nucleon in vector-spinor theory.[3] Here the presence of spin and nonplanar graphs, together with radiative corrections and renormalization, all play important roles in showing that the nucleon lies on a Regge trajectory through sixth order of perturbation theory. Attempts to extend this result to higher orders have met with great difficulty,[3] and perhaps our methods will prove more useful here than the usual techniques.

ACKNOWLEDGMENT

We are grateful to Alex Dragt for several useful conversations.

PHYSICAL REVIEW VOLUME 187, NUMBER 5 25 NOVEMBER 1969

Pion-Nucleon Charge-Exchange Scattering and the Crossover Phenomenon with $M=0$ ρ and $M=1$ ρ' Trajectories*

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(Received 26 May 1969)

A Regge-pole model with $(M=0)$ ρ and $(M=1)$ ρ' trajectories is used to fit the πN charge-exchange differential cross-section and polarization data. The model automatically predicts the crossover phenomenon around $t=-0.2$ GeV^2 in πN elastic scattering without assuming any zeros in the nonflip ρ residue. Thus it avoids difficulties of such zeros due to factorization. The parameters of the ρ and ρ' are consistent with the nucleon electromagnetic form factors and predict the mass of ρ' to be around 1.1 GeV.

I. INTRODUCTION

In high-energy πN, KN, and NN elastic scattering, it is found that the cross-section difference $d\sigma/dt(AB\to AB)-d\sigma/dt(\bar{A}B-\bar{A}B)$ changes sign[1] around $t=t_0\sim -0.2$ GeV^2 (here $\bar{A}$ means the antiparticle of A). This "crossover" phenomenon in the previous Regge-pole models had been attributed to the presence of a zero at $t=t_0$ in the helicity-nonflip residue functions of the ρ- and the ω-exchange amplitudes.[2–4] There are two objections to such a zero: (1) Factorization would imply such a zero to be present in the ω residue functions for all channels,[4,5] which contradicts the $\gamma p\to\pi^0 p$ data[6,7]; (2) factorization also implies (assuming simple zeros) that the $\rho\pi\pi$ residue vanishes at t_0. Since the same $\rho\pi\pi$ residue appears in both πN residues, this in turn would imply that in, addition to helicity nonflip, the helicity-flip πN residue is also zero at t_0. This is in contradiction with the experimental πN charge-exchange data. Alternative explanations to avoid the difficulty of a zero have been suggested.[7]

We propose that the relevant πN data can be explained in terms of the (usual) $M=0$ Regge trajectory together with a conspiring $M=1$ ρ' trajectory with otherwise the same quantum numbers as the ρ tra-

TABLE I. Summary of results.

Parameters			
A_ρ (GeV mb$^{1/2}$)	-5.34	$A_{\rho'}$ (GeV^{-1} mb$^{1/2}$)	-25.7
B_ρ (GeV^{-2})	3.92	$B_{\rho'}$ (GeV^{-2})	0.92
C_ρ (GeV^{-1} mb$^{1/2}$)	-5.50	$C_{\rho'}$ (GeV^{-1} mb$^{1/2}$)	-9.67
D_ρ (GeV^{-2})	1.81	$D_{\rho'}$ (GeV^{-2})	0.43
α_ρ	$0.60+0.83t$	$\alpha_{\rho'}$	$0.32+0.64t$

χ^2 comparison		This work	Ref. 8
$d\sigma/dt$	χ^2	84.4	97.4
Data points		70	57
$p(d\sigma/dt)$	χ^2	6.8	3.6
Data points		12	12
Total	χ^2	91.2	101
No. of points		82	69
No. of parameters		12	11

Crossover point		Residue at the ρ pole		
E_L (GeV)	t (GeV^2)		Calculated	Ref. 2
3	-0.19	nonflip	0.64	0.87
7	-0.21	flip	4.0	3.98
11	-0.22			
15	-0.23			

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* Work supported in part by the Atomic Energy Commission, under Contract No. AEC AT(11-1)34P107A.

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[1] K. J. Foley *et al.*, Phys. Rev. Letters **10**, 376 (1963).

[2] B. R. Desai, Phys. Rev. **142**, 1255 (1966).

[3] R. J. N. Phillips and W. Rarita, Phys. Rev. **139**, B1336 (1965).

[4] W. Rarita, R. J. Riddell, Jr., C. B. Chiu, and R. J. N. Phillips, Phys. Rev. **165**, 1615 (1968).

[5] W. Rarita and V. L. Teplitz, Phys. Rev. Letters **12**, 206 (1964).

[6] R. Alvarez *et al.*, Phys. Rev. Letters **12**, 707 (1964); M. Braunschweig *et al.*, Phys. Letters **22**, 705 (1966); G. C. Bolon *et al.*, Phys. Rev. Letters **18**, 926 (1967).

[7] V. Barger and L. Durand III, Phys. Rev. Letters **19**, 1295 (1967).

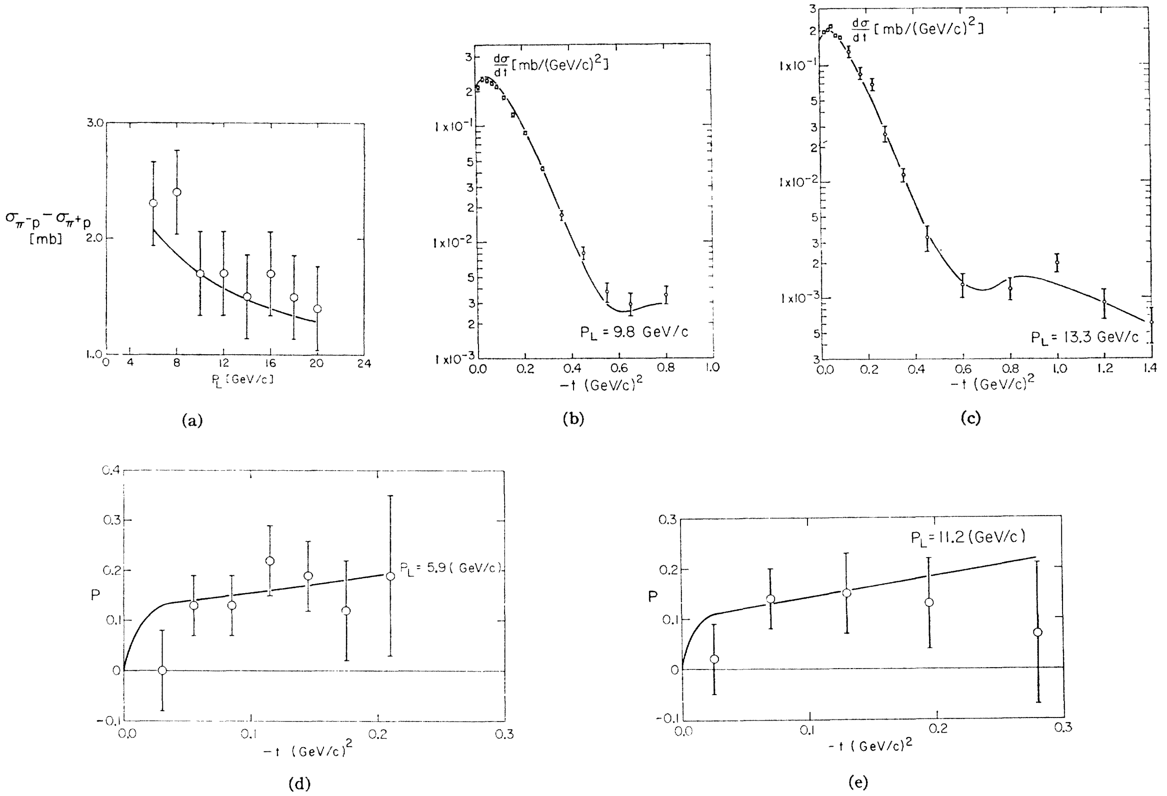

FIG. 1. Some sample graphs showing the quality of the fit: (a) π^+p, π^-p total cross-section difference; (b) and (c) $\pi^-p\to\pi^0n$ differential cross section at $P_L=9.8$ and 13.3 GeV/c; (d) and (e) neutron polarization in $\pi^-p\to\pi^0n$ at $P_L=5.9$ and 11.2 GeV/c.

jectory without a zero in the residue functions of either ρ or ρ' at the crossover point $t=t_0$. Since the conspiring ρ' contribution to the helicity-nonflip amplitude vanishes[8] $\propto t$ at $t=0$, for negative t away from $t=0$ its contribution will grow and can cancel the ρ contribution. The fact that a conspiring ρ' is needed in the πN finite-energy sum rule (FESR) has been known for some time.[9] The small but nonvanishing neutron polarization in the πN charge-exchange reaction gives further indication that some secondary contribution other than that of the ρ is required. In fact, Sertorio and Toller[8] have fitted the πN charge-exchange polarization data with the ρ and ρ'. However, they still assume that the helicity-nonflip residue of the ρ has a zero around $t\sim-0.25$ GeV^2.

We consider only the πN charge-exchange reaction. We find that the $\rho+\rho'$ model without a zero in the residue functions can give a very good fit to the high-energy πN charge-exchange data. The best fit we got has the following additional nice features: (a) The ρ and ρ' contributions to the imaginary part of the helicity-nonflip amplitude do indeed cancel each other around $t\sim-0.2$ GeV^2 (see Table I) as required by the crossover phenomenon in the elastic πN scattering, although the latter information is not put in; (b) the ρ nonflip and flip residues when extrapolated to the pole ($t=m_\rho^2$) are in good agreement with the values derived from the nucleon electromagnetic structure data.[2] The ρ' trajectory we get will pass through 1 around $t\sim1.1$ GeV^2, implying the existence of a particle with exactly the same quantum numbers of the ρ meson at about this mass. Although the experimental situation is not clear, it is interesting to notice that this ρ' trajectory is consistent with the suggestion of a conspiring B trajectory[8,10]; namely, ρ' and B form an $M=1$ parity doublet.

In Sec. II we shall set up all the formulas in the helicity formalism. In Sec. III we summarize all our results. In the final section we discuss the implications of our model and compare it with alternative models.

II. FORMULAS

The t-channel helicity amplitudes are used throughout this work. The conspiring ρ' nonflip residue vanishes $\propto t$ near $t=0$. The sense-choosing mechanism is assumed for ρ (ρ') at α_ρ $(\alpha_{\rho'})=0$. We have, using the notation

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[8] L. Sertorio and M. Toller, Phys. Rev. Letters **19**, 1146 (1967).

[9] R. Dolen, D. Horn, and C. Schmid, Phys. Rev. **166**, 1768 (1968).

[10] R. F. Sawyer, Phys. Rev. Letters **19**, 137 (1967).

of Ref. 11,

$$\sigma_{\pi^+p}-\sigma_{\pi^-p}=\frac{2\sqrt{2}}{q_s\sqrt{s}}\,\mathrm{Im}f_{++}|_{t=0},$$

$$\frac{d\sigma}{dt}(\pi^-p\to\pi^0n)=\frac{1}{4\pi sq_s^2}(|f_{++}|^2+|f_{+-}|^2),$$

$$P\left(\frac{d\sigma}{dt}\right)(\pi^-p\to\pi^0n)=-\frac{\mathrm{Im}(f_{++}f_{+-}^*)}{2\pi sq_s^2}.$$

The helicity amplitudes are parametrized as follows:

$$f_{++}=\frac{1}{2(4m_N^2-t)^{1/2}}\bigg\{A_\rho e^{B_\rho t}(1+\alpha_\rho)\times\frac{1-e^{-i\pi\alpha_\rho}}{\sin\pi\alpha_\rho}\left(\frac{E_L}{E_0}\right)^{\alpha_\rho}+tA_{\rho'}e^{B_{\rho'}t}(1+\alpha_{\rho'})\times\frac{1-e^{-i\pi\alpha_{\rho'}}}{\sin\pi\alpha_{\rho'}}\left(\frac{E_L}{E_0}\right)^{\alpha_{\rho'}}\bigg\},$$

$$f_{+-}=\sin\theta_t[t(t-4m_\pi^2)]^{1/2}\bigg\{C_\rho e^{D_\rho t}\alpha_\rho(1+\alpha_\rho)\times\frac{1-e^{-i\pi\alpha_\rho}}{\sin\pi\alpha_\rho}\left(\frac{E_L}{E_0}\right)^{\alpha_\rho-1}+C_{\rho'}e^{D_{\rho'}t}\alpha_{\rho'}(1+\alpha_{\rho'})\times\frac{1-e^{-i\pi\alpha_{\rho'}}}{\sin\pi\alpha_{\rho'}}\left(\frac{E_L}{E_0}\right)^{\alpha_{\rho'}-1}\bigg\},$$

where E_0 is chosen to be 1 GeV. The ρ and ρ' trajectories are parametrized as linear in t:

$$\alpha_\rho=a_\rho+b_\rho t,\quad \alpha_{\rho'}=a_{\rho'}+b_{\rho'}t.$$

The relations between the helicity amplitudes and the invariant amplitudes A' and B as defined by Singh[12] are

$$f_{++}=\tfrac{1}{4}(4m_N^2-t)^{1/2}A',$$

$$f_{+-}=\tfrac{1}{8}[t(t-4m_\pi^2)]^{1/2}\sin\theta_t\,B.$$

III. RESULTS

The results of our best fit are summarized in Table I. The χ^2 compares favorably with the fit by Sertorio and Toller.[8] Some examples showing the quality of the fit are plotted in Fig. 1. The cross-section data used are the same as those used in Ref. 11. The polarization data are the same as those used in Ref. 8 by Bonamy *et al.*[13] The values of t at which the contributions of the ρ and ρ' to the imaginary part of the helicity-nonflip amplitude cancel each other (the crossover point) are also listed as a function of the pion lab energy E_L. Notice that the crossover point moves to larger negative value of t as E_L increases. The values of the flip and nonflip residues when extrapolated to the ρ pole are compared with those obtained by Desai from the nucleon form factor.[2]

IV. DISCUSSIONS

We have presented a Regge-pole model with $M=0$, ρ and $M=1$, ρ' trajectories for the πN charge-exchange scattering without assuming a zero either in the ρ or the ρ' helicity-nonflip residues. It explains the crossover phenomenon in πN elastic scattering automatically by the cancellation between the ρ and ρ' contributions to the imaginary part of the nonflip amplitude. In contrast to a zero at fixed value of t for the ρ nonflip residue in the previous Regge-pole model, this point of cancellation will move out slowly to larger negative values of t as E_L increases. This eventually can be tested by experiment. The ρ and ρ' trajectory can pass through zero at different values of t (although, for the sense-choosing mechanism we assumed, they tend to pass through zero at about the same value in order that the dip not be filled). This will make possible to test whether the ρ trajectory can choose Chew's,[14] Gell-Mann's,[15] or the no-compensation[16] ghost-eliminating mechanism without having to make further assumptions about the background that serves as the bottom of the dip at $\alpha_\rho=0$. The detailed analysis of our fit using FESR and comparisons between various ghost-eliminating mechanisms will be the subject of another publication. We note that our ρ' trajectory passes through 1 around $t=1.1$ GeV^2. It is important that the prediction of a ρ' meson with a mass squared of around 1.1 GeV^2 be checked experimentally, especially in view of the recent attempt to explain dips, crossover phenomena, etc., by absorptive corrections to Regge-pole exchange.[17] In this latter model the existence of a dip depends crucially on the fact that only *one* Regge pole is contributing.[17] Thus, experimentally, whether a secondary ρ' exists should serve to distinguish the merit of the two models.

ACKNOWLEDGMENT

We would like to thank Dr. Jan Dash for many useful discussions and for his help with the fitting program.

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[11] F. Arbab, N. F. Bali, and J. W. Dash, Phys. Rev. **158**, 1515 (1967).

[12] V. Singh, Phys. Rev. **129**, 1889 (1963).

[13] Bonamy *et al.*, Phys. Letters **23**, 501 (1966).

[14] G. F. Chew, Phys. Rev. Letters **16**, 60 (1966).

[15] M. Gell-Mann, in *Proceedings of the International Conference on High-Energy Physics, CERN, 1962,* edited by J. Prentki (CERN, Geneva, 1962), p. 539.

[16] C. Chiu, S.-Y. Chu, and L. L. Wang, Phys. Rev. **161**, 1563 (1967).

[17] F. Henyey *et al.*, Phys. Rev. Letters **21**, 946 (1968); F. Henyey *et al.*, Phys. Rev. **182**, 1579 (1969).