Overview of post-quantum cryptography

D. J. Bernstein
University of Illinois at Chicago &
Technische Universiteit Eindhoven

Cryptography = "secret writing".

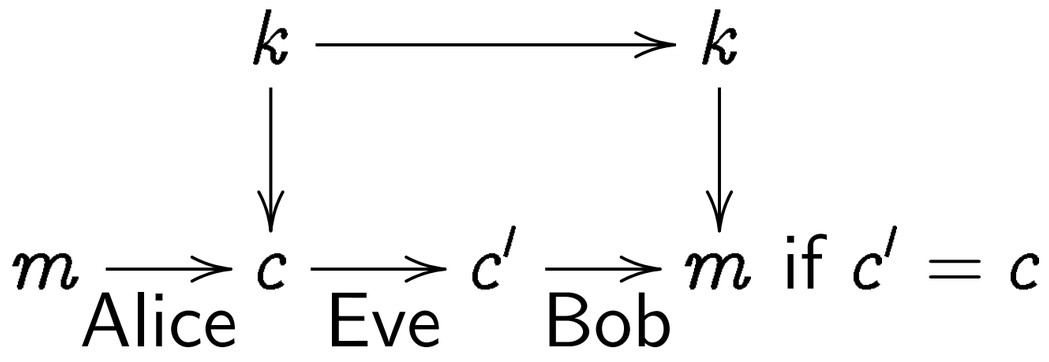
Achieve various security goals
by secretly transforming messages.

Major theme of research:
Users have cost constraints.
Can be challenging to reach
acceptable security levels.

Secret-key cryptography

Prerequisite: Alice and Bob
share a short secret key k
not known to eavesdropper Eve.

Security goals:
Confidentiality and integrity
for any number of messages
exchanged by Alice and Bob,
despite Eve's espionage+forgery.



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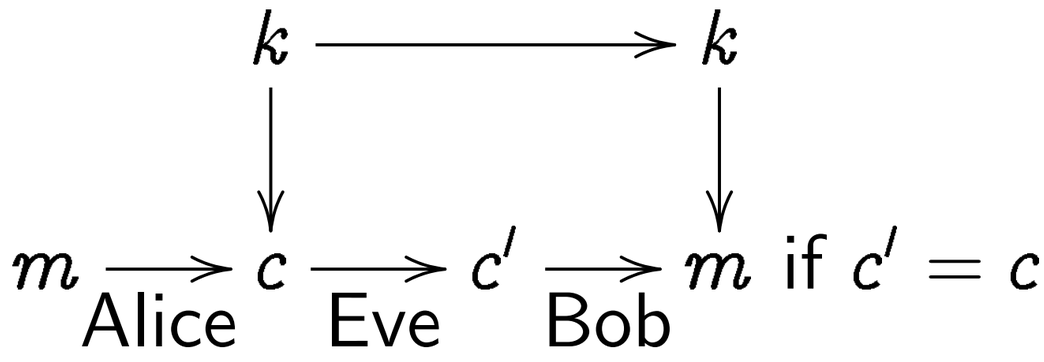

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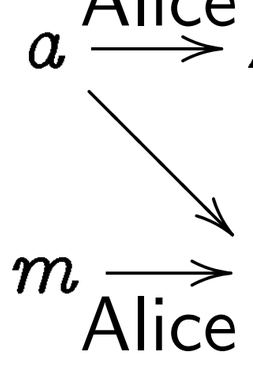

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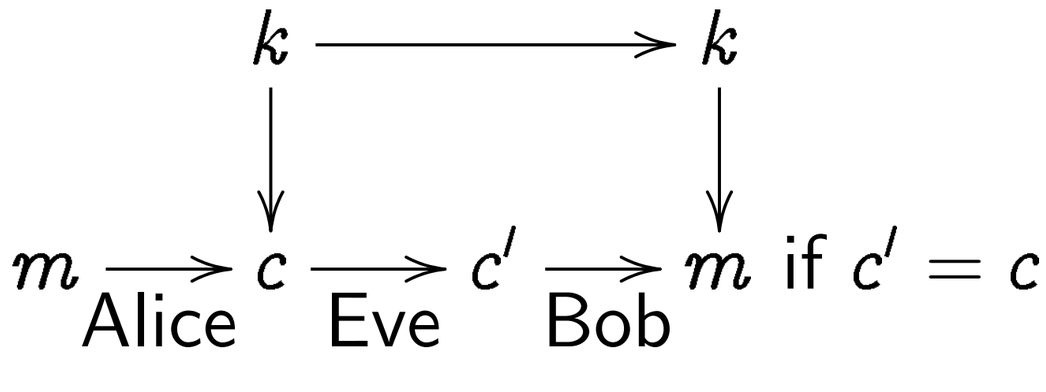

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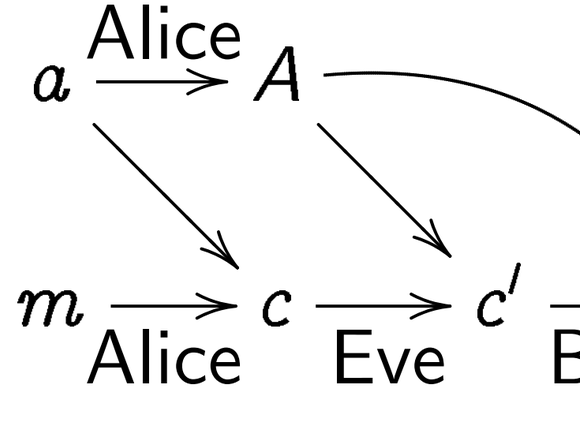

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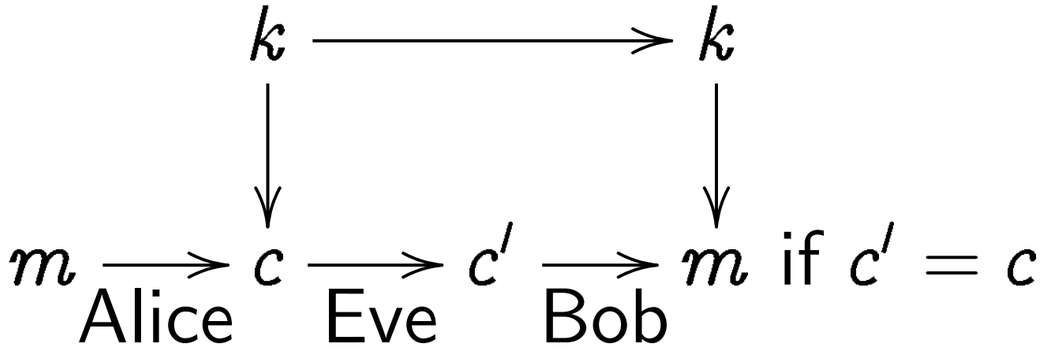

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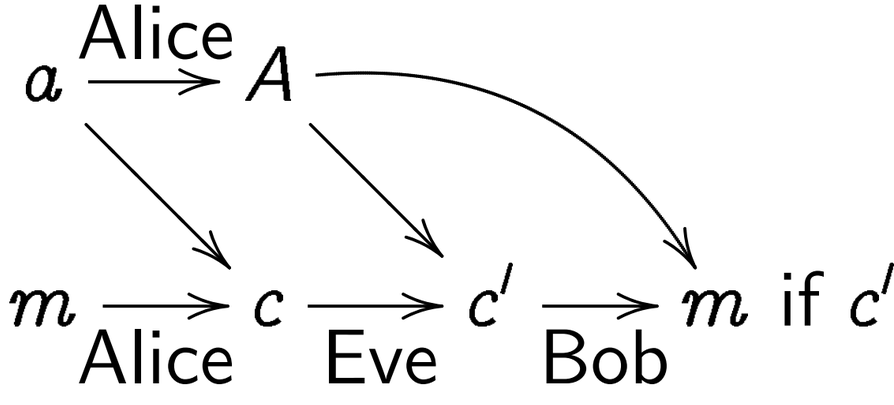

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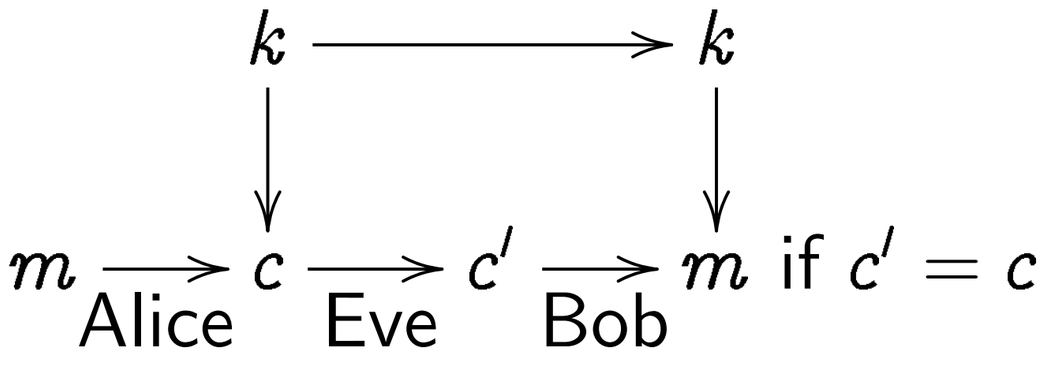

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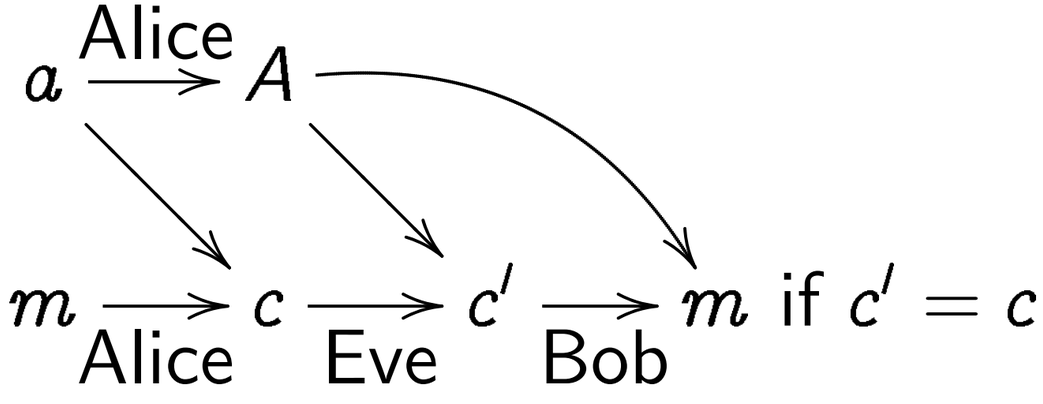

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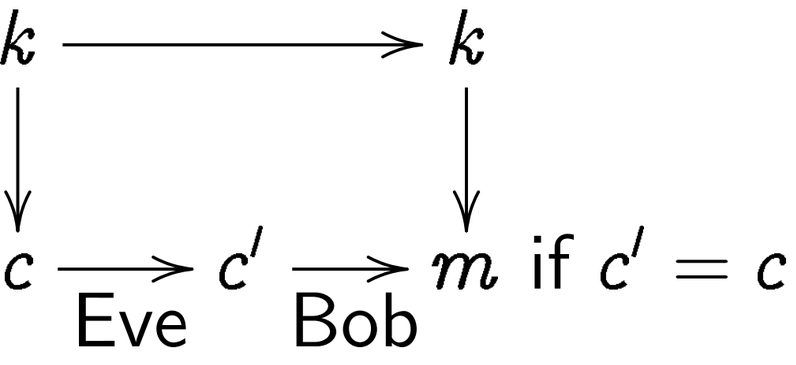

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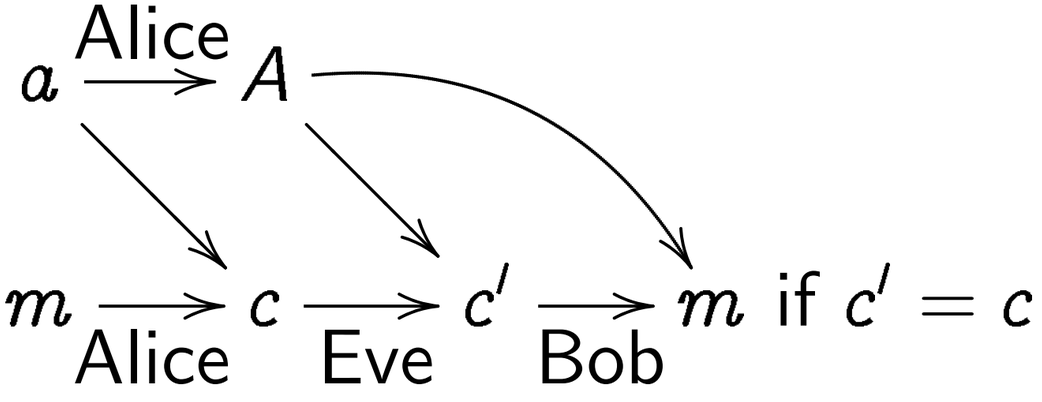

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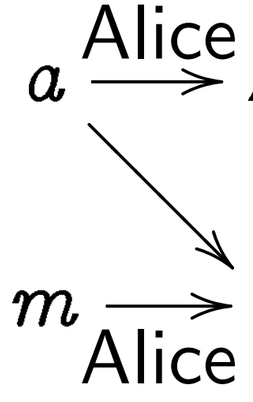

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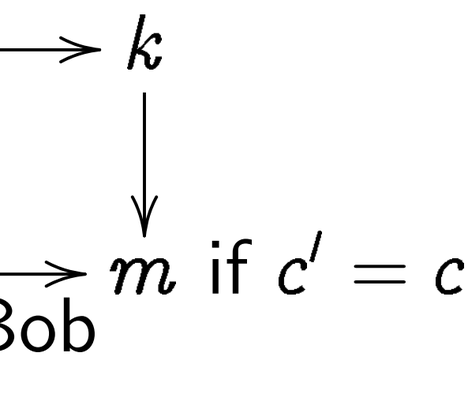

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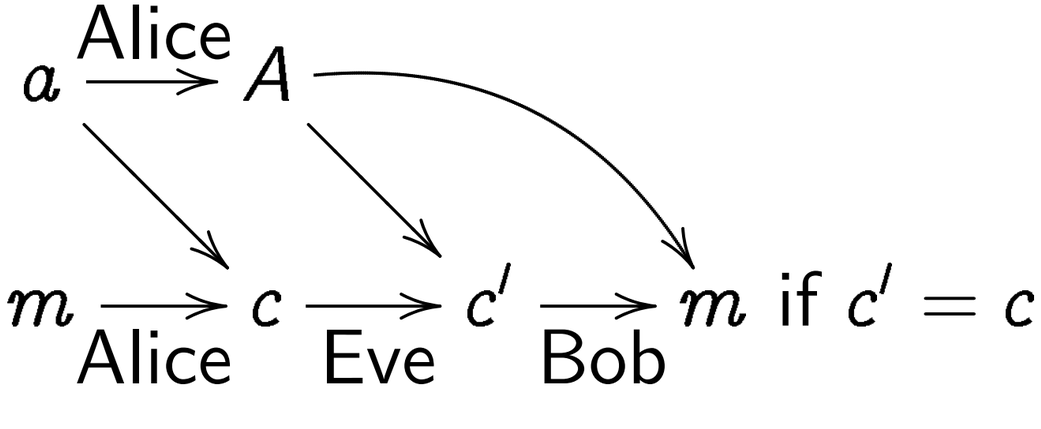

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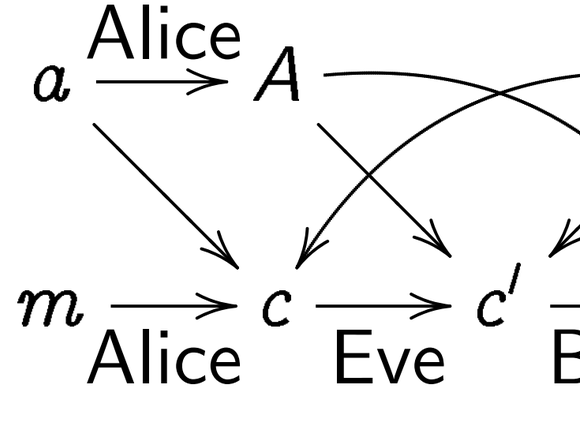

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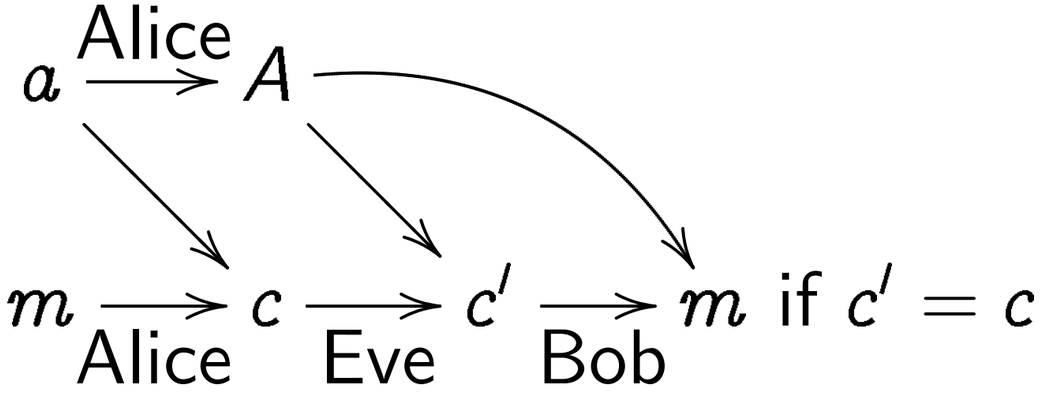

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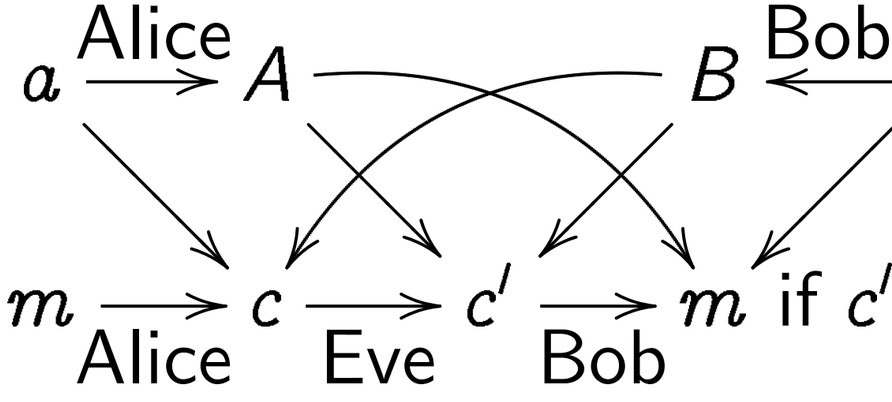

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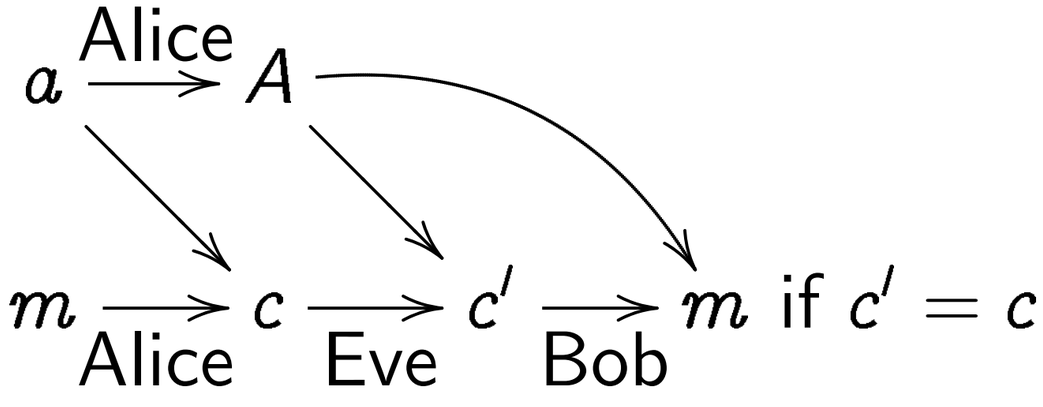

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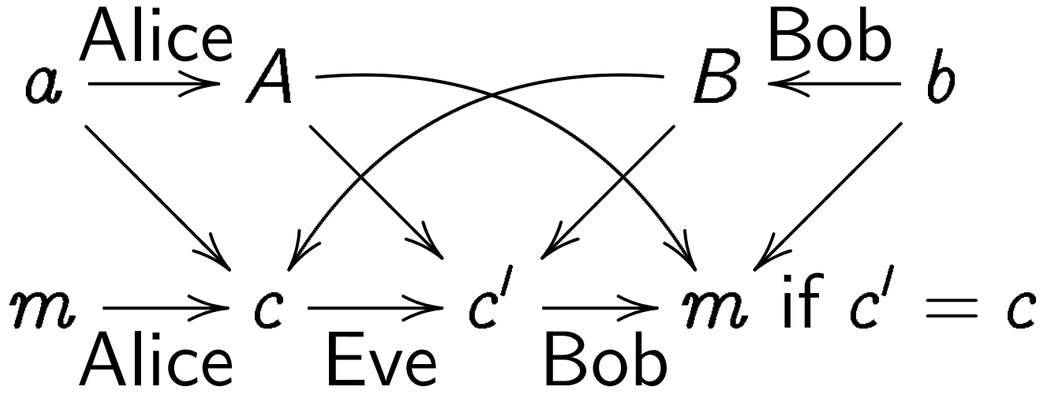

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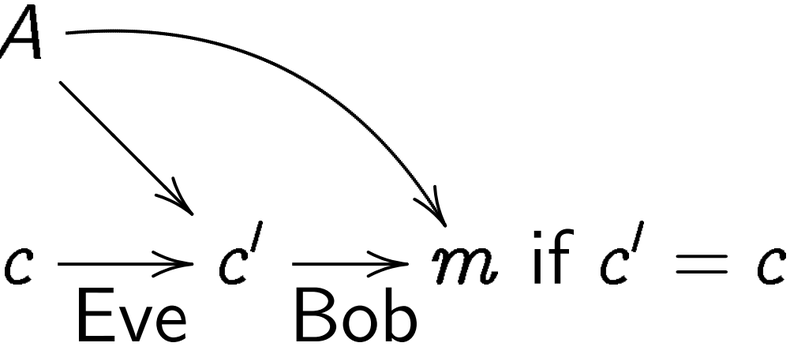

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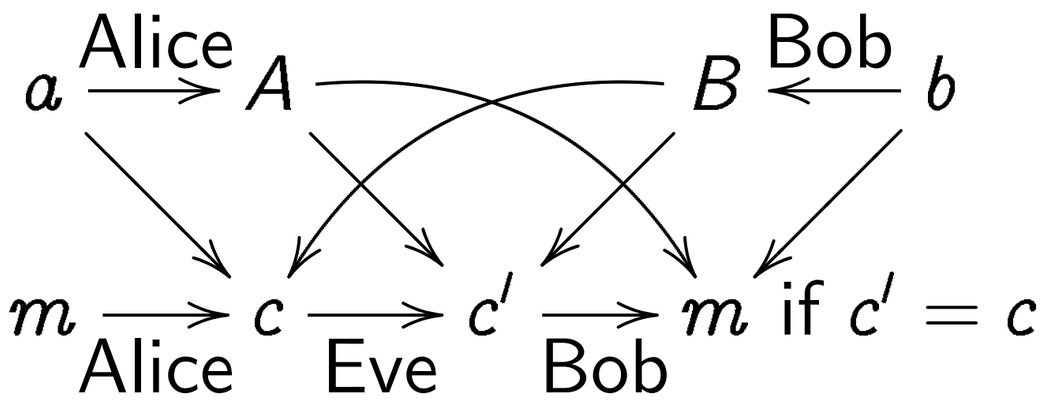

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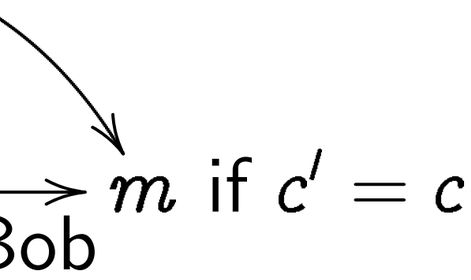

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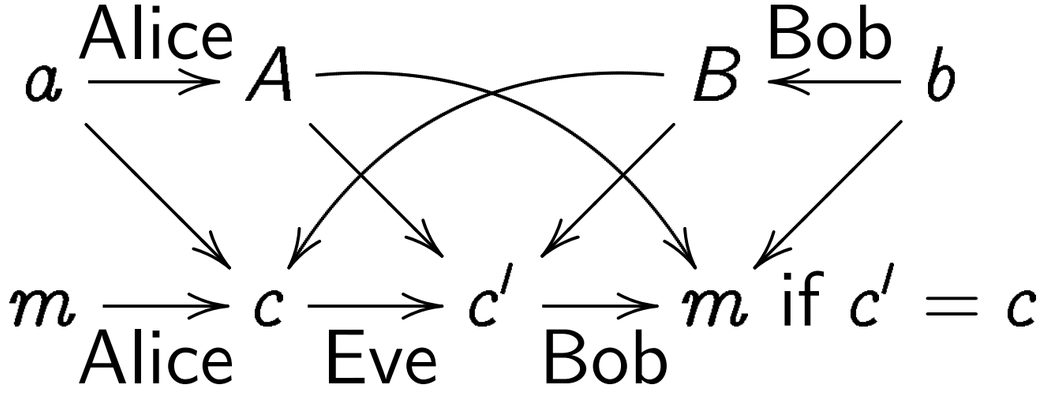

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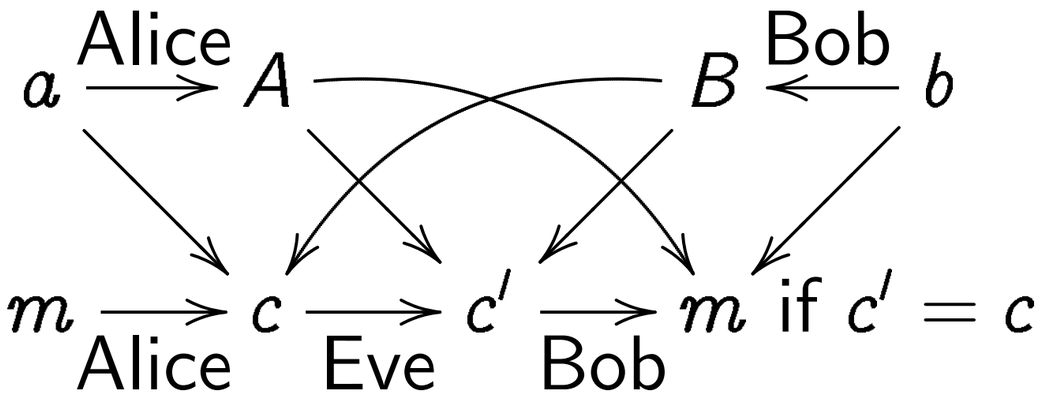

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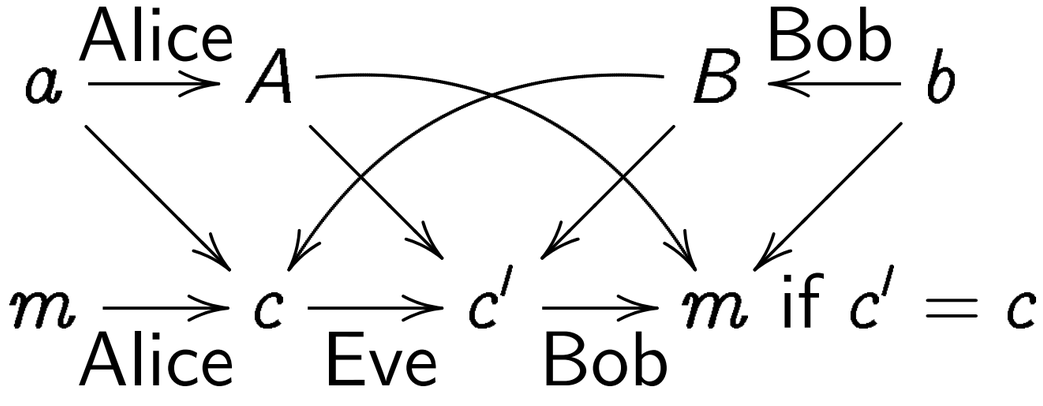

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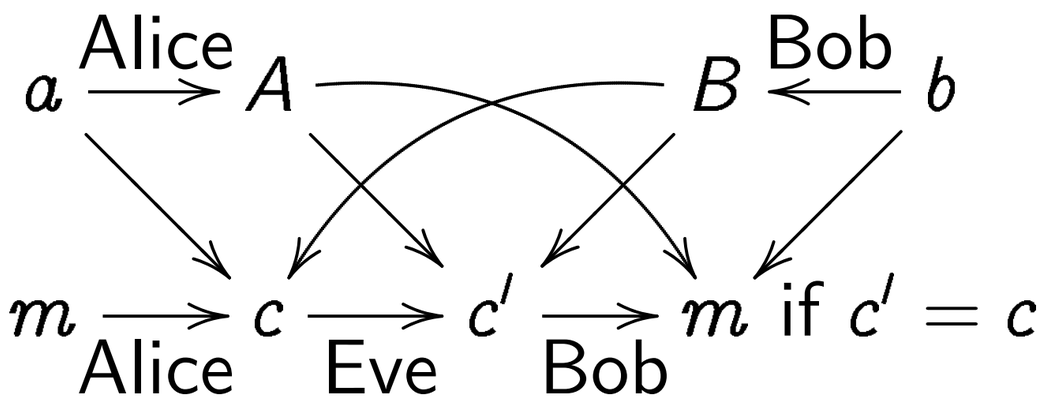

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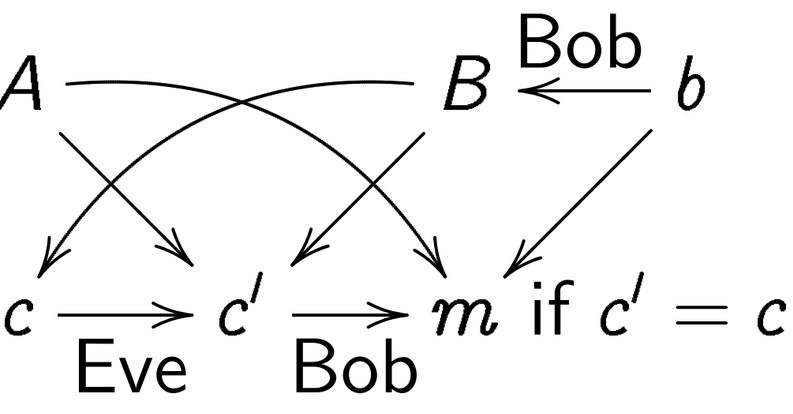

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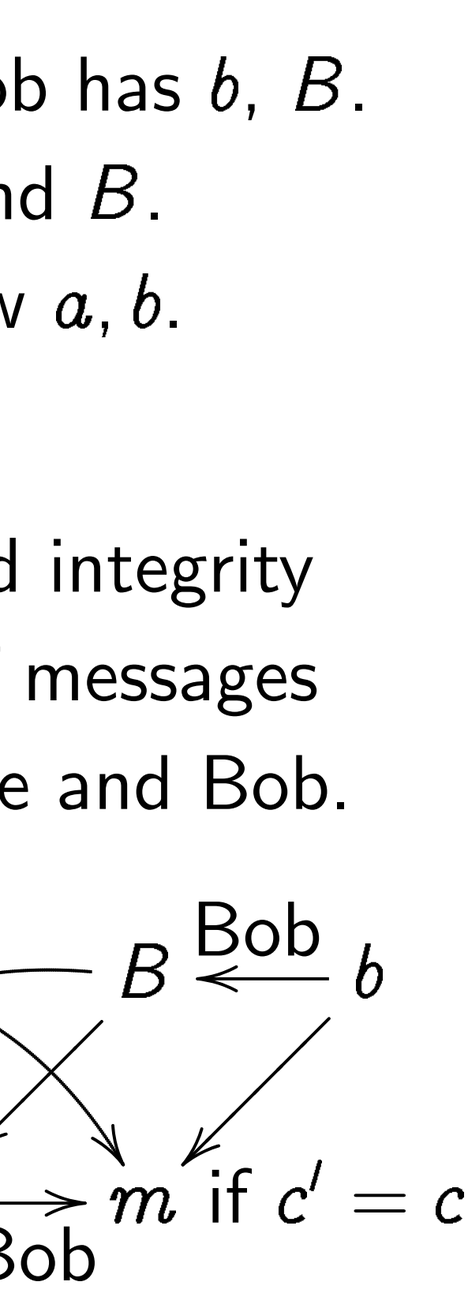

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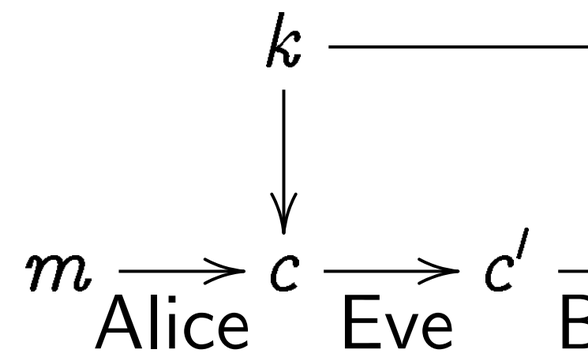

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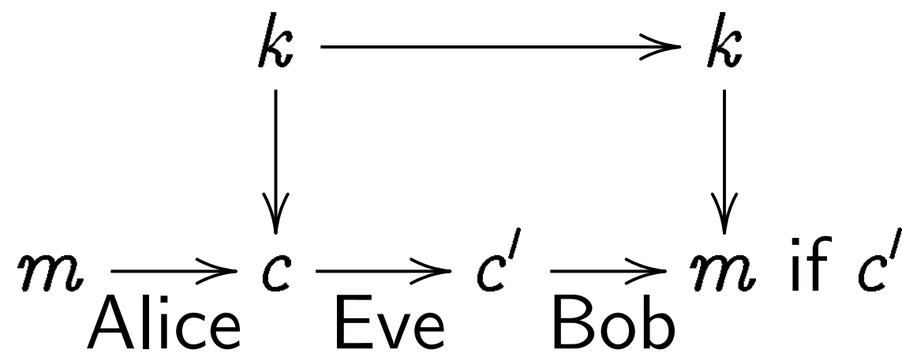

Very easy solutions if k is
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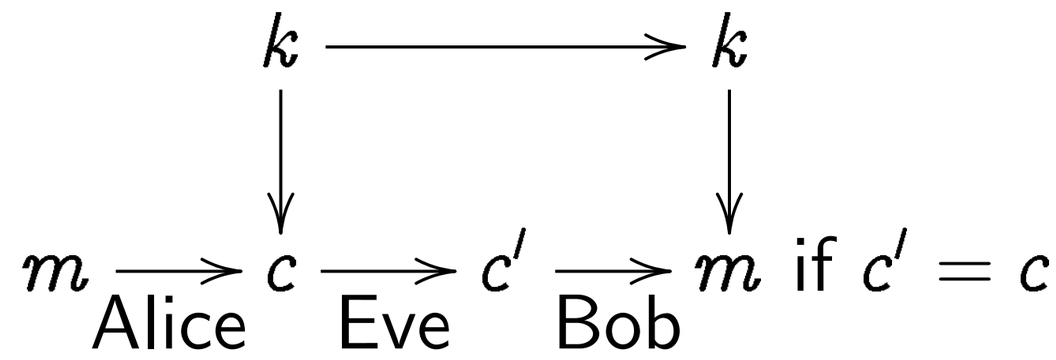

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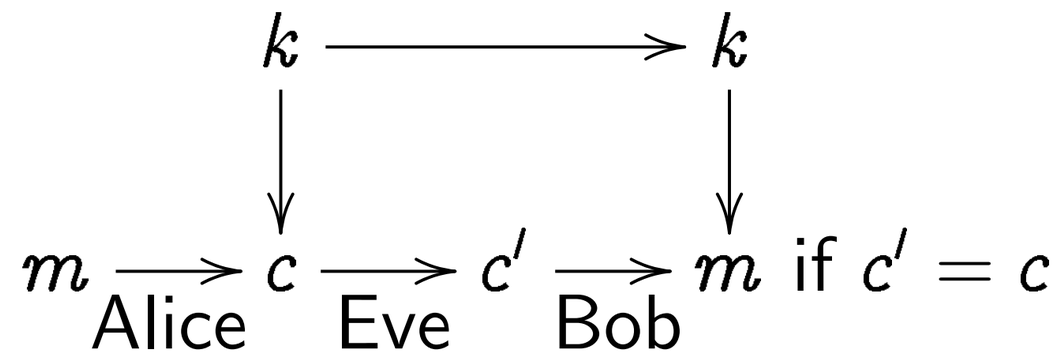

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Already standardized method
to expand short k into string
indistinguishable from long k:
1998 Daemen–Rijmen "Rijndael"
cipher ("AES") using 256-bit key.
Security analyzed in papers by
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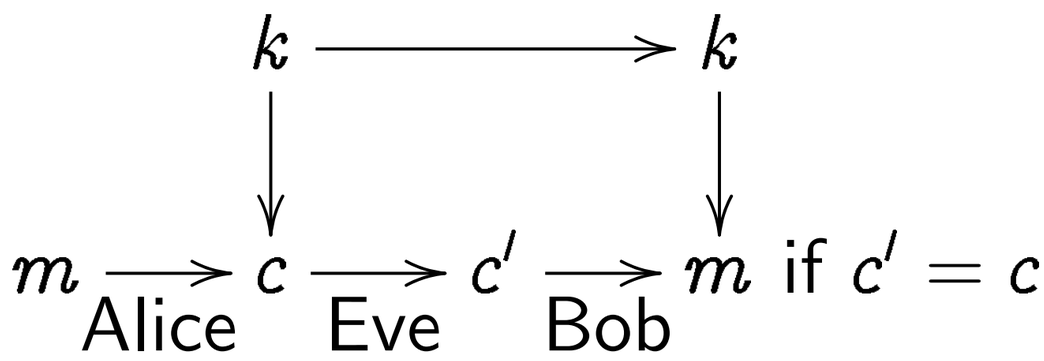
Post-quantum secret-key crypto



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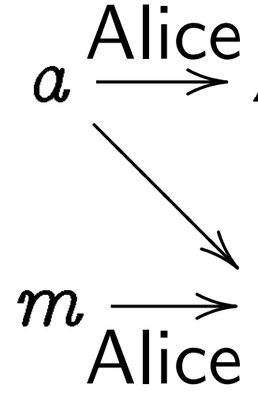

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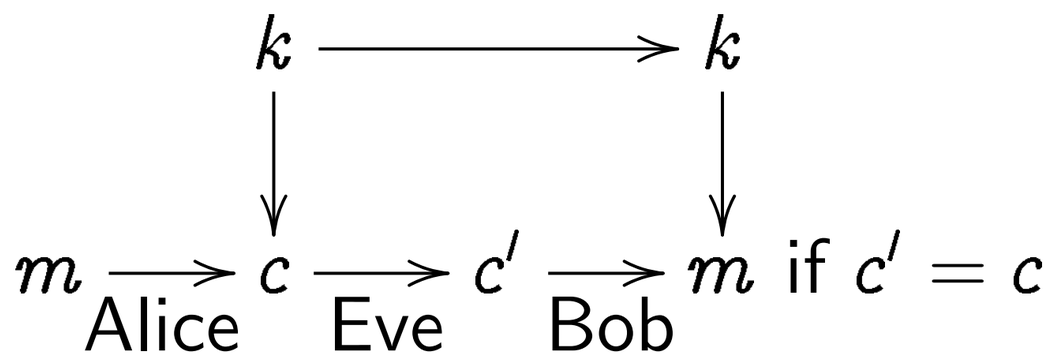

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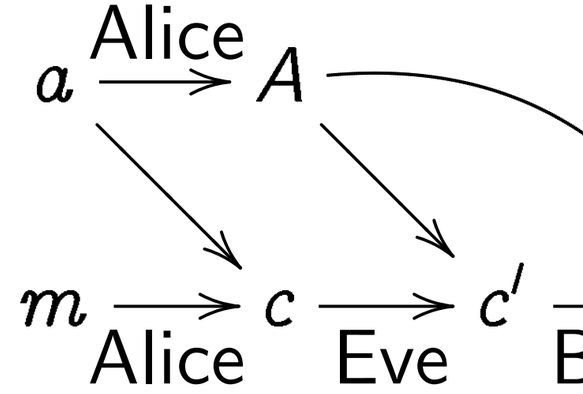

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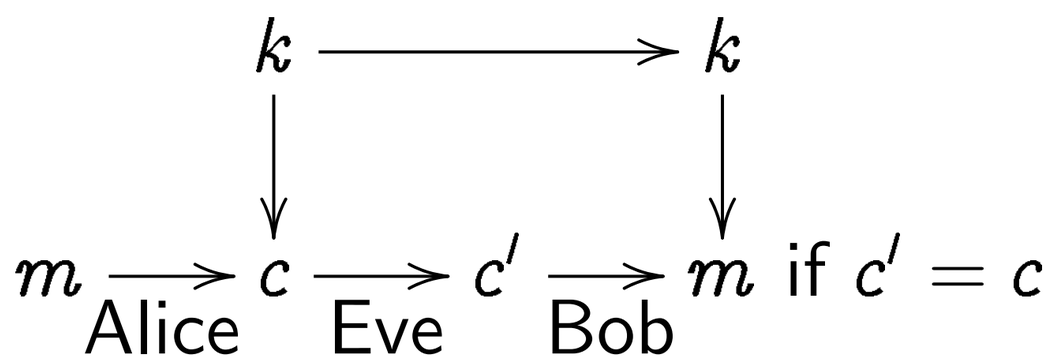

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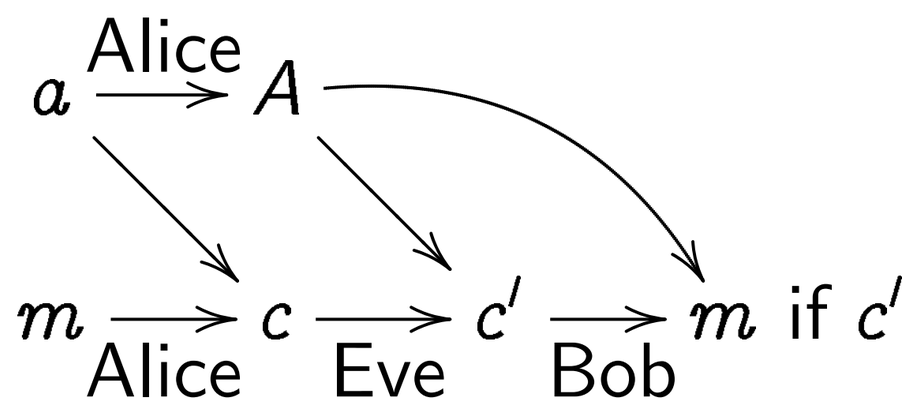

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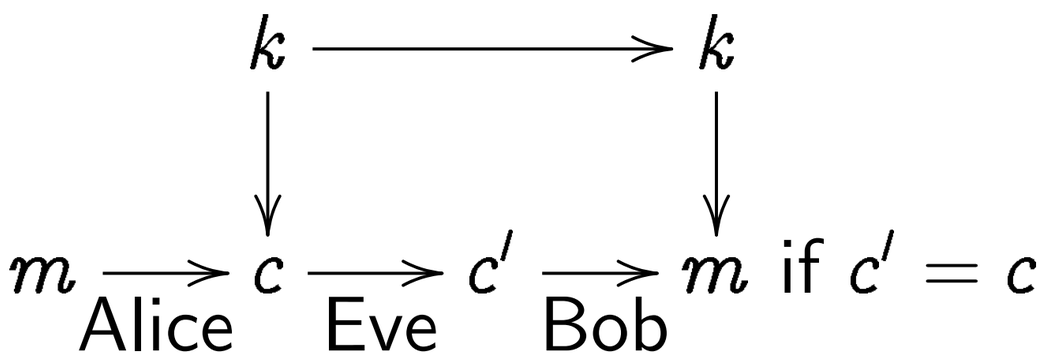

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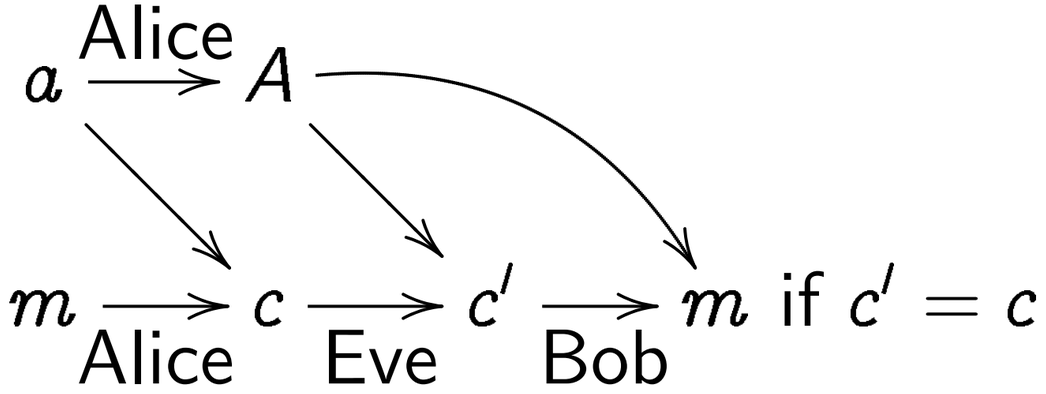

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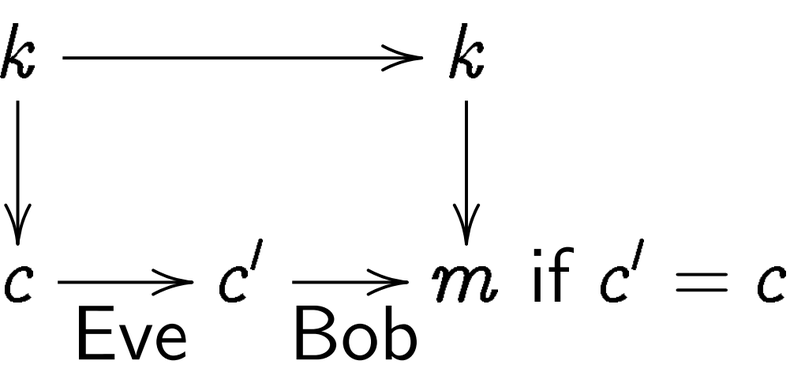

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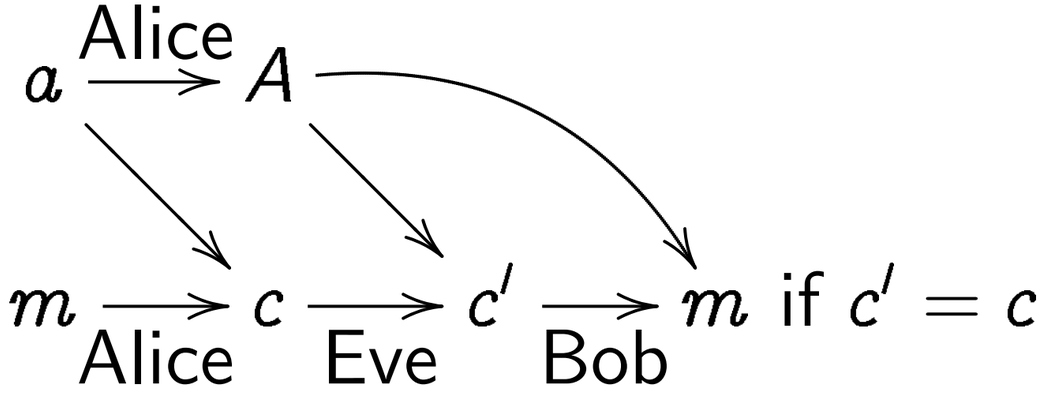

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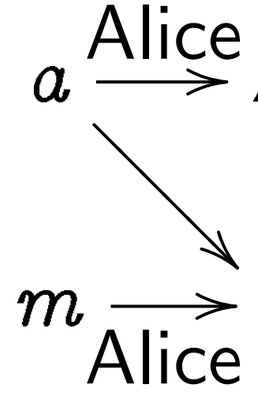

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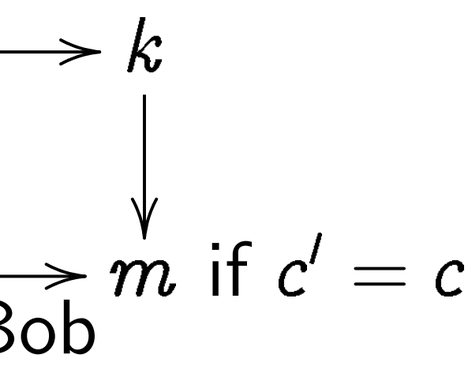

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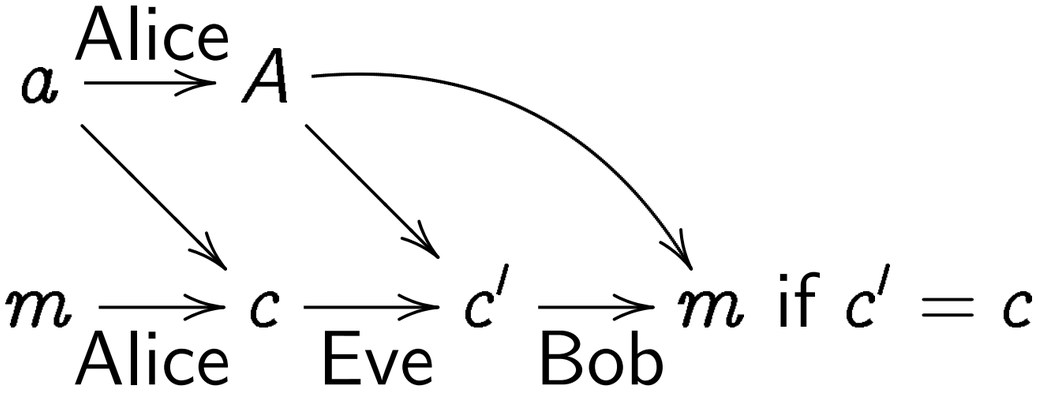

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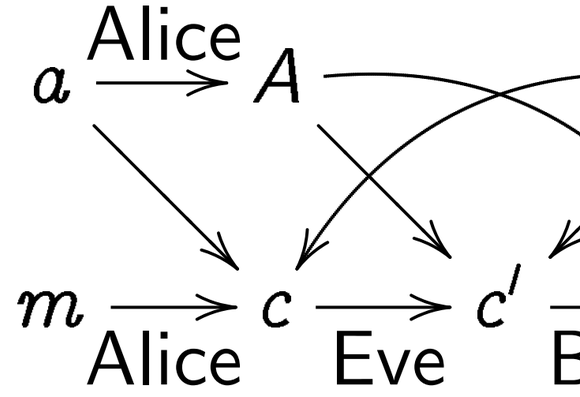

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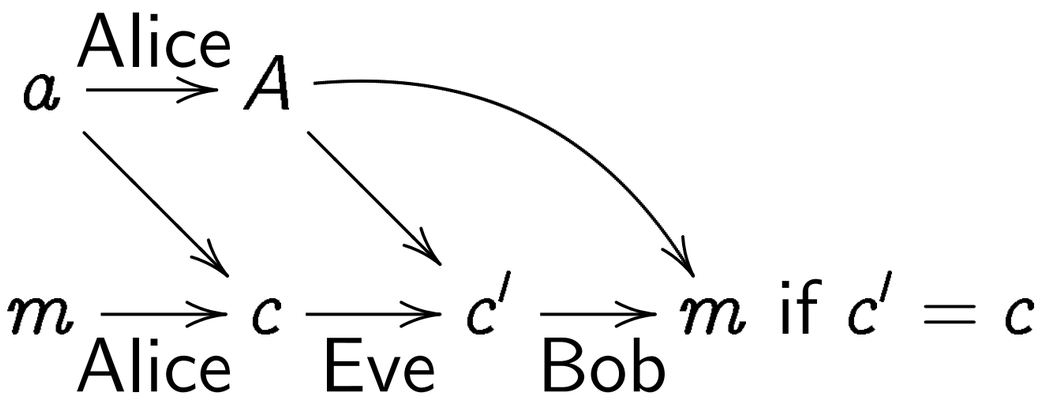

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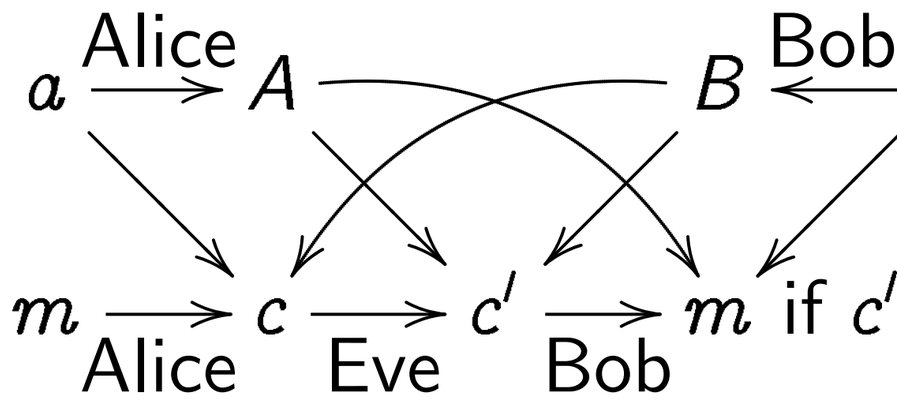

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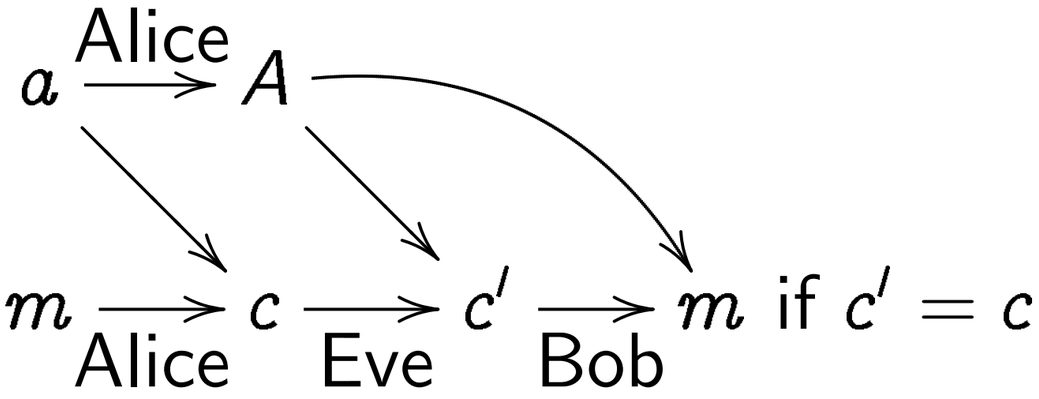

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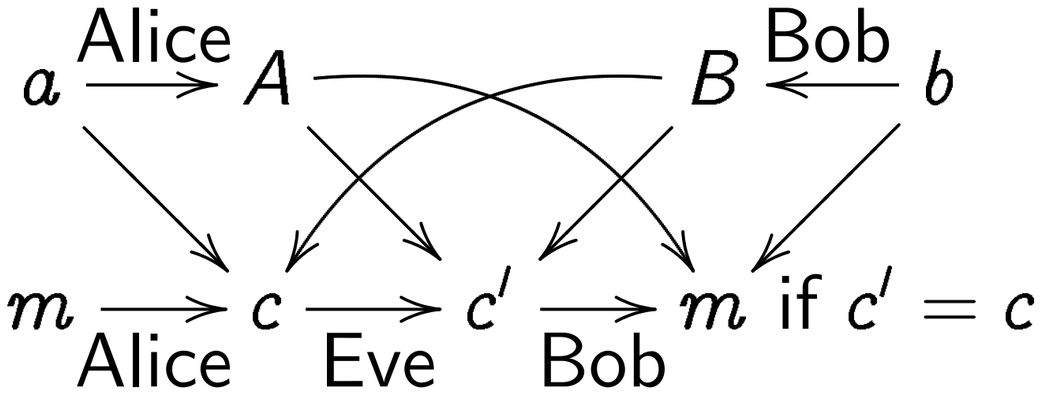

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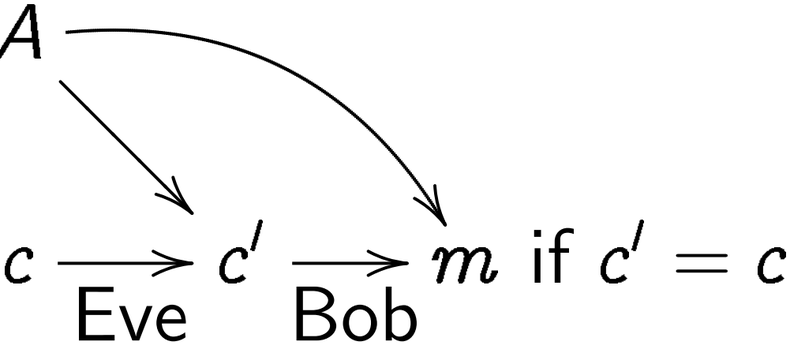

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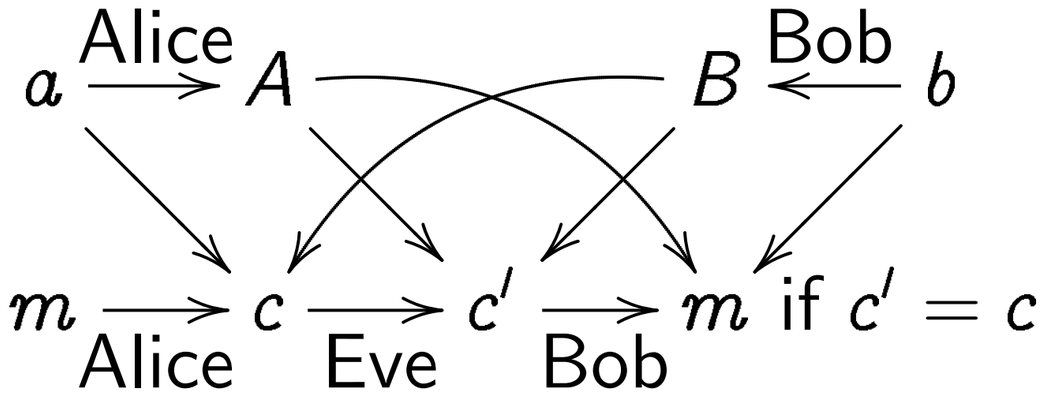

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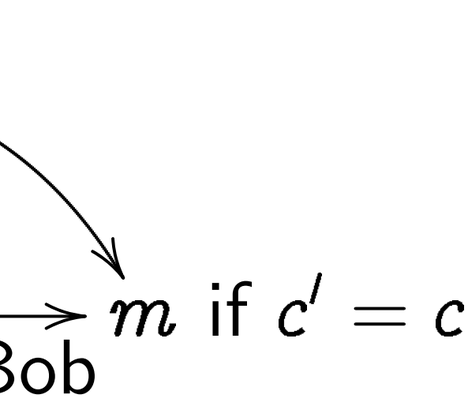

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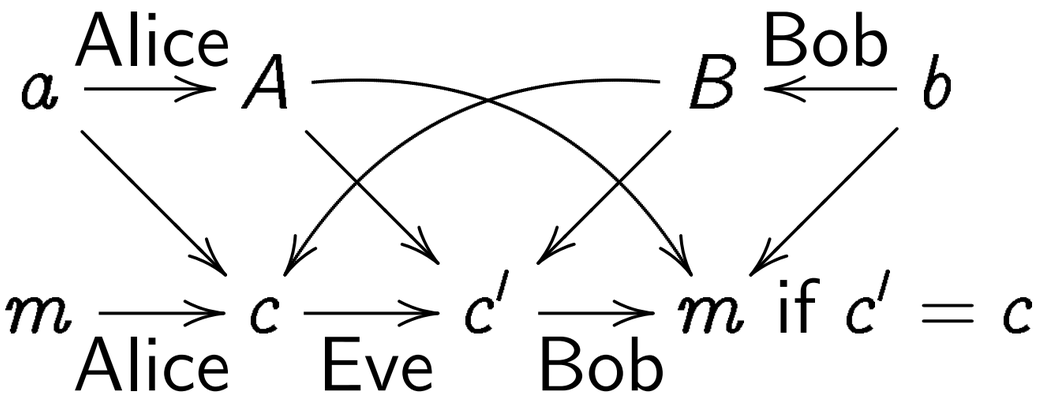

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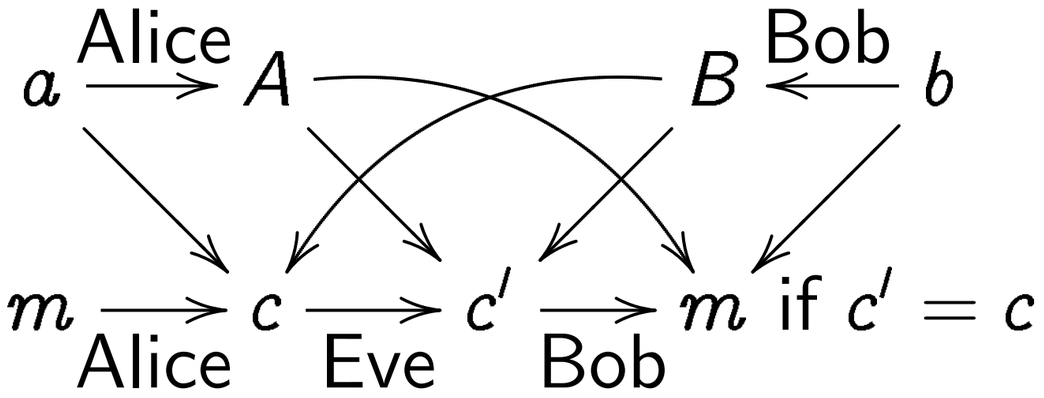

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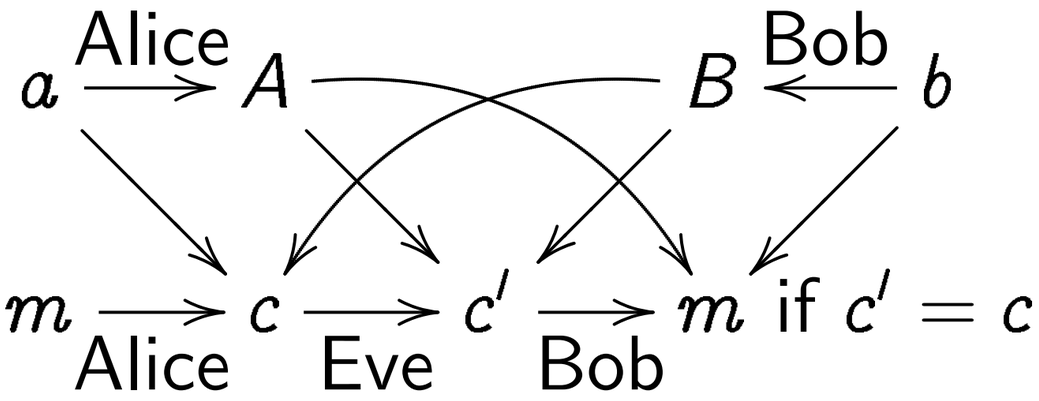

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