This must of course be compared with the nucleon point distribution, which for Ca^{40} is almost exactly twice the proton point distribution. (It is for that reason that in this case we are able to compare the proton distribution directly with the Hartree-Fock nucleon distribution plotted to half-scale.) It will be seen that the agreement is not unsatisfactory, and that both distributions depart very significantly from the so-called Fermi distribution which is flat in the central region. The Fermi distribution is of course a purely phenomenological one and has no deeper justification, in the way that one based on the Hartree-Fock approach may be considered to have. It will also be seen that the surface thickness of the point distribution is some 20% narrower than the surface thickness of the charge distribution. This effect is likely to be independent of A and indicates that the value of the surface thickness obtained from electron scattering should not be taken over uncritically into nuclear-structure calculations.

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He^3+p Elastic Scattering from 12.6 to 15.4 MeV*

P. F. Donovan, J. V. Kane,† and J. F. Mollenauer
Bell Telephone Laboratories, Murray Hill, New Jersey

and

D. Boyd
Rutgers University, New Brunswick, New Jersey

and

P. D. Parker‡
Brookhaven National Laboratory, Upton, New York

and

Č. Zupančič
Physics Division, CERN, Geneva, Switzerland

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The elastic scattering of protons from He^3 has been studied in a search for a narrow level in Li^4 reported to lie about 10.6 MeV above He^3+p. Data were obtained at laboratory angles of 120° and 150° at proton energies from 12.6–15.4 MeV in 100-keV steps and from 13.84 to 14.74 MeV in overlapping 10-keV steps. Measurements were made relative to the elastic scattering of protons from He^4 by using a mixture of He^3 and He^4 in the gas target. The ratio of the $He^3(p,p)$ yield to the $He^4(p,p)$ yield was smooth to ±0.75% over the entire energy range. In this region an experimental upper limit of 10^{-5} times the Wigner limit was determined for the reduced width of any narrow singlet s-wave resonance.

I. INTRODUCTION

The observation of nine events forming a narrow peak in the energy spectrum of π^- mesons from the decay of ${}_\Lambda He^4$ was interpreted by Beniston *et al.*[1] as evidence for the possible existence of a narrow level in Li^4 located 10.62±0.20 MeV above He^3+p with a width of 0.23±0.20 MeV. Because of the narrowness of this level, an assignment of $T=2$ was suggested although the events were observed to break up via the $T=1$ He^3+p channel rather than the available $T=2$ channel (see Fig. 1). No confirmation of the existence of such a state was obtained by studies of the $He^3(p,p)$-He^3 excitation function in this region by Dangle *et al.*[2] and by Igo and Leland.[3] However, both of these experiments used energy steps that were larger than their respective target thicknesses and could, therefore, have missed a very narrow resonance. Estimates described in the Appendix indicate that if the resonance were $T=2$, then because of the small kinetic energy available

* Work supported in part by the National Science Foundation and the U. S. Atomic Energy Commission.
† Present address: Physics Department, Michigan State University, East Lansing, Michigan.
‡ Present address: Physics Department, Yale University, New Haven, Connecticut.
[1] M. J. Beniston, B. Krishnamurthy, R. Levi-Setti, and M. Raymund, Phys. Rev. Letters 13, 553 (1964).
[2] R. L. Dangle, J. Jobst, and T. I. Bonner, Bull. Am. Phys. Soc. 10, 422 (1965).
[3] G. J. Igo and W. T. Leland, Bull. Am. Phys. Soc. 10, 1193 (1965).

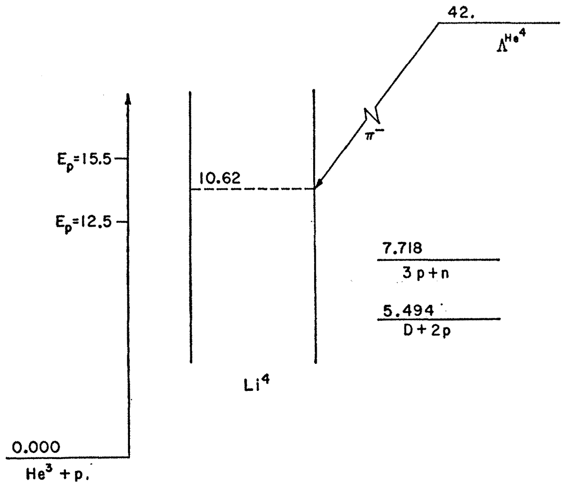

FIG. 1. Energy-level diagram for Li^4 indicating the energetics of the various channels and decays pertinent to this experiment.

for the allowed breakup into four nucleons and because of the isospin forbiddenness of the decay to He^3+p the width of such a resonance might well be of the order of 10 keV or less. The present experiment was therefore undertaken in order to cover this energy region of He^3+p elastic scattering with overlapping steps to determine if such a narrow level had been missed in the previous measurements.

II. EXPERIMENTAL PROCEDURE

Measurements were made using a 2-cm-diam gas target[4] with a 3.8×10^{-3} mm Havar steel window.[5] The gas target was filled with a mixture of He^3(30±3 psi absolute) and He^4(30±3 psi absolute) so that an accurate measurement could be obtained of the $He^3(p,p)$ yield relative to the $He^4(p,p)$ yield independent of such considerations as beam current integration, absolute beam geometry inside the gas target, variations in gas density along the beam path, analyzer deadtime, etc. A magnetically analyzed proton beam, typically 0.02 μA, from the Rutgers-Bell tandem Van de Graaff was collimated to a diameter of 2 mm before striking the entrance foil. Elastically scattered protons emerging from the gas target were measured at laboratory angles of 120° and 150° using a 3-mm-thick Si(Li) detector collimated so that the detector viewed 4 mm of the beam path in the gas. At the proton energies used in this experiment, and at a typical helium gas pressure of 60 psi absolute, this corresponds to a proton energy spread of 10 keV. Combining this with the energy spread in the incident beam of $\approx$10 keV (contributed by the finite resolution of the incident beam and by straggling in the entrance foil) yields an effective proton-energy resolution of $\approx$15 keV. Therefore, in order not to miss any very narrow resonance, measurements were taken in 10-keV steps over the range of proton energies from 13.86–14.76 MeV; from the results of Beniston *et al.*[1] a resonance would be expected at a proton energy of 14.16±0.27 MeV.

Figure 2 shows an example of the proton spectra obtained at a laboratory angle of 120°, indicating the peaks due to elastic scattering from He^3 and He^4. The area under each of these two peaks (typically 5×10^5 counts) was obtained by using a light-pen and an on-line computer[6] to fit a general quadratic to the background below, between and above these two peaks. The ratio of the He^3 yield to the He^4 yield (with a typical uncertainty of ±0.3%) was then examined as a function of the incident proton energy to find any anomaly which might be attributed to a resonance in He^3+p elastic scattering. This ratio is plotted as a function of proton energy in Figs. 3 and 4 for the measurements made at 120° and 150°, respectively. From these graphs, it is clear that there are no pronounced anomalies in the He^3+p scattering in this energy range, the energy dependence of the yield being smooth to ±0.75% (120°) and ±0.55% (150°), as indicated by the boundary lines in Figs. 3 and 4. These boundary lines define deviations from the mean cross section of ±0.75% and ±0.55%, respectively, and although a few points fall outside these limits, in every case a second point measured at the same energy lies well within the boundary lines.

Using the $He^4(p,p)$ differential cross-section measurements, $(d\sigma/d\Omega_{c.m.})_{He^4}$ of Brockman,[7] one can obtain $He^3(p,p)$ differential cross sections from the data in Figs. 3 and 4 as

$$\left(\frac{d\sigma}{d\Omega_{c.m.}}\right)_{He^3}=\frac{Y_{He^3}\,P_{He^4}}{Y_{He^4}\,P_{He^3}}\left(\frac{d\sigma}{d\Omega_{c.m.}}\right)_{He^4}\frac{(d\Omega_{c.m.}/d\Omega_{lab})_{He^4}}{(d\Omega_{c.m.}/d\Omega_{lab})_{He^3}},$$

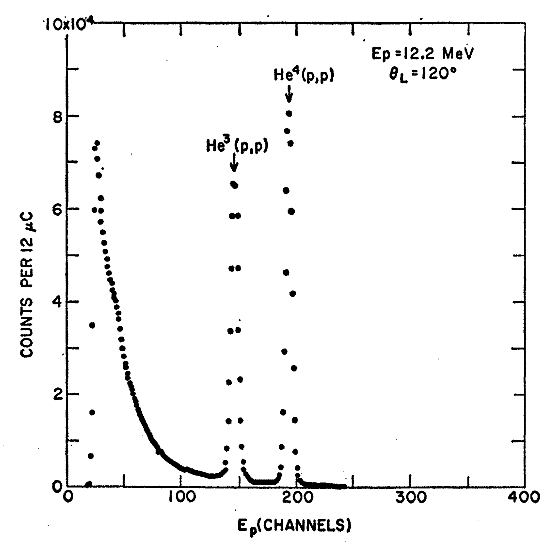

FIG. 2. Spectrum of protons elastically scattered from a gas target containing a mixture of He^3 and He^4.

[4] J. F. Mollenauer, Rev. Sci. Instr. **36**, 1044 (1965).
[5] Hamilton Watch Company, Lancaster, Pennsylvania.
[6] Scientific Data Systems, model 910, Santa Monica, California.
[7] K. W. Brockman, Phys. Rev. **108**, 1000 (1957).

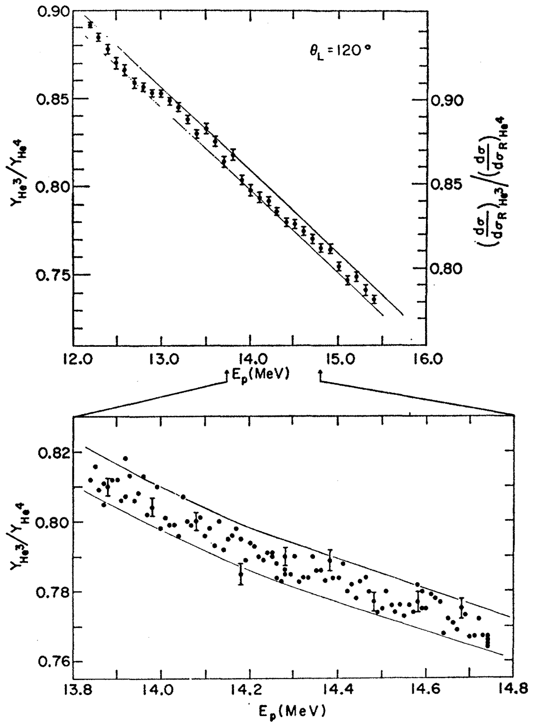

FIG. 3. Plot of the ratio of $He^3(p,p)$ yield to $He^4(p,p)$ yield as a function of incident proton energy at a laboratory angle of 120°. The upper graph shows the excitation function in 100-keV steps while the lower insert shows a more detailed excitation function in 10-keV steps over the center-of-mass energy region from 10.39–11.07 MeV. Note the expanded scale and suppressed zero on the vertical axis; typical error bars are only ±0.3%. The boundary lines define limiting deviations of ±0.75% from the mean cross section.

where Y_{He^3}/Y_{He^4} is the ratio of the yields of elastically scattered protons from He^3 and He^4 and P_{He^4}/P_{He^3} is the ratio of the He^4 and He^3 gas pressures in the target cell. For the present experiment $P_{He^4}/P_{He^3}=1.0\pm0.2$. Re-expressing the cross sections relative to their respective Rutherford cross section σ_R, one obtains

$$\theta_{\text{lab}}=120°,\quad \left(\frac{d\sigma}{d\sigma_R}\right)_{He^3} \Big/ \left(\frac{d\sigma}{d\sigma_R}\right)_{He^4} = (1.06\pm0.21)\frac{Y_{He^3}}{Y_{He^4}},$$

$$\theta_{\text{lab}}=150°,\quad \left(\frac{d\sigma}{d\sigma_R}\right)_{He^3} \Big/ \left(\frac{d\sigma}{d\sigma_R}\right)_{He^4} = (1.10\pm0.22)\frac{Y_{He^3}}{Y_{He^4}},$$

where the errors represent the uncertainty in the relative He^3, He^4 pressures. These scales are indicated in Figs. 3 and 4 where it should be noted that although the error bars still indicate the relative uncertainties in the points, there is an additional ±20% over-all uncertainty in the normalization of the data to the $(d\sigma/d\sigma_R)_{He^3}/(d\sigma/d\sigma_R)_{He^4}$ scales.

III. ANALYSIS

In order to set an upper limit on the width of such a state it is necessary to calculate the magnitude and shape of the anomaly that should have been observed. For this purpose it is useful to express the $He^3(p,p)$ scattering cross section as

$$\sigma(\theta)=\tfrac{3}{4}|{}^3f(\theta)|^2+\tfrac{1}{4}|{}^1f(\theta)|^2,$$

where ${}^3f(\theta)$ and ${}^1f(\theta)$ are the triplet and singlet scattering amplitudes defined by Schiff.[8] Since the purported 10.62-MeV state is supposed to be a $T=2$ configuration, it would presumably be the lowest $T=2$ state and consequently have $J^\pi=0^+$, occurring as a resonance in the singlet s-wave phase shift ${}^1\delta_0$. The s-wave, p-wave, and d-wave singlet and triplet scattering amplitudes were therefore evaluated for a resonant ${}^1\delta_0$ at a proton energy of 14.5 MeV ($E_{\text{c.m.}}=10.875$ MeV). (These calculations are quite insensitive to the actual resonant proton energy assumed.) Nonresonant phase shifts were extrapolated from the analysis of Tombrello,[9] as ${}^1\delta_0=-100°$, ${}^3\delta_0=-90°$, ${}^1\delta_1=+30°$, ${}^3\delta_1=+53°$, ${}^1\delta_2=-30°$, ${}^3\delta_2\approx0°$. Partial waves with $l\geq3$ were neglected because their actual phase shifts are not known and their hard-sphere phase shifts will be only a few degrees. The effects of the reaction chan-

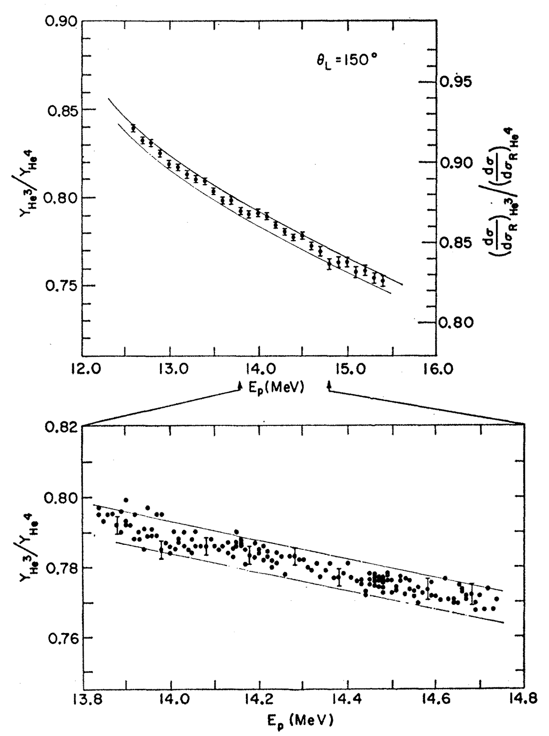

FIG. 4. Plot of the ratio of $He^3(p,p)$ yield to the $He^4(p,p)$ yield as a function of incident proton energy at a laboratory angle of 150°. The upper graph shows the excitation function in 100-keV steps while the lower insert shows a more detailed excitation function in 10-keV steps over the center-of-mass energy region from 10.39–11.07 MeV. Note the expanded scale and suppressed zero on the vertical axis; typical error bars are only ±0.3%. The boundary lines define limiting deviations of ±0.55% from the mean cross section.

[8] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949), Eq. (20.24).

[9] T. A. Tombrello, Phys. Rev. **138** B40 (1965).

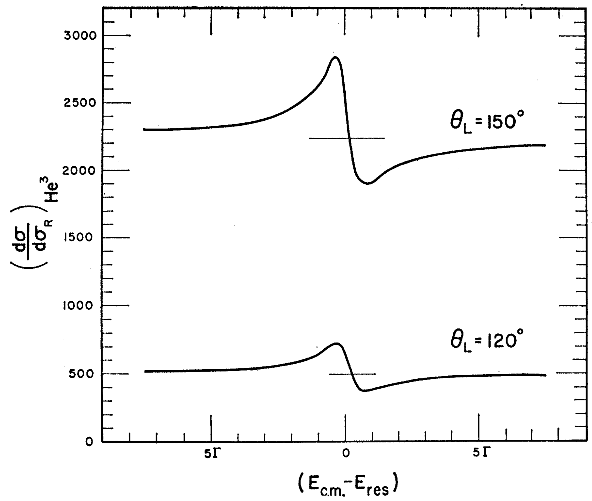

FIG. 5. The expected shape and magnitude of the contributions from a ${}^1S_0$ resonance to the $He^3(p,p)He^3$ cross section at laboratory angles of 120° and 150° plotted as functions of $(E_{c.m.} - E_{res})$ in units of the center-of-mass width of the resonance.

nels $He^3+p \rightarrow d+2p$ and $He^3+p \rightarrow 3p+n$ have been neglected; no evidence was found for the 4-body channel in Ref. 1, and measurements quoted in Ref. 9 indicate that at $E_p=10$ MeV at laboratory angles of 30° and 45° the cross section for the 3-body channel is less than 1% of the elastic scattering cross section.

The results of this evaluation of $\sigma(\theta)$ at laboratory angles of 120° and 150° are plotted in Fig. 5. A comparison of the expected anomalies with the experimental data (anomalies are *not* present with more than 0.75% of the observed cross sections) yields an upper limit for the width of such a state of $\Gamma_{c.m.} \lesssim 280$ eV. At these energies this is equivalent to a reduced width for s-wave protons of ~ 90 eV, compared to the Wigner limit of ≈ 7 MeV for this system. Thus, the work reported here is in agreement with the more recent ${}_{\Lambda}He^4$ results of Gajewski *et al.*[10] and does not support the existence of a narrow state (280 eV$<\Gamma<$1.0 MeV) at an excitation of 10.62 MeV in Li^4 with a large partial width (Γ_p/Γ) for decay into $p+He^3$.

APPENDIX: AN ESTIMATE OF THE WIDTH OF A $T=2$ STATE AT 10.6 MeV IN Li^4

It appears from the systematics of analog states in light nuclei that a $T=2$ state in Li^4 should have a partial width on the order of 100 eV–1 keV for the isospin forbidden decay into a proton and He^3. The question is whether it is reasonable to expect an even smaller width for the isospin allowed decay into three protons and a neutron, as is indicated by the experiment of Beniston *et al.*[1]

[10] W. Gajewski, J. Sacton, P. Vilain, G. Wilquet, D. Stanley, D. H. Davis, E. R. Fletcher, J. E. Allen, V. A. Bull, A. P. Conway, and P. V. March, Phys. Letters **21**, 673 (1966).

The ratio r of the four-body to the two-body decay probability may be expressed as $r=(|H_4|^2 p_4/|H_2|^2 p_2)$. Here, H_2 and H_4 are transition matrix elements, while p_2 and p_4 are the densities of final states for the two-body and four-body decay, respectively. The densities of final states may be expressed in the form $p_4=(V^3/h^9)F_4(U_4)$ and $p_2=(V/h^3)F_2(U_2)$, where V is a normalizing volume, h is Planck's constant, and $F_n(U_n)$ is given by[11]

$$F_n(U_n) = \frac{(2\pi)^{3(n-1)/2} \prod_1^n (m_i^{3/2})}{\Gamma[3(n-1)/2](\sum_1^n m_i)^{3/2}} U_n^{(3n/2-5/2)}.$$

In this expression, m_i is the mass of the ith emitted particle and U_n is the total kinetic energy available for the n-body decay. The transition matrix elements may be estimated to be on the order of

$$|H_4|^2 = (\Omega/V)^3 f_4$$

and

$$|H_2|^2 = (\Omega/V) f_2,$$

where Ω is the volume of Li^4, while f_2 and f_4 are the enhancement (or hindrance) factors arising from final-state interactions, penetration effects and isospin selection rules. The strong final-state interactions in the singlet pp and pn systems should enhance the four-body decay by about a factor of 100.[12] The penetration factor for four-body decay is expected to be on the order of 0.1 because of small kinetic energies of the outgoing particles. On the other hand, the two-body decay should be hardly hindered by penetration factors, but should be retarded by a factor on the order of 10^{-4} because of the isospin selection rule.

In this way one is led to the estimate

$$r = \frac{f_4}{f_2} \frac{\Omega^2}{h^6} \frac{(2\pi)^3 m^3}{\frac{3}{2}\times\frac{5}{2}\times\frac{7}{2}\times 3^{3/2}} \frac{U_4^{7/2}}{U_2^{1/2}} \approx 1,$$

m being the proton mass.

Obviously, the above estimates are so crude that a deviation from them of one order of magnitude in either direction would not be surprising. We thus conclude that the four-body decay of a $T=2$ state at 10.6 MeV in Li^4 may well be less probable than the two-body decay. Even if the four-body width should be an order of magnitude greater than the width for two-body decay, the total width of the state probably still should not exceed 10 keV.

[11] R. H. Milburn, Rev. Mod. Phys. **27**, 1 (1955); Č. Zupančič, Nuclear Institute J. Stefan Report No. R-429, Ljubljana, Yugoslavia, 1964 (unpublished); *Few Nucleon Problems* (Federal Nuclear Energy Commission of Yugoslavia, 1964), Vol. II, p. 18.

[12] Č. Zupančič, Rev. Mod. Phys. **37**, 330 (1965); Brookhaven National Laboratory Report No. BNL 948 (C-46), Vol. II, p. 622, 1965 (unpublished).