Archives

Ref.TH.1647-CERN

SOME PROPERTIES OF ABSORPTIVE CORRECTIONS [*]

A.D. Martin [+] and P. Estabrooks ⊕
CERN - Geneva

ABSTRACT

Absorptive corrections to Regge pole exchange are isolated in a non-flip $\pi^- p \to \pi^- \pi^+ n$ reaction amplitude. The strength of absorption is found to decrease rapidly as the mass of the produced $\pi^- \pi^+$ system increases. In particular, the Williams model, which has been found to give a good fit of the small t data in the ρ region, is no longer satisfactory in the f region.

---

[*] To be published in the Proceedings of the 1973 Rencontre de Moriond.

[+] On leave of absence from the University of Durham.

⊕ Supported by the National Research Council of Canada.

Ref.TH.1647-CERN
4 April 1973

An outstanding puzzle in the description of high energy scattering processes is the nature of absorptive corrections to Regge pole exchange. It is generally agreed that such corrections exist and several model dependent attempts at obtaining their general systematics have been made 1). However, experimental results show that the predictive power of existing absorption models is limited. We are forced to conclude that, as yet, there is no reliable theoretically based prescription for calculating the size or structure of the absorptive effects. Amplitude analyses of spin 0 - spin $\frac{1}{2}$ scattering data tend to confirm the expectations that the absorptive corrections to Regge pole exchange are more important in helicity non-flip than in helicity flip amplitudes 2). However, empirical estimates of their strength require an assumption about the structure of the Regge pole contributions.

Here we emphasize that the high statistics data for the reaction $\pi^- p \to \pi^- \pi^+ n$ offer an excellent opportunity for studying the phase, s, $M_{\pi\pi}$ (and to a lesser extent t and L) dependence of absorption in a much more model independent way. The variables are indicated in Fig. 1. Consider high s, small t, and a region of $M_{\pi\pi}$ where only S and P wave $\pi^-\pi^+$ production is dominant. Then the observable s channel moments of the $\pi^-\pi^+$ angular distribution can be expressed in terms of the production amplitudes as follows 3)

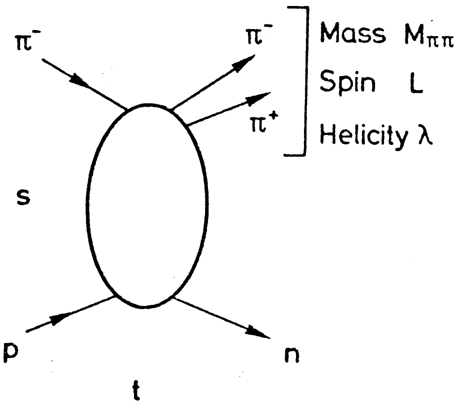


Figure 1 : Variables for the reaction $\pi^- p \to \pi^- \pi^+ n$.

$$
\begin{aligned}
\langle Y_0^0 \rangle &= |S|^2 + |P_0|^2 + |P_+|^2 + |P_-|^2 \\
\langle Y_0^1 \rangle &= 2\mathrm{Re}(S\, P_0^*) \\
\langle Y_1^1 \rangle &= \sqrt{2}\ \mathrm{Re}(S\, P_-^*) \\
\langle Y_0^2 \rangle &= \frac{1}{\sqrt{5}}\ (2|P_0|^2 - |P_+|^2 - |P_-|^2) \\
\langle Y_1^2 \rangle &= \sqrt{\frac{6}{5}}\ \mathrm{Re}(P_- P_0^*) \\
\langle Y_2^2 \rangle &= -\sqrt{\frac{3}{10}}\ (|P_+|^2 - |P_-|^2)
\end{aligned}
\qquad (1)
$$

In terms of the s channel helicity amplitudes $H^{L\lambda}_{\lambda_n\lambda_p}$ we have

$$S = H^{S}_{+-}, \quad P_o = H^{P,o}_{+-}, \quad P_- = \frac{1}{\sqrt{2}}\left(H^{P,1}_{+-} - H^{P,-1}_{+-}\right) \tag{2}$$

$$|P_+|^2 = \tfrac{1}{2}\left|H^{P,1}_{+-} + H^{P,-1}_{+-}\right|^2 + \tfrac{1}{2}\left|H^{P,1}_{++} + H^{P,-1}_{++}\right|^2 . \tag{3}$$

To order $1/s$ the amplitudes (2) arise from unnatural parity (π) exchange and Eq. (3) is due to natural parity (A_2) exchange. We have assumed that the (A_1 exchange) amplitudes

$$H^{S}_{++}, \quad H^{P,o}_{++} \qquad \text{and} \qquad H^{P,1}_{++} - H^{P,-1}_{++}$$

are negligible in comparison to the respective amplitudes in Eq. (2), apart from including the $O(1/s)$ contributions from π pole exchange as described in Ref. 3). The six observables of Eq. (1) determine $|P_+|$ and the magnitudes and relative phases of S, P_o and P_-. We work with s channel amplitudes since absorption is believed to be simpler in the s channel.

Before looking at the results of such an amplitude analysis we anticipate the t structure of the amplitudes and indicate how the absorptive correction can be isolated. For small t we expect the amplitudes of Eq. (2) to be dominated by π exchange and thus be of the form

$$P_o = M_{\pi\pi}\,\frac{\sqrt{-t'}}{t-\mu^2}, \qquad P_- = C - \frac{2t'}{t-\mu^2} .$$

For simplicity we have omitted the signature factor on the π pole terms and also slope factors of the form $\exp(bt)$. π exchange contributes equally to H^{-1}_{+-} and H^{1}_{+-}. The former is double helicity flip ($n=2$) and so is required to vanish in the forward direction like

$t' \equiv t - t_{min}$. Hence the t' in the π pole contribution to P_-. Similarly the A_2 exchange contributions to $H^{\pm 1}_{+-}$ must vanish like t'. However, for the $n = 0$ amplitude H^1_{+-} the π and A_2 absorptive corrections need not vanish in the forward direction and so can be distinguished from the vanishing pole contributions. Hence the term C in P_-. The cross-over zeros observed in the s channel $<Y^1_1>$ and $<Y^2_1>$ moments near $-t = \mu^2$ arise from the destructive interference between C and the π pole term in P_-. There may of course be absorptive corrections to H^0_{+-} and H^{-1}_{+-} but, since they are required to vanish in the forward direction just as their respective pole contributions, they are hard to isolate.

High statistics data exist for the reaction $\pi^- p \rightarrow \pi^- \pi^+ n$ at 17.2 GeV/c. An amplitude analysis of the observed s channel moments has been performed for successive 20 MeV $\pi^- \pi^+$ mass bins in order to extract $\pi\pi$ phase shifts $^{4)}$. We find two allowed solutions for the amplitude components. A typical example is shown in Fig. 2 ; the points are the results of amplitude analyses at successive t values for the mass bin $700 < M_{\pi\pi} < 720$ MeV. Rather than showing relative phases, S and P_- have been projected into components parallel and perpendicular to P_o on the Argand plot. The amplitudes S and P_o are relevant to $\pi\pi$ phase shift analysis $^{4)}$. However, here we are interested in the properties of C, the absorptive correction to H^1_{+-}. It occurs equally in P_- and P_+.

THE PHASE OF C

The absorptive correction C can be isolated with some confidence at small t because to a good approximation the π pole contribution to P_- is determined from a knowledge of P_o

$$P^{\pi}_{-} = \frac{2\sqrt{-t'}}{M_{\pi\pi}} \; P_o .$$

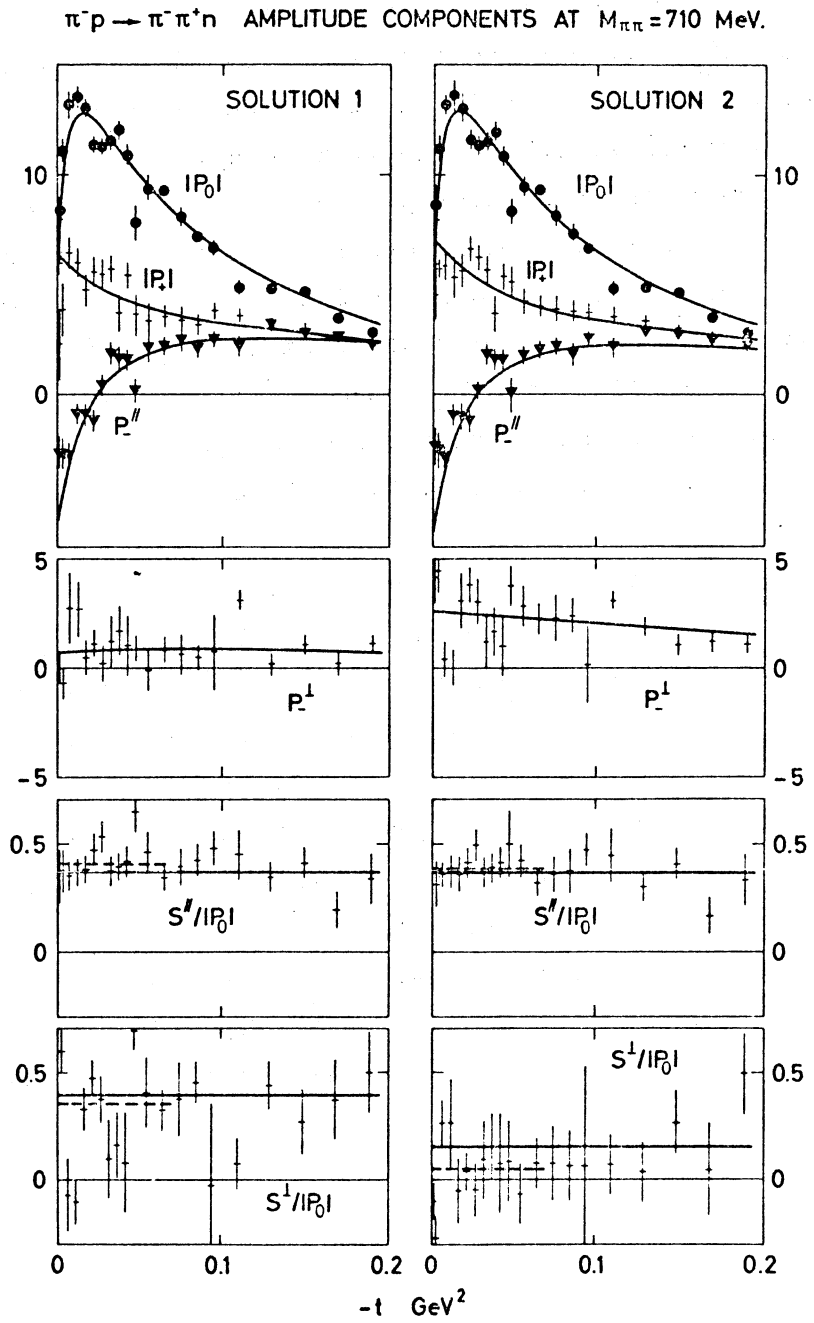


Figure 2 : The amplitude components for $\pi^- p \rightarrow \pi^- \pi^+ n$ at 17.2 GeV/c obtained from the observed s channel moments for the mass bin $700 < M_{\pi\pi} < 720$ MeV, taken from Ref. 4). Solution 1 is the physical solution

Thus the absorptive correction to H^1_{+-} is

$$C^{\parallel} = P^{\parallel}_{-} - P^{\pi}_{-}, \qquad C^{\perp} = P^{\perp}_{-}.$$

The underlying assumption is that H^1_{+-} is much more affected by absorption at small t than P_0. The results for $P^{\parallel}_{-}$ show the destructive interference of the π pole and absorptive contributions. Moreover $P^{\perp}_{-}$ specifies their relative phase. From Eqs. (1) we see that only the sign of the product $S^{\perp}P^{\perp}_{-}$ is determined and from Fig. 2 we notice that the magnitude of $P^{\perp}_{-}$ differs appreciably in the two solutions. It is here that the $\pi\pi$ phase shift analysis [4)] comes to our aid. For example at 710 MeV $S^{\perp}$ is found to be positive. Moreover, a comparison of the predictions of the $\pi^0\pi^0$ mass distribution with that available from $\pi^- p \rightarrow \pi^0 \pi^0 n$ data clearly selects solution 1 as the physical $\pi^- p \rightarrow \pi^- \pi^+ n$ amplitudes. The absorptive correction has a small positive component $C^{\perp}$. The components of P_- at the cross-over point $t \approx -\mu^2$ are sketched in Fig. 3. Such a relative phase would be obtained by the prescriptions [see for example Ref. 5)] which calculate C by convoluting a Regge P^{π}_{-} contribution with a dominantly imaginary elastic amplitude. However, absorptive corrections to A_2 exchange may also contribute to C. It is clear that reliable phase information at differing energies will be valuable for unravelling the π and A_2 absorptive corrections to C.

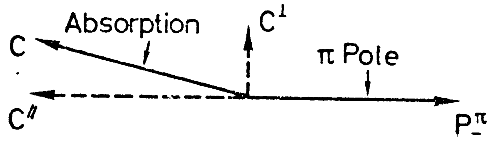


Figure 3 : the components of P_- for $-t \approx \mu^2$. P^{π}_{-} is the π pole contribution. C is the total absorptive correction to the π and A_2 contributions to H^1_{+-}.

s DEPENDENCE

Using $\pi^- p \rightarrow \pi^- \pi^+ n$ data [6)] at 4.5, 6.95, 15 and 17.2 GeV/c with the produced $\pi^- \pi^+$ system in the ρ mass band we calculate and compare the s dependence of the amplitudes P_0, P_- and P_+ as described in Ref. 7). The results are shown in Fig. 4. Since C contributes equally to P_+ and P_- we have two chances to study its energy dependence at small t. However, due to the destructive interference between the π pole and C contributions to P_-, the more reliable estimate will come from P_+. The results indicate that

$$\alpha^{C}_{\mathrm{eff}} \approx \alpha^{\pi}_{\mathrm{eff}}$$

at small t.

$M_{\pi\pi}$ DEPENDENCE

Rather than presenting the results of the general amplitude analysis as a function of $M_{\pi\pi}$ it is sufficient to consider the following simple analysis since it contains the essential features. Assume that $\pi^- p \rightarrow \pi^- \pi^+ n$ is described by π exchange except for the replacement

$$\frac{t'}{t-\mu^2} \rightarrow \frac{t'}{t-\mu^2} - C$$

in the evasive $H^{P,1}_{+-}$ amplitude. Take C to be a real constant parameter to be determined in each $M_{\pi\pi}$ bin. Make the same replacement in the evasive amplitudes for D wave $\pi^- \pi^+$ production. Assume all amplitudes have a common slope b, that is, contain a factor $\exp[b(t-\mu^2)]$. The choice $C = 1$ corresponds to the Williams model [8)] or Poor Man's Absorption [9)]. In total we have seven parameters C, b and the magnitudes and relative phases of the amplitudes S, P_0 and D_0. This simple parametrization gives a good description of all the

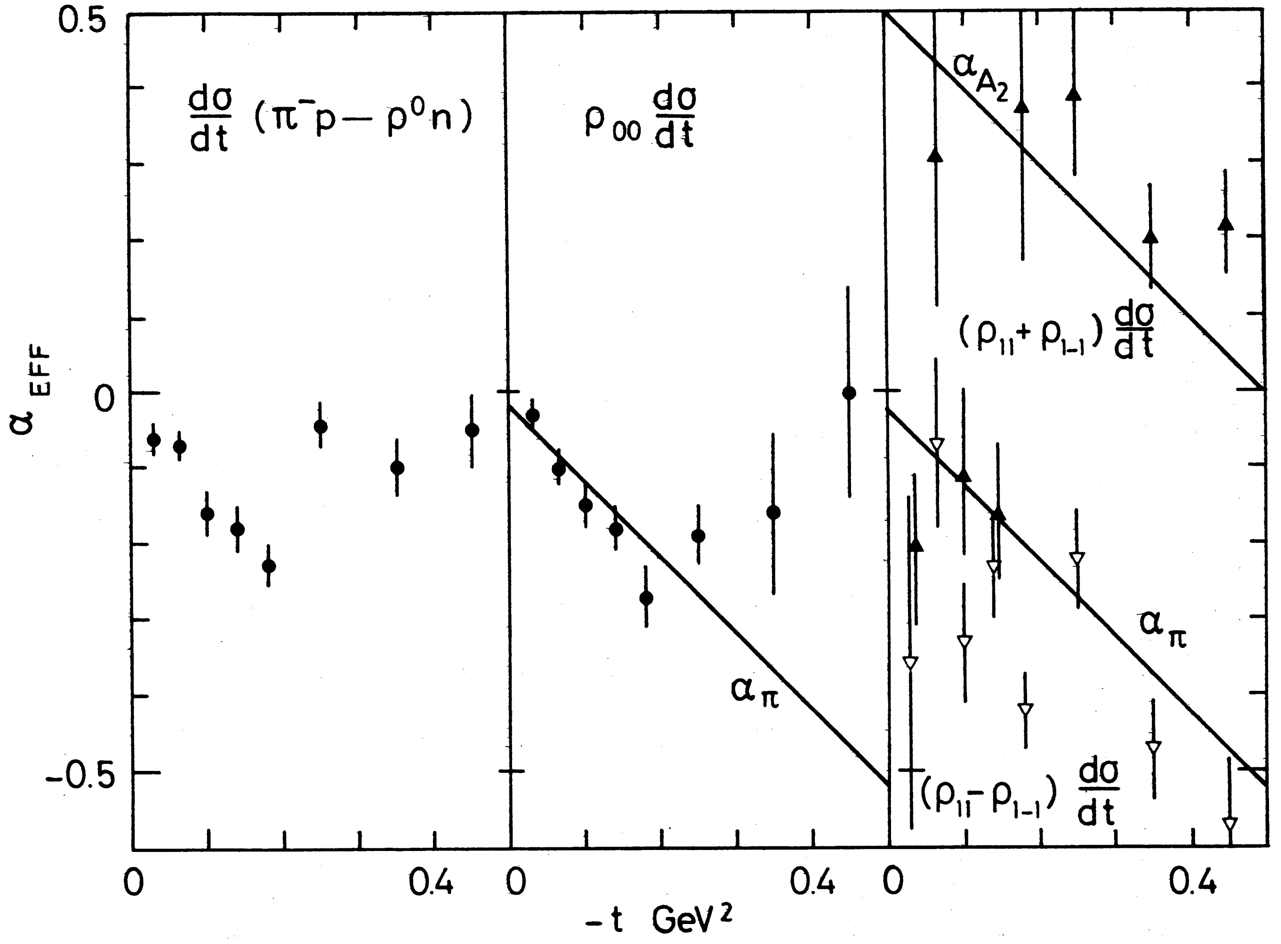


Figure 4 : The effective Regge trajectories for the various components of the $\pi^- p \rightarrow \rho^0 n$ cross-section obtained from data at 4.5, 6.95, 15, 17.2 GeV/c as described in Ref. 7). The solid and open triangles are for $|P_+|^2$ and $|P_-|^2$ respectively.

observed moments, $<Y^J_M>$ with $J \leq 4$, in the range $0 < -t < 0.1$ GeV^2. The values obtained for C and b are shown as a function of $M_{\pi\pi}$ in Fig. 5. A surprising result is the decrease of the strength of absorption with increasing $M_{\pi\pi}$.

Ochs and Wagner 10) independently obtain this result by taking suitably weighted ratios of the experimental t channel moments $<Y^J_1> / <Y^J_0>$ integrated over $0.01 < -t < 0.15$ GeV^2. Moreover they observe the same tendency for all J values.

Since the $\pi^+\pi^-$ system is produced in higher spin (L) states as $M_{\pi\pi}$ increases it could be that the strength of absorption really decreases with increasing L (and not $M_{\pi\pi}$). In an attempt to check this we allowed different absorption parameters, C_P and C_D, for the evasive P and D wave amplitudes. However within the errors no systematic dependence of C on L emerged.

The decrease of absorption with increasing $M_{\pi\pi}$ is hard to understand. The conventional absorption prescription could produce such an effect if the slope b_P of the unabsorbed Regge pole amplitude increased with $M_{\pi\pi}$. Assuming the dominant S, P_0, D_0 amplitudes are little affected by absorption then $b_P \approx b$, and no such trend is seen in Fig. 5.

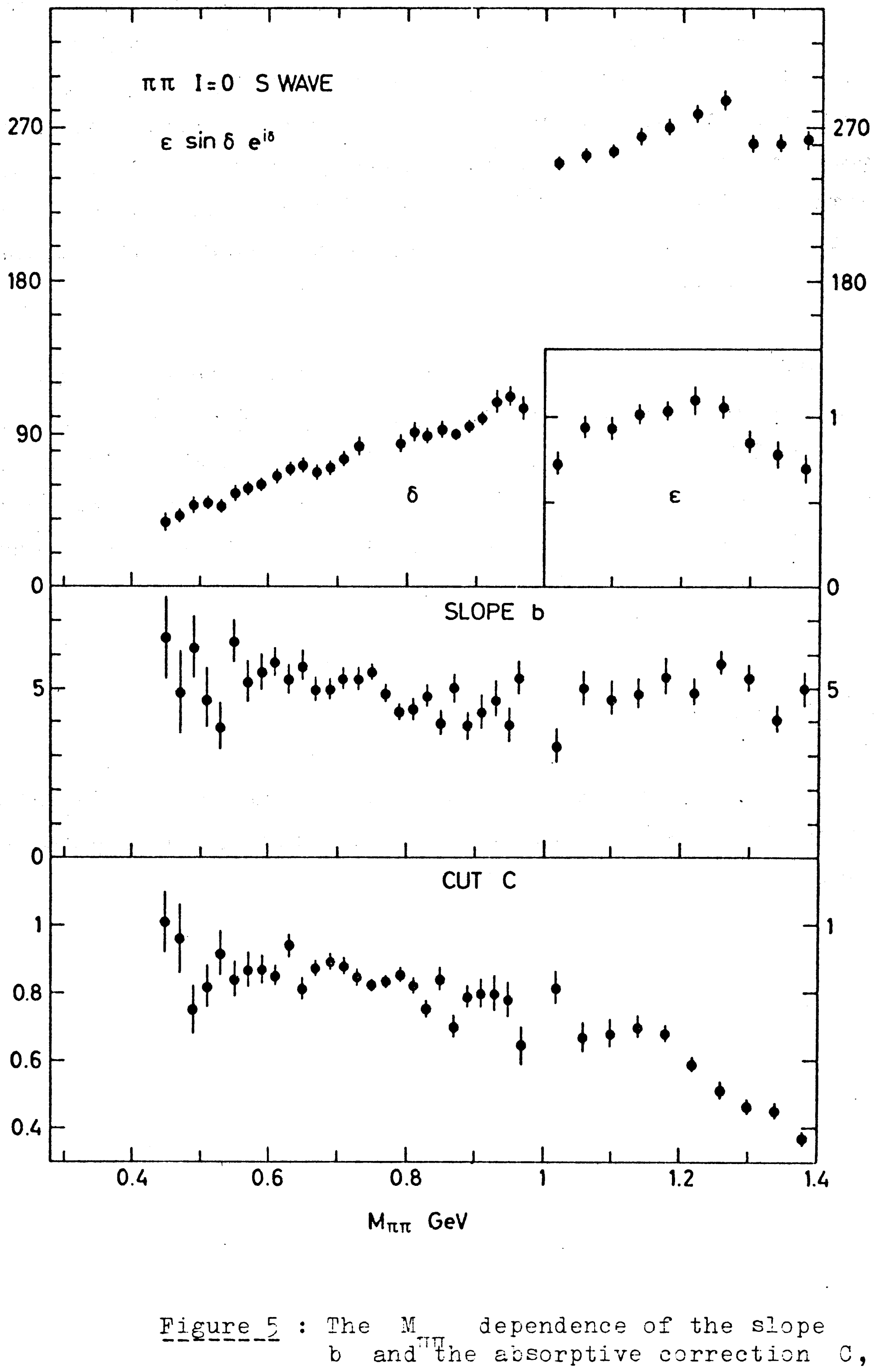
An alternative explanation is based on the use of finite mass sum rules and duality to extrapolate the triple Regge limit to low $M_{\pi\pi}$. In this way, Hoyer, Roberts and Roy 11) predict that on average the $M_{\pi\pi}$ dependence of the resonance production cross-section is

$$< \frac{d\sigma}{dt} > = < M^2_{\pi\pi} >^{\frac{1}{2} - 2\alpha_i(t)}$$

where α_i is the exchanged trajectory. As $M_{\pi\pi}$ increases they thus predict that on average :

(a) π exchange increases relative to A_2 exchange amplitudes as $M_{\pi\pi}$;

(b) antishrinkage : the slope decreases as log $M_{\pi\pi}$.



<u>Figure 5</u> : The $M_{\pi\pi}$ dependence of the slope b and the absorptive correction C, taken from Ref. 4.

The $\pi^- p \to \pi^- \pi^+ n$ data at 17.2 GeV/c show definite support for (a) but not for (b). In a finite energy sum rule and Regge analysis of pion photoproduction data Worden 12) estimates that 40% of C at small t is attributable to the absorptive corrections associated with A_2 exchange. Thus noting (a), he obtains a decrease of C with increasing $M_{\pi\pi}$. However, the amount of the observed decrease, and to a lesser extent, the phase and observed s dependence suggest the explanation is more complicated.

Of course, the $\pi^- p \to \pi^- \pi^+ n$ data have other important implications of absorption, such as the indication of a dip in $\rho^H_{00}\, d\sigma/dt$ at $-t \sim 0.5$ GeV^2 in the 4.5 and 6.95 GeV/c data which is not visible in the higher statistics 17.2 GeV/c data. However, here we prefer to emphasize how many of the empirical properties of absorption can be studied in a more model independent way.

ACKNOWLEDGEMENTS

It is a pleasure to thank Chris Michael for useful discussions and to acknowledge the collaboration of the members of the CERN-Munich experiment.

REFERENCES

1) For example :

M. Ross, F. Henyey and G.L. Kane - Nuclear Phys. B23, 269 (1970) ;

H. Harari - Ann.Phys. 63, 432 (1971).

2) C. Michael - Proceedings of the 4th International Conference on High Energy Collisions, Oxford (1972).

3) P. Estabrooks and A.D. Martin - Phys.Letters 41B, 350 (1972).

4) P. Estabrooks, A.D. Martin, G. Grayer, B. Hyams, C. Jones, P. Weilhammer, W. Blum, H. Dietl, W. Koch, E. Lorenz, G. Lütjens, W. Männer, J. Meissburger, W. Ochs and U. Stierlin - Invited talk by A.D. Martin, to be published in Proceedings of ππ International Conference, Tallahassee, Florida.

5) F. Henyey, G.L. Kane, J. Pumplin and M. Ross - Phys.Rev. 182, 1579 (1969).

6) W. Barish and W. Selove - to be published ;

J.A.J. Matthews et al. - Nuclear Phys. B32, 336 (1971) ;

F. Bulos et al. - Phys.Rev.Letters 26, 1453 (1971) ;

G. Grayer et al. - Proceedings of the 4th International Conference on High Energy Collisions, Oxford (1972).

7) P. Estabrooks and A.D. Martin - Phys.Letters 42B, 229 (1972).

8) P.K. Williams - Phys.Rev. D1, 1312 (1970).

9) G.C. Fox - Proceedings of Cal.Tech. Conference "Phenomenology in Particle Physics", p. 703 (1971).

10) W. Ochs and F. Wagner - M.P.I. Munich Preprint (February 1973).

11) P. Hoyer, R.G. Roberts and D.P. Roy - Rutherford Laboratory Preprint RPP/T/35 (1972).

12) R.P. Worden - Nuclear Phys. B37, 253 (1972) and to be published in Proceedings of Daresbury Meeting on Pion Exchange (1973).