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HOT HADRONIC AND QUARK MATTER IN $\bar{p}$ ANNIHILATION ON NUCLEI *)

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A B S T R A C T

A substantial amount of energy has to be delivered to a nucleus in order to study the properties of hadronic matter at high temperature and/or density. We are proposing to employ the annihilation of antiprotons at LEAR momenta of 0.5-1.5 GeV/c on heavy nuclei; a discussion of possible interesting reaction channels is presented. Theoretical models of hot hadronic matter and hot quark gluon plasma are outlined.

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ABSTRACT

A substantial amount of energy has to be delivered to a nucleus in order to study the properties of hadronic matter at high temperature and/or density. We are proposing to employ the annihilation of antiprotons at LEAR momenta of 0.5 - 1.5 GeV/c on heavy nuclei; a discussion of possible interesting reaction channels is presented. Theoretical models of hot hadronic matter and hot quark gluon plasma are outlined.

1. INTRODUCTION

The aim of this lecture is to show how antiproton annihilation on nuclei can be used to study the equations of state of hot hadronic matter and to understand the space-time properties of the annihilation process. Quite early in the beginning of antiproton physics, the group of Segrè[1] has investigated the gross characteristics of the $\overline{p}$-annihilation in photographic emulsions. For us the relevant result to remember is the possibility of the transfer of a large fraction of the annihilation energy to nuclear fragments. Although such nuclear explosion events are rather rare - perhaps 1-2% of all annihilations, at low energies - they

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document the ability of the annihilation process to transfer the available energy to the spectator nucleons. In the next section we will try to give a qualitative explanation why this reaction channel is at all possible and what can be learned about the annihilation from such observations.

However, as these events do occur, the possibility arises to study the properties of heated nuclear matter - hadronic matter - experimentally in $\bar{p}$ - annihilations on nuclei. In this case we use $\bar{p}$ to deliver a substantial amount of energy to the target nucleus[2], quite in the same manner as is the case in high energy heavy ion collisions ($E/A \geq 1.5$GeV/c) at the BEVELAC[3]. Now however we have a quite different entrance channel to the reaction thus new and complementary information may be obtained.

In order to put these considerations on a sufficiently firm ground we then briefly describe a theory of hot hadronic matter developed recently[4,5], that can be used to interpret such experiments. The properties of hot hadronic matter that are in our judgement relevant are: thermal equilibrium, chemical equilibrium between mesons and baryons, resonance dominance of hadron-hadron interactions and finite size of a hadron. The latter is an essential element of the quark bag model of hadrons[6] and leads naturally to the occurrence of a new phenomenon: hot hadronic matter may 'condensate' into quark matter when individual quark bags are so close that they 'touch' each other[7]. The common way to perceive this phase transition is through a high baryon number density perhaps such as may be found in the case of a neutron star[8]. We will argue here that in relativistic physics quark bag condensation can also occur at high temperatures and low baryon densities - that is when a sufficient meson density is attained and all available space is filled by quark - antiquark pairs. In $\bar{p}$-annihilation, when it occurs inside a nucleus, a very 'hot spot' develops (assuming equilibration) - we expect that there exists a probability for the creation of quark plasma in such reactions.

In order to understand this phase transition we must describe in section 4 our present understanding of strong interactions in terms of quantum chromodynamics of quarks and gluons.

We are aware that most of the annihilation events on nuclei are of little interest to us as they proceed into other channels; in most cases a nucleus just provides a more 'dense' target for the annihilation than hydrogen. Therefore already at this point we emphasize that an experiment to study the properties of hadronic matter must be triggered by the occurrence of collective effects, such as an excess in the number of charged medium energy particles.

From now on we shall refer to a reaction channel in which a large fraction of the annihilation energy is found in a multiplicity of nuclear fragments as a 'nuclear explosion'. As mentioned, evidence for this reaction channel can be derived from emulsion experiments. In the article of Segrè[1] it is emphasized that among 220 annihilation stars one showed a nuclear multiplicity of 16. Since only 40% of these stars were annihilations in flight, this event corresponds to 1% of all eligible events. In our opinion, this high nuclear multiplicity is the most characteristic signature for the nuclear explosion. Thus the simplest trigger to discriminate against the 100-fold higher background of other events is a charged particle multiplicity counter. We observe that thin targets must be used to allow slow multi-charged nuclear fragments to arrive in the detector - this requires an intense beam of (slow) antiprotons that only the planned LEAR facility[9] at CERN can currently offer. Also the other experimental problems in such experiments: rescattering, spallation and pion contamination, can then be handled.

The question arises under what conditions we may expect the highest yield of the collective annihilation events. There is, most likely, an optimum antiproton momentum at which a particularly high yield of nuclear explosions can be expected. With higher $\bar{p}$ momenta the chance to penetrate deeply into a nucleus increases, but the required condition for pions to be found in the desired energy window restrains the experiment perhaps to less than 2.5 GeV/c. Another very important reason to use as small a P_{lab} as experimentally possible is the resulting smaller β_c for the centre-of-mass of the compound annihilation fireball (see Fig.1).

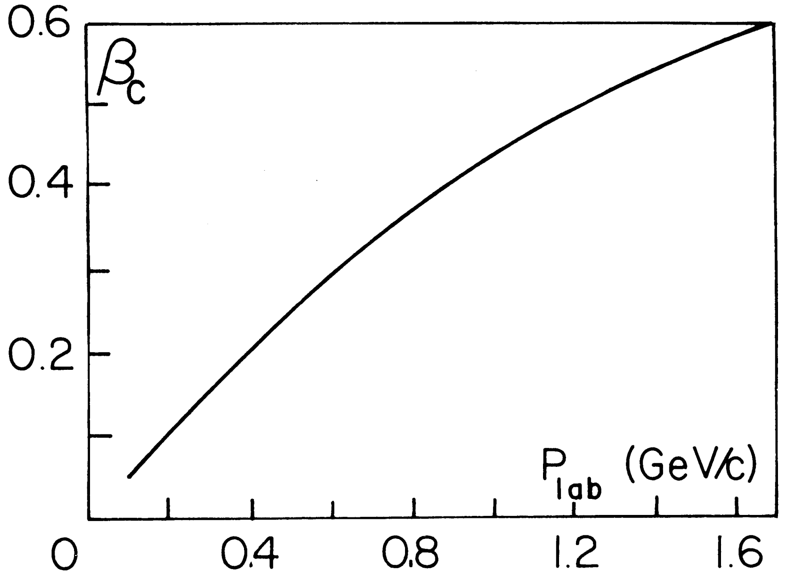

Figure 1 - The velocity of the compound p-$\bar{p}$ fireball as function of laboratory $\bar{p}$ momentum.

This 'hot spot' then travels relatively slowly through the surrounding matter and has significantly more time to undergo reactions, as compared with highly relativistic projectiles.

2. ASPECTS OF $\bar{p}$ - ANNIHILATION

When $\bar{p}$ impinges on a heavy nucleus, it will almost always eventually annihilate on one of the nucleons. The annihilation cross-section is very substantial at small P_{lab} and falls to about 50 mb above $\sim$1.5 GeV/c[10]. The range of antiprotons in homogeneous nuclear matter of density ν_N=1/6 nucleon/fm^3 is

$$\lambda \cong \frac{1}{\nu_N \bar{\sigma}_a} = 1.4 \text{ fm} \left[50/\bar{\sigma}_a [\text{mb}]\right] \qquad (2.1)$$

where $\bar{\sigma}_a$ is the average $\bar{p}p$ and $\bar{p}n$ annihilation cross-section. λ is a small fraction of the nucleon diameter at small momenta ($\lesssim$ 200 MeV/c) : most annihilations will occur at the nuclear surface. This is in fact known from the $\bar{p}$ atomic data[11]; for higher momenta (P_{lab} > 1 GeV/c) λ reaches the magnitude of one nucleon diameter. Then there is a sizeable chance that an antiproton can penetrate deeply into a heavy nucleus. The exponential absorption law allows about 2% of annihilations to occur four (inelastic) interaction lengths into the nuclear matter which is practically in the middle of heavy nuclei for $P_{lab} \gtrsim 1$ GeV/c.

Many yet unknown reaction channels govern the $\bar{p}p$ and $\bar{p}n$ interactions; aside from direct annihilations we can also have peripheral reactions in which pions are produced, while the baryons are not destroyed. Loss of momentum in such reactions assures that the annihilation follows soon in a subsequent reaction in a heavy nucleus. A similar remark is true for the charge-exchange process $\bar{p}p \leftrightarrow \bar{n}n$. A qualitative measure for the reaction channel is the number of pions produced in the annihilation. In our investigations the total number of mesons in the annihilation is very relevant, including the neutrals. Some information on neutrals can be extracted from the missing mass spectra. For example, the total pion multiplicity distribution has been obtained at 4.6 GeV/c[12]. A Gaussian fits the relative yield quite well, with the average number of pions $\langle n \rangle = 6.8 \pm 2$. At smaller momenta $\langle n \rangle$ seems to decrease slightly, data for annihilation at rest in deuterium[13] indicate three charged mesons on the average. Including about two neutrals we expect that $\langle n \rangle \gtrsim 5$ at rest, averaged over isospin.

In annihilations on nuclear targets pions in the momentum range of 200-400 MeV/c in the nuclear frame are most interesting to us : such pions would have high reaction cross-sections in the peak of the Δ_{33} resonance of about 200 mb (at P_π=280 MeV/c). Such pions have zero range in nuclear matter and we could expect that a substantial part of the annihilation energy is converted into the kinetic energy of nucleons. In this context we must also keep in mind the broadening of the momentum distribution of pions by the Doppler effect - the annihilation centre-of-mass moves with $0.2 < \beta < 0.5$ (for $0.4 < P_{lab} < 1.3$ GeV/c) through the nuclear target. Not necessarily all annihilations will yield pions in the desired energy window - we estimate that in about 10% of reactions pions would carry the required energy of 315 ± 70 MeV (≙ 280 ± 100 MeV/c). This estimate is based on the requirement that a sufficient number of pions is available so that their average energy is of the required magnitude.

Other information on reaction channels comes from the evaluation of pion correlations in annihilation[14,15]. This indicates that the pion source has a size[14] in space of $r = 1.89 \pm 0.06$ fm and time $c\tau = 1.52 \pm 0.14$ fm, compatible with the intuitive assumption of a very short-lived, hadron-sized, fireball as the intermediate pion emitting state. If baryonium exists as a stable state, a part of the cross-section could be attributed to the creation of such compound $\bar{p}p$ states. If the width of such a state is about 100 MeV, it could live long enough to escape from the nucleus at $\beta \sim 0.6$. In that way direct subthreshhold baryonium production would be feasible. We will not follow here this line of thought.

To summarize the essential points of the above discussion:

a) a few percent of antiprotons with 0.4 GeV/c < P_{lab} < 1.7 GeV/c will penetrate deeply into a heavy nucleus;

b) about 10% will decay into numerous pions with an average energy near resonance;

c) the average annihilation domain in space-time has hadronic sizes.

Consequently we expect[2] that about one to one-half percent of all annihilations on heavy nuclei should lead to the conversion of a large fraction of annihilation energy into the kinetic energy of nucleons, in qualitative accordance with the experiment[1].

3. HOT HADRONIC MATTER

One of the possible reaction channels as outlined above is thus the thermalization of the available energy among several nucleons in the vicinity of the annihilation spot. This will lead to a nuclear 'fireball' formed at about normal nuclear baryon density: $\nu_N = .17$ baryon/fm^3, but at high mean kinetic energy (temperature). Distributing naively the available energy among $\sim$ 5 baryons and 5 pions we find $T \sim 140$ MeV $\sim m_\pi$ which will lead to the critical phenomena outlined below. Thus in the case of $\bar{p}$ - annihilations high temperatures at lower densities than in heavy ion collisions can be most likely achieved. The high temperature has been the subject of intense recent investigations[4,5]. We are looking forward to the possibility of the observation of dilute and hot nuclear gases. We now outline the main theoretical ideas required to treat the properties of hot hadronic matter.

In order to derive the physical properties of hot hadronic matter which are independent of a particular choice of the effective hadronic interaction, we employ a technique -'bootstrap'- developed for similar problems in elementary particle physics - here, however, sufficiently modified to suit the different physical environment[4,5]. It is possible to view the bootstrap technique only as a convenient way of introducing the relevant physical properties which cannot be so easily defined by the choice of a specific particle interaction but which globally might even be more important than details of a two-body force. We will concentrate on the gross features of hadronic matter made out of two different kinds of hadrons - mesons and baryons, both considered implicitly as bound states of confined quarks. We ensure that in the thermodynamical description the baryon number is conserved, and all available particle production channels are open. The hadronic matter can consist of all different types of resonances. A further crucial property is that all hadrons have a finite volume. In the bag model of hadrons with almost massless, light, confined quarks, the hadronic mass M_H and volume V_H are related at zero external pressure by[6]

$$M_H = 4BV_H \qquad (3.1)$$

where B is the bag constant ; its value obtained from the hadronic spectrum[6] is of the order of $(145\ \mathrm{MeV})^4$. Other values proposed recently[17] go as high as $(235\ \mathrm{MeV})^4$.

If we consider the hadronic matter enclosed in a finite volume, then at fixed baryon density there will be a maximum hadronic temperature T_o at which all the available volume will have been filled with matter. At this limiting temperature our current description must be complemented with the theory of quark gluon plasma (see next section). The interpretation of this limiting temperature as belonging to a phase transition (hadron matter to quark gluon matter) has been first proposed by Cabibbo and Parisi[16]. In our approach we use the ideas of the statistical bootstrap model in which the statement: 'hadrons consist of hadrons and their mutual interaction is dominated by new hadron production' is converted into an integral equation for the mass spectrum $\tau(m^2)$ that characterizes how many hadrons are found in the mass interval (m^2, m^2+dm^2). Thus our initial aim is to derive the properties of hadronic matter, that is the equations of state, in terms of the mass spectrum. Then with the bootstrap postulate we have accomplished our aim.

The description of the thermodynamic properties of hot nuclear (hadronic) matter begins with the grand partition function $Z(\beta, V, \lambda)$ as obtained from the level density $\sigma(p,V,b)$

$$Z(\beta,V,\lambda) = \sum_{b=-\infty}^{\infty} \lambda^b \int e^{-\beta\cdot p}\, \sigma(p,V,b)\, d^4p \qquad (3.2)$$

We use here the covariant generalization[18] of thermodynamics with the inverse temperature four vector β_μ and the parallel volume four vector V_μ. In the rest frame of the system we have $(\hbar=c=k=1)$:

$$\beta\cdot p = \beta_\mu p^\mu = \beta_0 E = E/T \qquad \text{a)}$$

$$V\cdot\beta = V_\mu \beta^\mu = V/T \qquad \text{b)} \qquad (3.3)$$

$$V^\mu = (V, \vec{0}) \qquad \text{c)}$$

$$\beta^\mu = \left(\frac{1}{T}, \vec{0}\right) \qquad \text{d)}$$

λ is the fugacity, related in the rest frame to the relativistic chemical potential μ

$$\lambda = \exp(\mu/T) \tag{3.4}$$

and introduced here to allow the conservation of the baryon number in the statistical ensemble. As is well known, all quantities of interest can be obtained in a straightforward manner differentiating ln Z with respect to three independent variables

Pressure: $$P = T\frac{\partial}{\partial V} \ln Z(\beta, V, \lambda) \qquad \text{a)}$$

Energy: $$\langle E \rangle = -\frac{\partial}{\partial \beta} \ln Z(\beta, V, \lambda) \qquad \text{b)(3.5)}$$

Baryon number: $$\langle b \rangle = \lambda\frac{\partial}{\partial \lambda} \ln Z(\beta, V, \lambda) \qquad \text{c)}$$

The grand microcanonical level density σ is given by the invariant phase space integral:

$$\sigma(p, V, b) = \delta^4(p)\,\delta_K(b) + \\ + \sum_{N=1}^{\infty} \frac{1}{N!} \int \delta^4\Big(p - \sum_{i=1}^{N} p_i\Big) \sum_{\{b_i\}} \delta_K\Big(b - \sum_{i=1}^{N} b_i\Big) \prod_{i=1}^{N} \frac{2\Delta_\mu p_i^\mu}{(2\pi)^3}\, \tau(p_i^2, b_i)\, d^4 p_i \tag{3.6}$$

Above, Boltzman statistics has been used for simplicity. This is possible, since at sufficiently high temperatures the quantum statistics effects are negligible. The first term above is the contribution of the vacuum state for $p_\mu = b = 0$. The N-th term in the above formula includes the sum $\{b_i\}$ over all possible partitions of the total baryon number b and an integral over all partitions of the total momentum p into N different contributions, each having an internal number of available states given by the mass spectrum $\tau(p^2, b)$. These Boltzmanions of four momentum $p_{i,\mu}$ and baryon number b_i are (possibly) excited hadronic states that occupy their proper volume $V_{h,i}$ and move in the 'available volume' Δ_μ *)

$$\Delta^\mu = V^\mu - \sum_{i=1}^{N} V^\mu_{i,H} = V^\mu - \sum_{i=1}^{N} p_i^\mu / 4B \tag{3.7}$$

Here in the last term we have used the covariant generalization of Eq. (3.1)

$$V^\mu_{i,H} = p_i^\mu / 4B \tag{3.8}$$

We now turn to the determination of the mass spectrum $\tau(p^2,b)$. Experimental knowledge of τ is limited to low masses and/or low baryon number. We therefore have to use the statistical bootstrap model in order to obtain a complete mass spectrum, consistent with both direct and indirect experimental evidence. The basic idea is very simple: when the total volume V in Eqs. (3.6)-(3.7) is just the proper volume of a hadronic cluster, then σ in Eq. (3.6), is up to a normalization factor, the mass spectrum. Indeed, how could we distinguish between a composite system (as described by Eq. (3.6)), compressed to its proper volume and an elementary hadronic cluster with the same quantum numbers p,b ? Thus we demand the bootstrap relation

$$\sigma(p, V, b)\Big|_{V=V_H} \sim H\tau(p^2, b) \tag{3.9}$$

where H is a free parameter used to fix the limiting temperature. We postulate the integral equation for the mass spectrum τ by analogy to Eq. (3.6):

$$H\tau(p^2, b) = Hz_b\delta_0(p^2 - M_b^2) + \\ + \sum_{N=2}^{\infty} \frac{1}{N!}\int \delta^4\Big(p - \sum_{i=1}^{N} p_i\Big) \sum_{\{b_i\}} \delta_K\Big(b - \sum_{i=1}^{N} b_i\Big) \prod_{i=1}^{N} H\tau(p_i^2, b_i)\, d^4p_i \tag{3.10}$$

*) If V^μ is given, Δ^μ depends on N. We will treat Δ^μ as free parameter, hence V^μ depends on N and after summing becomes an expectation value Eq. (3.13).

Here we have omitted the vacuum contribution and have separated the one hadron part. The index 'o' restricts the δ-function to the positive root, z_b is the multiplicity $(2I+1)(2J+1)$ of the lowest baryon multiplets ($b=0, b=\pm 1$) made of light quarks and M_b is their average mass. We easily see that only the three pions ($b=0$) and two nucleons ($|b|=1$) contribute. We note that the solution of the above equation is well suited to be used in Eq. (3.6) as it is consistent with the assumptions made there about statistics and interactions of hadronic lumps.

To find the physical solution of this integral equation we perform a double transform:

$$\varphi(\beta,\lambda) := \sum_{b=-\infty}^{\infty} \lambda^b \int H z_b \delta_0(p^2 - M_b^2)\, e^{-\beta\cdot p}\, d^4p \qquad (3.11)$$
$$= 2\pi H T\left[3 m_\pi K_1\left(\frac{m_\pi}{T}\right) + 4\left(\lambda + \frac{1}{\lambda}\right) m_N K_1\left(\frac{m_N}{T}\right)\right]$$

$$\phi(\beta,\lambda) := \sum_{b=-\infty}^{\infty} \lambda^b \int H \tau(p^2, b)\, e^{-\beta\cdot p}\, d^4p$$

We find applying the same β-λ transform to the entire Eq. (3.10)

$$\varphi(\beta,\lambda) = 2\phi(\beta,\lambda) - \exp[\phi(\beta,\lambda)] + 1\,. \qquad (3.12)$$

The known properties of the inverse function of Eq. (3.12) and in particular the root singularity at $\varphi \to \varphi_0 = \ln(4/e)$ corresponding on inspection of Eq. (3.11) to $T \to T_0 \simeq 1.4\, m_\pi$ (at $\lambda = 1$ and $H = 0.724\ \mathrm{GeV}^{-2}$) lead to the occurrence of the critical temperature[5] of hadronic interactions associated with the exponential growth of the mass spectrum: $\tau \sim e^{m/T_0}$. The point $\varphi(\beta,\lambda) = \varphi_0$ defines in the μ-T plane a critical curve shown in Fig. 2. Beyond the full line the expression for ϕ diverges (and as we shall see, so does that for $\ln Z$). However, although the presence of this limit already signals the end of the hadronic world, the details of the phase transition to quark matter are more intricate.

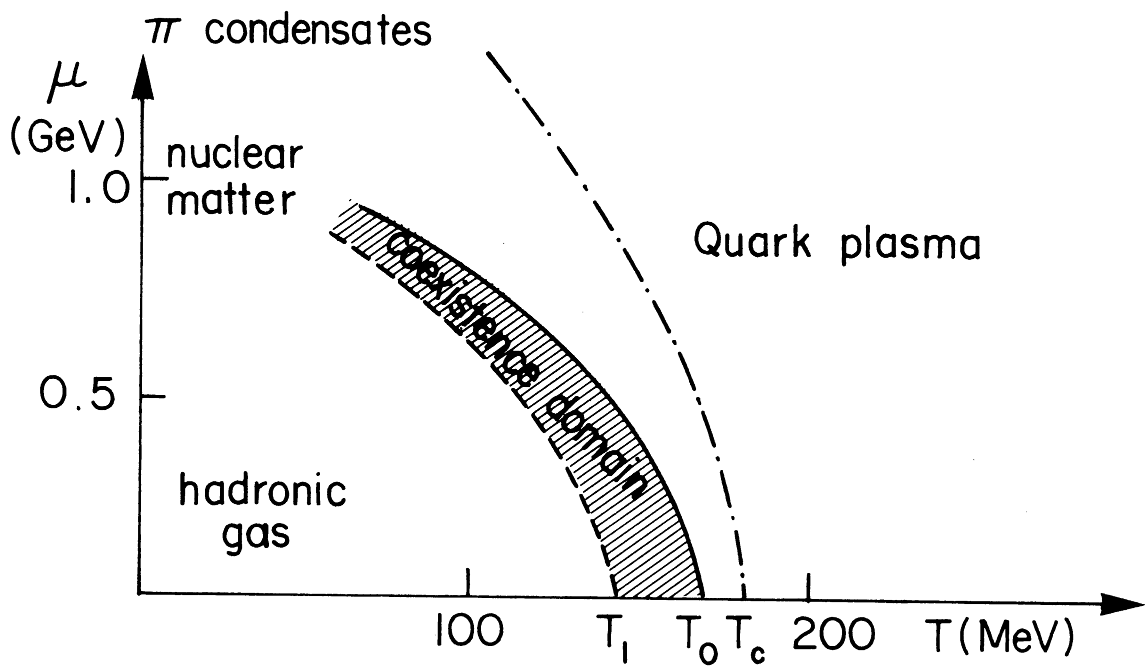

Figure 2 - Phase diagram for hadronic matter (qualitative) Full line: limit of existence. Dashed line: boiling point of incompressible hadrons. Dash-dotted lines: phase boundaries between hadronic matter and quark plasma.

With $\tau(p^2,b)$ given implicitly by its λ-β transform Eq. (3.12) we can now compute ln Z, Eq. (3.2) without needing to invert the transforms. However, we do need to explain the way we are going to treat the volume: The external volume has, through the introduction of the available volume, become an expectation value as other quantities and in the rest frame we find

$$\langle V \rangle = \Delta + \langle E \rangle / 4B =: V_{ex} \tag{3.13}$$

where the expectation value of the total energy is just the mass of the system. Thus we can first derive all quantities of interest for a system of mathematical point particles enclosed in Δ

$$\ln Z_p(\beta,\Delta,\lambda) \equiv \ln Z(\beta, V_{ex}, \lambda) \tag{3.14}$$

as obtained from Eqs (3.2) and (3.6). Thereafter we use this identity to compute quantities of physical interest.

Using the form similarity between the integral transforms Eq. (3.2) and (3.11) we find

$$\ln Z_p(\beta,\Delta,\lambda) = -\frac{2\Delta_\mu}{H(2\pi)^3}\cdot\frac{\partial}{\partial\beta_\mu}\phi(\beta,\lambda) \qquad (3.15)$$

With 'point' densities

$$\varepsilon_p = -\frac{1}{\Delta}\frac{\partial}{\partial\beta}\ln Z_p(\beta,\Delta,\lambda) = \frac{2}{(2\pi)^3}\frac{1}{H}\frac{\partial^2}{\partial\beta^2}\phi(\beta,\lambda) \qquad \text{a)}$$

$$\nu_p = \frac{1}{\Delta}\lambda\frac{\partial}{\partial\lambda}\ln Z_p(\beta,\Delta,\lambda) = -\frac{2}{(2\pi)^3}\frac{1}{H}\lambda\frac{\partial}{\partial\lambda}\frac{\partial}{\partial\beta}\phi(\beta,\lambda) \qquad \text{b)} \qquad (3.16)$$

$$P_p = \frac{T}{\Delta}\ln Z_p(\beta,\Delta,\lambda) = -\frac{2}{(2\pi)^3}\frac{1}{H}\frac{\partial}{\partial\beta}\phi(\beta,\lambda) \qquad \text{c)}$$

we find the equations of state of hot hadronic matter

$$\varepsilon := \langle E\rangle/\langle V\rangle = \varepsilon_p/(1+\varepsilon_p/4B) \qquad \text{a)}$$

$$\nu := \langle b\rangle/\langle V\rangle = \nu_p/(1+\varepsilon_p/4B) \qquad \text{b)}$$

$$V_{ex} = \langle V\rangle = \Delta\cdot(1+\varepsilon_p/4B) \qquad \text{c)} \qquad (3.17)$$

$$P = P_p/(1+\varepsilon_p/4B) \qquad \text{d)}$$

We note that near to the critical curve of Fig. 2 the expression for ε_p which is a second derivative at the root singularity of ϕ diverges and therefore

$$\varepsilon \xrightarrow[\text{critical}]{} 4B \qquad \text{a)}$$

$$P \xrightarrow[\text{critical}]{} 0 \qquad \text{b)} \qquad (3.18)$$

which confirms our expectation that the hadronic matter has become a giant quark bag at this point.

What we find is that as the hadron density (that is the chemical potential or temperature) increases to the critical value, the hadronic matter begins to cluster and forms ultimately a big lump when all the available volume is occupied. At this point we also find that the pressure P vanishes. It is interesting to draw a pressure - volume diagram as is usual in the theory of van der Waals gases. In order to eliminate the chemical potential in favour of volume from our considerations we choose a fixed baryon number <b> of a hadronic fireball (<b> must be sufficiently large to allow the statistical approach). Then for a given <b> we have from Eq. (3.17b)

$$\langle V \rangle = \langle b \rangle \left[\nu_p(\mu,T) / (1 + \varepsilon(\mu,T)/4B) \right]^{-1} \tag{3.19}$$

Now we can draw (full line in Fig. 3) the pressure, Eq. (3.17d) as

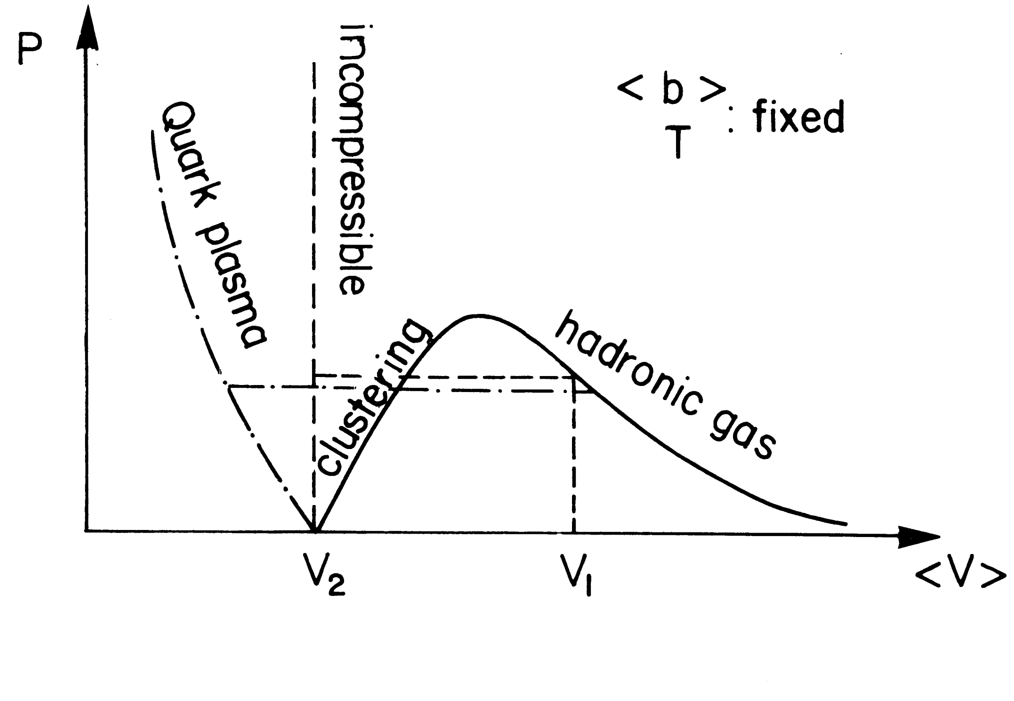

Figure 3 - P-V diagram of hadronic matter with Maxwell construction indicated for incompressible clusters (dashed lines) and quark plasma clusters (dash-dotted lines). T and <b> are fixed.

a function of $\langle V \rangle$ Eq. (3.19). When $\langle V \rangle$ is large, we see that pressure rises as we decrease the external volume of the fireball. But at some point the effect of lumping takes over and then the pressure drops rapidly to zero. At this point $\Delta = 0$ and there is only one hadronic cluster left as mentioned before.

In order to compress further we must compress the internal hadronic structure. The simplest approximation consists in the assumption (to be abandoned in the next section) of incompressibility of hadronic matter. Then, as indicated by the dashed lines in Fig. 3, by the Maxwell construction a pair of points for fixed T connecting the two branches can be found such that

$$\int_{V_2}^{V_1} P\,dV = P_{vap} \cdot (V_1 - V_2) \qquad (3.20)$$

where P_{vap} (horizontal dashed line) is the volume-independent vapour pressure over the condensed hadron matter at the given temperature. Thus in this region of V the two phases: hadronic 'gas' and hadronic 'fluid' (that is quark matter as we will see below) can coexist with each other. In the μ-T diagram the region of coexistence is shown shaded in Fig. 2.

We note here that, like is the case with a van der Waals gas, most particles are 'boiled' off with a kinetic energy corresponding to the boiling temperature - that is the dashed line in Fig. 2. Thus it is the 'boiling' temperature T_1 at $\lambda = 0$ of hadronic matter that must be about 150 MeV as determined in many experiments[5]. Consequently, with incompressible hadronic clusters the maximal hadronic temperature T_0 is of the order of 190 MeV, which fixes the value of the constant H to about 0.724 GeV^{-2}. Just below we will allow the quark phase to be compressed thus leading to slightly larger T_0 since the phase coexistence occurs then sooner at high temperatures.

As we can see, the properties of hot hadronic matter can be computed theoretically. The different experimental observables that emerge, are ratios of particle multiplicities (prongs) for the boiled off mesons and baryons. The corresponding experimental temperature can be derived from the measurement of the inclusive pion and proton spectra. We shall not go into these phenomenological details here yet, except to note that more than inclusive measurement of $\bar{p}$ annihilation on nuclei is needed in order to obtain a test of the theory of hot hadronic matter.

4. QUARK PHASE

As we saw in the last section, at the critical hadronic curve in the μ-T diagram, Fig. 2, the properties of hadronic matter

$$\varepsilon = 4B \qquad \text{a)}$$
$$P = 0 \qquad \text{b)} \qquad (4.1)$$

are precisely the same as of a large quark bag. Not only for this reason it is very suggestive to consider in the μ-T diagram the domain beyond the critical curve as the quark matter phase. In our approach to hadronic matter we have made several approximations that exclude small temperatures $T \lesssim 40$ MeV from our current discussion. We cannot describe those phenomena of hadronic matter that depend on the details of N-N interactions or/and quantum statistics: normal nuclear matter, pion condensates, etc. But we can consider here the properties of hot quark matter at $T > 40$ MeV. At this temperature it is a quark-antiquark gluon plasma, as light quark pairs may easily be excited.

In order to understand these remarks and the properties of quark plasma in general, we must begin with a summary of the relevant postulates and results that characterize the current understanding of strong interactions in quantum chromodynamics (QCD). The most important postulate is that the proper vacuum state in QCD is not the (trivial) perturbative state that we (naively) imagine to exist everywhere and which is little changed when the interactions are turned on/off. In QCD the true vacuum state is believed to have a complicated structure which originates in the glue ('photon') sector of the theory. The perturbative vacuum is an excited state with an energy density B above the true vacuum.

It is to be found inside hadrons where therefore perturbative quanta of the theory, in particular quarks, can exist. The occurrence of the true vacuum state is intimately connected to the glue-glue interaction; unlike QED the photons of QCD also carry a charge - here colour - that is responsible for the quark-quark interaction.

In the above discussion the confinement of quarks is a natural feature of the hypothetical structure of the true vacuum. If it is for example a colour superconductor, then an isolated charge cannot occur as it is the case with charge superconductors. Another way to look at this is to realize that a single coloured object would, according to Gauss' theorem, have an electric field that can only end on other colour charges. In the region penetrated by this field the true vacuum is displaced, thus effectively rising the mass of a quasi-isolated quark by the amount $B \cdot V_{field}$.

Another feature of the true vacuum is that it exercises a pressure on the surface of the region of the perturbative vacuum to which quarks are confined. Indeed this is just the idea of the original MIT bag model[6]. The Fermi pressure of almost massless light quarks is in equilibrium with the vacuum pressure B. The total energy

$$E = \frac{n\varepsilon_0 - z_0}{R} + BV \qquad (4.2)$$

has a minimum R_H as function of the bag size R. n is the number of enclosed quarks and antiquarks, while ε_0 is the eigenvalue of the Dirac equation with proper boundary conditions that confine the quark density inside V. For a sphere in three space dimensions the lowest s state is at $\varepsilon_0 = 2.04$. The first term in Eq. (4.2) is the quark energy, while the last term is the energy difference between the perturbative and true vacuum. From $(\partial E/\partial R) = 0$, Eq. (4.1) is obtained. z_0 is a contribution of the zero point energy and may be neglected when n becomes large.

When many quarks are combined to form a giant quark bag, then the computation of the first term in Eq. (4.2) must be carried out in a many-body theory. In particular this also allows to include the effect of the finite temperature, that is, of internal excitation. As u and d quarks are almost massless inside, they are easily produced in pairs and so at $T > 40$ MeV many $q\bar{q}$ pairs will be present. Also some $s\bar{s}$ pairs will be produced, but we will not follow this point further here. Furthermore, gluons can be present when $T \neq 0$ and will be included here in our considerations. However, until we properly understand the vacuum problem, this term must be distrusted - fortunately it is of marginal importance in our considerations.

A further effect that must be taken into consideration is the quark-quark interaction. We shall use here the first order contribution in the QCD coupling constant $\alpha_s = g^2/4\pi$. However, as $\alpha_s(q^2)$ decreases when the average momentum exchanged between quarks increases this will be only good at relatively high densities or/and temperatures. We will not follow this point further here, except to say that important contributions are still expected in higher order in α_s. The collective contributions such as plasma effects are known to be of comparable order of magnitude. Also the dependence of the running coupling constant $\alpha_s(q^2)$ on the density and temperature is known only qualitatively.

As we have outlined in the last section a complete description of the thermodynamical behaviour of a many-particle system can be derived from the grand partition function Z or equivalently from the related thermodynamic potential[20] $\Omega(T,V,\mu) := -\beta^{-1}\ln Z(\beta,V,\lambda)$. Our first objective is to allow for chemical equilibrium between particles and antiparticles; we consider a mixture of two ideal Fermi gases consisting of quarks and antiquarks, respectively. However, since we can only regulate the difference in number between these two species - proportional to the baryon number - the two chemical potentials have to satisfy the relation

$$\mu_{\bar{q}} = -\mu_q \tag{4.3}$$

As the quarks carry only 1/3 of the baryon number, the chemical potential μ of the preceding section, Eq. (3.4), is related to μ_q by

$$\mu = 3\mu_q \qquad (4.4)$$

After a partial integration of a well-known result[20] we obtain for the free Fermi-antiFermi gas the result

$$\Omega_q^0(T,V,\mu) = -\frac{1}{3}\frac{gV}{2\pi^2}\int_0^\infty dp \cdot p^3 \left[\frac{1}{\exp\beta(\omega(p)-\mu_q)+1} + \frac{1}{\exp\beta(\omega(p)+\mu_q)+1}\right] \qquad (4.5)$$

where the integration over all allowed momenta of the states with energy $\omega(p)$ is carried out. The coefficient g*) in Eq. (4.5) counts the degeneracy of each single particle state - in case of two light spin 1/2 coloured quarks we have $g = 12$. V is the volume filled with quarks. Since

$$\omega(p) \pm \mu_q = \sqrt{p^2 + m_q^2} \pm \mu_q \approx |p| \pm \mu_q \quad ; \quad \mu_q \gg m_q$$

we can evaluate the integral Eq. (4.5) as a power series in (m/μ_q) for arbitrary $\beta^{-1} = T$. Including interaction to first order[22] in α_s leads to the following analytic expression[21] in the limit $m/\mu \to 0$:

$$\Omega_q^1(T,V,\mu) = -\frac{1}{3}\frac{gV}{2\pi^2}\left[\left(1-\frac{2\alpha_s}{\pi}\right)\left(\frac{\mu^4}{4\cdot 3^4} + \frac{\mu^2}{2\cdot 3^2}(\pi T)^2\right) + \right.$$

$$\left. + \left(1 - \frac{50}{21}\frac{\alpha_s}{\pi}\right)\frac{7}{60}(\pi T)^4\right] \qquad (4.6)$$

(For $\alpha_s = 0$ this reduces to Ω_q^0, Eq. (4.5).) Now we can compute the equations of state through this order in α_s.

$$\nu = \frac{\langle b \rangle}{V} = \frac{1}{V}\left(-\frac{\partial}{\partial\mu}\Omega_q(T,V,\mu)\right)$$

$$= \frac{g}{6\pi^2}\left(1-\frac{2\alpha_s}{\pi}\right)\left(\frac{\mu^3}{81} + \frac{\mu}{9}(\pi T)^2\right) \qquad (4.7)$$

---

*) Not to be confounded with the coupling constant.

The above equation fixes μ for a given baryon density, temperature and interaction strength α_s. This third order equation for μ can be solved analytically. We note here that at normal nuclear density $\mu_q > 50$ MeV for $T < 200$ MeV, which justifies our approach.

For the energy density of the quark-gluon phase we find

$$\varepsilon = \frac{\langle E \rangle}{V} = \frac{1}{V}\left(\frac{\partial}{\partial\beta}\beta\Omega(\beta,V,\mu) - \mu\frac{\partial}{\partial\mu}\Omega(\beta,V,\mu)\right)$$

$$= \frac{g}{2\pi^2}\left[\left(1-\frac{2\alpha_s}{\pi}\right)\left(\frac{\mu^4}{324} + \frac{\mu^2}{18}(\pi T)^2\right)\right.$$

$$\left. + \left(1-\frac{50}{21}\frac{\alpha_s}{\pi}\right)\frac{7}{60}(\pi T)^4\right] + \frac{8}{15\pi^2}\left(1-\frac{15\alpha_s}{4\pi}\right)(\pi T)^4 \tag{4.8}$$

where we have added the contribution of massless gluons (last term in Eq. (4.8)). We further verify that the quark-gluon energy is related by the ultra-relativistic relation to the pressure :

$$P = \frac{1}{3}\frac{\langle E \rangle}{V} \tag{4.9}$$

even when the perturbative interaction (α_s) and internal excitation (T) are present (in the limit $m_q/\mu_q \to 0$).

The total energy (mass) of the large quark bag is now :

$$M_H = \langle E \rangle + BV \tag{4.10}$$

while the pressure on the surface is :

$$P = \frac{1}{3}\frac{\langle E \rangle}{V} - B \tag{4.11}$$

This leads to the standard equations of state[8]

$$P = \frac{1}{3}\left(\frac{M_H}{V} - 4B\right) \tag{4.12}$$

Thus when $P = 0$ we find $M_H = 4BV$ as expected.

It is easy to study the properties of quark matter with the above equations: we will focus here only on the joining of the hadronic phase with quark plasma and the Maxwell construction in the P-V diagram. Ideally both the critical curve for quark plasma corresponding to $P = 0$ in Eq. (4.12) and that for hadronic matter should be identical. However, as long as the quark matter critical curve lies within the boundary set by the hadronic critical curve, the quark pressure curve will cut the hadronic P-V curve at $P \geq 0$ (dash-dotted line in Fig. 3). Let us assume now that the relevant constants lead to this behaviour (which is indeed the case).

The so-introduced compressibility of quark matter allows now to complete the Maxwell construction, leading to the quark matter limit for the domain of coexistence of hadronic and quark matter, shown by the dash-dotted line in Fig. 2. We see that the accessibility of quark plasma eases as the chemical potential and therefore the baryon density decreases and the critical temperature at that density increases. Unfortunately a definite theoretical prediction concerning the attainability of the phase transition in $\bar{p}$ annihilations must await clarification for the values of different constants used, e.g., α_s, B[6,17] and other parameters that enter into the integral equation, Eq. (3.10) for the mass spectrum $\tau(p^2,b)$. What is very remarkable is that the actually accepted values of these parameters allow to match both phases quite nicely and lead to results quite compatible with our intuition. In particular, the great difference of the theoretical description of the two neighbouring phases is recalled. Still both lead to the same result at the phase boundary.

5. SUMMARY AND OUTLOOK

In the last two sections we have seen the ingredients that must enter into an adequate theory of hot hadronic matter and the quark gluon plasma. It is very important to realize that the adequate theoretical tools exist today to make an eventual comparison between theory and experiment feasible and to shed new light on mechanisms and channels in $\bar{p}$ annihilation on nuclei. As is apparent from the discussion presented here, many fundamental theoretical uncertainties can be probably settled thus making the study of hot hadronic matter, and its equations of state, a very rewarding subject.

In the first part of this report we have shown that $\bar{p}$ annihilation on nuclei may be a useful tool in the study of the properties of hot hadronic matter. It appears both from initial experimental results and qualitative theoretical considerations that in about 1% of annihilations at low momenta total disintegration of the target nucleus can be expected with a large fraction of the annihilation energy carried by the nuclear fragments. This peculiar reaction mechanism demands the study and comprehension of the properties of hot hadronic matter.

We then introduce a model of hot hadronic matter based on the most important global aspects of strong interactions - in particular we assume that the hadronic interactions are dominated by hadronic resonance production, while the thermodynamic properties follow from the assumption of thermal and chemical equilibrium between baryons and mesons. Another crucial property is the finite size of hadrons that enters through a limit on the hadronic energy density from quark bag models of hadrons. We find that the hot hadronic gas of quark bags lumps when the empty space between the bags disappears as either new baryons are added or when through increase in temperature new mesons are created. At a certain maximum temperature T_1 for a given baryon density (or equivalently chemical potential) the lumping of hadronic matter begins to dominate, making the gas of quark bags unstable against a collapse to a state consisting of one hadronic lump. By performing the standard Maxwell construction in the P-V diagram, we find the phase limits and in particular the baryon density dependent boiling point of hadronic matter.

As our implicit assumption has been that hadrons are separated quark bags, the properties of hadronic lumps must be studied within a quark plasma theory, including the perturbative QCD interactions. Thus in the preceding section we have described the properties of this state and shown that it is quite naturally also the continuation of the hadronic phase discussed before.

We do not need to stress the importance of an eventual discovery of quark plasma: it would at once test our understanding of strong interactions. What remains is to recognize through close interaction between theory and experiment the characteristic experimental signatures that carry the information about the formation of quark matter. Any strongly coupled observable is most likely unsuitable as it will 'forget' during the evolution of the reaction about the different stages of matter that might have been reached in transit. Only the weakly coupled quantum numbers are suitable 'thermometers'. They are: (anti-) strangeness, electron and muon number. Clearly the discovery of quark matter will be difficult.

The study of hot hadronic matter is in itself a very interesting subject: all the concepts outlined here, the boiling temperature, the exponential hadronic spectrum, the phase coexistence region, the chemical and thermal equilibrium need experimental clarification which will significantly deepen our current understanding of strong interactions and in particular of the annihilation mechanism.

The presence of spectator matter in $\bar{p}$ annihilations on nuclei may allow to decide if there is a compound p-$\bar{p}$ state, a highly excited 'meson' that could be formed in the initial stage of the annihilation reaction. Its existence would favour the production cross-section for quark plasma, since once created it could collide with and absorb the spectator baryons.

The path to the proper understanding of the annihilation on nuclei is very long. We will have to learn which of the reaction products come from the hot zone, which are knocked-on spectators or evaporation nucleons from nuclear fragments. How will the number of hot pions compare with that of hot baryons for different targets ? What controls the strangeness production ?

As is apparent from this discussion, we will need to have a complete record of the annihilation reaction in order to draw the required conclusions about the annihilation channel and the observed phase of hadronic matter. Thus it is necessary to consider complex detector systems - both data gathering and evaluation will be very complex but feasible. However, from the type of experiments described here a wealth of important information about hadronic matter will be gathered that will fully justify the effort undertaken in this direction as the intense low energy $\bar{p}$ beams[9] become available.

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