CONSPIRING REGGE TRAJECTORIES AND THE POLARIZATION IN HIGH-ENERGY, CHARGE-EXCHANGE, PION-NUCLEON SCATTERING*

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In a recent paper[1] it was suggested that the polarization observed in high-energy, charge-exchange, pion-nucleon scattering is due to the interference between the contribution of the ρ trajectory and the contribution of another new trajectory ρ' with the same quantum numbers. The introduction of a second asymptotic term in the expansion of the spin-flip amplitude and of the nonforward spin-nonflip amplitude seems to be required also by sum rules derived from dispersion relations.[2] However, the sum rule for the forward spin-nonflip amplitude is perfectly satisfied by the ρ contribution alone.[3,4] It has also been shown[5] that if the last sum rule is introduced as a constraint and the ρ' residues are assumed to be slowly varying functions of t, it is not possible to obtain a sufficiently large polarization from the ρ-ρ' interference. This apparent contradiction between the sum rule and the actually available polarization measurements[6] can be avoided if we assume that the spin-nonflip residue of the ρ' pole has a zero for $t=0$. This is exactly what happens if the ρ' trajectory is of the type β according to the classification given by Gribov and Volkov.[7]

In order to show this, we write the residues of the contribution of the ρ' pole to nucleon-nucleon scattering in the factorized form $\beta_{\lambda_1\lambda_3}{}^N(t) \times\beta_{\lambda_2\lambda_4}{}^N(t)$. The subscripts $\lambda_1\cdots\lambda_4$ are the helicities of the nucleons in the t channel. In a similar way we write the residues which appear in pion-nucleon scattering in the form $\beta^\pi(t)\beta_{\lambda_2\lambda_4}{}^N(t)$. Taking into account the kinematic singularities of the t-channel helicity amplitudes,[8] we may write these residues in the form

$$\beta_{++}{}^N(t)\beta_{++}{}^N(t)=\gamma_1(t),\quad \beta^\pi(t)\beta_{++}{}^N(t)=\gamma_4(t),$$
$$\beta_{++}{}^N(t)\beta_{+-}{}^N(t)=t^{\frac{1}{2}}\gamma_2(t),\quad \beta^\pi(t)\beta_{+-}{}^N(t)=t^{\frac{1}{2}}\gamma_5(t),$$
$$\beta_{+-}{}^N(t)\beta_{+-}{}^N(t)=\gamma_3(t), \tag{1}$$

where the functions $\gamma_i(t)$ are free of kinematic singularities in a neighborhood of $t=0$. We omit in the preceding formulas threshold factors and kinematic singularities for $t\neq 0$.

There are two simple ways to satisfy the conditions (1), which correspond to the trajectories of the type α and β of Gribov and Volkov.[7] The first solution is

$$\beta_{++}{}^N(t)\sim C_1,\quad \beta_{+-}{}^N(t)\sim t^{\frac{1}{2}}C_2,$$
$$\beta^\pi(t)\sim C_3, \tag{2}$$

and the second is

$$\beta_{++}{}^N(t)\sim t^{\frac{1}{2}}C_1,\quad \beta_{+-}{}^N(t)\sim C_2,$$
$$\beta^\pi(t)\sim t^{\frac{1}{2}}C_3. \tag{3}$$

The first is the behavior usually assumed for the "classical" Regge trajectories, e.g., the ρ trajectory. If we consider only pion-nucleon scattering, the second solution differs from the first by an additional factor t which appears in the residue $\beta^\pi(t)\beta_{++}{}^N(t)$. This is proportional to the residue which appears in the amplitude A of the Singh formalism.[9] This is exactly the factor t which we need in order to avoid the contradiction with the dispersion-theoretical sum rule.

In order to test the model we are proposing by means of a fit of the experimental points, it is necessary to make some assumptions about the behavior of the ρ' residues when $\alpha_{\rho'}(t)=0$. In absence of more reliable information, some suggestion can be derived from the Lorentz or the O(4) symmetry of the amplitude at $t=0$.[10-12] Using this symmetry, the behavior of the Regge trajectories for $t\to 0$ can be classified by means of the representations of the Lorentz group. Using the notation of Ref. 11, the ρ trajectory would belong to a family of trajectories with the Lorentz quantum numbers $M=0$, $\sigma=1$, $\tau=-1$, while in the model we are proposing, the ρ' would belong to a family with Lorentz quantum numbers $M=1$, $\tau=-1$. In order to determine the powers of $\alpha(t)$ contained in the residues, we assume that it is possible to build a dynamical model in which a parameter, e.g., the coupling constant, can be varied in such a way that $\alpha(0)=0$. In this case the powers of $\alpha(t)$ contained in the residues are suggested by the Lorentz symmetry, combined with some simplicity assumptions. For the ρ trajectory, this method gives the usual result, i.e., a factor $\alpha(t)$ in the B amplitude and no $\alpha(t)$ factor in the A amplitude. For the ρ' trajectory, no $\alpha(t)$ factor is suggested. This last case corresponds to a singular behavior of the Regge residues, exactly in the way suggested by Mandelstam and Wang.[13] These authors suggest that this behavior should be followed also by the residues of the ρ trajectory while the behavior usually assumed is only approximately valid. We are assuming (consistently with the Lorentz symmetry) that this approximation is no more possible for the ρ' trajectory.

Following the above mentioned considerations, we modify the expressions used in Refs. 1 and 5 in the following way:

$$A=C(\alpha_\rho+1)\zeta(\alpha_\rho)(E/E_0)^{\alpha_\rho}[(1+H)\exp(C_1t)-H]$$
$$+C't(\alpha_{\rho'}+1)\zeta(\alpha_{\rho'})(E/E_0)^{\alpha_{\rho'}}\exp(C_1't),$$
$$B=D\alpha_\rho(\alpha_\rho+1)\zeta(\alpha_\rho)(E/E_0)^{\alpha_\rho-1}\exp(D_1't)$$
$$+D'(\alpha_{\rho'}+1)\zeta(\alpha_{\rho'})E/E_0)^{\alpha_{\rho'}-1}\exp(D_1't), \quad (4)$$

where

$$\zeta(\alpha)=-\frac{\Gamma(\alpha+\frac{3}{2})}{\Gamma(\alpha+1)}\frac{e^{-i\pi\alpha}-1}{\sin\pi\alpha}.$$

A fit of the experimental data has been done in the same way and with the same data as in Ref. 5. Of course in this case the ρ' contribution does not appear in the dispersion-theoretical sum rule which is used as a constraint. The results are given in Table I and in Figs. 1 and 2. For the sake of comparison we show in Figs. 1 and 2 also the fit obtained in Ref. 5 by means of a nonconspiring ρ' model. The $\alpha_\rho(t)$ trajectory has been taken to be $\alpha_\rho(t)=0.58+t$ while for the $\alpha_{\rho'}(t)$ trajectory we have chosen the following parametrization:

$$\alpha_{\rho'}(t)=\alpha_{\rho'}(0)+[1-\alpha_{\rho'}(0)]t/t_0 \text{ with } t_0=1(\text{GeV}/c)^2.$$

Table I. ρ + conspiring ρ' fit.

$\alpha_{\rho'}(0)$	0.27
C	2.048 $\text{mb}^{1/2}$
C'	28.36 $\text{mb}^{1/2}\,\text{BeV}^{-2}$
D	78.34 $\text{mb}^{1/2}$
D'	-4.78 $\text{mb}^{1/2}$
H	2.40
C_1	1.36 BeV^{-2}
D_1	0.118 BeV^{-2}
D_1'	-0.544 BeV^{-2}
C_1'	4.80 BeV^{-2}
$\chi^2(d\sigma/dt)=97.44$	
$\chi^2(\text{polarization})=3.6$	
$\chi^2(\text{total})=101.04$	
69 points	

In Figs. 1 and 2 the data are taken from Ref. 6. The continuous curve represents the calculated polarization corresponding to the fit I (Table I); the dashed curve represents the polarization corresponding to the fit II, i.e., the fit of Ref. 5. Comparing fits I and II, we see that the introduction of the conspiring ρ' has permitted a decrease of the total χ^2 from 116.0 to 101.0. Still more significant is the decrease of the part of the χ^2 due to the polarization data, which has decreased from 24.2 to 3.6.

It seems therefore reasonable to assume that the ρ' trajectory is of the β type. In this case, both the Gribov-Volkov arguments and the group-theoretical formalism[10,11] require that at $t=0$ the ρ' trajectory crosses another trajectory with opposite parity; this is the phenomenon called "conspiracy." This trajectory can be

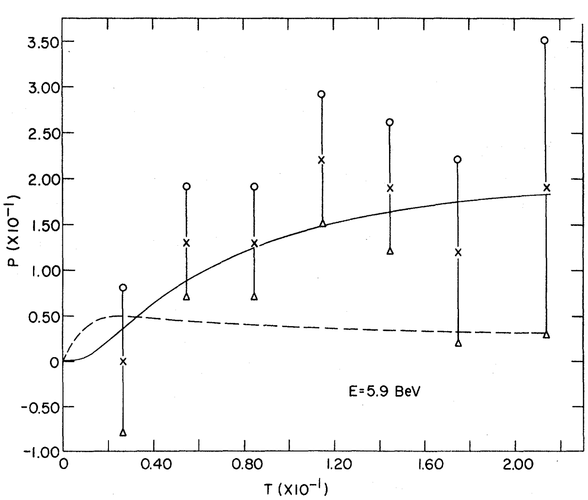

FIG. 1. On the abscissa is the momentum transfer t expressed in $(\text{GeV}/c)^2$; on the ordinate is the neutron polarization.

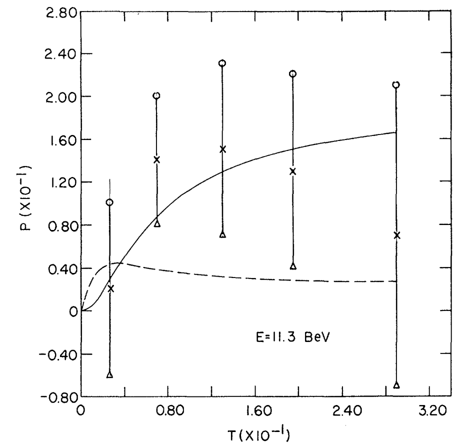

FIG. 2. On the abscissa is the momentum transfer t expressed in $(\text{GeV}/c)^2$; on the ordinate is the neutron polarization.

identified with the B trajectory which must be necessary for the explanation of the data available about the reaction $\pi+N\rightarrow\omega+\Delta$.[14] Because of the lack of normalization for $d\sigma/dt$ at 6-GeV/c pion momentum, the evaluation of the B-trajectory parameters is still affected by uncertainty.

Until new experimental data permit a quantitative evaluation of $\alpha_B(0)$ and consequent support or rejection of the choice $\alpha_B(0)=0.27$, we can keep the B trajectory as a candidate for the ρ' parity doubling.

Starting from completely different considerations Sawyer has suggested recently[15] that the B is a conspiring trajectory. Our analysis seems to be a support to this suggestion.

A more detailed picture of the conspiring Regge trajectories can be suggested if we assume an approximate exchange degeneracy.[16] In this case, two other conspiring trajectories with opposite signature and G parity would correspond to the conspiring trajectories ρ' and B. One of these could be the pion trajectory. However, there are strong arguments against a conspiring pion trajectory.[12 17] Alternatively, these trajectories could be identified with the trajectories d and d', as introduced by Arbab and Dash,[17] in order to explain the peak in the proton-neutron charge-exchange differential cross section if the pion does not conspire.

Note that the d' trajectory has the same quantum numbers as the A_2 trajectory and therefore could contribute to the reaction $\pi^-+p\rightarrow\eta+n$ giving rise to a polarization which could be theoretically treated in a way very similar to the one used in the present paper. It seems, therefore, that a measurement of this polarization at various energies above 4 GeV would be very helpful for the understanding of the conspiracy phenomenon.

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