What do quantum computers do?

Daniel J. Bernstein
University of Illinois at Chicago

"Quantum algorithm" means an algorithm that a quantum computer can run.

i.e. a sequence of instructions, where each instruction is in a quantum computer's supported instruction set.

How do we know which instructions a quantum computer will support?

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Data stored in 4 qubits: a list of 16 numbers, not all zero. e.g.:
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e.g.: $(0, 1, 1)$.

Data stored in 64 bits:
a list of 64 elements of $\{0, 1\}$.
e.g.: $(1, 1, 1, 1, 1, 0, 0, 0, 1,$
$0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0,$
$0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1,$
$1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1,$
$0, 0, 1, 1, 1, 0, 0, 1, 0, 0, 0,$
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The state of a quantum computer

Data stored in 3 qubits:
a list of 8 numbers, not all zero.
e.g.: $(3, 1, 4, 1, 5, 9, 2, 6)$.
e.g.: $(-2, 7, -1, 8, 1, -8, -2, 8)$.
e.g.: $(0, 0, 0, 0, 0, 1, 0, 0)$.

Data stored in 4 qubits: a list of 16 numbers, not all zero. e.g.:
$(3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3)$.

Data stored in 64 qubits:
a list of 2^{64} numbers, not all zero.

The state of a computer

Data ("state") stored in 3 bits:
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e.g.: $(0, 0, 0)$.
e.g.: $(1, 1, 1)$.
e.g.: $(0, 1, 1)$.

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"Quantum RNG."

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Warning: Quantum RNGs sold today are measurably biased.

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NOT gates

NOT_0 gate on 3 q

$(3, 1, 4, 1, 5, 9, 2, 6$

$(1, 3, 1, 4, 9, 5, 6, 2$

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e.g.: Say 3 qubits have state $(0, 0, 0, 0, 0, 1, 0, 0)$.

Measurement produces

$000 = 0$ with probability 0;

$001 = 1$ with probability 0;

$010 = 2$ with probability 0;

$011 = 3$ with probability 0;

$100 = 4$ with probability 0;

$101 = 5$ with probability 1;

$110 = 6$ with probability 0;

$111 = 7$ with probability 0.

5 is guaranteed outcome.

NOT gates

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$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto (1, 3, 1, 4, 9, 5, 6, 2)$.

NOT_0 gate on 4 qubits:
$(3,1,4,1,5,9,2,6,5,3,5,8,9,7,9,3) \mapsto (1,3,1,4,9,5,6,2,3,5,8,5,7,9,3,9)$.

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NOT_1 gate on 3 qubits:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(4, 1, 3, 1, 2, 6, 5, 9)$.

e.g.: Say 3 qubits have state $(0,0,0,0,0,1,0,0)$.

Measurement produces
$000 = 0$ with probability 0;
$001 = 1$ with probability 0;
$010 = 2$ with probability 0;
$011 = 3$ with probability 0;
$100 = 4$ with probability 0;
$101 = 5$ with probability 1;
$110 = 6$ with probability 0;
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$(3,1,4,1,5,9,2,6) \mapsto (4,1,3,1,2,6,5,9)$.

NOT_2 gate on 3 qubits:
$(3,1,4,1,5,9,2,6) \mapsto (5,9,2,6,3,1,4,1)$.

y 3 qubits have state

$0, 0, 1, 0, 0)$.

ement produces

with probability 0;

with probability 0;

with probability 0;

with probability 0;

with probability 0;

with probability 1;

with probability 0;

with probability 0.

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NOT gates

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$(5, 9, 2, 6, 3, 1, 4, 1)$.

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$(0, 0, 1,$

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NOT gates

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state
(1, 0, 0, 0, 0, 0, 0, 0
(0, 1, 0, 0, 0, 0, 0, 0
(0, 0, 1, 0, 0, 0, 0, 0
(0, 0, 0, 1, 0, 0, 0, 0
(0, 0, 0, 0, 1, 0, 0, 0
(0, 0, 0, 0, 0, 1, 0, 0
(0, 0, 0, 0, 0, 0, 1, 0
(0, 0, 0, 0, 0, 0, 0, 1

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NOT gates

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$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(5, 9, 2, 6, 3, 1, 4, 1)$.

state	measure
$(1, 0, 0, 0, 0, 0, 0, 0)$	000
$(0, 1, 0, 0, 0, 0, 0, 0)$	001
$(0, 0, 1, 0, 0, 0, 0, 0)$	010
$(0, 0, 0, 1, 0, 0, 0, 0)$	011
$(0, 0, 0, 0, 1, 0, 0, 0)$	100
$(0, 0, 0, 0, 0, 1, 0, 0)$	101
$(0, 0, 0, 0, 0, 0, 1, 0)$	110
$(0, 0, 0, 0, 0, 0, 0, 1)$	111

Operation on quantum state
NOT_0, swapping pairs.
Operation after measuremen
flipping bit 0 of result.
Flip: output is not input.

NOT gates

NOT_0 gate on 3 qubits:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(1, 3, 1, 4, 9, 5, 6, 2)$.

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$(3,1,4,1,5,9,2,6,5,3,5,8,9,7,9,3) \mapsto$
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$(5, 9, 2, 6, 3, 1, 4, 1)$.

state	measurement
$(1, 0, 0, 0, 0, 0, 0, 0)$	000
$(0, 1, 0, 0, 0, 0, 0, 0)$	001
$(0, 0, 1, 0, 0, 0, 0, 0)$	010
$(0, 0, 0, 1, 0, 0, 0, 0)$	011
$(0, 0, 0, 0, 1, 0, 0, 0)$	100
$(0, 0, 0, 0, 0, 1, 0, 0)$	101
$(0, 0, 0, 0, 0, 0, 1, 0)$	110
$(0, 0, 0, 0, 0, 0, 0, 1)$	111

Operation on quantum state:
NOT_0, swapping pairs.
Operation after measurement:
flipping bit 0 of result.
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ate on 3 qubits:

1, 5, 9, 2, 6) ↦

4, 9, 5, 6, 2).

ate on 4 qubits:

5,9,2,6,5,3,5,8,9,7,9,3) ↦

9,5,6,2,3,5,8,5,7,9,3,9).

ate on 3 qubits:

1, 5, 9, 2, 6) ↦

1, 2, 6, 5, 9).

ate on 3 qubits:

1, 5, 9, 2, 6) ↦

6, 3, 1, 4, 1).

state	measurement
$(1,0,0,0,0,0,0,0)$	000
$(0,1,0,0,0,0,0,0)$	001
$(0,0,1,0,0,0,0,0)$	010
$(0,0,0,1,0,0,0,0)$	011
$(0,0,0,0,1,0,0,0)$	100
$(0,0,0,0,0,1,0,0)$	101
$(0,0,0,0,0,0,1,0)$	110
$(0,0,0,0,0,0,0,1)$	111

Operation on quantum state:

NOT_0, swapping pairs.

Operation after measurement:

flipping bit 0 of result.

Flip: output is not input.

Controll

e.g. CN

(3, 1, 4, 1

(3, 1, 1, 4

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3,5,8,9,7,9,3) $\mapsto$

5,8,5,7,9,3,9).

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ubits:

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).

state	measurement
$(1,0,0,0,0,0,0,0)$	000
$(0,1,0,0,0,0,0,0)$	001
$(0,0,1,0,0,0,0,0)$	010
$(0,0,0,1,0,0,0,0)$	011
$(0,0,0,0,1,0,0,0)$	100
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$(0,0,0,0,0,0,1,0)$	110
$(0,0,0,0,0,0,0,1)$	111

Operation on quantum state:

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Operation after measurement:

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Controlled-NOT g

e.g. $\text{CNOT}_{1,0}$:

$(3, 1, 4, 1, 5, 9, 2, 6$

$(3, 1, 1, 4, 5, 9, 6, 2$

$,3) \mapsto$

$,9)$.

state	measurement
$(1,0,0,0,0,0,0,0)$	000
$(0,1,0,0,0,0,0,0)$	001
$(0,0,1,0,0,0,0,0)$	010
$(0,0,0,1,0,0,0,0)$	011
$(0,0,0,0,1,0,0,0)$	100
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$(3,1,1,4,5,9,6,2)$.

state	measurement
$(1,0,0,0,0,0,0,0)$	000
$(0,1,0,0,0,0,0,0)$	001
$(0,0,1,0,0,0,0,0)$	010
$(0,0,0,1,0,0,0,0)$	011
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$(0,1,0,0,0,0,0,0)$	001
$(0,0,1,0,0,0,0,0)$	010
$(0,0,0,1,0,0,0,0)$	011
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state	measurement
$(1,0,0,0,0,0,0,0)$	000
$(0,1,0,0,0,0,0,0)$	001
$(0,0,1,0,0,0,0,0)$	010
$(0,0,0,1,0,0,0,0)$	011
$(0,0,0,0,1,0,0,0)$	100
$(0,0,0,0,0,1,0,0)$	101
$(0,0,0,0,0,0,1,0)$	110
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e.g. $\text{CNOT}_{2,0}$: $(3,1,4,1,5,9,2,6) \mapsto (3,1,4,1,9,5,6,2)$.

state	measurement
$(1,0,0,0,0,0,0,0)$	000
$(0,1,0,0,0,0,0,0)$	001
$(0,0,1,0,0,0,0,0)$	010
$(0,0,0,1,0,0,0,0)$	011
$(0,0,0,0,1,0,0,0)$	100
$(0,0,0,0,0,1,0,0)$	101
$(0,0,0,0,0,0,1,0)$	110
$(0,0,0,0,0,0,0,1)$	111

Operation on quantum state: NOT_0, swapping pairs.

Operation after measurement: flipping bit 0 of result.

Flip: output is not input.

Controlled-NOT gates

e.g. $\text{CNOT}_{1,0}$:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,1,4,5,9,6,2)$.

Operation after measurement: flipping bit 0 *if* bit 1 is set; i.e., $(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. $\text{CNOT}_{2,0}$:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,4,1,9,5,6,2)$.

e.g. $\text{CNOT}_{0,2}$:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,9,4,6,5,1,2,1)$.

state	measurement
0, 0, 0, 0, 0)	000
0, 0, 0, 0, 0)	001
0, 0, 0, 0, 0)	010
1, 0, 0, 0, 0)	011
0, 1, 0, 0, 0)	100
0, 0, 1, 0, 0)	101
0, 0, 0, 1, 0)	110
0, 0, 0, 0, 1)	111

on on quantum state:
swapping pairs.
on after measurement:
bit 0 of result.
tput is not input.

Controlled-NOT gates

e.g. $\text{CNOT}_{1,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 1, 4, 5, 9, 6, 2)$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. $\text{CNOT}_{2,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 9, 5, 6, 2)$.

e.g. $\text{CNOT}_{0,2}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 9, 4, 6, 5, 1, 2, 1)$.

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e.g. CCN
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(3, 1, 4, 1

measurement

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0)	001
0)	010
0)	011
0)	100
0)	101
0)	110
1)	111

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Controlled-NOT gates

e.g. $\text{CNOT}_{1,0}$:

$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$

$(3, 1, 1, 4, 5, 9, 6, 2)$.

Operation after measurement:

flipping bit 0 *if* bit 1 is set; i.e.,

$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. $\text{CNOT}_{2,0}$:

$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$

$(3, 1, 4, 1, 9, 5, 6, 2)$.

e.g. $\text{CNOT}_{0,2}$:

$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$

$(3, 9, 4, 6, 5, 1, 2, 1)$.

Toffoli gates

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e.g. $\text{CCNOT}_{2,1,0}$:

(3, 1, 4, 1, 5, 9, 2, 6

(3, 1, 4, 1, 5, 9, 6, 2

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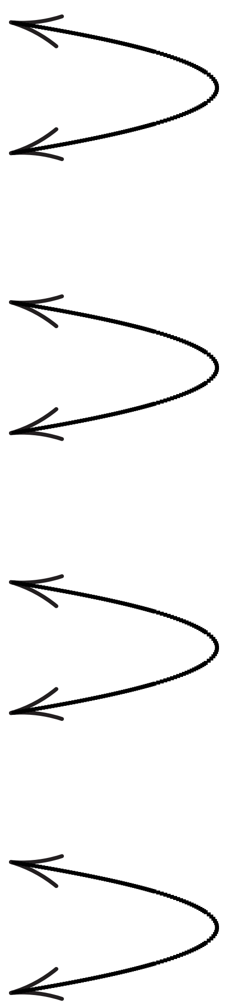

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Controlled-NOT gates

e.g. $\text{CNOT}_{1,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
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Operation after measurement:
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Toffoli gates

Also known as
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Controlled-NOT gates

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Toffoli gates

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Controlled-NOT gates

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Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
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e.g. $\text{CNOT}_{2,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
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e.g. $\text{CNOT}_{0,2}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
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Toffoli gates

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e.g. $\text{CCNOT}_{2,1,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
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Operation after measurement:
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1 q_2)$.

Controlled-NOT gates

e.g. $\text{CNOT}_{1,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 1, 4, 5, 9, 6, 2)$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. $\text{CNOT}_{2,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 9, 5, 6, 2)$.

e.g. $\text{CNOT}_{0,2}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 9, 4, 6, 5, 1, 2, 1)$.

Toffoli gates

Also known as
controlled-controlled-NOT gates.

e.g. $\text{CCNOT}_{2,1,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 5, 9, 6, 2)$.

Operation after measurement:
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1 q_2)$.

e.g. $\text{CCNOT}_{0,1,2}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 6, 5, 9, 2, 1)$.

ed-NOT gates

OT$_{1,0}$:

1, 5, 9, 2, 6) $\mapsto$

4, 5, 9, 6, 2).

on after measurement:

bit 0 *if* bit 1 is set; i.e.,

$q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

OT$_{2,0}$:

1, 5, 9, 2, 6) $\mapsto$

1, 9, 5, 6, 2).

OT$_{0,2}$:

1, 5, 9, 2, 6) $\mapsto$

6, 5, 1, 2, 1).

Toffoli gates

Also known as controlled-controlled-NOT gates.

e.g. CCNOT$_{2,1,0}$:

$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$

$(3, 1, 4, 1, 5, 9, 6, 2)$.

Operation after measurement:

$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1 q_2)$.

e.g. CCNOT$_{0,1,2}$:

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$(3, 1, 4, 6, 5, 9, 2, 1)$.

More sh

Combine

to build

ates

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, $q_1, q_0 \oplus q_1$).

) $\mapsto$

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) $\mapsto$

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Toffoli gates

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$(3, 1, 4, 6, 5, 9, 2, 1)$.

More shuffling

Combine NOT, CN

to build other perr

nt:
i.e.,
q_1).

Toffoli gates

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More shuffling

Combine NOT, CNOT, Toff
to build other permutations.

Toffoli gates

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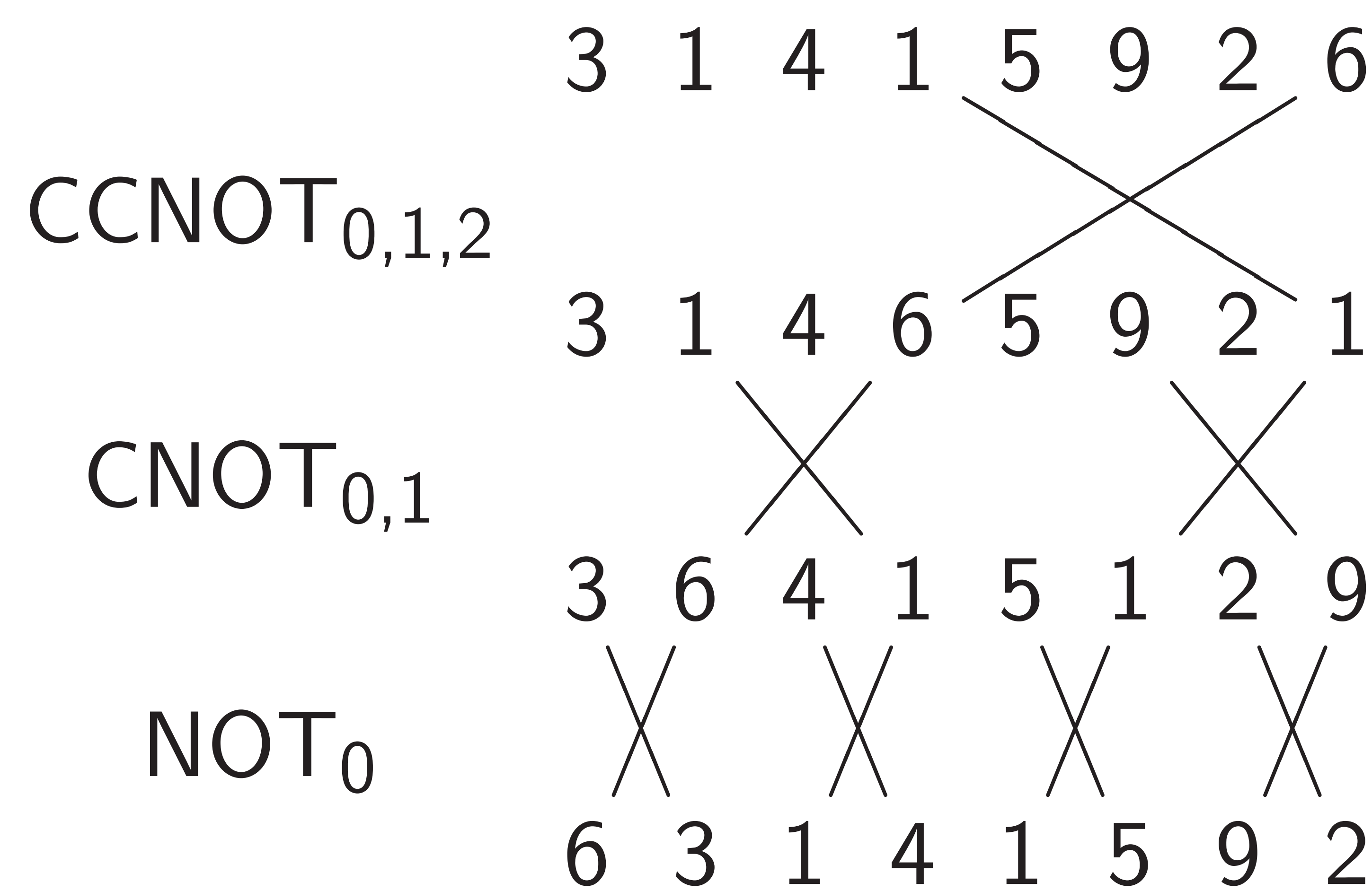
More shuffling

Combine NOT, CNOT, Toffoli
to build other permutations.

e.g. series of gates to
rotate 8 positions by distance 1:



ates

own as

ed-controlled-NOT gates.

$\text{NOT}_{2,1,0}$:

$1, 5, 9, 2, 6) \mapsto$

$1, 5, 9, 6, 2)$.

on after measurement:

$q_0) \mapsto (q_2, q_1, q_0 \oplus q_1 q_2)$.

$\text{NOT}_{0,1,2}$:

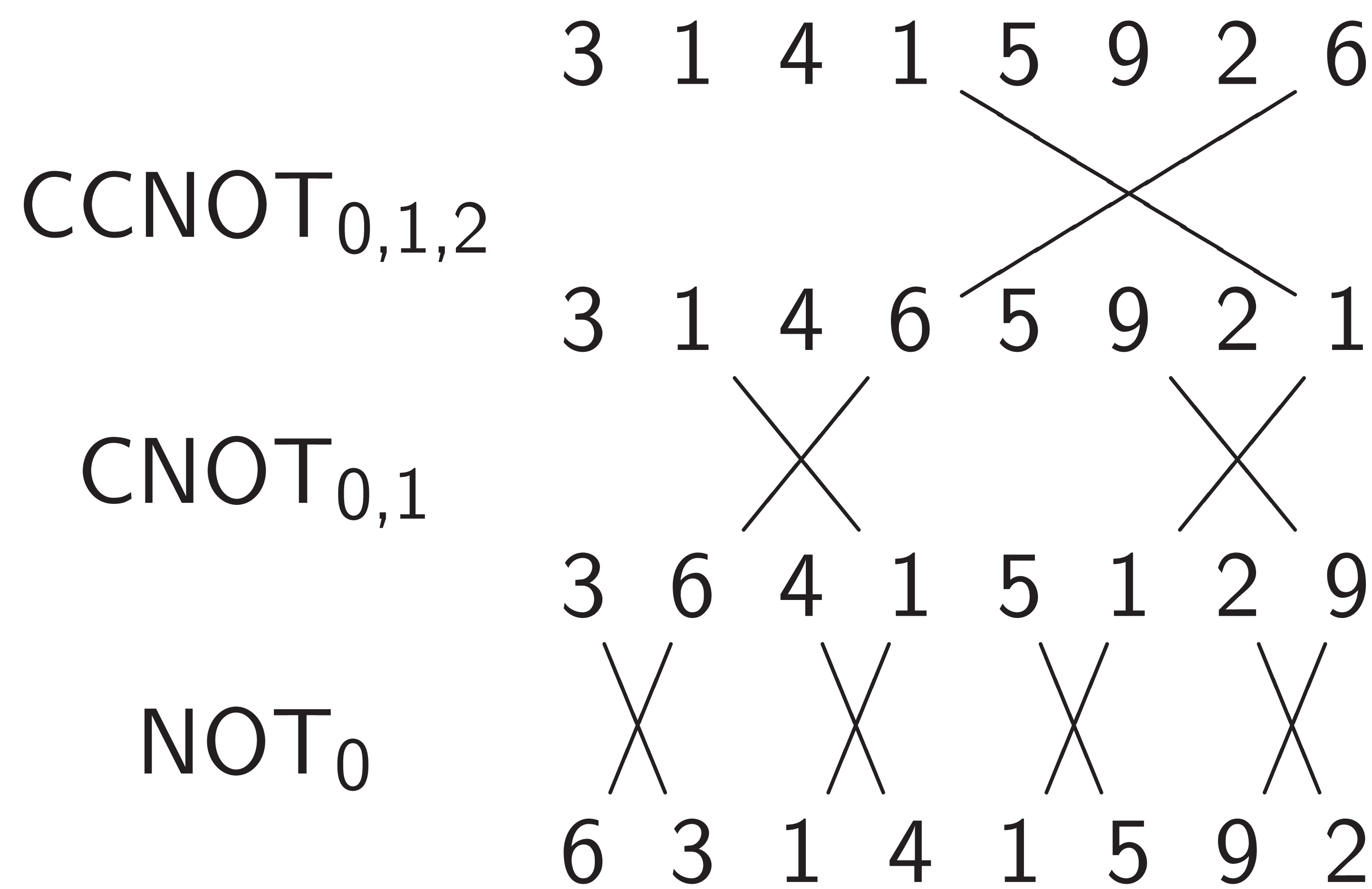
$1, 5, 9, 2, 6) \mapsto$

$6, 5, 9, 2, 1)$.

More shuffling

Combine NOT, CNOT, Toffoli to build other permutations.

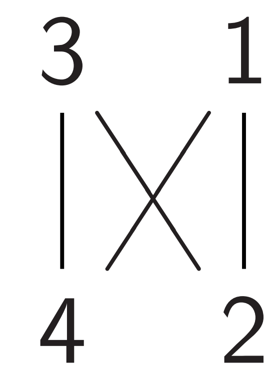
e.g. series of gates to rotate 8 positions by distance 1:



Hadama

Hadama

$(a, b) \mapsto$



ed-NOT gates.

$) \mapsto$

$).$

easurement:

$, q_1, q_0 \oplus q_1 q_2).$

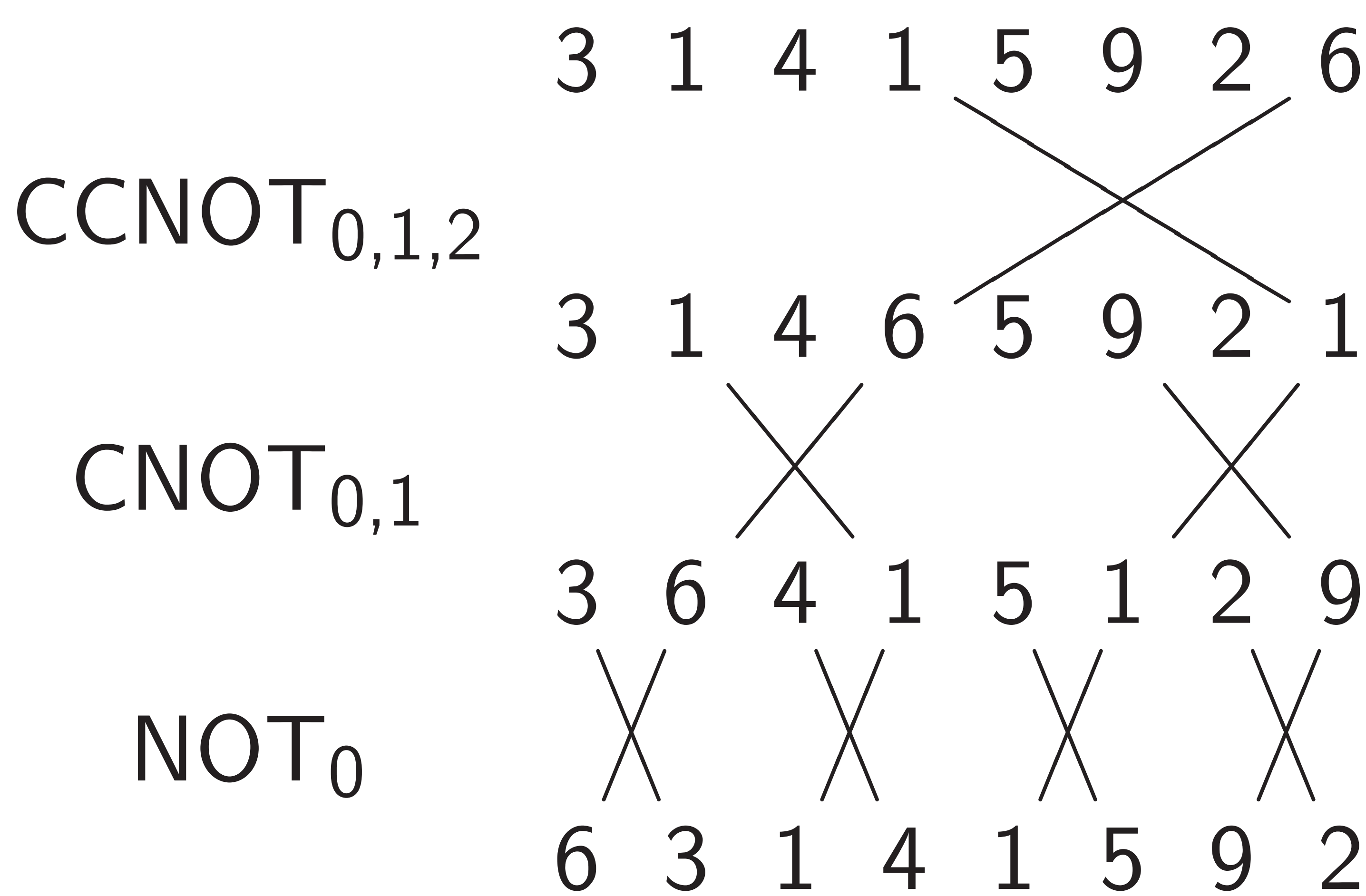
$) \mapsto$

$).$

More shuffling

Combine NOT, CNOT, Toffoli
to build other permutations.

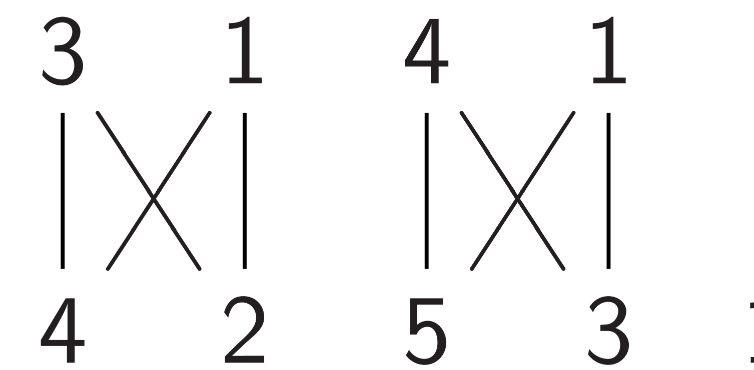
e.g. series of gates to
rotate 8 positions by distance 1:



Hadamard gates

$Hadamard_0$:

$(a, b) \mapsto (a + b, a$



ates.

nt:

$q_1 q_2$).

More shuffling

Combine NOT, CNOT, Toffoli to build other permutations.

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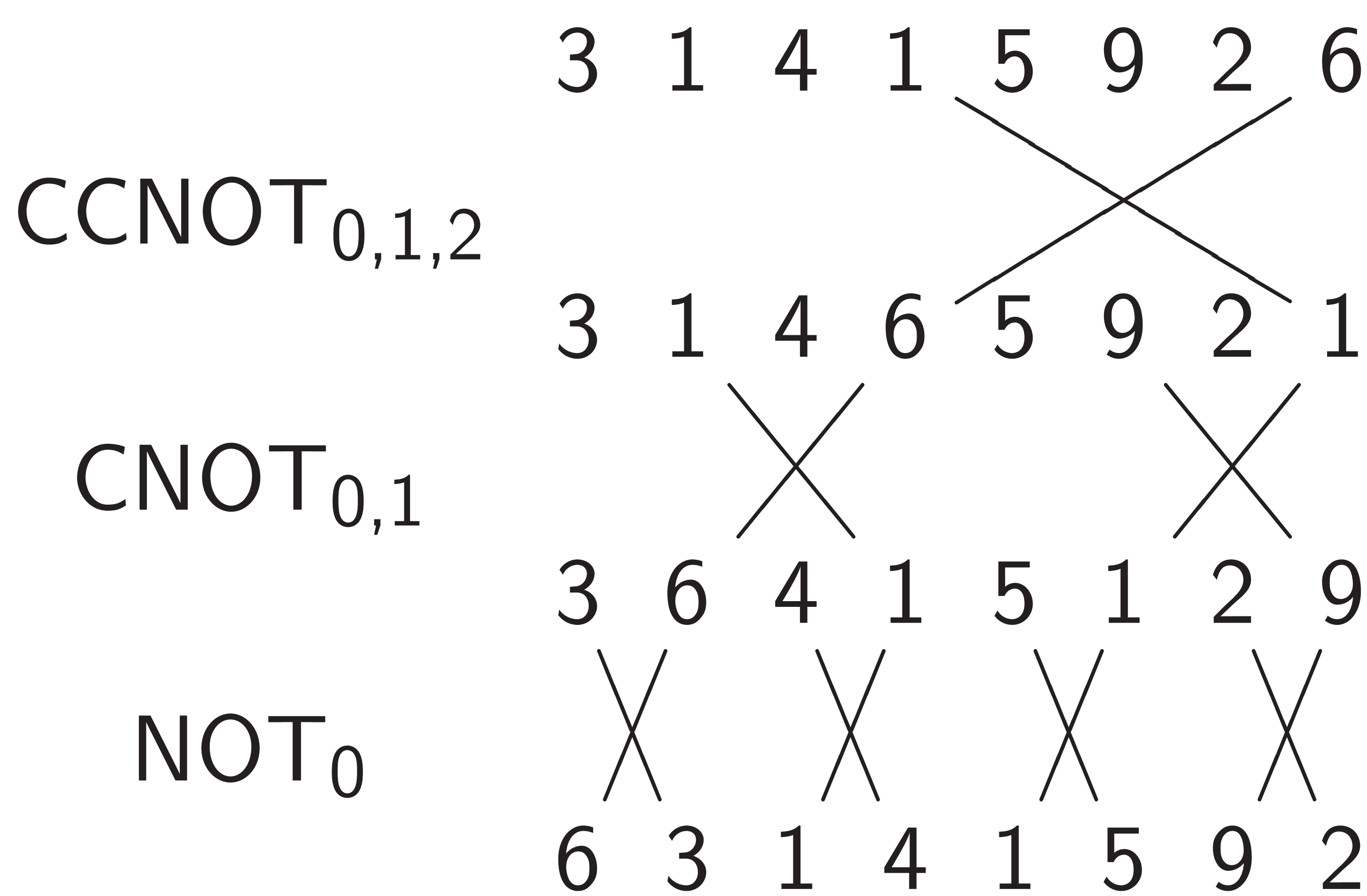
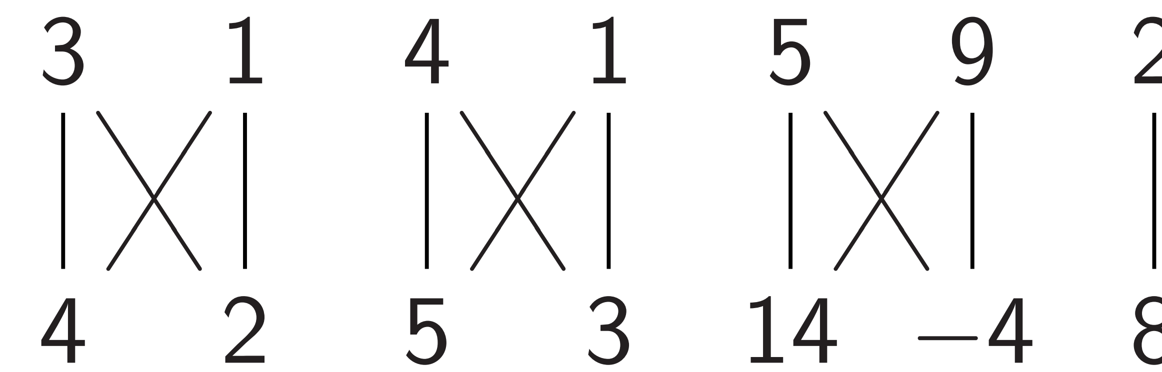


Hadamard gates

Hadamard$_0$:

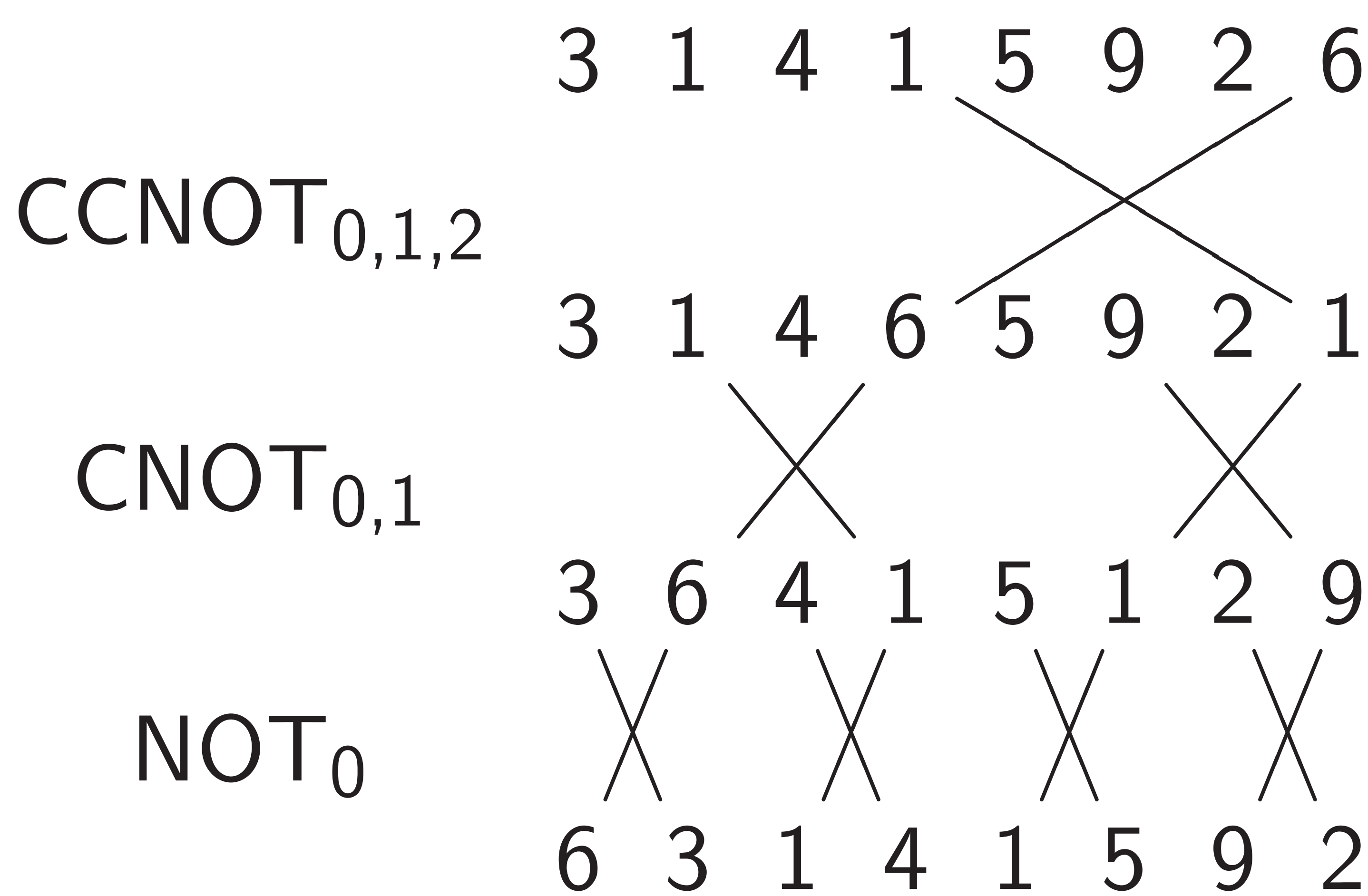
$(a, b) \mapsto (a + b, a - b)$.



More shuffling

Combine NOT, CNOT, Toffoli to build other permutations.

e.g. series of gates to rotate 8 positions by distance 1:



Hadamard gates

Hadamard_0:

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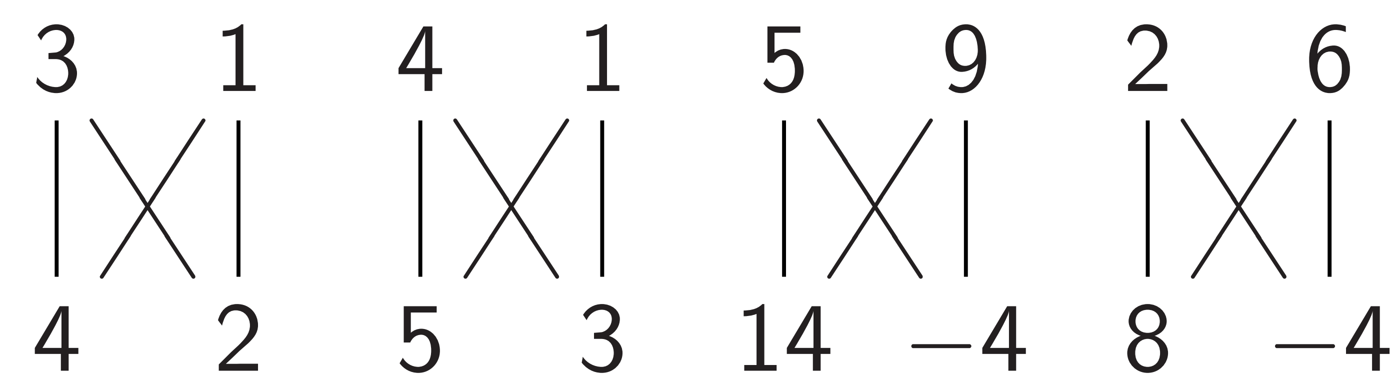

More shuffling

Combine NOT, CNOT, Toffoli to build other permutations.

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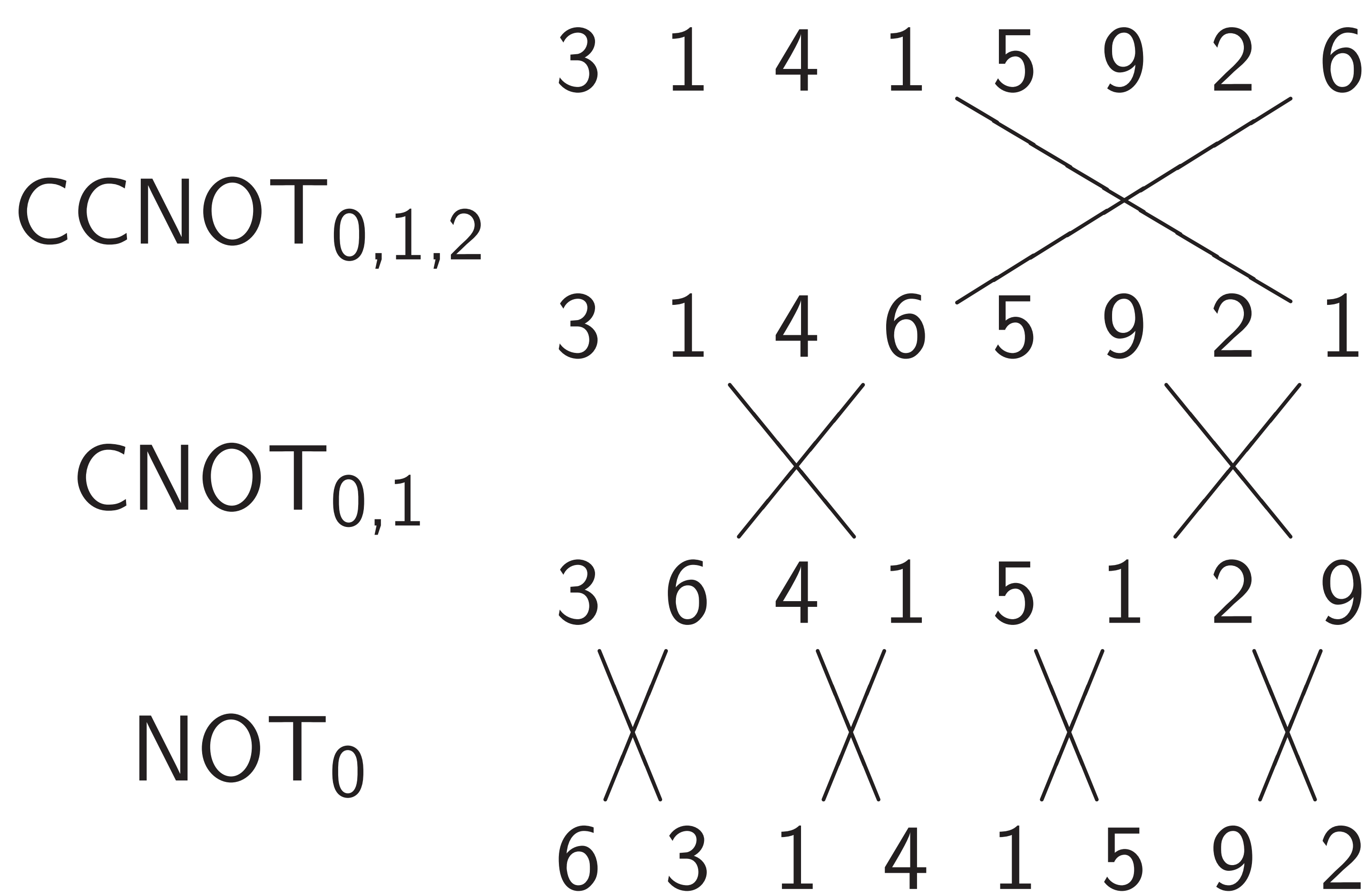

Hadamard gates

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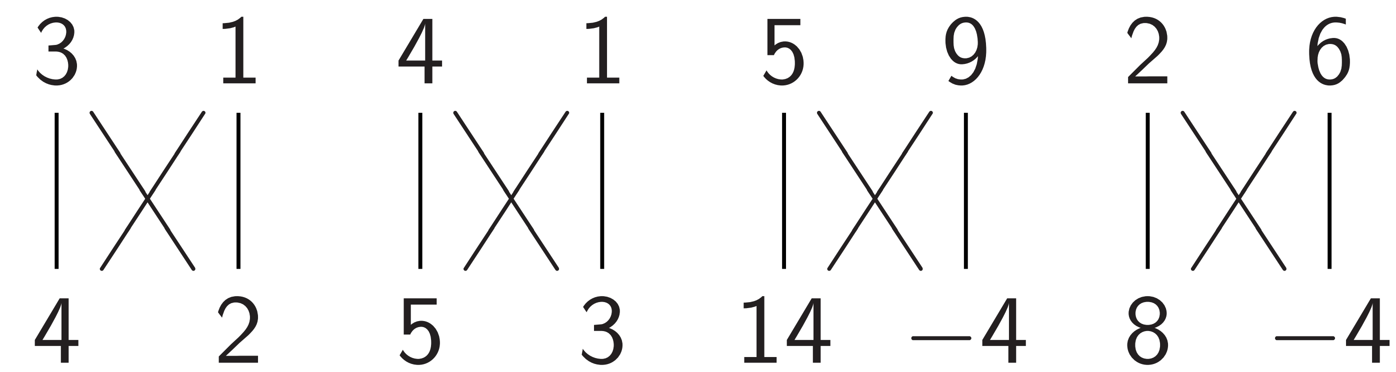
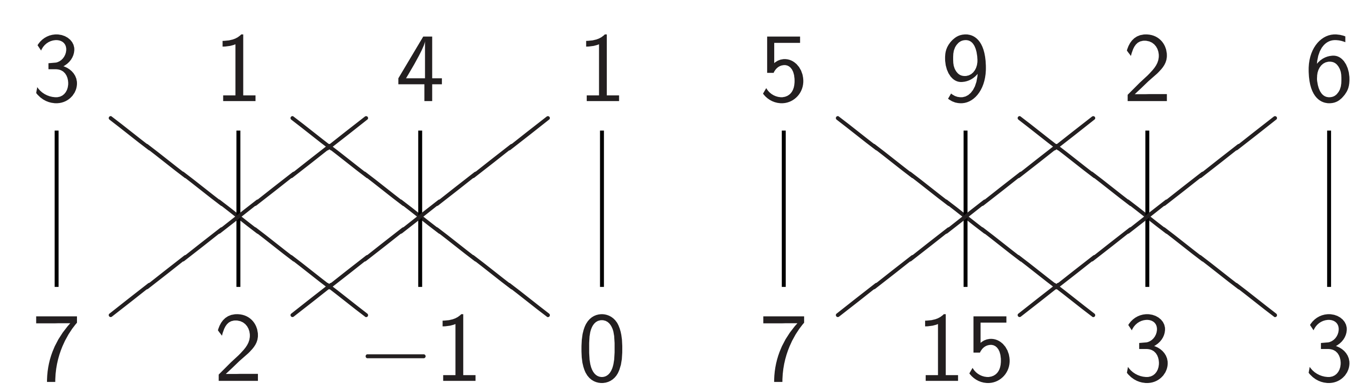


Hadamard$_1$:

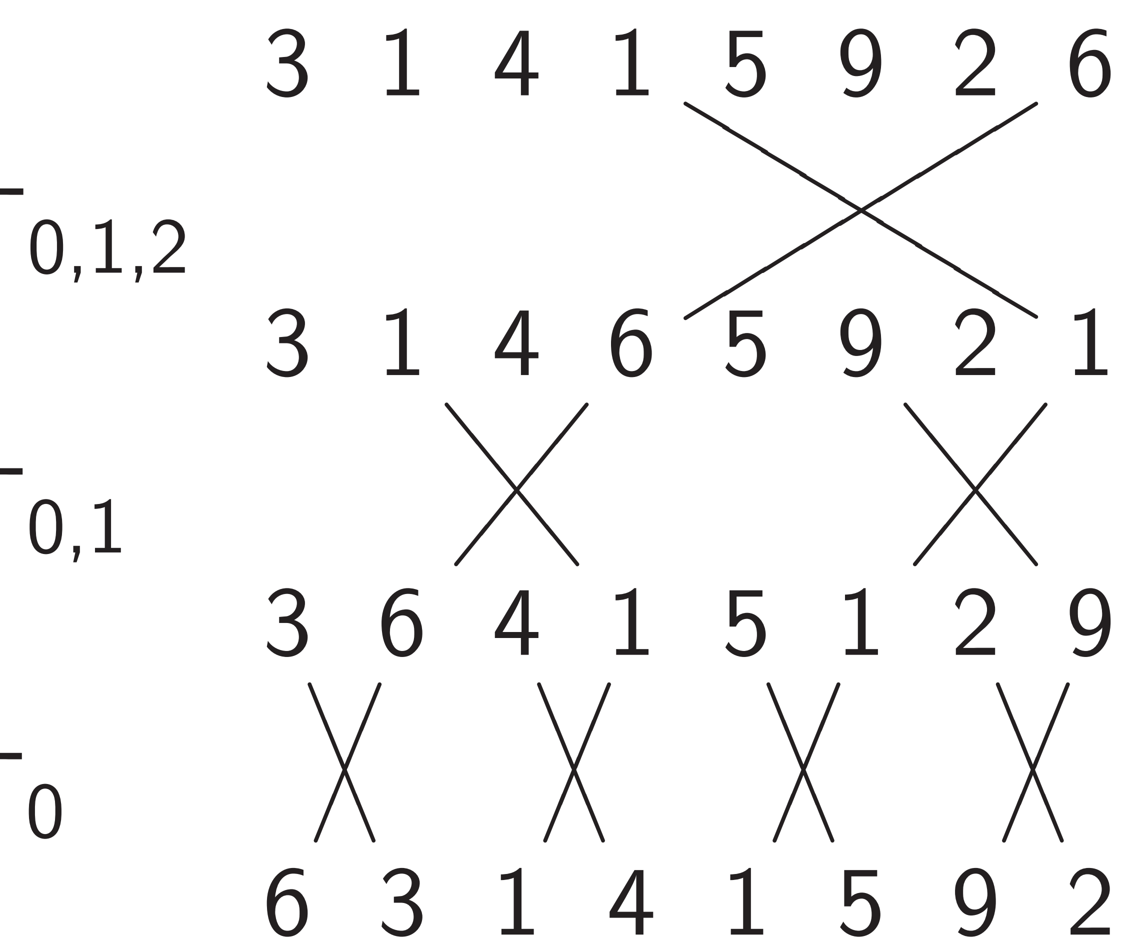
$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



uffling

e NOT, CNOT, Toffoli
other permutations.

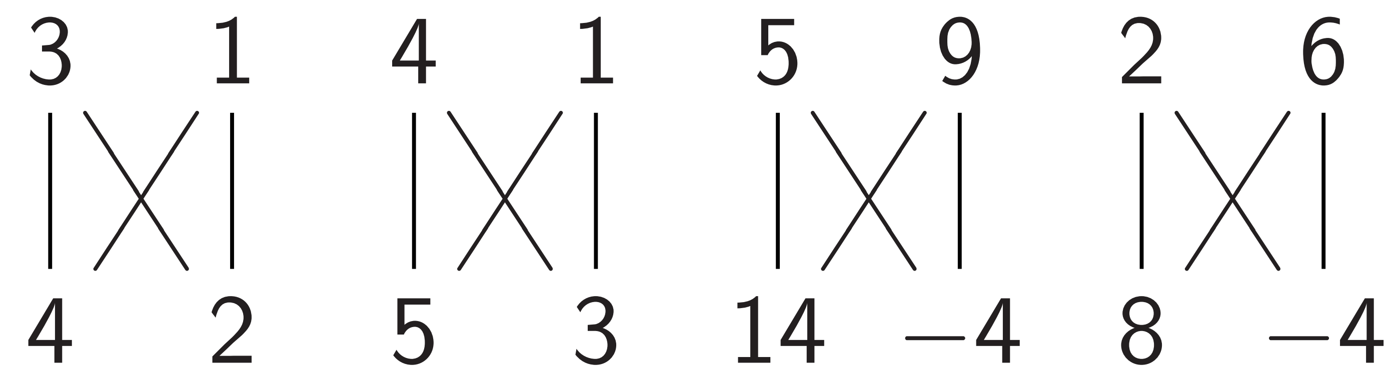
es of gates to
positions by distance 1:



Hadamard gates

Hadamard_0:

$(a, b) \mapsto (a + b, a - b)$.



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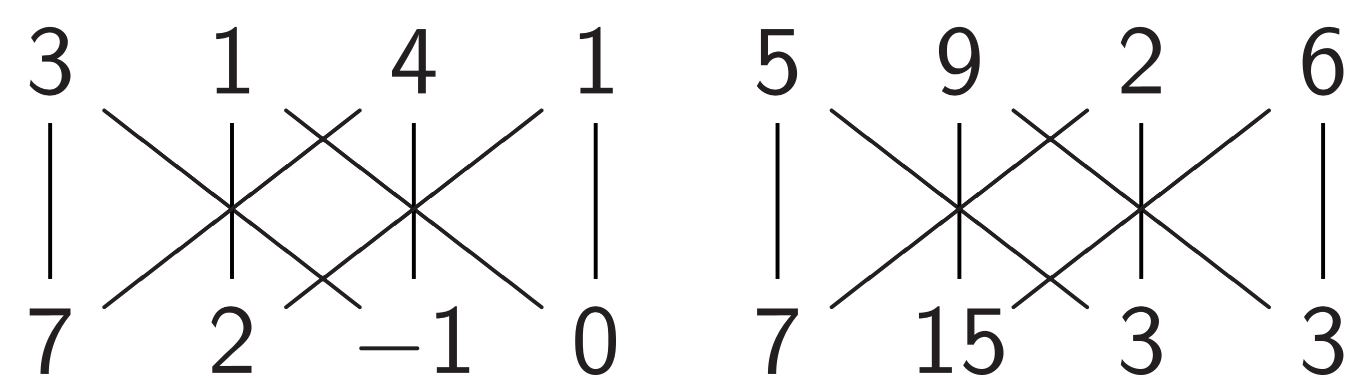
$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's

Step 1.

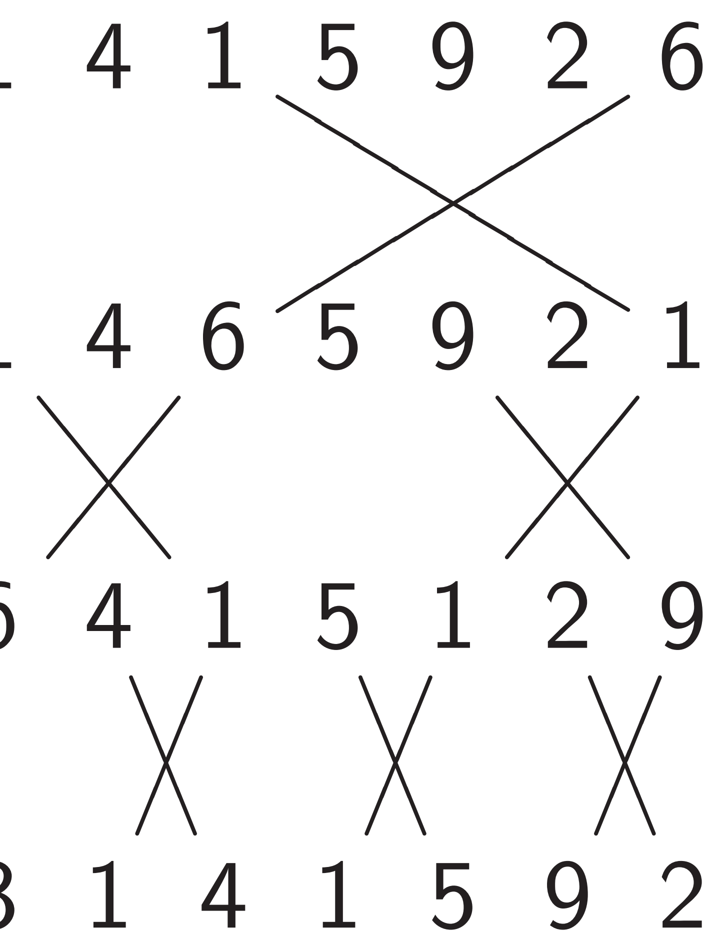
1, 0, 0, 0
0, 0, 0, 0
0, 0, 0, 0
0, 0, 0, 0
0, 0, 0, 0
0, 0, 0, 0
0, 0, 0, 0
0, 0, 0, 0

NOT, Toffoli

mutations.

s to

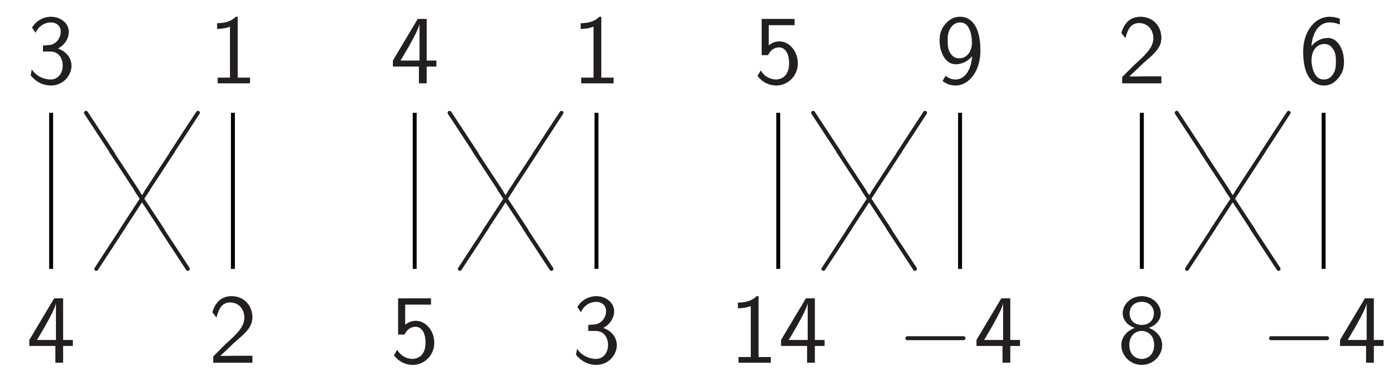
by distance 1:



Hadamard gates

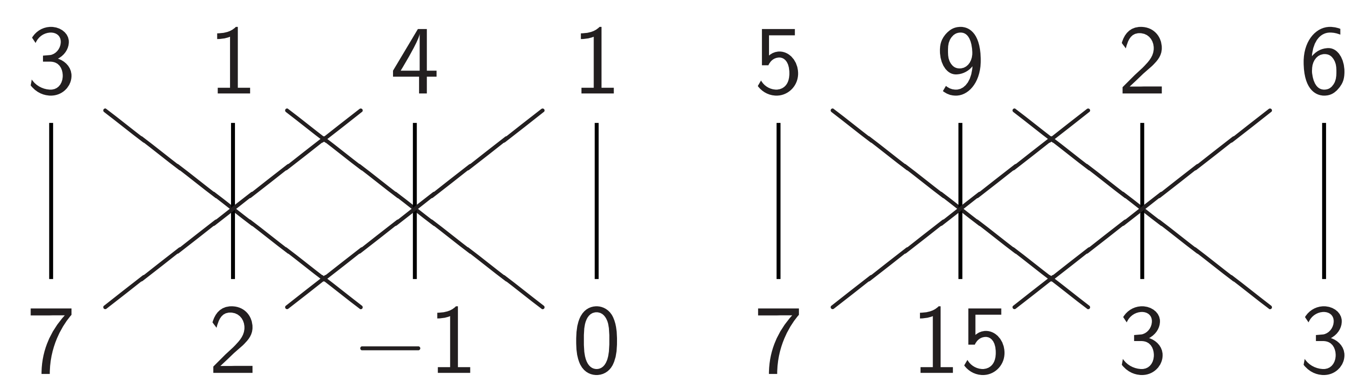
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



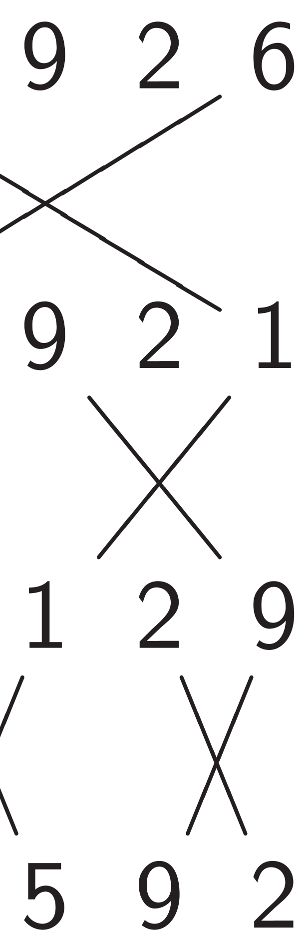
Simon's algorithm

Step 1. Set up pu

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

foli

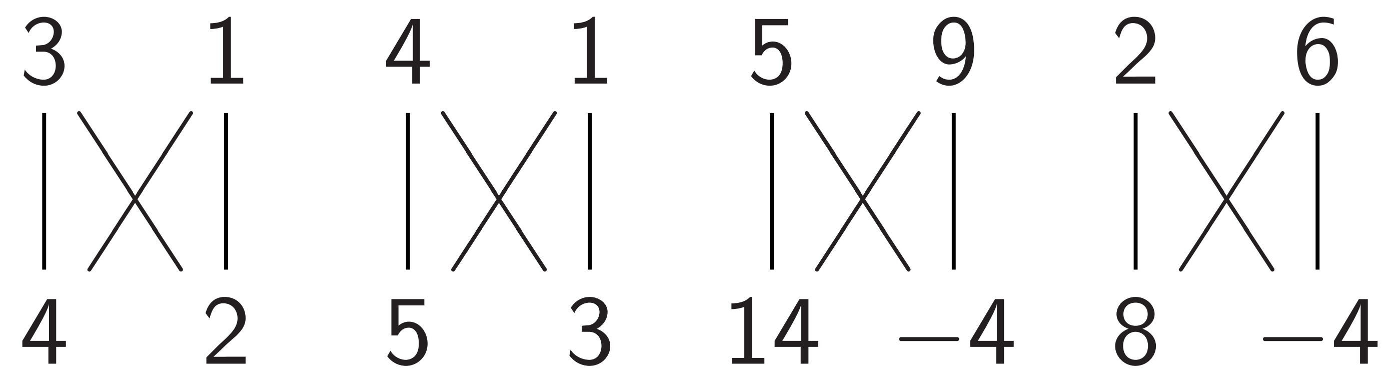
ce 1:



Hadamard gates

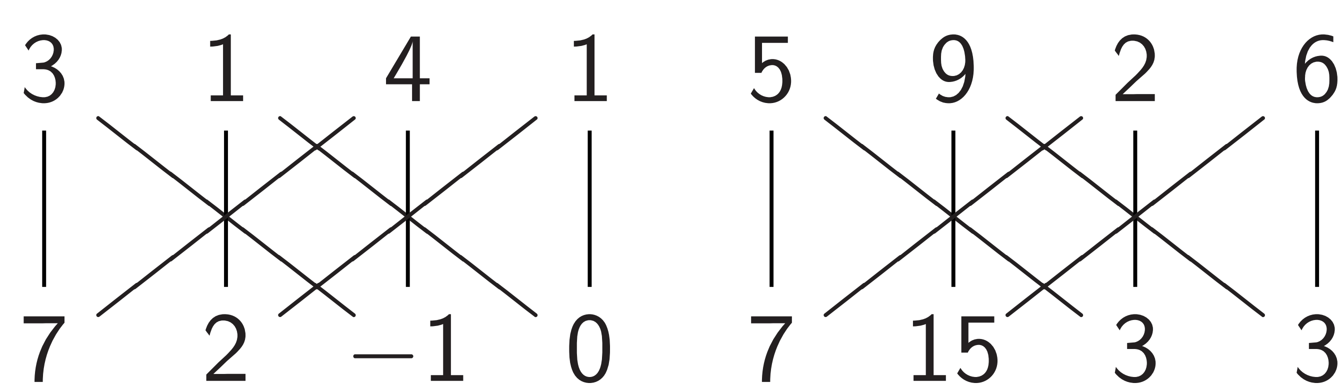
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

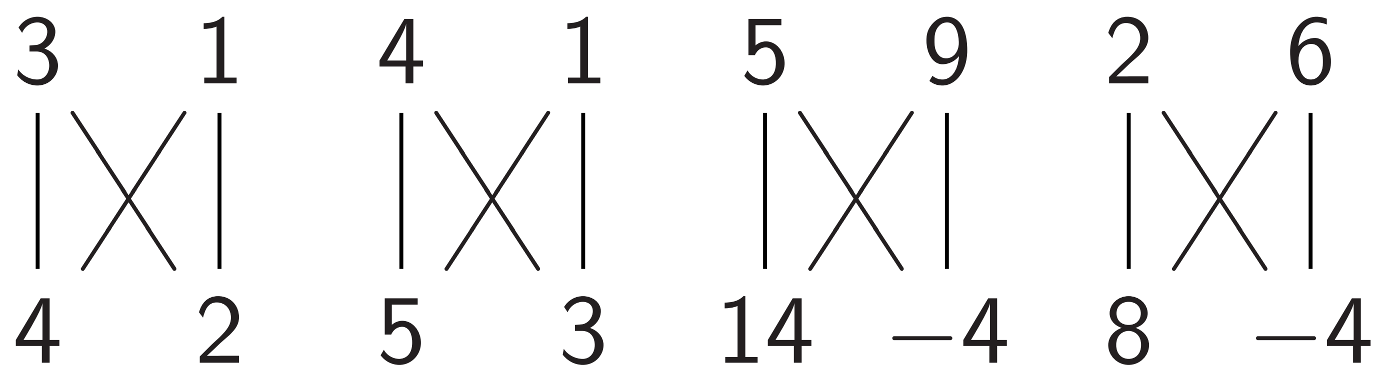
Step 1. Set up pure zero sta

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Hadamard gates

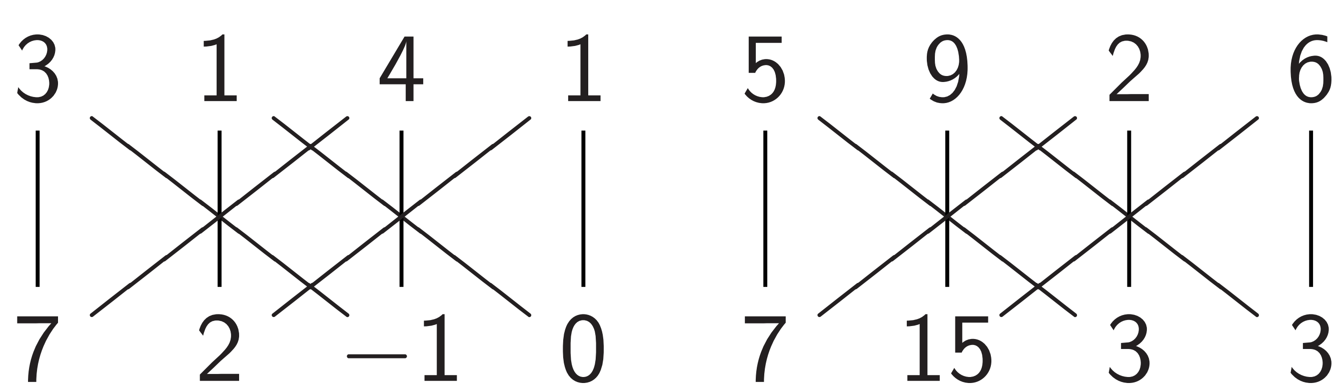
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

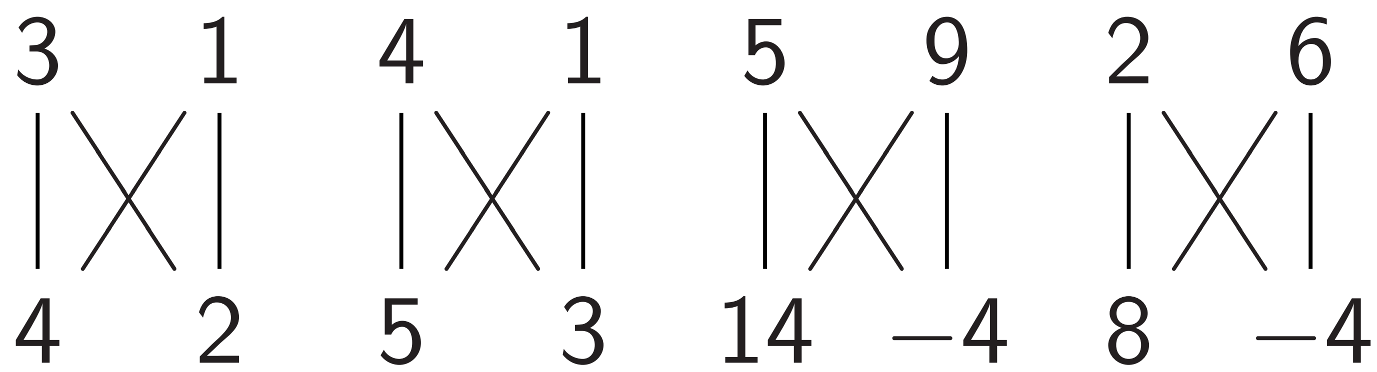
Step 1. Set up pure zero state:

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Hadamard gates

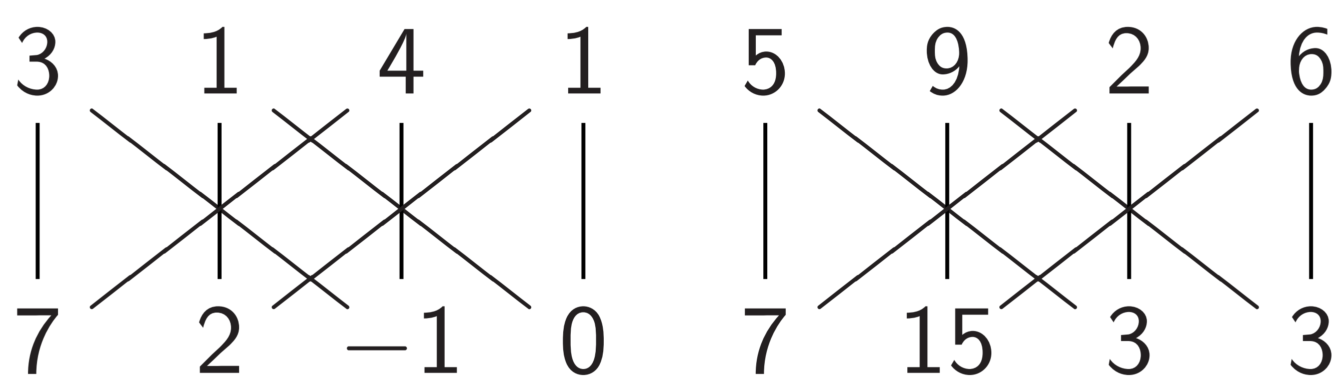
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

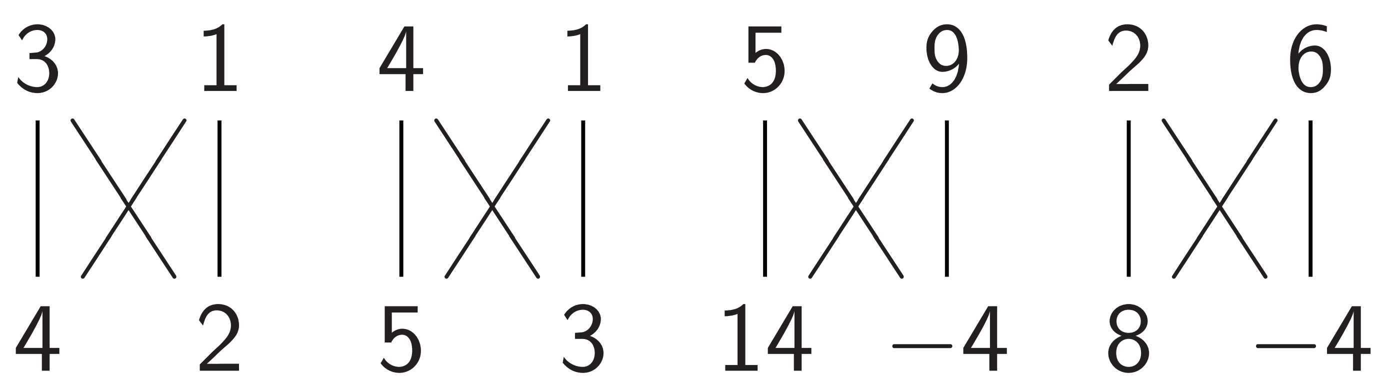
Step 2. Hadamard$_0$:

1, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Hadamard gates

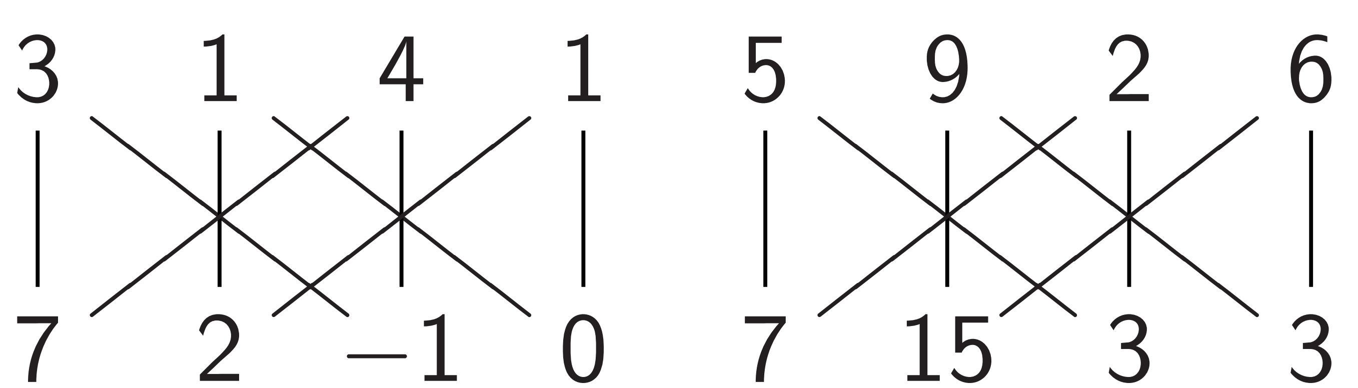
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

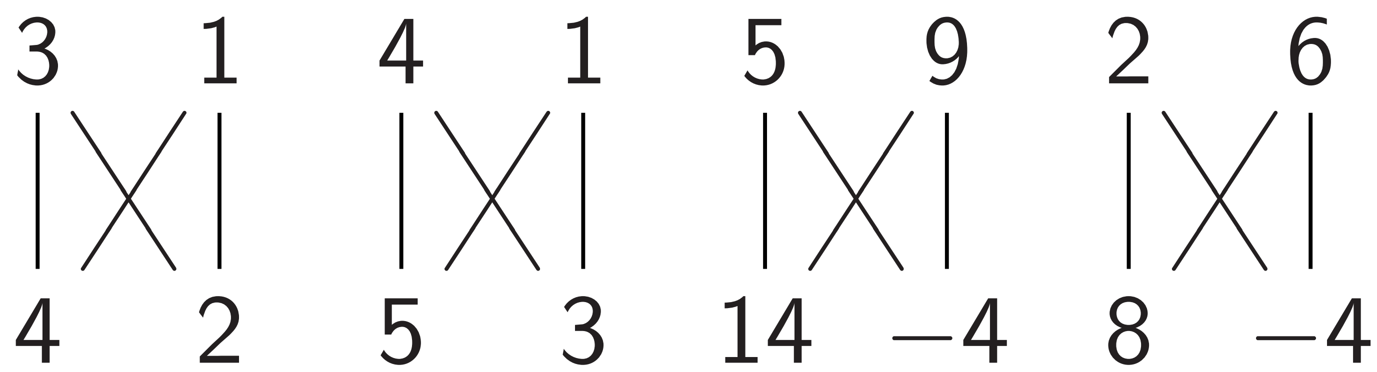
Step 3. Hadamard$_1$:

1, 1, 1, 1, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Hadamard gates

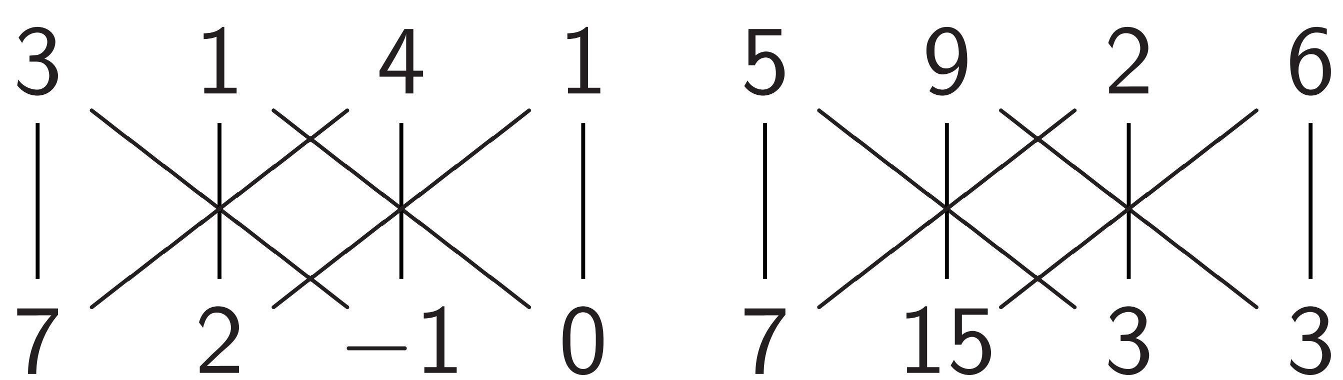
Hadamard_0:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard_1:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 4. Hadamard_2:

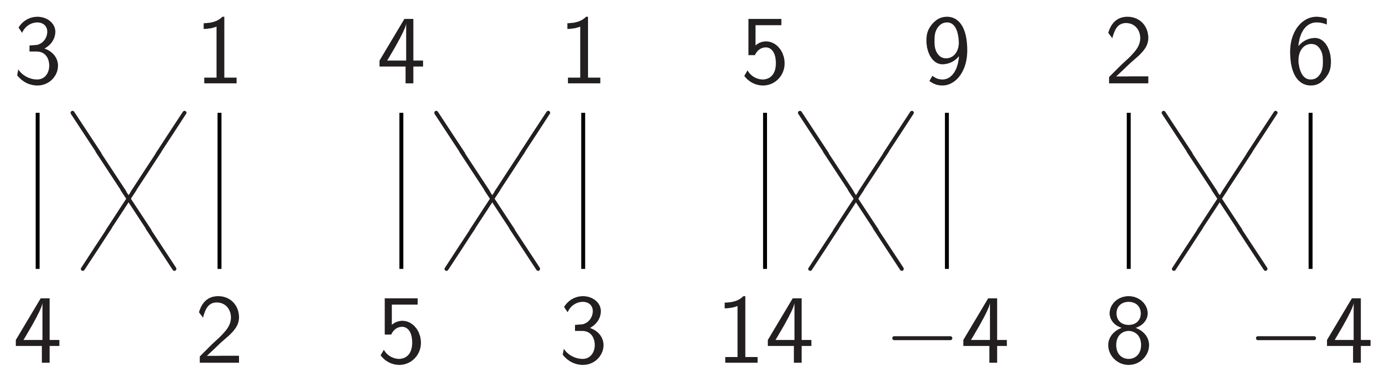
1, 1, 1, 1, 1, 1, 1, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe.

Hadamard gates

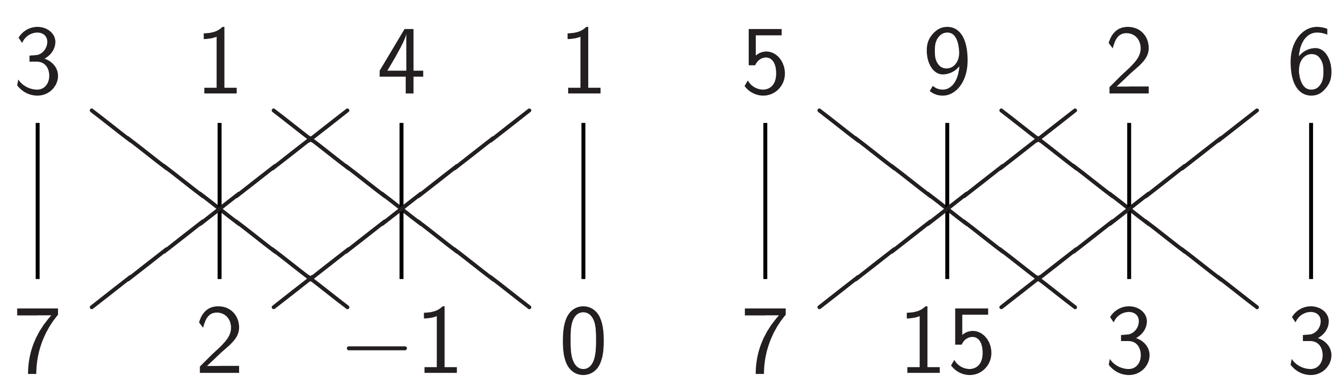
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 5. CNOT$_{0,3}$:

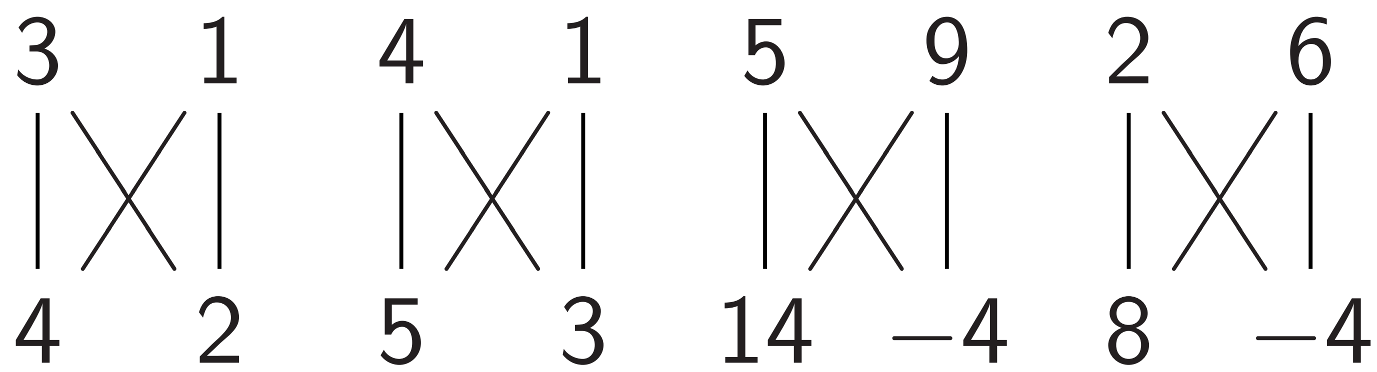
1, 0, 1, 0, 1, 0, 1, 0,
0, 1, 0, 1, 0, 1, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Hadamard gates

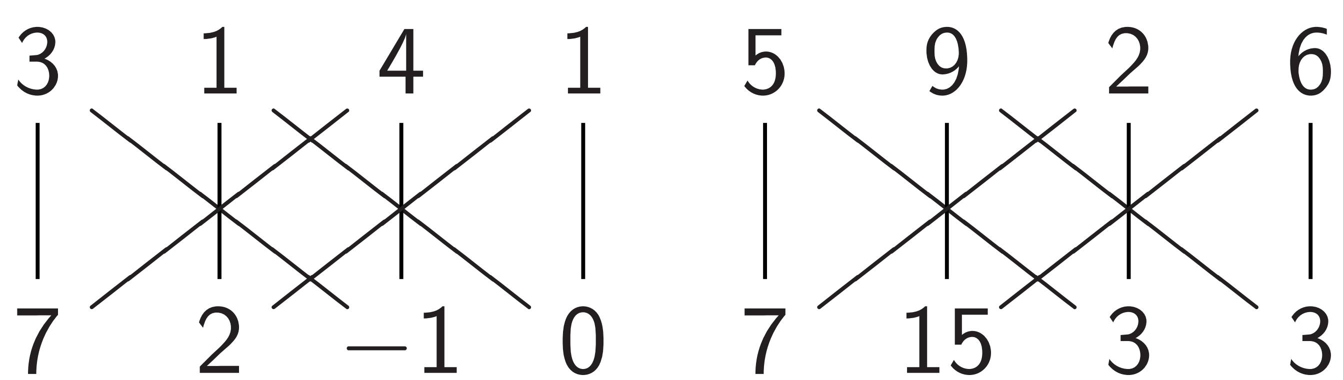
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 5b. More shuffling:

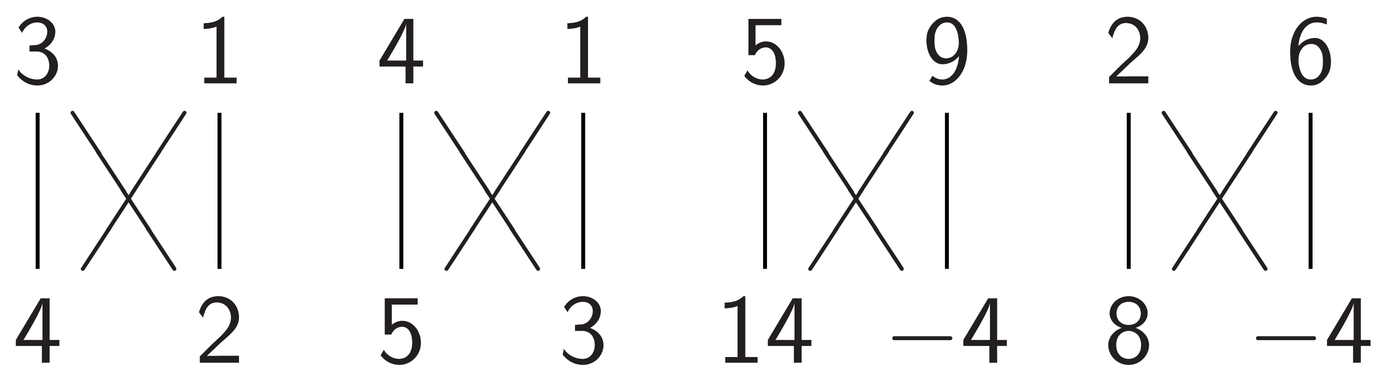
1, 0, 0, 0, 1, 0, 0, 0,
0, 1, 0, 0, 0, 1, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 1, 0,
0, 0, 0, 1, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Hadamard gates

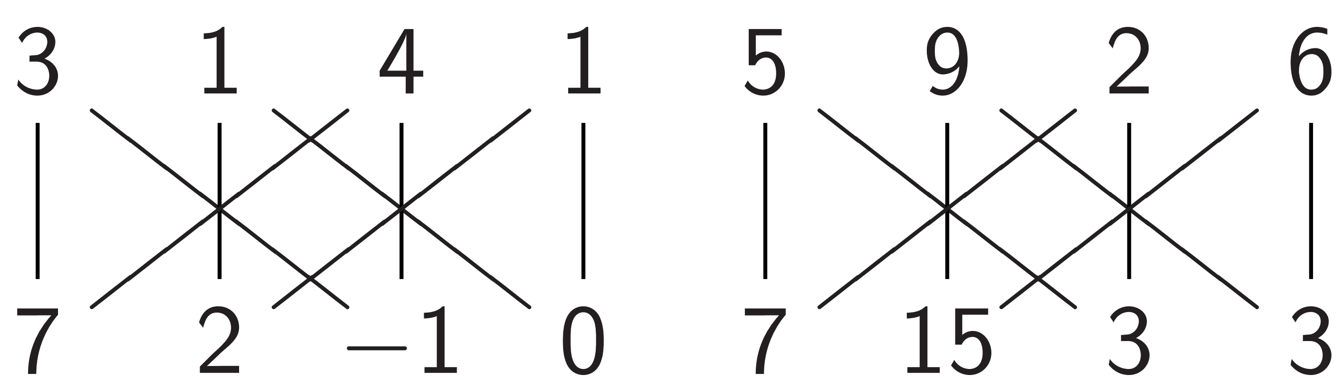
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 5c. More shuffling:

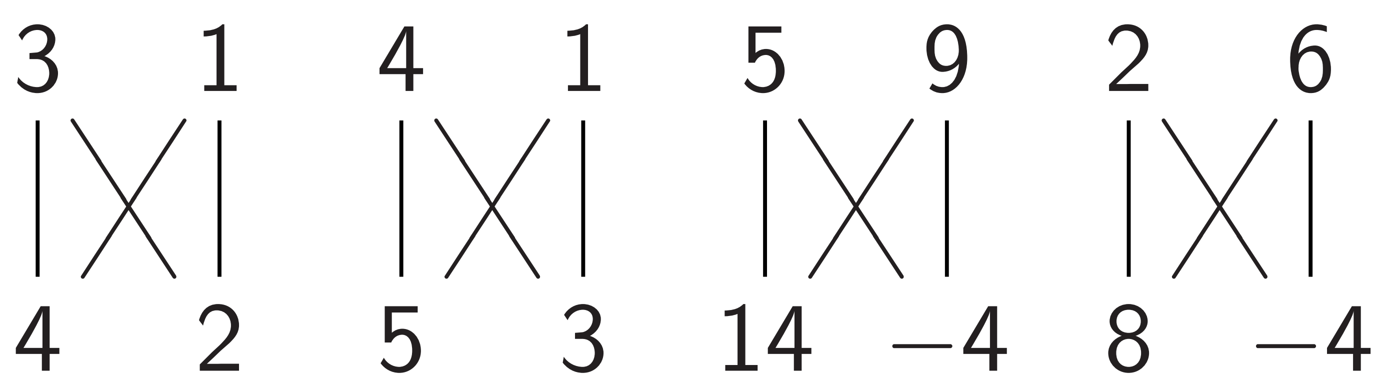
1, 0, 0, 0, 0, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
0, 0, 1, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 1.

Each column is a parallel universe performing its own computations.

Hadamard gates

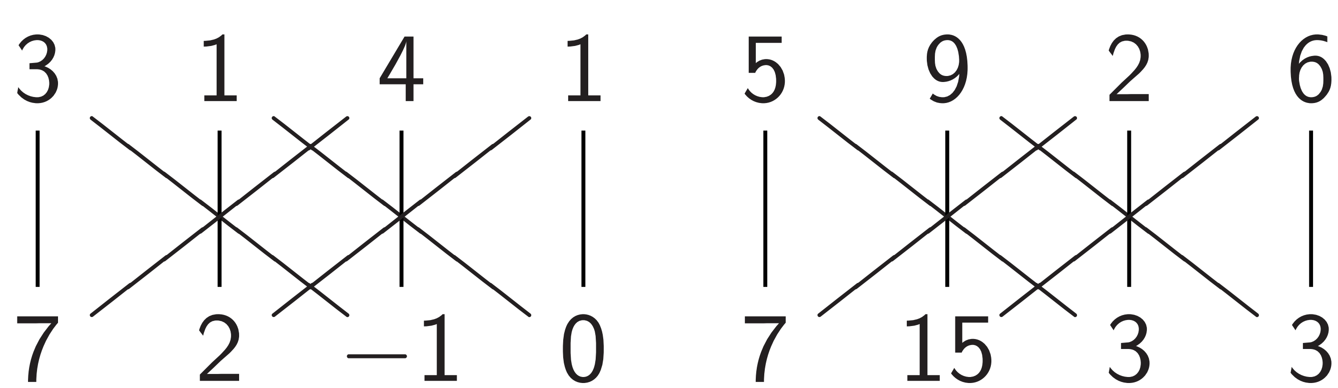
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 5d. More shuffling:

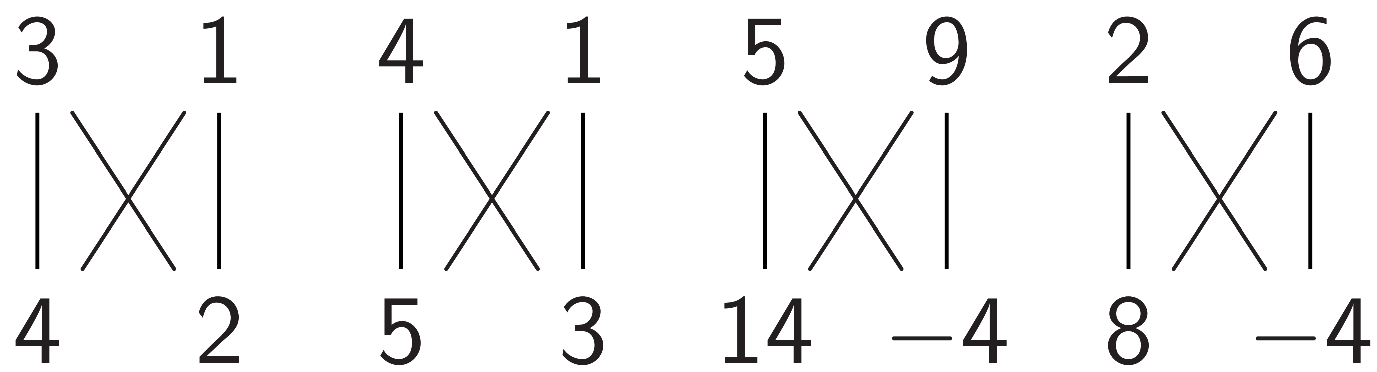
1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 0, 1, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Hadamard gates

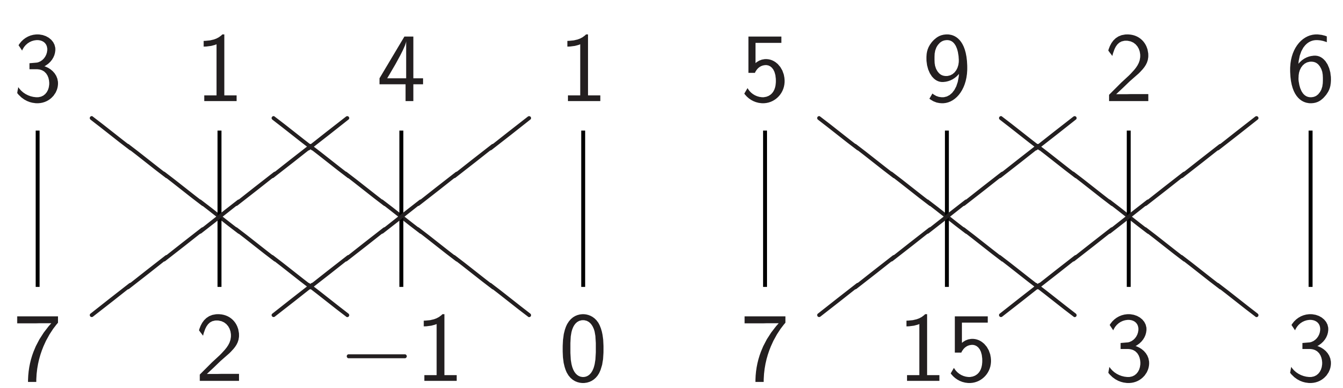
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 5e. More shuffling:

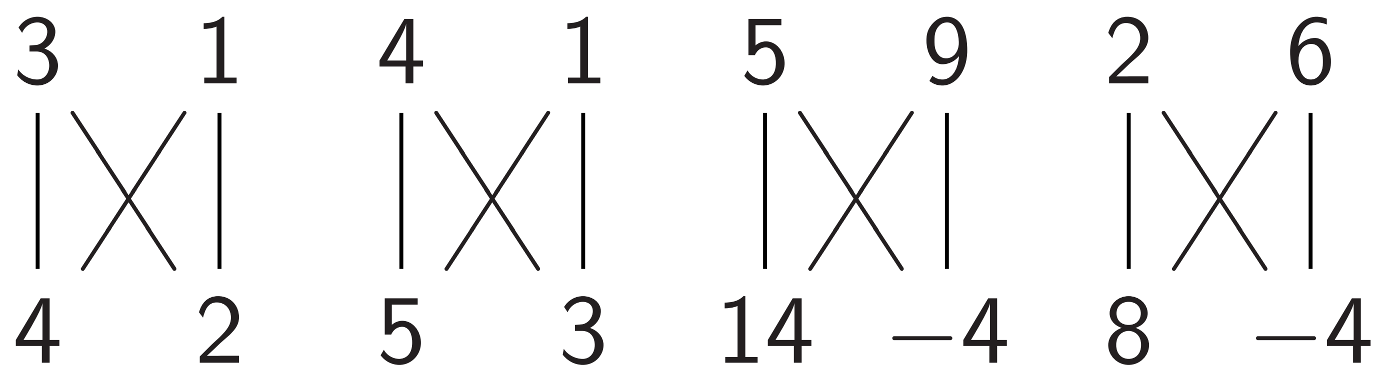
1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Hadamard gates

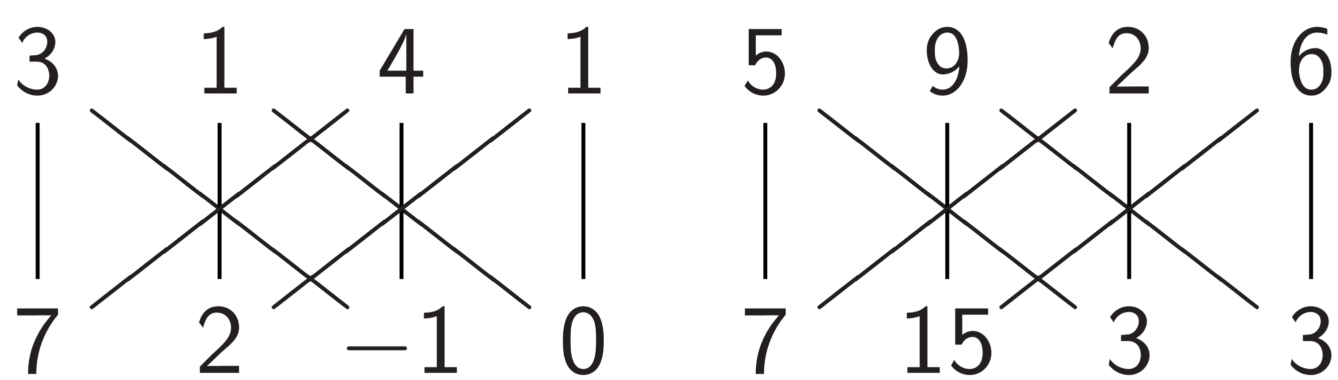
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 5f. More shuffling:

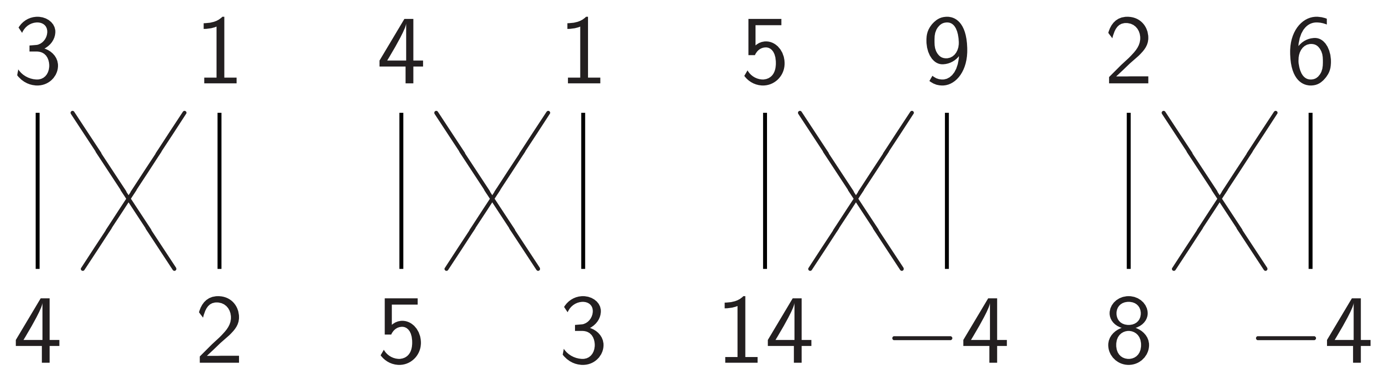
0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0.

Each column is a parallel universe performing its own computations.

Hadamard gates

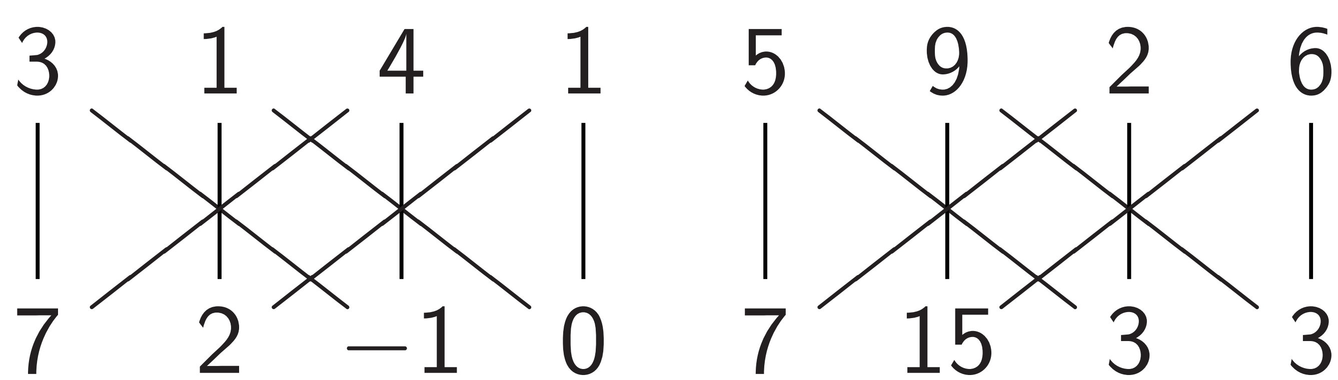
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 5g. More shuffling:

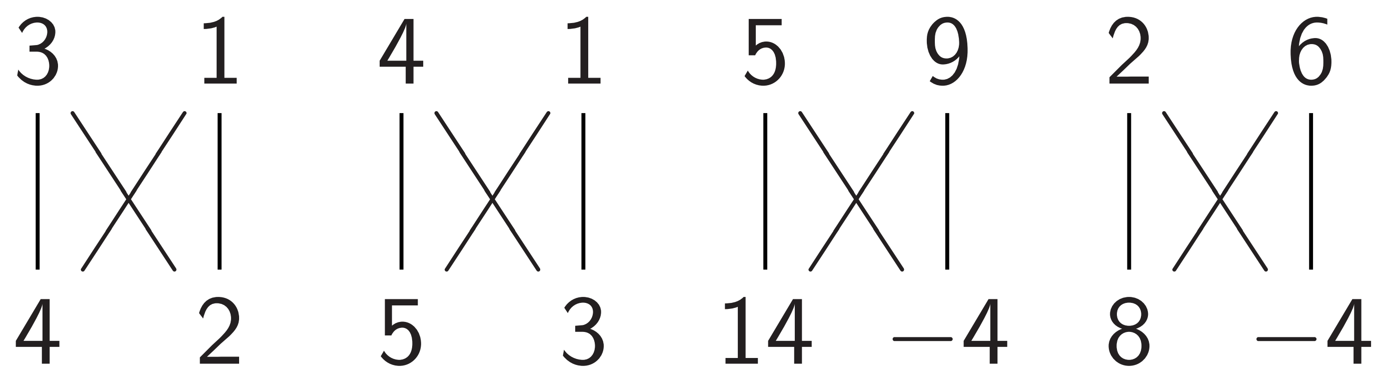
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1.

Each column is a parallel universe performing its own computations.

Hadamard gates

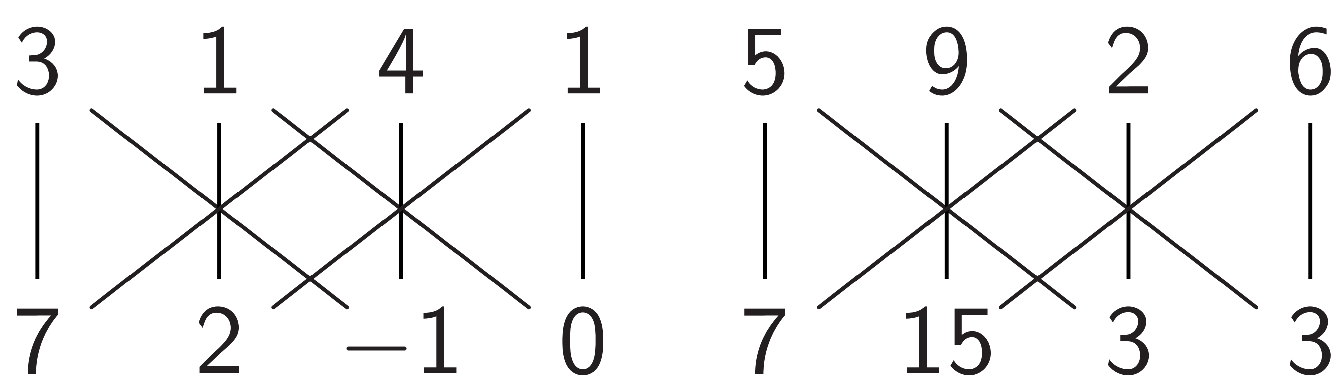
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 5h. More shuffling:

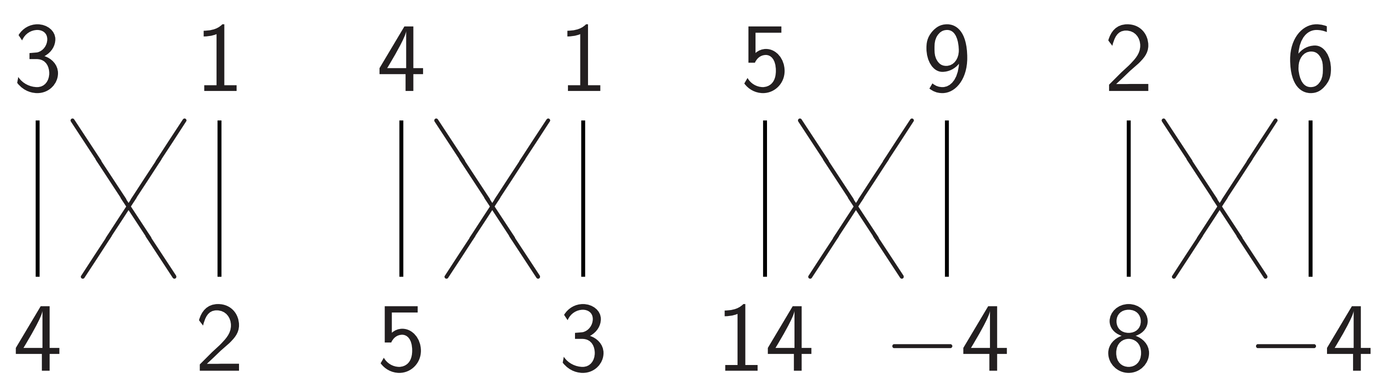
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Hadamard gates

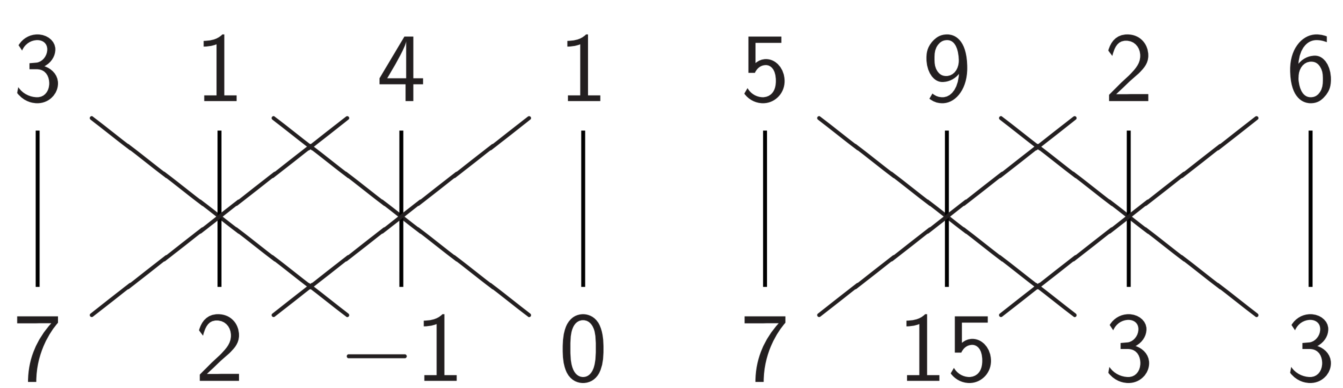
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 5i. More shuffling:

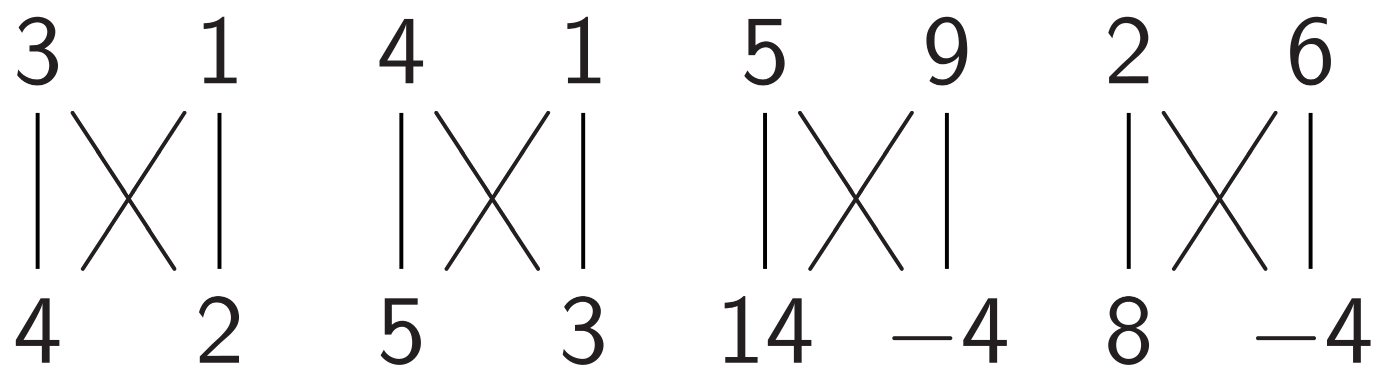
0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 0, 1, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 1,
0, 0, 1, 0, 0, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Hadamard gates

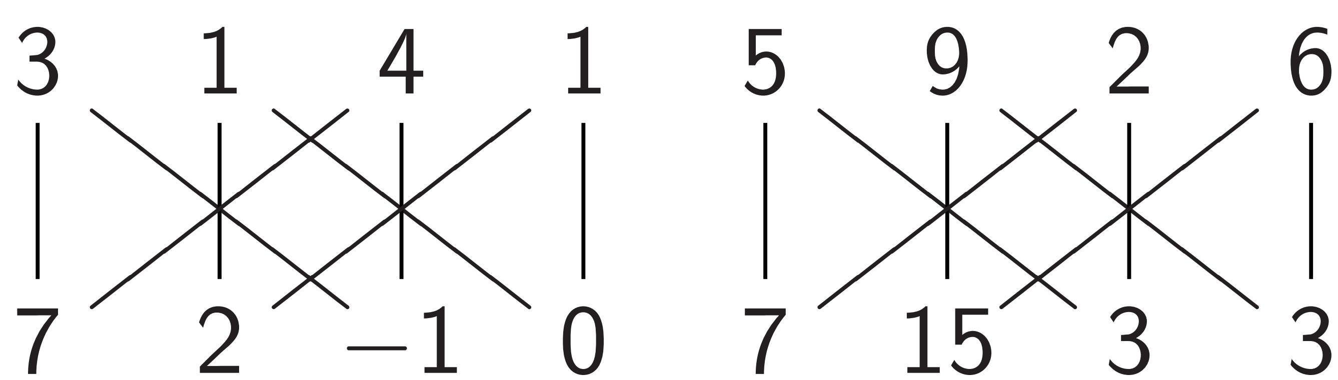
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 5j. Final shuffling:

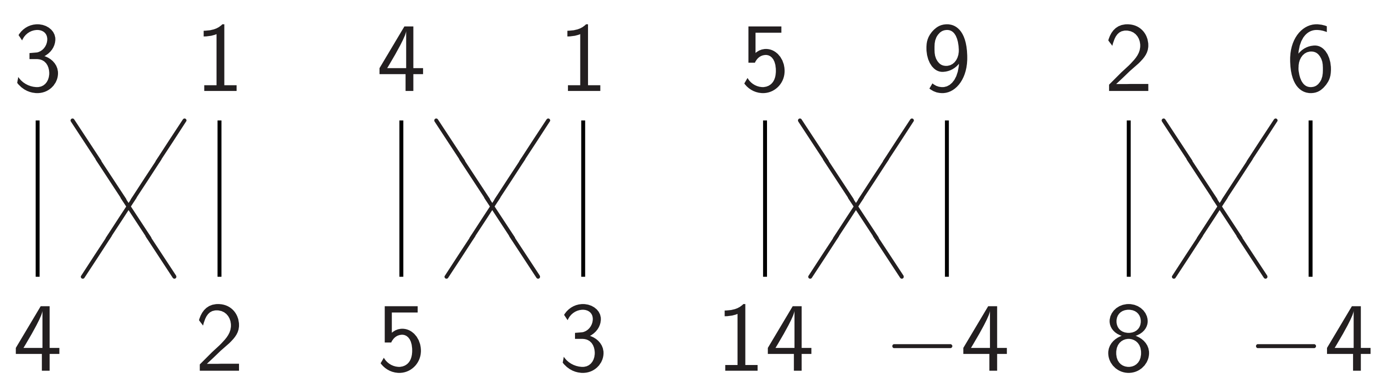
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 1, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
1, 0, 0, 0, 0, 1, 0, 0.

Each column is a parallel universe performing its own computations.

Hadamard gates

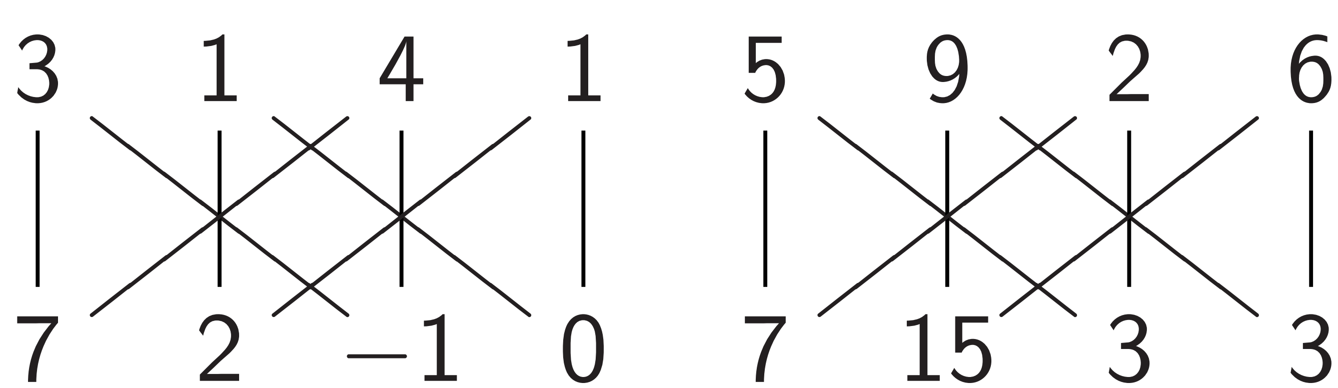
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 5j. Final shuffling:

0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 1, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
1, 0, 0, 0, 0, 1, 0, 0.

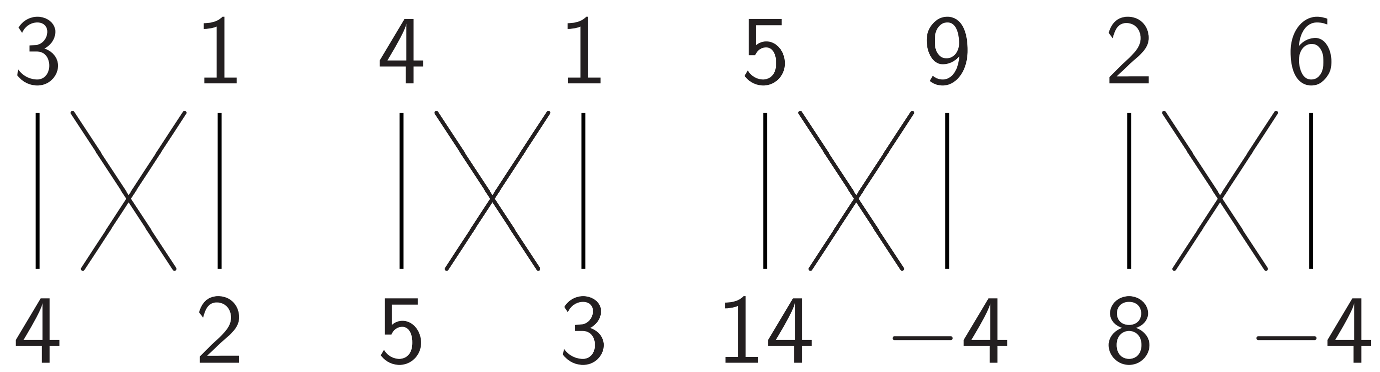
Each column is a parallel universe performing its own computations.

Surprise: u and $u \oplus 101$ match.

Hadamard gates

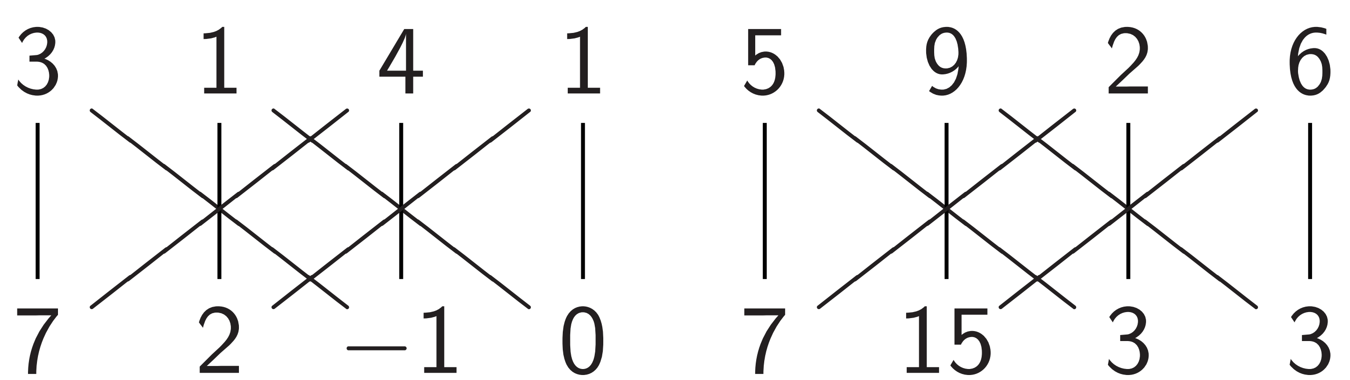
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 6. Hadamard$_0$:

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 1, \overline{1}, 0, 0, 1, 1,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 1, 1, 0, 0, 1, \overline{1},$

$1, \overline{1}, 0, 0, 1, 1, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

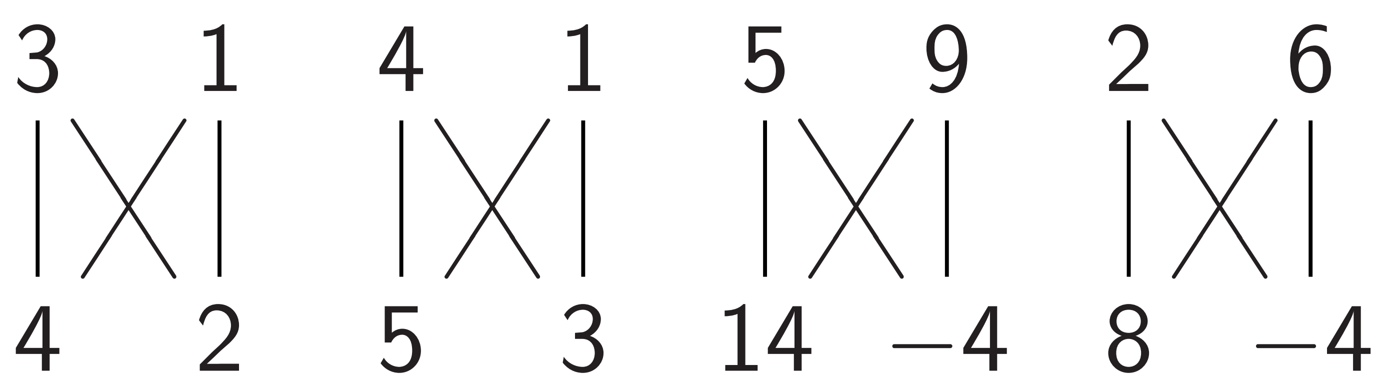
$0, 0, 0, 0, 0, 0, 0, 0,$

$1, 1, 0, 0, 1, \overline{1}, 0, 0.$

Hadamard gates

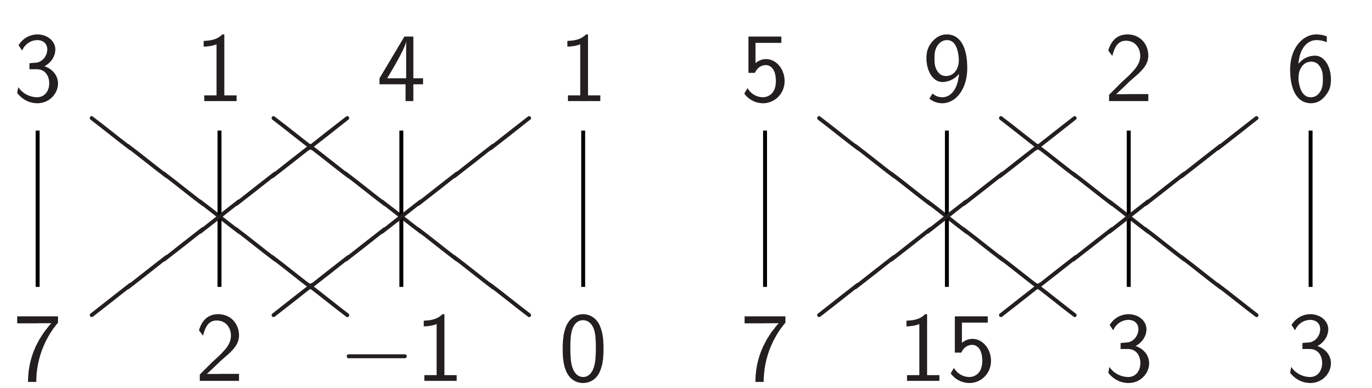
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

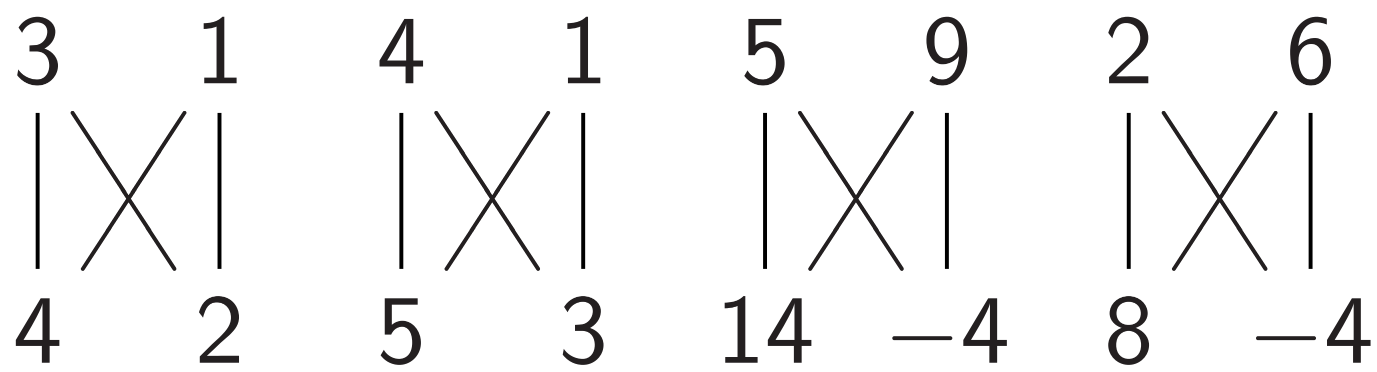
Step 7. Hadamard$_1$:

$0, 0, 0, 0, 0, 0, 0, 0,$
$1, \overline{1}, \overline{1}, 1, 1, 1, \overline{1}, \overline{1},$
$0, 0, 0, 0, 0, 0, 0, 0,$
$1, 1, \overline{1}, \overline{1}, 1, \overline{1}, \overline{1}, 1,$
$1, \overline{1}, 1, \overline{1}, 1, 1, 1, 1,$
$0, 0, 0, 0, 0, 0, 0, 0,$
$0, 0, 0, 0, 0, 0, 0, 0,$
$1, 1, 1, 1, 1, \overline{1}, 1, \overline{1}.$

Hadamard gates

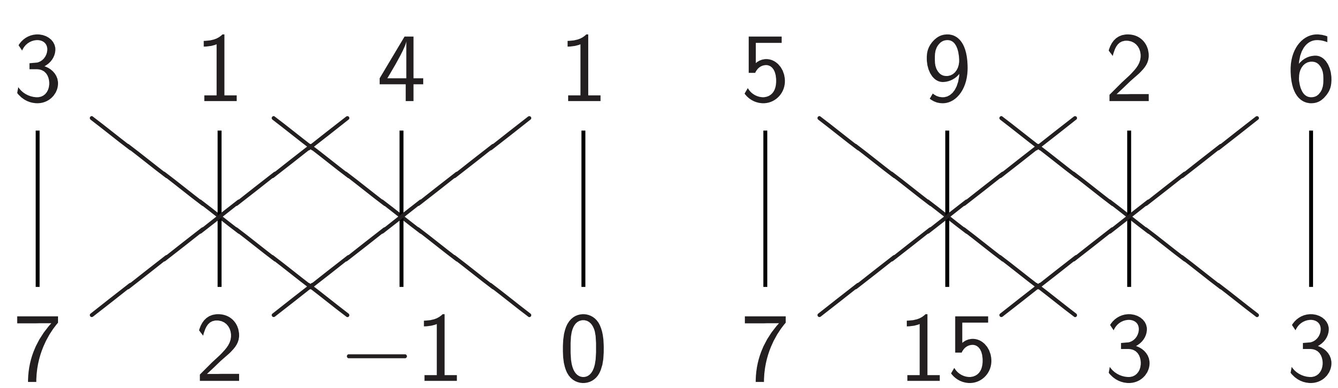
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

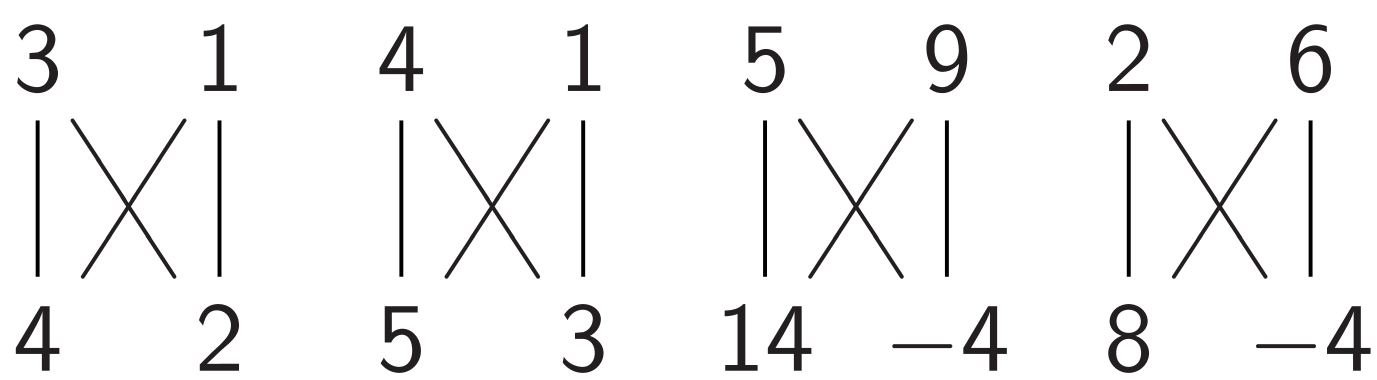
Step 8. Hadamard$_2$:

0, 0, 0, 0, 0, 0, 0, 0,
2, 0, $\overline{2}$, 0, 0, $\overline{2}$, 0, 2,
0, 0, 0, 0, 0, 0, 0, 0,
2, 0, $\overline{2}$, 0, 0, 2, 0, $\overline{2}$,
2, 0, 2, 0, 0, $\overline{2}$, 0, $\overline{2}$,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
2, 0, 2, 0, 0, 2, 0, 2.

Hadamard gates

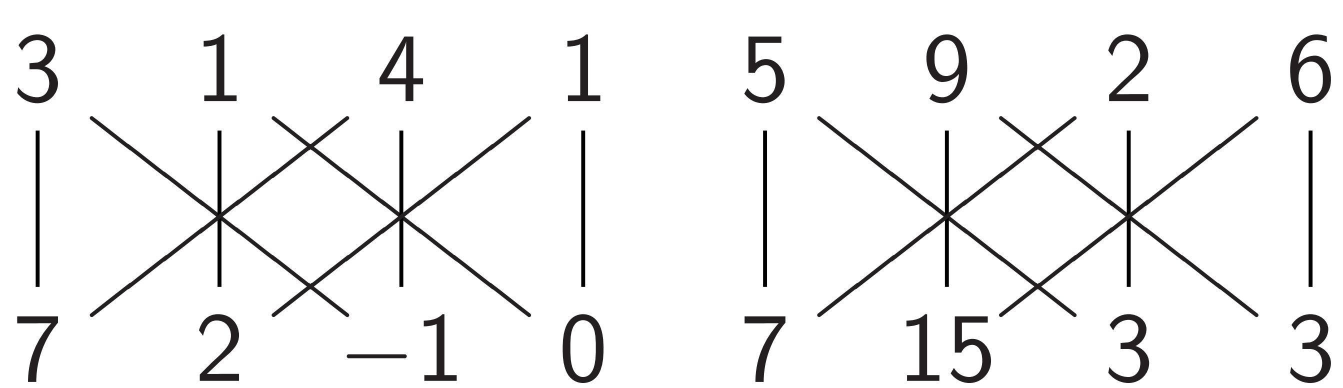
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 8. Hadamard$_2$:

$0, 0, 0, 0, 0, 0, 0, 0,$

$2, 0, \overline{2}, 0, 0, \overline{2}, 0, 2,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$2, 0, \overline{2}, 0, 0, 2, 0, \overline{2},$

$2, 0, 2, 0, 0, \overline{2}, 0, \overline{2},$

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$2, 0, 2, 0, 0, 2, 0, 2.$

Step 9: Measure. Obtain some information about the surprise: a random vector orthogonal to 101.