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An analysis of super-high energy cosmic-ray propagation in the Galaxy

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Calculations of the cosmic-ray energy spectrum and the propagation pathlength in the energy range $10^{14} - 10^{19}$ eV have been performed within the framework of a combined approach based on a diffusion model and a simulation of particle trajectories in the Galaxy. The obtained escape pathlength and the interaction probability for nuclei in the Galaxy are discussed. The resulting spectrum for protons at Earth is compared to experimental data.

1. Introduction

The origin of super-high energy cosmic rays (CRs) is one of the most important problems in astrophysics. Since the spectrum at the sources is not identical to the spectrum observed at Earth, a study of sources is closely connected to the investigation of CR propagation processes in the Galaxy. This necessitates knowledge about the structure of the galactic magnetic fields. Unfortunately, there is no standard field configuration, different magnetic field models are able to describe available experimental data [1, 2, 3, 4]. How CRs are accelerated to extremely high energies is another open question. Although the popular model of CR acceleration by shock waves in expanding shells of supernovae (for example [5, 6, 7]) is almost recognized as "standard theory", there are still a number of unresolved problems in this model. Furthermore, the question about the role of other acceleration mechanisms is not quite clear, and could lead to different CR energy spectra at the sources [1].

The validity of various concepts is verified by the calculation of the primary CR energy spectrum, making assumptions on the density distribution of CR sources, the energy spectrum at the sources, and the configuration of the galactic magnetic fields. The diffusion model can be used in the energy range up to 10^{17} eV, where the spectrum is calculated using the diffusion equation for the density of CRs in the Galaxy. At higher energies this model ceases to be valid, and it becomes necessary to carry out numerical calculations of particle trajectories for the propagation in the magnetic fields. This method works best for the highest-energy particles, since the time for the calculations required is inversely proportional to the particle energy. Therefore, the calculation of the CR spectrum in the energy range $10^{14} - 10^{19}$ eV has been performed within the framework of a combined approach, the use of a diffusion model and the numerical integration of particle trajectories.

2. Assumptions

High isotropy and a comparatively long retention of CRs in the Galaxy ($\sim 10^7$ years for the disk model) reveal the diffusion nature of particle motion in the interstellar magnetic fields. This process is described by a corresponding diffusion tensor [1, 3, 8]. The steady-state diffusion equation for the CR density $N(r)$ is (neglecting nuclear interactions and energy losses)

$$-\nabla_i D_{ij}(r) \nabla_j N(r) = Q(r), \tag{1}$$

with the source term $Q(r)$ and the diffusion tensor $D_{ij}(r)$.

Under the assumption of azimuthal symmetry and taking into account the predominance of the toroidal component of the magnetic field, eq. 1 is presented in cylindrical coordinates as

$$\left[-\frac{1}{r}\frac{\partial}{\partial r}rD_\perp\frac{\partial}{\partial r}-\frac{\partial}{\partial z}D_\perp\frac{\partial}{\partial z}-\frac{\partial}{\partial z}D_A\frac{\partial}{\partial r}+\frac{1}{r}\frac{\partial}{\partial r}rD_A\frac{\partial}{\partial z}\right]N(r,z)=Q(r,z), \quad (2)$$

where $N(r,z)$ is the CR density averaged over the large-scale fluctuations with the characteristic scale $L \sim$ 100 pc [3]. $D_\perp \propto E^m$ is the diffusion coefficient, where m is much less than one ($m \approx 0.2$), and $D_A \propto E$ is the Hall diffusion coefficient. Thus, the influence of Hall diffusion becomes predominant at sufficiently high energies ($> 10^{15}$ eV). The sharp enhancement of the diffusion coefficient leads to the excessive leakage of CRs from the Galaxy at energies exceeding 10^{17} eV. For investigations of the CR propagation at such energies it is necessary to carry out numerical calculations of the trajectories for individual particles.

The calculation is based on the solution of the equation of motion for a charged particle in the magnetic field of the Galaxy. In this work the calculation was carried out using a fourth order Runge-Kutta method. Trajectories of CRs were calculated until they left the Galaxy. While testing the differential scheme that is used in the calculation, it was found that the accuracy of the obtained trajectories for protons with an energy of 10^{15} eV after passing a distance of 1 pc amounts to $5 \cdot 10^{-8}$ pc. The retention time of a proton with such an energy averages to about 10 million years, hence, a total error for the trajectory approximation by the differential scheme used is about 0.5 pc.

The magnetic field of the Galaxy consists of a large-scale (regular) and a chaotic component $\vec{B} = \vec{B}_{reg} + \vec{B}_{irr}$. A purely azimuthal magnetic field was assumed for the regular field $B_z = 0$, $B_r = 0$, $B_\phi = 1\ \mu\text{G} \cdot \exp(-z^2/z_0^2 - r^2/r_0^2)$, where $z_0 = 5$ kpc and $r_0 = 10$ kpc are constants [3]. The irregular field was constructed according to an algorithm used in [9], that takes into account the correlation of magnetic field intensity in adjacent cells. The radius of the Galaxy is assumed to be 15 kpc and the galactic disk had a half-thickness of 200 pc. The position of the Solar system was defined at $r = 8.5$ kpc, $\phi = 0°$, and $z = 0$ kpc. A radial distribution of supernovae remnants along the galactic disk was considered as sources [10].

3. Results

The obtained pathlength in the Galaxy for protons as function of energy is presented in Fig. 1 (left). The interstellar matter density was taken as $n_d = 1$ cm^{-3} for the galactic disk and $n_h = 0.01$ cm^{-3} for the halo. For heavier nuclei with charge Z the pathlength scales with the rigidity, i.e. is related to the values for protons $\lambda(E)$ as $\lambda(E,Z) = \lambda(E/Z)$. At the corresponding knees, the amount of traversed material is less than 1 g/cm^2. The dashed dotted line indicates a trend at lower energies according to $\lambda \propto E^{-\delta}$. To reach values of about 10 g/cm^2 as obtained around 1 GeV [11], one needs a relatively small slope $\delta \approx 0.2$ — much lower than the value usually assumed ($\delta \approx 0.6$).

Measurements of the ratio of secondary to primary CR nuclei at energies in the GeV regime are successfully described using leaky box models, e.g. [11, 12]. Both examples are compared to the predictions of the diffusion model in Fig. 1 (left). Extrapolating these relations to higher energies, the strong dependence of the pathlength on energy ($\propto E^{-0.6}$) leads to extremely small values at PeV energies. Above 10^5 GeV the traversed matter would be smaller than the pathlength accumulated on a straight line from the galactic center to the solar system $\lambda_{gc} = 8$ kpc $\cdot$ 1 proton/cm^3 ≈ 0.04 g/cm^3. This value is indicated in the figure as dotted line. A similar conclusion can be derived from anisotropy measurements. Leaky box models, with their extremely steep decrease of the pathlength $\lambda \propto E^{-0.6}$, yield relative large anisotropies even at modest energies, which seem to be ruled out by the measurements [13].

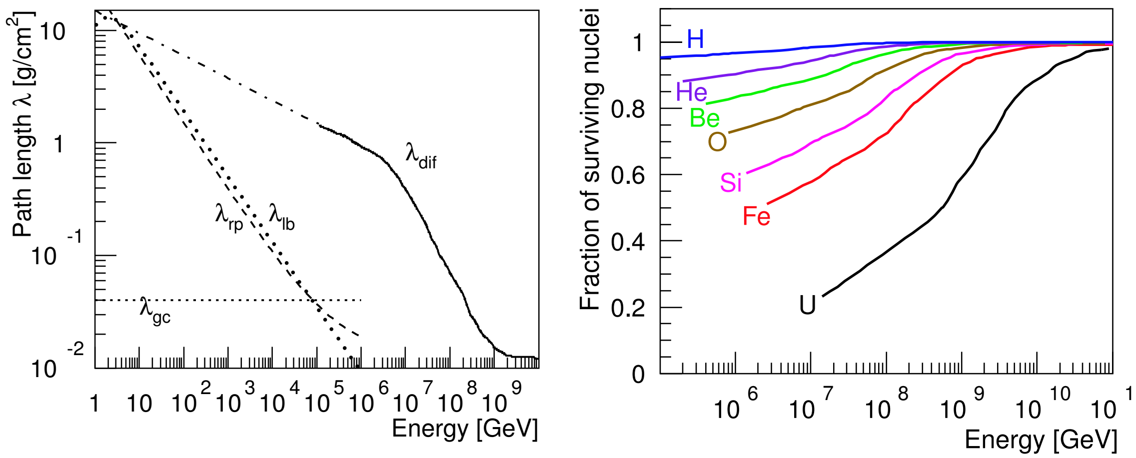

Figure 1. *Left:* Pathlength in the Galaxy for protons. The values for the diffusion model (λ_{dif}) are indicated by the solid line. They are extrapolated to lower energies by the dashed dotted line. The dashed and dotted lines indicate a leaky box model (λ_{lb} [11]) and a residual pathlength model (λ_{rp} [12]). The dotted line indicates the matter accumulated along a straight line from the galactic center (λ_{gc}). *Right:* Fraction of nuclei surviving without interaction in the Galaxy as function of energy for different elements.

The interaction probability for different nuclei has been calculated using the obtained pathlength and interaction parameters according to the QGSJET model [14]. Nuclear fragmentation is taken into account in an approximate approach [15]. It should be pointed out that a nuclear fragment conserves the trajectory direction of its parent if Z/A in question is the same as for the primary nucleus and for most stable nuclei the ratio Z/A is close to $1/2$. The resulting fraction of nuclei which survive without an interaction is presented in Fig. 1 (right) for selected elements. It turns out that at the respective knees ($\sim Z \cdot 4.5$ PeV) more than $\sim 50\%$ of the nuclei survive without interactions, even for the heaviest elements. This is an important result, since the *poly gonato* model relates the contribution of ultra-heavy CRs to the second knee in the all-particle spectrum around 400 PeV [16].

The results of the calculations of the proton spectrum are shown in Fig. 2. They were obtained using the diffusion model and numerical calculations of trajectories. Both methods give identical results up to about $3 \cdot 10^{16}$ eV. At higher energies there is a continuous decrease of the intensity in the "diffusion" spectrum, which corresponds to the excessive increase in the diffusion coefficient that results in a large leakage of particles from the Galaxy. An energy of 10^{17} eV can be accepted as the boundary for the applicability of the diffusion model. At this energy the results obtained with the two methods differ by a factor of 2 and for higher energies the diffusion approximation of CR propagation in the Galaxy becomes invalid.

The results from air shower experiments [18, 19] indicate a relatively pronounced knee at an energy of about 4 PeV. The spectra are compatible with the *poly gonato* model, see Fig. 2, with a change of the spectral index at the knee of $\Delta\gamma = 2.10 \pm 0.24$ [16]. The observed steepening of the energy spectrum should be compared with a value of $1 - m \approx 0.8$ as predicted by the diffusion model [3]. It is obvious that the experimental value of $\Delta\gamma$ is larger, hence, at least a part of the observed steepening should be related to a change of the shape of the spectrum at the sources.

The maximum energy reached during the acceleration process and the corresponding shape of the spectrum at these energies depend on the intensity of the magnetic fields in the acceleration zone and on assumptions for the feedback of CRs to the shock front. The uncertainties of the parameters used yield differences in the

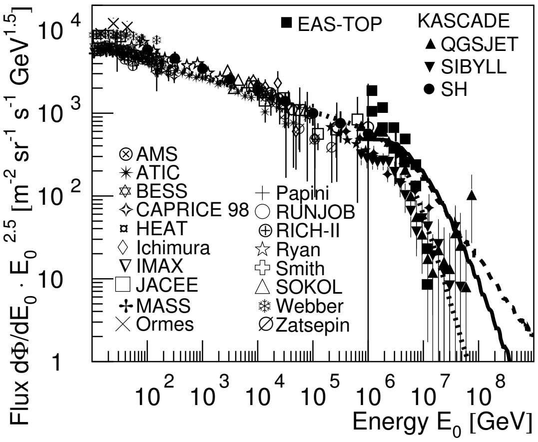

Figure 2. Calculated spectra for protons for the diffusion model (—) and the numerical trajectory calculations (– – –) compared to the flux obtained by various experiments, for references see [17], and the *poly gonato* model (· · ·) [16].

maximum energy attained in the order of ± one decade [6, 20]. In addition, the situation is complicated by the dependence of the escape pathlength on energy. As discussed above, the dependence $\lambda \propto E^{-0.6}$ can not be extrapolated to knee energies. On the other hand, a dependence $\propto E^{-0.2}$ requires additional assumptions for the spectral shape at the sources to explain the observed energy spectra. Thus, there is no agreement about a "standard model" scenario. At present, it is difficult to draw definite conclusions from the comparison of the observed spectra for different elemental groups with the standard model of CR acceleration at ultra high energies.

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