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THE IMPORTANCE OF THE STRUCTURE OF THE POMERON \*)

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#### ABSTRACT

The derivation of the two-Pomeron cut discontinuity and decoupling results for Pomeron vertices from both the t channel and the s channel is discussed. It is emphasized that it is essential to consider the structure of the Pomeron in terms of its s channel intermediate states, if consistency between the s and t channel results is to be achieved.

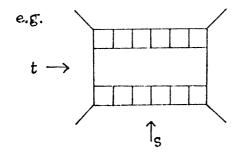
<sup>\*)</sup> Presented to the VIII Rencontre de Moriond, Méribel-les-Allues, March 4-16, 1973.

## THE IMPORTANCE OF THE STRUCTURE OF THE POMERON

The whole of this talk will be based on the assumption that the Pomeron is a simple factorisable Regge pole (with intercept one) accompanied by the associated multi-Pomeron cuts which t-channel unitarity certainly requires. This is the only j-plane structure for the Pomeron which is known to be consistent with t-channel unitarity. I shall talk about both the sign of the two-Pomeron cut and the decoupling results for the Pomeron which are derived from s-channel unitarity. The common message will be that it is essential to consider the "structure" of the Pomeron in terms of its s-channel intermediate states if consistency between s-channel and t-channel results is to be achieved.

## Regge cuts from t-channel unitarity

Because of its technical complexity this is a difficult subject to explain simply. It was a long time ago that Mandelstam<sup>1)</sup> originally emphasized that the Feynman graphs which are associated with the exchange of two Regge poles, and which might be thought to give rise to Regge cuts, have a simpler intermediate state structure in the t-channel than in the s-channel.



The sum over graphs of this form is associated with many s-channel states but essentially only with the four-particle state in the t-channel. More specifically since

it is the asymptotic limit  $s \to \infty$  with t fixed which we want to consider, we can clearly consider this limit in the neighbourhood of

the lowest intermediate state in the t-channel, which is the fourparticle state. In fact it was by considering the t-channel discontinuity that Mandelstam was able to show that the set of graphs shown
above does not give rise to a two-Reggeon cut but the set of graphs
where the ladders are joined by non-planar couplings does.

Mandelstam's work suggested therefore that the two-Reggeon cut could be studied through the four-particle unitarity integral. In principle the asymptotic behaviour of this integral can be studied in terms of its Froissart-Gribov continuation to complex j. Not long after Mandelstam's work Gribov, Pomeranchuk and Ter-Martirosyan<sup>2</sup>) suggested a general S-matrix approach to this problem which was based on Mandelstam's analysis. In fact I shall illustrate this approach by considering the three-particle integral

in which the Reggeon-particle cut is generated. This is simpler to consider than the four-particle integral and although the Reggeon-particle cut does not contribute to the high-energy behaviour for negative t it is otherwise analogous to the two-Reggeon cut.

We consider first the partial wave projection of the integral

$$I_{j}(t) = \frac{1}{16(2\pi)^{3}3!} \int_{4m^{2}}^{(t^{\frac{1}{2}}-m)^{2}} dt_{1} \frac{\lambda^{\frac{1}{2}}(t,t_{1},m^{2})}{t} \left(\frac{t_{1}-4m^{2}}{t_{1}}\right)^{\frac{1}{2}}$$

$$\times \sum_{|n| \leq j} \sum_{\ell \geq |n|} a_{j\ell n}(t,t_{1}) a_{j\ell n}^{-}(t,t_{1})$$
(1)

where  $\ell$ , n,  $t_1$  label the angular momentum, helicity, and energy of a selected pair of the three intermediate state particles. Gribov et al. then showed that if it was assumed that the multiple partial—wave projections  $a_{j\ell n}$  had Froissart-Gribov continuations to complex j,  $\ell$ , n and that the Froissart-Gribov continuation of (1) could be obtained by performing Sommerfeld-Watson transformations on the sums over n and  $\ell$ , then a cut would be generated as expected.

Unfortunately, subsequent investigation<sup>3,4)</sup> of these technical assumptions suggested that this approach could not be carried through and Gribov moved back to perturbation theory to justify the construction of a Reggeon Field Theory description of Regge cuts<sup>5)</sup>. In fact, I have now shown that these technical problems can be overcome. Ignoring odd signature contributions the continuation of (1) to complex j can be written in the form<sup>6)</sup>

$$I(j,t) = \frac{-1}{64(2\pi)^{3}3!} \int_{4m^{2}}^{(t^{\frac{1}{2}}-m)^{2}} dt_{1} \frac{\lambda^{\frac{1}{2}}(t,t_{1},m^{2})}{t} \left(\frac{t_{1}-4m^{2}}{t_{1}}\right)^{\frac{1}{2}}$$

$$\times \sin \frac{\pi}{2} j \int_{\sin \frac{\pi}{2} n}^{dn} \sum_{\substack{\ell=n=0\\ \ell+n=0}}^{\infty} \frac{a(j,\ell,n) \ a^{-}(j,\ell,n)}{\sin \frac{\pi}{2} \ (j + n)}.$$
(2)

The basic points in deriving this result were the use of multiple integral representations of amplitudes to define the continuations  $a(j,\ell,n)$  and the proper treatment of the helicity sum. This sum is written as a helicity contour integral with the contour in the form shown in Fig. 1.

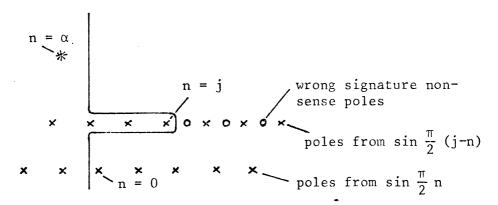


Fig. 1 Helicity contour

At even integer j the two sets of poles shown in the figure pinch the contour to give a pole whose residue is a finite sum over helicities and which is isolated by the factor  $\sin \pi/2$  j in (2). The proper treatment of helicity continuations and sums is also important in the development of a Sommerfeld-Watson approach to multi-Regge theory<sup>6</sup>,<sup>7</sup>).

We can now discuss the generation of a cut in (2)  $^8$ . A Regge pole at  $\ell=\alpha$  in a(j, $\ell$ ,n) will give poles in the n-plane at  $n=\alpha$   $\alpha-1$ , ... in the sum over  $(\ell-n)$  in (2). The amplitudes a(j, $\ell$ ,n) and a  $(j,\ell,n)$  will have nonsense wrong-signature inverse square root branch-points at j=n-1. The product of these will give a pole at j=n-1 which can pinch with the pole at  $n=\alpha$  to give a pole in the integral over  $\ell$  in (2) at  $j=\alpha(\ell_1)-1$ . Using  $N_{\alpha}(j,\ell)$  to denote the "fixed-pole residue" of a(j, $\ell$ ,n) at j=n-1,  $n=\ell=\alpha$  we can write the relevant part of (2) in the form

$$\sin \frac{\pi}{2} j \frac{i}{16(2\pi)^{3}3!}$$

$$(t^{\frac{1}{2}-m})^{2} dt_{1} \frac{\lambda^{\frac{1}{2}}(t,t_{1},m^{2})}{t} \left(\frac{t_{1}-4m^{2}}{t_{1}}\right)^{\frac{1}{2}} \frac{N_{\alpha}(j,t)N_{\alpha}^{-}(j,t)}{\sin \frac{\pi}{2} \alpha(t_{1}) \left[j-\alpha(t_{1})+1\right]}$$

Now we can see that when the pole at  $j = \alpha(t_1) - 1$  hits the end point of the integral at  $t_1 = (t^{\frac{1}{2}} - m)^2$  there will be a singularity at

$$j = \alpha [(t^{\frac{1}{2}} - m)^2] - 1$$
 (4)

This is the Mandelstam or Reggeon-particle cut. Further manipulation of the unitarity equations satisfied by  $N_{\alpha}(j,t)$  gives the discontinuity across the cut in the form

$$\frac{i \sin \frac{\pi}{2} j}{8} \int_{-\pi}^{\pi} dt_1 \frac{\lambda^{\frac{1}{2}}(t,t_1,m^2)\delta[j-\alpha(t_1)+1]N_{\alpha}(j^+,t)N_{\alpha}(j^-,t)}{t \sin \frac{\pi}{2}\alpha(t_1)}.$$
(5)

Pictorially we can represent the generation of the cut by

$$\longrightarrow \longrightarrow \bigcirc (6)$$

So the discontinuity is given in terms of the fixed-pole residue of the three-particle/Reggeon amplitude.

The generation of the two-Reggeon cut in the four-particle unitarity integral can be discussed in exactly the same way.

The discontinuity now involves the fixed-pole residue of the two particle/two Reggeon amplitude  $N_{\alpha_1}$ ,  $\alpha_2$  (j,t)

disc a(j,t) =
2 Reggeon
 cut

$$-\frac{i \sin \frac{\pi}{2} j}{2^5} \int_{t_1}^{t_1} dt_1 dt_2 \frac{\left[-\lambda(t,t_1,t_2)\right]^{\frac{1}{2}} \delta\left[j - \alpha_1(t_1) - \alpha_2(t_2) + 1\right]}{t \sin \frac{\pi}{2} \alpha_1 \sin \frac{\pi}{2} \alpha_2}$$

$$\times N_{\alpha_1\alpha_2}(j^+,t)N_{\alpha_1\alpha_2}(j^-,t) . \qquad (8)$$

The real analyticity property of  ${\rm N}_{\alpha_1\alpha_2}$  gives that

$$\frac{\lambda}{t} N(j^{+}) N(j^{-}) = \left| \left( \frac{\lambda}{t} \right)^{\frac{1}{2}} N(j^{+}) \right|^{2}$$
 (9)

and the sign of the imaginary part of (8) is determined to be negative.

So if  $\alpha_1 = \alpha_2 = \text{Pomeron}$  we have the result that the two Pomeron cut contributes negatively to the total cross-section. It is very difficult to give a simple explanation of why the sign comes out negative except to point out that the method is clearly independent of the strength of the couplings in the theory and so is the sign. This means that the sign must be the same as that obtained from the Mandelstam graphs in weak-coupling perturbation theory. It is not possible therefore to look for a positive sign<sup>9)</sup> from the complicated analytic continuation from positive to negative t which we have not discussed. A complete discussion of this continuation will be given in a future paper 10).

# Regge cuts from the s-channel

There have been many attempts to understand Regge cuts through the s-channel and the most recent are the papers by Abarbanel 11) and Chew9) which obtain a positive sign for the two-Reggeon discontinuity. This positive sign is obtained by considering only the contributions to the two-Reggeon cut coming from multi-Reggeon production amplitudes. However, once an s-channel structure for a Reggeon is admitted a second method of generation of the cut becomes available and it is this which ultimately gives the negative sign for the cut. This point has recently been emphasized by Halliday and Sachrajda 12) in a detailed study of weak-coupling perturbation theory. Essentially the same point has been made in the past by Caneschi 13), and also by Dash, Fulco and Pignotti<sup>14)</sup> and more recently by Blankenbecler<sup>15)</sup> and by Neff<sup>16)</sup>. The argument has usually been presented in terms of unitarity corrections to multi-Regge exchange changing the sign of the AFS cut<sup>17</sup>). Neff has phrased the argument in terms of the currently popular two-component model of particle production and this is probably the easiest to understand.

Two production mechanisms are assumed which can be pictorially represented as

The first mechanism is some general multiperipheral "single fireball" production mechanism which builds up the constant total cross-section

$$\frac{1}{2} + \frac{1}{2} - \frac{1}{2} \sim I_m$$

where Trepresent the contribution of the Pomeron to the four-particle amplitude. The second mechanism corresponds to "diffractive" production of two fireballs and gives positive contributions to the two-Pomeron cut contribution to the total cross-section. Amongst this contribution is the original AFS contribution from the elastic cross-section<sup>17</sup>)

$$\left(\begin{array}{c} \\ \\ \\ \\ \end{array}\right) \begin{array}{c} \\ \\ \\ \end{array} \left(\begin{array}{c} \\ \\ \\ \end{array}\right) \begin{array}{c} \\ \\ \\ \end{array} \begin{array}{c} \\ \\ \\ \end{array} \begin{array}{c} \\ \\ \end{array}$$

The positivity of this cut contribution is, of course, obvious to everyone. However, if we have two production mechanisms the first thing unitarity does is to introduce absorptive corrections of one by the other. So to the multiperipheral production mechanism we must also add a rescattering effect which we can represent as

There are now interference terms in the total cross-section of the form

$$\equiv T i \text{ Im } T$$
 (15)

Since the Pomeron is pure imaginary this gives a negative contribution to the two Pomeron cut. Since there are two interference terms of this sort they give exactly twice the AFS contribution and so exactly reverse the sign of this contribution.

The positivity of the total cross-section is maintained by adding

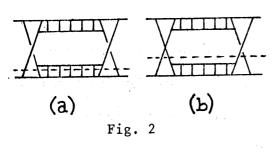
$$\left| \frac{1}{2} + \frac{1}{2} \right|^2 \sim \frac{1}{2}$$

$$(16)$$

which gives a three Pomeron cut. We, therefore, see that the negative sign of the two-Pomeron cut is explicitly compensated for by the positive sign of the three-Pomeron cut.

The above argument is clearly over-simplified and incomplete but it can be made part of an iterative solution of the unitarity equations 13-15) and the reversal of the sign of the two-Pomeron cut remains. The above argument also describes in essence the way an overall negative sign emerges for the Mandelstam graphs in perturbation theory. Of course, the contribution from the two-particle intermediate state is exactly cancelled in perturbation theory and we have to consider at least the four-particle intermediate state so

that non-planar couplings of the Pomerons are allowed. However, Halliday and Sachrajda have shown that for the Mandelstam graphs the overall negative sign comes from s-channel "cuts" through the graph of the form shown in Fig. 2a which exactly reverse the sign of the con-



tributions from cuts through the graphs of the form of Fig. 2b.

Therefore cuts through a Reggeon exactly reverse the sign of contributions from cuts which leave the Reggeon untouched.

So the negative sign of the two-Pomeron cut is directly related to the vital equation (11) above. This equation says simply that the Pomeron has a structure in terms of s-channel intermediate states<sup>9)</sup>. This effect is neglected in the s-channel discussion of cuts by Abarbanel<sup>11)</sup> and by Chew<sup>9)</sup> and also in the sum rule treatment of cuts given by Kang, Moore and Nicolescu<sup>18)</sup>. This is why these authors obtain a positive sign. Since it is t-channel iteration which forces a Regge pole to look something like a Feynman ladder graph and so have many s-channel intermediate states it is not surprising that it is necessary to take this structure into account to obtain consistency between the s- and t-channel.

## Pomeron decoupling theorems

We now want to discuss the relevance of the structure of the Pomeron to the decoupling results that have been derived from s-channel unitarity expressed in the form of inclusive sum rules. The final consequence of these arguments is that a Pomeron pole with intercept

one must decouple from the total cross-section. Since the total cross-section would then be given by the two-Pomeron cut it is clear that this decoupling cannot be consistent with the negative sign cut. Since the negative sign of the cut is connected with the structure of the Pomeron it must be important to consider this structure in the decoupling arguments.

The first constraint derived from the unitarity sum rules was the triple Pomeron zero.

## The triple Pomeron zero

If the triple Pomeron coupling > which appears in the triple

Pomeron contribution to the one-particle inclusive cross-section does

not vanish when all three Pomerons have zero mass, then integrating

this contribution will give a total cross-section increasing like

log log s. It is therefore concluded that the zero mass triple

Pomeron coupling must vanish<sup>19)</sup>. However, once a structure is allowed

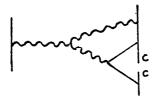
for the Pomeron the situation becomes much more subtle. Pictorially

the integral of the triple-Pomeron coupling can be represented as



The loop integral is now analogous to that considered above for the AFS cut. This suggests that if we take account of the structure of the Pomeron then there may be cancellations analogous to the cancellation of the AFS cut that we have already discussed. In fact Halliday and Sachrajda have shown that in the Feynman ladder graph model this

contribution would be exactly cancelled by diagrams of the form (the momentum of particle c being integrated over)



where the Reggeon has been "cut open" at one end. The ladder graph model cannot give a Pomeron with unit intercept but Halliday and Sachrajda show that in general a contribution of this form will produce a negative log log s contribution to the total cross-section. Of course, if this diagram does cancel the triple Pomeron contribution then since it is non-leading relative to the triple Pomeron contribution in the internal triple Pomeron limit, it must give a negative contribution to the inclusive cross-section in some region of phase space. Again this negative contribution may be cancelled by yet further contributions-possibly associated with multi-Pomeron cuts. We can say therefore that the ladder model does not give a concrete example of how the vanishing of the triple-Pomeron coupling may be avoided but it does suggest that the structure of a Pomeron will be such as to make the whole discussion of the asymptotic behaviour of sum rule integrals highly non-uniform. As a result it is not possible to make any statement about how the sum rules are satisfied once contributions to multiparticle amplitudes are allowed which do not correspond to any simple Regge diagrams but rather to unitarity cuts through Reggeons. These contributions cannot be considered in detail without a knowledge of the structure of a Reggeon.

Although the sum rule cannot be said to require the triple Pomeron zero the zero does seem to be required by t-channel unitarity. t-channel unitarity gives the Regge cut discontinuity formulae, and these formulae can be regarded as unitarity equations governing the interaction of a Reggeon pole with its cuts. These equations can be solved using either S-matrix or field theory techniques. The zero mass triple Pomeron vertex which appears in the inclusive cross-section can be identified with the triple Pomeron vertex governing the interaction of a Pomeron pole with its cuts. In a Reggeon field theory the decoupling comes about from requiring that the bare Pomeron intercept of one not be renormalized. In this context the triple Pomeron decoupling was discovered four years ago by Gribov and Migdal<sup>20</sup>). Bronzan<sup>21</sup>) has considered a model containing just sums over Pomeron ladder graphs which simply illustrates this point. In

where the lines represent Pomerons. The integral equation for  $\Gamma_{ppp}$  given by this model has an effective singular potential coming from the unrenormalized Pomeron exchange in the rungs of the ladders. This singular potential leads directly to  $\Gamma_{ppp}$  vanishing at the zero mass point.

From the S-matrix point of view the triple Pomeron decoupling can be understood as resulting from the collision of the Pomeron pole

and two-Pomeron cut<sup>22</sup>). The Pomeron amplitudes  $\approx 0 \approx$ ,  $\approx 0 \approx$  and  $\equiv 0 \equiv$  have the N/D representations<sup>2</sup>,<sup>8</sup>)

$$\approx 0 \approx = \frac{1}{B(t,j) + \frac{1}{\alpha' \left(\frac{1}{4}t\right)} \log \left[j - \alpha_c(t)\right]}$$
(17)

$$\approx 0 \approx = \frac{A^{\frac{1}{2}}}{B(t,j) + \frac{1}{\alpha'(\frac{1}{4}t)} \log [j - \alpha_c(t)]}$$
 (18)

$$= \frac{A(t,j)}{B(t,j) + \frac{1}{\alpha'\left(\frac{1}{4}t\right)} \log\left[j - \alpha_{c}(t)\right]}$$
(19)

where A and B do not have the two-Pomeron cut at  $j = 2\alpha_c(t) = 2\alpha_p(t/4) - 1$ . To obtain a self-consistent Pomeron trajectory from the denominator in these equations and to obtain the negative sign for the two-Pomeron cut it is necessary<sup>23</sup> to have a second order CDD pole in A and B which passes through j = 1 at t = 0. This pole is arranged to give a finite Pomeron contribution to T at t = 0, but because  $A^{\frac{1}{2}}$  appears in (18) in contrast to the A in (19) the Pomeron contribution to T appears at T appears at T and T are vanishes at T and T are vanishes at T and T are required triple Pomeron decoupling. The presence of CDD poles in the N/D representation is directly related to the singular potential that appears in the field theory approach to the problem.

Cardy<sup>24)</sup> has recently shown that this mechanism which produces the triple Pomeron zero also suppresses cut contributions to the inclusive triple Pomeron limit by  $(\log M^2)^{-2}$  instead of the single logarithm characteristic of cut contributions to two-body processes.

Finally, we consider the implications of the triple Pomeron zero.

## The Reggeon-Pomeron-particle vertex

There have been two arguments given that the zero in the triple Pomeron vertex has implications for the Reggeon-Pomeron-particle vertex. The aim is to show that the vertex  $\stackrel{P}{\longleftarrow}$  vanishes when the Pomeron has zero mass. This result can then be continued to the particle pole on the Reggeon trajectory to conclude that the Pomeron must decouple from the total cross-section<sup>25)</sup>.

Abarbanel, Gribov and Kanchelli $^{26}$ ) considered the subenergy discontinuity formula

$$disc_{M^2} = \sum_{n}$$
(20)

Using the Schwartz inequality together with the Mueller discontinuity formula gives

$$\begin{vmatrix} disc \\ M^2 \end{vmatrix} \le \begin{vmatrix} disc \\ M^2 - 1 \end{vmatrix} = \begin{vmatrix} disc \\ M^2 \end{vmatrix}$$
 (21)

Taking the multi-Regge limit and choosing the quantum numbers of the particles so that the two outer exchanges are Pomerons but the central one is a Reggeon, and going to zero mass of the Pomerons, gives an inequality which can be written in the form

$$|A \text{ Disc } (R \text{ p})|^2 \leq |P| |P| |B|$$
 (22)

The vanishing of the right-hand side then requires the left-hand side to vanish and this appears to give the required result. However, it was shown by Moen and White<sup>27)</sup> that taking the discontinuity of Removes that part of the vertex giving the total cross-section. In this case taking account of the singularity structure of the Reggeon is sufficient to weaken the otherwise strong result.

Another derivation of the vanishing of the R vertex has been given by Jones, Low, Tye, Veneziano and Young<sup>28</sup>). They consider the inclusive sum rule relating the one and two-particle inclusive cross-sections and obtain an inequality which pictorially we can represent as

The vanishing of P at zero Pomeron mass then requires of to vanish similarly. Taking the internal Regge limit so that

and using factorisation then requires \( \ldots \) to vanish. Since particle d can carry quantum numbers the internal exchange can be a Reggeon and this gives the required result. If both the amplitudes involved here contained only Regge poles so that the Pomeron contributions gave the leading asymptotic behaviour even in t < 0 then this result could only be avoided if non-uniformities were present at t = 0 which we do not expect to be the case for Regge pole amplitudes. However, once cuts are taken into account, together with the structure of

Reggeons and Pomerons, then non-uniformity of the asymptotic limits involved can again be expected to prevent this argument going through for analogous reasons to those we discussed when considering the vanishing of the triple Pomeron vertex. This has been discussed by Cardy and White in the context of the Feynman ladder graph model<sup>10</sup>,<sup>29</sup>).

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