nonuplet is J. Schwinger, Phys. Rev. 135, B816 (1964). That discussion did not involve the speculation of a wider symmetry group [W(3)] as is occasionally stated.

[6]I am conscious of the honorable roles played by other semiempirical formulas, such as the one that produced the birth of the quantum theory.

REGGE-POLE MODEL FOR PHOTOPRODUCTION OF PIONS AND K MESONS

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High-energy data on photoproduction of pions and K mesons are fitted with a Regge-pole model involving conspiring trajectories.

The recent data of Boyarski et al.[1] on the reaction $\gamma p \to n\pi^+$ display a sharp peak in the forward direction (Fig. 1). We shall show that this peak, as well as a similar peak in the reaction $np \to pn$, can be understood quantitatively in terms of the exchange of the pion Regge trajectory, provided that a second trajectory, π_C, is assumed to be degenerate with the pion trajectory at $t = 0$. Such a model also fits the forward K^+ photoproduction data if K^* exchange is included.

The forward peak in the π^+ photoproduction data shown in Fig. 1 demands interpretation in terms of pion exchange, since it has a slope $(d\sigma/dt)\sigma^{-1} \sim m_\pi^{-2}$, a very large slope compared with the usual forward diffraction peaks. Note its similarity to the forward np charge-exchange (backward np scattering, or $np \to pn$) data at 8 GeV, which has been fitted by Phillips and by Arbab and Dash in terms of π^+ exchange.[2,3]

In both $np \to pn$ and pion photoproduction, naïve one-pion-exchange models do not give forward peaks, but instead predict zero cross section at $t = 0$ (which lies just outside the physical region in photoproduction). In fact, it has been shown by Drell and Sullivan and by Frautschi and Jones that no single-particle exchange contributes to photoproduction at $t = 0$.[4] This can easily be seen by decomposing the invariant amplitudes A_i, which have been shown by Ball[5] to be free from kinematical singularities, in terms of regularized, parity-conserving,

t-channel helicity amplitudes F_i:

$$A_1 = -(tF_1 + 2mF_3)/(t - 4m^2), \tag{1}$$

$$A_2 = \frac{F_1}{t - 4m^2} + \frac{F_2}{t(t - \mu_\pi^2)} + \frac{2mF_3}{t(t - 4m^2)}, \tag{2}$$

$$A_3 = -F_4, \tag{3}$$

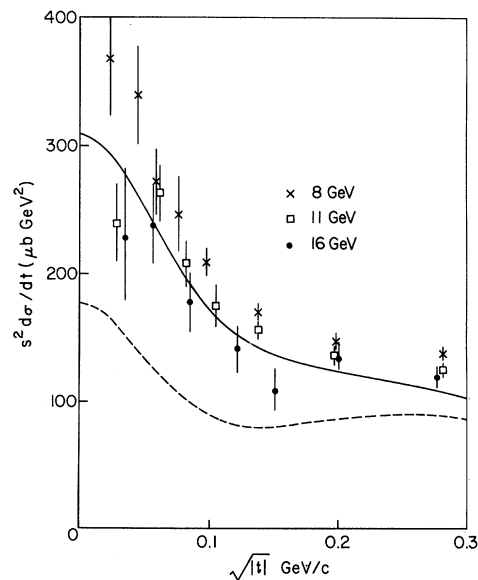$$A_4 = -(2mF_1 + F_3)/(t - 4m^2), \tag{4}$$

FIG. 1. Pion-conspiracy fits with the forward peak in π^+ photoproduction data of Boyarski et al. (Ref. 1). Note the expanded scale. The dashed curve has $\lambda = 0.54$; the solid curve, $\lambda = 0.4$.

where m is the nucleon mass, μ is the pion mass, and where the F_i, defined by

$$F_1(s,t) = K_1(t)[\bar{f}_{01,++}{}^t + \bar{f}_{01,--}{}^t]; \quad K_1(t) = (t - \mu_\pi{}^2)^{-1}, \tag{5a}$$

$$F_2(s,t) = K_2(t)[\bar{f}_{01,++}{}^t - \bar{f}_{01,--}{}^t]; \quad K_2(5)\left(\frac{t}{t-4m^2}\right)^{1/2}, \tag{5b}$$

$$F_3(s,t) = K_3(t)[\bar{f}_{01,+-}{}^t + \bar{f}_{01,-+}{}^t]; \quad K_3(t) = t^{1/2}(t - \mu_\pi{}^2)^{-1}, \tag{5c}$$

$$F_4(s,t) = K_4(t)[\bar{f}_{01,+-}{}^t - \bar{f}_{01,-+}{}^t]; \quad K_4(t) = (t - \mu_\pi{}^2)^{-1}(t-4m^2)^{-1/2}, \tag{5d}$$

are free from kinematical singularities. The helicity amplitudes $\bar{f}_{cd,ab}{}^t$ are related to the usual Jacob-Wick amplitudes $f_{cd,ab}$ in the t channel according to

$$f_{cd,ab} = (\sin\tfrac{1}{2}\theta_t)^{|a-b-c+d|}$$

$$\times (\cos\tfrac{1}{2}\theta_t)^{|a-b+c-d|}\bar{f}_{cd,ab}{}^t.$$

The amplitudes F_1 and F_3 receive contributions from normal-parity trajectories such as ρ and A_2. The pion contributes only to F_2, whereas the A_1 trajectory contributes to F_4.

In order that the amplitudes A_i be regular at $t=0$ and $t=4m^2$, the F's must be related at these points. The pole in A_2 coming from pion exchange is required by gauge invariance.[5,6] The point $t=4m^2$ will be regular if the condition $2mF_1(4m^2) + F_3(4m^2) = 0$ is satisfied. This is simply a threshold condition which can be understood by analyzing the helicity amplitudes in terms of orbital angular momenta.

The condition at $t=0$,

$$F_2(0) = -(\mu_\pi{}^2/2m)F_3(0), \tag{6}$$

is much more interesting, since F_2 and F_3 receive contributions from different trajectories. If only a <u>single</u> trajectory is involved, then one is forced to the conclusion that $F_2(0) = F_3(0) = 0$. Since the asymptotic form of the cross section continued to $t=0$ is

$$\frac{d\sigma}{d\Omega} = \frac{s}{(8\pi)^2}|A_1(0)|^2 = \frac{s}{(16\pi m)^2}|F_3(0)|^2, \tag{7}$$

it follows that any single-trajectory exchange gives a cross section which vanishes at $t=0$.

In order to satisfy Eq. (6) without forcing both sides to vanish, one needs to postulate "conspiring" trajectories. Since the steepness of the forward peak suggests pion exchange, we look for a conspiracy involving the pion.

The only such possibility, if one leaves aside branching points and fixed poles in the J plane, involves a parity doublet.[7] The pion contributes only to F_2:

$$F_2(s,t) = \beta_\pi(t)\alpha_\pi(t)\zeta_\pi(t)(s/s_0)^{\alpha_\pi - 1}, \tag{8}$$

where

$$\zeta_i = [1 + \exp(-i\pi\alpha_i)]/\sin\pi\alpha_i. \tag{9}$$

A Regge pole π_C contributing to F_3 has the form

$$F_3(s,t) = \beta_C(t)\alpha_C(t)\zeta_C(t)(s/s_0)^{\alpha_C - 1}. \tag{10}$$

Then if Eq. (6) is to be satisfied at all s, it follows that $\beta_\pi(0) = -(\mu_\pi{}^2/2m)\beta_C(0)$ and $\alpha_C(0) = \alpha_\pi(0)$. That is, the pion and its conspirator must have degenerate trajectories at $t=0$. The conspirator must have opposite parity to the pion, but all other quantum numbers the same.

This conspiracy is the same as that used by Phillips and by Arbab and Dash to fit the $np \to pn$ data.[3] The embarrassing prediction of a 0^+ meson degenerate with the pion was avoided by having the π_C trajectory "choose nonsense" at $J=0$, which was also required to fit the $np \to pn$ data.

It was also necessary to introduce a linear variation of the pion residue function in order to reconcile the height of the forward peak with the known value of the pion-nucleon coupling constant g^2. The form used was $g^2[1 - \lambda(t - \mu_\pi{}^2)/\mu_\pi{}^2]$, where $\lambda = 0.54 \pm 0.07$ for Phillips' best fit. This function is zero at $t = -0.85\mu_\pi{}^2$. Factorization then implies that such a zero must occur universally in all reactions involving pion exchange in order to avoid a singularity in the residue function. Thus we are led to the following pion residue function in π^+ pho-

toproduction:

$$\beta_\pi(t) = -\sqrt{2}\pi e_\gamma g_\gamma \mu_\pi{}^2[1 + \lambda(t - \mu_\pi{}^2)/\mu_\pi{}^2]. \quad (11)$$

On the other hand, β_C is taken to be a constant, determined by Eq. (6).

Arbab and Dash have argued that the zero in the pion residue is expected on the basis of the O(4) or O(3, 1) symmetry of the scattering amplitude at zero four-momentum transfer.[3,8] If the pion participates in the type of conspiracy we have discussed, then it belongs to an $M = 1$ representation of O(4). But if the pion mass were zero, it would then have to decouple from the $N\bar{N}$ amplitude because $M = 1$ representations with integral J do not couple to $J = 0$. Thus the residue of the pion pole would be zero at $t = 0$. Experience with partially conserved axial-vector currents indicates that it is meaningful to assume that amplitudes vary smoothly as the pion mass is varied from zero to its physical value; so presumably the zero moves to some point in the neighborhood of $t = 0$. Such arguments have been extended in a recent work by Mandelstam on partially conserved axial-vector currents and conspiracies.[9] In light of his work we regard the zero in the pion's residue function in Eq. (11) as a very interesting and crucial aspect of the fits we are presenting.

The fit to the π^+ photoproduction data with the parameter λ fixed at the value obtained from $np \to pn$ is shown by the dashed line in Fig. 1. No additional parameters have been introduced.[10] A better fit, the solid line, is obtained if λ is readjusted to $\lambda = 0.4$, as compared with $\lambda = 0.54 \pm 0.07$ in $np \to pn$. We regard this as satisfac-
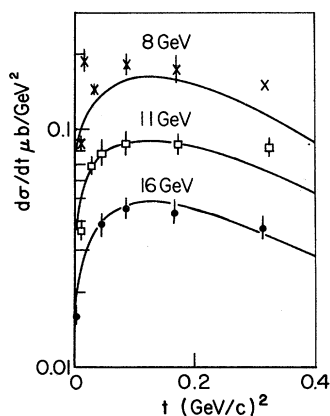
tory agreement, in view of the simplicity of the assumptions. In particular, one could tinker with the fit by taking for $\beta_\pi(t)$ more complicated functions which have a zero in the right place and take on the prescribed value at $t = \mu^2$.

The Stanford Linear Accelerator Center preliminary data on $\gamma p \to \Lambda K^+$ and $\gamma p \to \Sigma^0 K^+$ are shown in Figs. 2 and 3.[11] Since the pion and the K meson belong to the same octet, we assume that the K meson also has a conspirator K_C. The unequal-mass kinematics modify the details of the conspiracy.

The connection to the invariant amplitudes now becomes

$$A_1 = -(tF_1 + 2MF_3)/(t - 4M^2), \quad (12)$$

$$A_2 = \frac{F_2}{(t - \Delta^2)(t - \mu_K{}^2)} + \frac{F_1}{t - 4M^2} + \frac{2MF_3}{t(t - 4M^2)}$$

$$-\frac{\Delta F_4}{t(t - \Delta^2)}, \quad (13)$$

$$A_3 = -\frac{1}{t}\left[F_4 + \frac{2M\Delta F_3}{t - 4M^2}\right] - \frac{\Delta F_1}{t - 4M^2}, \quad (14)$$
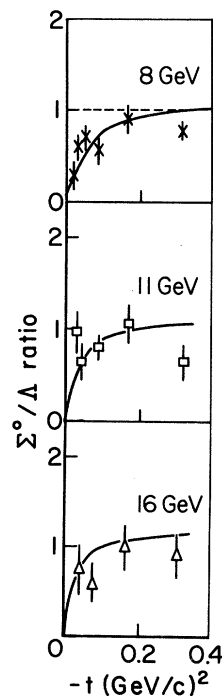
$$A_4 = -(F_3 + 2MF_1)/(t - 4M^2), \quad (15)$$

FIG. 2. K-meson–conspiracy fits to $\gamma p \to \Lambda K^+$ (Ref. 11), with K^* exchange included.

FIG. 3. K-meson–conspiracy plus K^*-exchange fits to Σ^0/Λ ratio. The K^* couplings have been adjusted to fit the data around $t = -0.3$, but the falloff around $t = 0$ is a prediction.

where $\Delta = m_N - m_Y$, $M = \frac{1}{2}(m_N + m_Y)$, and μ_K now represents the K mass. The kinematic factors defined in Eq. (5) now become

$$K_1(t) = (t - \mu_K{}^2)^{-1}[t/(t - \Delta^2)]^{1/2},$$

$$K_2(t) = [t/(t - 4M^2)]^{1/2},$$

$$K_3(t) = t/(t - \mu_K{}^2)(t - \Delta^2)^{1/2},$$

$$K_4(t) = t/(t - \mu_K{}^2)(t - 4M^2)^{1/2}.$$

The condition at $t = 0$ now reads

$$F_4(0) = (\Delta/2M)F_3(0), \tag{16}$$

whereas we have a new condition at the "pseudothreshold" $t = \Delta^2$,

$$F_4(\Delta^2) = [\Delta/(\Delta^2 - \mu_K{}^2)]F_2(\Delta^2). \tag{17}$$

In the limit $\Delta \to 0$ these reduce to the equal-mass condition, Eq. (6).[12] That the amplitude F_4 is now involved adds no further trajectories to the conspiracy, since the K contributes to both F_2 and F_4 (the pion is forbidden to contribute to F_4 by G parity). For simplicity we assume that Eq. (17) holds for all t in the forward region, and we take the K_C residue to be a constant determined by Eq. (16) as in the pion case. We take the residue function $\beta_K(t)$ to have the form of Eq. (11), where λ_K must be determined from the data. In view of the large π-K mass difference, it is unreasonable to assume that $\lambda_K = \lambda_\pi$.

We take the coupling constants $g_{p\Lambda K}{}^2 = 16.0 \pm 2.5$ and $g_{p\Sigma^0 K}{}^2 + 0.3 \pm 0.5$ from the recent dispersion-relation analysis by Kim.[13] Comparison of these couplings with Fig. 3 shows that either these couplings are grossly in error, or K photoproduction is not dominated by $K + K_C$ exchange. Having no reason to choose the former alternative, we choose the latter and add K^* exchange to the model. Before proceeding to this further complication, we point out an unambiguous prediction of our conspiracy model: The ratio of the Σ^0 and Λ photoproduction cross sections extrapolated to $t = 0$ should be in the ratio of the K coupling constants. This follows from the fact that all well-known trajectories (and therefore other members of their SU(3) multiplets) are known not to conspire at $t = 0$. The present data are not sufficiently accurate to check this prediction, but the 8-GeV results, which are the most precise in the forward region, do show a deep dip in the Σ/Λ ratio.

The parameter λ_K can be fixed from the forward points in Λ photoproduction, where the contributions of nonconspiring trajectories are small. We find that $\lambda_K = 0.73$ gives a good fit at all three energies if we take the K and K_C trajectories to be linear with the canonical slope $d\alpha/dt \simeq 1$ GeV. Then we add enough K^* exchange to fit the near-forward data, with the result shown in Figs. 2 and 3.[14] Since our main concern in this paper is the conspiracy in π and K exchange, we will not attempt a detailed fit of the high-t data, where additional parameters will be required.

We have shown that the conspiracy model of pseudoscalar meson exchange provides a consistent one-parameter fit to the near-forward data in both π^+ photoproduction and $np \to pn$. It also fits the K^+ photoproduction near-forward data if K^* exchange is included and makes a definite prediction for the Σ^0/Λ ratio. The striking difference between π and K forward photoproduction comes about because of the large π-K mass difference. The nearby pion pole dominates in a region $t < m_\pi{}^2$, producing a forward peak, whereas the relatively far away K pole dominates over K^* exchange only at the forward point. The K^* exchange vanishes at $t = 0$, thus producing the forward dip.

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[7]We look for a model in terms of Regge poles only, on grounds of the simplicity and greater predictive power of such a model. It is undoubtedly possible to fit these data with branch points (see the fits at lower energy by J. Fryland and D. Gordon, to be published). Another possibility which we find less attractive than the π, π_c conspiracy is that the pion does not conspire, but interferes with two other conspiring trajectories (of π, π_c type).

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J. M. Wang, Phys. Rev. 160, 1560 (1967).

[9]S. Mandelstam, in the Conference on πN Scattering, at Irvine, California, 1-2 December 1967 (unpublished).

[10]The pion and π_c trajectories have been taken to be degenerate, passing through $\mu_\pi{}^2$ and having the canonical slope 1 GeV. Since the fit presented covers a very narrow range of t, it is insensitive to the slopes of these trajectories. Moreover, we have neglected the possible contribution of π_c to F_1, since it is not needed to fit the near-forward data.

[11]B. Richter, Stanford Linear Accelerator Report No. SLAC-PUB-353, 1965 (unpublished).

[12]Our conclusion differs from that of J. P. Ader, M. Capdeville, and Ph. Salin [CERN Report No. TH. 803 (unpublished)], who ignored the $t = 0$ condition. Independent confirmation of this condition can easily be deduced from the method used by E. Leader, to be published.

[13]J. K. Kim, Phys. Rev. Letters 19, 1074 (1967).

[14]The details of the K^* exchange are unimportant here. We chose $K^*(1420)$, a member of the A_2 octet, because the A_2 seems to have a rather flat trajectory [R. Phillips and W. Rarita, Phys. Letters 19, 598 (1965)] which avoids a dip in the region explored.

DECAY RATES OF $\Delta S = -\Delta Q$ TRANSITIONS AND POSSIBLE $\Delta S = 2$ LEPTONIC DECAYS*

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Relations between possible $\Delta S = -\Delta Q$ decay rates of K^0-$\bar{K}^0$, K^+, Σ^+, and Ξ^0 are given. A connection between $\Delta S = -\Delta Q$ and $\Delta S = 2$ leptonic decays is also suggested.

The degree of validity of the selection rule $\Delta S = \Delta Q$ remains one of the most interesting questions in weak-interaction physics. Much effort has been dedicated to the search for possible decays which violate this rule, e.g., $K^0 \to \pi^+ e^- \bar{\nu}_e$; $K^+ \to \pi^+ \pi^+ e^- \bar{\nu}_e$; $\Sigma^+ \to n e^+ \nu_e$. The experimental information, summarized below, is becoming increasingly significant, and it is likely to improve considerably in the near future. In this note we present some possible relations between the $\Delta S = -\Delta Q$ decay rates of K^0-$\bar{K}^0$ and Σ^+, and the rate of the still undiscovered $\Xi^0 \to \Sigma^- e^+ \nu_e$ decay. Also, we discuss a possible connection between $\Delta S = -\Delta Q$ and $\Delta S = 2$ leptonic decays. As a result, we find that decays of the type[1] $\Xi^0 \to p l^- \bar{\nu}_l$ and $\Xi^- \to n l^- \bar{\nu}_l$ might exist with detectable branching ratios.

We shall first summarize the present experimental knowledge on $\Delta S = -\Delta Q$ decays:

(i) Results on $K_{l3}{}^0$ decays are usually given in terms of a parameter which is the ratio of the $\Delta S = -\Delta Q$ amplitude $A(\bar{K}^0 \to \pi^- l^+ \nu_l) \equiv g$ to the $\Delta S = \Delta Q$ amplitude $A(K^0 \to \pi^- l^+ \nu_l) \equiv f$; i.e., $g/f = |x|e^{i\varphi}$. A weighted average of world data[2] gives the result

$$|x| = 0.22 \pm 0.08, \quad \varphi \sim 60°. \tag{1}$$

(ii) No events of the type $K^+ \to \pi^+ \pi^+ e^- \bar{\nu}_e$ have been observed to date. Based on 208 $K^+ \to \pi^+ \pi^- \times e^+ \nu_e$ events which have been reported,[3] one

can set the following upper limit:

$$\Gamma(K^+ \to \pi^+ \pi^+ e^- \bar{\nu}_e)/\Gamma(K^+ \to \pi^+ \pi^- e^+ \nu_e) < 2\%. \tag{2}$$

(iii) There are two reported events which are candidates[4] for $\Sigma^+ \to n \mu^+ \nu_\mu$ and one reported event which is a candidate for $\Sigma^+ \to n e^+ \nu_e$.[5] If these three candidates are taken as certain, then we have the following branching ratio[6]:

$$\frac{\Gamma(\Sigma^+ \to n e^+ \nu_e) + \Gamma(\Sigma^+ \to n \mu^+ \nu_\mu)}{\Gamma(\Sigma^+ \to \text{all modes})}$$

$$\sim (4 \pm 3) \times 10^{-5}, \tag{3}$$

consistent with an independent upper limit recently published[7]:

$$\frac{\Gamma(\Sigma^+ \to n e^+ \nu_e) + \Gamma(\Sigma^+ \to n \mu^+ \nu_\mu)}{\Gamma(\Sigma^- \to n e^- \bar{\nu}_e) + \Gamma(\Sigma^- \to n \mu^- \bar{\nu}_\mu)}$$

$$\lesssim 3.4\% \ (90\% \text{ confidence level}). \tag{4}$$

We shall now discuss some theoretical considerations concerning these processes. By analogy to the usual weak processes, we shall assume that $\Delta S = -\Delta Q$ decays are described by an effective Hamiltonian of the current × cur-