

## Constraints on the minimal supergravity model from the $b \rightarrow s\gamma$ decay

Jizhi Wu and Richard Arnowitt

*Department of Physics, Texas A&M University, College Station, Texas 77843*

Pran Nath

*Theoretical Physics Division, CERN, CH-1211 Geneva 23, Switzerland and Department of Physics, Northeastern University, Boston, Massachusetts 02115\**

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The constraints on the minimal supergravity model from the  $b \rightarrow s\gamma$  decay are studied. A large domain in the parameter space for the model satisfies the CLEO bound  $B(b \rightarrow s\gamma) < 5.4 \times 10^{-4}$ . However, the allowed domain is expected to diminish significantly with an improved bound on this decay. The dependence of the  $b \rightarrow s\gamma$  branching ratio on various parameters is studied in detail. It is found that, for  $A_t < 0$  and the top quark mass within the vicinity of the center of the CDF value  $m_t^{\text{pole}} = 174 \pm 17$  GeV, there exists only a small allowed domain because the light top squark is tachyonic for most of the parameter space. A similar phenomenon exists for a lighter top and  $A_t$  negative when the GUT coupling constant is slightly reduced. For  $A_t > 0$ , however, the branching ratio is much less sensitive to small changes in  $m_t$  and  $\alpha_G$ .

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The extensive analyses of the high precision data from the CERN  $e^+e^-$  collider LEP in the last few years have indicated that the idea of grand unification is only valid when combined with supersymmetry [1]. One of most promising and most studied models is the minimal supergravity model (MSGM) [2]. Supersymmetry (SUSY) is naturally and softly broken by a hidden sector. In addition to Yukawa couplings, the gauge coupling constant  $\alpha_G$ , and the unification scale  $M_{\text{GUT}} \simeq 10^{16}$  GeV, there are only five free parameters in this model: the four soft breaking terms (the universal scalar mass  $m_0$ , the universal gaugino mass  $m_{1/2}$ , the cubic scalar coupling  $A_0$ , and the quadratic scalar coupling  $B_0$ ), and a supersymmetric Higgs boson mixing  $\mu_0$ . The common approach to constraining the grand unified theory (GUT) model is to utilize the renormalization group equations (RGE) to make contact with physics at the electroweak scale  $M_{\text{EW}}$  [3]. Remarkably, the evolution of RGE's from  $M_{\text{GUT}}$  to  $M_{\text{EW}}$  produces a Higgs potential with a negative  $m_{H_2}^2$  if the top quark is heavy, signaling a spontaneous breaking of the electroweak gauge symmetry. As a consequence of this radiative breaking, two constraints arise which relate the GUT parameters to the electroweak parameters. One may then eliminate two GUT parameters  $B_0$  and  $\mu_0$  in favor of two electroweak parameters: the Higgs vacuum expectation value (VEV) ratio  $\tan\beta \equiv v_2/v_1$ , and the  $Z$  boson mass  $M_Z$ . Therefore the low-energy physics depends only on four parameters

$$m_0, m_{\tilde{g}}, A_t, \tan\beta, \quad (1)$$

and the sign of  $\mu$  (since the renormalization group equa-

tions determine only  $\mu^2$ ). Here, we have replaced  $m_{1/2}$  by the gluino mass  $m_{\tilde{g}}$  and  $A_0$  by  $A_t$ , their values at the electroweak scale  $M_{\text{EW}}$ . In most analyses, only the masses of the third generation of leptons and quarks are important. For small  $\tan\beta$ , one needs to retain only the top quark mass. Hence, the four free parameters in Eq. (1) plus the top quark mass  $m_t$  suffice to parametrize the MSGM.

The constraints on the parameter space of the MSGM may be classified into three major categories. First, various theoretical considerations put stringent constraints on the parameter space. For example, the color  $SU(3)_C$  group should remain unbroken when discussing the radiative breaking;  $\mu^2$  should also stay positive to guarantee this breaking; all scalar particles must be nontachyonic; and the allowed parameter space should be such that theory remains in the perturbative domain. Some of these issues will be discussed in a separate paper [4]. Second, cosmological considerations and the proton stability also strongly constrain the model [3]. Thus, for  $SU(5)$ -type models, proton stability requires that  $\tan\beta$  should not be too large, i.e.,  $\tan\beta \lesssim 10$  [5]. There still exists a large domain in the parameter space which satisfies both the proton decay and the relic density bounds [6]. Third, there exist a vast amount of data from the electroweak physics in the low-energy domain. It is thus very interesting to use these data to constrain the MSGM. One of the interesting processes is the  $b \rightarrow s\gamma$  decay. This decay is very sensitive to the structure of fundamental interactions at  $M_{\text{EW}}$ , because its rate is of order  $G_F^2 \alpha$ , while most other flavor-changing neutral current (FCNC) processes are of order  $G_F^2 \alpha^2$ . We shall study the constraint coming from the  $b \rightarrow s\gamma$  decay in the MSGM in this paper. The combined constraints from  $b \rightarrow s\gamma$ , proton decay, and relic density will be discussed elsewhere [7].

The recent CLEO II experiment gives the following measurement for the branching ratio of the exclusive  $B \rightarrow K(892)^*\gamma$  decay [8]:  $B(B \rightarrow K(892)^*\gamma) = (4.5 \pm$

\*Permanent address.

$1.5 \pm 0.9) \times 10^{-5}$ , and an upper bound on the inclusive  $B \rightarrow X_s \gamma$  decay,  $B(B \rightarrow X_s \gamma) < 5.4 \times 10^{-4}$ . Much work has been devoted to the determination of the ratio  $B(B \rightarrow K(892)^* \gamma) / B(B \rightarrow X_s \gamma)$  from QCD calculations [9–11]. An accurate calculation is of interest since the inclusive  $B$  decay is not as easy to study experimentally as the exclusive  $B$  decay. The earlier calculations [9] made use of the constituent-quark model and the two-point QCD sum rules, and the results for this ratio ranged from 0.05 to 0.40. More recently, there have appeared calculations based on some new developments in QCD. The method based on the three-point QCD sum rules predicts a value of  $0.17 \pm 0.05$  [10]. The method that incorporates the chiral symmetry for the light degrees of freedom and the heavy quark spin-flavor symmetries for the heavy quarks gives a value of  $0.90 \pm 0.03$  for this ratio [11]. Although these results are in accord with the CLEO II measurement, more study is needed to fully utilize the CLEO result. Because the  $b$  quark mass is much greater than the QCD scale, the dominant contributions to the inclusive  $B$  decay are from the short-range interactions. Barring a large interference between the long- and short-range contributions, one may model the inclusive

$B$  transitions, such as  $B \rightarrow X_s \gamma$  and  $B \rightarrow X_c e \bar{\nu}_e$ , by the decay of a free  $b$  quark to a free light quark, such as a free  $s$  or  $c$  quark. In fact, a calculation based on the heavy quark effective theory shows that  $B(B \rightarrow X_s \gamma)$  agrees with the free quark results,  $B(b \rightarrow s \gamma)$  up to corrections of order  $1/m_b^2$  [12]. Thus, the bound on  $B(B \rightarrow X_s \gamma)$  transfers to an upper bound,  $B(b \rightarrow s \gamma) < 5.4 \times 10^{-4}$ . It is a common practice to use the ratio defined as

$$R = \frac{B(b \rightarrow s \gamma)}{B(b \rightarrow c e \bar{\nu}_e)} \simeq \frac{B(B \rightarrow X_s \gamma)}{B(B \rightarrow X_c e \bar{\nu}_e)} \quad (2)$$

to constrain various models, utilizing the well-determined value of  $(10.7 \pm 0.5)\%$  for  $B(B \rightarrow X_c e \bar{\nu}_e)$ . The advantage of using  $R$ , instead of  $B(b \rightarrow s \gamma)$ , is that the latter is dependent upon  $m_b^5$  and certain elements of the Kobayashi-Maskawa matrix, while the former only depends on  $z = m_c/m_b$ , the ratio between the  $c$  and  $b$  quark masses, which is much better determined than both masses, i.e.,  $z = 0.316 \pm 0.013$  [13].

The ratio  $R$  defined in Eq. (2) has been calculated as [14]

$$R = \frac{6\alpha \{ \eta^{16/23} C_7(M_W) + \frac{8}{3} [\eta^{14/23} - \eta^{16/23} C_8(M_W)] + C_2(M_W) \}^2}{\pi I(z) \{ 1 - (2/3\pi) [\alpha_s(M_W)/\eta] f(z) \}}, \quad (3)$$

where  $\eta = \alpha_s(M_W)/\alpha_s(m_b) = 0.548$ , and  $M_W$  is the  $W$  boson mass. Here,  $I(z) = 1 - 8z^2 + 8z^6 - z^8 - 24z^4 \ln z$  is the phase-space factor, and  $f(z) = 2.41$ , a QCD correction factor, for the semileptonic process,  $b \rightarrow c e \bar{\nu}_e$ .  $C_7(M_W)$  and  $C_8(M_W)$  are the coefficients of the photonic and gluonic penguin operators for the  $bs$  transition at the electroweak scale. These coefficients are model dependent and sensitive to the underlying fundamental interactions at  $M_W$ . For the standard model (SM), only the penguin diagram induced by the  $W$ - $t$  loop contributes to  $C_7(M_W)$  and  $C_8(M_W)$ , whereas, for the MSGM, many SUSY particles contribute.  $C_2(M_W)$  is a coefficient coming from a mixing between the photonic penguin operator and many four-quark operators present at  $M_W$ . The form and number of these four-quark operators differ depending on the model. Fortunately, for the MSGM, they are the same as those for the SM [15]. The calculation of  $C_2(M_W)$  is an involved procedure and a number of evaluations exist [14,16,17]. We shall use the results of Ref. [17], which takes into account the full-leading-order logarithmic contributions, i.e.,  $C_2(M_W) = \sum_{i=1}^8 a_i \eta^{b_i} = -0.1795$ , where  $a_i$  satisfy  $\sum_i a_i = 0$  (since at the electroweak scale, only the photonic penguin diagram has a contribution to the  $b \rightarrow s \gamma$  decay). The numerical values for  $a_i$  and  $b_i$  have been a matter of debate, in the sense that different calculations give different results. Nevertheless, the overall effect on  $C_2(M_W)$  is insignificant, the difference being only about 1%. A comment on the accuracy of Eq. (3), however, is in order. There exist several uncertainties in using this equation to calculate  $B(b \rightarrow s \gamma)$ . For instance,

Eq. (2) is based on the spectator quark model. The error in determining the strong interaction constant,  $\alpha_s(M_W)$ , is still large. The most significant uncertainties come from the absence of a complete evaluation of the next-to-leading short-distance QCD corrections to  $B(b \rightarrow s \gamma)$ , causing about a 25% inaccuracy. To better determine the theoretical predictions for  $B(B \rightarrow X_s \gamma)$ , it would be necessary to calculate certain three-loop mixings and two-loop penguin diagrams [18].

In the rest of this paper, we will concentrate on the MSGM predictions for  $B(b \rightarrow s \gamma)$ . As mentioned above, this model contains many particles not present in the standard model. Thus, besides the  $W$  boson contributions, there exist the penguin diagrams induced by the charged Higgs bosons, the chargino squarks, the gluino, and the neutralinos. The coefficients  $C_7(M_W)$  and  $C_8(M_W)$  were calculated for the minimal supersymmetric model (MSSM) in Ref. [19]. (We have re-derived these coefficients confirming their results.) It is found that the gluino and neutralino contributions are small compared to other sources. We will hence ignore their contributions below. A large contribution from the charged Higgs boson in the MSSM was found in Ref. [20] and these authors thus concluded that a slight improvement in the experimental bound on  $B(b \rightarrow s \gamma)$  will exclude the search for the charged Higgs boson via the  $t \rightarrow b H^+$  decay channel. However, these papers did not consider the chargino-squark penguin diagrams. As it turned out, although the charged Higgs boson enhances the standard model amplitude, the chargino-squark loops

may contribute to the amplitude constructively or destructively, depending on the parameters chosen. In fact, as shown in Ref. [21], in the exact supersymmetric limit, the coefficients for  $b\bar{s}\gamma$  and  $b\bar{s}g$  transition operators,  $C_7(M_W)$  and  $C_8(M_W)$ , vanish exactly. Other papers on the  $b \rightarrow s\gamma$  decay in SUSY models can be found in [22]. We will follow the notation of Ref. [21], and assume that the first two generations of squarks are degenerate in mass. We then expect the contributions from these degenerate squarks to  $B(b \rightarrow s\gamma)$  to be small, since their masses are proportional to  $m_0 \lesssim 1$  TeV. On the other hand, the scalar top squarks,  $\hat{t}_1$  and  $\hat{t}_2$ , are badly split in their masses, due to the large top mass, implied by the recent Collider Detector at Fermilab (CDF) data [23]. The top squark mass matrix is given by

$$\begin{pmatrix} m_{\hat{t}_L}^2 & m_t(A_t + \mu \cot\beta) \\ m_t(A_t + \mu \cot\beta) & m_{\hat{t}_R}^2 \end{pmatrix}, \quad (4)$$

where  $m_{\hat{t}_L}^2$  and  $m_{\hat{t}_R}^2$  are given in Ref. [24]. Thus, the light top squark mass is

$$m_{\hat{t}_L}^2 = \frac{1}{2}[m_{\hat{t}_L}^2 + m_{\hat{t}_R}^2 - \sqrt{(m_{\hat{t}_L}^2 - m_{\hat{t}_R}^2)^2 + 4m_t^2(A_t + \mu \cot\beta)^2}]. \quad (5)$$

One can demonstrate that, for a large portion of the parameter space, the light top squark (mass)<sup>2</sup> may turn negative, signaling either the breaking of the color  $SU(3)_C$  group or the existence of tachyons in the theory. The requirement that  $m_{\hat{t}_L}^2$  be positive is very stringent and could eliminate a considerably large domain in the parameter space [4].

Our strategy is to use the one-loop RGE's to calculate the mass spectrum of the model relevant to  $b \rightarrow s\gamma$  (all the up-type squarks, the charginos, and the charged Higgs bosons). We then use this spectrum to evaluate the ratio  $R$  via Eq. (3) and Eq. (2) to obtain  $B(B \rightarrow X_s\gamma) = R \times B(B \rightarrow X_c e \bar{\nu}_e)$ , and compare the results with the CLEO II bound. Of the five parameters,  $m_t$  is restricted by the CDF bound,  $m_t^{\text{pole}} \simeq 174 \pm 17$  GeV [23]. In our analysis,  $m_t$  is the running top mass at  $M_Z$ , which is related to the pole mass by [25]

$$m_t^{\text{pole}} = m_t \rho_z^t \left[ 1 + \frac{4\alpha_s(M_Z)}{3\pi} + 11 \left( \frac{\alpha_s(M_Z)}{\pi} \right)^2 \right], \quad (6)$$

where  $\rho_z^t \simeq h_t(m_t)/h_t(M_Z)$  is the ratio of the top Yukawa couplings at  $m_t$  and  $M_Z$  (we assume that the Higgs VEV's,  $v_1$  and  $v_2$ , and hence  $\tan\beta$ , do not change significantly between  $m_t$  and  $M_Z$ ). The pole mass given by Eq. (6) is about 3–5% larger than the running mass at  $M_Z$ . The naturalness condition restricts  $m_0$  and  $m_{\tilde{g}}$  to be less than  $\sim 1$  TeV, and we allow them to lie between 100 GeV and 2 TeV. We parametrize  $A_t$  in units of  $m_0$ , and restrict  $|A_t/m_0| \leq 2.0$ . Notice also that the top mass is very close to its Landau pole,  $m_t^{\text{Landau}} = C \sin\beta$ , where  $C \sim 195$  GeV. This implies that the error in  $\alpha_s(M_Z)$  plays an important role in the RGE analysis of the spectrum. Here, instead of varying  $\alpha_s(M_Z)$ , we

let  $\alpha_G$  vary between 1/24.11 and 1/24.5 with a fixed  $M_{\text{GUT}} = 10^{16.187}$ , since a slight change in  $M_{\text{GUT}}$  does not affect the coupling unification as significantly as a change in  $\alpha_G$ . It turns out that  $\alpha_G = 1/24.11$  gives, at the one-loop level and for  $m_s = M_Z$ , the best fit to  $\alpha_s(M_Z) = 0.118$ ,  $\alpha_2(M_Z) = 0.03358 \pm 0.00011$ , and  $\alpha_1(M_Z) \equiv (5/3)\alpha_Y = 0.016985 \pm 0.000020$  for the  $M_{\text{GUT}}$  cited above, and the two-loop corrections are small [4]. Decreasing  $\alpha_G$  corresponds to decreasing  $\alpha_s(M_Z)$ . We will discuss below the consequences of varying  $\alpha_G$ .

We have surveyed a large domain in the parameter space described by Eq. (1). The branching ratio depends importantly on the values of the parameters in this four-dimensional space. Significant deviations from the SM value can occur on certain regions of the parameter space. Characteristically, the region where large deviations from the SM arise occurs when  $m_0, m_{\tilde{g}}$  are much smaller than their naturalness limits and  $\tan\beta$  gets large, i.e., typically larger than 10. Specifically, in this region of the parameter space  $B(b \rightarrow s\gamma)$  can be significantly below its SM predictions, and for certain points an almost perfect cancellation is observed. This is because the chargino-squark penguin diagrams contribute destructively to the total amplitude at these points, with a coefficient  $\sim 1/\cos\beta \sim \tan\beta$ . This destructive interference between various sources in the MSSM has also been observed previously [26] (the symmetric distribution of the branching ratios around the SM values found in Ref. [26] is because those authors allowed  $\tan\beta$  to be as large as 60). Similar deviations from the SM value can also occur for smaller  $\tan\beta$ , although this is less frequent. For  $\tan\beta < 10.0$ , the deviations of  $B(b \rightarrow s\gamma)$  from the SM values are less dramatic. In this region, the current CLEO bound is not stringent enough to strongly constrain the MSGM. However, with a moderate, e.g., about 30%, improvement in the CLEO bound constraints on the model will emerge.

Figures 1, 2, and 3 show plots of  $B(b \rightarrow s\gamma)$  as a function of the light chargino masses  $m_{\tilde{W}_1}$  and the soft supersymmetry breaking parameter  $m_0$ , for  $\tan\beta = 5.0$ ,  $|A_t/m_0| = 0.5$ ,  $m_t = 160, 170$  GeV, and  $\alpha_G^{-1} = 24.11, 24.5$ . In these graphs, all the masses are in units of GeV. The graphs for  $m_t = 150$  GeV are similar to figures 1, 2, and 3, except that the branching ratios are smaller. We also impose a phenomenological lower bound on the light chargino mass, i.e.,  $m_{\tilde{W}_1} > 45$  GeV. These figures are characterized by four parameters,  $m_t, \alpha_G^{-1}$ , and the signs of  $A_t$  and  $\mu$ . The figures labeled by (a) have  $A_t < 0, \mu > 0$ , those by (b) have  $A_t < 0, \mu < 0$ , those by (c) have  $A_t > 0, \mu > 0$  and those by (d) have  $A_t > 0, \mu < 0$ . These figures contain the following results. (1) The  $b \rightarrow s\gamma$  branching ratio at the points at which  $A_t$  and  $\mu$  have the same sign is in general larger than that at the points where  $A_t$  and  $\mu$  have the opposite sign. This can be seen by comparing Figs. 1(a) and 1(d) with Figs. 1(b) and 1(c). This is a generic feature of the MSGM, because Eq. (5) gives a smaller light top squark mass when  $A_t$  and  $\mu$  have the same sign. The gaps in the lines for  $m_0 = 1000$  GeV in Fig. 1(a) and for  $m_0 = 600$  and 1000 GeV in Fig. 1(b) are due to the light top squark turning tachyonic. These gaps occur only for  $A_t$  nega-

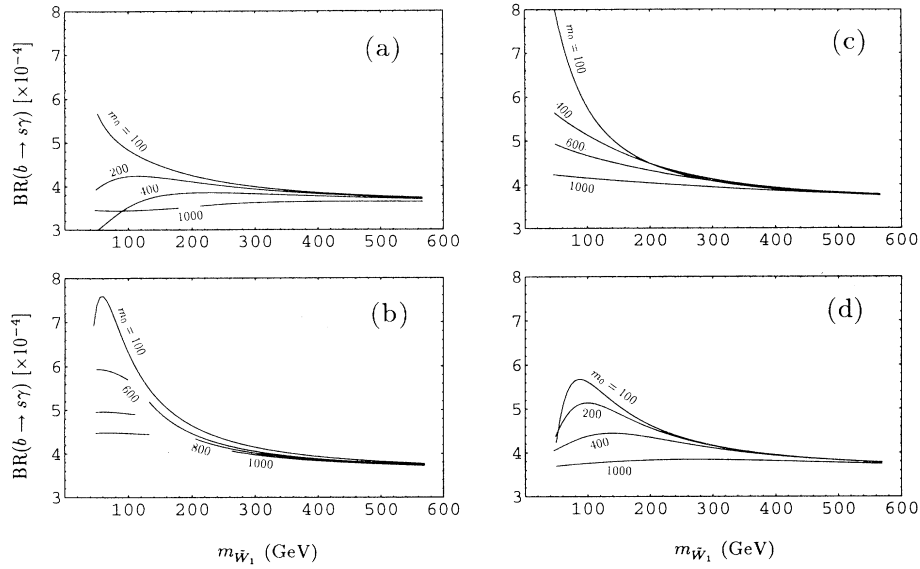


FIG. 1. Branching ratio  $B(b \rightarrow s\gamma)$  plotted with respect to the light chargino mass  $m_{\tilde{W}_1} > 45$  GeV for  $\tan\beta = 5.0$ , the running mass  $m_t = 160$  GeV,  $|A_t/m_0| = 0.5$ , and  $\alpha_G^{-1} = 24.11$ , and for (a)  $A_t < 0, \mu > 0$ ; (b)  $A_t < 0, \mu < 0$ ; (c)  $A_t > 0, \mu > 0$ ; and (d)  $A_t > 0, \mu < 0$ . All masses are in units of GeV. The standard model gives a branching ratio  $B(b \rightarrow s\gamma) = 3.55 \times 10^{-4}$ . The discontinuity in the  $m_0 = 1000$  GeV line for (a) and in the  $m_0 = 600, 800,$  and  $1000$  GeV lines for (b) is due to the light top squark turning tachyonic.

tive for the region in parameter space we have studied. Generally, the constraint that the light top squark remains nontachyonic is most stringent for  $A_t < 0$ . (2) The  $b \rightarrow s\gamma$  branching ratio increases when  $m_t$  increases. The dependence on  $m_t$  is correlated with other parameters. The parameters for Fig. 1 ( $m_t = 160, m_t^{\text{pole}} \simeq 165$ )

and those for Fig. 3 ( $m_t = 170, m_t^{\text{pole}} \simeq 175$ ) differ only in  $m_t$ . Remarkable is that a slight change in  $m_t$  (about 6%) for negative  $A_t$  [Figs. 1(a) and 1(b) vs Figs. 3(a) and 3(b)] eliminates a large part of the allowed parameter space. For example, while the values of  $m_0$  ranging from 100 to 1000 GeV are all allowed for  $m_t = 160$  GeV,

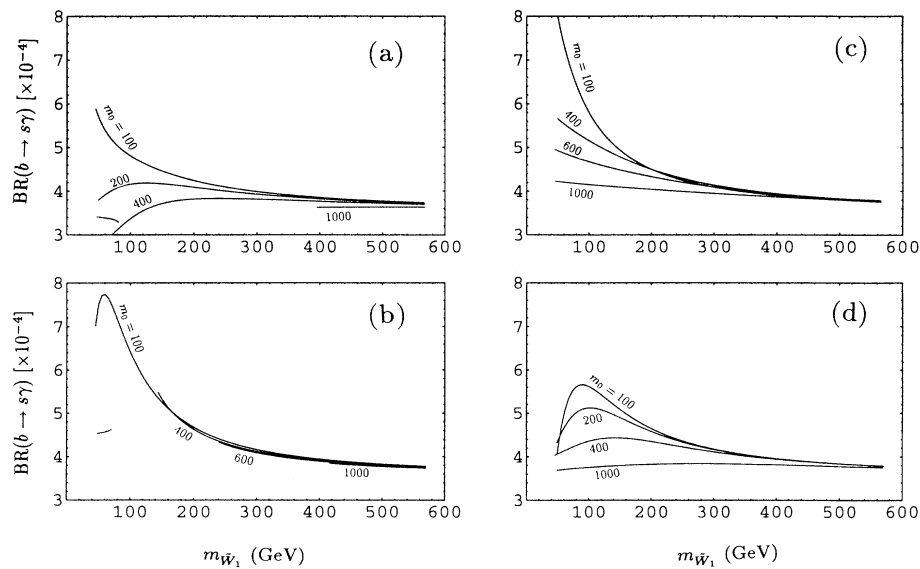


FIG. 2. Same as Fig. 1 for  $\alpha_G = 1/24.5$ . The discontinuity in the  $m_0 = 1000$  GeV line for (a) and for (b) is also due to the light top squark turning tachyonic, as in Fig. 1.

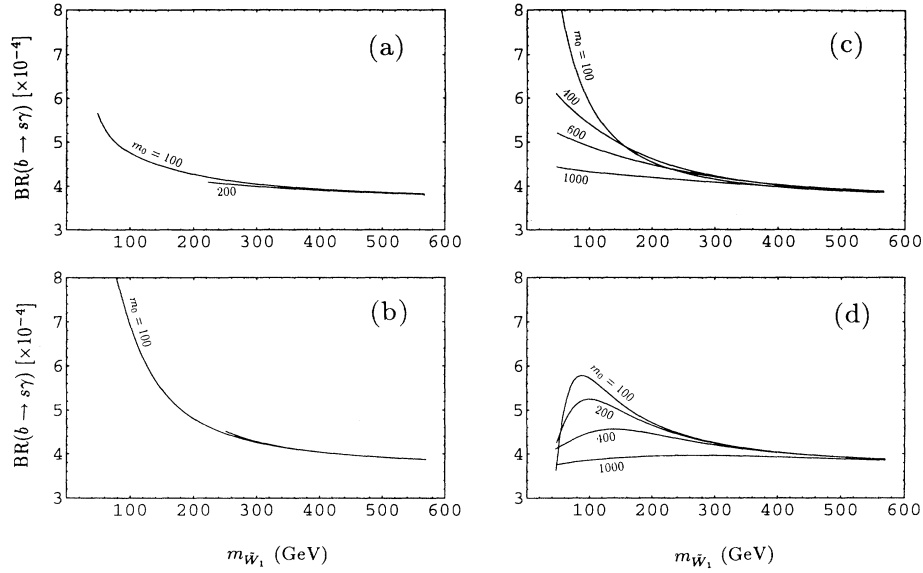


FIG. 3. Same as Fig. 1 for the running mass  $m_t = 170$  GeV. The standard model gives a branching ratio  $B(b \rightarrow s\gamma) = 3.66 \times 10^{-4}$ .

for both positive [1(a)] and negative [1(b)]  $\mu$ , the only allowed values of  $m_0$  are below 200 GeV for  $m_t = 170$  GeV for both positive [3(a)] and negative [3(b)]  $\mu$ . However, if  $A_t$  is positive [Figs. 1(c), 1(d), 3(c) and 3(d)], the same change in  $m_t$  does not significantly affect the allowed parameter domain. The reduction in parameter space can again be explained by the light top squark turning tachyonic. (3) Most interesting is the  $\alpha_G$  dependence of the branching ratio. To see this effect, let us compare Fig. 1 [ $\alpha_G^{-1} = 24.11$ , for which  $\alpha_s(M_Z) = 0.118$ ] with Figs. 2 [ $\alpha_G^{-1} = 24.5$ , for which  $\alpha_s(M_Z) = 0.113$ ]. For negative  $A_t$ , we again find that a larger part of the parameter space is excluded [1(a) vs 2(a), and 1(b) vs 2(b)], while for positive  $A_t$ , the allowed parameter space remains almost the same [1(c) vs 2(c), and 1(d) vs 2(d)]. Although the light top squark turning tachyonic is the reason for this, the physics involved is quite different from the above. Qualitatively, one can attribute this phenomenon to the fact that  $h_t$  is very close to its Landau pole, namely, a reduction in  $\alpha_G$  modifies various form factors defined in Ref. [24], making  $h_t$  closer to its Landau pole. This in turn is reflected in the light top squark turning tachyonic. The combined effect of simultaneous change in  $m_t$  and  $\alpha_G$  is very dramatic—the only allowed parameter space is  $m_0 = 100$  GeV for  $m_t = 170$  and  $\alpha_G^{-1} = 24.5$  when  $A_t$  is negative. This is because the simultaneous changes in both  $m_t$  and  $\alpha_G$  add up almost multiplicatively to aggravate the closeness to the Landau pole [4]. For positive  $A_t$ , the change is not as large, and the allowed domain

is still large. A similar reduction of the allowed domain also exists for  $A_t$  with other negative values—the more negative  $A_t$  is, the smaller the allowed domain remains.

In conclusion, we have performed a detailed study of the constraints from the  $b \rightarrow s\gamma$  decay on the MSGM. There are regions of the parameter space where the branching ratio exceeds the CLEO II bound, and this region is excluded. However, there still exists a large domain that satisfies the CLEO bound. A more accurate determination of the branching ratio would further constrain this model. An interesting result of the analysis is that very little allowed domain of the parameter space was found for  $A_t < 0$  and  $m_t$  in the vicinity of the CDF central value of 174 GeV for  $A_t < 0$ . Thus the allowed domain resides mostly in the  $A_t > 0$  region.

*Note added in proof.* Notice that in Eq. (3), we have left out a CKM factor  $\sim 0.95$ . To find the leading order (LO) QCD approximation, we need to leave out the braces in the denominator of Eq. (3) since this is a next-to-leading order (NLO) correction to  $b \rightarrow c\bar{\nu}_e$  process. Therefore, to find the LO QCD values for the branching ratio  $B(b \rightarrow s\gamma)$ , we need to reduce the branch ratios calculated in the paper by a factor of 0.8436. Thus, the LO QCD value for  $B(b \rightarrow s\gamma)$  for the standard model is  $2.99 \times 10^{-4}$ , compared to the most recent measurement for  $B(b \rightarrow s\gamma)$  of  $(2.3 \pm 0.67) \times 10^{-4}$  from CLEO, which came after this paper was finished and submitted.

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