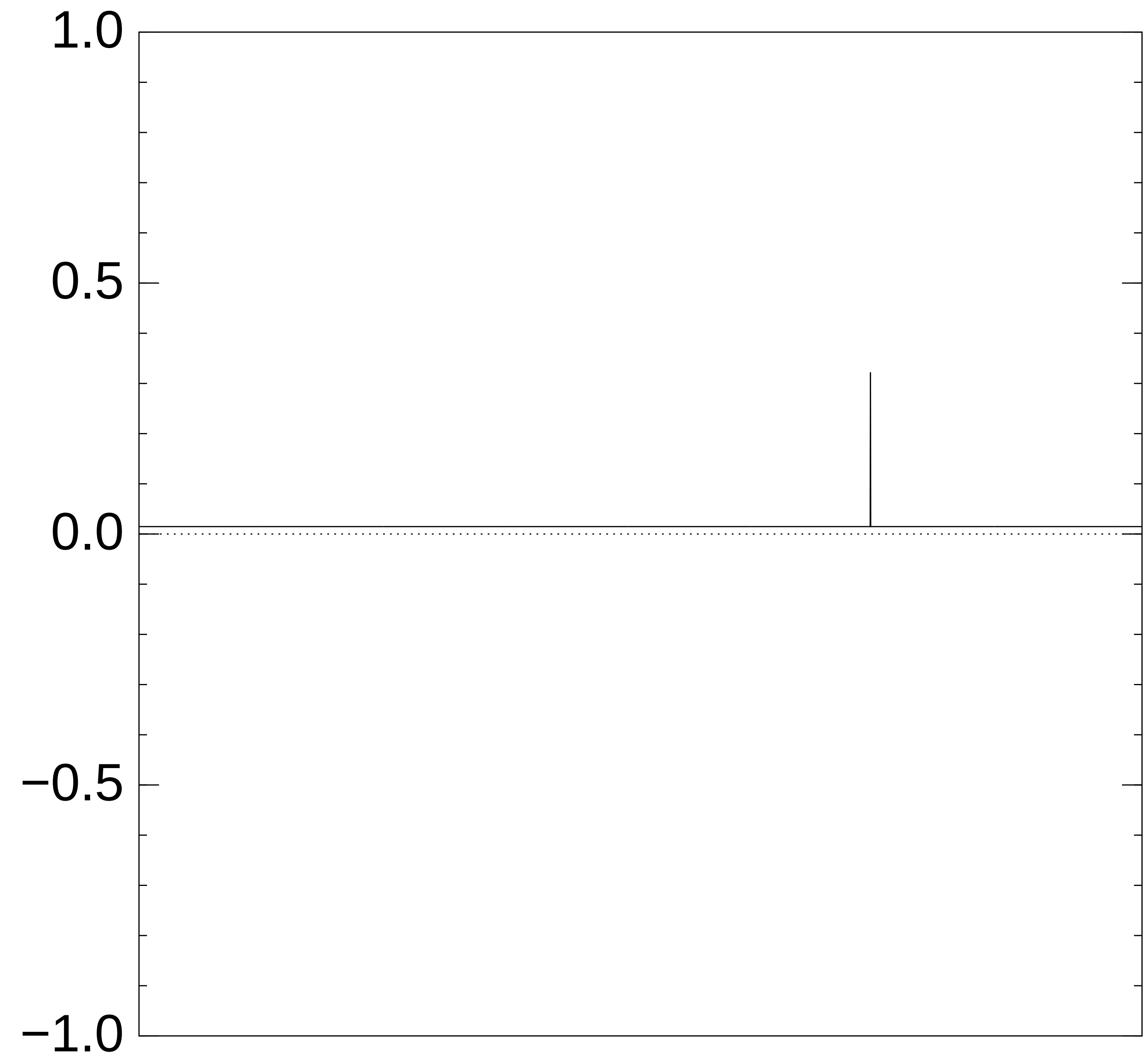
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $10 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

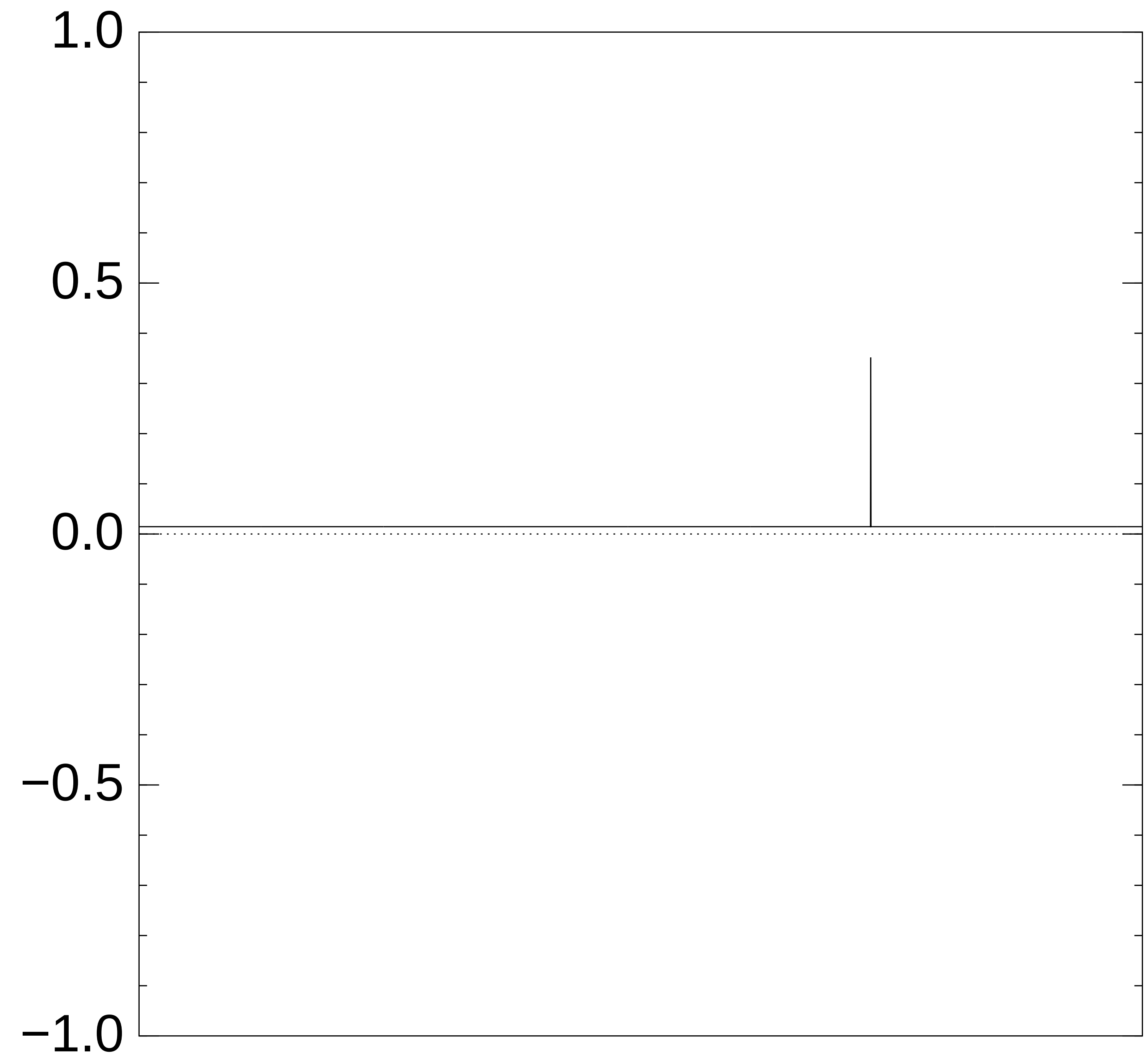
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $11 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

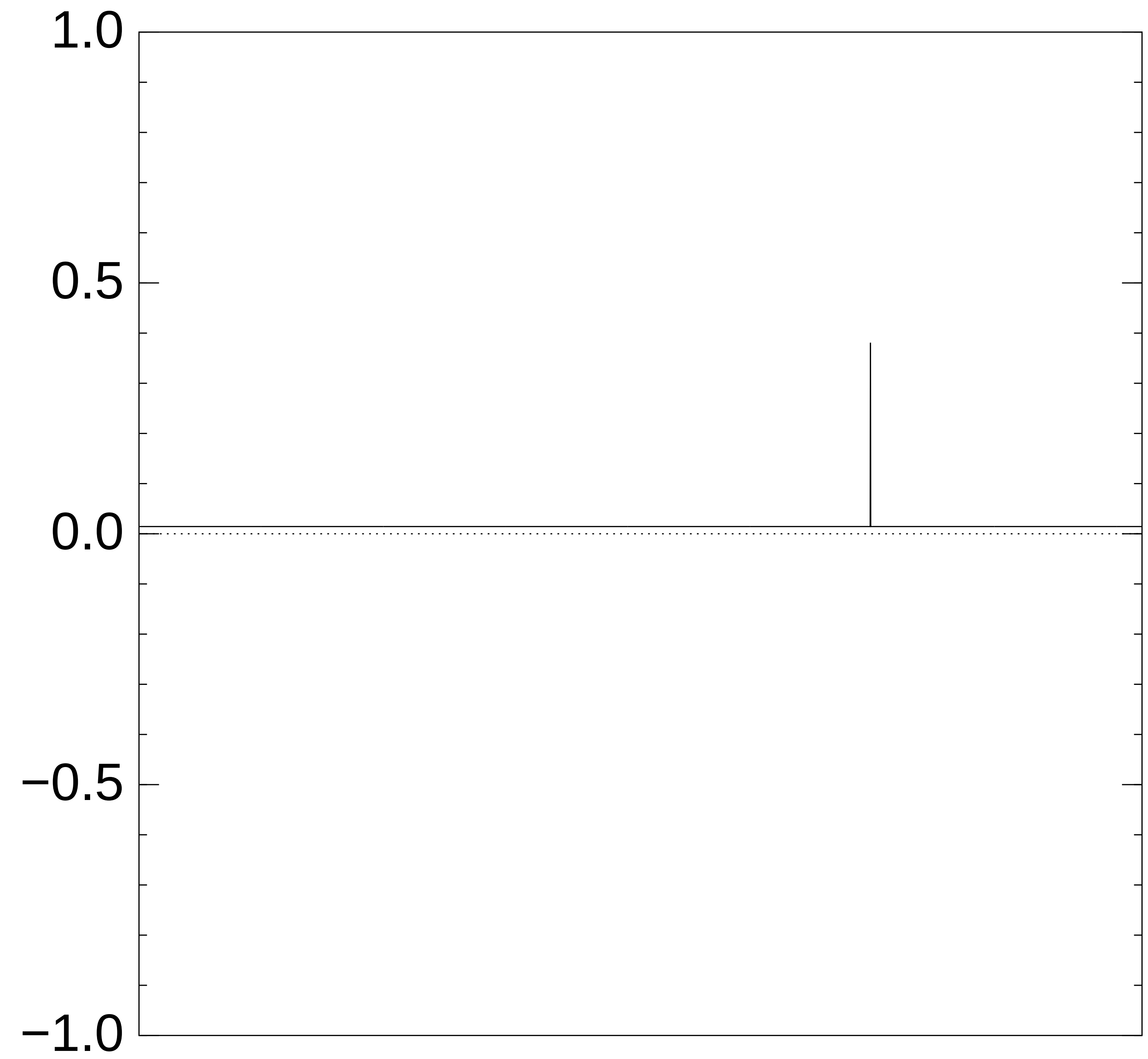
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $12 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

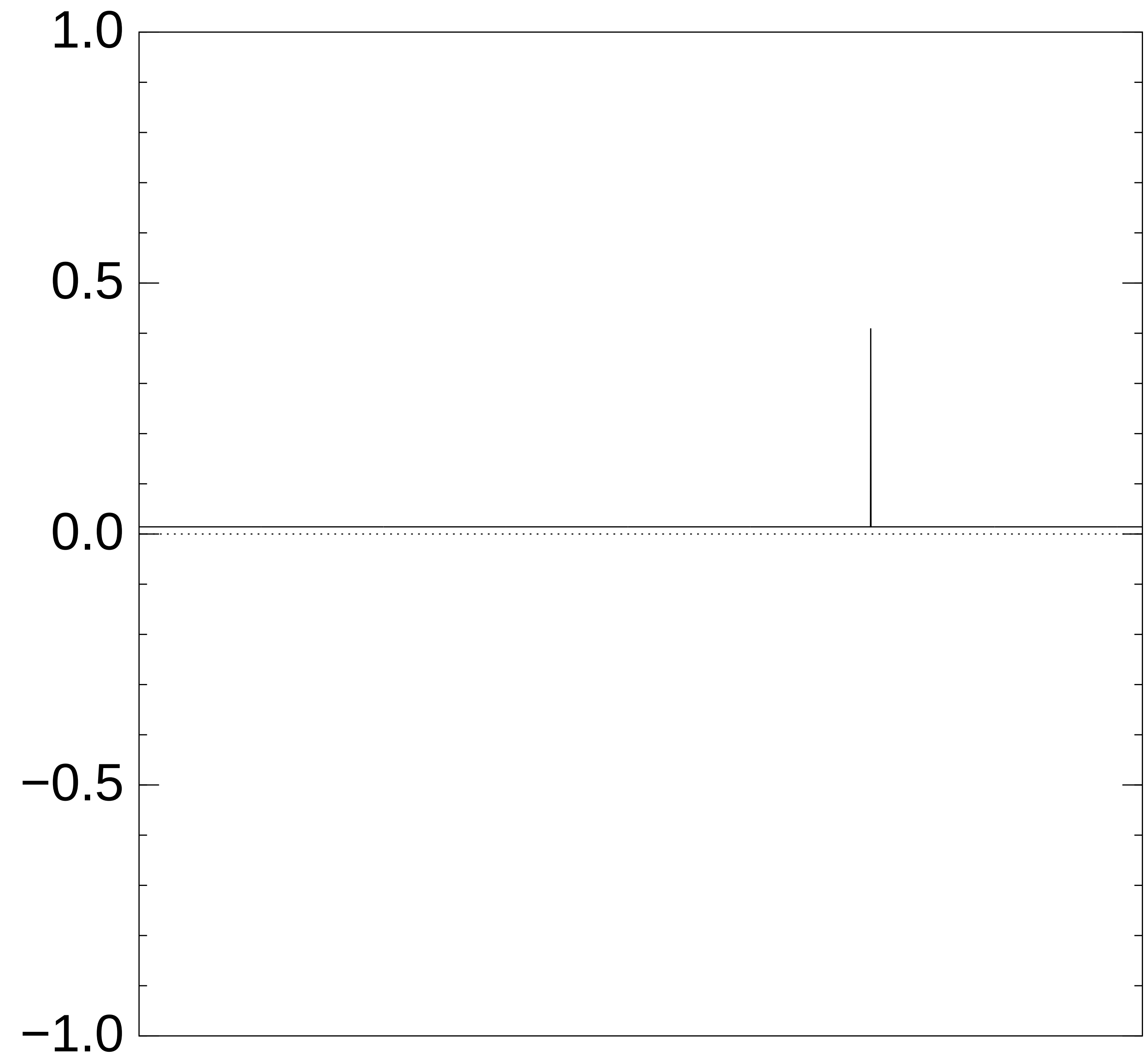
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $13 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

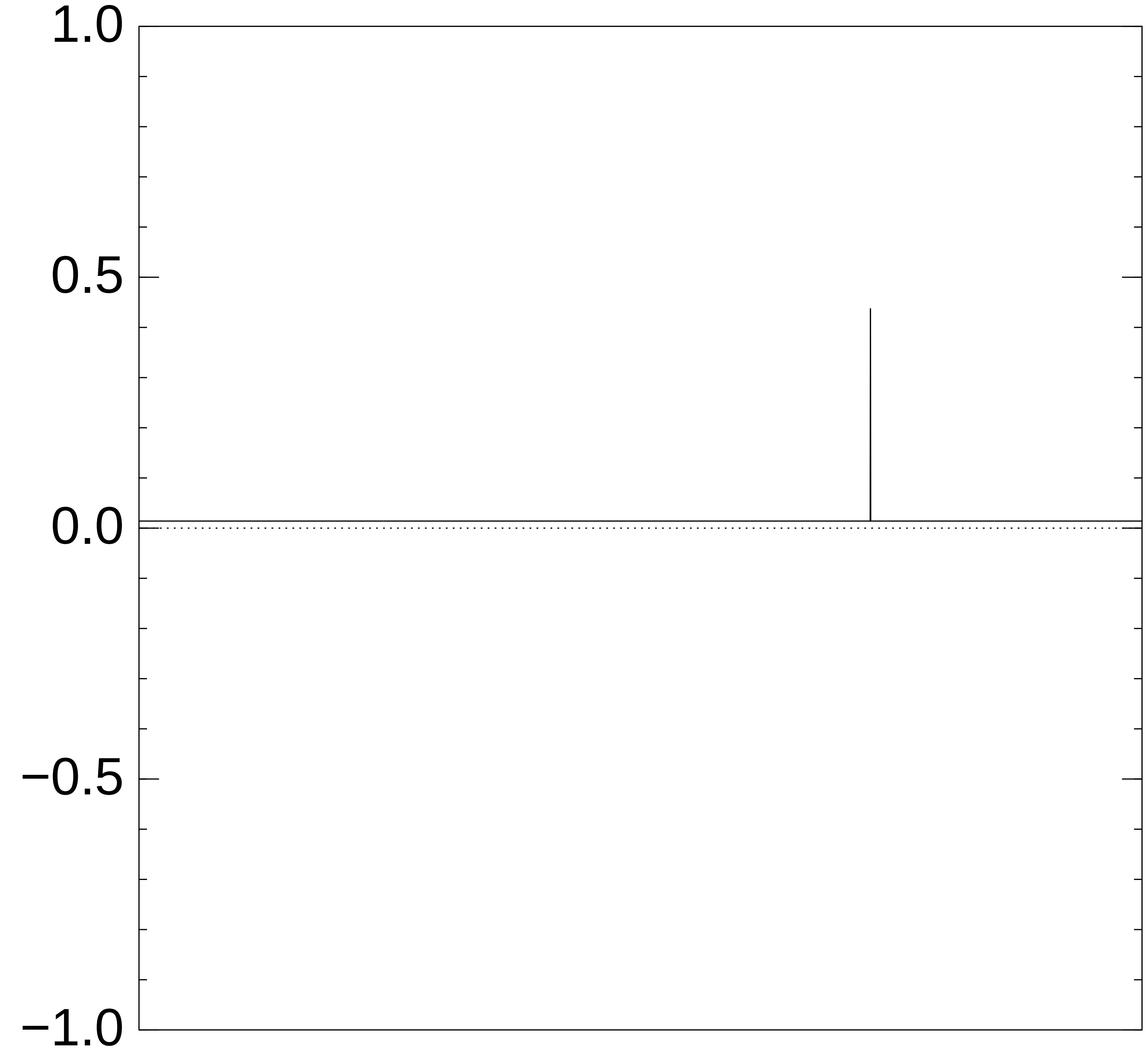
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $14 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

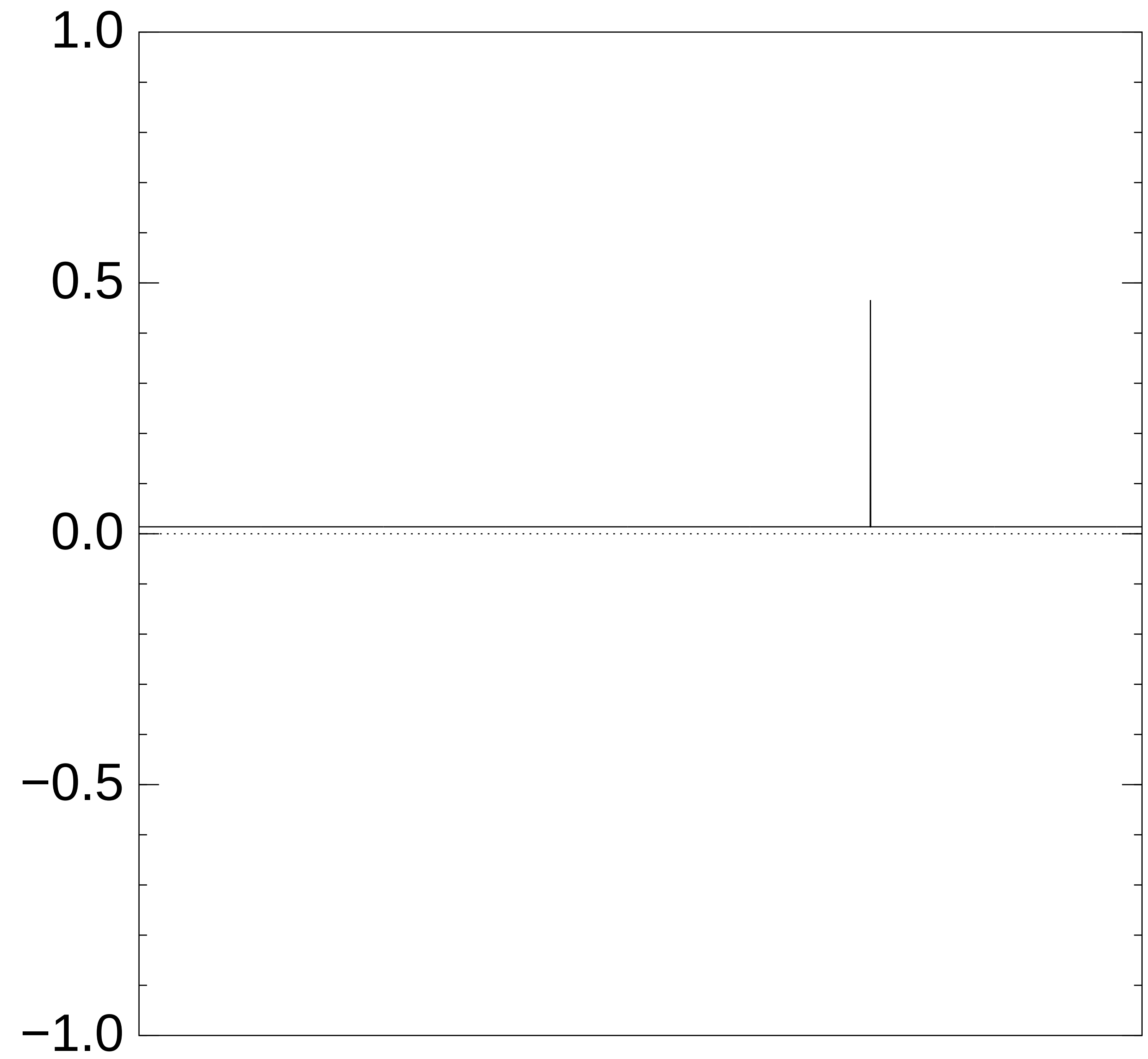
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $15 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

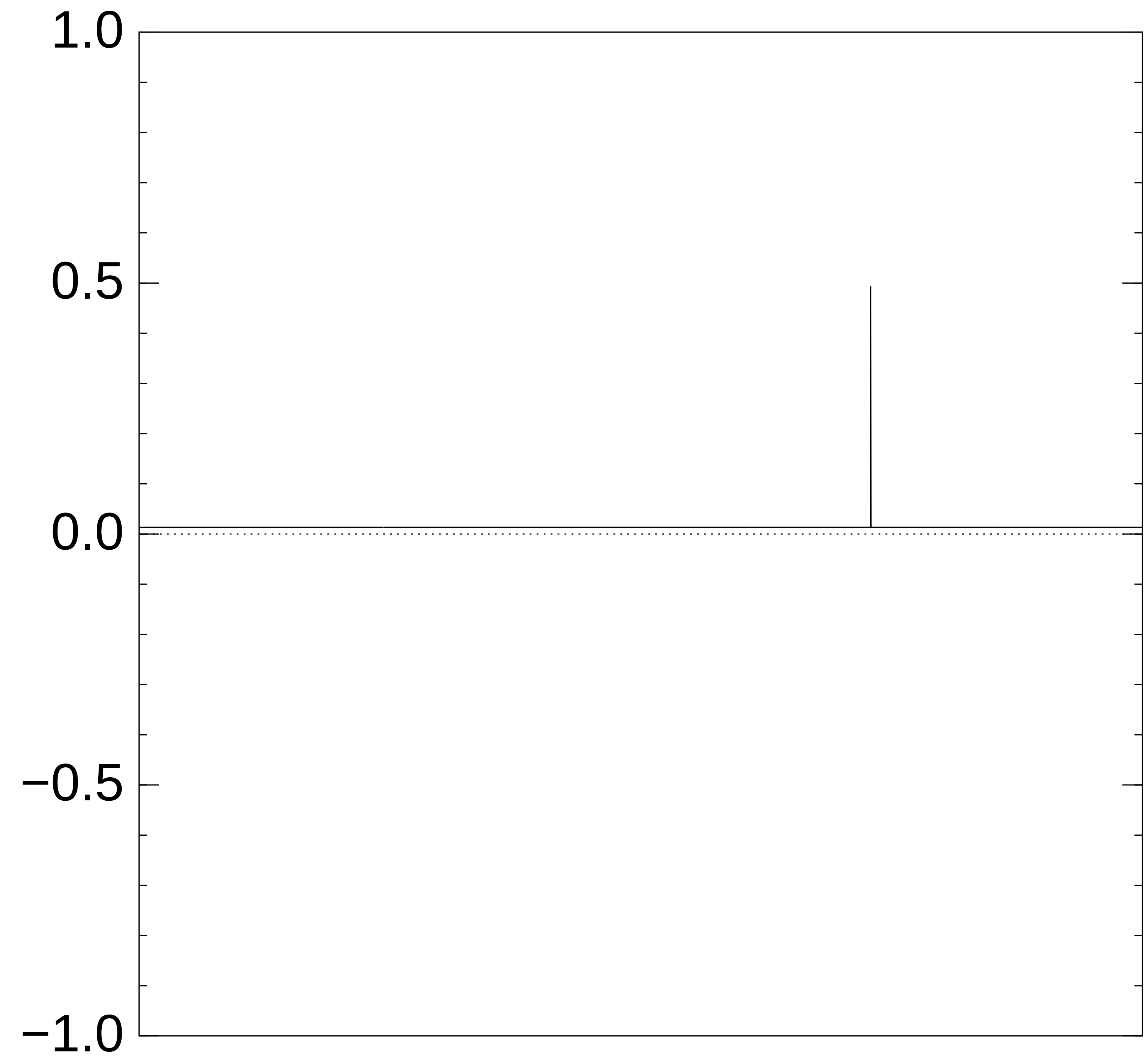
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $16 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

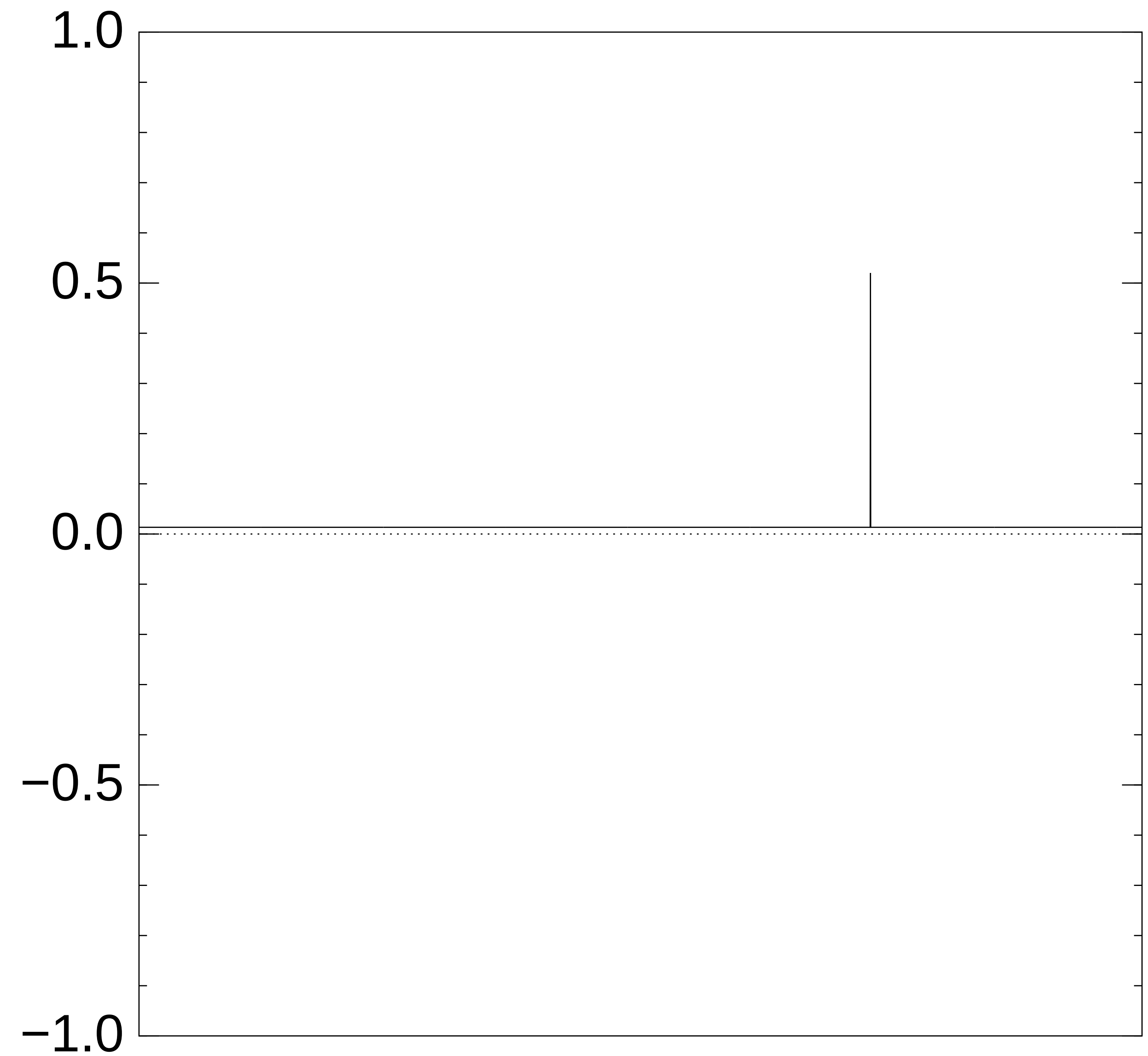
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $17 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

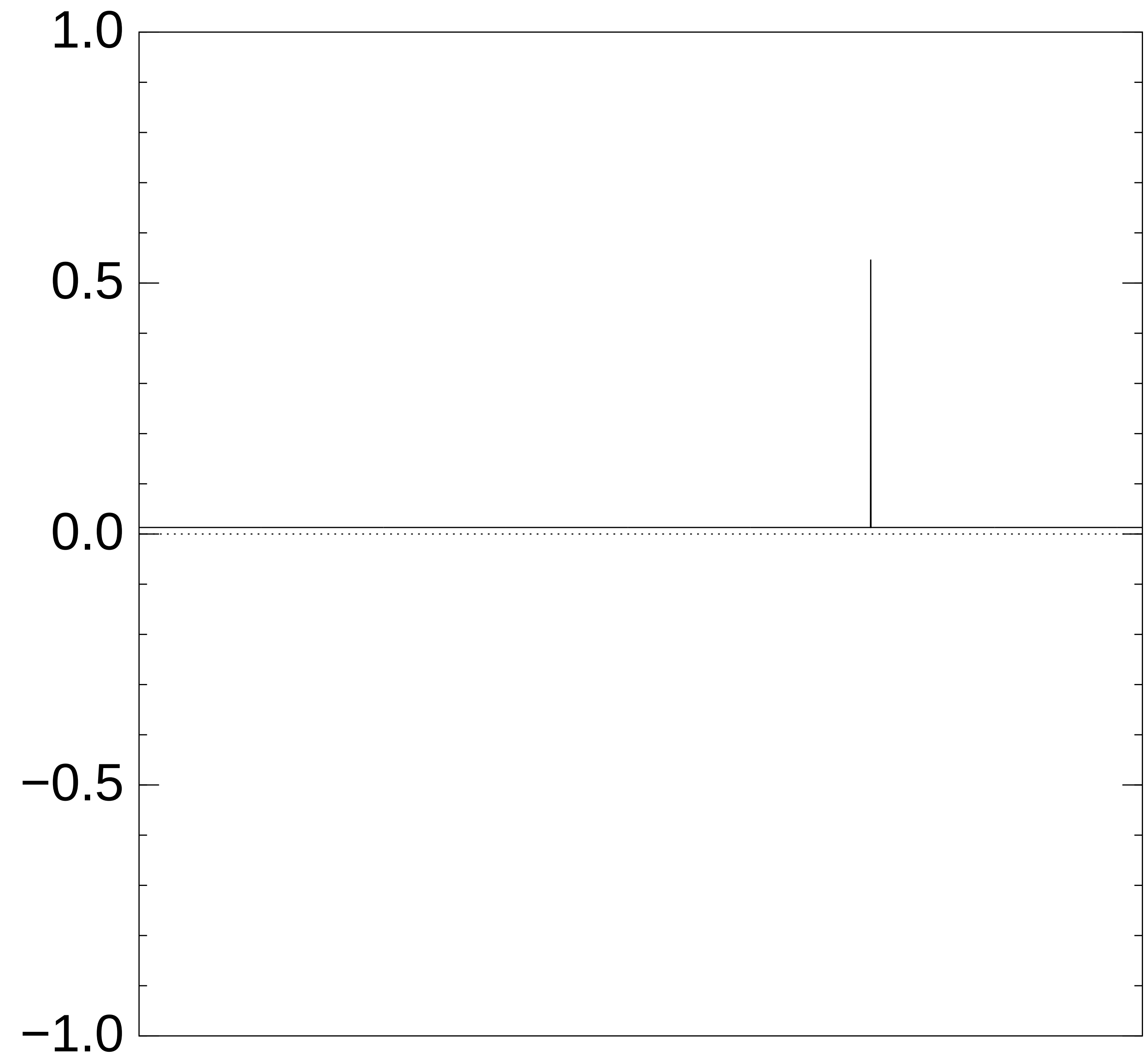
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $18 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

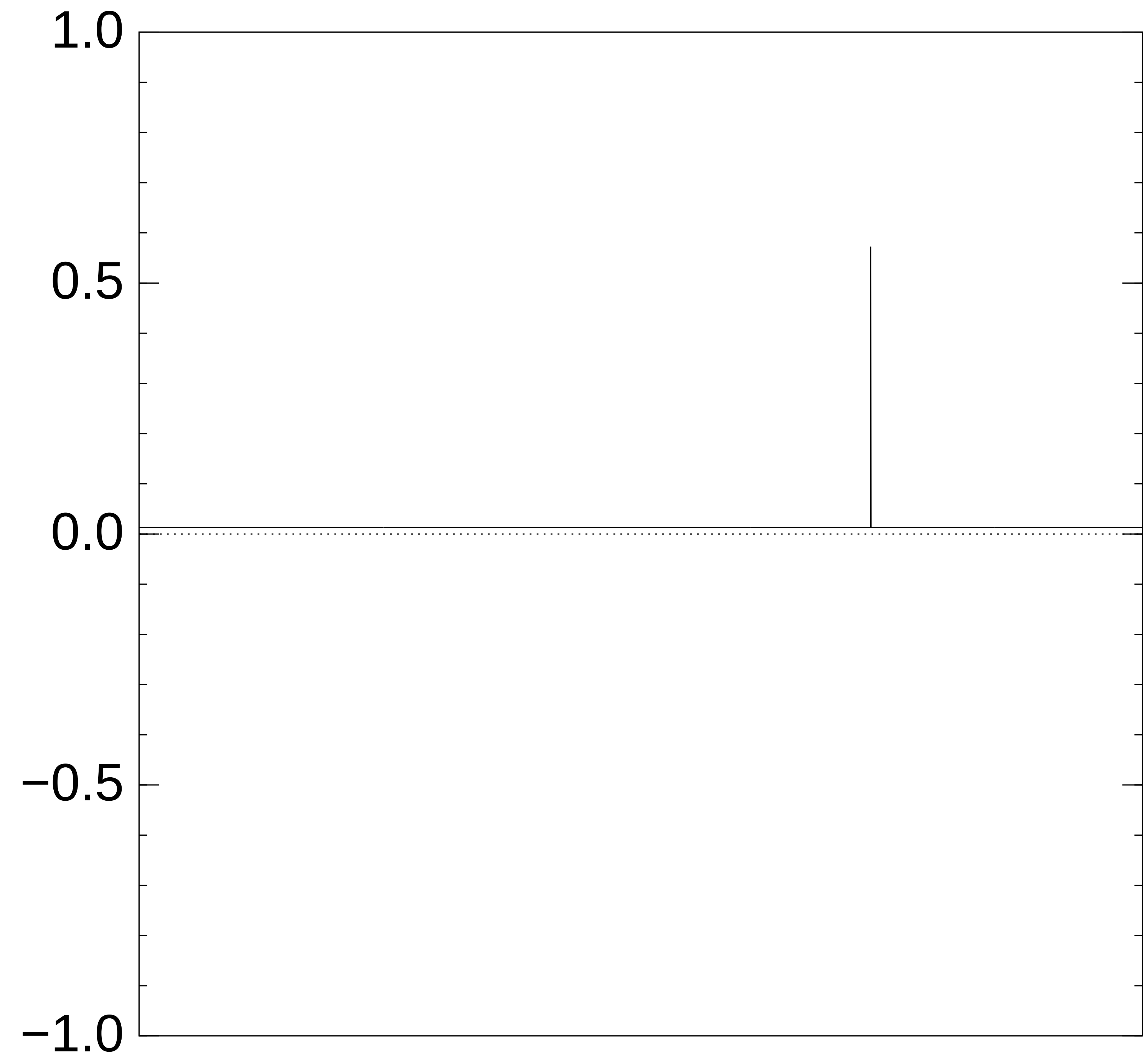
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $19 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

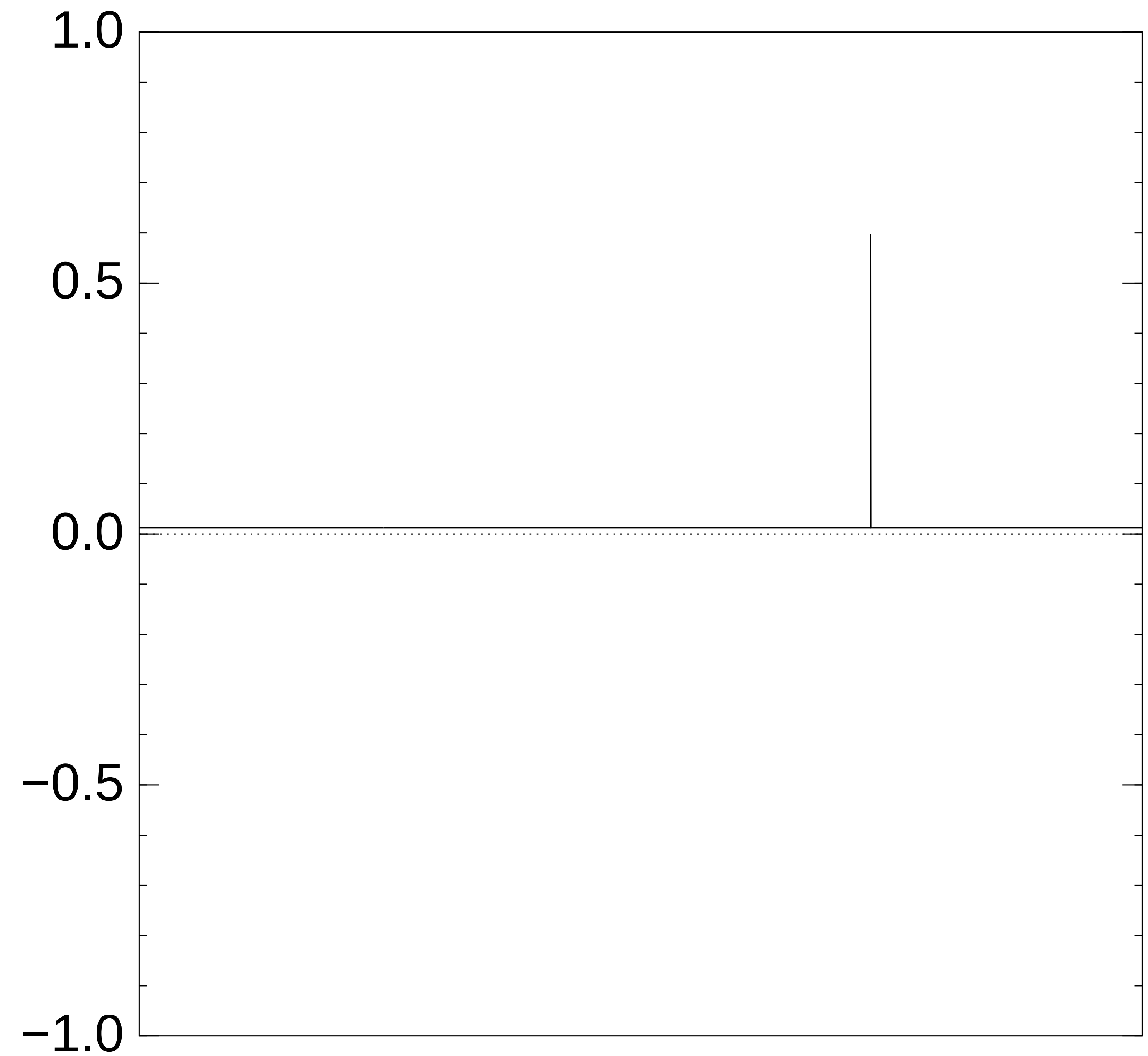
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $20 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

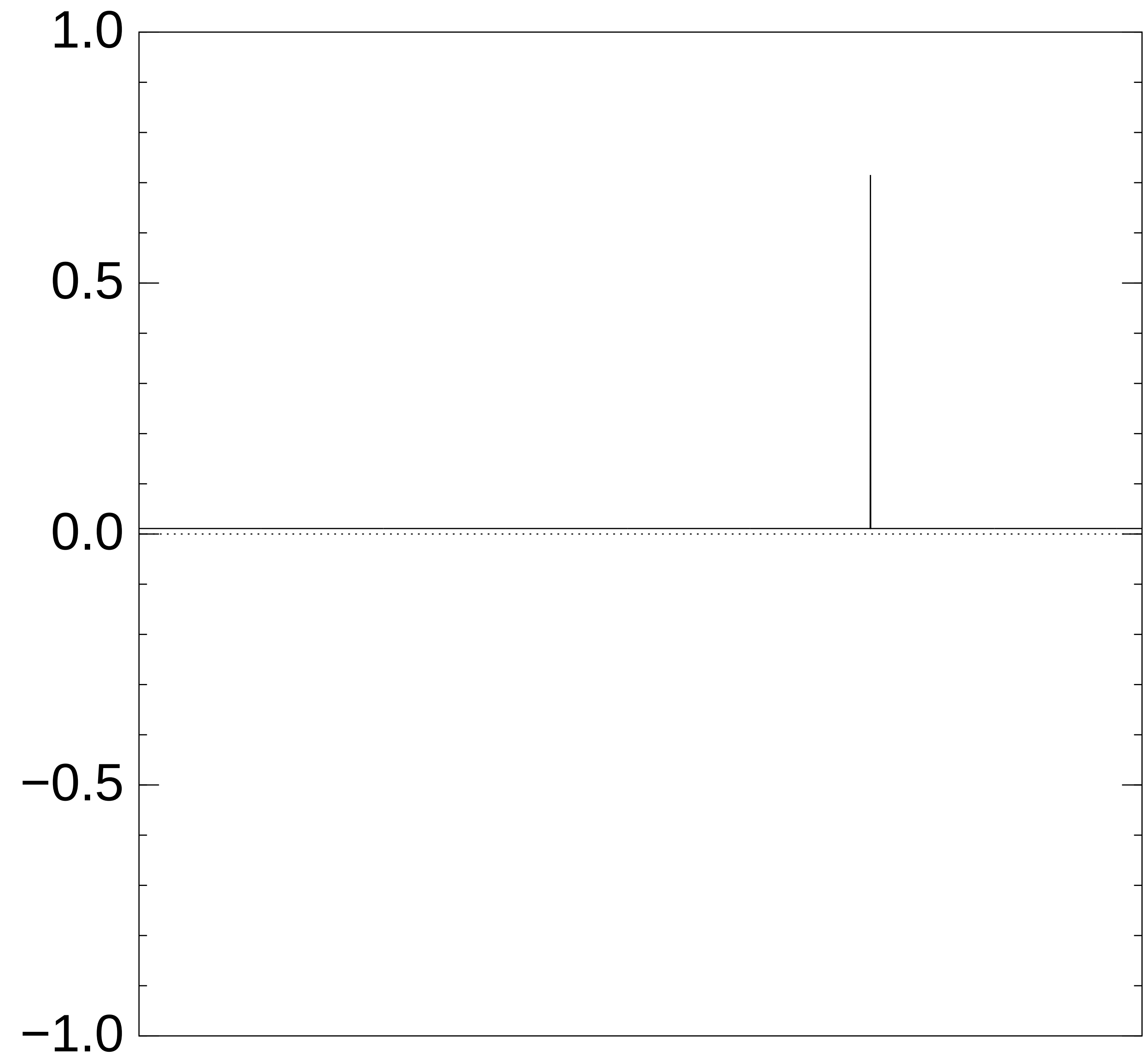
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $25 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

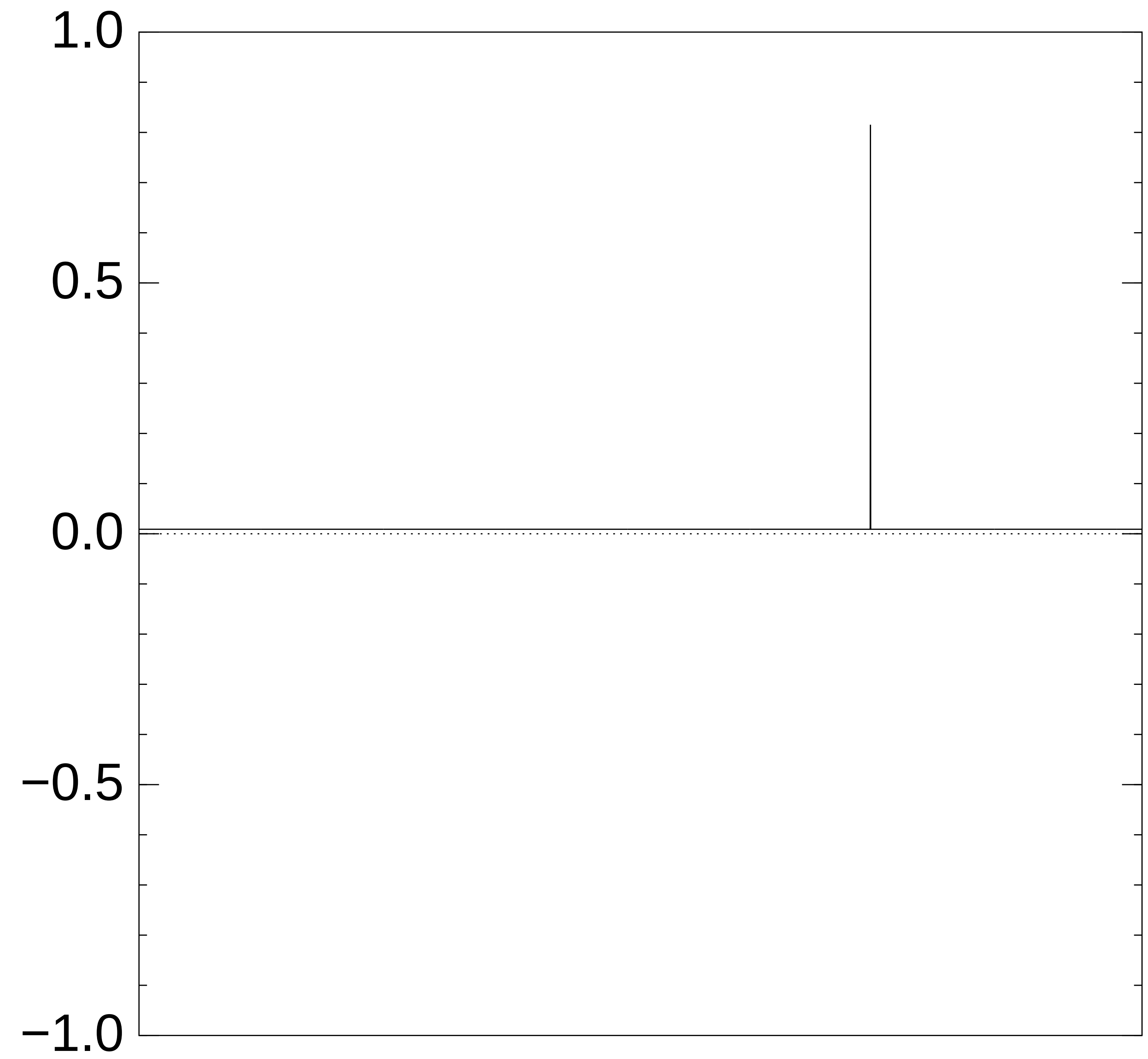
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $30 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

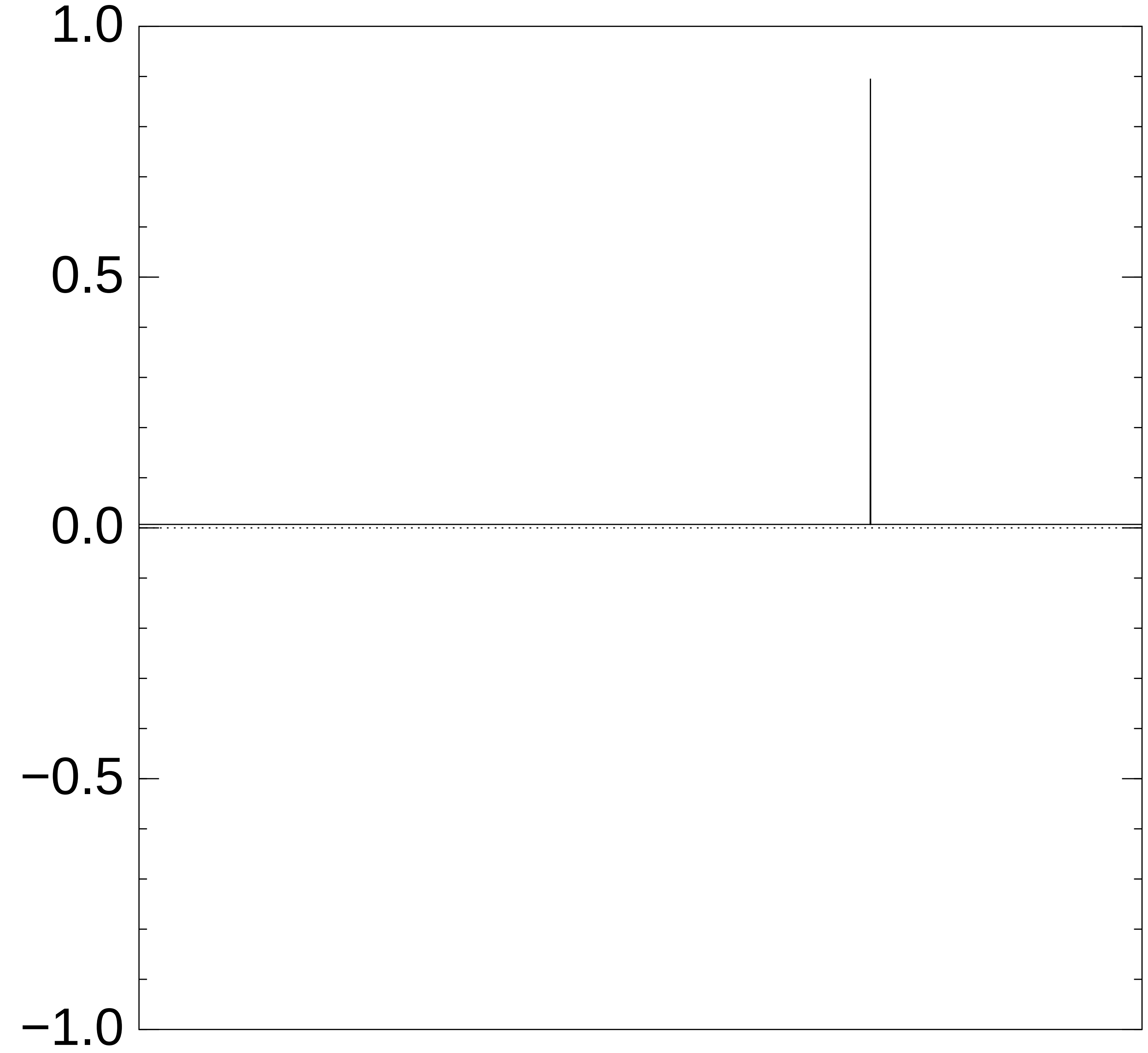
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $35 \times$ (Step 1 + Step 2):



Good moment to stop, measure.

Start from uniform superposition over all n-bit strings q.

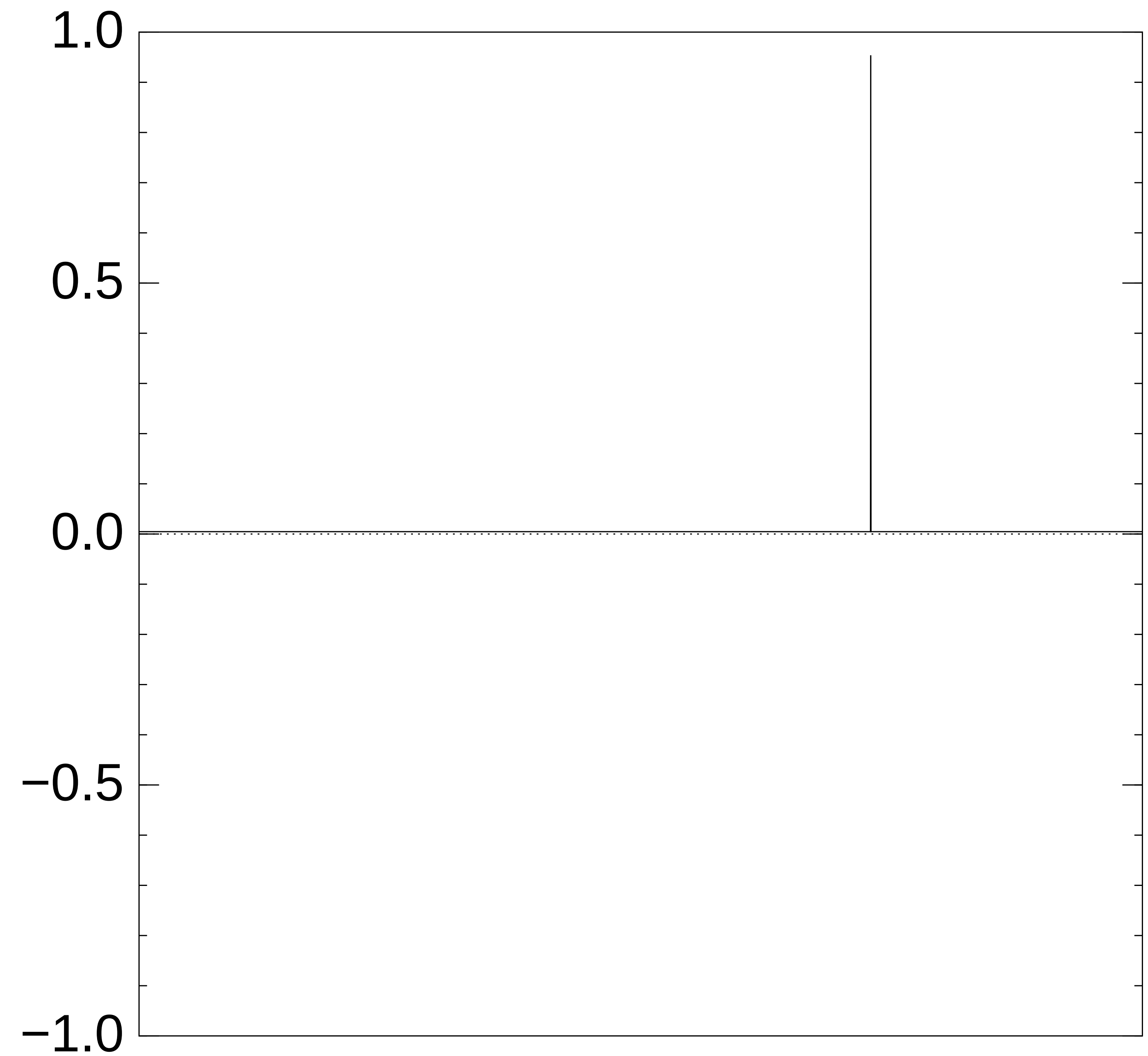
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $40 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

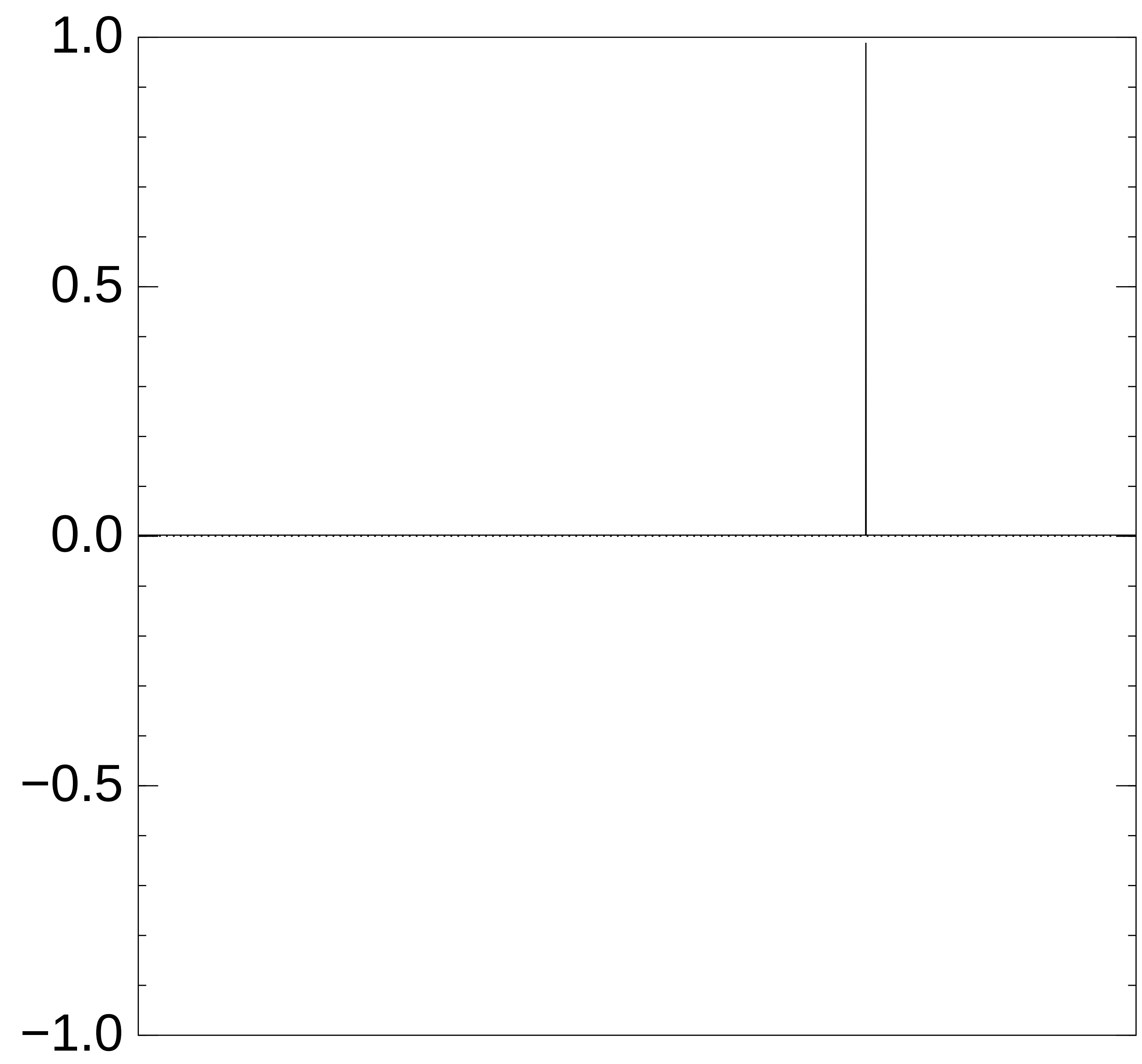
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $45 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

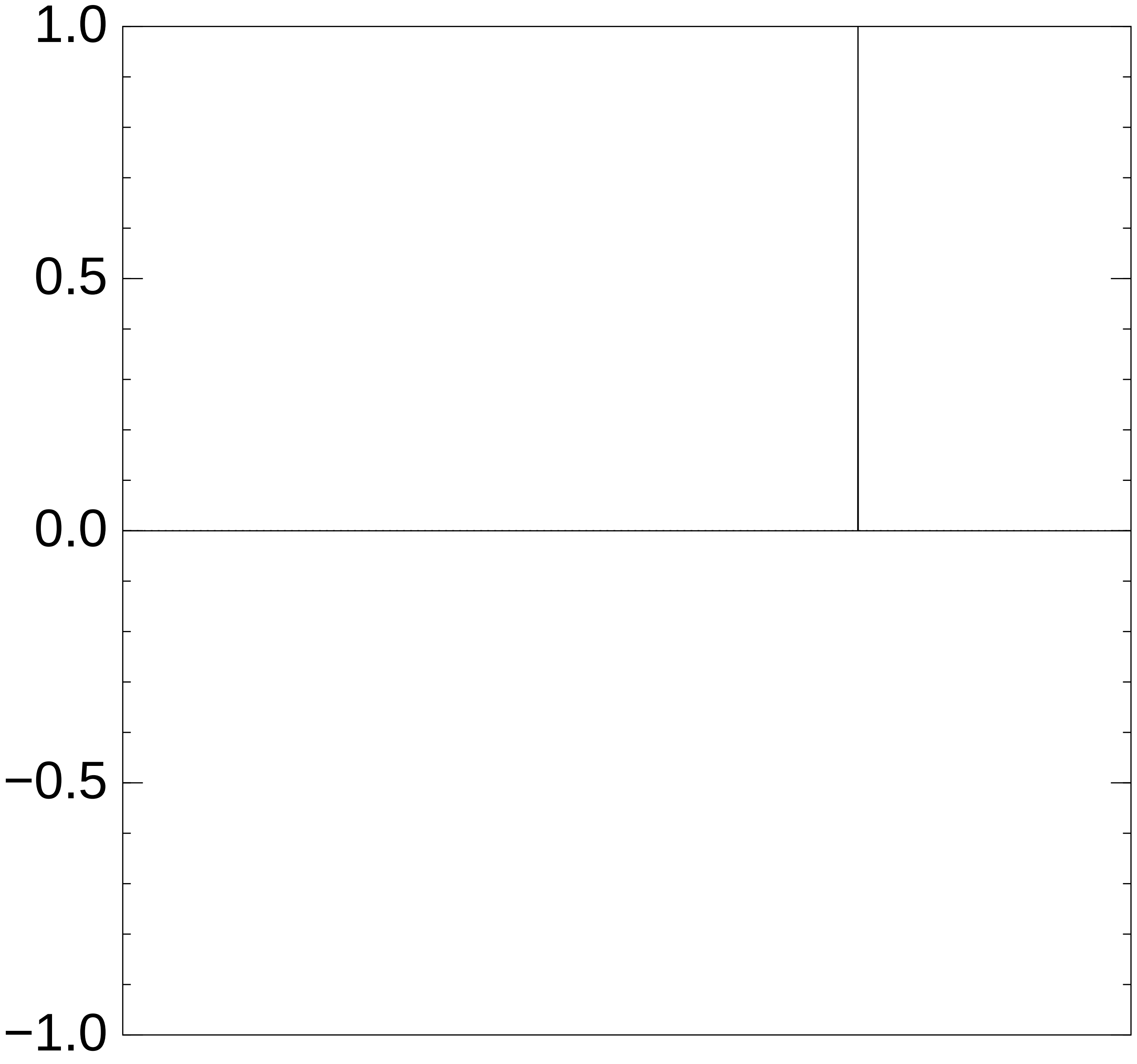
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $50 \times$ (Step 1 + Step 2):



Traditional stopping point.

Start from uniform superposition over all n-bit strings q.

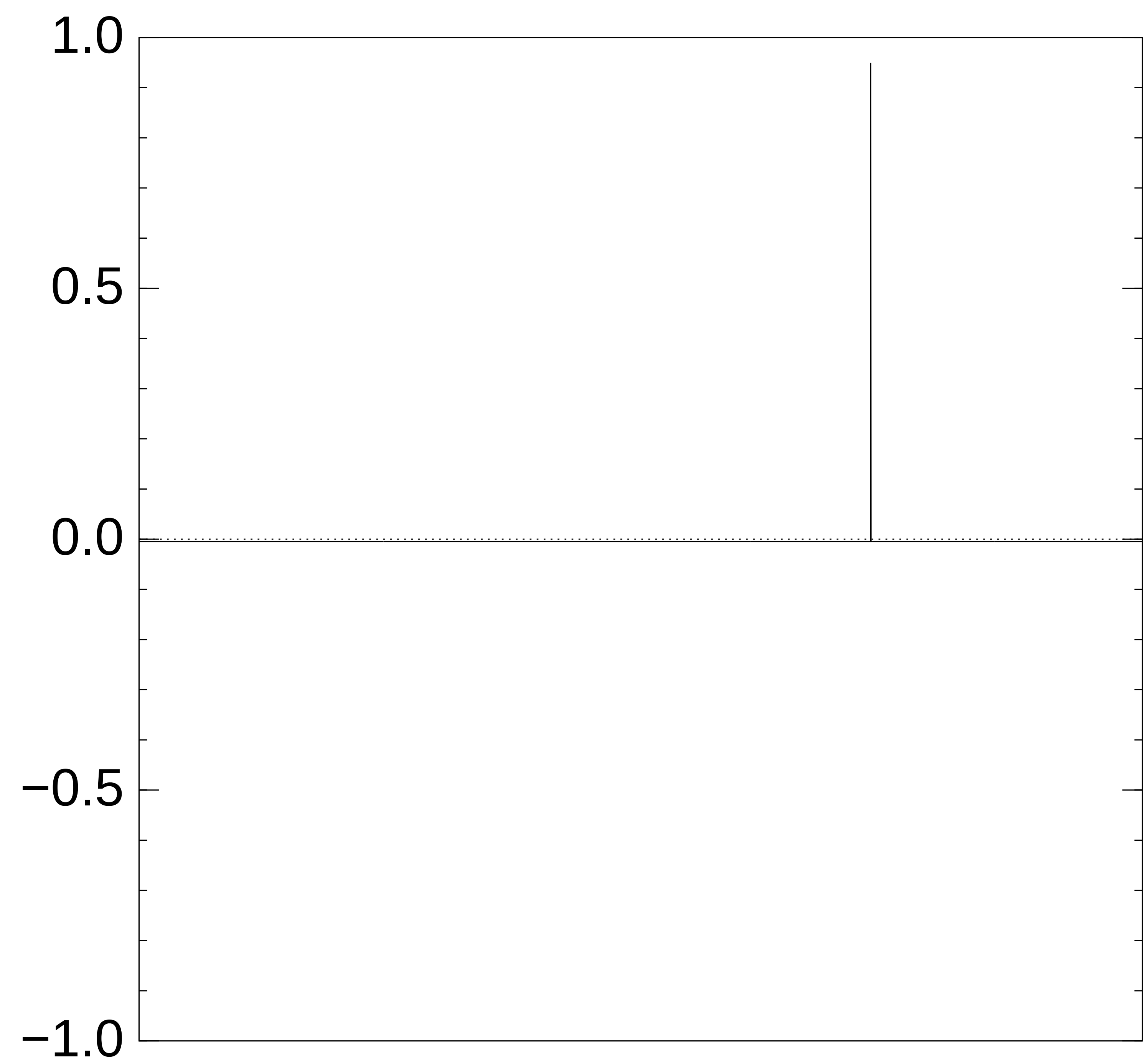
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $60 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

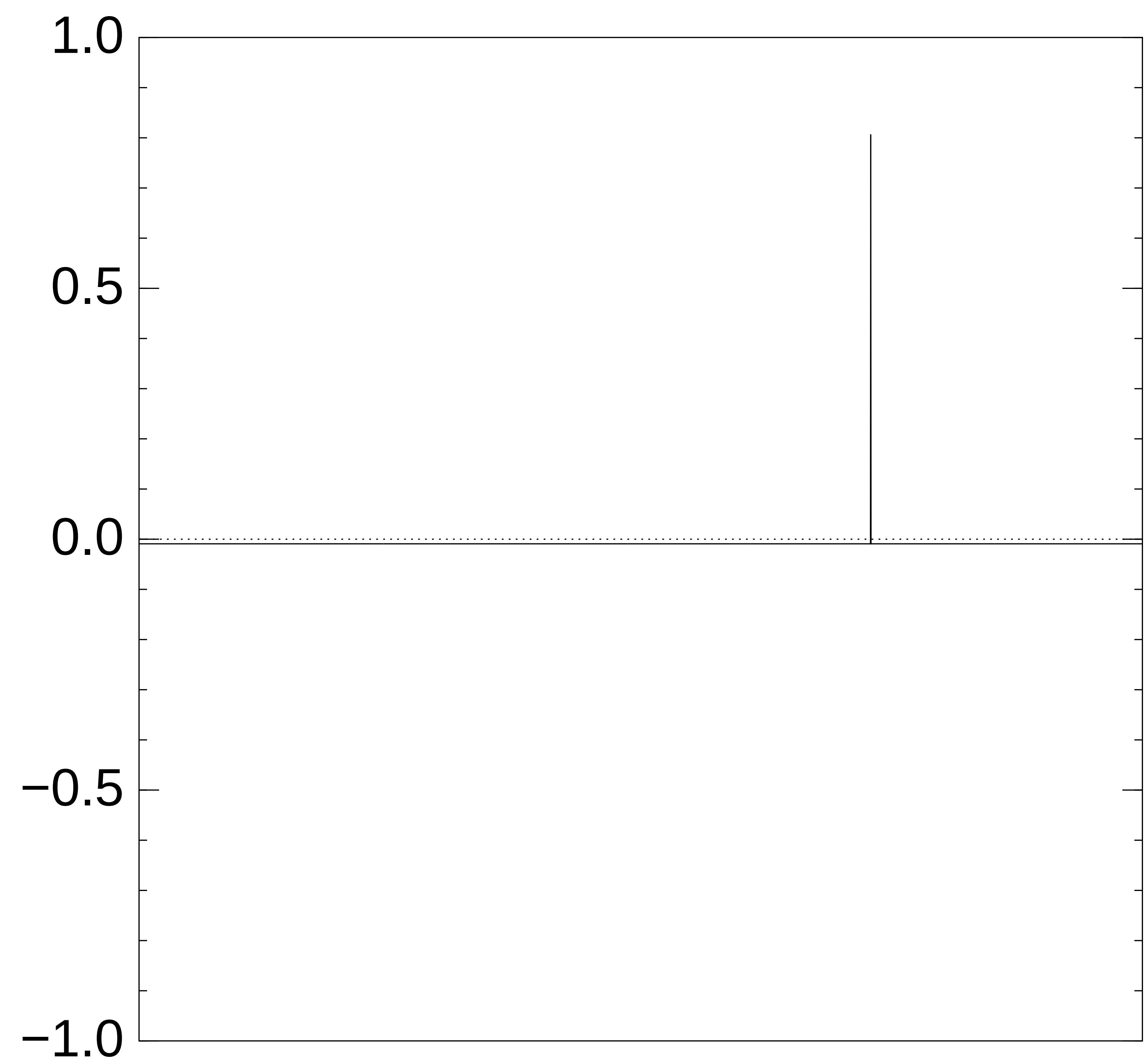
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $70 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

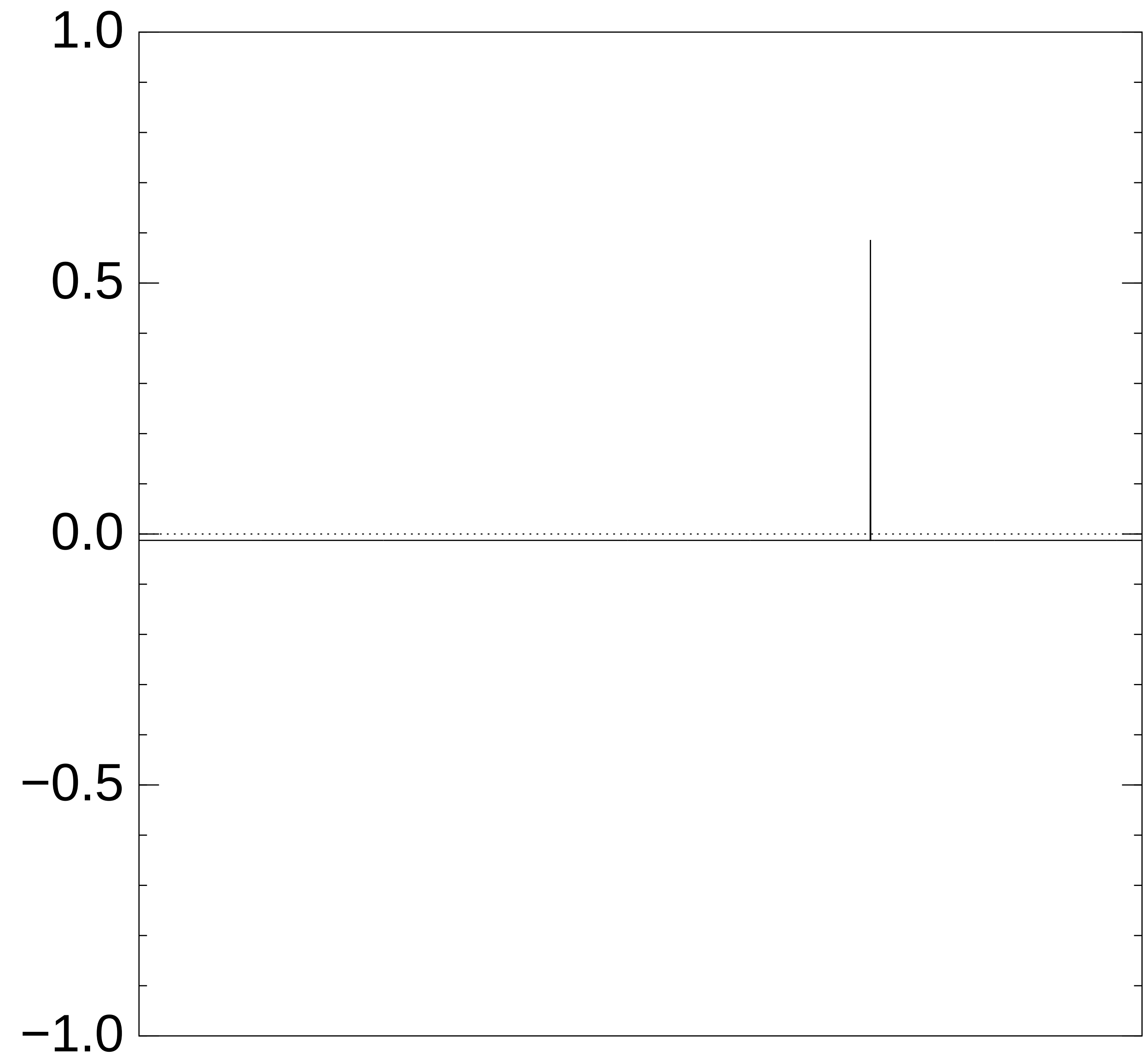
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $80 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

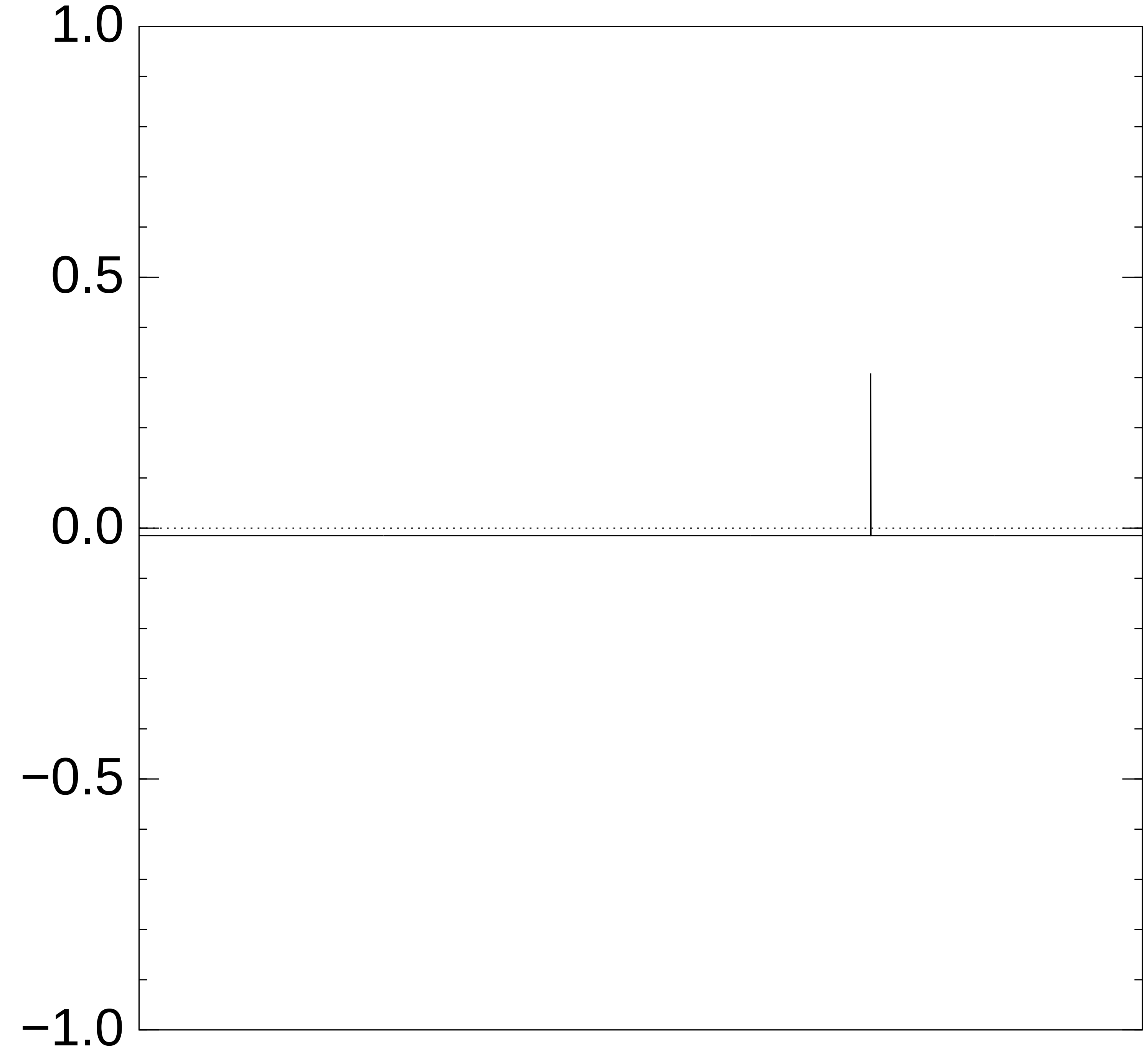
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $90 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

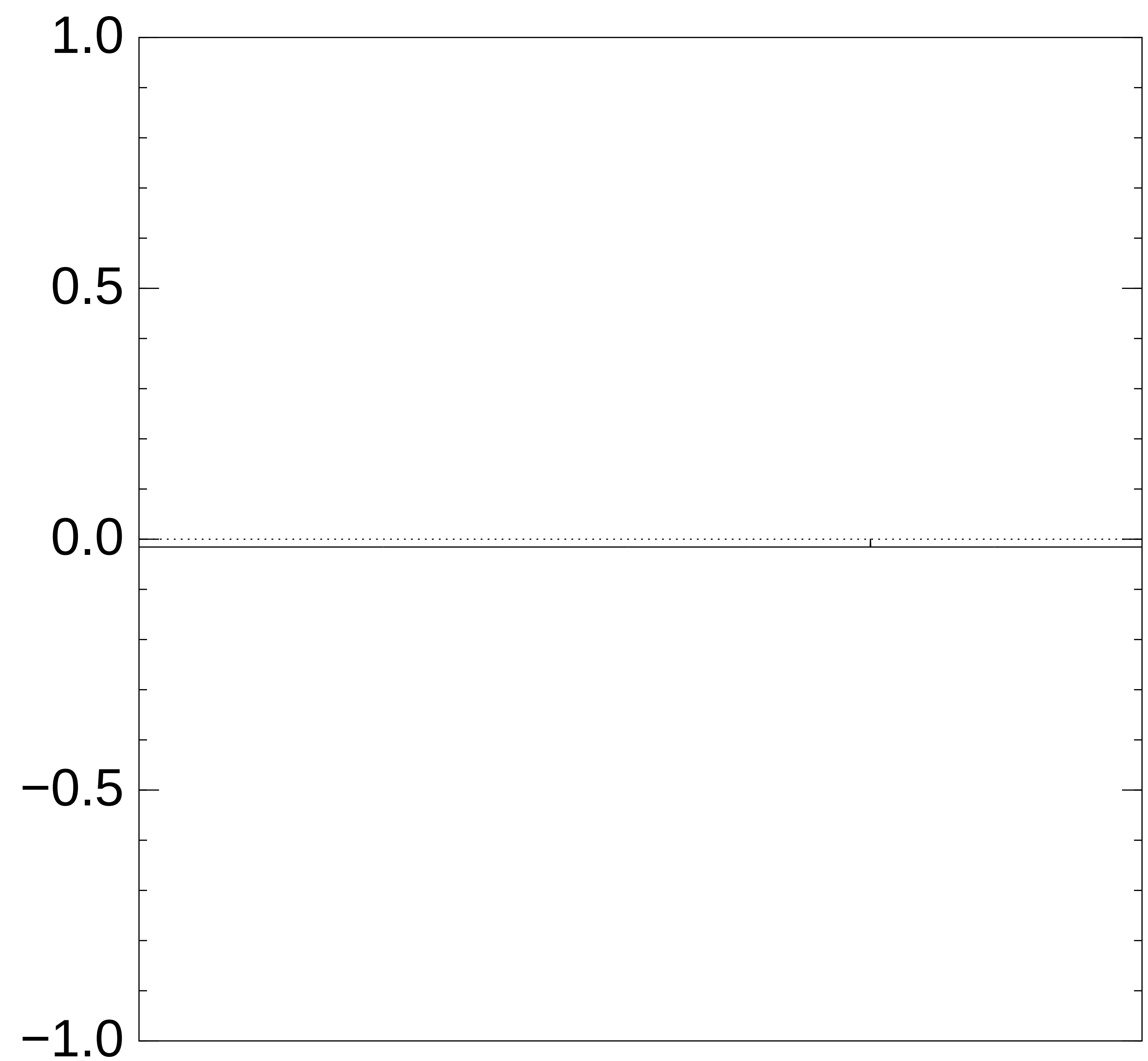
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $100 \times$ (Step 1 + Step 2):



Very bad stopping point.

om uniform superposition
n-bit strings q.

Set $a \leftarrow b$ where
a_q if $f(q) = 0$,
otherwise.
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"Grover diffusion".
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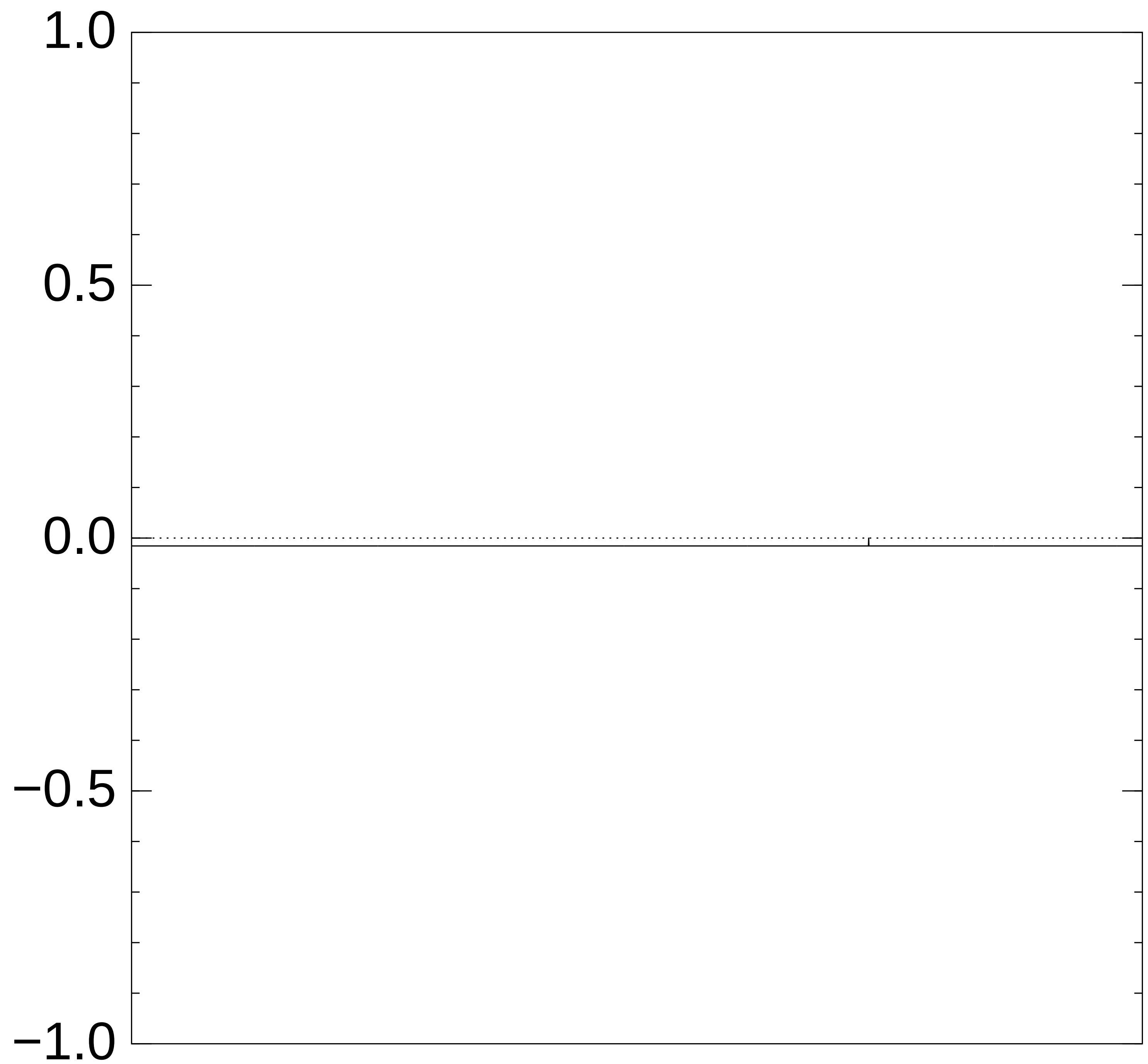
Step 1 + Step 2
$58 \cdot 2^{0.5n}$ times.

the n qubits.
gh probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $100 \times$ (Step 1 + Step 2):



Very bad stopping point.

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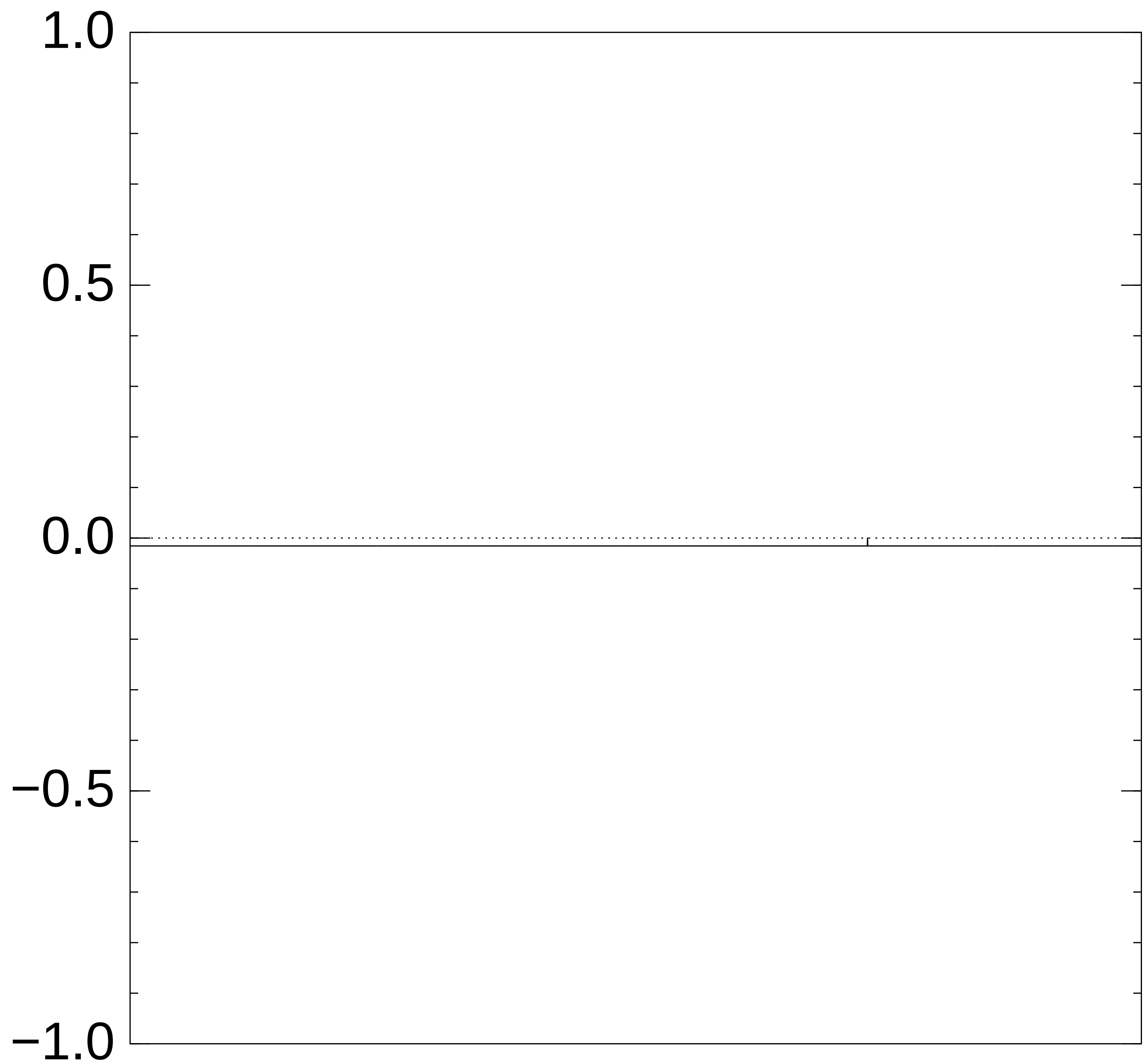
Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $100 \times (\text{Step } 1 + \text{Step } 2)$:



Very bad stopping point.

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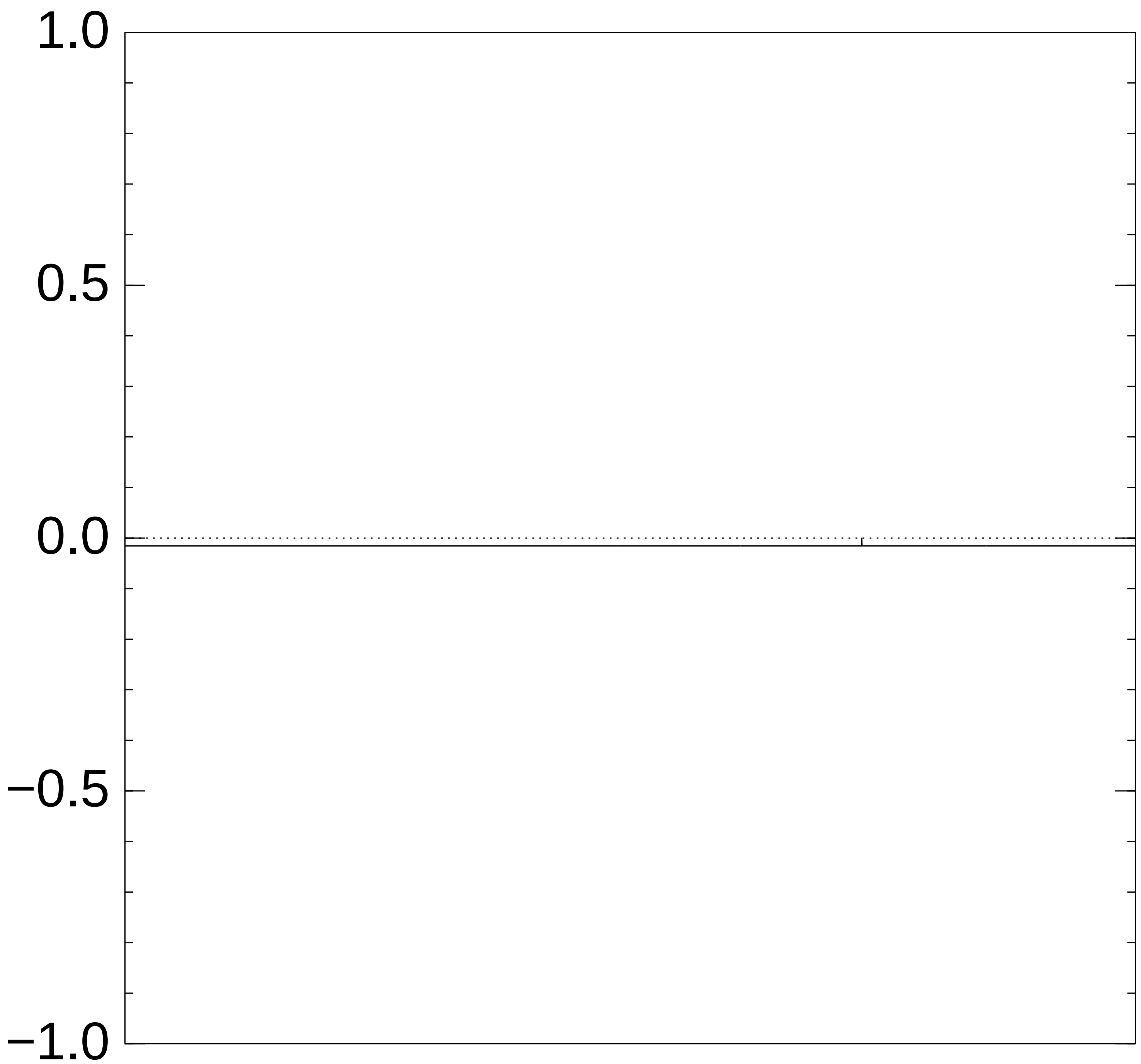

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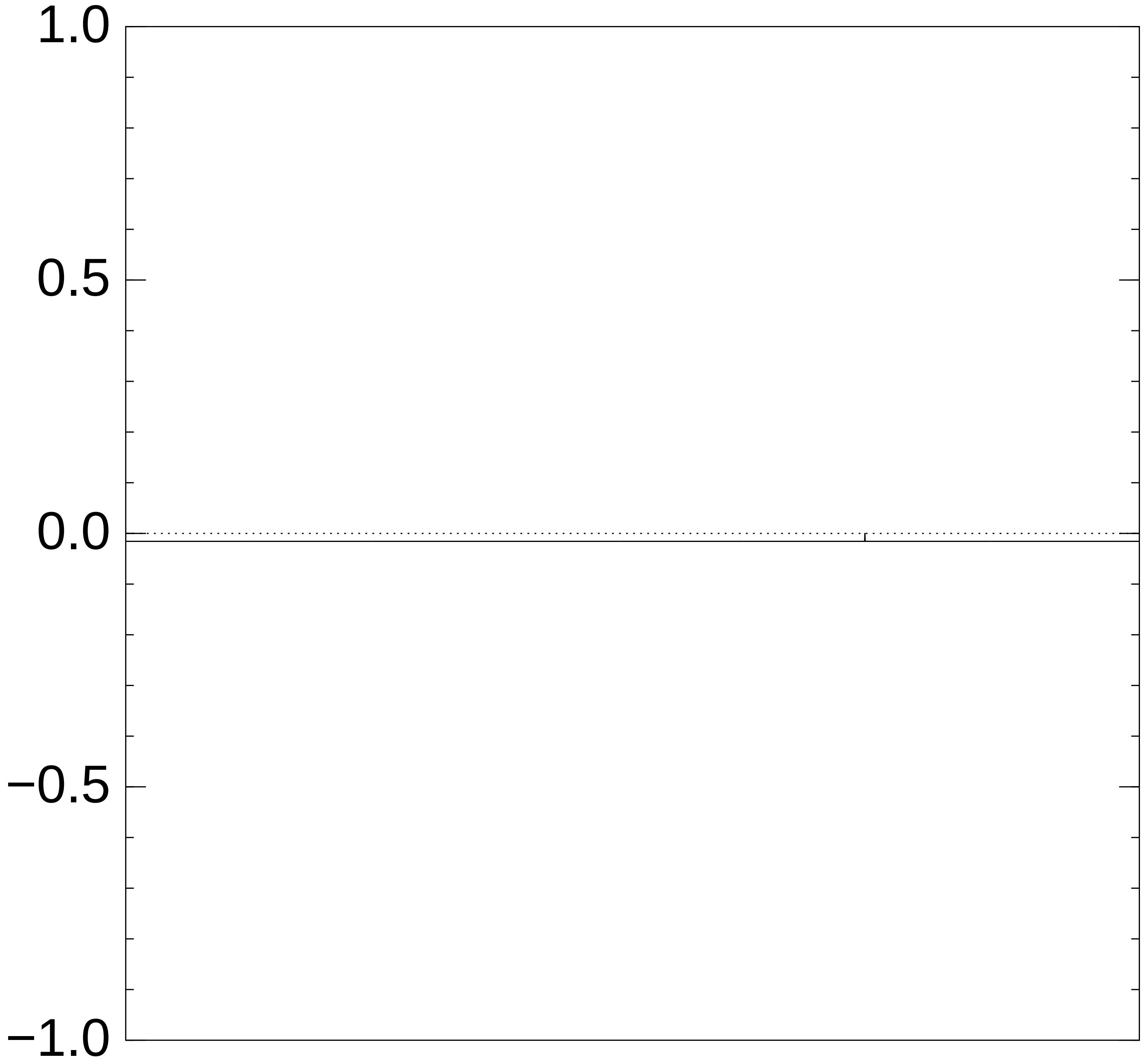

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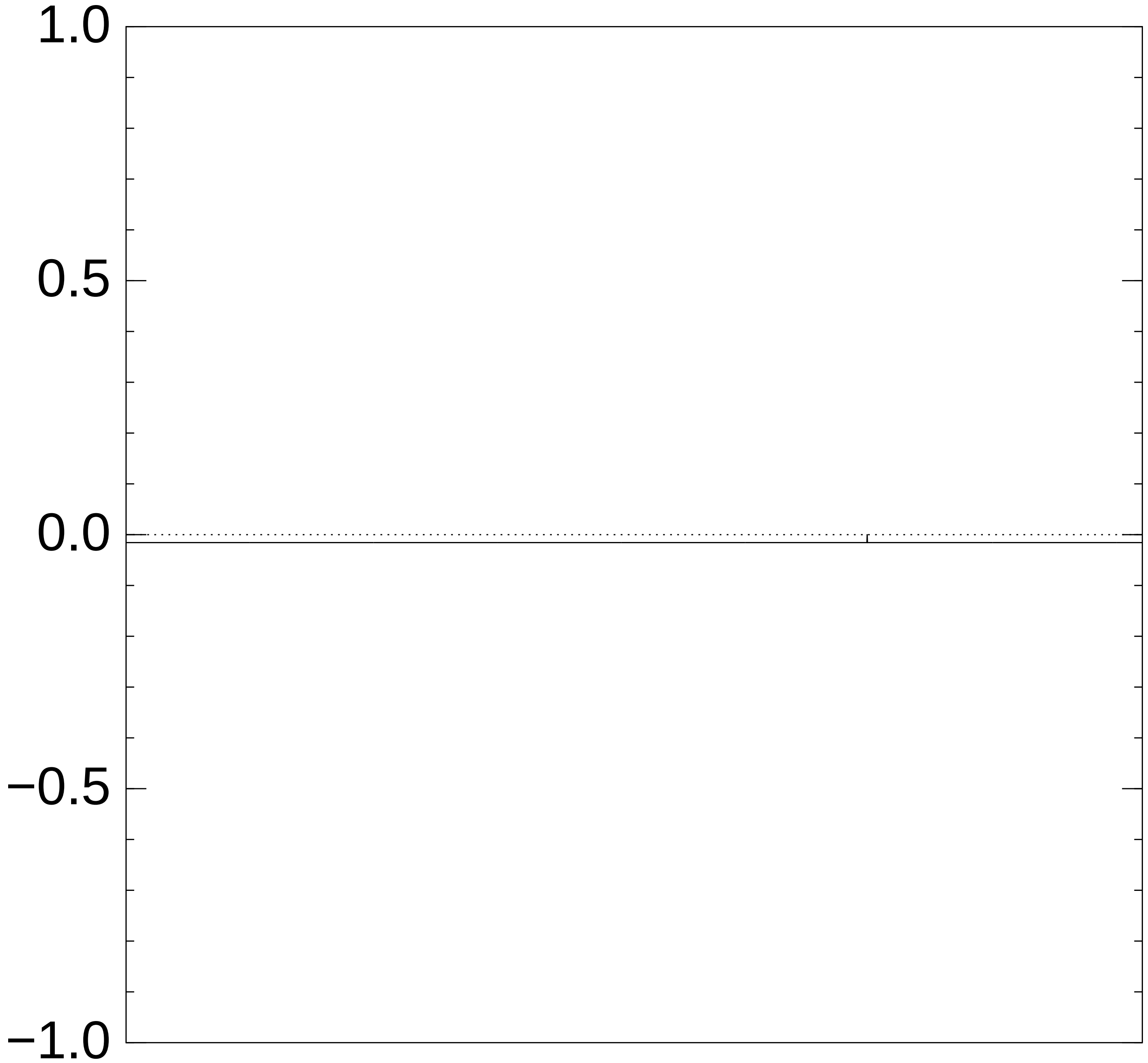

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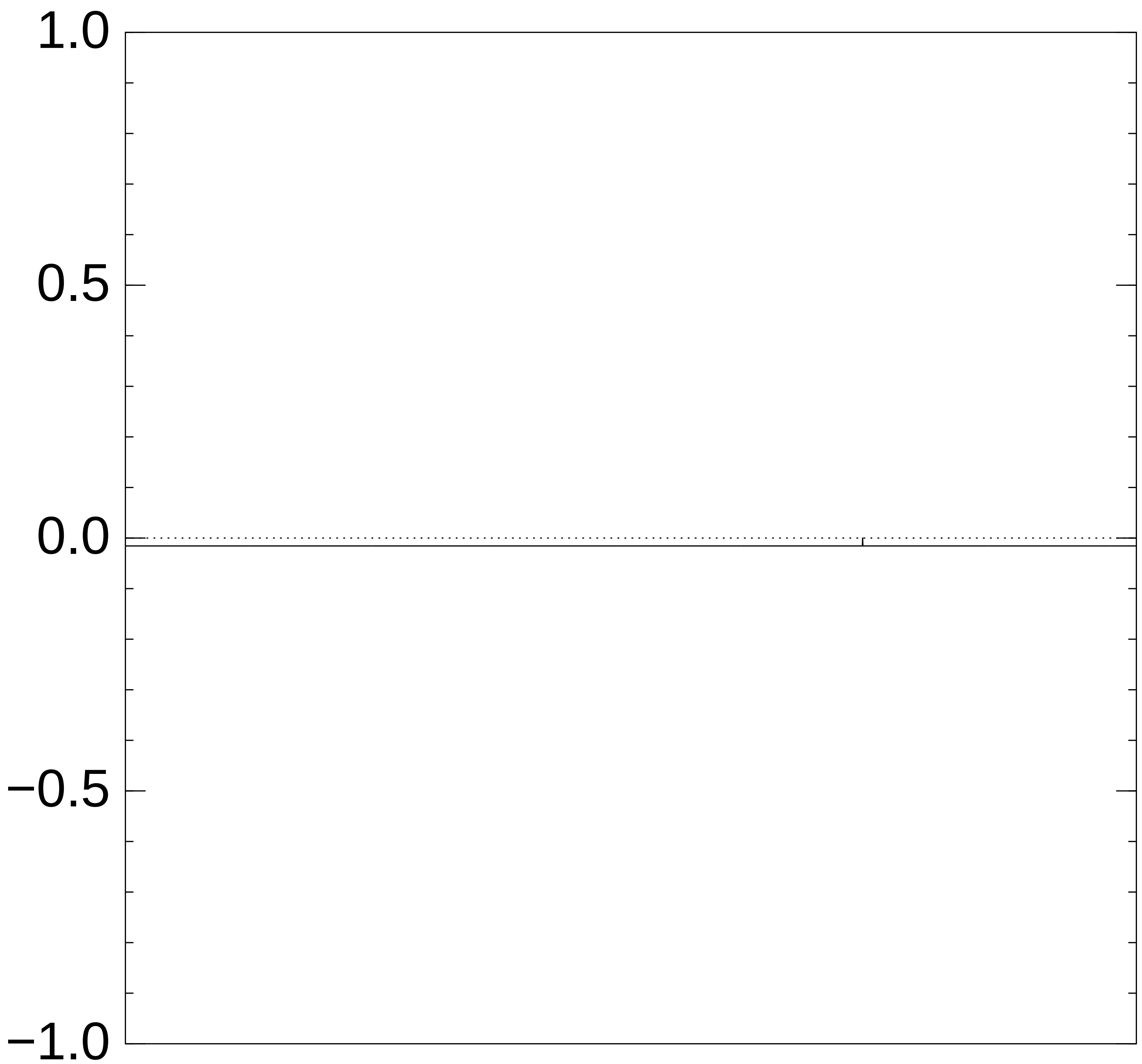

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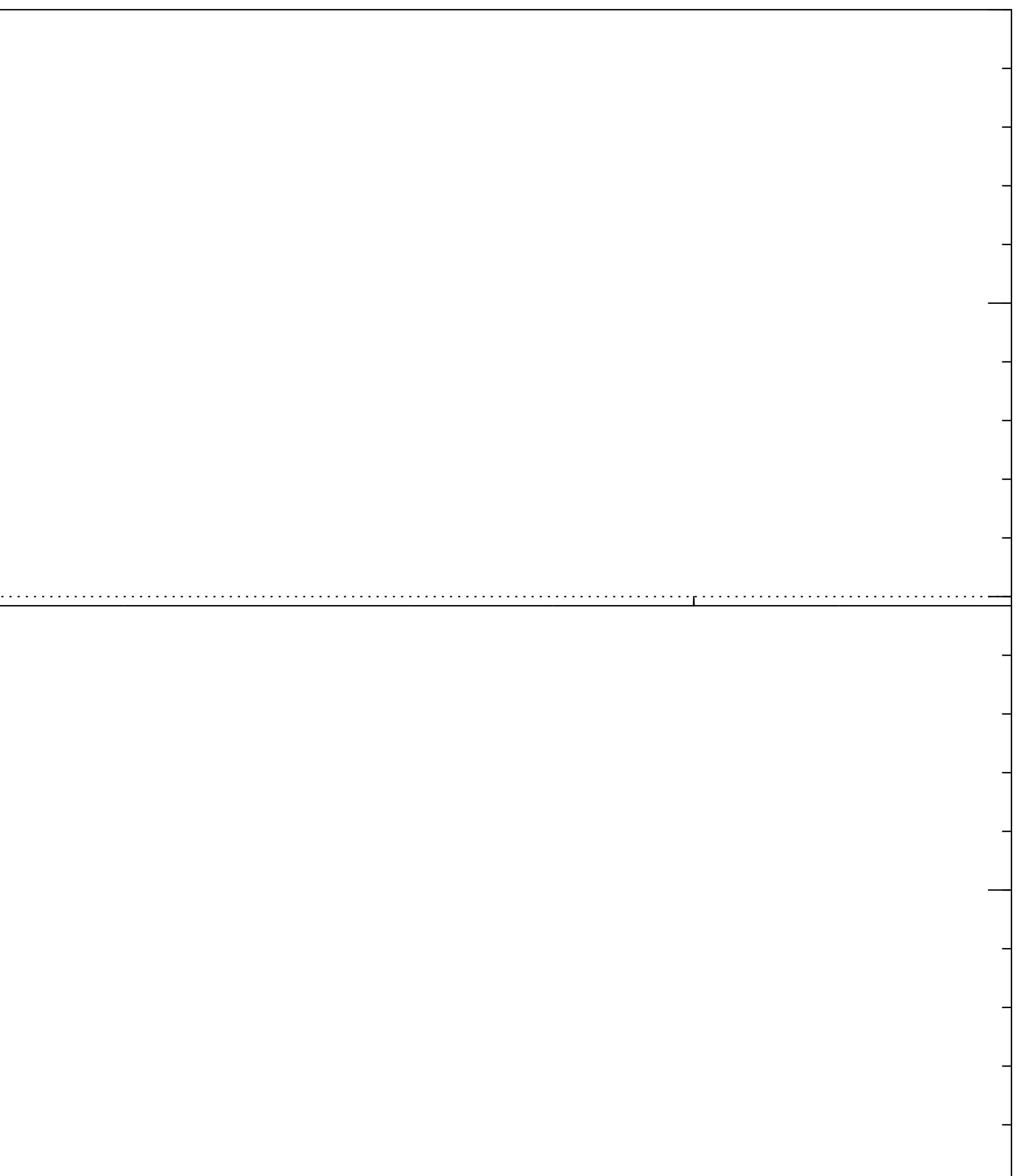

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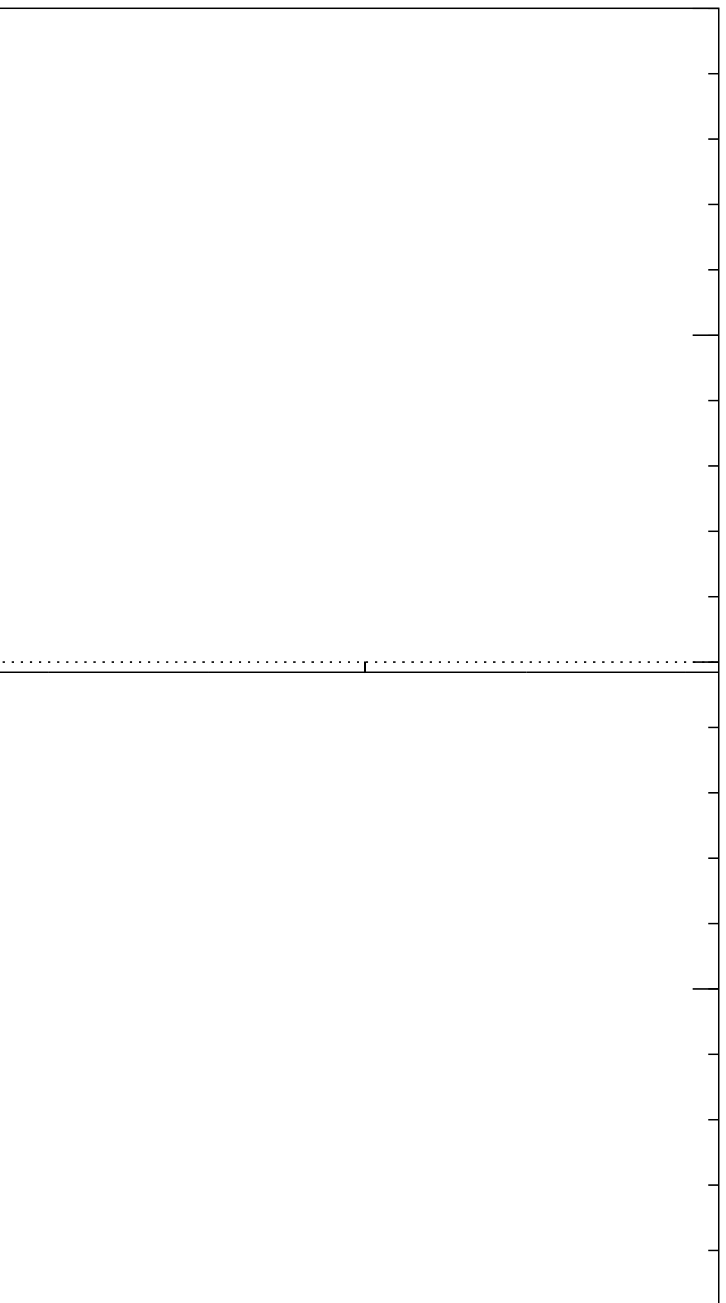

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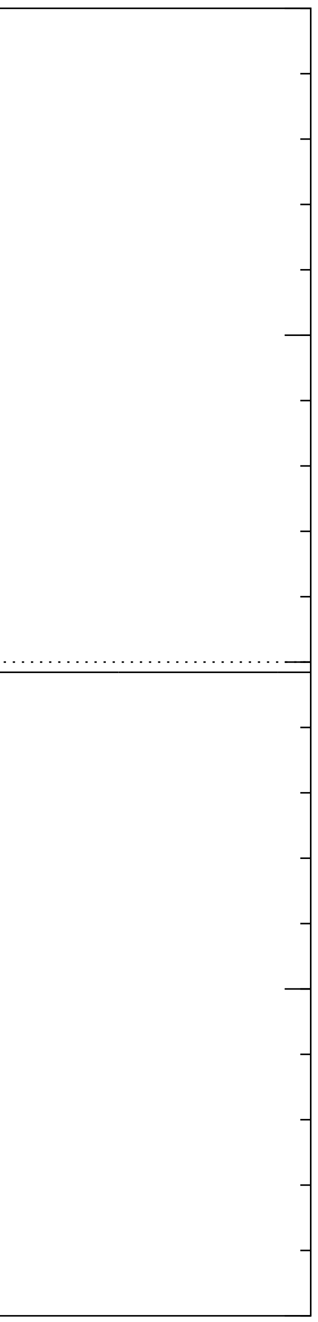

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