



# Improved measurement of the $b$ -quark mass at LEP

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## Abstract

An improved measurement of the  $b$ -quark mass  $m_b(M_Z)$  has been obtained using the data sets collected by the DELPHI experiment at the  $Z$  peak from 1994 to 2000. The normalised three-jet rate of  $b$ -quarks with respect to light  $\ell$ -quarks ( $\ell = u, d, s$ ),  $R_3^{b\ell}$ , defined using the DURHAM and CAMBRIDGE jet finding algorithms, has been measured and compared with the NLO massive calculation in order to extract  $m_b(M_Z)$ . The increase in statistics and better understanding of the experimental flavour tagging, of effects from the  $b$ -quark fragmentation and from gluon splitting rates into  $b$  and  $c$ -quark pairs lead to a reduction of both the statistical and systematic uncertainties. The overall uncertainty is now 385 MeV.

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# 1 Introduction

The radiation of gluons by quarks depends not only on the strong coupling constant but also on the mass of the emitting quark. The latter effect is analogous to that affecting photon emission by fermions and can reach sizeable effects for specific processes, for instance the production of three-jet and four-jet event in  $e^+e^-$  annihilations [1, 2].

For the case of three-jet final states in  $e^+e^-$  collisions, Next-to-Leading Order (NLO) calculations including massive terms have been computed, [3, 4, 5, 6], allowing for new experimental studies with reduced theoretical uncertainties. In fact, these mass effects have been observed at the  $Z^0$  peak in the production of three-jet events initiated by  $b$ -quarks in the LEP and SLC collaborations [7, 8, 9, 10, 11, 12]. The results obtained by these analyses have improved the precision of previous experimental tests on the universality of strong interactions [7, 9, 10, 11] and have enabled the measurement of the  $b$ -quark mass far from the  $b\bar{b}$  production threshold [7, 9, 11, 12].

The observable used by DELPHI to quantify these effects is,

$$R_3^{b\ell}(y_c) = \frac{[\Gamma_3(y_c)/\Gamma_{tot}]^{Z \rightarrow b\bar{b}}}{[\Gamma_3(y_c)/\Gamma_{tot}]^{Z \rightarrow \ell\bar{\ell}}} \quad (1)$$

for two jet finding algorithms, DURHAM [13] and CAMBRIDGE [14], being  $[\Gamma_3(y_c)/\Gamma_{tot}]^{Z \rightarrow b\bar{b}}$  and  $[\Gamma_3(y_c)/\Gamma_{tot}]^{Z \rightarrow \ell\bar{\ell}}$  the normalized three-jet cross sections for  $b$ - and  $\ell \equiv uds$ -quarks.

The previous DELPHI measurement of  $m_b(M_Z)$  used data collected at a center of mass energy  $\sqrt{s} \approx M_Z$  from 1992 to 1994 and jets were only reconstructed with DURHAM [7]. The result obtained was:

$$m_b(M_Z) = 2.67 \pm 0.25 \text{ (stat)} \pm 0.34 \text{ (had)} \pm 0.27 \text{ (theo)} \text{ GeV}/c^2 \quad (2)$$

In this paper a new analysis to measure  $m_b(M_Z)$  performed with data collected by DELPHI from 1994 to 2000 is presented. DURHAM and CAMBRIDGE algorithms are used to form jets. The CAMBRIDGE algorithm has the advantage of having a smaller theoretical uncertainty [6]. A detailed study of how mass effects and the hadronization process are implemented in the generators has been done. The effect of the  $g$  splitting rates,  $g \rightarrow b\bar{b}$  and  $g \rightarrow c\bar{c}$ , on the measurement has also been taken into account.

## 2 Experimental strategy

First of all, from the raw data collected by the DELPHI detector at the  $Z^0$  peak, a sample of  $\sim 1.4 \times 10^6$  hadronic  $Z^0$  decays was selected (the procedure described in [7] was used). The next step was to separate the  $b$  quark initiated events from those originated by  $\ell$  quarks.

As it will be shown in section 2.3, the hadronization process is better controlled if a cut on the  $b$  quark jet energies is done for  $b$  events,  $x_E^b(\text{jet}) \geq 0.55$ , where  $x_E^b(\text{jet}) = E_{b\text{-jet}}/E_{\text{beam}}$ . A method to distinguish the  $b$  quark jets from the gluon jet was then used.

The measured  $R_3^{b\ell}$  observable was then corrected for detector and acceptance effects, kinematic bias introduced in the tagging procedure and the hadronization process to bring it to parton level (see section 2.2). Finally, a comparison with the NLO massive calculations was done and  $m_b(M_Z)$  extracted.

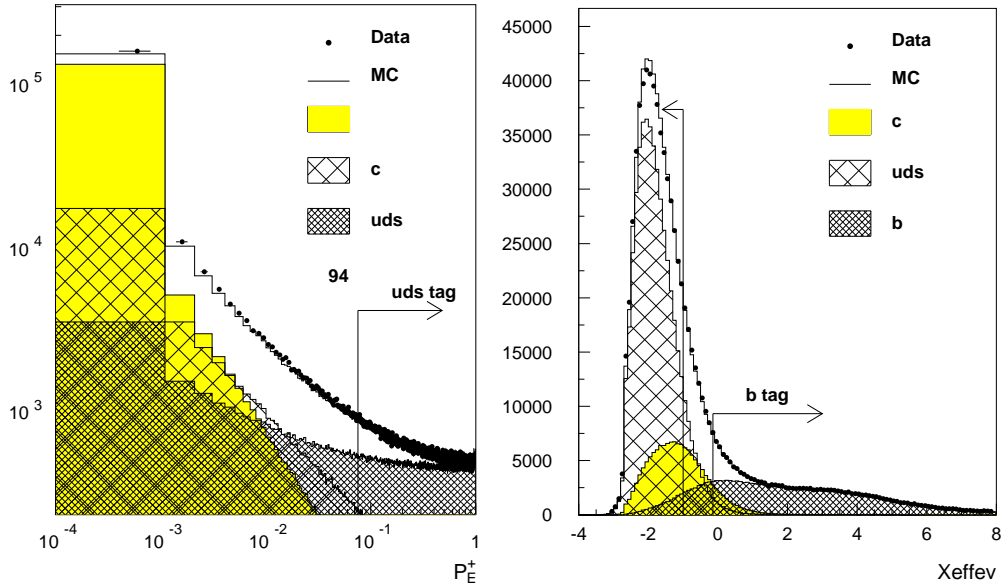


Figure 1: Event distribution of  $P_E^+$  (left) and  $X_{effev}$  (right). The real (points) and simulated (histogram) data are compared. The specific contribution of each quark flavour is displayed as derived from the DELPHI simulated data. The cuts used to tag the  $b$ -quark and  $\ell$ -quark samples are also indicated.

## 2.1 Flavour tag

Two algorithms to select  $\ell$  and  $b$ -quark initiated events are available in DELPHI: the lifetime-signed impact parameter of charged particles in the event [15] and the combined tagging technique [16].

The former method discriminates the flavour of the event by calculating the probability,  $P_E^+$ , of having all particles compatible with being generated in the event interaction point (Impact Parameter method). For the second technique, an optimal combination of a set of discriminating variables defined for each reconstructed jet was performed, leading to a single variable per event,  $X_{effev}$  (Combined method).

The  $P_E^+$  and the  $X_{effev}$  distributions obtained are presented in figure 1, where the contribution of each flavour for the simulated data is also shown.

Regarding the stability of the final result (see figure 2 left), the Combined technique was used to select  $b$  events by requiring  $X_{effev} > -0.15$  ( $P_b = 86\%$ ) while the Impact Parameter method was used for  $\ell$  tagging by  $P_E^+ > 0.07$  ( $P_\ell = 82\%$ ). For  $b$  tagging both techniques were observed to be equally stable and the Combined method was chosen because higher purities could be reached with the same efficiency. Table 1 shows the purities and contaminations obtained with these cuts for both tagged samples.

In order to perform the cut on the  $b$  quark energies an identification of the gluon and  $b$  quark jets is required for the  $b$  tagged events. The two tagging techniques can also provide a discriminative variable per jet. Again, based on an stability argument (see figure 2 right), the Combined technique was used to identify the pair of jets most likely to come from  $b$  quarks, with a jet purity of 81% and an efficiency of 90%.

Figure 3 shows the  $x_E^b(\text{jet})$  distribution for real and simulated data. The cut  $x_E^b(\text{jet}) \geq 0.55$  was then applied for both  $b$  jets.

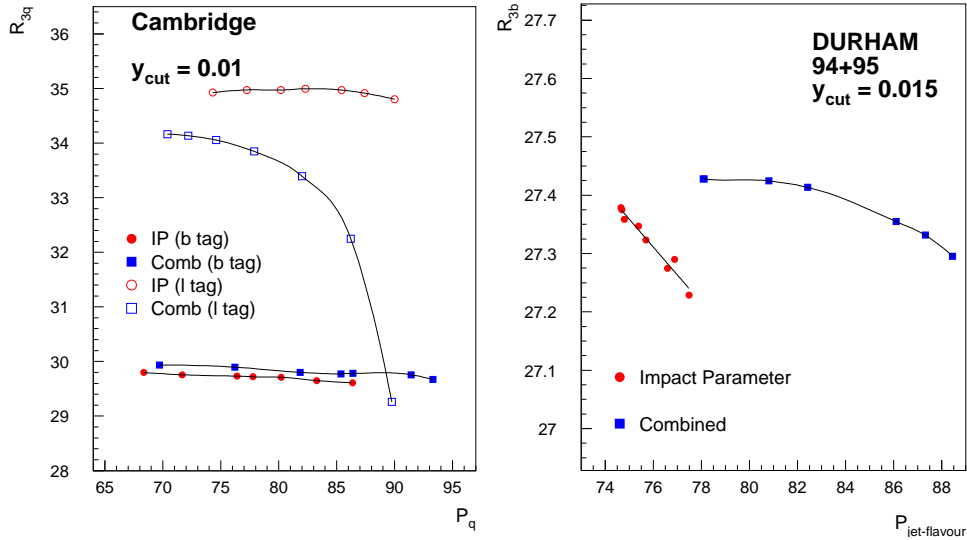


Figure 2: (Left)  $R_{3q}$  at parton level as a function of the purity of the  $q$  tagged sample,  $P_q$ , for  $q = b, \ell$ , when CAMBRIDGE is used to form jets. (Right)  $R_{3b}$  as a function of the jet identification purity at a  $y_c = 0.015$  for the DURHAM algorithm.

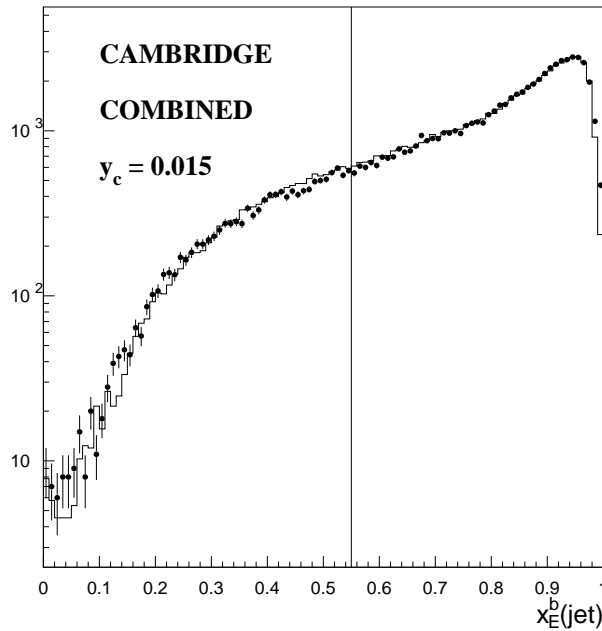


Figure 3:  $x_E^b(\text{jet})$  distribution for real and simulated data for three jets  $b$  tagged events at a  $y_c = 0.015$  for CAMBRIDGE. The simulated data was obtained by generating events with Jetset 7.3 and passing through the DELPHI detector simulation.

| Method           | Type $q$ | $l \rightarrow q$ (%) | $c \rightarrow q$ (%) | $b \rightarrow q$ (%) |
|------------------|----------|-----------------------|-----------------------|-----------------------|
| Impact Parameter | $\ell$   | 82                    | 15                    | 3                     |
| Combined         | $b$      | 4                     | 10                    | 86                    |

Table 1: Flavour compositions of the samples tagged as  $\ell$ -quark and  $b$ -quark events.

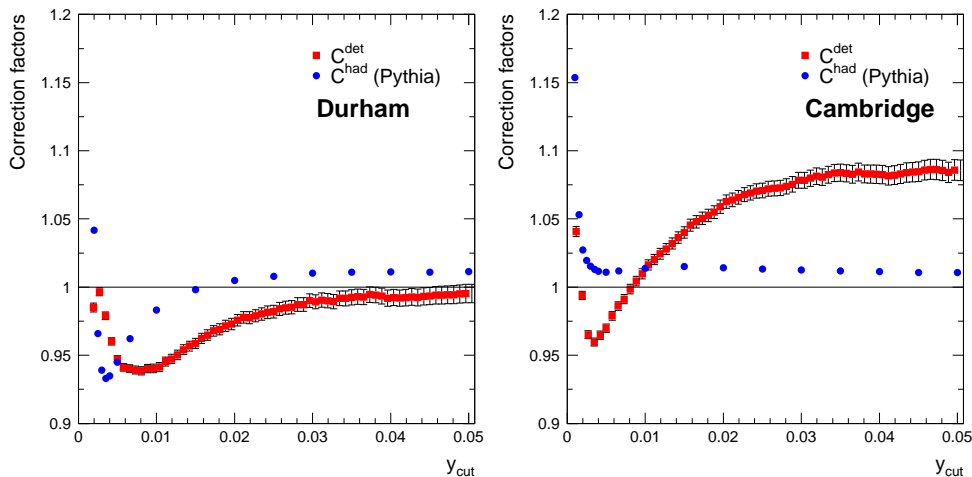


Figure 4: Detector and hadronization corrections applied to the measured  $R_3^{b\ell}$  for DURHAM and CAMBRIDGE.

## 2.2 Correction procedure

Once the observable was measured at detector level, a correction procedure was used to bring the observable at parton level to compare with the theoretical calculations.

The correction expression used was described in detail in [7]. The complete DELPHI simulation (DELSIM), which uses JETSET 7.3 [17] to generate the events that go through the detector simulation, was used to correct for detector effects. A recent version of JETSET, PYTHIA 6.131 [18], previously tuned to DELPHI data [19], was used to get the hadronization factors.

Figure 4 shows the size of the detector and hadronization corrections for  $R_3^{b\ell}$ . For instance, the detector correction is of about 2% for both algorithms while the fragmentation correction is about 5‰ for DURHAM and 1‰ for CAMBRIDGE at a  $y_c = 0.02$  and  $y_c = 0.01$  respectively.

Further correction factors were applied in order to correct for the gluon splitting rates of the DELSIM simulation and the hadronization model and mass parameter used in PYTHIA.

- Gluon splitting: In events containing a  $g \rightarrow b\bar{b}$  or  $g \rightarrow c\bar{c}$ , the quarks emerging from the gluon have an impact on the tagging procedure. A reweighting of the DELSIM simulation was then needed in order to have the measured rates [21]:

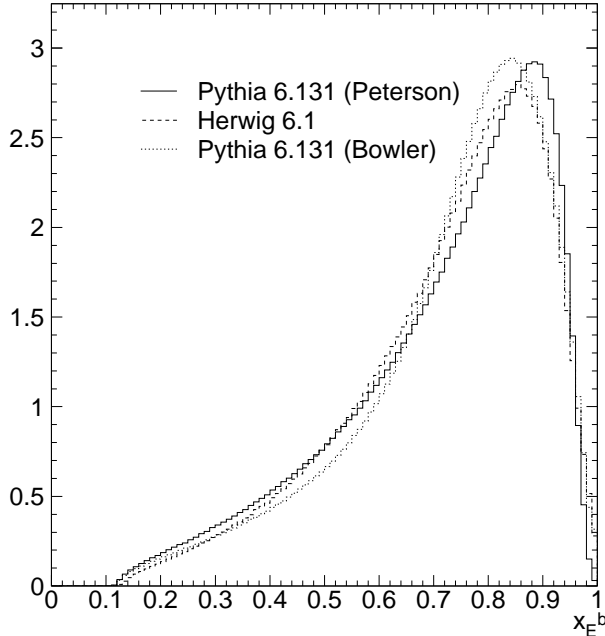


Figure 5:  $x_E^b$  distribution for different Monte Carlo generators.

$$\begin{aligned}
 g_{c\bar{c}} &= 0.0296 \pm 0.0038 \\
 g_{b\bar{b}} &= 0.00254 \pm 0.00051
 \end{aligned}
 \tag{3}$$

which means a reweighting factor of 1.91 for the  $g_{c\bar{c}}$  events and of 1.64 for the  $g_{b\bar{b}}$  ones.

The difference on the measured observable between switching on and off the gluon splitting rates in the generator is of about 1% for DURHAM at a  $y_c = 0.02$  and of 6‰ for CAMBRIDGE at a  $y_c = 0.01$ .

- **Hadronization model:** Three hadronization models were considered, the cluster one implemented in HERWIG [22] and two other models based on the string fragmentation of PYTHIA but with different heavy quark fragmentation functions, Peterson [23] and Bowler [24]. Figure 5 shows the  $b$  fragmentation function of the three models. The two generators used, PYTHIA 6.131 and HERWIG 6.1, were previously tuned to DELPHI data.

Since the three models describe the data reasonable well and none of them perfectly [20], the mean of the three fragmentation corrections was used to correct the data.

- **The mass used in the MC:** In the version of PYTHIA used to correct for the hadronization process, the kinematical  $b$  quark mass,  $M_b$ , is set to  $5 \text{ GeV}/c^2$ .

In order to describe mass effects, the PYTHIA generator uses three sets of quark masses, the kinematical,  $M_b$ , used at parton level, the constituent,  $M_b^{const}$ , used during the hadronization process and to derive not yet found B hadrons masses, and the known B hadron masses. These three sets of masses are not connected to

each other. Therefore, this situation is not appropriate to study mass effects because the changes made for parton masses don't propagate as they should during the hadronization process. If this connection is physically imposed in the generator the hadronization correction is independent of the  $b$  quark mass value.

The kinematical  $b$  quark mass used in PYTHIA at parton level can be identified as the pole mass with a precision of about  $\Lambda_{\text{QCD}}$  [26]. This means that  $M_b$  can be set to  $4.9 \pm 0.2 \text{ GeV}/c^2$ .

Therefore, a correction to go from a mass of  $M_b = 5 \text{ GeV}/c^2$  to  $4.9 \text{ GeV}/c^2$  was applied, having the effect of increasing the measured  $R_3^{b\ell}$  in about 2-3‰.

## 2.3 Theoretical and systematic uncertainties

Different sources of theoretical and systematic uncertainties have been considered:

- Theoretical: The theoretical uncertainty, evaluated from the  $\mu$  scale dependence and the mass ambiguity [6, 7], obtained for  $m_b(M_Z)$  from the NLO calculations was found to be smaller for CAMBRIDGE, 60 MeV, than for DURHAM, 280 MeV.
- Hadronization: The error due to the hadronization procedure has the following contributions:
  - Tuning: Uncertainty on the tuned parameters of PYTHIA that are relevant in the fragmentation process. This contribution was evaluated by varying these parameters ( $\Lambda_{\text{QCD}}, Q_0, \sigma_q, \epsilon_b, a$ ) within  $\pm\sigma$  from their central tuned values and taking into account that they are not completely independent [19].
  - Hadronisation model: It was calculated as the standard deviation of the three hadronization corrections used. Figure 6 shows this uncertainty as a function of the  $y_c$  for the present case in which a cut on  $x_E^b(\text{jet})$  is done compared to the case in which no cut is made.
  - PYTHIA mass parameter ambiguity: The uncertainty on the mass parameter used in the generator,  $M_b = 4.9 \pm 0.2 \text{ GeV}/c^2$ , was propagated to the effect on the hadronization correction.
- Physics and detector modelling:
  - Gluon splitting: The gluon splitting rates of equation 3 were varied within twice their uncertainty and the effect on the measured observable was taken as the gluon splitting error.
  - Tagging: It has been evaluated by varying the purities and contamination factors of the correction formulae [7] within their uncertainty. The errors on  $P_b$  and  $P_\ell$  were obtained from the known relative error of the  $b$  tagging and mistagging efficiencies,  $\Delta\epsilon_b^b/\epsilon_b^b = 3\%$  and  $\Delta\epsilon_\ell^b/\epsilon_\ell^b = \Delta\epsilon_c^b/\epsilon_c^b = 8\%$  [27], taking into account that  $\ell$  tagging is really anti- $b$  tagging, ie.  $\Delta\epsilon_q^\ell = \Delta\epsilon_q^b$  for  $q = b, c, \ell$  for the same cut value.
  - Jet identification: It was evaluated by varying the cuts applied to distinguish the  $b$  quark jets from the gluon jet in a  $b$  tagged event within efficiencies of 80% to 100%.

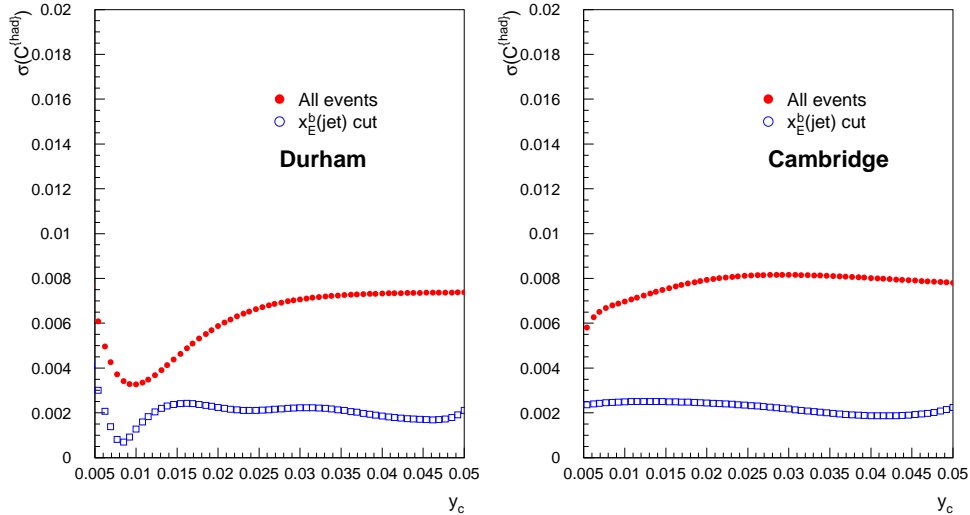


Figure 6: Hadronization model uncertainty as a function of the  $y_c$  for the DURHAM (left) and CAMBRIDGE (right).

### 3 Final results and discussion

After all the corrections described in 2.2, the final  $R_3^{bl}$  obtained at parton level, as a function of the  $y_c$ , is shown in figure 7. The LO and NLO theoretical curves in terms of the pole and the running mass are also shown in the plot, for  $M_b = 4.8 \text{ GeV}/c^2$  and  $m_b(M_Z) = 3 \text{ GeV}/c^2$ .

The final result was obtained by averaging the results of each separate year since, as figure 8 shows for a given  $y_c$ , all of them give compatible results.

The jet resolution parameters of  $y_c = 0.02$  and  $y_c = 0.009$  were chosen to give the final result for DURHAM and CAMBRIDGE respectively, so to keep the correction, the total uncertainty and the four jet rate small (figure 9 shows the the total error on  $m_b(M_Z)$  and the statistical, systematic and theoretical contributions as a function of  $y_c$ ).

Table 2 shows, for the jet resolution parameters at which the two measurements are performed, the results obtained for  $R_3^{bl}$  and the extracted  $m_b(M_Z)$ , as well as the contributions of all different sources of errors.

The results obtained by this analysis for  $m_b(M_Z)$  are then,

$$m_b(M_Z) = 2.99_{-0.27}^{+0.25} \text{ (stat)}_{-0.37}^{+0.34} \text{ (syst)} \pm 0.28 \text{ (theo)} \text{ GeV}/c^2 \quad (4)$$

when DURHAM is used to reconstruct jets and,

$$m_b(M_Z) = 2.82 \pm 0.19 \text{ (stat)}_{-0.33}^{+0.31} \text{ (syst)} \pm 0.06 \text{ (theo)} \text{ GeV}/c^2 \quad (5)$$

when CAMBRIDGE is the algorithm used.

Both results are compatible within their error. The measurement performed with CAMBRIDGE is more precise than the one obtained with DURHAM, with a total error of  $^{+370}_{-390} \text{ MeV}$  instead of  $^{+500}_{-540} \text{ MeV}$ .

Apart from  $m_b(M_Z)$ , the NLO calculations in terms of the pole mass can be used to get a direct value of  $M_b$ . All these values are shown in tables 3 and 4 for DURHAM and



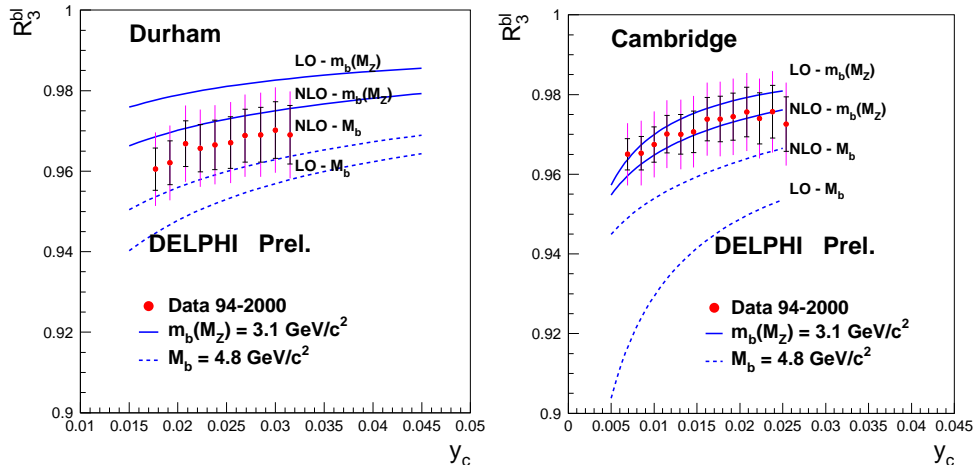


Figure 7: Final  $R_3^{b\ell}$  as a function of  $y_c$  obtained at parton level compared with the LO and NLO theoretical predictions calculated in terms of a pole mass of  $M_b = 4.8 \text{ GeV}/c^2$  and in terms of a running mass of  $m_b(M_Z) = 3 \text{ GeV}/c^2$ .

CAMBRIDGE respectively. The mass obtained by fitting to the LO calculations is also shown in the tables.

The result can also be interpreted as a test of flavour independence of the strong coupling constant by using the relation introduced in [7]. For the case of CAMBRIDGE, neglecting the  $\mathcal{O}(\alpha_s)$  terms and taking as an input the average  $b$  quark mass from low energy measurements [28],  $m_b(m_b) = 4.23 \pm 0.07 \text{ GeV}/c^2$ , we get,

$$\alpha_s^b/\alpha_s^\ell = 1.001 \pm 0.004(\text{stat}) \pm 0.007(\text{syst}) \pm 0.002(\text{theo}) \quad (6)$$

which verifies  $\alpha_s$  universality with a precision of less than 1%.

The  $R_3^{b\ell}$  was also measured at hadron level in order to compare with the PYTHIA and HERWIG generators and see how well mass effects are modelled. Figure 10 shows, as a function of the  $y_c$ , the measurement obtained together with the curves predicted for the two generators. For large values of  $y_c$ , both Monte Carlo describe the data while for small values HERWIG is better.

As a summary, a new measurement of the  $b$  quark mass has been done using  $Z^0$  data collected by DELPHI from 1994 until 2000. The same observable than in the previous measurement [7] has been used and the total uncertainty is now  $^{+500}_{-540} \text{ MeV}$  for the case of DURHAM. A precision of  $^{+370}_{-390} \text{ MeV}$  has been reached when CAMBRIDGE is used to define the jets.

The present measurement has been performed in a restricted region of the phase space to have a better control of the fragmentation process. In order to compensate for the loss of statistics, data from 1994 until 2000 has been analysed. New versions of the generators, PYTHIA 6.131 and HERWIG 6.1, where mass effects are much better reproduced, have been used to correct the data. A better understanding of the way mass effects are implemented in the generators have led to a more reliable hadronization correction. The error due to the mass parameter ambiguity in the generators is the dominant contributions to the total error for both jet algorithms. An investigation towards understanding the error associated to the  $b$  quark mass used in PYTHIA is being performed.

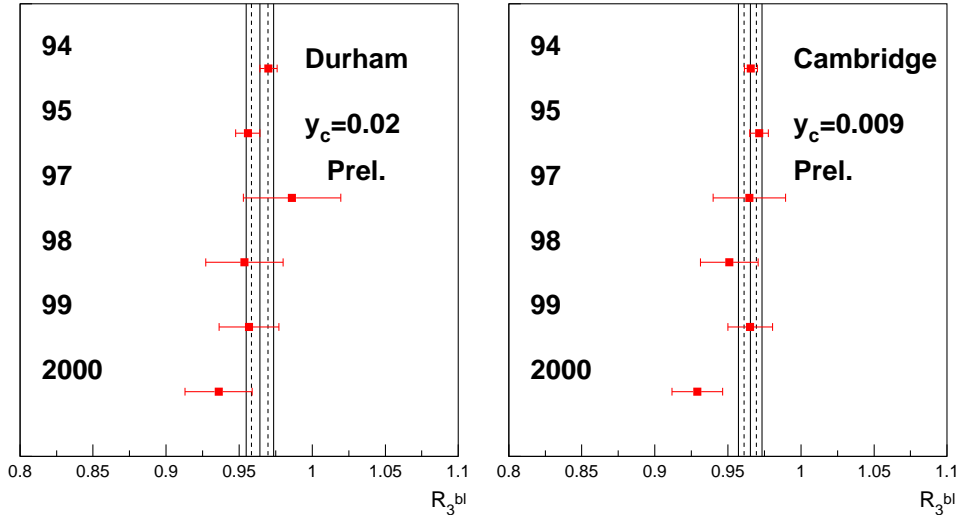


Figure 8:  $R_3^{bl}$  at parton level obtained for each analysed year for a fixed  $y_c$  for DURHAM (left) and CAMBRIDGE (right). The error bars represent the statistical error. The vertical lines show the average value with its statistical and total error.

The effect of the gluon splitting rates of the Monte Carlo on the  $b$  and  $\ell$  tagging were also taken into account in the final result. For the DURHAM algorithm, the correction and the error induced on the measurement by this effect are sizeable.

The present result of DELPHI is compatible with the measurements performed at low energy [28] and with the ones done at  $M_Z$  at the other LEP experiments and at SLC, as shown in figure 11.

A net change in the value of the running  $b$ -quark mass between the scales  $\mu_1 = M_Z$  and  $\mu_2 = M_{\Upsilon/2}$  is observed with about 3.5 standard deviations when CAMBRIDGE is used,  $m_b(M_{\Upsilon/2}) - m_b(M_Z) = 1.31^{+0.37}_{-0.40} \text{ GeV}/c^2$ . Universality of the strong coupling constant has also been verified at the level of less than 1%.

## 4 Acknowledgements

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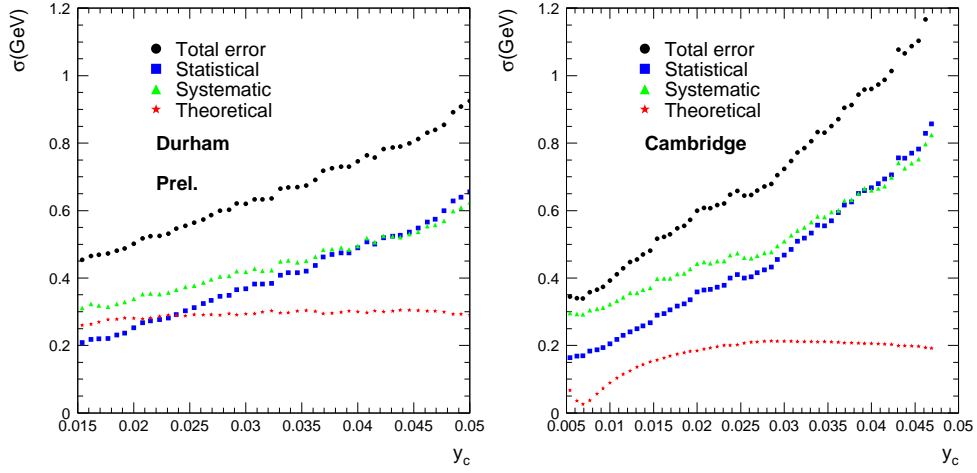


Figure 9: Total, statistical, systematic and theoretical errors on  $m_b(M_Z)$  as a function of  $y_c$  for DURHAM (left) and CAMBRIDGE (right) algorithms.

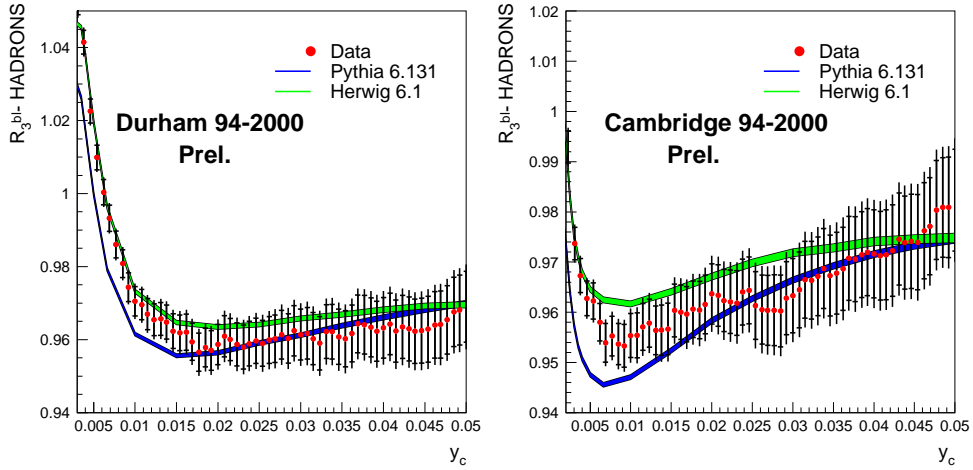


Figure 10: Final  $R_3^{bl}$  as a function of  $y_c$  obtained at hadron level compared with the PYTHIA 6.131 and HERWIG 6.1 predictions.

| DURHAM (94-2000)                                 | $R_3^{bl}(y_c = 0.02)$  | $m_b(M_Z)$ GeV/c <sup>2</sup> |
|--|-------------------------|-------------------------------|
| Value  | 0.9641                  | 2.99                          |
| Statistical                                      | $\pm 0.0045$            | $+0.20$<br>$-0.22$            |
| Simulation                                       | $\pm 0.0033$            | $+0.15$<br>$-0.16$            |
| Fragmentation Tuning                             | $\pm 0.0015$            | $\pm 0.07$                    |
| Fragmentation Model                              | $\pm 0.0022$            | $\pm 0.10$                    |
| Mass parameter                                   | $\pm 0.0051$            | $+0.23$<br>$-0.24$            |
| Gluon Splitting                                  | $\pm 0.0031$            | $+0.14$<br>$-0.15$            |
| Tagging  | $+0.0034$<br>$-0.0035$  | $+0.16$<br>$-0.17$            |
| Jet identification                               | $\pm 0.0018$            | $\pm 0.08$                    |
| Mass Ambiguity error                             | –                       | $\pm 0.26$                    |
| $\mu$ -scale error ( $0.5 \leq \mu/M_Z \leq 2$ ) | –                       | $\pm 0.11$                    |
| CAMBRIDGE (94-2000)                              | $R_3^{bl}(y_c = 0.009)$ | $m_b(M_Z)$ GeV/c <sup>2</sup> |
| Value  | 0.9653                  | 2.82                          |
| Statistical                                      | $\pm 0.0034$            | $+0.15$<br>$-0.16$            |
| Simulation                                       | $\pm 0.0025$            | $\pm 0.11$                    |
| Fragmentation Tuning                             | $\pm 0.0010$            | $\pm 0.05$                    |
| Fragmentation Model                              | $\pm 0.0025$            | $+0.11$<br>$-0.12$            |
| Mass parameter                                   | $\pm 0.0056$            | $+0.25$<br>$-0.26$            |
| Gluon Splitting                                  | $\pm 0.0014$            | $\pm 0.06$                    |
| Tagging  | $\pm 0.0021$            | $+0.09$<br>$-0.10$            |
| Jet identification                               | $\pm 0.0020$            | $\pm 0.09$                    |
| Mass Ambiguity error                             | –                       | $\pm 0.06$                    |
| $\mu$ -scale error ( $0.1 \leq \mu/M_Z \leq 1$ ) | –                       | $\pm 0.001$                   |

Table 2: Values of  $R_3^{bl}$  and  $m_b(M_Z)$  obtained with DURHAM and CAMBRIDGE algorithms and break-down of their associated errors (statistical and systematic) for  $y_c = 0.02$  and  $y_c=0.009$  respectively.

| DURHAM | $m_b(M_Z)$ (GeV) | $M_b$ (GeV) |
|--------|------------------|-------------|
| NLO    | 2.99             | 4.12        |
| LO     | 4.00             |             |

Table 3: Results of  $m_b(M_Z)$  and  $M_b$  obtained directly from the NLO calculations and the mass extracted from the LO for the DURHAM algorithm.

| CAMBRIDGE | $m_b(M_Z)$ (GeV) | $M_b$ (GeV) |
|-----------|------------------|-------------|
| NLO       | 2.82             | 4.17        |
| LO        | 3.15             |             |

Table 4: Results of  $m_b(M_Z)$  and  $M_b$  obtained directly from the NLO calculations and the mass extracted from the LO for the CAMBRIDGE algorithm.

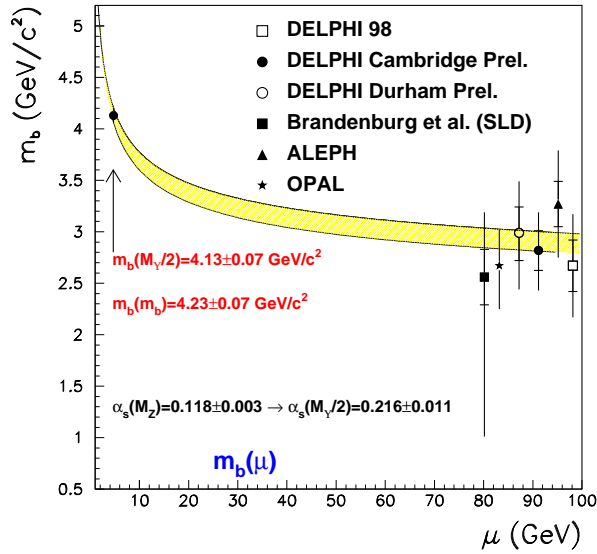


Figure 11: The running of  $m_b(\mu)$ . The shaded area corresponds to the band associated to  $m_b(\mu)$  when running  $m_b(\mu)$  at  $M_\Upsilon/2$  up to the  $M_Z$  scale using the QCD renormalization group equations and  $\alpha_s(M_Z) = 0.118 \pm 0.003$ . The values measured at LEP and SLC are displayed together with the statistical and total errors.

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