Quantum algorithms

Daniel J. Bernstein
University of Illinois at Chicago

"Quantum algorithm"
means an algorithm that
a quantum computer can run.

i.e. a sequence of instructions,
where each instruction is
in a quantum computer's
supported instruction set.

How do we know which instructions a quantum computer will support?

Quantum computer type 1 (QC1):
stores many "qubits";
can efficiently perform
"Hadamard gate", "T gate",
"controlled NOT gate".

Making these instructions work is the main goal of quantum-computer engineering.

Combine these instructions
to compute "Toffoli gate";
... "Simon's algorithm";
... "Shor's algorithm";
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Partial proof: see, e.g.,
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Measurement produces element q
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Measurement produces
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with probability $|a_q|^2 / \sum_r |a_r|^2$.

$(a_0, a_1, a_2, a_3, a_4, a$
$(a_4, a_5, a_6, a_7, a_0, a$
is "complementing
$(q_0, q_1, q_2) \mapsto (q_0$

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$+ 8|6\rangle$:

Fast quantum operations, part 1

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_1, a_0, a_3, a_2, a_5, a_4, a_7, a_6)$
is complementing index bit 0,
hence "complementing qubit 0".

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7)$
is measured as (q_0, q_1, q_2),
representing $q = q_0 + 2q_1 + 4q_2$,
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$(a_4, a_5, a_6, a_7, a_0, a_1, a_2, a_3)$
is "complementing qubit 2":
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- ∘ complementing qubit 0
- ∘ swapping qubits 0 and 2.

Similarly: swapping qubits i, j.

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"control
$(q_0, q_1, q$

erations, part 1

$a_5, a_6, a_7) \mapsto$

$a_4, a_7, a_6)$

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$a_5, a_6, a_7)$

$_0, q_1, q_2)$,

$q_0 + 2q_1 + 4q_2$,

$_q|^2 / \sum_r |a_r|^2$.

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$_0 \oplus 1, q_1, q_2)$,

,

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"controlled NOT g
$(q_0, q_1, q_2) \mapsto (q_0$

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pping qubits 0 and 2":

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menting qubit 2

ping qubits 0 and 2

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9

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$a_2, a_3, a_4, a_5, a_6, a_7,$

$_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$

$a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$

$a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$

$_1, a_3, a_2, a_4, a_5, a_7, a_6,$

$_{11}, a_{10}, a_{12}, a_{13}, a_{15}, a_{14},$

$a_{19}, a_{18}, a_{20}, a_{21}, a_{23}, a_{22},$

$a_{27}, a_{26}, a_{28}, a_{29}, a_{31}, a_{30})$.

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$

$(a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6)$

is a "Toffoli gate" $=$

"controlled controlled NOT gate":

$(q_0, q_1, q_2) \mapsto (q_0 \oplus q_1 q_2, q_1, q_2)$.

Example with more qubits:

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7,$

$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$

$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$

$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$

$\mapsto (a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6,$

$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{15}, a_{14},$

$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{23}, a_{22},$

$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{31}, a_{30})$.

Reversib

Say p is

of $\{0, 1,$

General

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$(a_0, a_1, .$

$(a_{p^{-1}(0)}$

$a_5, a_6, a_7) \mapsto$

$a_5, a_7, a_6)$

OR gate" $=$

gate":

$\oplus\, q_1, q_1, q_2)$.

e qubits:

$a_5, a_6, a_7,$

$, a_{13}, a_{14}, a_{15},$

$20, a_{21}, a_{22}, a_{23},$

$28, a_{29}, a_{30}, a_{31})$

$_4, a_5, a_7, a_6,$

$, a_{13}, a_{15}, a_{14},$

$20, a_{21}, a_{23}, a_{22},$

$28, a_{29}, a_{31}, a_{30})$.

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$

$(a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6)$

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$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$

$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$

$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$

$\mapsto (a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6,$

$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{15}, a_{14},$

$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{23}, a_{22},$

$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{31}, a_{30})$.

Reversible comput

Say p is a permuta

of $\{0, 1, \ldots, 2^n -$

General strategy t

these fast quantur

to obtain index pe

$(a_0, a_1, \ldots, a_{2^n-1})$

$(a_{p^{-1}(0)}, a_{p^{-1}(1)}, \cdot$

$\longmapsto$

$=$

$q_2)$.

a_{15},

$_2, a_{23}$,

$_0, a_{31})$

$_6$,

a_{14},

$_3, a_{22}$,

$_1, a_{30})$.

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6)$
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Example with more qubits:

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$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
$\mapsto (a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{15}, a_{14},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{23}, a_{22},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{31}, a_{30})$.

Reversible computation

Say p is a permutation
of $\{0, 1, \ldots, 2^n - 1\}$.

General strategy to compose
these fast quantum operatio
to obtain index permutation
$(a_0, a_1, \ldots, a_{2^n-1}) \mapsto$
$(a_{p^{-1}(0)}, a_{p^{-1}(1)}, \cdots, a_{p^{-1}(2^n}$

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6)$
is a "Toffoli gate" $=$
"controlled controlled NOT gate":
$(q_0, q_1, q_2) \mapsto (q_0 \oplus q_1 q_2, q_1, q_2)$.

Example with more qubits:
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
$\mapsto (a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{15}, a_{14},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{23}, a_{22},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{31}, a_{30})$.

Reversible computation

Say p is a permutation
of $\{0, 1, \ldots, 2^n - 1\}$.

General strategy to compose
these fast quantum operations
to obtain index permutation
$(a_0, a_1, \ldots, a_{2^n-1}) \mapsto$
$(a_{p^{-1}(0)}, a_{p^{-1}(1)}, \cdots, a_{p^{-1}(2^n-1)})$:

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6)$
is a "Toffoli gate" =
"controlled controlled NOT gate":
$(q_0, q_1, q_2) \mapsto (q_0 \oplus q_1 q_2, q_1, q_2)$.

Example with more qubits:
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
$\mapsto (a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{15}, a_{14},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{23}, a_{22},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{31}, a_{30})$.

Reversible computation

Say p is a permutation
of $\{0, 1, \ldots, 2^n - 1\}$.

General strategy to compose
these fast quantum operations
to obtain index permutation
$(a_0, a_1, \ldots, a_{2^n-1}) \mapsto$
$(a_{p^{-1}(0)}, a_{p^{-1}(1)}, \cdots, a_{p^{-1}(2^n-1)})$:

1. Build a traditional circuit
to compute $j \mapsto p(j)$
using NOT/XOR/AND gates.

2. Convert into reversible gates:
e.g., convert AND into Toffoli.

$a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$

$a_2, a_3, a_4, a_5, a_7, a_6)$

ffoli gate" $=$

led controlled NOT gate":

$q_2) \mapsto (q_0 \oplus q_1 q_2, q_1, q_2)$.

e with more qubits:

$a_2, a_3, a_4, a_5, a_6, a_7,$

$_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$

$a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$

$a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$

$_1, a_2, a_3, a_4, a_5, a_7, a_6,$

$_{10}, a_{11}, a_{12}, a_{13}, a_{15}, a_{14},$

$a_{18}, a_{19}, a_{20}, a_{21}, a_{23}, a_{22},$

$a_{26}, a_{27}, a_{28}, a_{29}, a_{31}, a_{30})$.

Reversible computation

Say p is a permutation of $\{0, 1, \ldots, 2^n - 1\}$.

General strategy to compose these fast quantum operations to obtain index permutation $(a_0, a_1, \ldots, a_{2^n-1}) \mapsto$ $(a_{p^{-1}(0)}, a_{p^{-1}(1)}, \ldots, a_{p^{-1}(2^n-1)})$:

1. Build a traditional circuit to compute $j \mapsto p(j)$ using NOT/XOR/AND gates.

2. Convert into reversible gates: e.g., convert AND into Toffoli.

Example

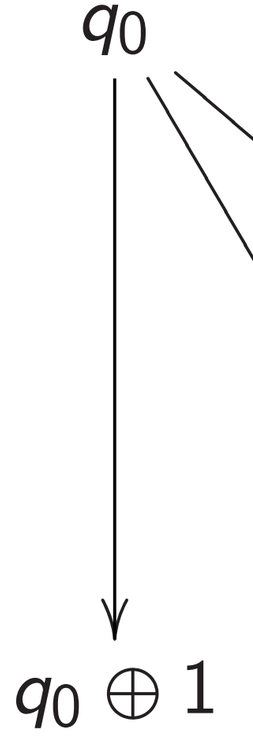
$(a_0, a_1, a$

$(a_7, a_0, a$

permuta

1. Build

to comp



$a_5, a_6, a_7) \mapsto$

$a_5, a_7, a_6)$

$=$

lled NOT gate":

$\oplus\, q_1 q_2, q_1, q_2)$.

e qubits:

$a_5, a_6, a_7,$

$, a_{13}, a_{14}, a_{15},$

$20}, a_{21}, a_{22}, a_{23},$

$28}, a_{29}, a_{30}, a_{31})$

$_4, a_5, a_7, a_6,$

$, a_{13}, a_{15}, a_{14},$

$20}, a_{21}, a_{23}, a_{22},$

$28}, a_{29}, a_{31}, a_{30})$.

Reversible computation

Say p is a permutation of $\{0, 1, \ldots, 2^n - 1\}$.

General strategy to compose these fast quantum operations to obtain index permutation $(a_0, a_1, \ldots, a_{2^n-1}) \mapsto (a_{p^{-1}(0)}, a_{p^{-1}(1)}, \ldots, a_{p^{-1}(2^n-1)})$:

1. Build a traditional circuit to compute $j \mapsto p(j)$ using NOT/XOR/AND gates.

2. Convert into reversible gates: e.g., convert AND into Toffoli.

Example: Let's co

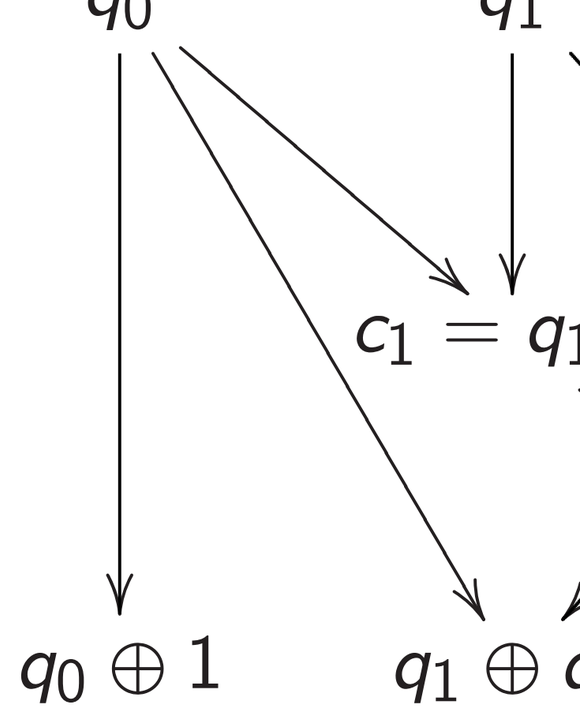
$(a_0, a_1, a_2, a_3, a_4, a$

$(a_7, a_0, a_1, a_2, a_3, a$

permutation $q \mapsto$

1. Build a traditio

to compute $q \mapsto q$



$\longmapsto$

gate":

$_1, q_2)$.

a_{15},

$_2, a_{23}$,

$_0, a_{31})$

$_6$,

a_{14},

$_3, a_{22}$,

$_1, a_{30})$.

Reversible computation

Say p is a permutation of $\{0, 1, \ldots, 2^n - 1\}$.

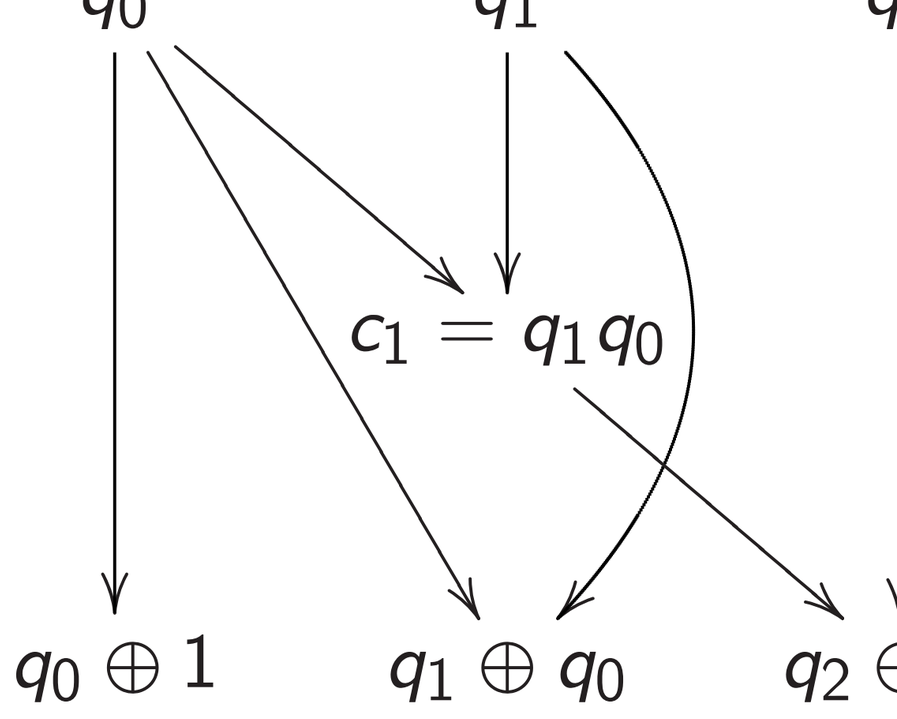
General strategy to compose these fast quantum operations to obtain index permutation $(a_0, a_1, \ldots, a_{2^n-1}) \mapsto (a_{p^{-1}(0)}, a_{p^{-1}(1)}, \ldots, a_{p^{-1}(2^n-1)})$:

1. Build a traditional circuit to compute $j \mapsto p(j)$ using NOT/XOR/AND gates.

2. Convert into reversible gates: e.g., convert AND into Toffoli.

Example: Let's compute $(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto (a_7, a_0, a_1, a_2, a_3, a_4, a_5, a_6)$; permutation $q \mapsto q + 1$ mod

1. Build a traditional circuit to compute $q \mapsto q + 1$ mod



Reversible computation

Say p is a permutation of $\{0, 1, \ldots, 2^n - 1\}$.

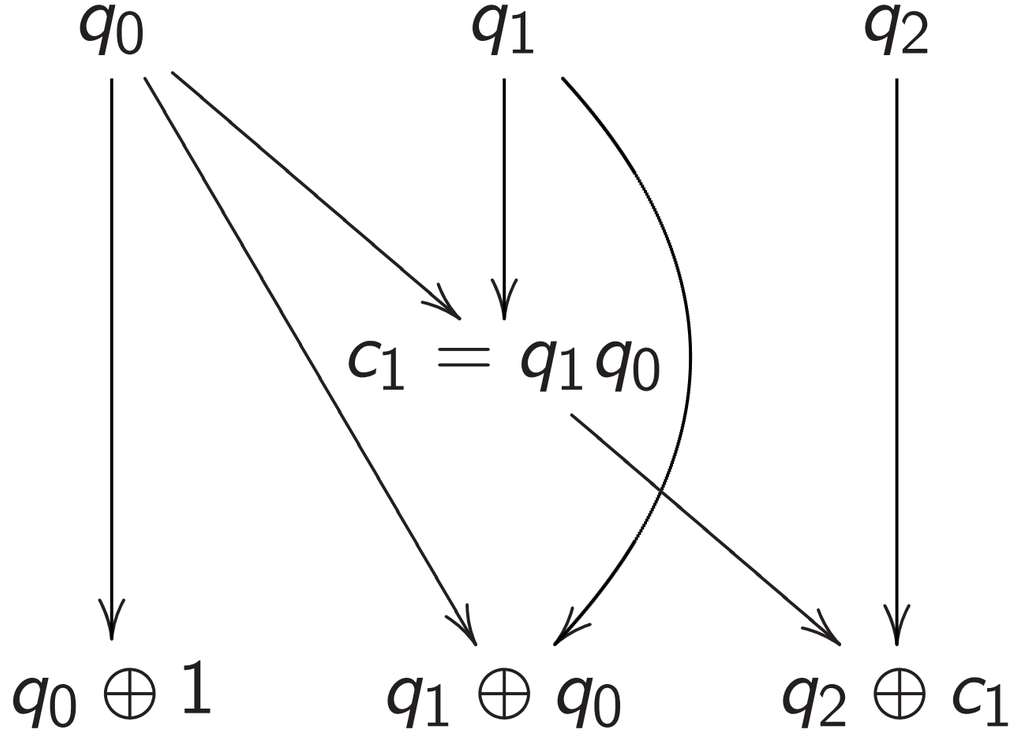
General strategy to compose these fast quantum operations to obtain index permutation $(a_0, a_1, \ldots, a_{2^n-1}) \mapsto (a_{p^{-1}(0)}, a_{p^{-1}(1)}, \ldots, a_{p^{-1}(2^n-1)})$:

1. Build a traditional circuit to compute $j \mapsto p(j)$ using NOT/XOR/AND gates.

2. Convert into reversible gates: e.g., convert AND into Toffoli.

Example: Let's compute $(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto (a_7, a_0, a_1, a_2, a_3, a_4, a_5, a_6)$; permutation $q \mapsto q + 1 \bmod 8$.

1. Build a traditional circuit to compute $q \mapsto q + 1 \bmod 8$.



le computation

a permutation
$\ldots, 2^n - 1\}$.

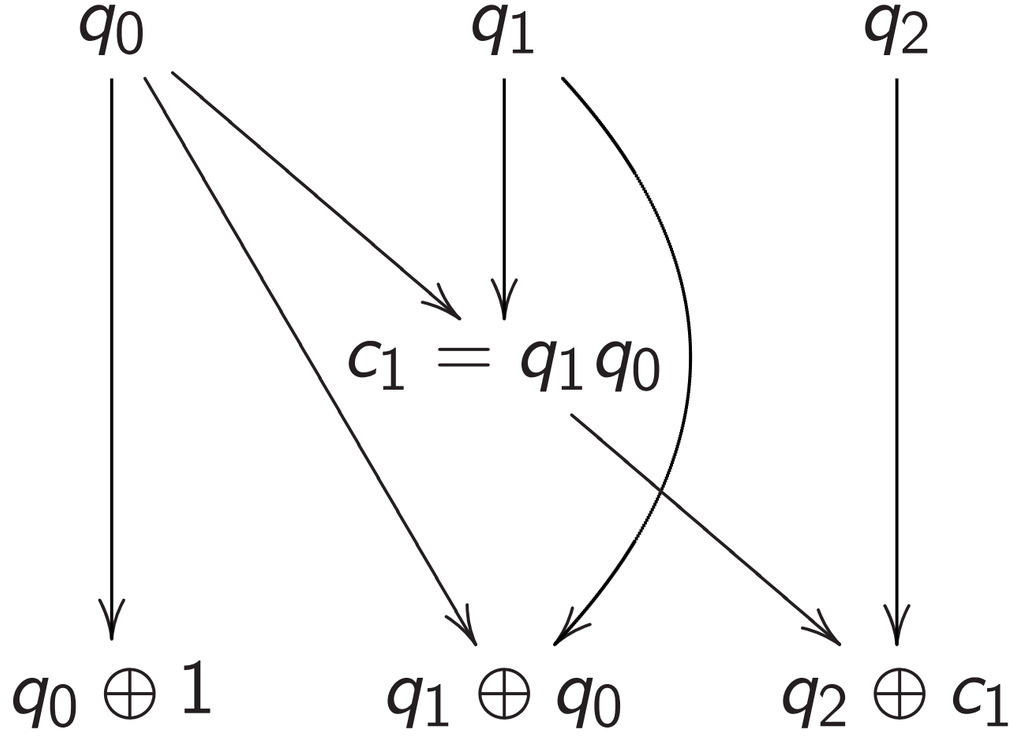
strategy to compose
st quantum operations
n index permutation
$\ldots, a_{2^n-1}) \mapsto$
$, a_{p^{-1}(1)}, \ldots, a_{p^{-1}(2^n-1)})$:

a traditional circuit
ute $j \mapsto p(j)$
OT/XOR/AND gates.

ert into reversible gates:
nvert AND into Toffoli.

Example: Let's compute
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_7, a_0, a_1, a_2, a_3, a_4, a_5, a_6)$;
permutation $q \mapsto q + 1 \bmod 8$.

1. Build a traditional circuit
to compute $q \mapsto q + 1 \bmod 8$.



2. Conv

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$(a_0, a_1, a$

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$\ldots, a_{p^{-1}(2^n-1)}$):

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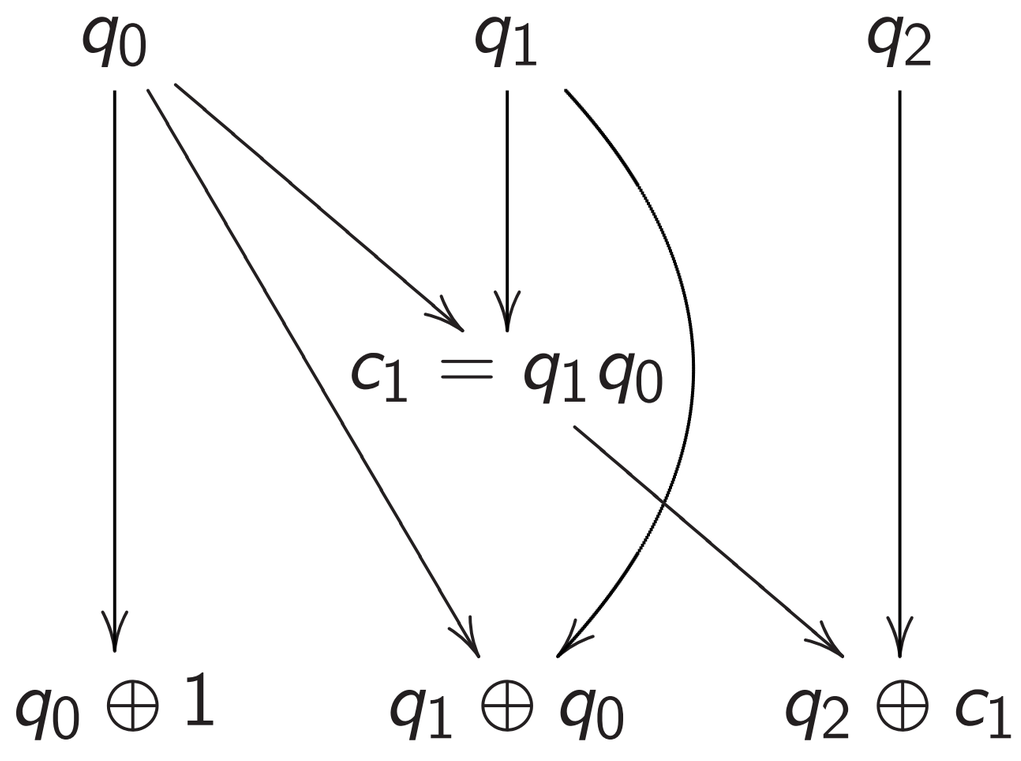
versible gates:
into Toffoli.

Example: Let's compute
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_7, a_0, a_1, a_2, a_3, a_4, a_5, a_6)$;
permutation $q \mapsto q + 1 \bmod 8$.

1. Build a traditional circuit
to compute $q \mapsto q + 1 \bmod 8$.

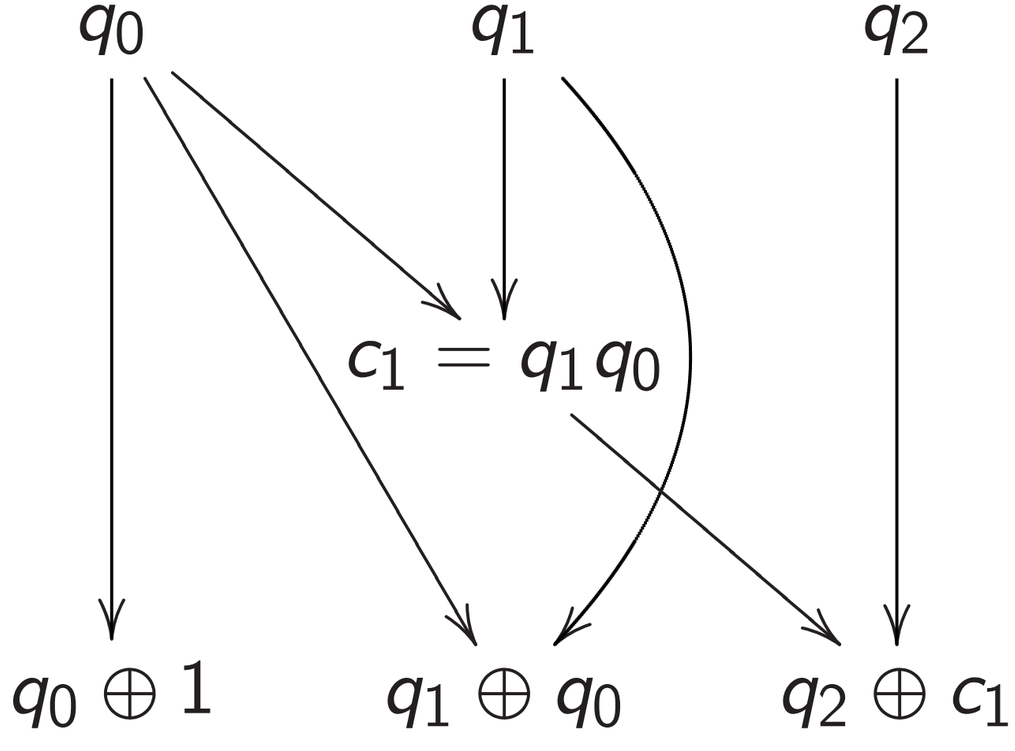


2. Convert into re

Toffoli for $q_2 \leftarrow q$
$(a_0, a_1, a_2, a_3, a_4, a$
$(a_0, a_1, a_2, a_7, a_4, a$

Example: Let's compute
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_7, a_0, a_1, a_2, a_3, a_4, a_5, a_6)$;
permutation $q \mapsto q + 1 \bmod 8$.

1. Build a traditional circuit
to compute $q \mapsto q + 1 \bmod 8$.



2. Convert into reversible ga

Toffoli for $q_2 \leftarrow q_2 \oplus q_1 q_0$:
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7)$ ↦
$(a_0, a_1, a_2, a_7, a_4, a_5, a_6, a_3)$.

Example: Let's compute
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
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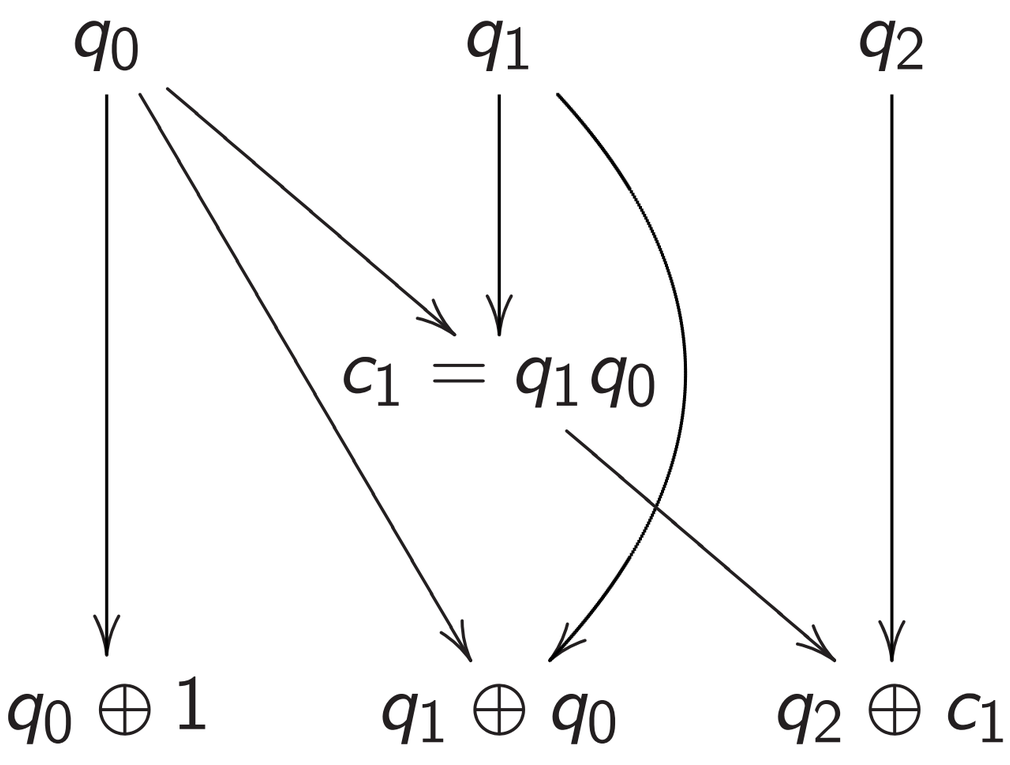
permutation $q \mapsto q + 1 \bmod 8$.

1. Build a traditional circuit
to compute $q \mapsto q + 1 \bmod 8$.

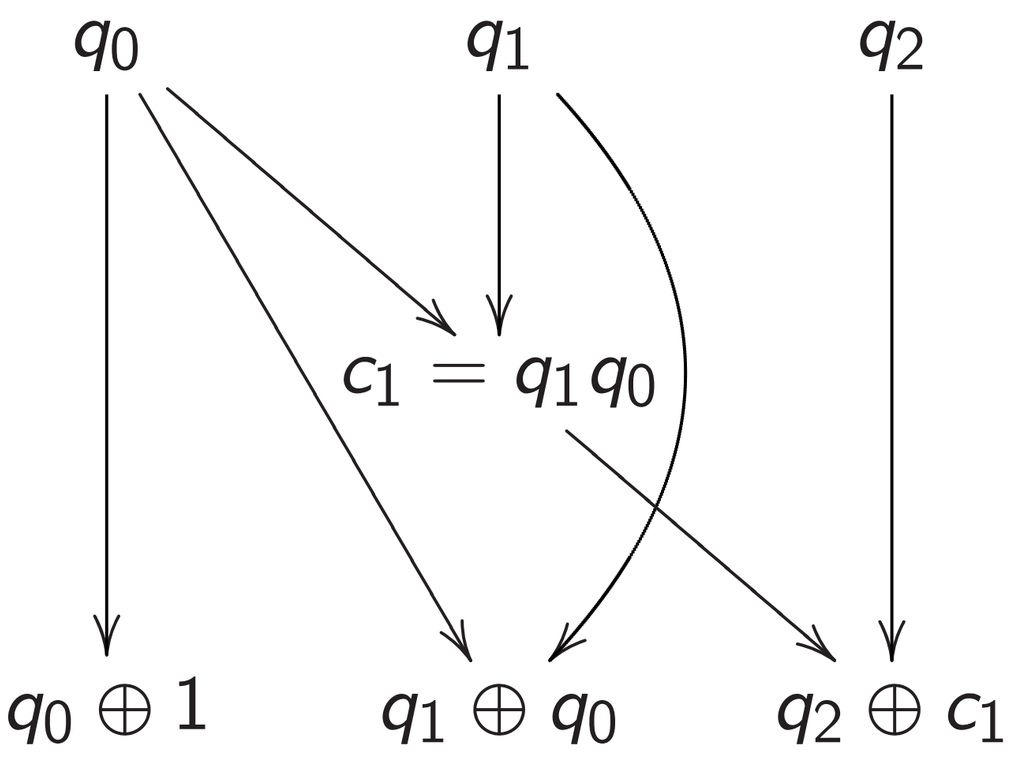


2. Convert into reversible gates.

Toffoli for $q_2 \leftarrow q_2 \oplus q_1 q_0$:
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_1, a_2, a_7, a_4, a_5, a_6, a_3)$.

Example: Let's compute
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_7, a_0, a_1, a_2, a_3, a_4, a_5, a_6)$;
permutation $q \mapsto q + 1 \bmod 8$.

1. Build a traditional circuit
to compute $q \mapsto q + 1 \bmod 8$.



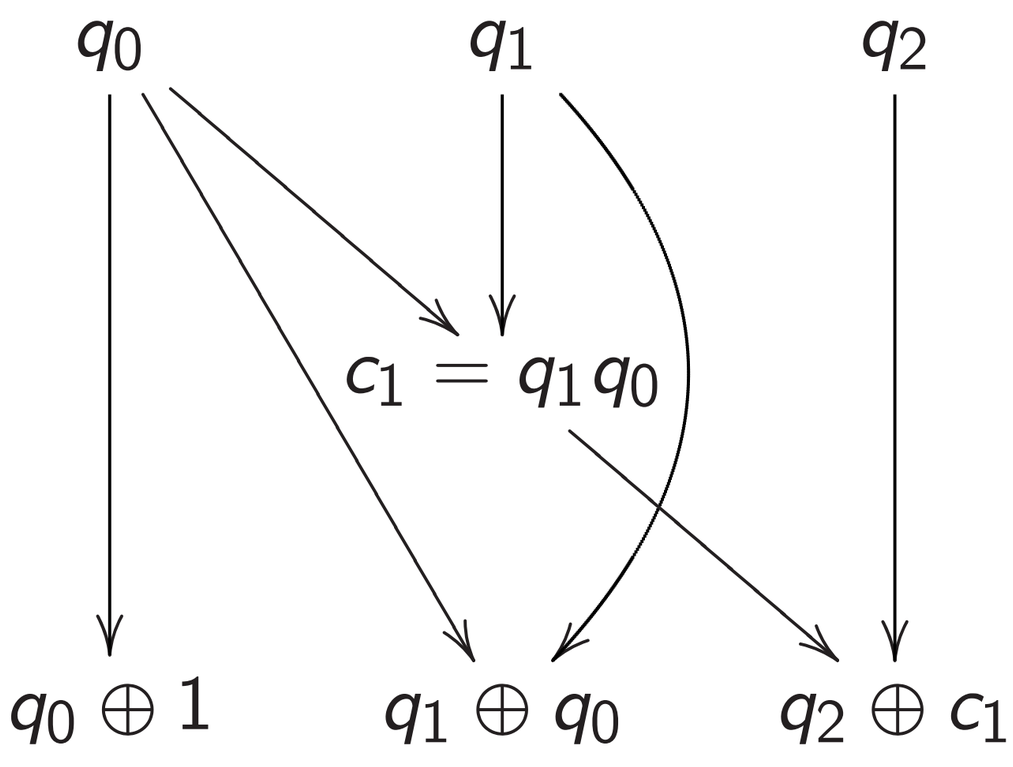
2. Convert into reversible gates.

Toffoli for $q_2 \leftarrow q_2 \oplus q_1 q_0$:
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_1, a_2, a_7, a_4, a_5, a_6, a_3)$.

Controlled NOT for $q_1 \leftarrow q_1 \oplus q_0$:
$(a_0, a_1, a_2, a_7, a_4, a_5, a_6, a_3) \mapsto$
$(a_0, a_7, a_2, a_1, a_4, a_3, a_6, a_5)$.

Example: Let's compute
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_7, a_0, a_1, a_2, a_3, a_4, a_5, a_6)$;
permutation $q \mapsto q + 1 \bmod 8$.

1. Build a traditional circuit
to compute $q \mapsto q + 1 \bmod 8$.



2. Convert into reversible gates.

Toffoli for $q_2 \leftarrow q_2 \oplus q_1 q_0$:
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_1, a_2, a_7, a_4, a_5, a_6, a_3)$.

Controlled NOT for $q_1 \leftarrow q_1 \oplus q_0$:
$(a_0, a_1, a_2, a_7, a_4, a_5, a_6, a_3) \mapsto$
$(a_0, a_7, a_2, a_1, a_4, a_3, a_6, a_5)$.

NOT for $q_0 \leftarrow q_0 \oplus 1$:
$(a_0, a_7, a_2, a_1, a_4, a_3, a_6, a_5) \mapsto$
$(a_7, a_0, a_1, a_2, a_3, a_4, a_5, a_6)$.

e: Let's compute

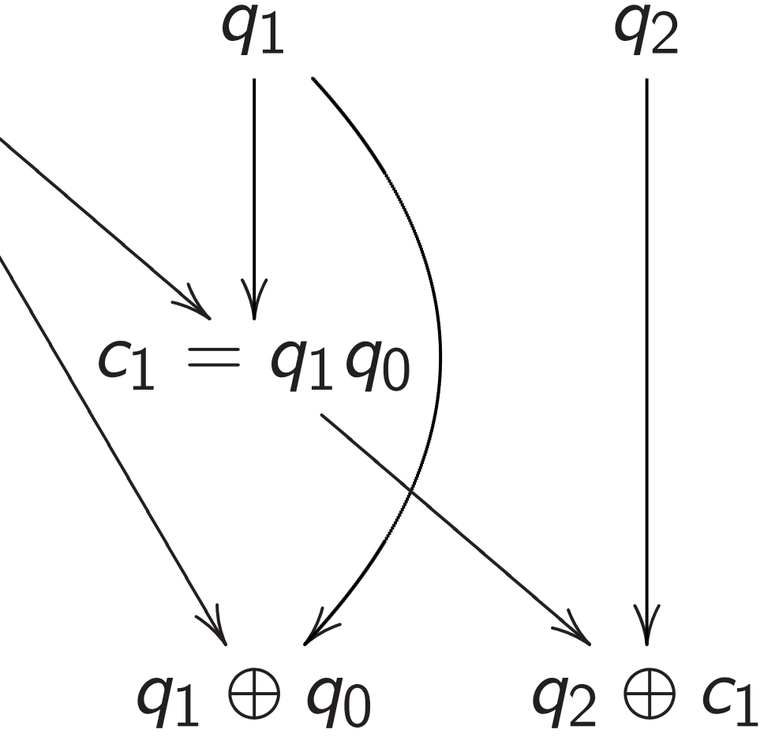
$a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$

$a_1, a_2, a_3, a_4, a_5, a_6)$;

tion $q \mapsto q + 1 \bmod 8$.

a traditional circuit

ute $q \mapsto q + 1 \bmod 8$.



2. Convert into reversible gates.

Toffoli for $q_2 \leftarrow q_2 \oplus q_1 q_0$:

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$

$(a_0, a_1, a_2, a_7, a_4, a_5, a_6, a_3)$.

Controlled NOT for $q_1 \leftarrow q_1 \oplus q_0$:

$(a_0, a_1, a_2, a_7, a_4, a_5, a_6, a_3) \mapsto$

$(a_0, a_7, a_2, a_1, a_4, a_3, a_6, a_5)$.

NOT for $q_0 \leftarrow q_0 \oplus 1$:

$(a_0, a_7, a_2, a_1, a_4, a_3, a_6, a_5) \mapsto$

$(a_7, a_0, a_1, a_2, a_3, a_4, a_5, a_6)$.

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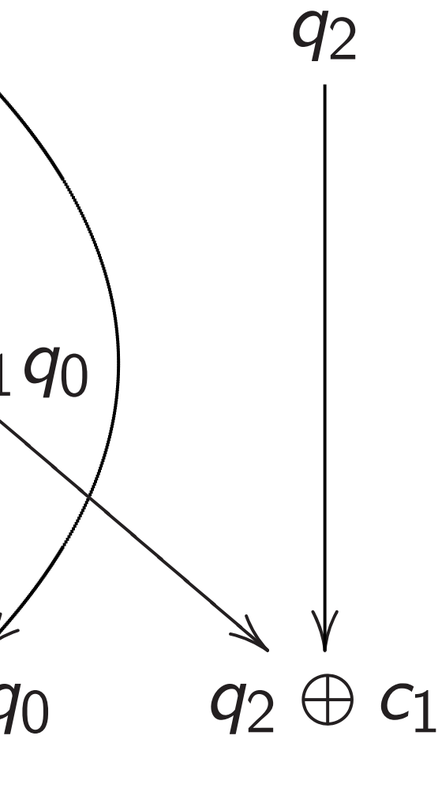

2. Convert into reversible gates.

Toffoli for $q_2 \leftarrow q_2 \oplus q_1 q_0$:

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Same outputs if all of
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Given fast circuit f
and fast circuit for
build fast reversibl
$(x, \text{zeros}) \mapsto (p(x)$

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Given fast circuit for p
and fast circuit for p^{-1},
build fast reversible circuit f
$(x, \text{zeros}) \mapsto (p(x), \text{zeros})$.

$g(i+1)$;
$g(i+2)$;

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Given fast circuit for p
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$(x, \text{zeros}) \mapsto (p(x), \text{zeros})$.

Replace reversible bit operations
with Toffoli gates etc.
permuting $\mathbf{C}^{2^{n+z}} \to \mathbf{C}^{2^{n+z}}$.

Permutation on first 2^n entries is
$(a_0, a_1, \ldots, a_{2^n-1}) \mapsto$
$(a_{p^{-1}(0)}, a_{p^{-1}(1)}, \ldots, a_{p^{-1}(2^n-1)})$.

Typically prepare vectors
supported on first 2^n entries
so don't care how permutation
acts on last $2^{n+z} - 2^n$ entries.

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Warning: Number of **qubits**
$\approx$ number of **bit operations**
in original p, p^{-1} circuits.

This can be much larger
than number of **bits stored**
in the original circuits.

Given fast circuit for p
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$(x, \text{zeros}) \mapsto (p(x), \text{zeros})$.

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Many useful techniques
to compress into fewer qubits,
but often these lose time.
Many subtle tradeoffs.

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Crude "poly-time" analyses
don't care about this,
but serious cryptanalysis
is much more precise.

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This can be much larger
than number of **bits stored**
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Many useful techniques
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but often these lose time.
Many subtle tradeoffs.

Crude "poly-time" analyses
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Aside from "normalization"
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have Hadamard $=$ Hadamard^{-1},
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from "uniform superposition"
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Simon's

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Simon's algorithm

Assume: nonzero $s \in \{0, 1\}^n$
satisfies $f(x) = f(x \oplus s)$
for every $x \in \{0, 1\}^n$.
Can we find this period s,
given a fast circuit for f?

Repeat n times: e.g.,
$(1, 0, 0, \ldots, 0) \mapsto (1, 1, 1, \ldots, 1)$.

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Traditional solution:
Compute f for many inputs,
sort, analyze collisions.
Success probability is very low
until #inputs approaches $2^{n/2}$.

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Say f maps n bits to m bits using
z "ancilla" bits for reversibility.

Prepare $n + m + z$ qubits
in pure zero state:
vector $(1, 0, 0, \ldots)$.

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Use n-fold Hadamard
to move first n qubits
into uniform superposition:
$(1, 1, 1, \ldots, 1, 0, 0, \ldots)$
with 2^n entries 1, others 0.

algorithm

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$f(x) = f(x \oplus s)$
$x \in \{0,1\}^n$.
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Apply fast vector
for reversible f co
1 in position $(q, 0,$
moves to position

Note symmetry be
1 at $(q, f(q), 0)$ a
1 at $(q \oplus s, f(q),$

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Simon's algorithm uses
far fewer qubit operations
if n is large and
reversibility overhead is low.

Say f maps n bits to m bits using
z "ancilla" bits for reversibility.

Prepare $n + m + z$ qubits
in pure zero state:
vector $(1, 0, 0, \ldots)$.

Use n-fold Hadamard
to move first n qubits
into uniform superposition:
$(1, 1, 1, \ldots, 1, 0, 0, \ldots)$
with 2^n entries 1, others 0.

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Apply fast vector permutatio
for reversible f computation
1 in position $(q, 0, 0)$
moves to position $(q, f(q), 0$

Note symmetry between
1 at $(q, f(q), 0)$ and
1 at $(q \oplus s, f(q), 0)$.

Simon's algorithm uses
far fewer qubit operations
if n is large and
reversibility overhead is low.

Say f maps n bits to m bits using
z "ancilla" bits for reversibility.

Prepare $n + m + z$ qubits
in pure zero state:
vector $(1, 0, 0, \ldots)$.

Use n-fold Hadamard
to move first n qubits
into uniform superposition:
$(1, 1, 1, \ldots, 1, 0, 0, \ldots)$
with 2^n entries 1, others 0.

Apply fast vector permutation
for reversible f computation:
1 in position $(q, 0, 0)$
moves to position $(q, f(q), 0)$.

Note symmetry between
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output is orthogonal to s.

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Use Gaussian elimination
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Example

$f(0) = 4$
$f(1) = 7$
$f(2) = 2$
$f(3) = 3$
$f(4) = 7$
$f(5) = 4$
$f(6) = 3$
$f(7) = 2$

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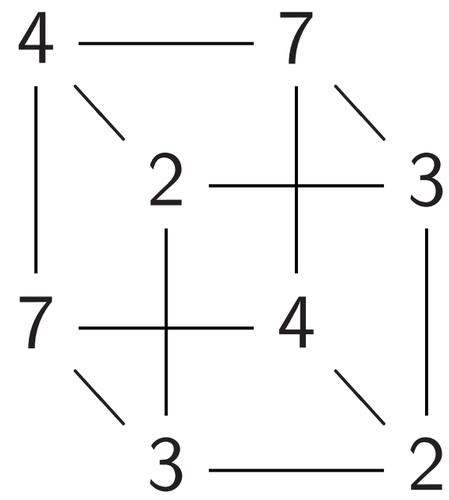

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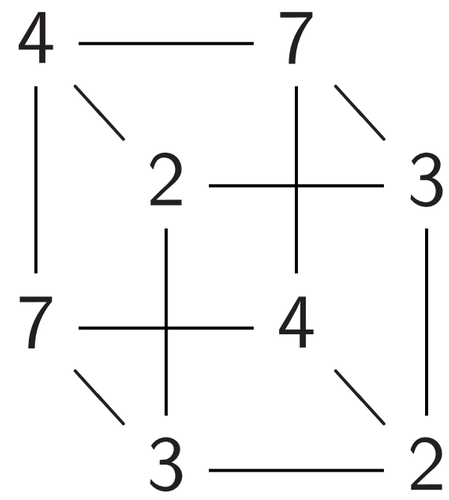

Complete table shows that
$f(x) = f(x \oplus 5)$ for all x.

Let's watch Simon's algorithm
for f, using 6 qubits.

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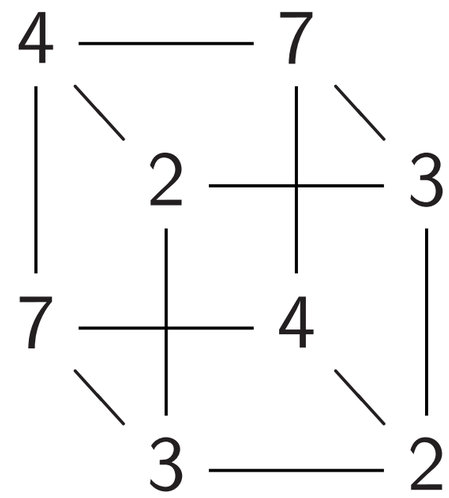

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Step 1.

1, 0, 0, 0

0, 0, 0, 0

0, 0, 0, 0

0, 0, 0, 0

0, 0, 0, 0

0, 0, 0, 0

0, 0, 0, 0

0, 0, 0, 0

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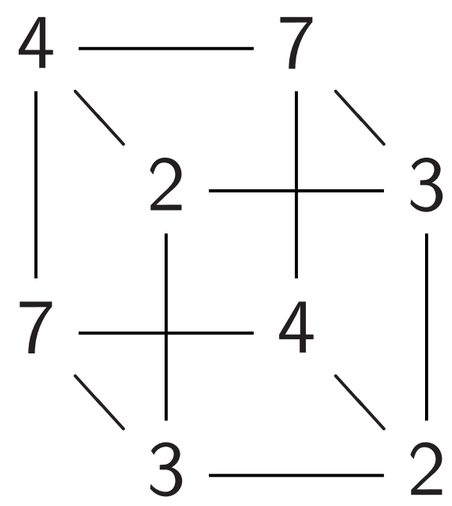

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Step 1. Set up pu

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

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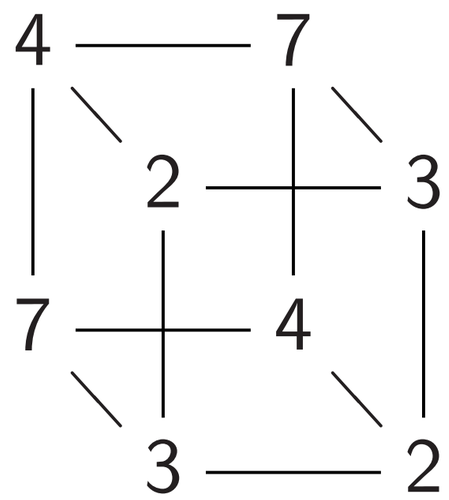

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Let's watch Simon's algorithm for f, using 6 qubits.

Step 1. Set up pure zero sta

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Example, 3 bits to 3 bits:

$f(0) = 4.$
$f(1) = 7.$
$f(2) = 2.$
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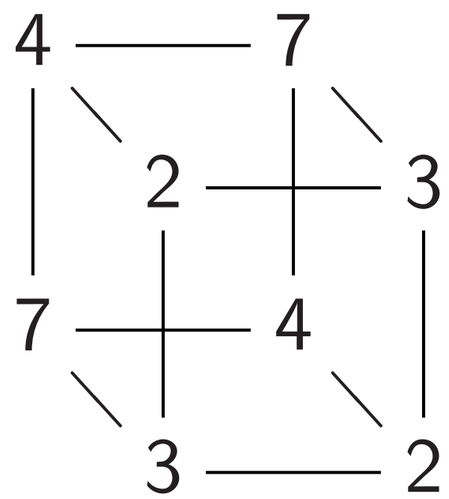

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Let's watch Simon's algorithm
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Step 1. Set up pure zero state:

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

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$f(0) = 4.$
$f(1) = 7.$
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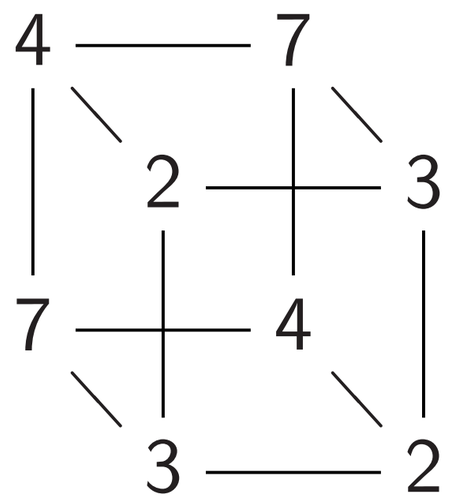

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Step 2. Hadamard on qubit 0:

1, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

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$f(0) = 4.$
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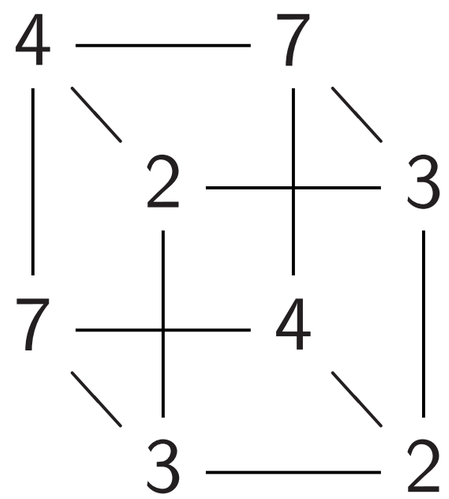

Complete table shows that
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Let's watch Simon's algorithm
for f, using 6 qubits.

Step 3. Hadamard on qubit 1:

1, 1, 1, 1, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Example, 3 bits to 3 bits:

$f(0) = 4.$
$f(1) = 7.$
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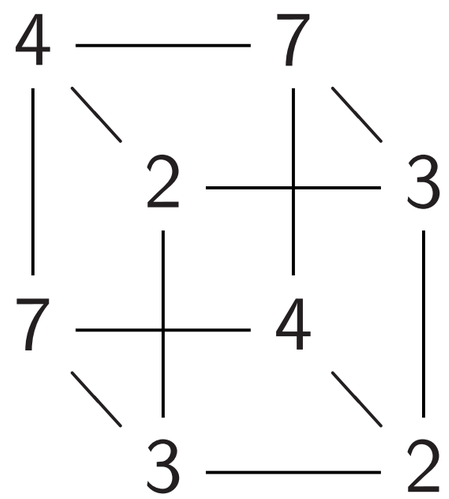

Complete table shows that
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Let's watch Simon's algorithm
for f, using 6 qubits.

Step 4. Hadamard on qubit 2:

1, 1, 1, 1, 1, 1, 1, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Example, 3 bits to 3 bits:

$f(0) = 4.$
$f(1) = 7.$
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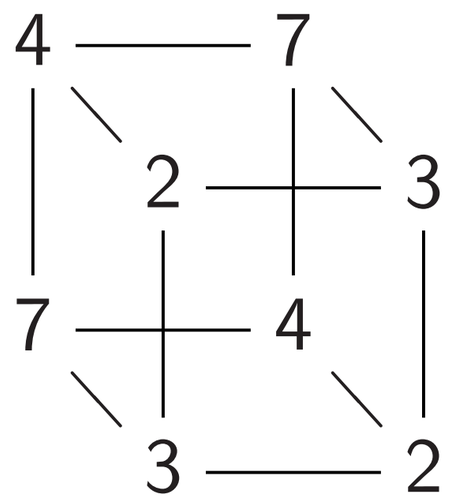

Complete table shows that
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Let's watch Simon's algorithm
for f, using 6 qubits.

Step 5. $(q, 0) \mapsto (q, f(q))$:

0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 0, 0, 1, 0, 0, 1, 0,
1, 0, 0, 0, 0, 1, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 1, 0, 0, 1, 0, 0, 0.

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$f(0) = 4.$
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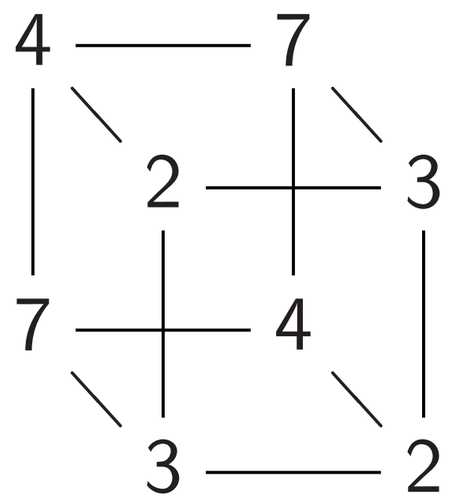

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Let's watch Simon's algorithm
for f, using 6 qubits.

Step 6. Hadamard on qubit 0:

$0, 0, 0, 0, 0, 0, 0, 0,$
$0, 0, 0, 0, 0, 0, 0, 0,$
$0, 0, 1, 1, 0, 0, 1, \overline{1},$
$0, 0, 1, \overline{1}, 0, 0, 1, 1,$
$1, 1, 0, 0, 1, \overline{1}, 0, 0,$
$0, 0, 0, 0, 0, 0, 0, 0,$
$0, 0, 0, 0, 0, 0, 0, 0,$
$1, \overline{1}, 0, 0, 1, 1, 0, 0.$

Notation: $\overline{1} = -1.$

Example, 3 bits to 3 bits:

$f(0) = 4.$
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$f(4) = 7.$
$f(5) = 4.$
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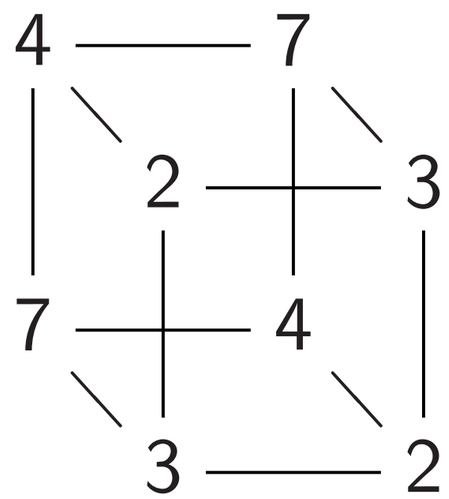

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Let's watch Simon's algorithm
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Step 7. Hadamard on qubit 1:

$0, 0, 0, 0, 0, 0, 0, 0,$
$0, 0, 0, 0, 0, 0, 0, 0,$
$1, 1, \overline{1}, \overline{1}, 1, \overline{1}, \overline{1}, 1,$
$1, \overline{1}, \overline{1}, 1, 1, 1, \overline{1}, \overline{1},$
$1, 1, 1, 1, 1, \overline{1}, 1, \overline{1},$
$0, 0, 0, 0, 0, 0, 0, 0,$
$0, 0, 0, 0, 0, 0, 0, 0,$
$1, \overline{1}, 1, \overline{1}, 1, 1, 1, 1.$

Example, 3 bits to 3 bits:

$f(0) = 4.$
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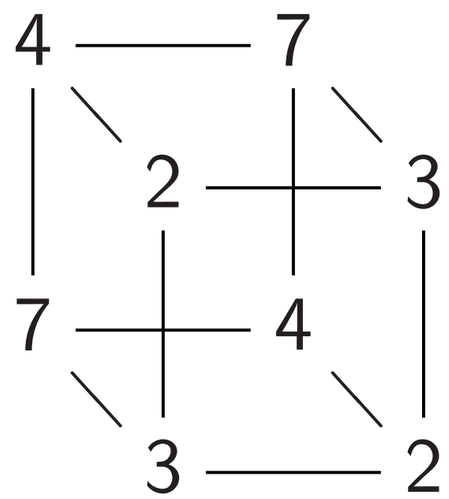

Complete table shows that
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Let's watch Simon's algorithm
for f, using 6 qubits.

Step 8. Hadamard on qubit 2:

$0, 0, 0, 0, 0, 0, 0, 0,$
$0, 0, 0, 0, 0, 0, 0, 0,$
$2, 0, \overline{2}, 0, 0, \overline{2}, 0, \overline{2},$
$2, 0, \overline{2}, 0, 0, \overline{2}, 0, 2,$
$2, 0, 2, 0, 0, 2, 0, 2,$
$0, 0, 0, 0, 0, 0, 0, 0,$
$0, 0, 0, 0, 0, 0, 0, 0,$
$2, 0, 2, 0, 0, \overline{2}, 0, \overline{2}.$

Example, 3 bits to 3 bits:

$f(0) = 4.$
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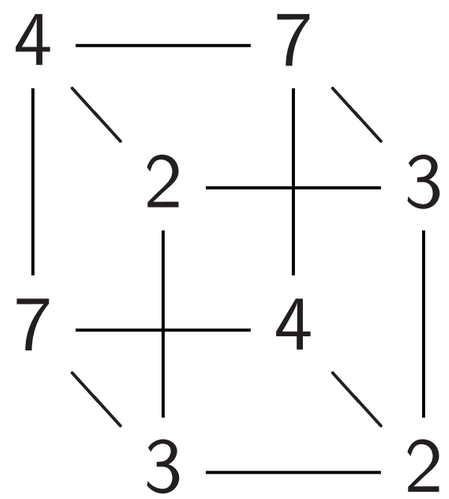

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$0, 0, 0, 0, 0, 0, 0, 0,$
$2, 0, \overline{2}, 0, 0, \overline{2}, 0, \overline{2},$
$2, 0, \overline{2}, 0, 0, \overline{2}, 0, 2,$
$2, 0, 2, 0, 0, 2, 0, 2,$
$0, 0, 0, 0, 0, 0, 0, 0,$
$0, 0, 0, 0, 0, 0, 0, 0,$
$2, 0, 2, 0, 0, \overline{2}, 0, \overline{2}.$

Step 9. Measure.
First 3 qubits are uniform random vector orthogonal to 101: i.e., 000, 010, 101, or 111.

, 3 bits to 3 bits:

4.

7.

2.

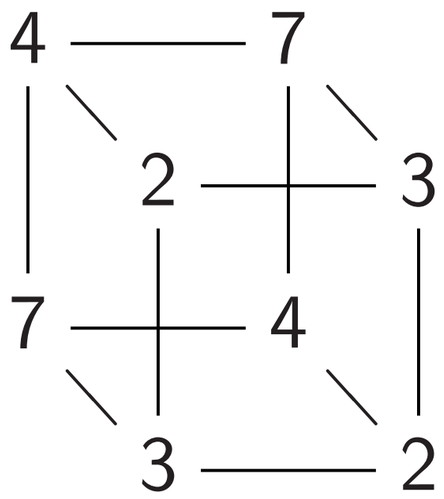
3.

7.

4.

3.

2.



e table shows that
$f(x \oplus 5)$ for all x.

tch Simon's algorithm
sing 6 qubits.

Step 8. Hadamard on qubit 2:

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$2, 0, \overline{2}, 0, 0, \overline{2}, 0, \overline{2},$

$2, 0, \overline{2}, 0, 0, \overline{2}, 0, 2,$

$2, 0, 2, 0, 0, 2, 0, 2,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$2, 0, 2, 0, 0, \overline{2}, 0, \overline{2}.$

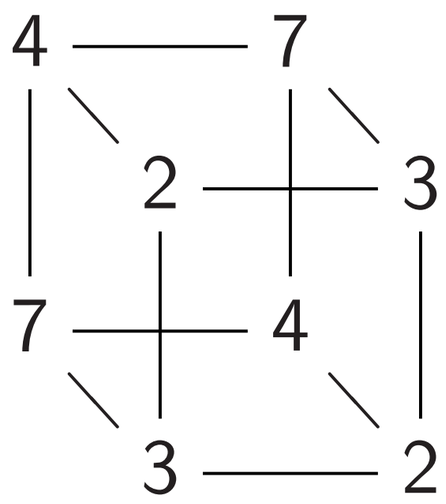
Step 9. Measure.
First 3 qubits are uniform random vector orthogonal to 101: i.e., 000, 010, 101, or 111.

Grover's

Assume:
has $f(s)$

Traditio
compute
hope to
Success
until #i

3 bits:



ows that
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n's algorithm
its.

Step 8. Hadamard on qubit 2:

$0, 0, 0, 0, 0, 0, 0, 0,$
$0, 0, 0, 0, 0, 0, 0, 0,$
$2, 0, \overline{2}, 0, 0, \overline{2}, 0, \overline{2},$
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$2, 0, 2, 0, 0, 2, 0, 2,$
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Step 9. Measure.
First 3 qubits are uniform random vector orthogonal to 101: i.e., 000, 010, 101, or 111.

Grover's algorithm

Assume: unique s
has $f(s) = 0$.

Traditional algorit
compute f for ma
hope to find outpu
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until #inputs appr

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Step 8. Hadamard on qubit 2:

$0, 0, 0, 0, 0, 0, 0, 0,$

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Step 9. Measure.

First 3 qubits are uniform random vector orthogonal to 101: i.e., 000, 010, 101, or 111.

Grover's algorithm

Assume: unique $s \in \{0,1\}^n$ has $f(s) = 0$.

Traditional algorithm to find
compute f for many inputs,
hope to find output 0.
Success probability is very lo
until #inputs approaches 2^n

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Step 9. Measure.

First 3 qubits are uniform random vector orthogonal to 101: i.e., 000, 010, 101, or 111.

Grover's algorithm

Assume: unique $s \in \{0,1\}^n$ has $f(s) = 0$.

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Grover's algorithm takes only $2^{n/2}$ reversible computations of f. Typically: reversibility overhead is small enough that this easily beats traditional algorithm.

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Measure.

qubits are uniform random

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Start fro
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Step 1:

$b_q = -a$

$b_q = a_q$

This is f

Step 2:

Negate

This is a

Repeat S
about 0.

Measure
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uniform random
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111.

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$b_q = -a_q$ if $f(q) =$
$b_q = a_q$ otherwise
This is fast.

Step 2: "Grover d
Negate a around i
This is also fast.

Repeat Step 1 + S
about $0.58 \cdot 2^{0.5n}$

Measure the n qub
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Step 2: "Grover diffusion".
Negate a around its average
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this fi

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Measure the n qubits.
With high probability this finds s.

algorithm

unique $s \in \{0,1\}^n$
$= 0$.

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probability is very low
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algorithm takes only $2^{n/2}$
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Normaliz
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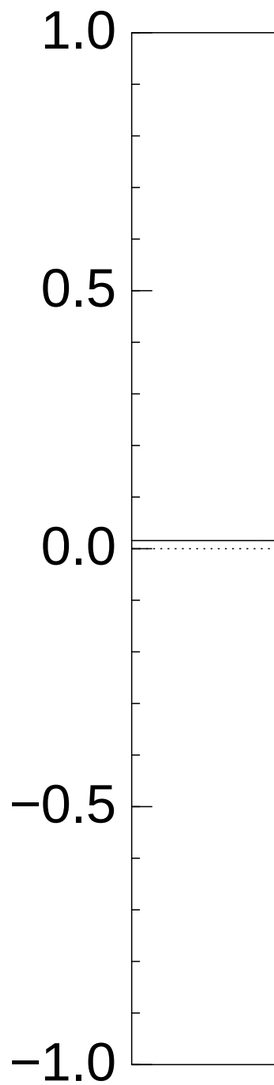

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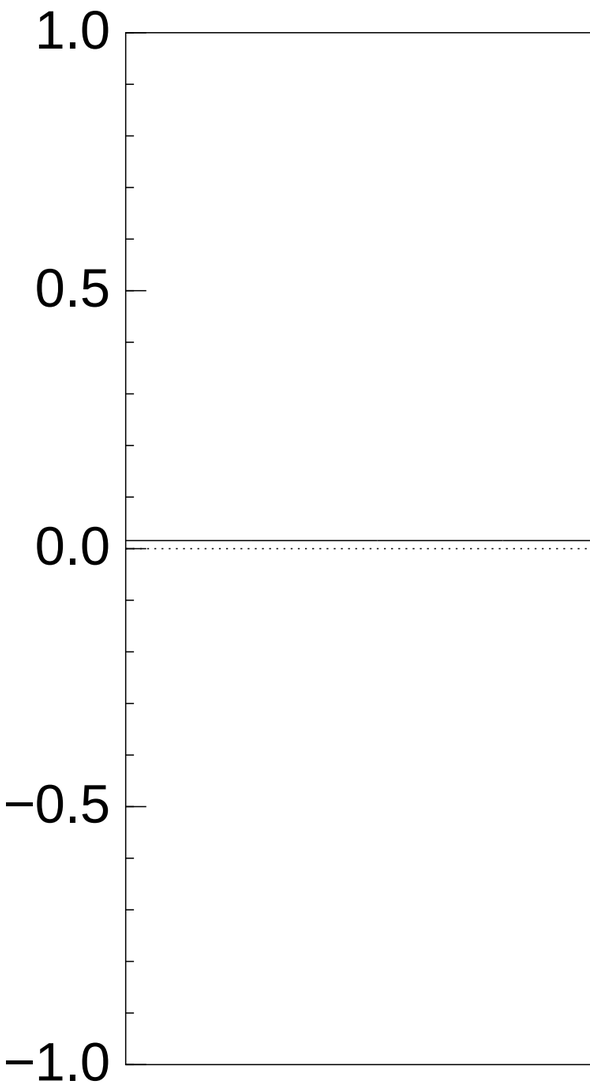
With high probability this finds s.

Normalized graph
for an example wit
after 0 steps:



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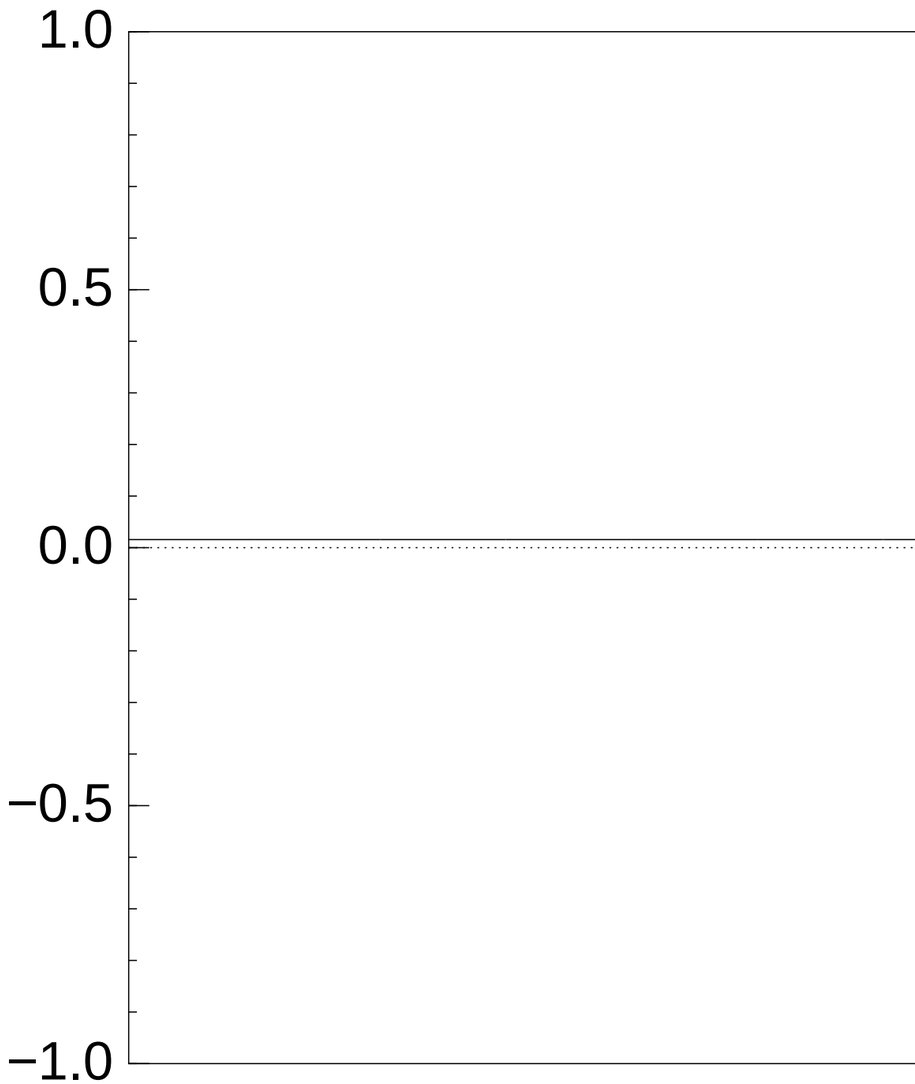
Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after 0 steps:



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Measure the n qubits.
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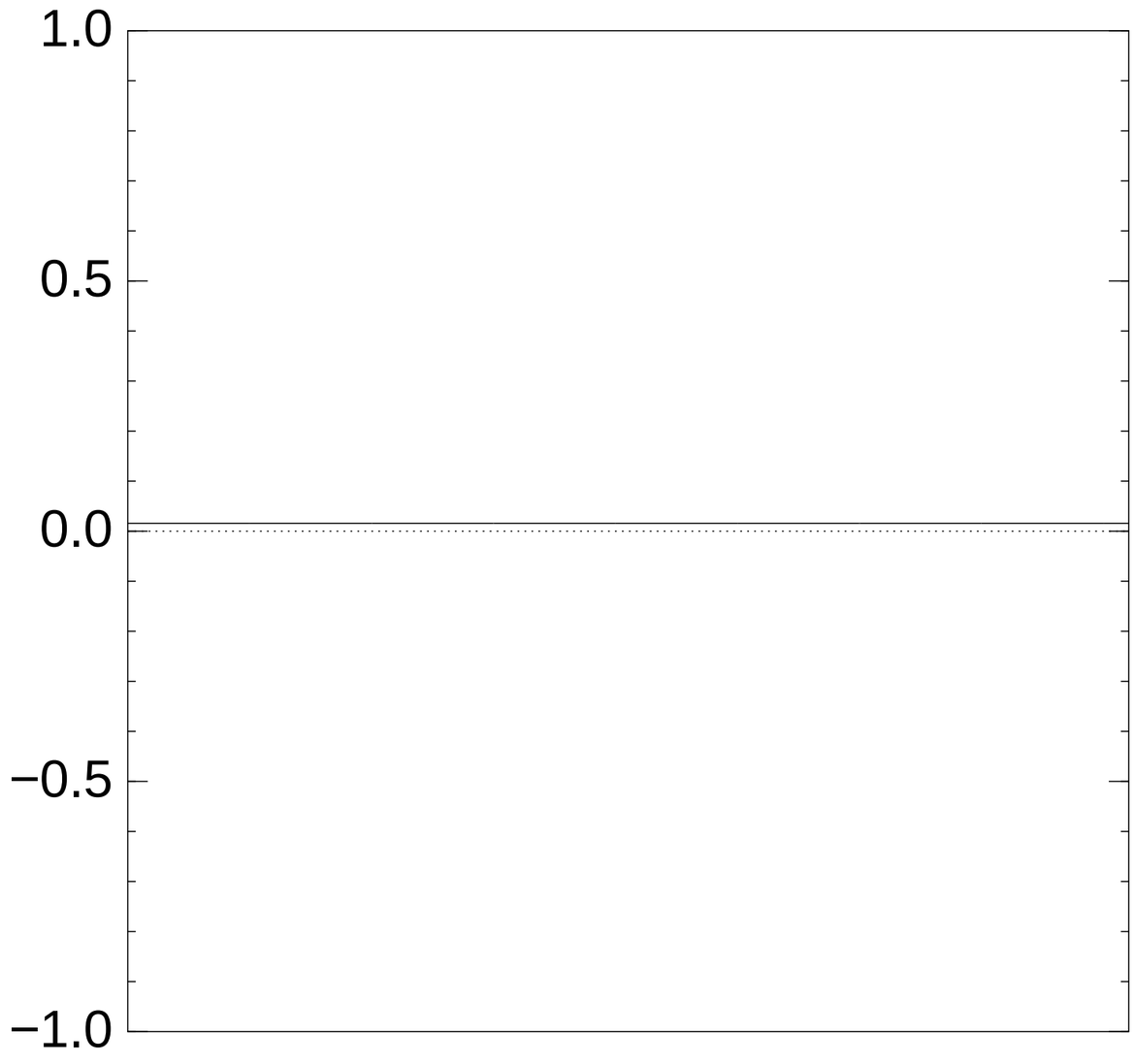
for an example with $n = 12$
after 0 steps:



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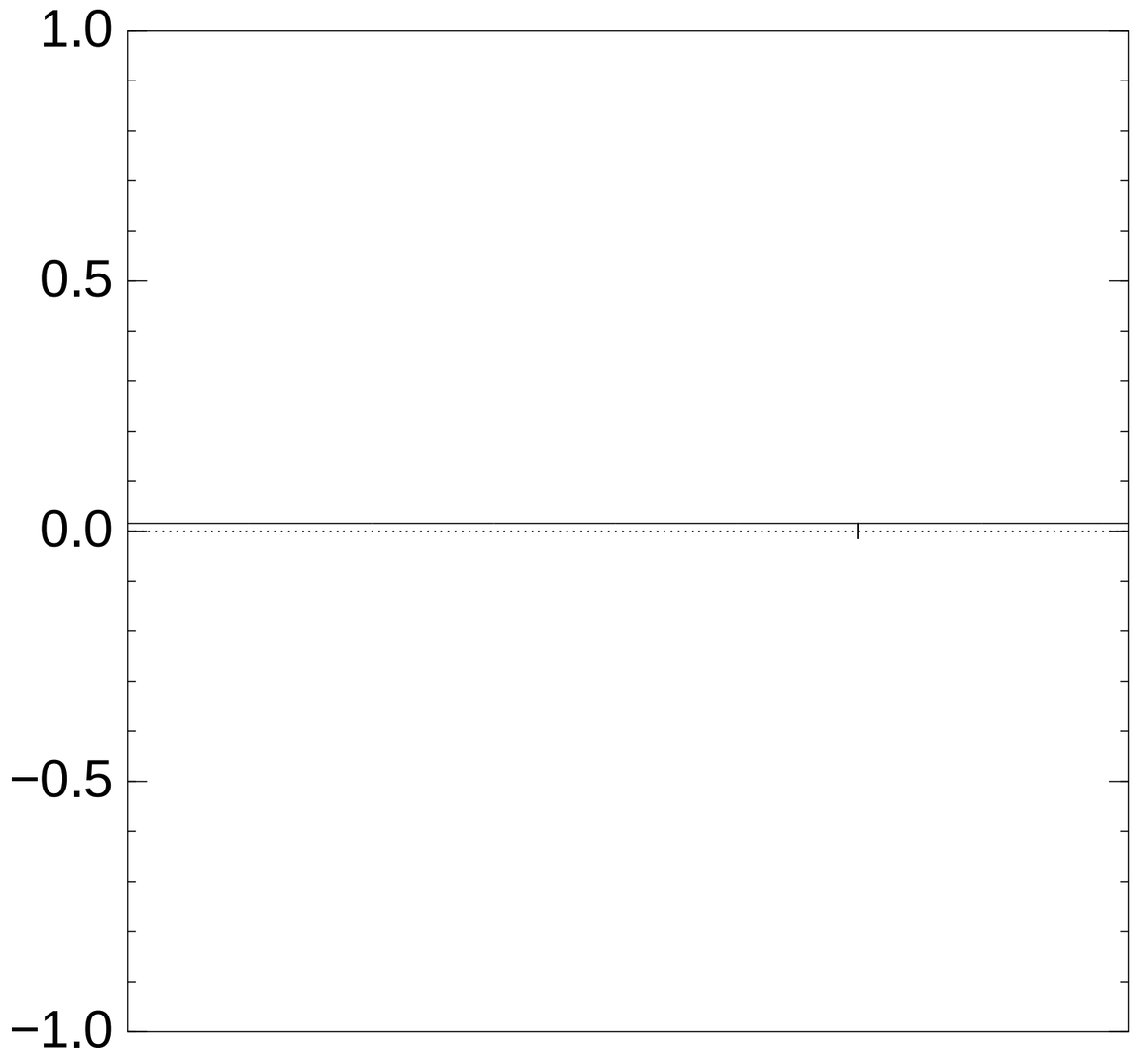
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after Step 1:



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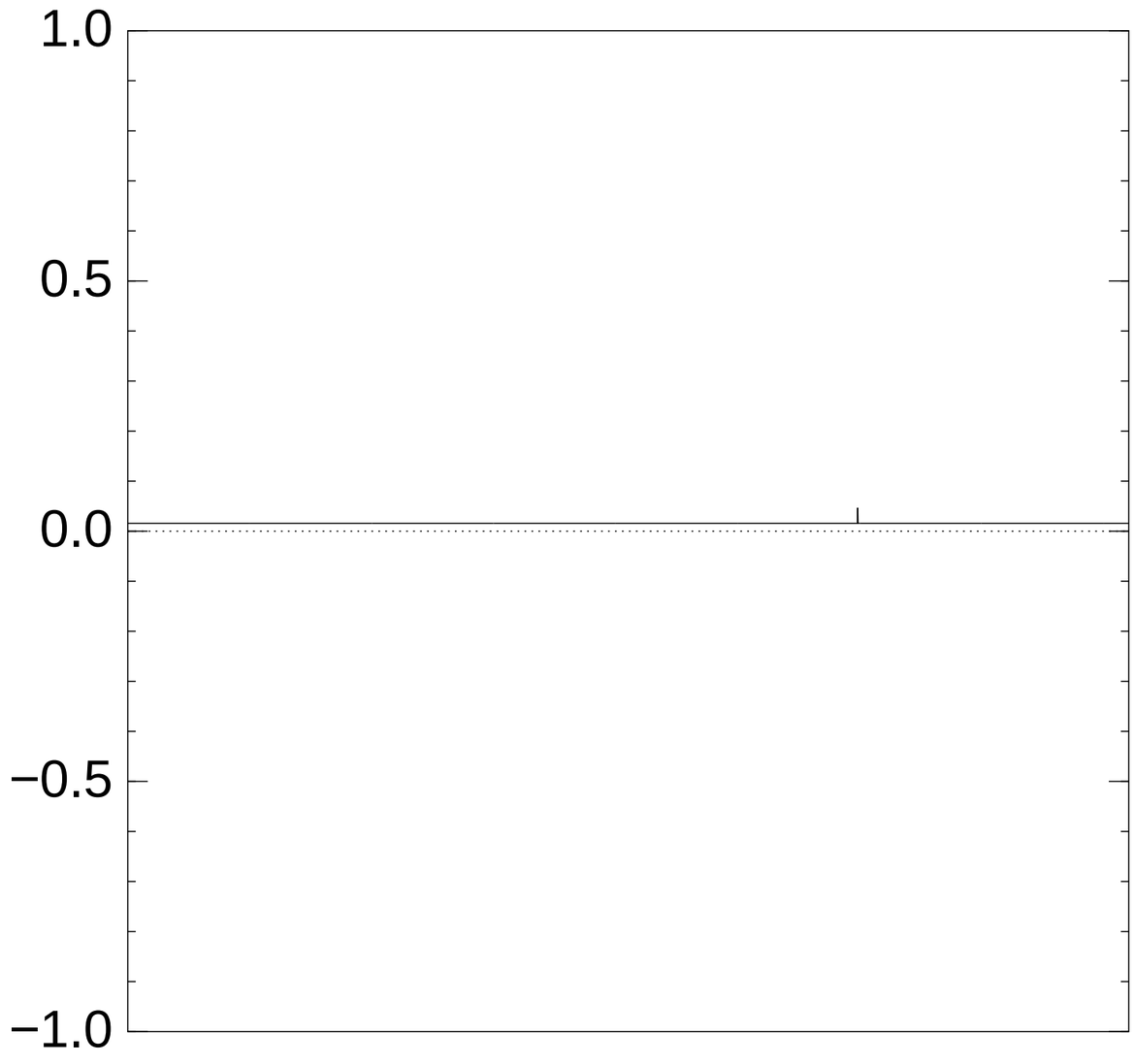
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after Step 1 + Step 2:



Start from uniform superposition
over all n-bit strings q.

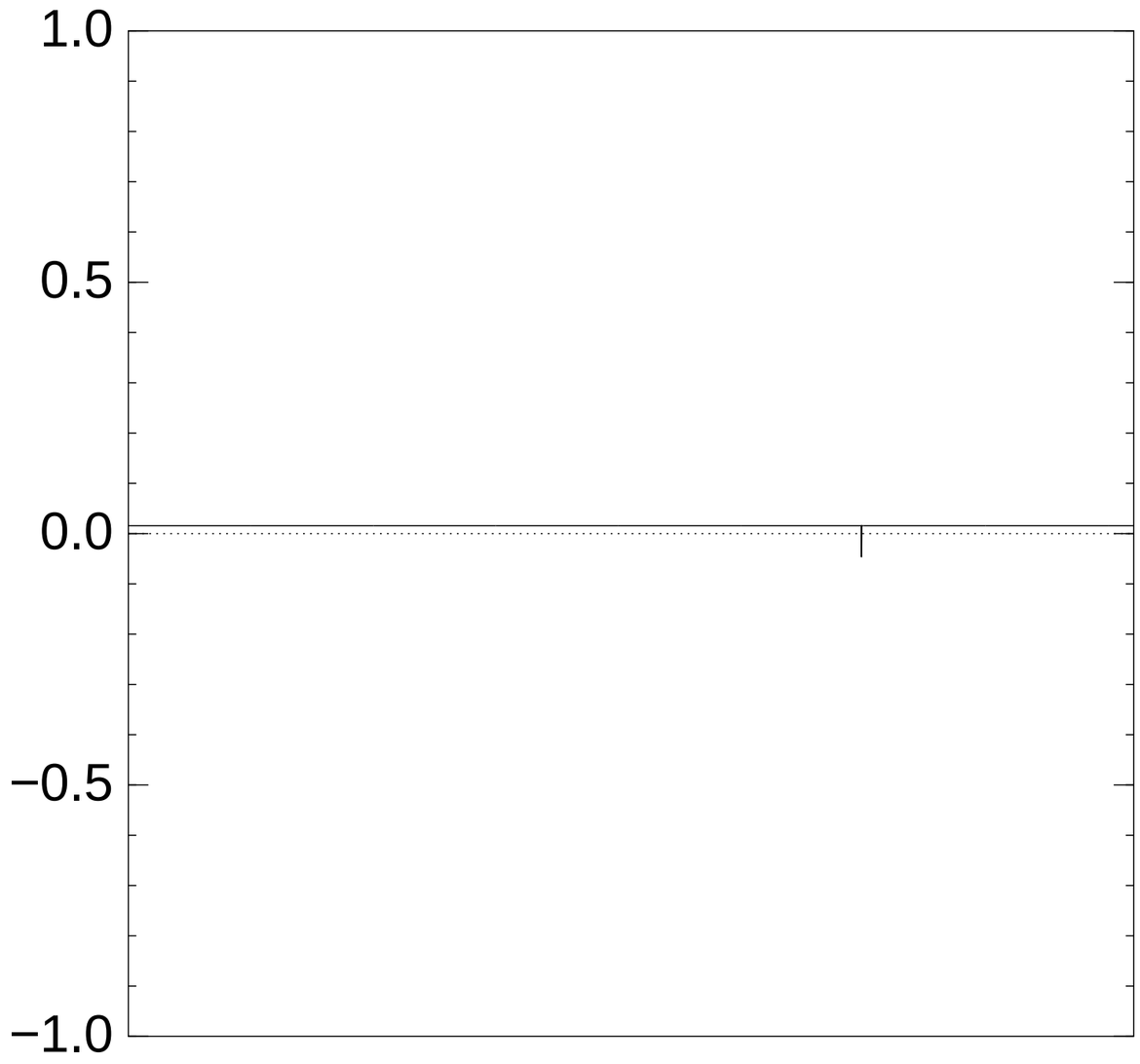
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after Step 1 + Step 2 + Step 1:



Start from uniform superposition over all n-bit strings q.

Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
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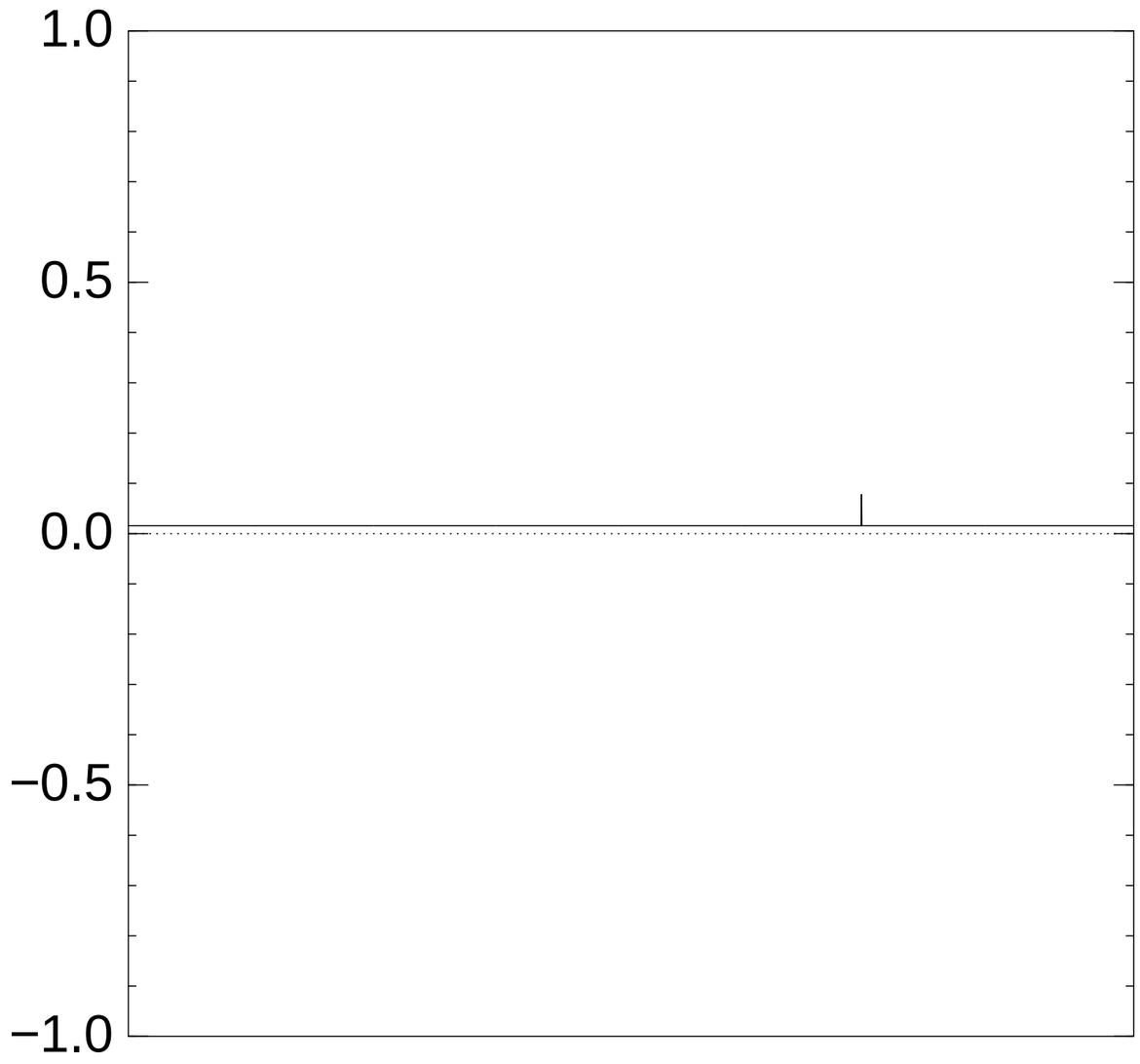
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $2 \times$ (Step 1 + Step 2):



Start from uniform superposition
over all n-bit strings q.

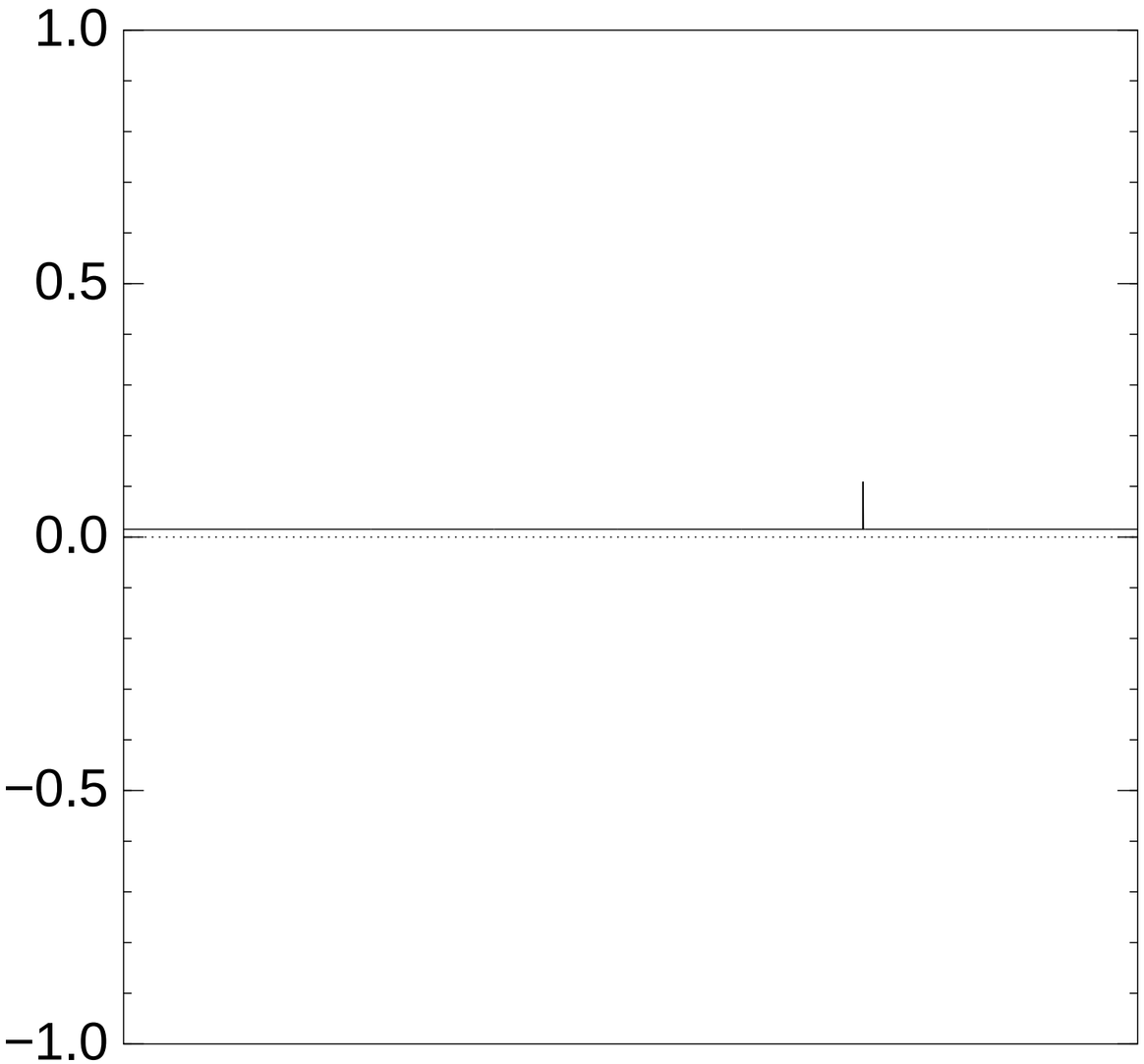
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $3 \times$ (Step 1 + Step 2):



Start from uniform superposition
over all n-bit strings q.

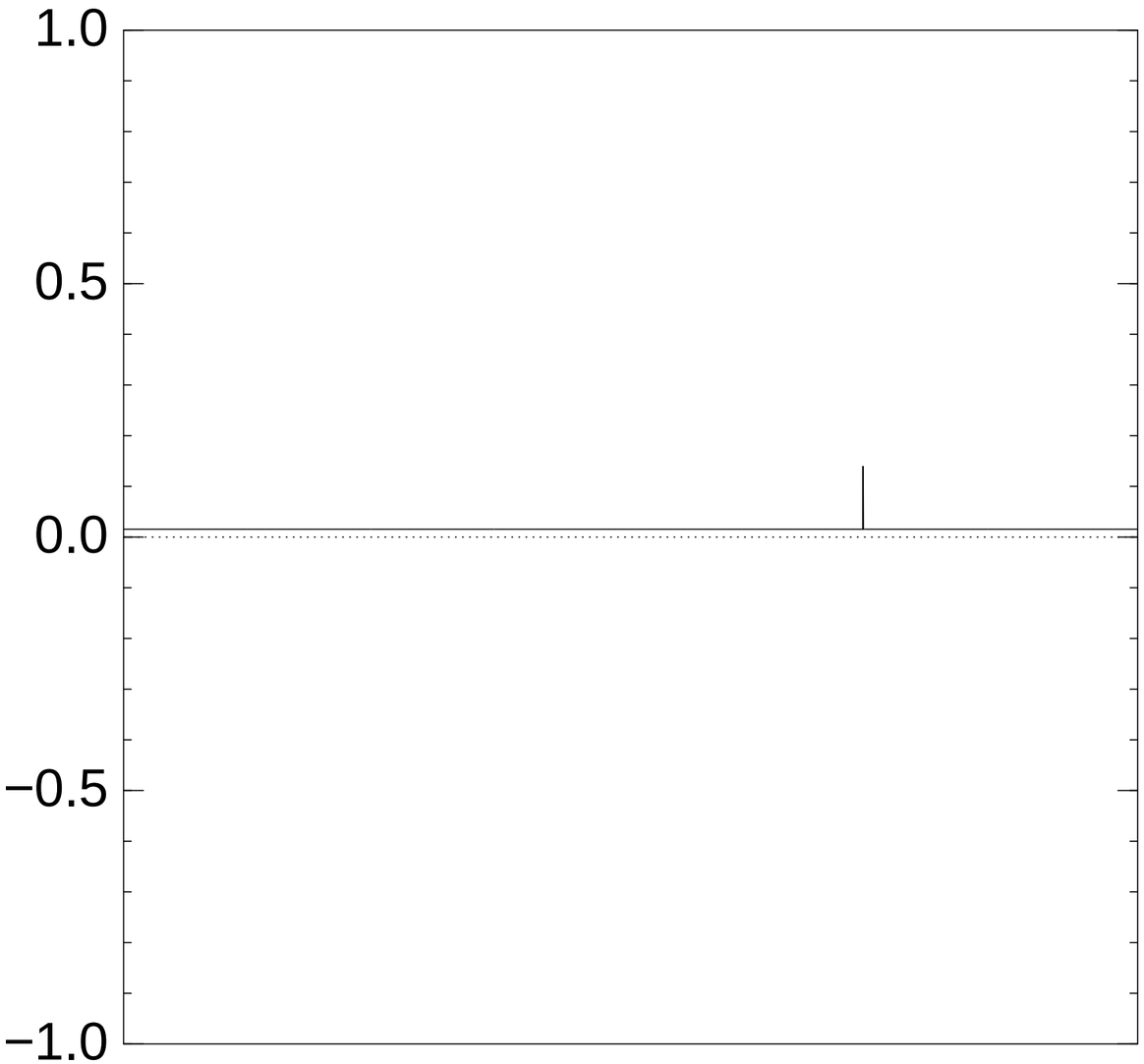
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $4 \times$ (Step 1 + Step 2):



Start from uniform superposition
over all n-bit strings q.

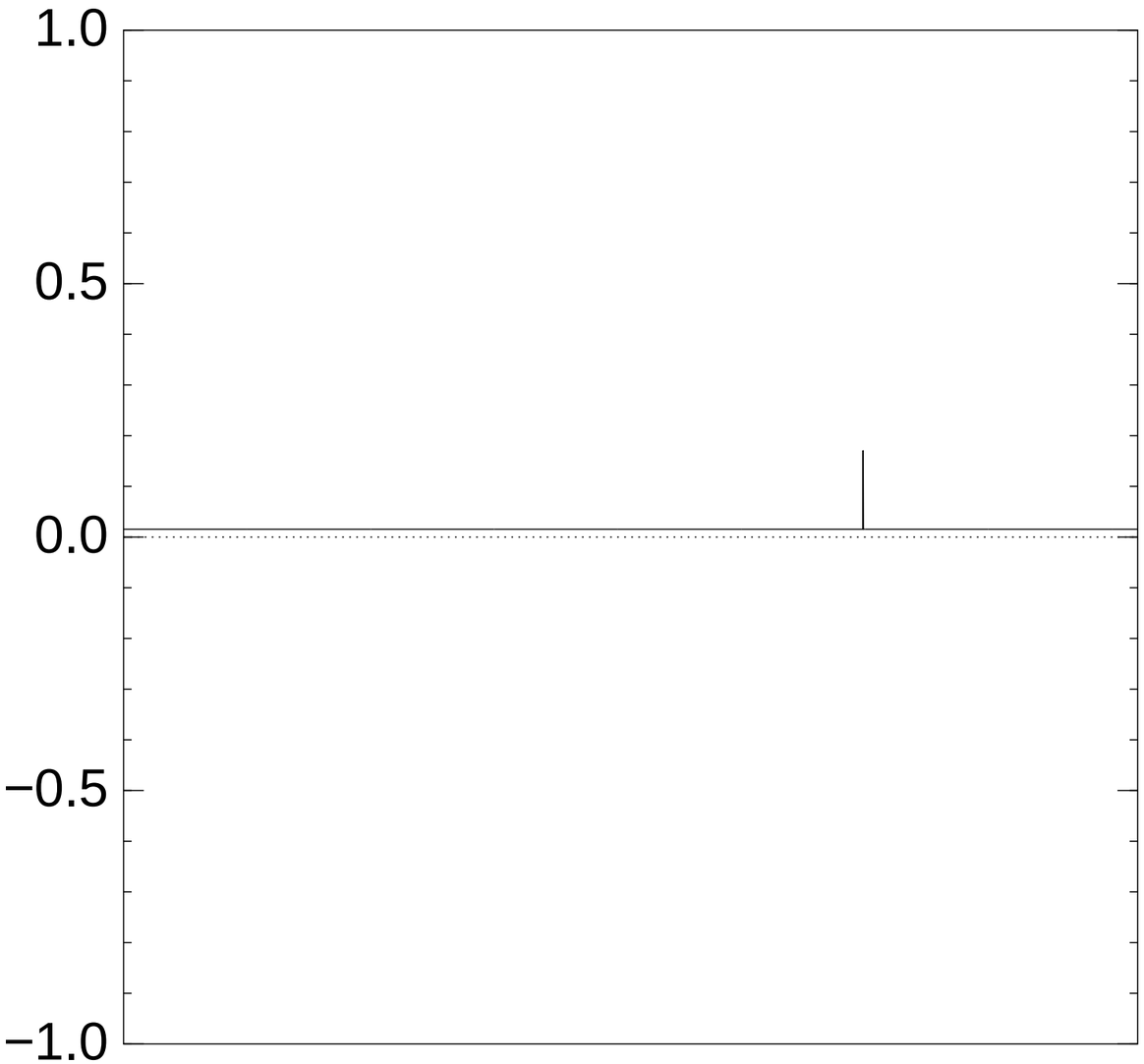
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $5 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

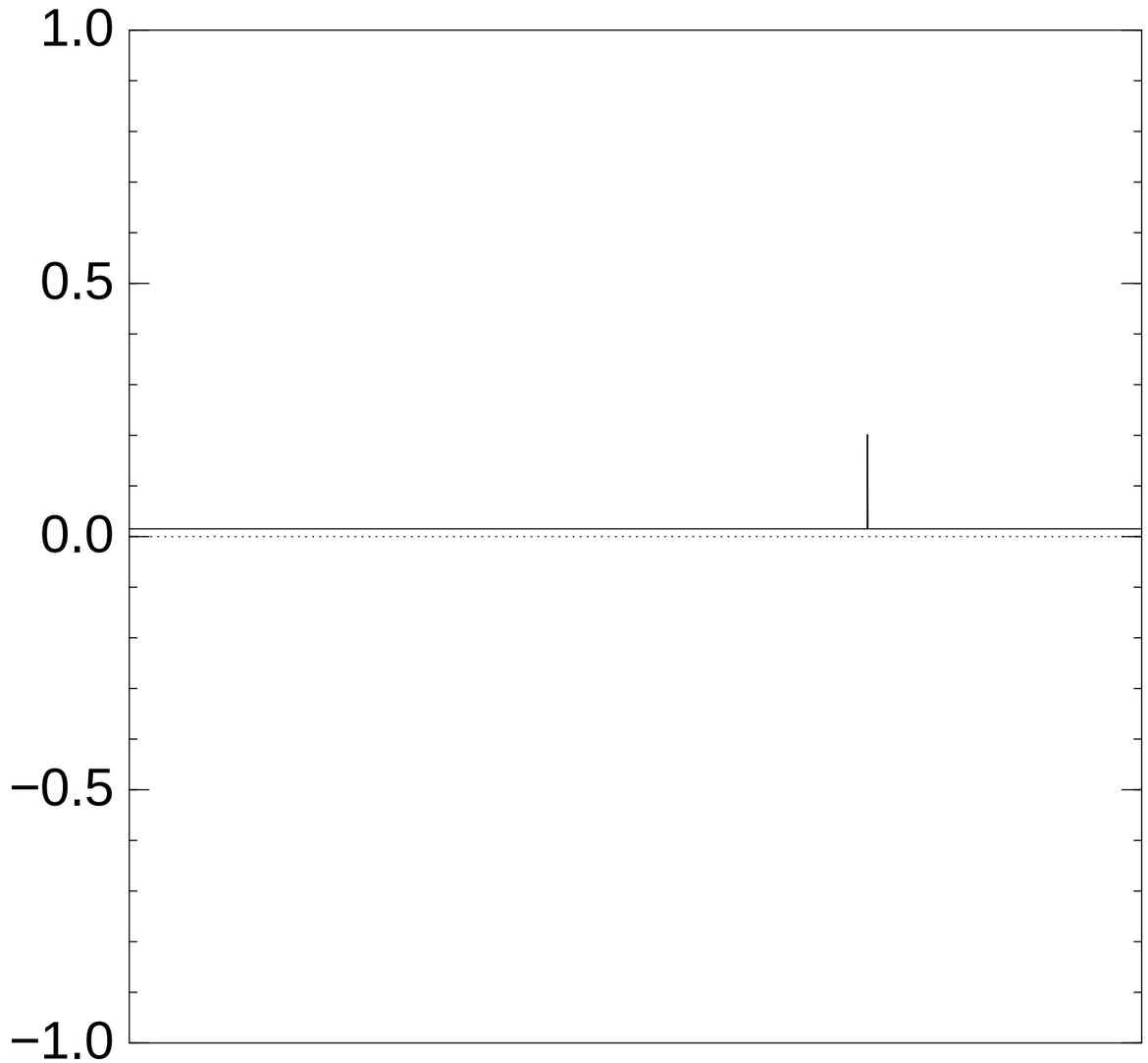
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $6 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
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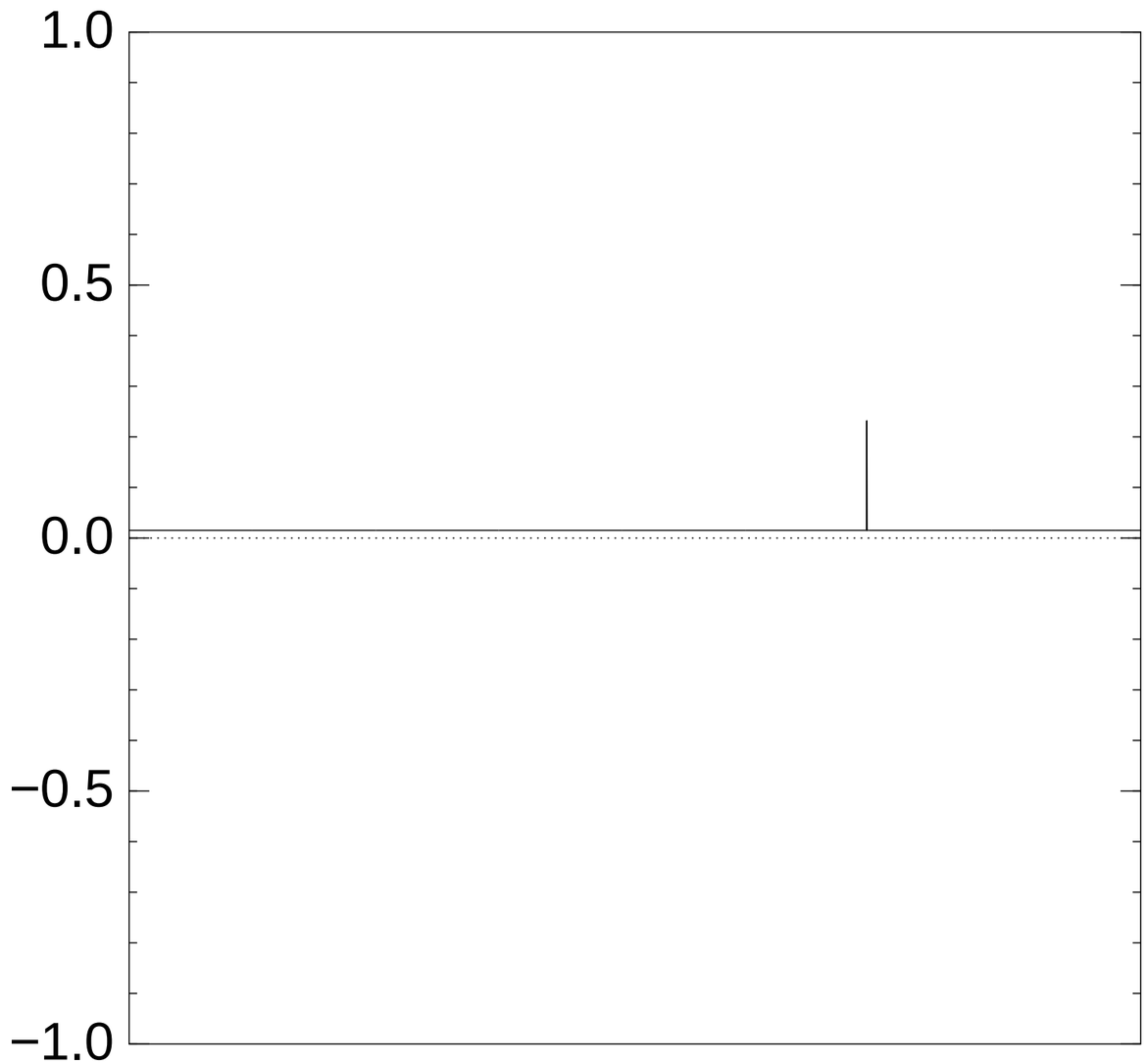
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $7 \times$ (Step 1 + Step 2):



Start from uniform superposition
over all n-bit strings q.

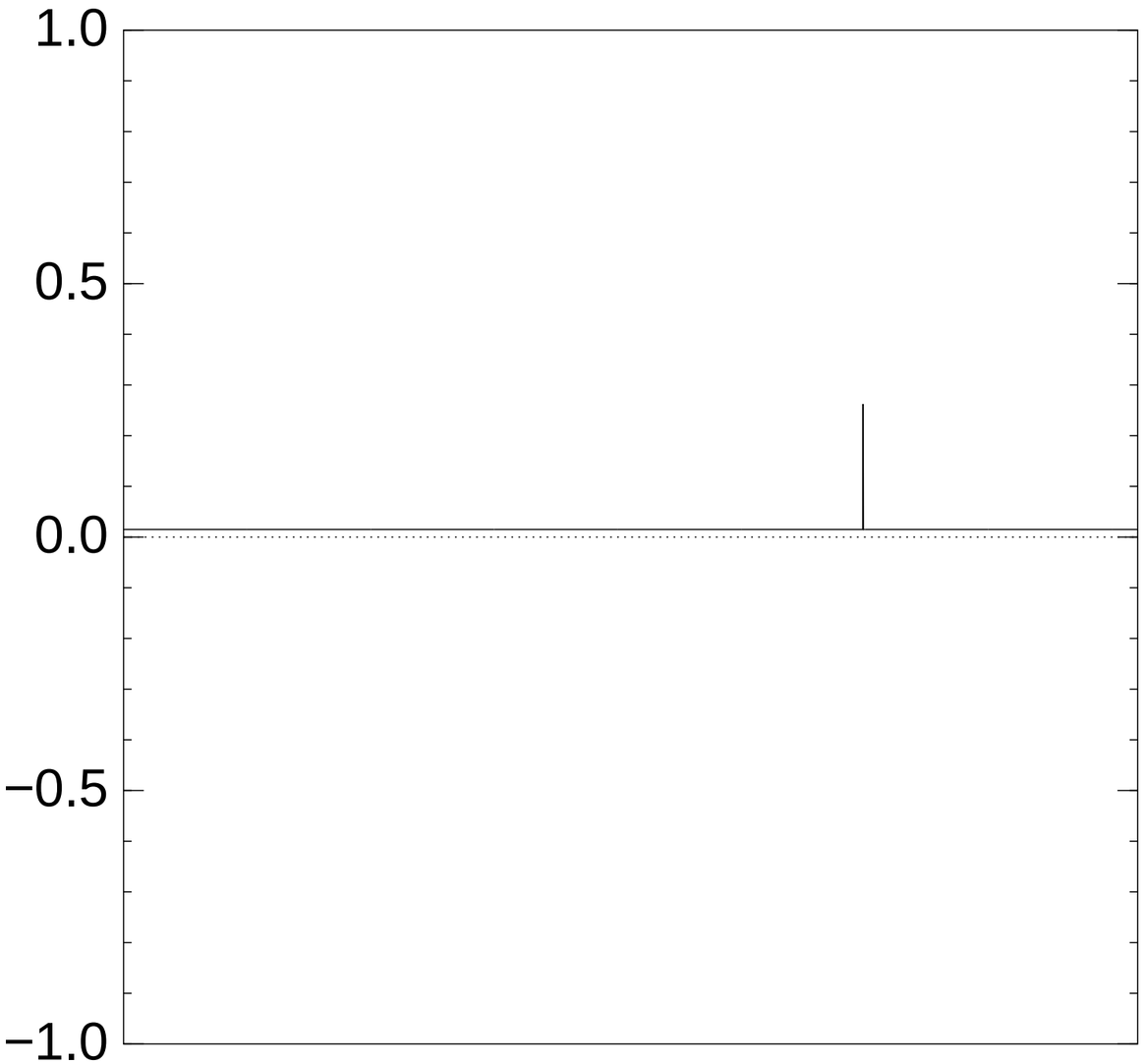
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $8 \times$ (Step 1 + Step 2):



Start from uniform superposition
over all n-bit strings q.

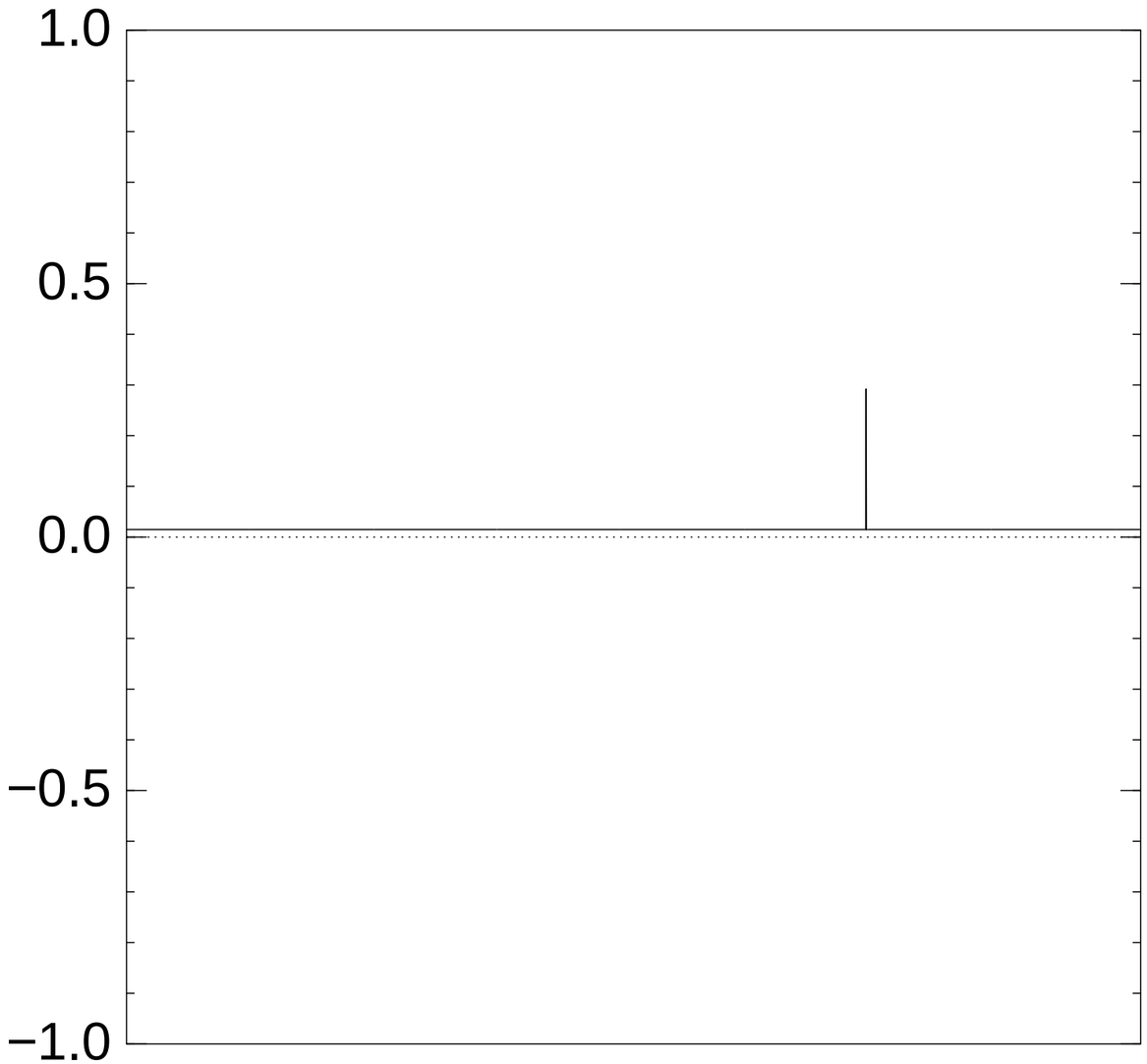
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $9 \times$ (Step 1 + Step 2):



Start from uniform superposition
over all n-bit strings q.

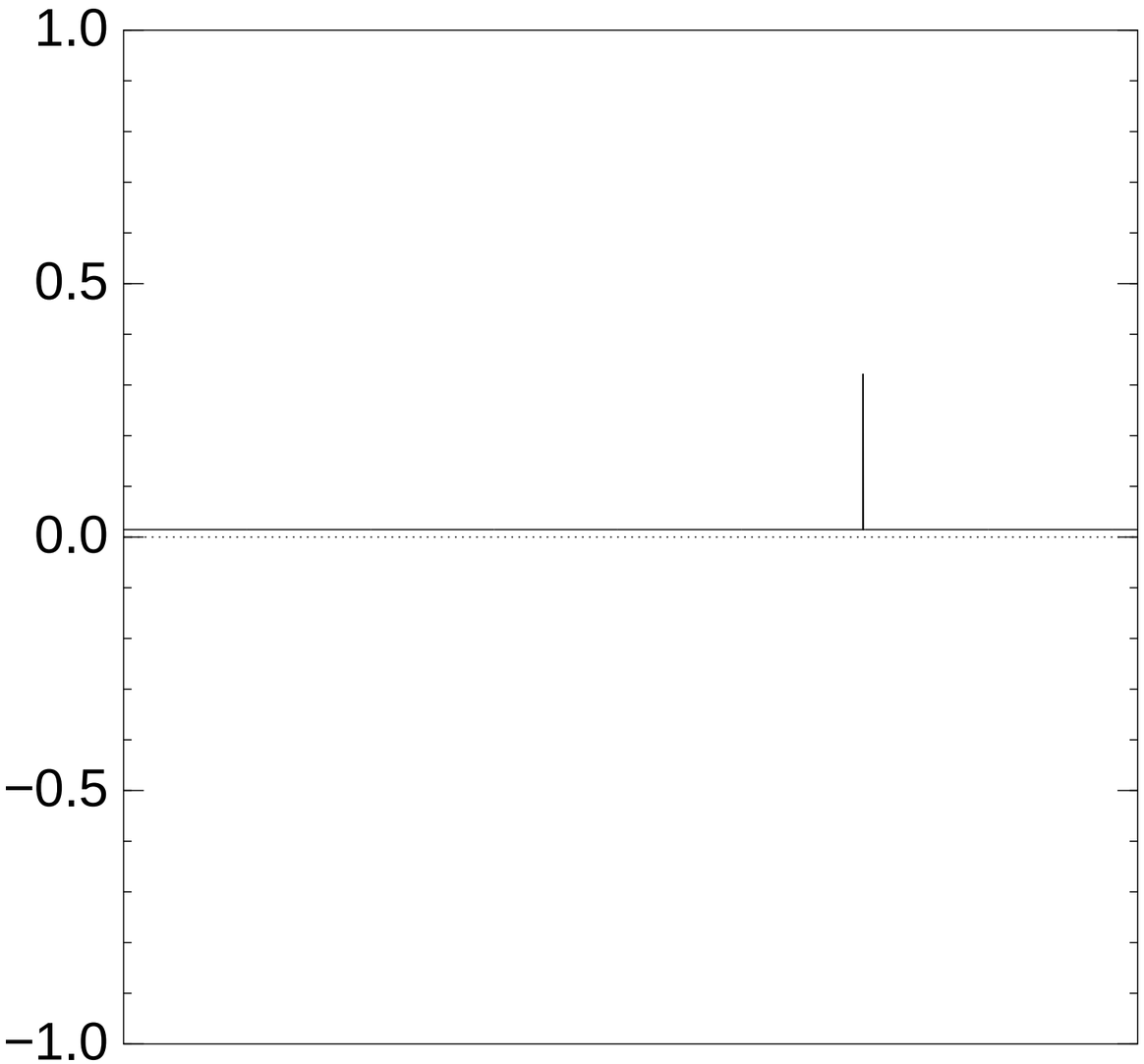
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $10 \times$ (Step 1 + Step 2):



Start from uniform superposition
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$b_q = -a_q$ if $f(q) = 0$,
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This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
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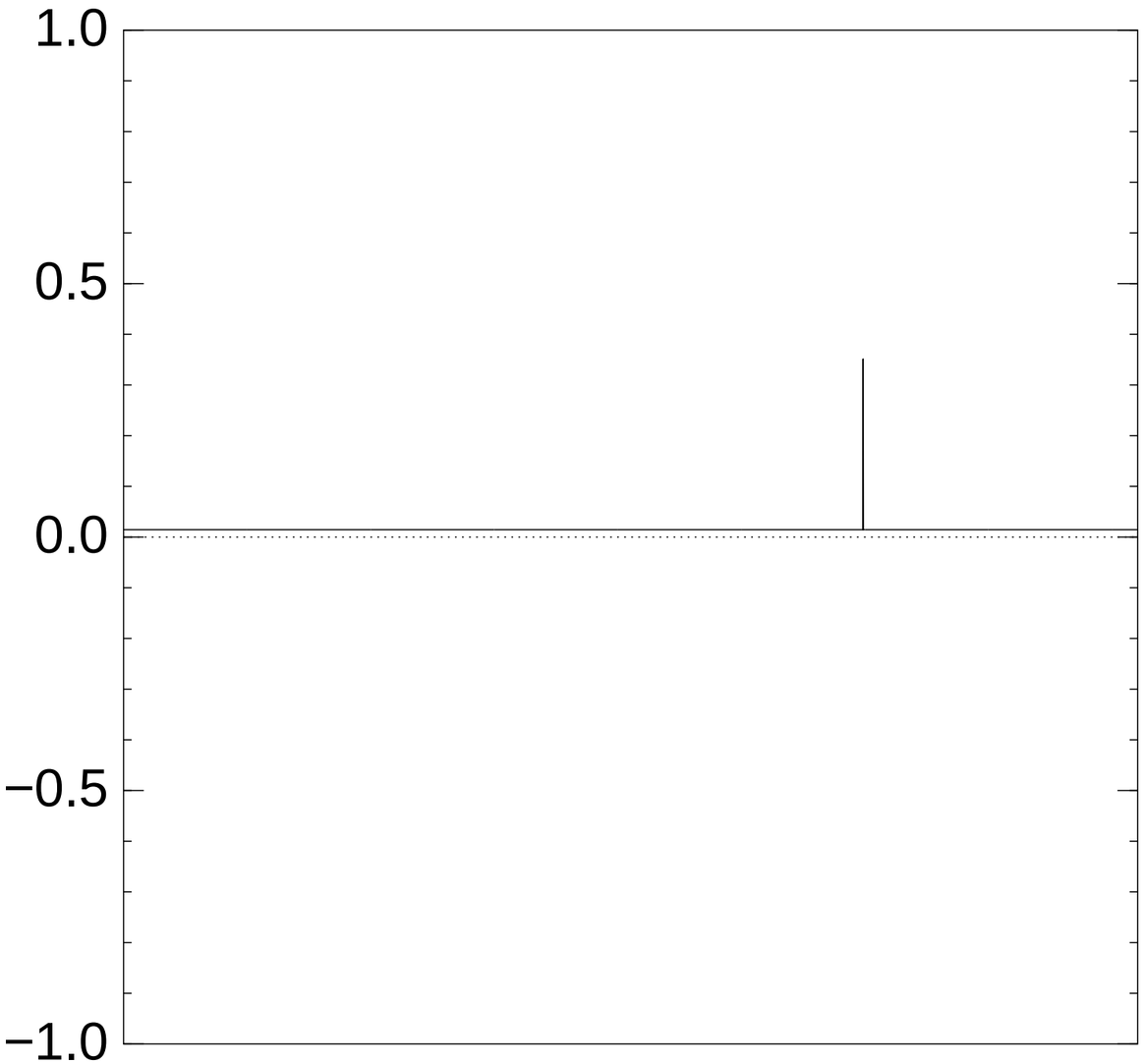
Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $11 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

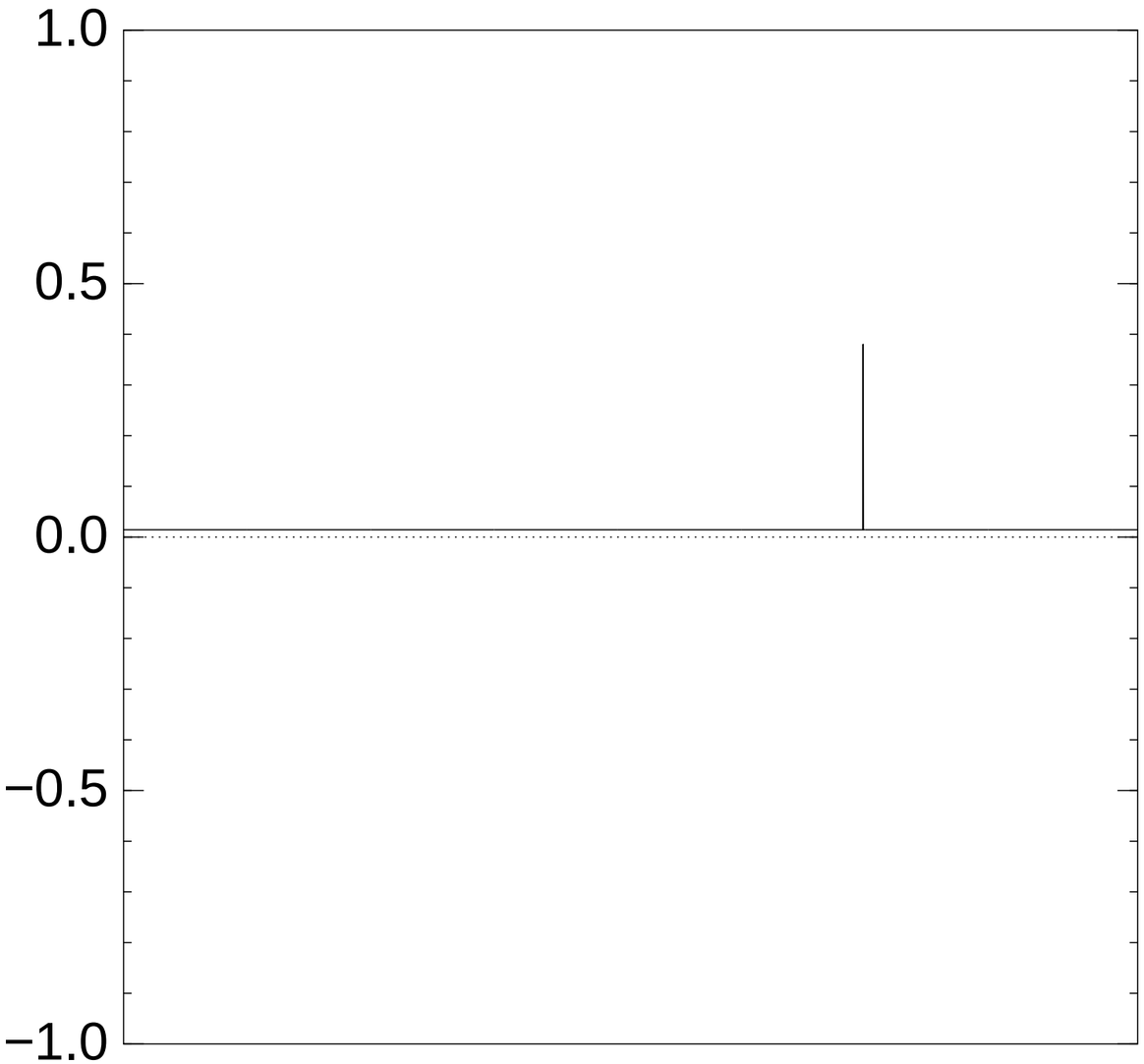
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $12 \times$ (Step 1 + Step 2):



Start from uniform superposition
over all n-bit strings q.

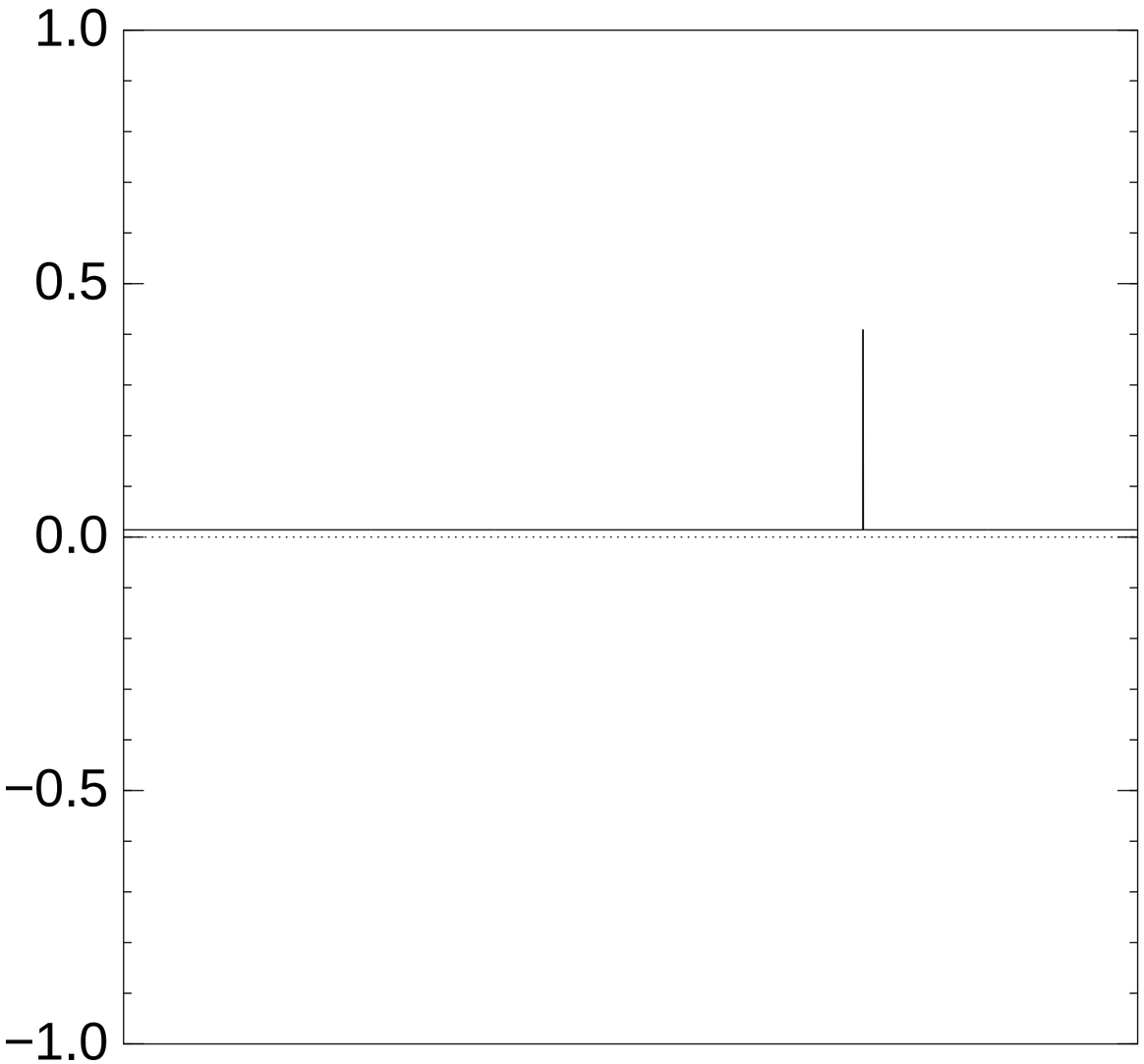
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $13 \times$ (Step 1 + Step 2):



Start from uniform superposition
over all n-bit strings q.

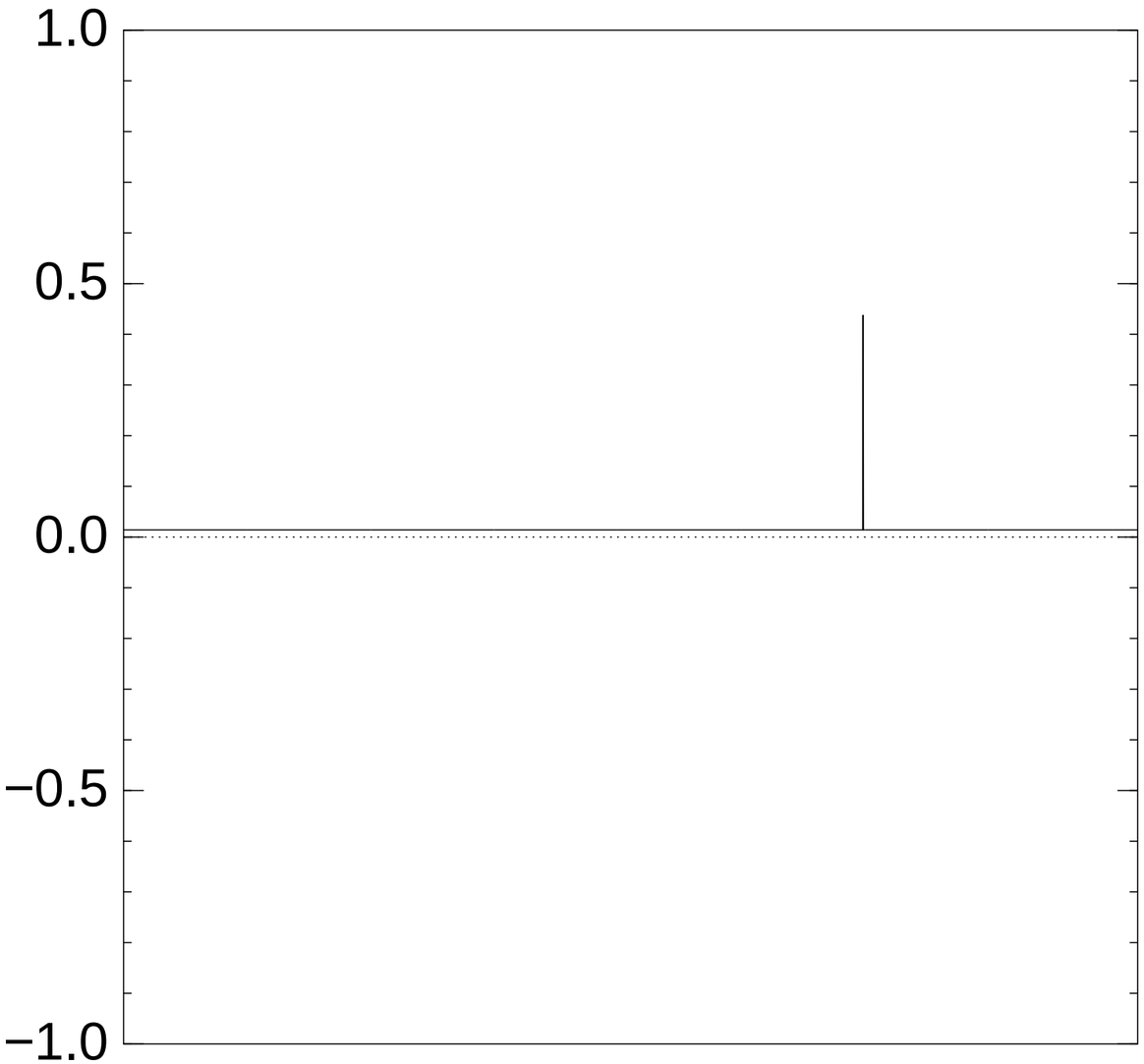
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $14 \times$ (Step 1 + Step 2):



Start from uniform superposition
over all n-bit strings q.

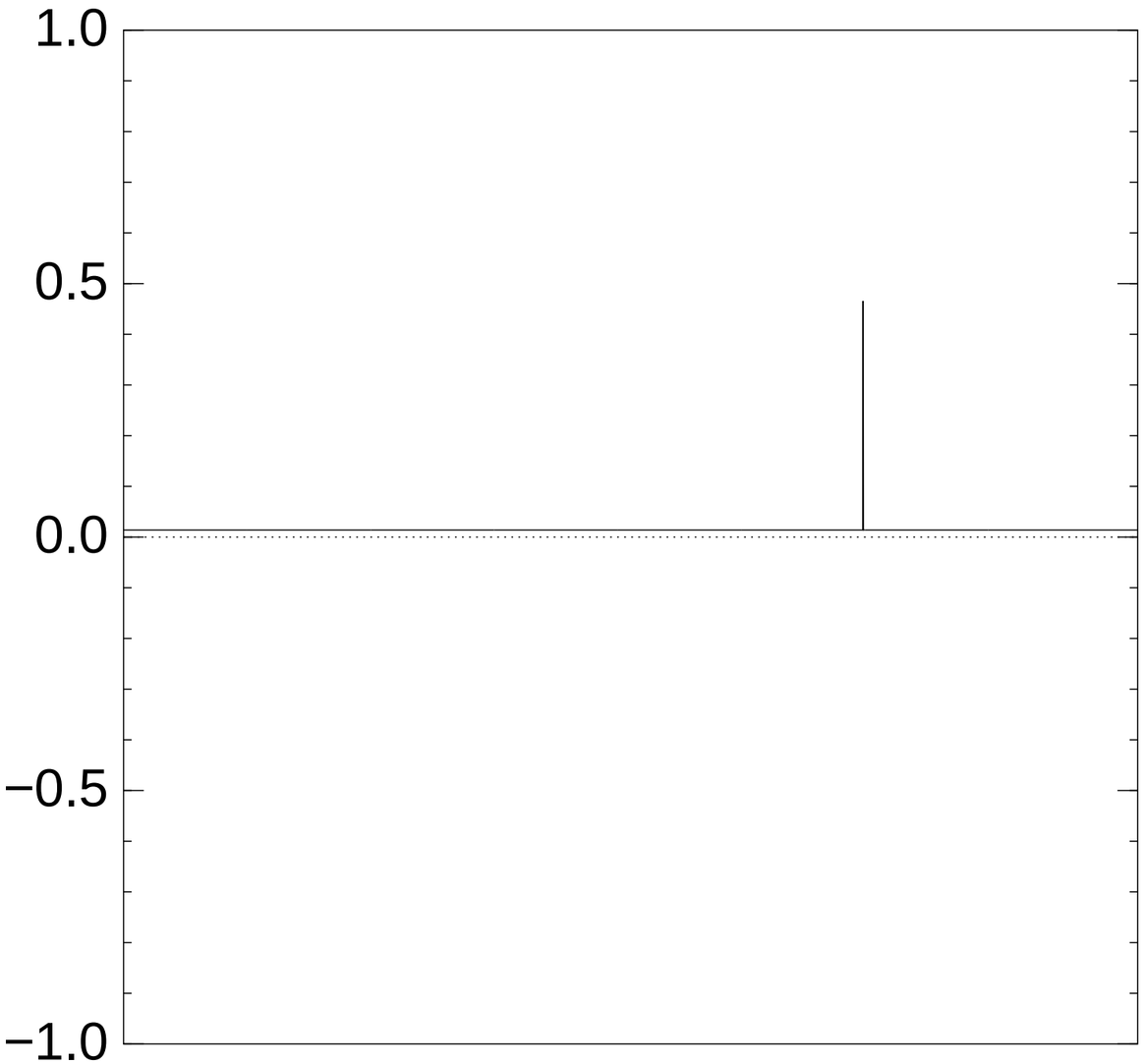
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $15 \times$ (Step 1 + Step 2):



Start from uniform superposition
over all n-bit strings q.

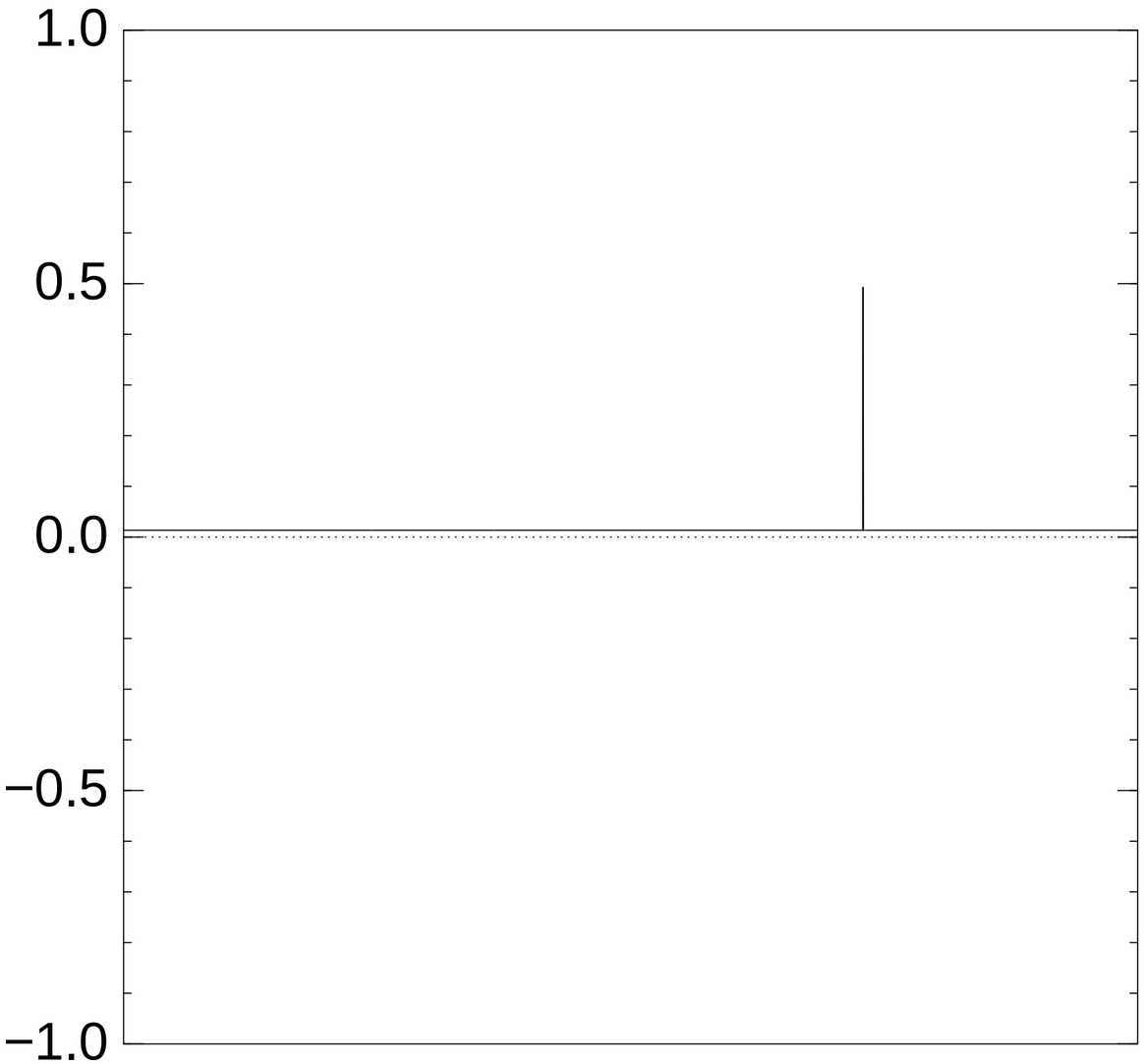
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $16 \times$ (Step 1 + Step 2):



Start from uniform superposition
over all n-bit strings q.

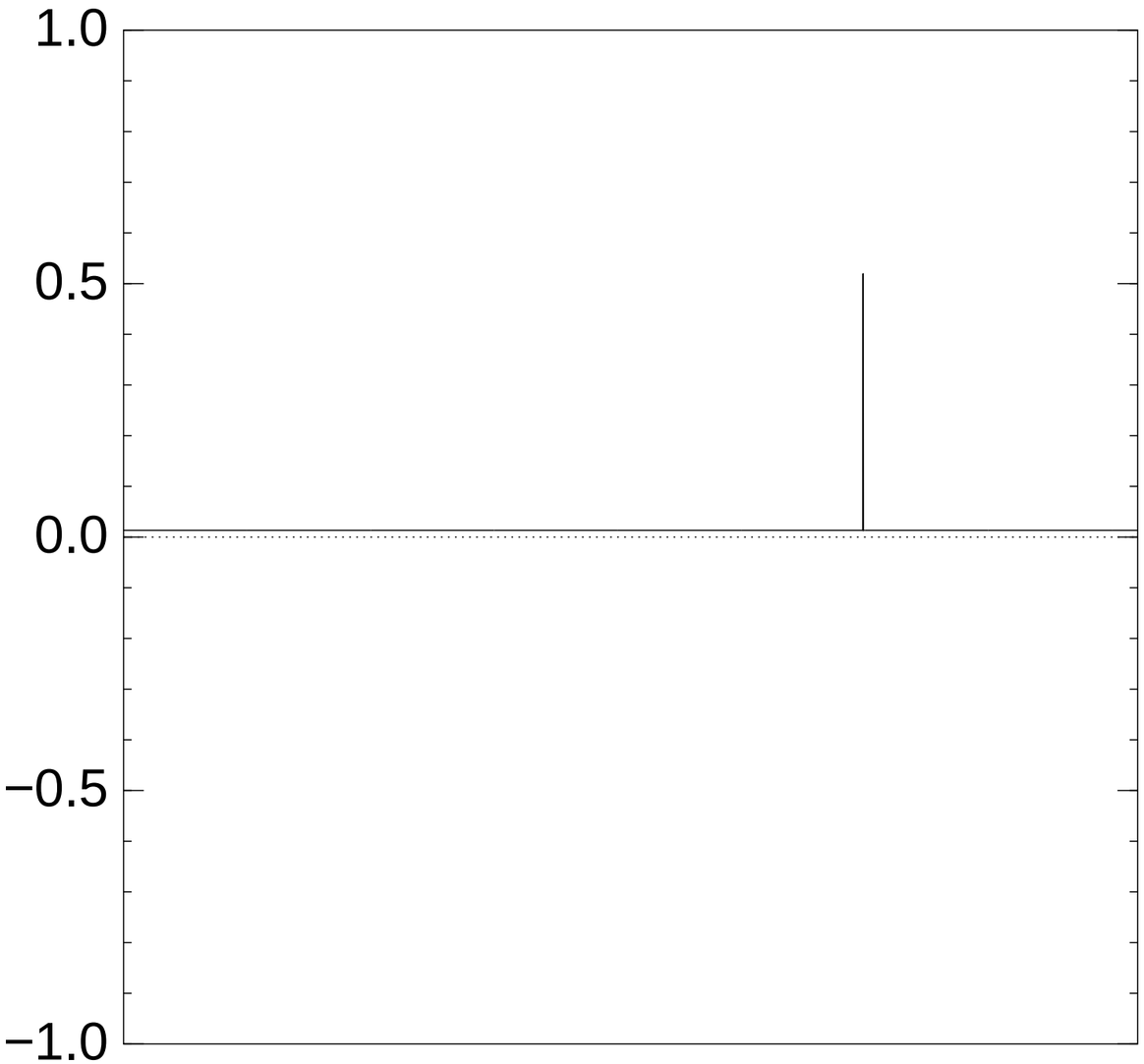
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $17 \times$ (Step 1 + Step 2):



Start from uniform superposition
over all n-bit strings q.

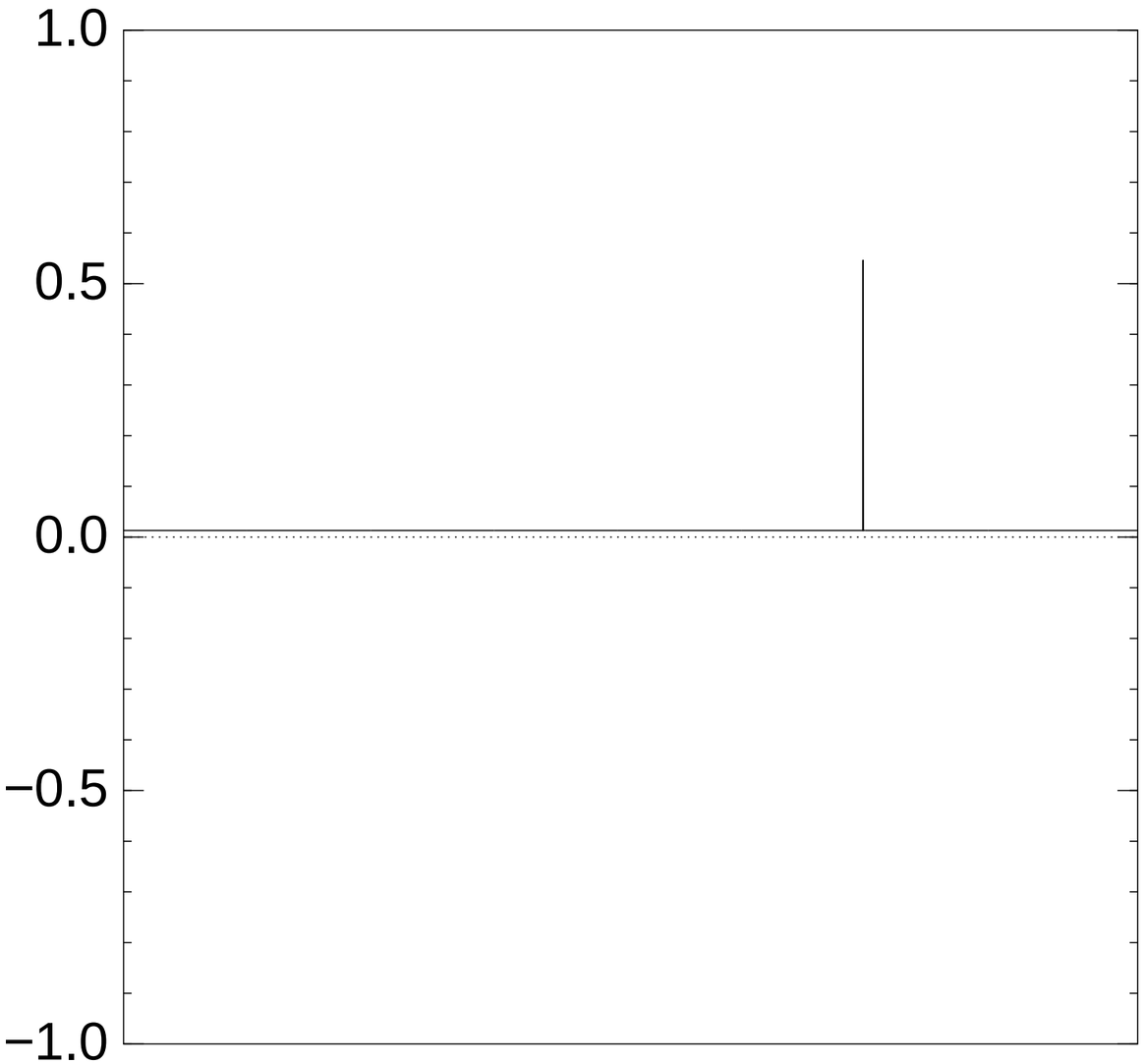
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $18 \times$ (Step 1 + Step 2):



Start from uniform superposition
over all n-bit strings q.

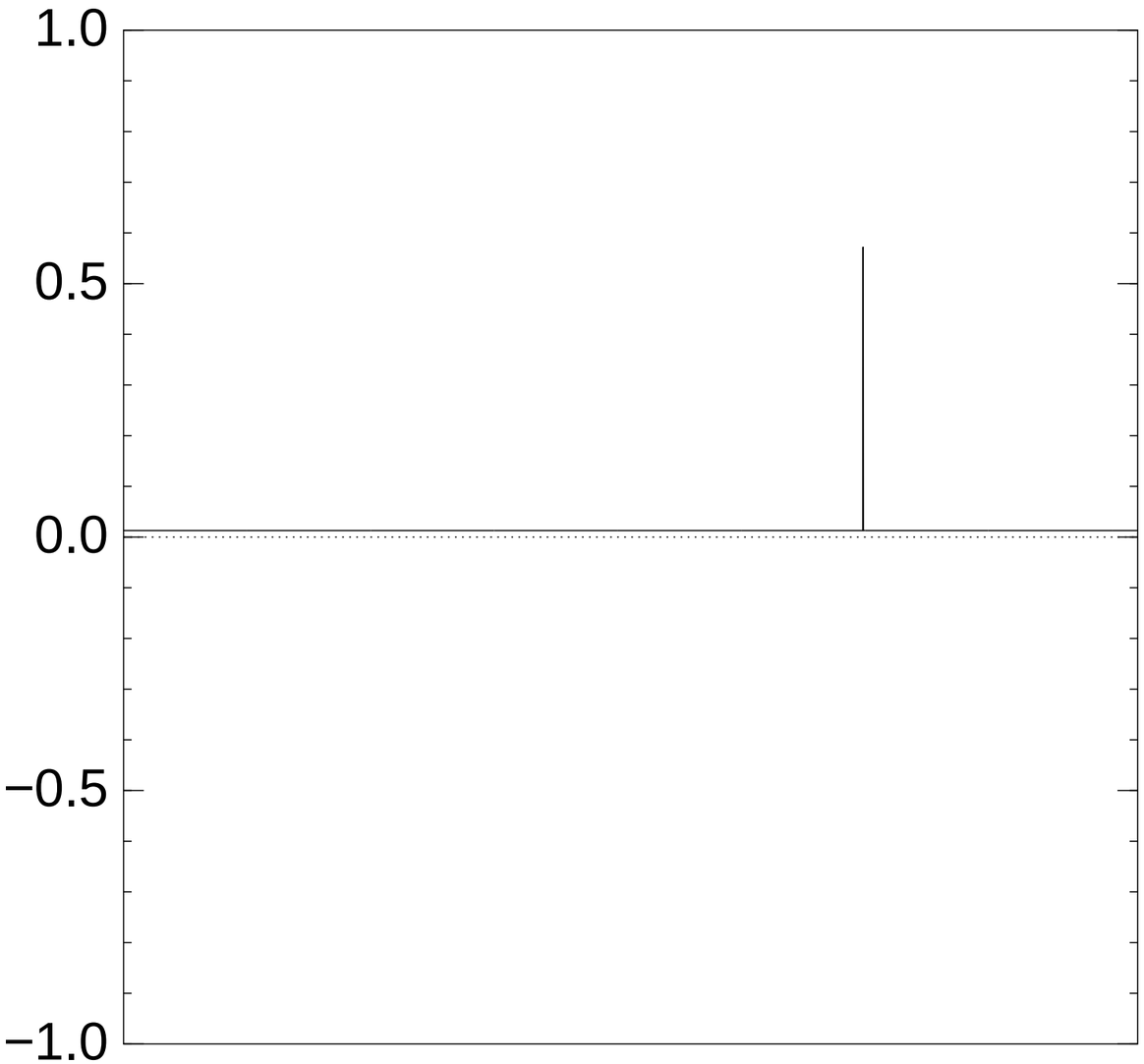
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $19 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

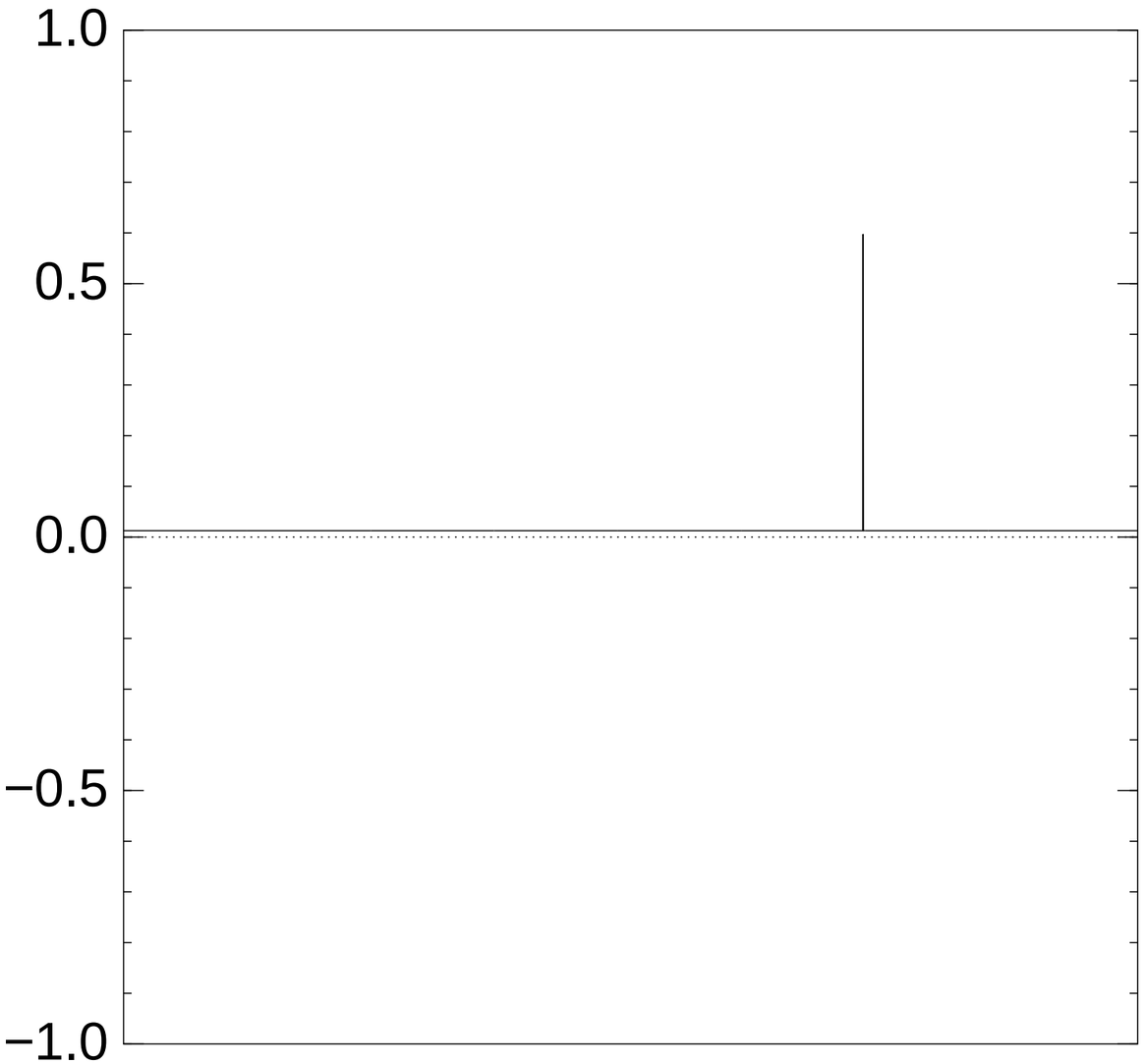
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $20 \times$ (Step 1 + Step 2):



Start from uniform superposition
over all n-bit strings q.

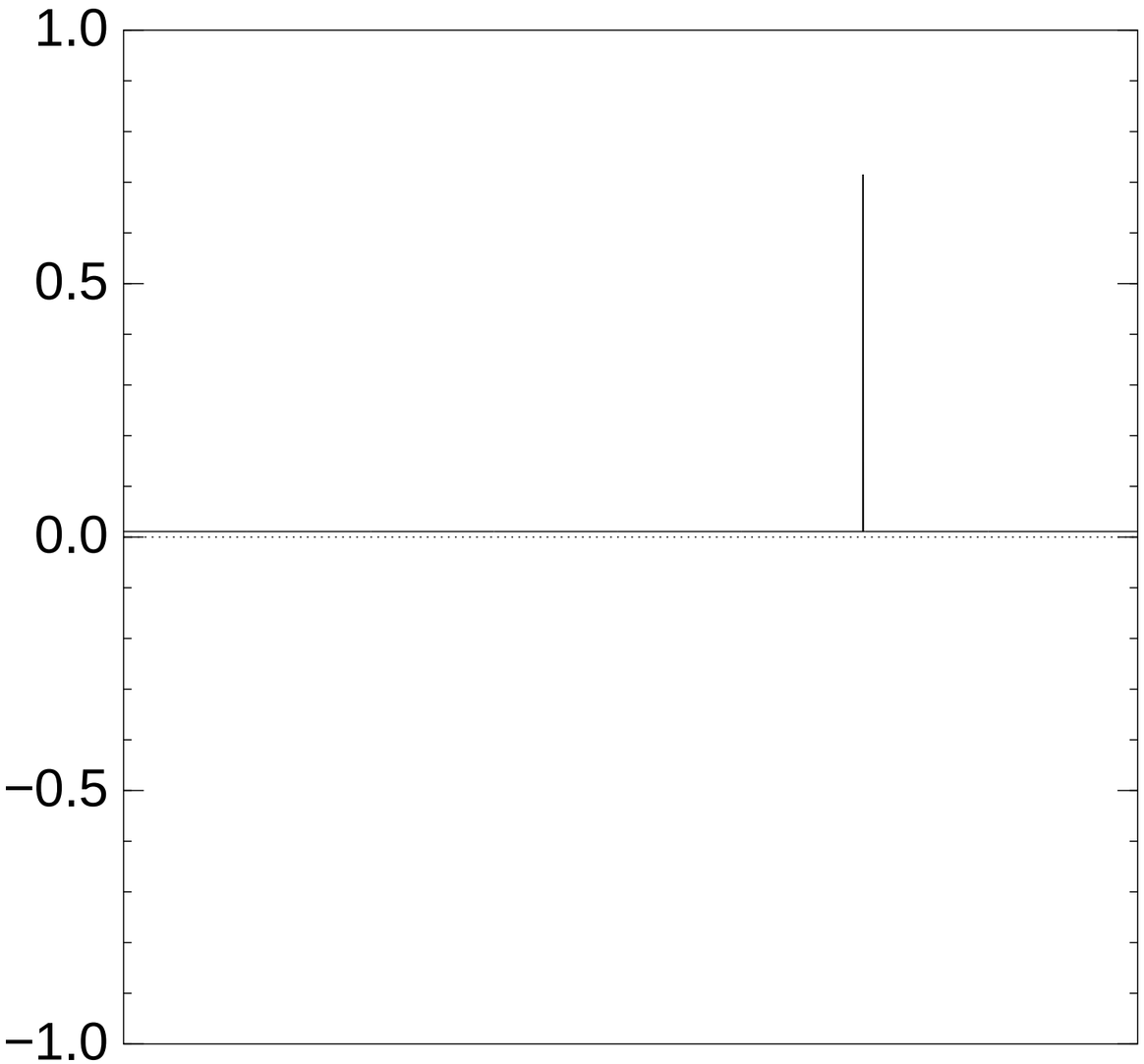
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $25 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

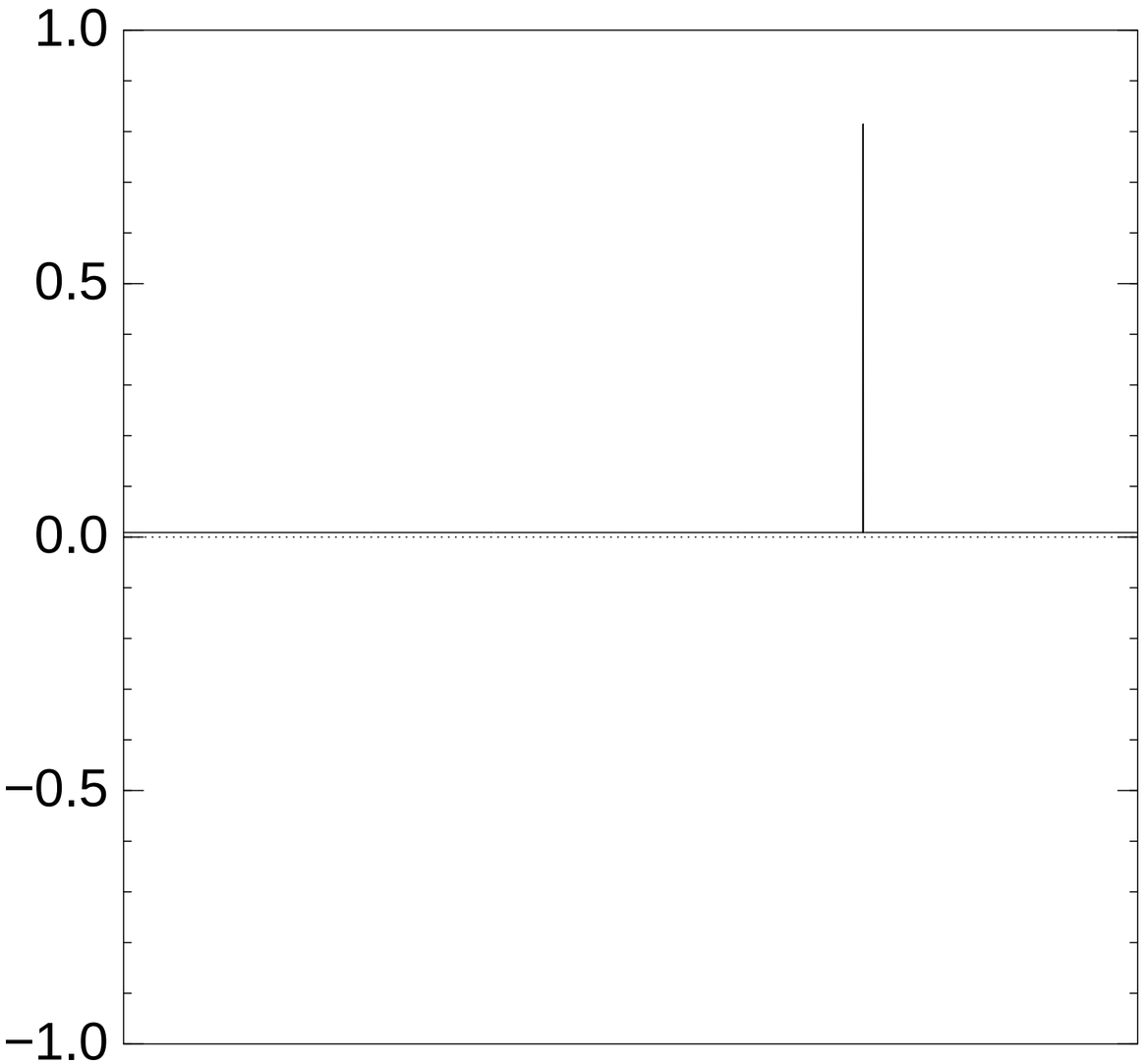
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $30 \times$ (Step 1 + Step 2):



Start from uniform superposition
over all n-bit strings q.

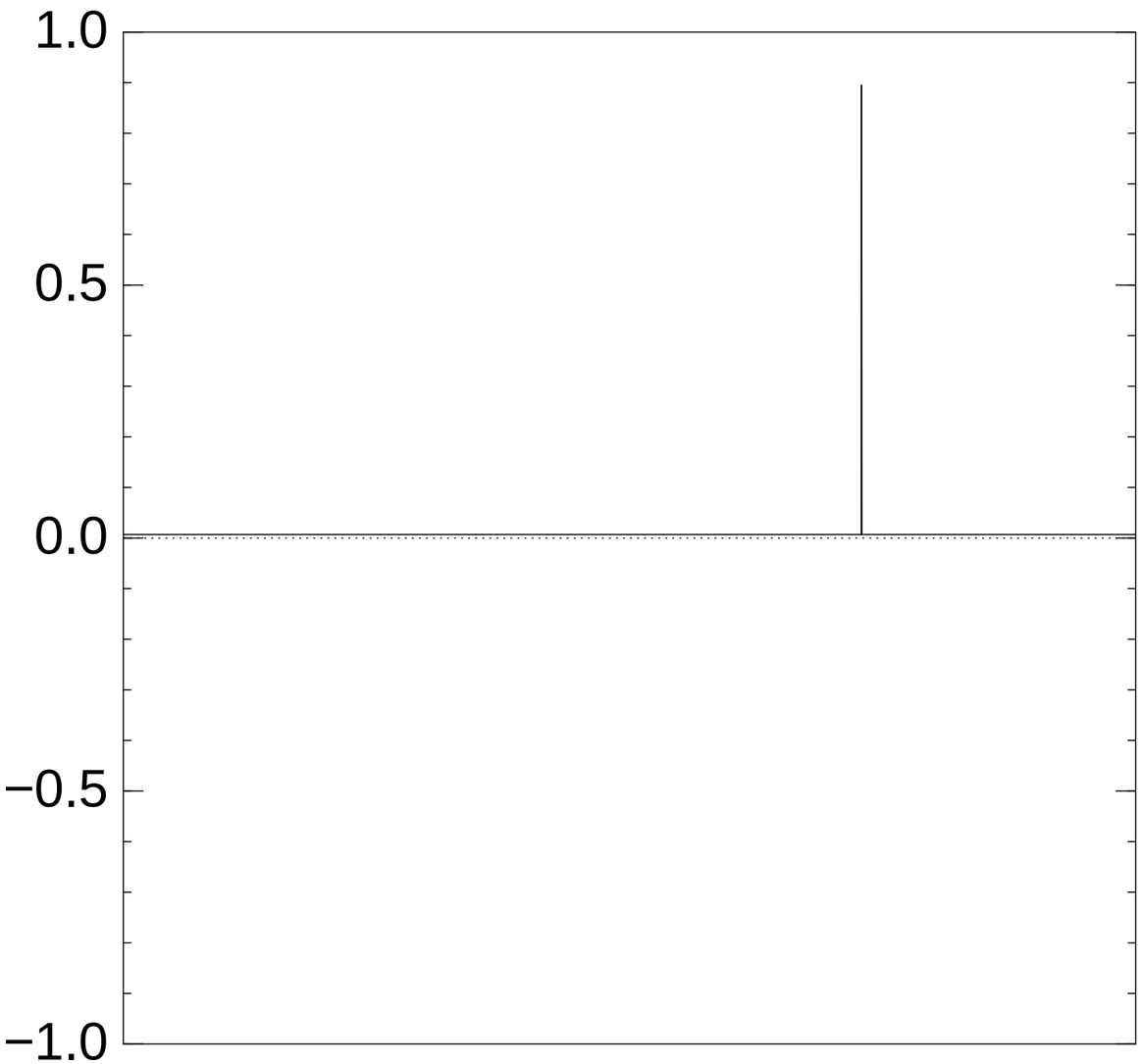
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $35 \times$ (Step 1 + Step 2):



Good moment to stop, measure.

Start from uniform superposition
over all n-bit strings q.

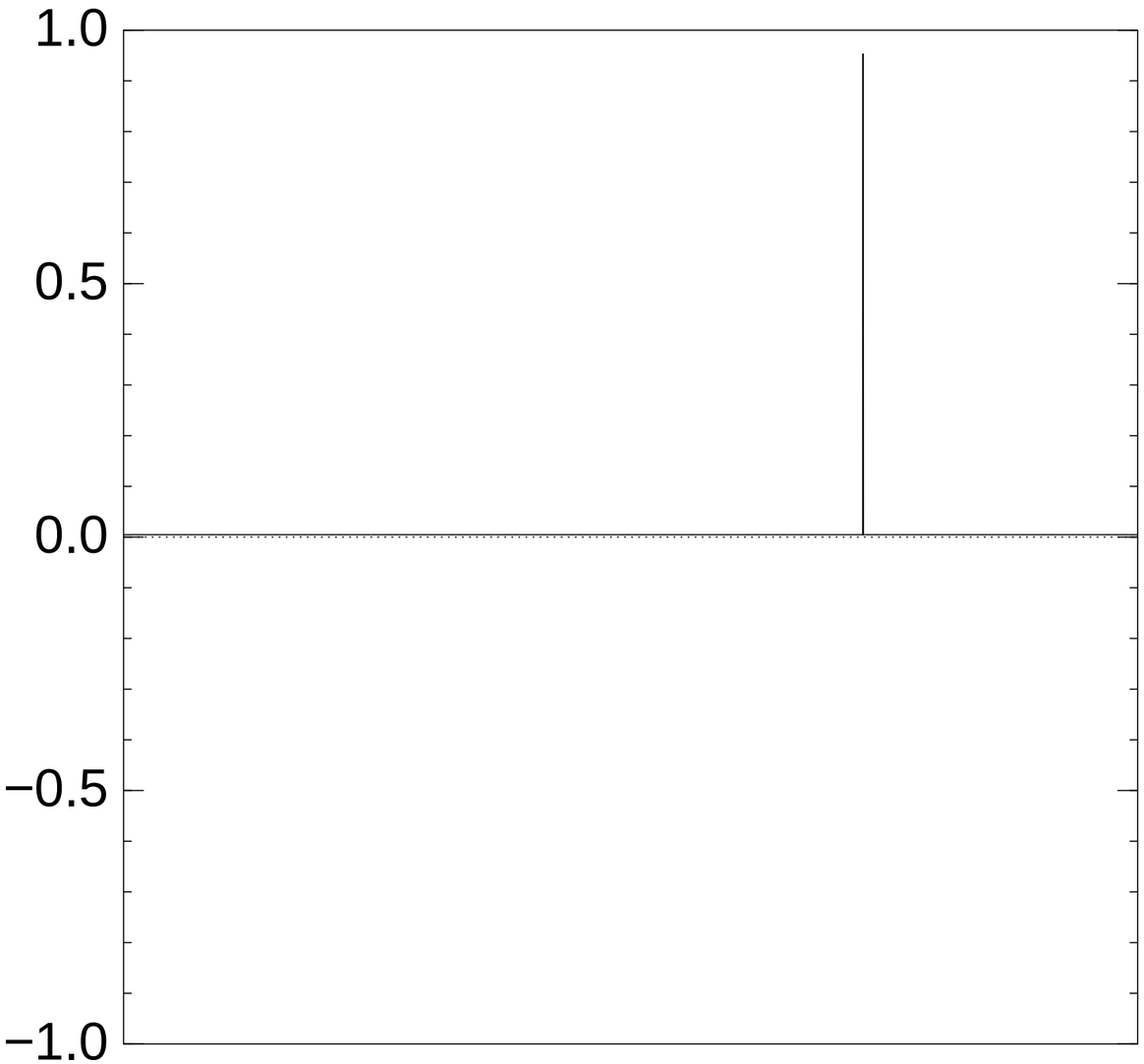
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $40 \times$ (Step 1 + Step 2):



Start from uniform superposition
over all n-bit strings q.

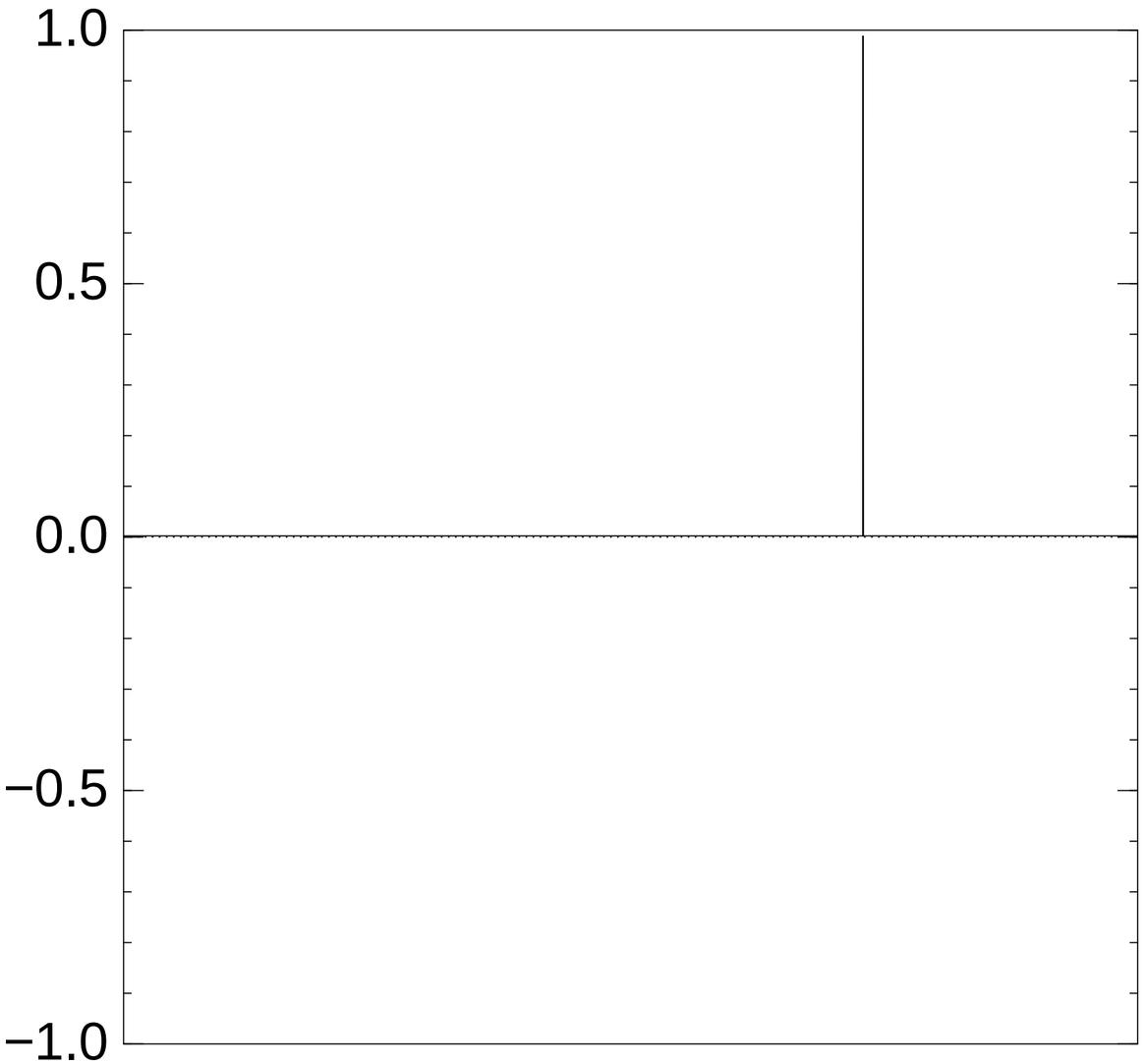
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $45 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

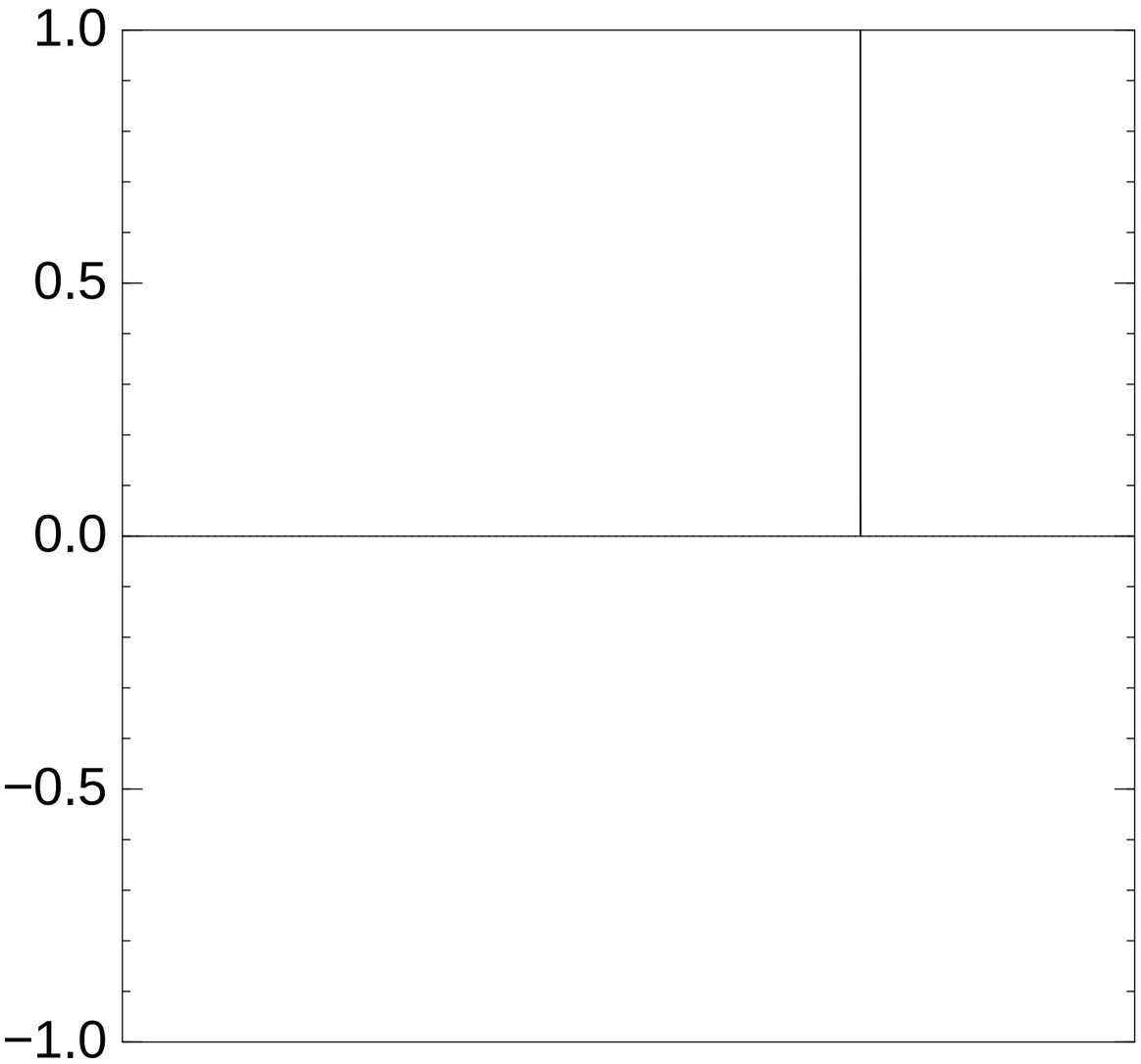
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $50 \times$ (Step 1 + Step 2):



Traditional stopping point.

Start from uniform superposition
over all n-bit strings q.

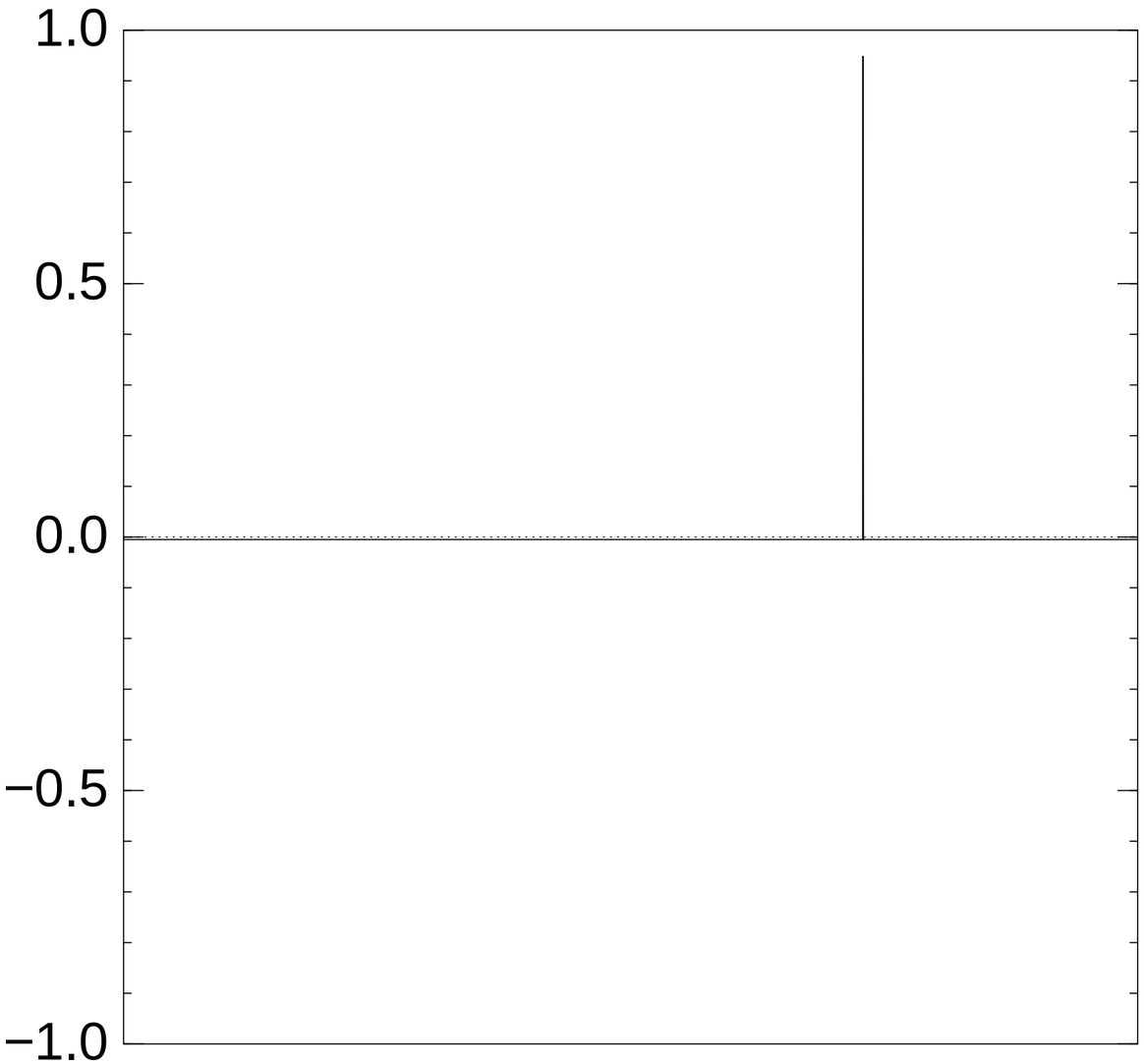
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $60 \times$ (Step 1 + Step 2):



Start from uniform superposition
over all n-bit strings q.

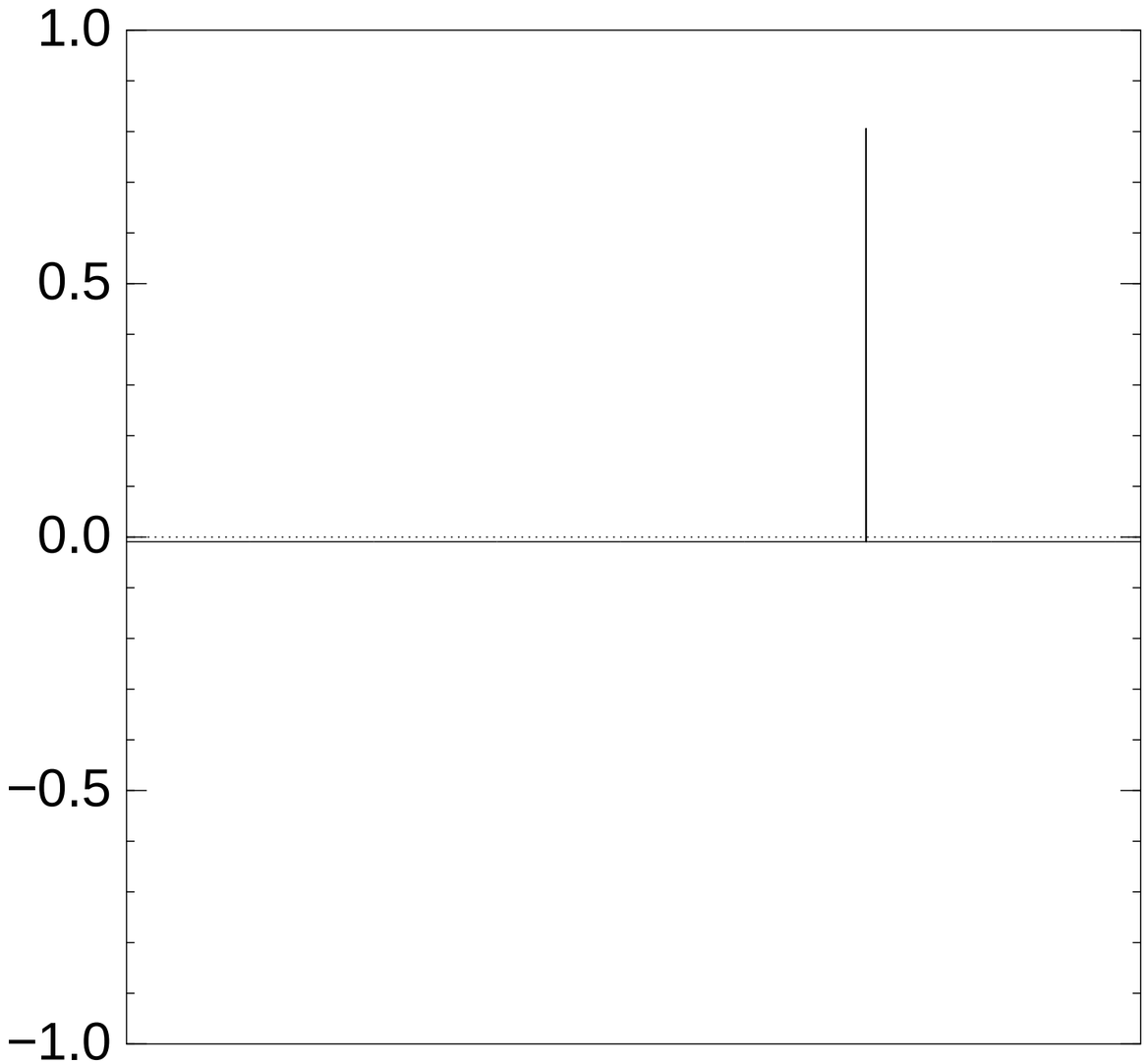
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $70 \times$ (Step 1 + Step 2):



Start from uniform superposition
over all n-bit strings q.

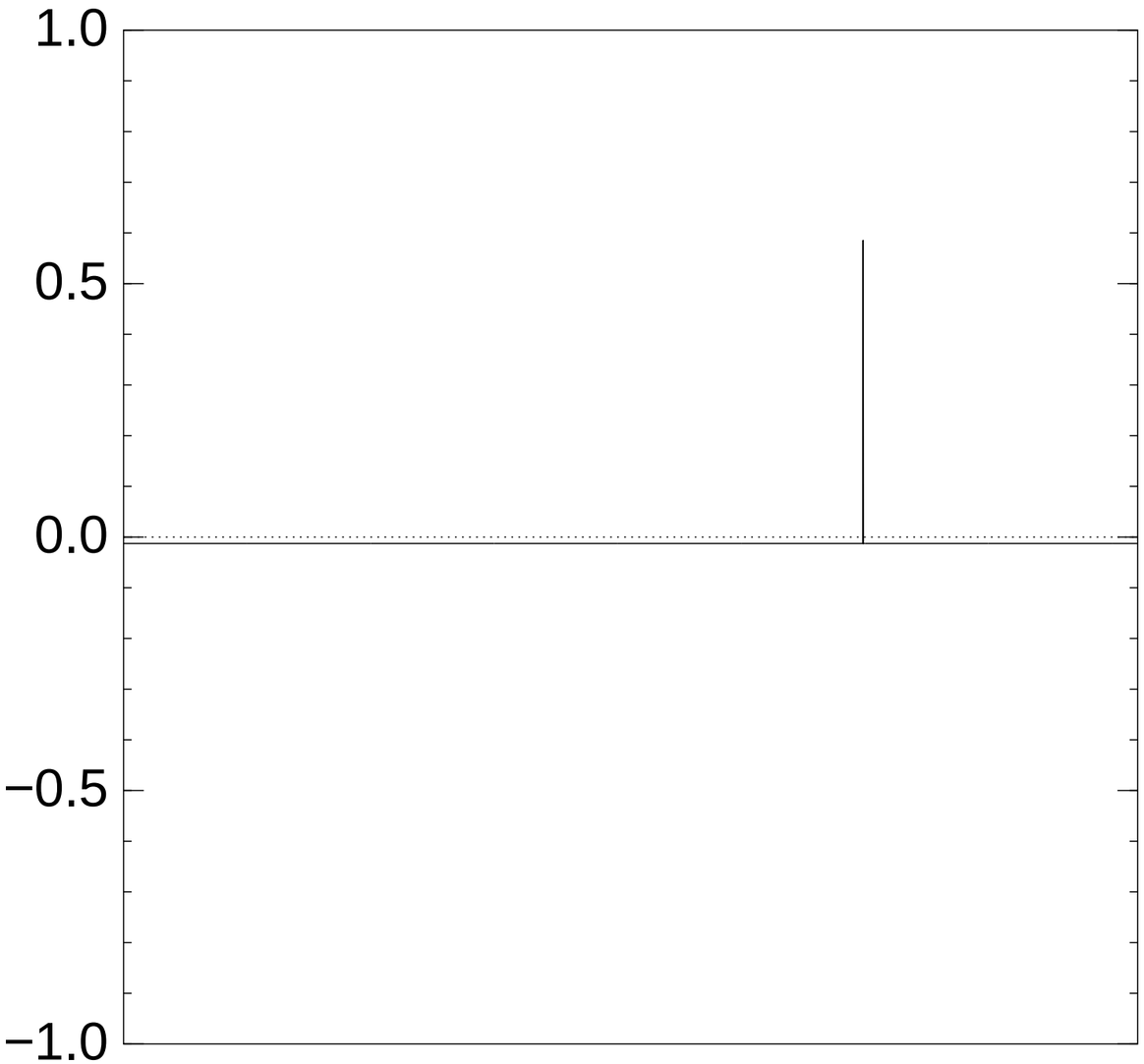
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $80 \times$ (Step 1 + Step 2):



Start from uniform superposition
over all n-bit strings q.

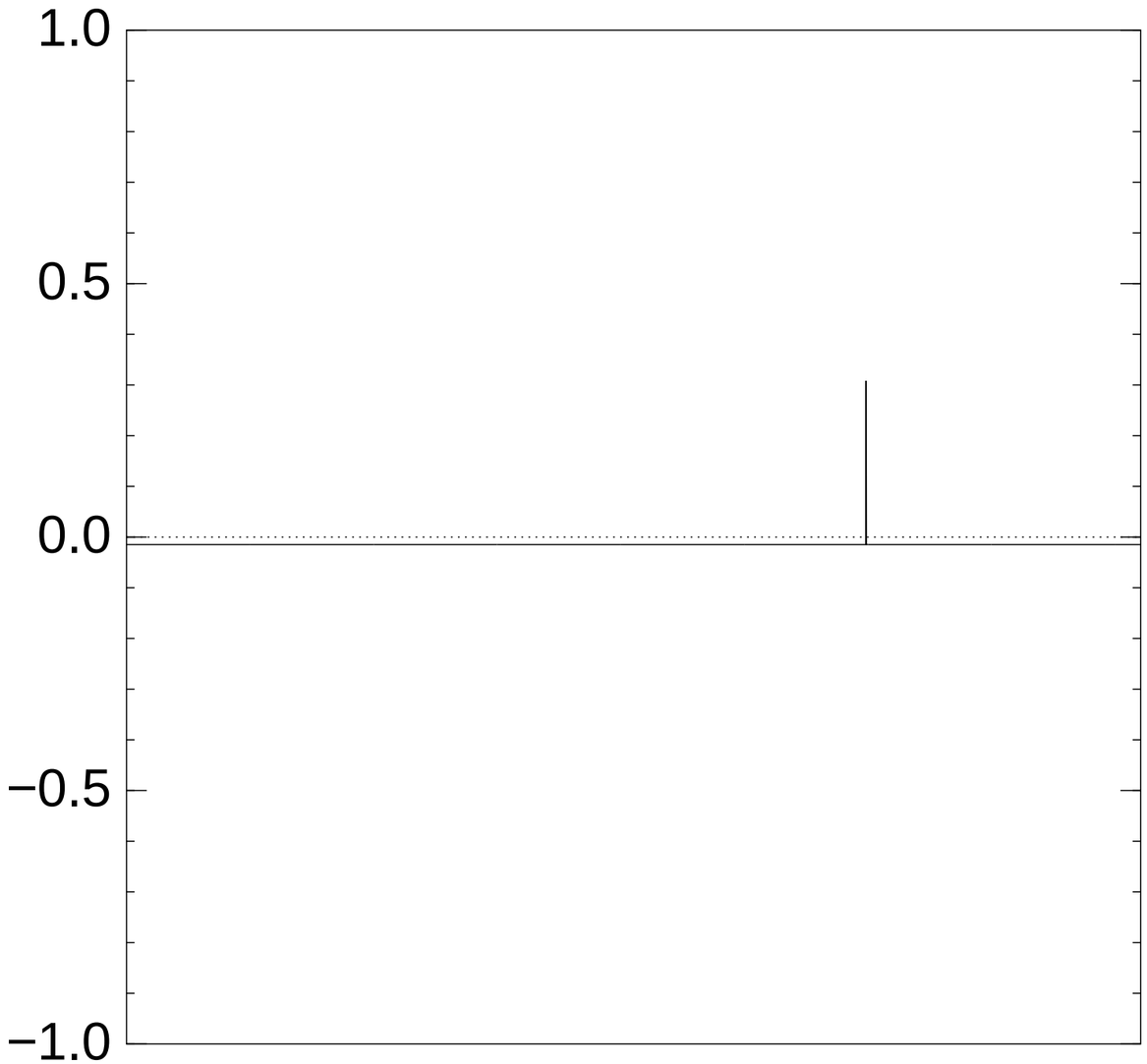
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $90 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

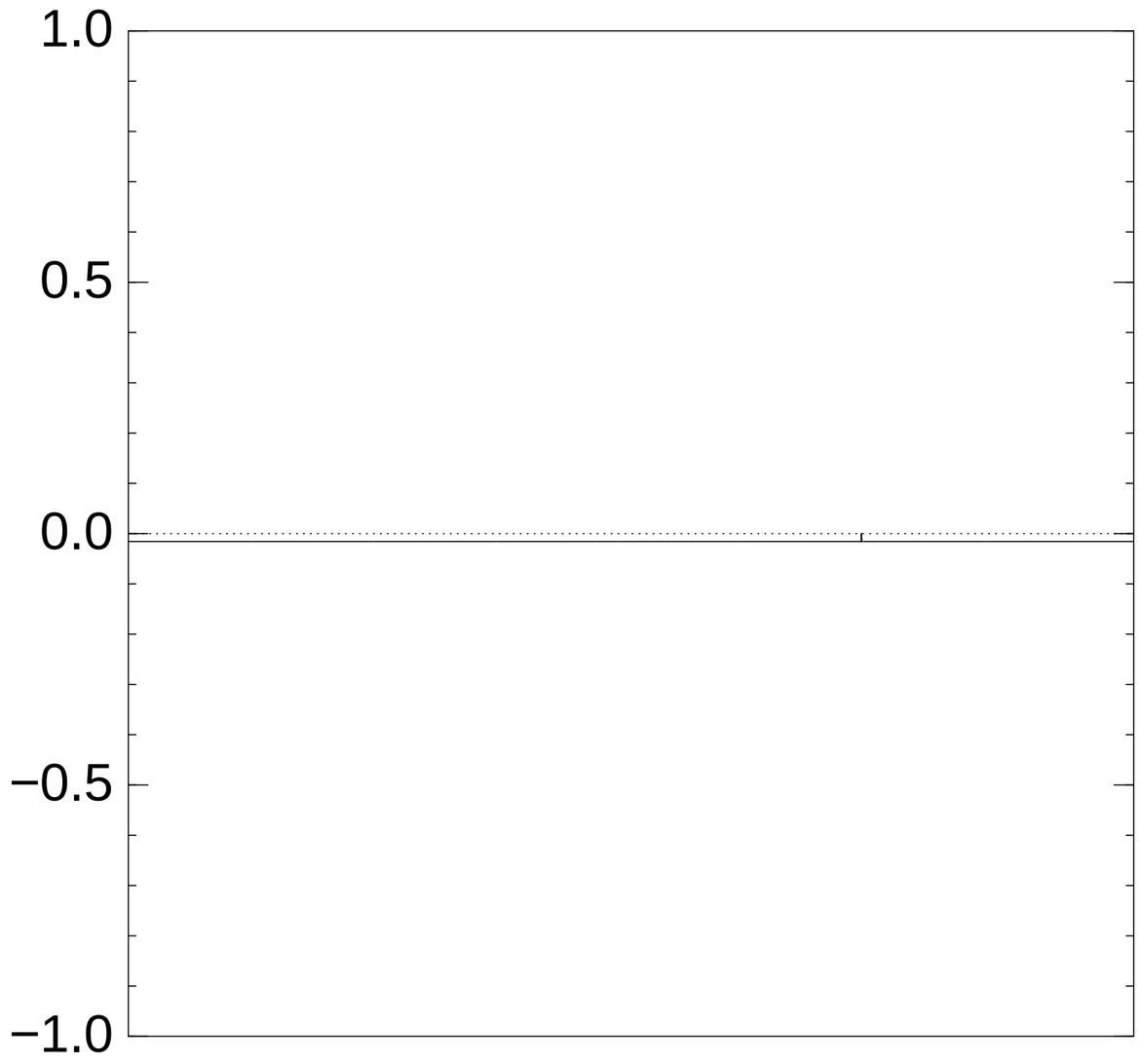
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $100 \times$ (Step 1 + Step 2):



Very bad stopping point.

om uniform superposition
n-bit strings q.

Set $a \leftarrow b$ where
a_q if $f(q) = 0$,
otherwise.
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"Grover diffusion".
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also fast.

Step 1 + Step 2
$58 \cdot 2^{0.5n}$ times.

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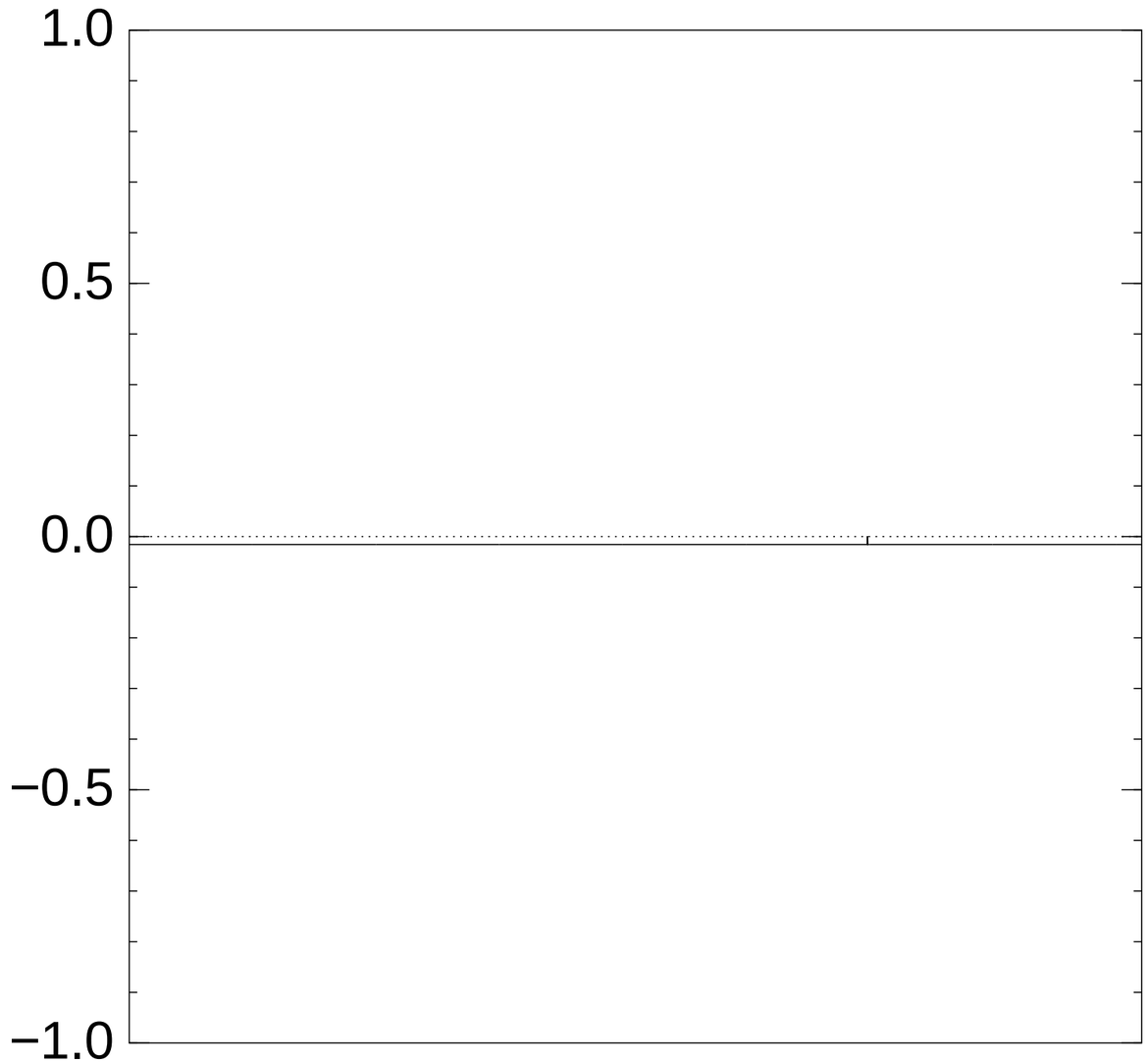
gh probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $100 \times$ (Step 1 + Step 2):



Very bad stopping point.

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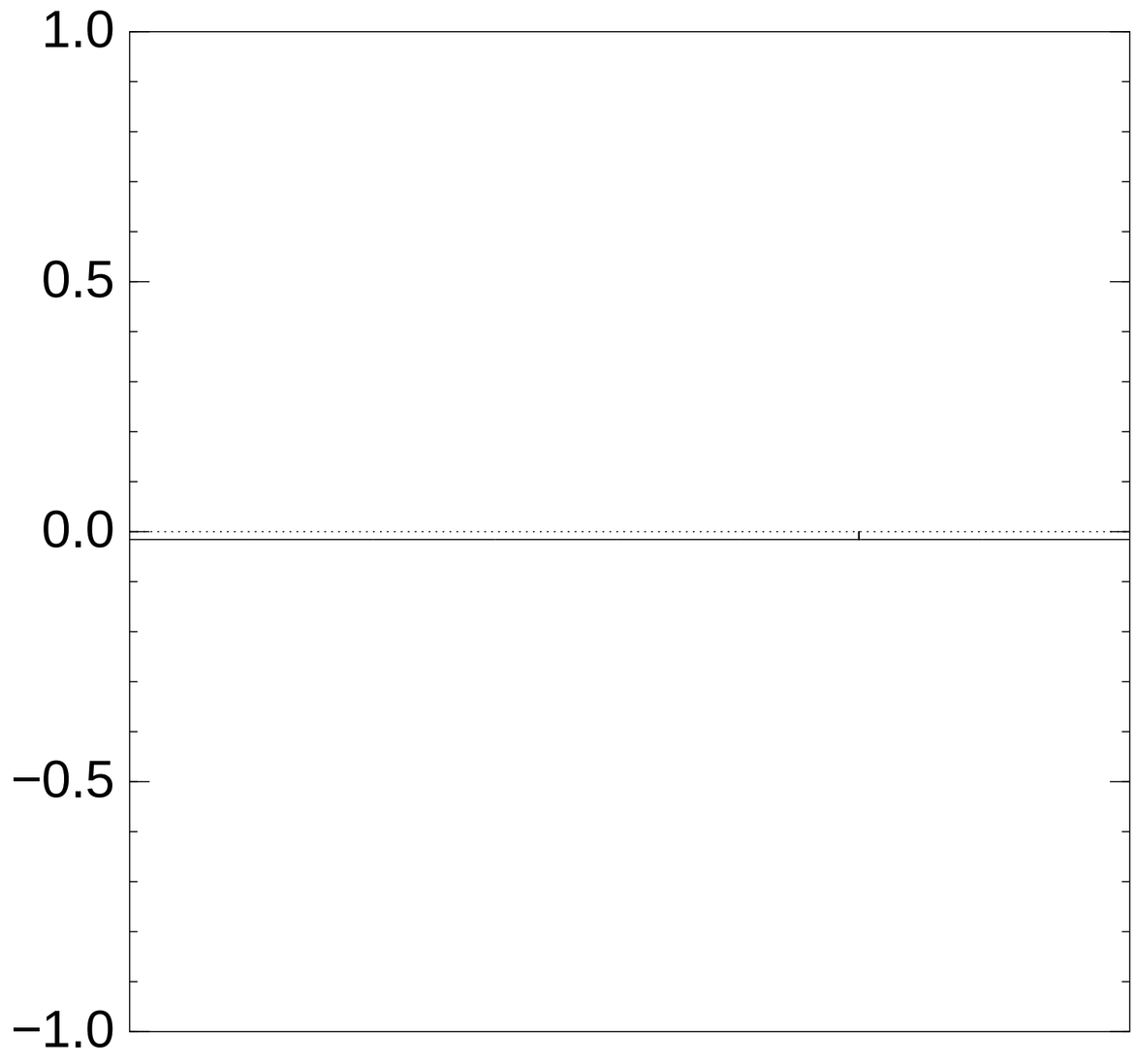
Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $100 \times$ (Step 1 + Step 2):



Very bad stopping point.

$q \mapsto a_q$ is complet
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(2) a_q for non-roo

Step 1 + Step 2
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to understand evo
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$\Rightarrow$ Probability is $\approx$
after $\approx(\pi/4)2^{0.5n}$

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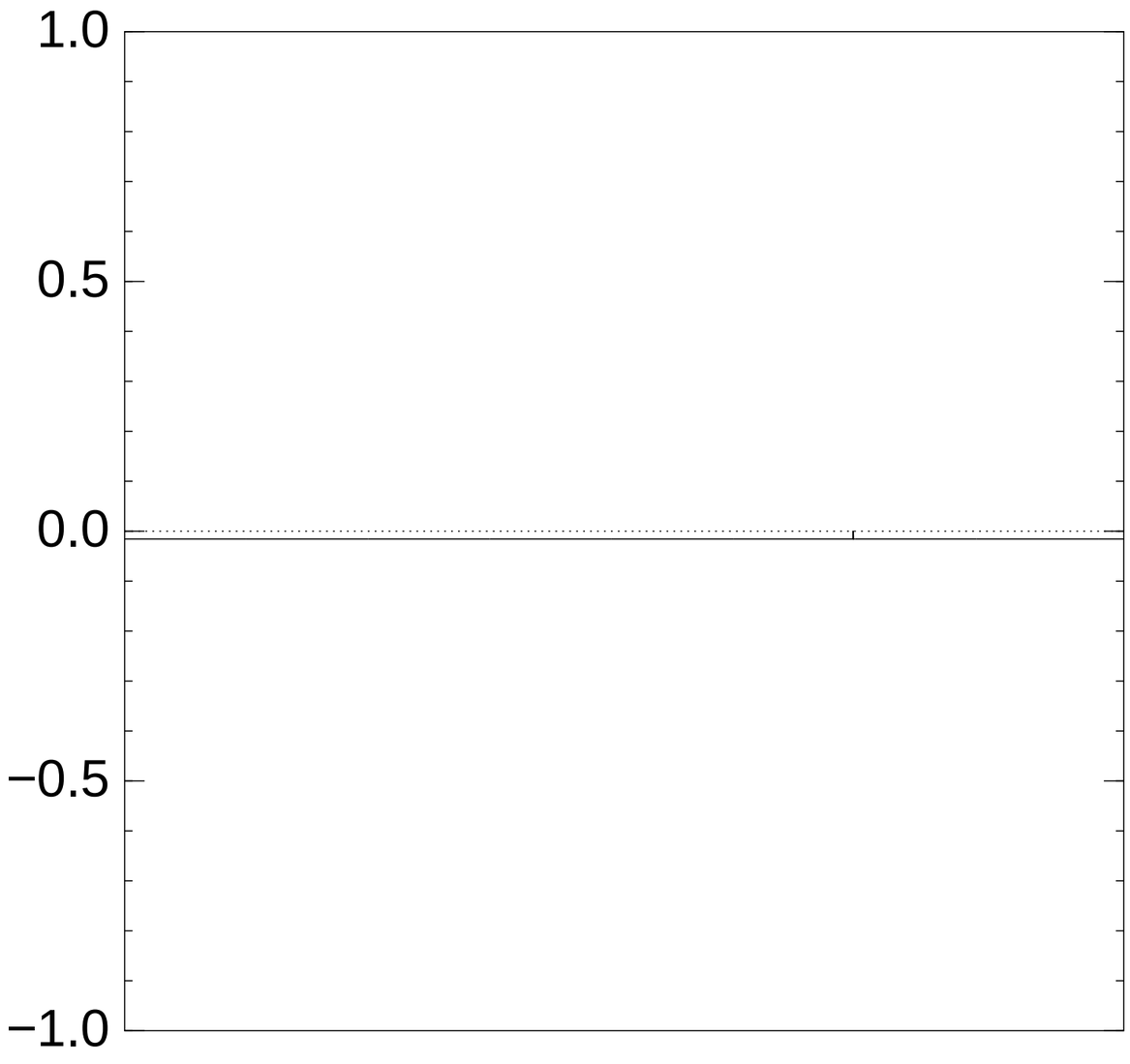
Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $100 \times$ (Step 1 + Step 2):



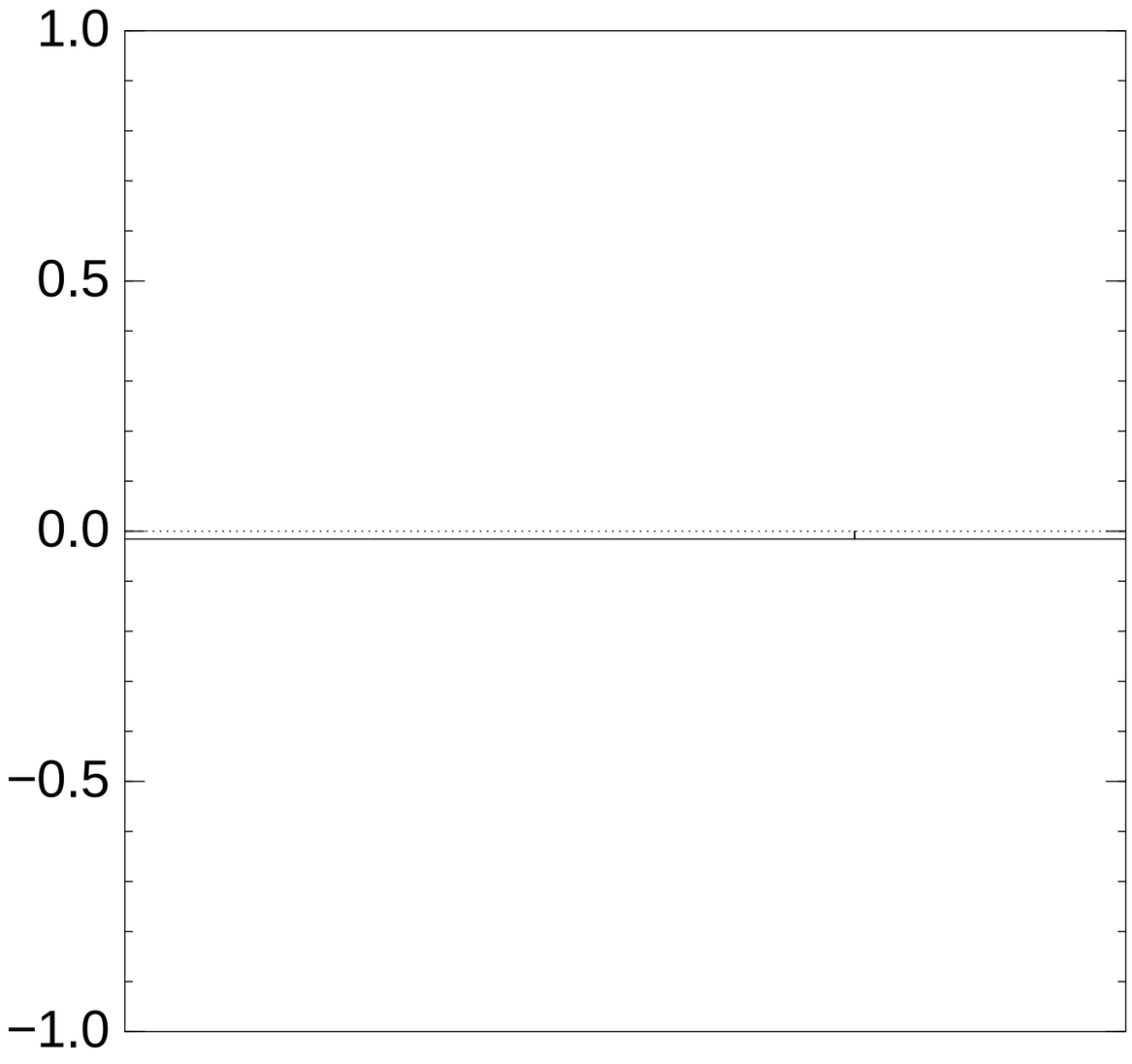
Very bad stopping point.

$q \mapsto a_q$ is completely describ
by a vector of two numbers
(with fixed multiplicities):
(1) a_q for roots q;
(2) a_q for non-roots q.

Step 1 + Step 2
act linearly on this vector.

Easily compute eigenvalues
and powers of this linear ma
to understand evolution
of state of Grover's algorithm
$\Rightarrow$ Probability is ≈ 1
after $\approx(\pi/4)2^{0.5n}$ iterations

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $100 \times$ (Step 1 + Step 2):



Very bad stopping point.

$q \mapsto a_q$ is completely described
by a vector of two numbers
(with fixed multiplicities):
(1) a_q for roots q;
(2) a_q for non-roots q.

Step 1 + Step 2
act linearly on this vector.

Easily compute eigenvalues
and powers of this linear map
to understand evolution
of state of Grover's algorithm.
$\Rightarrow$ Probability is ≈ 1
after $\approx (\pi/4)2^{0.5n}$ iterations.

zed graph of $q \mapsto a_q$
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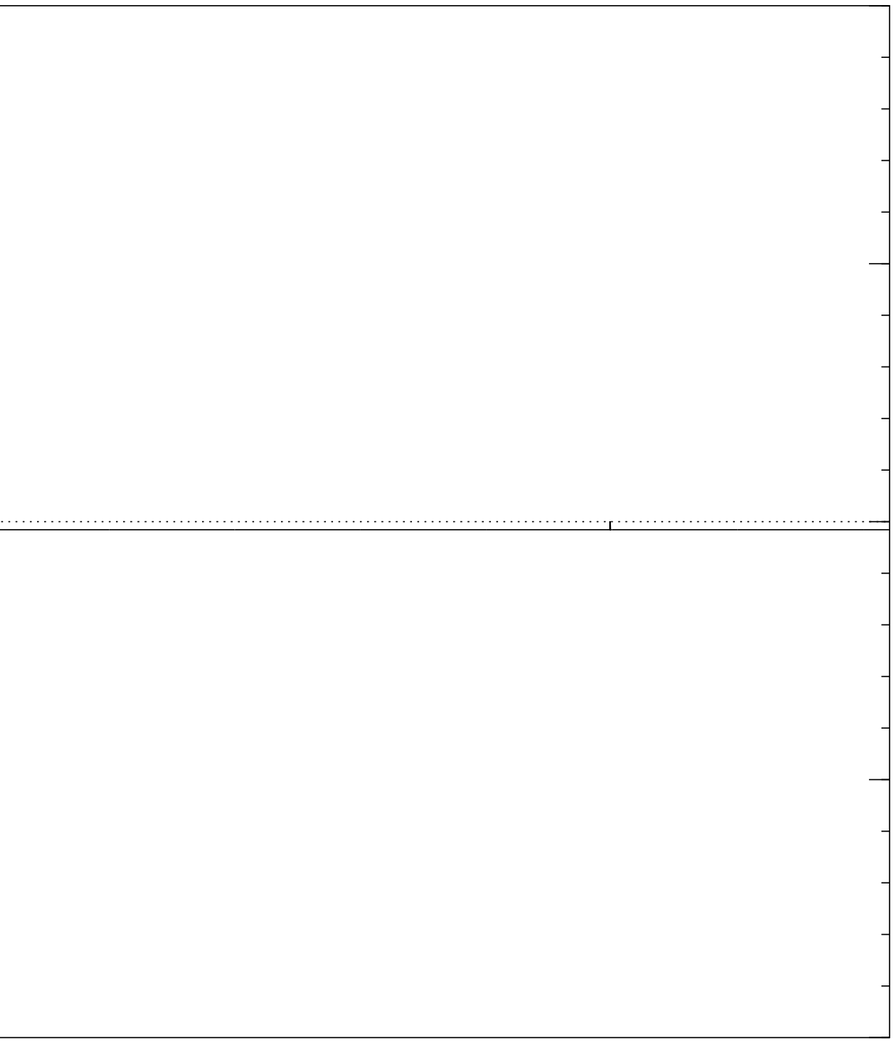
$0 \times$ (Step 1 + Step 2):



d stopping point.

$q \mapsto a_q$ is completely described
by a vector of two numbers
(with fixed multiplicities):
(1) a_q for roots q;
(2) a_q for non-roots q.

Step 1 + Step 2
act linearly on this vector.

Easily compute eigenvalues
and powers of this linear map
to understand evolution
of state of Grover's algorithm.
$\Rightarrow$ Probability is ≈ 1
after $\approx (\pi/4) 2^{0.5n}$ iterations.

Textboo

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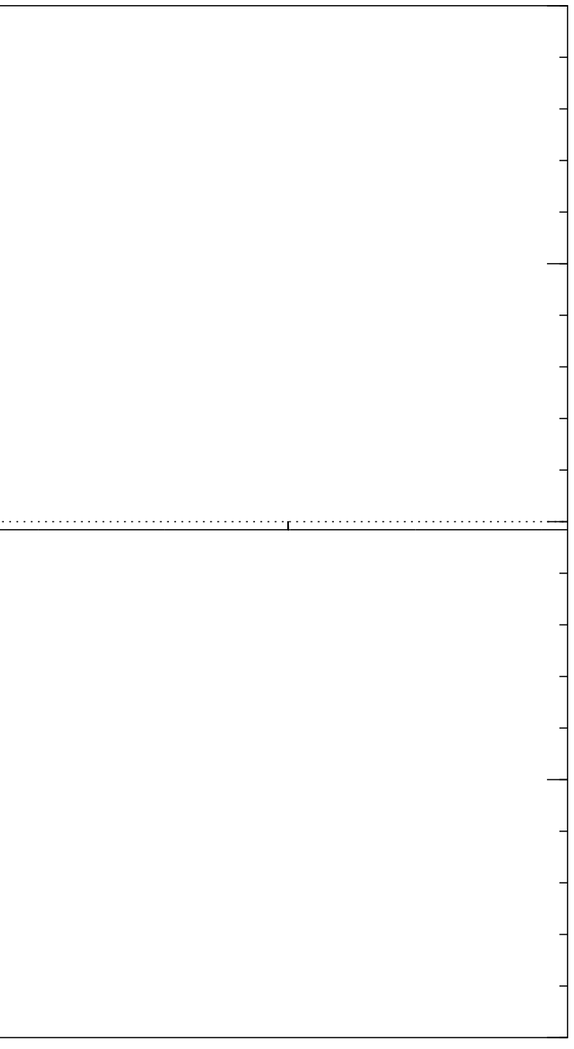
of $q \mapsto a_q$
th $n = 12$
$1 +$ Step 2):



point.

$q \mapsto a_q$ is completely described
by a vector of two numbers
(with fixed multiplicities):
(1) a_q for roots q;
(2) a_q for non-roots q.

Step 1 + Step 2
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Easily compute eigenvalues
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$\Rightarrow$ Probability is ≈ 1
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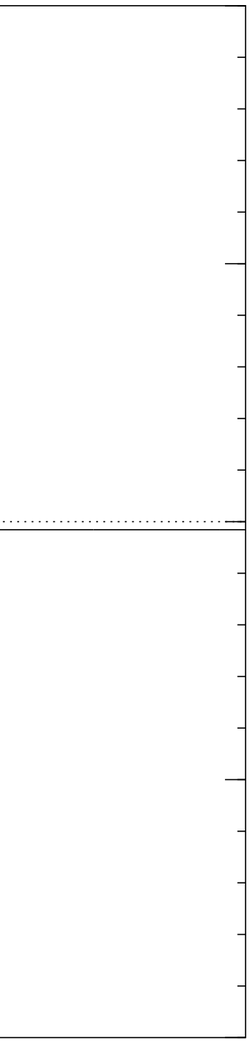

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Real-world security systems
cannot avoid these questions.