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# Glueball spectrum in the bag model and in lattice gauge theories

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Existing bag-model calculations for the masses of lowest-lying glueball states are extended and reviewed and compared to the present lattice Monte Carlo calculations. The predicted spectra agree remarkably well.

## I. INTRODUCTION

Since the advent of quantum chromodynamics there has been interest in hadronic states containing gluons only, so-called glueballs or gluonic states.[1] There are as yet no positively identified glueball states, although there exist some promising candidates.[2] Theoretical predictions for the spectrum are available from the MIT bag model[3–5] and lattice Monte Carlo calculations.[6–8] In the bag model the glueball states are constructed from a few valence gluons (cavity modes) whereas in the lattice approach, one extracts masses from correlation functions measured at large imaginary times. The purpose of this note is to elaborate somewhat on the bag description and to compare the results of the two different approaches.

## II. BAG-MODEL PREDICTIONS

The MIT bag model is successful in explaining masses and other dimensional quantities for the lowest-lying mesons and baryons.[9,10,3,4] As free parameters one typically has the bag constant $B$, the strong coupling constant $\alpha_s$ (or alternatively $\Lambda_{\rm QCD}$ entering a running $\alpha_s$), the quark self-energy parameter $e_q$, and/or the geometrical or Casimir energy parameter $Z_0$. The lowest-lying glueball states are constructed from two or three valence gluons in the lowest transverse electric (TE) and transverse magnetic (TM) cavity modes. The only additional parameters entering are the corresponding self-energies $e_{\rm TE}$ and $e_{\rm TM}$. We now discuss the results of two different bag model fits to the glueball mass spectrum.

In Ref. 4 $e_{\rm TE}$ and $e_{\rm TM}$ were fitted to $\iota(1440)$, which has the (TE)(TM) quantum numbers $0^{-+}$, under the three different *ad hoc* assumptions $e_{\rm TE}=e_{\rm TM}$, $e_{\rm TE}=2e_{\rm TM}$, and $e_{\rm TE}=\frac{1}{2}e_{\rm TM}$. In our discussion we restrict ourselves to the latter since the two others correspond to the situation of having the TE gluon self-energy larger than the TE kinetic energy. Corrections for center-of-mass motion are made by including them in the Casimir parameter $Z$. The resulting spectrum is shown in Fig. 1.

Bag-model predictions of the glueball masses are also given in Ref. 5. The $({\rm TE})^2$ and (TE)(TM) mass splittings are calculated with the goal of showing that the $\iota(1440)$ and $\theta(1640)$ ($J^{PC}=2^{++}$) could be accommodated as glueball states in the bag model. That the separate gluon modes may have different self-energies was not considered. However, the model could fit the $\iota$ and $\theta$ for a variety of $\alpha_s$ and $R$, and choosing $R=4$ GeV$^{-1}$ puts the $0^{++}$ and $2^{-+}$ masses about where the quoted fit of Ref. 4 puts them.

In Ref. 3 a different bag approach, based on a specific model[11] for the QCD vacuum, is employed. The basic assumption of this model is that the QCD vacuum is locally well described as a color- and spin-singlet $({\rm TE})^2$ gluon state. Over long distance scales, these condensed "glueballs" are

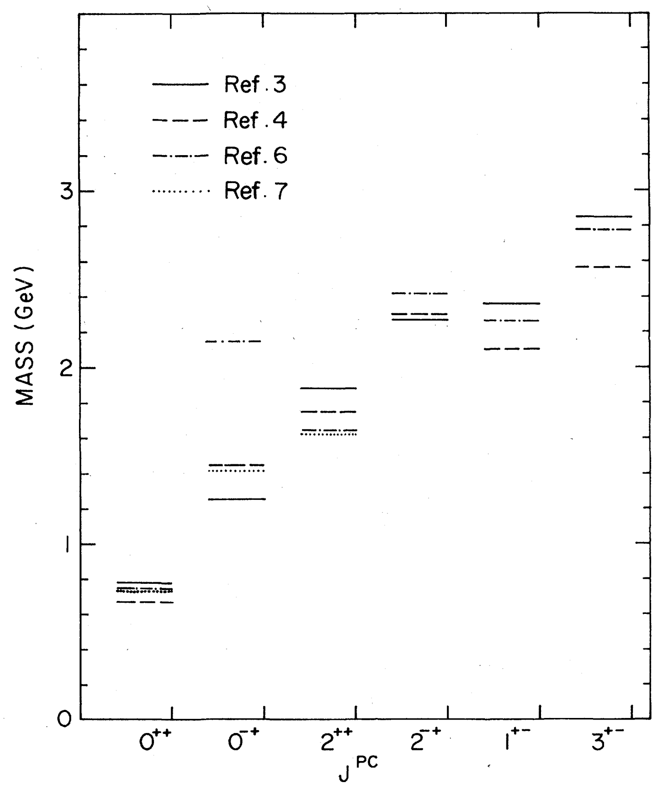


FIG. 1. The lowest-lying glueball states for each of the indicated quantum numbers. Lines without dots are bag-model calculations (our work in Ref. 3 is supplemented by the $0^{++}$ calculation reported here) and lines with dots are Monte Carlo calculations on a lattice. We have used fit I with $C_{\rm TE}/C_{\rm TM}=1/2$ from Ref. 4 and in plotting Ref. 6 we used $M_{0^{++}}=750$ MeV following the third article of that reference.

 

described by an effective Lagrangian

$$L=\frac{1}{2}(\partial_\mu\phi)^2-\frac{m^2}{2}\phi^2-\frac{\lambda}{4}\phi^4 \quad . \tag{1}$$

By minimizing with respect to $\phi$ and assuming that the vacuum is densely filled by glueballs of radius $R$, one obtains

$$B=\frac{3[-m^2(R)]^{1/2}}{8\pi R^3} \quad , \tag{2}$$

where $m^2(R)<0$ because of the instability of the vacuum. So once $B^{1/4}$ and $\Lambda_{QCD}$ are determined from fits to the meson and baryon spectra Eq. (2) gives $e_{TE}$ through $m^2$. There will also be an *observable* $J^{PC}=0^{++}$ state which is an excitation of $\phi$. The mass of this object can be estimated by expanding around $\phi_0$, and finding a term

$$(\sqrt{2}\mu)^2\tfrac{1}{2}(\phi-\phi_0)^2 \quad ,$$

where $\mu^2=-m^2$. Thus we get a *real* scalar particle with mass $\sqrt{2}\mu$. Using Eq. (2) and the parameters of Ref. 3 we obtain

$$\sqrt{2}\mu=2\sqrt{2B}\frac{4\pi R^3}{3}\approx 780\text{ MeV} \quad . \tag{3}$$

The remaining states involving $(TE)^2$ $2^{++}$, (TE)(TM), and $(TE)^3$ modes are computed with the *ad hoc* assumption $e_{TM}=e_{TE}$ in Ref. 3. The resulting spectrum is shown in Fig. 1 and labeled CHP. As is seen from this figure, the two bag-model approaches agree well despite the different algorithms [results from Ref. 4 were not given for $(TE)^3$; we have included them here using their parameters and methods]. The agreement for the $(TE)^2$ $0^{++}$ state is perhaps a coincidence. In Ref. 4 this is a conventional hadronic state whereas as shown above it is a collective excitation in Ref. 3.

## III. LATTICE GAUGE THEORIES

During the last two years several attempts have been made to extract glueballs masses in lattice QCD using Monte Carlo methods.[6–8] The technique is to choose appropriate corrected two point correlation functions that have the relevant quantum numbers (e.g., between plaquettes for the $0^{++}$ state). This correlation function is then evaluated numerically and from its large-imaginary-time falloff the mass of the lowest state with the quantum numbers in question can be extracted. The concept of valence gluons does not enter in lattice QCD calculations,[12] although in Ref. 13 an attempt is made to measure $\langle G|\vec{B}^2|G\rangle$ inside the $2^{++}$ glueballs with a suggestive dipole structure as a result, perhaps indicating a few-valence-gluons structure.

Glueball masses predicted from lattice gauge calculations in Refs. 6 and 7 are also plotted in Fig. 1. A number of remarks should be made.

Only for the $0^{++}$ and $2^{++}$ states (and the latter only in Ref. 7) does an appropriate "scaling window" exist. Hence only for these states is it safe to go to the continuum limit and quote a definite mass. Other lattice-gauge-theory predictions should be viewed with caution.

The masses we use from Ref. 7 come directly from the concluding section of the last listed article. In Ref. 6, only the $0^{++}$ mass is quoted in MeV. To try to get an idea of their mass spectrum, while remembering the above caveat, we have taken, using the $\beta=5.7$ column of their Table 5,[14] the ratio of the minimum mass (given in lattice units) that they calculate using single operators from the set of operators relevant for a given state to the minimum mass similarly obtained for the $0^{++}$ state, and then suppose the latter to weigh the 750 MeV that they quote.[6]

Some additional states which are not amenable to bag descriptions are amenable to latttice-gauge-theory calculations. Of special interest is the $1^{-+}$ state that cannot be made as a $q\bar{q}$ pair, for which Refs. 6 and 7 predict masses of 2.44 GeV and 1.73 GeV, respectively.

We have not thus far mentioned error estimates. We cannot, unfortunately, argue that they are small. It is not clear how to estimate the errors for the bag-model calculations and the errors quoted for the lattice-gauge-theory calculations are statistical only.[15] Also, the overall scale of the lattice-gauge-theory mass predictions depends directly on one's choice for the SU(3) string tension (which for Refs. 6 and 7 is 440 and 400 MeV, respectively).

We conclude by noting from Fig. 1 that the lattice predictions and the bag-model predictions agree substantially for both the absolute mass scale and the spin-splitting pattern. In addition, this may itself suggest that a two- or three-valence-gluon structure for the glueballs is substantially correct.

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[1]A review is given by S. Meshkov, in *Experimental Meson Spectroscopy,* proceedings of the International Conference, Brookhaven National Laboratory, 1983, edited by S. Lindenbaum (AIP, New York, 1984).

[2]See, e.g., D. Hitlin, in *Proceedings of the 1983 International Symposium on Lepton and Photon Interactions, Ithaca, New York,* edited by D. G. Cassel and D. L. Kreinick (Cornell University, Ithaca, New York, 1984).

[3]C. E. Carlson, T. H. Hansson, and C. Peterson, Phys. Rev. D **27**, 1556 (1983); **28**, 2895 (E) (1983).

[4]M. Chanowitz and S. Sharpe, Nucl. Phys. **B222**, 211 (1983).

[5]T. E. Barnes, F. E. Close, and S. Monaghan, Phys. Lett. **110B**, 159 (1982); Nucl. Phys. **B198**, 380 (1982).

[6]B. Berg and A. Billoire, Phys. Lett. **113B**, 65 (1982); **114B**, 324 (1982); Nucl. Phys. **B221**, 109 (1983); **B226**, 405 (1983).

[7]K. Ishihawa, M. Teper, and G. Schierholz, Phys. Lett. **116B**, 429 (1982); Z. Phys. C **21**, 167 (1983).

[8]H. W. Hamber and U. M. Heller, Phys. Rev. D **29**, 928 (1984).

[9]T. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D **12**, 2060 (1975).

[10]For a recent discussion see H. Flensburg, C. Peterson, and L. Shöld, Z. Phys. C **22**, 293 (1984).

[11]T. H. Hansson, K. Johnson, and C. Peterson, Phys. Rev. D **26**, 2069 (1982).


[12]This is in contrast to meson and baryon Monte Carlo mass calculations in the so-called quenched approximation [see, e.g., H. Lipps *et al.*, Phys. Lett. **126B**, 250 (1983)], where correlation functions between static quark sources are considered.


[13]K. Ishikawa, G. Schierholz, H. Schneider, and M. Teper, Nucl. Phys. **B222**, 221 (1983).


[14]From the last article of Ref. 6. In all but one case, the mass gotten using $\beta=5.7$ lies between the mass got from the other listed values of $\beta$ (5.4 and 5.6). The $0^{++}$ mass is quoted in the penultimate article. One should note that the masses are somewhat revised from those found in earlier articles of this series.

[15]From Ref. 7, we have errors $\pm 40$, $^{+240}_{-170}$, $\pm 100$, $\pm 300$, and $\pm 220$ MeV for the $0^{++}$, $0^{-+}$, $2^{++}$, $1^{+-}$, and $1^{-+}$, respectively; and from Ref. 6 we can find $\pm 80$ MeV for the $0^{++}$.