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THE LIGHT CONE AND SYMMETRY BREAKING

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I. INTRODUCTION

In this lecture I will review some work done this past year by Preparata and myself [1),2)] on an application of the canonical light cone (LC) operator product expansion formalism [3),4),5),6)] we had previously developed. The question we addressed ourselves to was : what are the observable consequences of a given operator algebra ? If one is given an exact operator algebra which does not correspond to an exact symmetry, then the physical states will not form representations of the algebra and so it is not straightforward to calculate measurable quantities of interest, such as matrix elements of the operators in question. I will try to show here how the use of LC techniques together with several dynamical assumptions enable one to deduce approximate relations between physical parameters from the exact operator relations embodied in current algebra and its generalization to the LC.

Consider, for example, current algebra. The (presumably) exact operator statements are the well-known Gell-Mann [7)] equal-time $SU(3) \times SU(3)$ commutation relations among the vector V_μ^a and axial-vector A_μ^a currents :

$$\delta(x_0)\left[V_0^a(x), V_0^b(0)\right] = i f^{abc} V_0^c(0)\, \delta^4(x) \tag{1.1a}$$

$$\delta(x_0)\left[V_0^a(x), A_0^b(0)\right] = i f^{abc} A_0^c(0)\, \delta^4(x) \tag{1.1b}$$

$$\delta(x_0)\left[A_0^a(x), A_0^b(0)\right] = i f^{abc} V_0^c(0)\, \delta^4(x) . \tag{1.1c}$$

The consequences of this algebra are exact zero energy and mass theorems [8),9)], the most famous being the Adler-Weisberger theorem

$$T^{(-)\prime}(\nu = 0, q^2 = 0) = -\frac{1}{f_\pi^2}\left(g_A^2 - 1\right) \tag{1.2}$$

for the energy derivative of the isospin odd off-shell pion-nucleon scattering amplitude at zero energy ν and squared pion mass q^2. Thus current algebra gives $T^{(-)\prime}(\nu, q^2)$ at the unphysical mass $q^2 = 0$ and the unphysical energy $\nu = 0$. This would not be much of a problem if only the energy were unphysical since energy dispersion relations (DR's) can be used to calculate (in principle exactly) the amplitude at $\nu = 0$ from the physical region data. The fact that q^2 is unphysical has, on the contrary, always presented a much more serious problem since off-shell data does not exist at present. Our approach is a step in the direction of resolving this problem. We will use mass DR's to effect the q^2 extrapolation in the same way that energy DR's provide the ν extrapolation.

A second way to pose the problem under consideration is in terms of corrections to pole dominance. If pole dominance were exact, then off-shell extrapolation would be trivial and there would be no problem. Thus, if pion pole dominance were exact in the above example, then $T(\nu,0)$ would equal the on-shell amplitude $T(\nu,\mu^2)$. In this language, what we are proposing to do is to compute corrections to pole dominance. For example, $T(\nu,0) = T(\nu,\mu^2)$ + (correction to pion pole dominance). We emphasize that our use of the word "correction" should not be taken to imply that the corrections are in any sense assumed to be small compared to the pole contributions. Indeed, in many examples the corrections will turn out to be more important than the pole contributions. In particular, our calculations of corrections will be strictly non-perturbative in character. We will discuss here corrections to vector meson (ρ,ω) and particularly pseudoscalar meson (π,K) pole dominance.

A third perspective on our problem is provided by an assumed underlying broken $SU(3) \times SU(3)$ symmetry of the hadrons generated by the charges of V_o^a and A_o^a [7),9),10),11)]. If this symmetry were exact [i.e., exact in the multiplet sense for vector $SU(3)$ and in the Goldstone sense for axial $SU(3)$], then the pseudoscalar mesons would be massless and so $q^2 = 0$ would be a physical point. Since the symmetry is, in fact, broken, the mesons acquire mass (and mass differences) and the problem becomes that of calculating (perhaps large) symmetry breaking effects. This approach to the problem makes it clear that a $SU(3) \times SU(3)$ symmetry breaking interaction should be specified. Given this interaction, and even given the magnitudes of the symmetry breaking parameters, the problem is, however, still far from being solved. What is further needed is a method for relating symmetry breaking parameters in Lagrangians to breaking effects in states. This is a difficult task, especially if large symmetry breaking parameters and/or dynamical enhancements are involved, so that a perturbative approach is not useful. We will see that our LC techniques offer the possibility of connecting physical and Lagrangian symmetry breaking and provide a framework for a non-perturbative approach to the problem.

We assume here the simplest symmetry breaking scheme in which $SU(3) \times SU(3)$ is broken only by a mass term

$$\mathcal{M} = 2\alpha_0 S^0 + 2\alpha_8 S^8 \qquad (1.3)$$

where the scalar nonet S^a, together with a pseudoscalar P^a, constitute a $(3,\bar{3}) + (\bar{3},3)$ representation of $SU(3) \times SU(3)$ [7),12),13)]:

$$\delta(x_0)[V_0^a(x),\ S^b(0)] = if^{abc}S^c(0)\delta^4(x) \tag{1.4a}$$

$$\delta(x_0)[V_0^a(x),\ P^b(0)] = if^{abc}P^c(0)\delta^4(x) \tag{1.4b}$$

$$\delta(x_0)[A_0^a(x),\ S^b(0)] = id^{abc}P^c(0)\delta^4(x) \tag{1.4c}$$

$$\delta(x_0)[A_0^a(x),\ P^b(0)] = -id^{abc}S^c(0)\delta^4(x)\ . \tag{1.4d}$$

In this paper, we shall, in fact, be more specific, and abstract additional algebraic relations from the gluon Lagrangian model [14)]

$$\mathcal{H} = \psi^\dagger(-i\underset{\sim}{\alpha}\cdot\underset{\sim}{\nabla} + g\beta\gamma^\mu B_\mu)\psi + \mathcal{M} + \mathcal{H}_B\ , \tag{1.5}$$

in which the quark fields ψ interact via a neutral vector meson B_μ. The currents are then simply Dirac bilinears and we have the additional relations

$$\delta(x_0)[S^a(x),\ P^b(0)] = if^{abc}V_0^c(0)\delta^4(x) \tag{1.6a}$$

$$\delta(x_0)[S^a(x),\ P^b(0)] = id^{abc}A_0^c(0)\delta^4(x) \tag{1.6b}$$

$$\delta(x_0)[P^a(x),\ P^b(0)] = if^{abc}V_0^c(0)\delta^4(x)\ . \tag{1.6c}$$

The divergences of the vector and axial vector currents are respectively thus

$$\mathcal{D}^a \equiv \partial^\mu V_\mu^a = 2\alpha_8 f^{8ab}S^b \tag{1.7a}$$

$$D^a \equiv \partial^\mu A_\mu^a = 2(\alpha_0 d^{0ab} + \alpha_8 d^{8ab})P^b. \tag{1.7b}$$

The SU(2) × SU(2) symmetry breaking parameter is $\varepsilon_2 \equiv \sqrt{2}\alpha_0 + \alpha_8$ and the SU(3) one is $\varepsilon_3 \equiv \alpha_8$. The axial divergences of interest are

$$D^{\pi^-} \equiv \partial^\mu A_\mu^{\pi^-} = \frac{2}{\sqrt{3}}\left(\sqrt{2}\,\alpha_0 + \alpha_8\right) P^{\pi^-} \equiv \varepsilon_\pi P^{\pi^-} \tag{1.8a}$$

$$D^{K^-} \equiv \partial^\mu A_\mu^{K^-} = \frac{2}{\sqrt{3}}\left(\sqrt{2}\,\alpha_0 - \tfrac{1}{2}\alpha_8\right) P^{K^-} \equiv \varepsilon_K P^{K^-} . \tag{1.8b}$$

In the limit of exact SU(2) × SU(2), $\varepsilon_\pi = 0$, and in the limit of exact SU(3), $\varepsilon_\pi = \varepsilon_K$ or $\alpha_8 = 0$.

The nature of the symmetry breaking term (1.3) is specified by the ratio

$$c \equiv \frac{\alpha_8}{\alpha_0} \tag{1.9}$$

The popular values are :

$$\text{SU(2)} \times \text{SU(2) much better than SU(3)} \Rightarrow \varepsilon_\pi \ll \varepsilon_K \Rightarrow \frac{c}{\sqrt{2}} \sim -1 , \tag{1.10a}$$

$$\text{SU(3) much better than SU(2)} \times \text{SU(2)} \Rightarrow \varepsilon_\pi \sim \varepsilon_K \Rightarrow \frac{c}{\sqrt{2}} \sim 0 . \tag{1.10b}$$

The numerical values in the two cases are :

$$\text{Meson spectrum}^{(12)} \Longrightarrow \frac{c}{\sqrt{2}} \simeq -0.9 , \tag{1.11a}$$

$$\text{Baryon spectrum}^{(15)} \Longrightarrow \frac{c}{\sqrt{2}} \simeq -0.2 . \tag{1.11b}$$

We will consider each of these possibilities below.

2. CANONICAL LIGHT CONE EXPANSIONS

The effectiveness of the use of energy DR's to compute the energy extrapolation is due to a comprehensive phenomenology, namely Regge theory, of high energy behaviour, in addition to the detailed knowledge of the low energy data. We have used canonical LC operator product expansions [3),6)] to provide a similar description of high mass behaviour in order to control the mass DR's. This canonical framework will be briefly reviewed in this Section.

To introduce the LC expansion, let us consider the scalar currents $j(x) \equiv :\varphi(x)\varphi(x):$ in $\lambda\varphi^4$ theory. We assume canonical (i.e., free field) dimensions for all currents. Thus, for example, $\dim j = 2$ and $\dim :\varphi\partial_{\alpha_1}\cdots\partial_{\alpha_n}\varphi: = n+2$. It then follows that the LC behaviour is [3)]

$$j(x)\,j(0) \xrightarrow[x^2\to 0]{} \frac{1}{x^2}\sum_n x^{\alpha_1}\cdots x^{\alpha_n}\, O_{\alpha_1\cdots\alpha_n}(0) \,, \tag{2.1}$$

where $\dim O_{\alpha_1\cdots\alpha_n} = n+2$. As discussed in detail in Ref. 3), the specific form of the O's can be obtained from consideration of the short distance $(x_\mu \to 0)$ expansions of all products $\partial_{\alpha_1}\cdots\partial_{\alpha_n} j(x)j(0)$ or, alternatively, from the highest spin contributions to the equal-time commutators $[\partial_{\alpha_1}\cdots\partial_{\alpha_n} j(x), j(0)]\,\delta(x_0)$. These commutators can be formally evaluated by using the canonical commutation relations for the fields $\varphi(x)$ and the equation of motion $(\Box + \mu^2)\varphi(x) = \lambda :\varphi(x)^3:$ to eliminate higher time derivatives. The results for the leading LC singularity are obviously independent of the interaction term since, for a given dimension, the leading LC singularity is carried by fields with the most Lorentz indices and these come from the kinetic term $\varphi\partial_\alpha\partial^\alpha\varphi$ [e.g., the interaction contribution $:\varphi\varphi:\lambda:\varphi\varphi:$ cannot carry a LC singularity whereas the free contribution $:\varphi\partial_\alpha\partial_\beta\varphi:$ of the same dimension (four) carries a LC singularity $x^\alpha x^\beta/x^2$] and so the free field expansion remains formally valid. This free field expansion follows simply from Wick's theorem

$$\begin{aligned} j(x)\,j(y) \;=\; & 4\Delta_+(x-y\,;\mu^2) :\varphi(x)\varphi(y): \\ & + \text{c-number} \;+\; \text{non-singular} \,, \end{aligned} \tag{2.2}$$

to be

$$j(x)\,j(y) \xrightarrow[\xi^2\to 0]{} \Delta_+(\xi)\sum_n \frac{1}{n!}\,\xi^{\alpha_1}\cdots\xi^{\alpha_n} :\varphi(\eta)\overleftrightarrow{\partial}_{\alpha_1}\cdots\overleftrightarrow{\partial}_{\alpha_n}\varphi(\eta): \,, \tag{2.3}$$

where

$$\xi = \frac{x-y}{2}, \qquad \eta = \frac{x+y}{2}, \qquad \overleftrightarrow{\partial} = \overleftarrow{\partial} - \overrightarrow{\partial}, \tag{2.4}$$

and

$$\Delta_+(\xi) = -\frac{1}{4\pi^2}\,\frac{1}{\xi^2 - i\varepsilon\xi_0} \tag{2.5}$$

in the massless free field Wightman function. Expansions of the type (2.3), which uniquely follow from the assumptions of canonical commutators and field equations, will be referred to as canonical LC expansions. They have been extensively used in Refs. 1)-6),16),17).

Comparison of (2.1) and (2.3) gives

$$O_{\alpha_1 \cdots \alpha_m} = -\frac{1}{4\pi^2}\,\frac{1}{m!}\,:\varphi\,\overleftrightarrow{\partial}_{\alpha_1} \cdots \overleftrightarrow{\partial}_{\alpha_m}\,\varphi: \quad .$$

The highest spin component of the equal-time commutators immediately follow from (2.3). For example

$$\delta(x_0 - y_0)\left[\partial_0 j(x), j(y)\right] = j(\eta)\,\delta(\underset{\sim}{\xi}) \ .$$

For actual physical applications, we will use the canonical gluon model (1.5). The interesting currents are the vector, axial-vector, scalar and pseudoscalar ones

$$V_\mu^a = :\bar\psi \gamma_\mu \frac{\lambda^a}{2}\psi: \tag{2.6a}$$

$$A_\mu^a = :\bar\psi \gamma_\mu \gamma_5 \frac{\lambda^a}{2}\psi: \tag{2.6b}$$

$$S^a = :\bar\psi \frac{\lambda^a}{2}\psi: \tag{2.6c}$$

$$P^a = :\bar\psi \gamma_5 \frac{\lambda^a}{2}\psi: \ . \tag{2.6d}$$

of dimension three. The free field expansions, for example

$$V_\mu^0(x)V_\nu^0(y) \xrightarrow[\xi^2\to 0]{} \frac{\partial}{\partial\xi_\alpha}\Delta_+(\xi)\Big[g_{\mu\alpha}O_\nu^{[-]}(\xi,\eta) + g_{\nu\alpha}O_\mu^{[-]}(\xi,\eta) - g_{\mu\nu}O_\alpha^{[-]}(\xi,\eta) + i\varepsilon_{\mu\nu\alpha\beta}O_{5\beta}^{[+]}(\xi,\eta)\Big], \tag{2.7}$$

$$O_\mu^{[\pm]}(\xi,\eta) = \sum_{ij}\frac{1}{i!j!}\xi^{\alpha_1}\cdots\xi^{\alpha_i}\xi^{\beta_1}\cdots\xi^{\beta_j} : \bar\Psi(\eta)\gamma_\mu\overleftarrow{\partial}_{\alpha_1}\cdots\overleftarrow{\partial}_{\alpha_i}\overrightarrow{\partial}_{\beta_1}\cdots\overrightarrow{\partial}_{\beta_j}\Psi(\eta): \pm(x\leftrightarrow y), \tag{2.8}$$

are now, however, altered by the interaction term since, for example $:\bar\Psi\partial_\alpha\partial_\beta\Psi:$ and $g^2:\bar\Psi B_\alpha B_\beta\Psi:$ can carry the same (leading) LC singularity. The effect of the interaction can, however, be simply accounted for by invoking the invariance of the theory under the gauge transformation [3]

$$\Psi(x) \longrightarrow e^{ig\Lambda(x)}\Psi(x), \qquad B_\mu(x) \longrightarrow B_\mu(x) + \partial_\mu\Lambda(x), \qquad (\Box + \mu^2)\Lambda(x) = 0. \tag{2.9}$$

The result [3] is simply to replace the derivatives ∂_ν in (2.8) by the gauge invariant derivatives

$$\Delta_\nu = \partial_\nu - igB_\nu \;. \tag{2.10}$$

The resulting expansions are then the unique ones which follow from the assumptions of canonical commutators and field equations.

An example of a canonical expansion, to be used below, is [1]

$$P^a\left(\tfrac{x}{2}\right)S^b\left(-\tfrac{x}{2}\right) \xrightarrow[x^2\to 0]{} \partial_\mu\left(\frac{1}{x^2}\right)\sum_n d^{abc}O^{\mu,c}_{\alpha_1\cdots\alpha_n}(0)\,x^{\alpha_1}\cdots x^{\alpha_n} + \partial_\mu\partial_\nu(\ln x^2)\sum_n f^{abc}O^{\mu\nu,c}_{\alpha_1\cdots\alpha_n}(0)\,x^{\alpha_1}\cdots x^{\alpha_n} + \left(\frac{1}{x^2}\right)\sum_n d^{abc}O^{c}_{\alpha_1\cdots\alpha_n}(0)\,x^{\alpha_1}\cdots x^{\alpha_n} \;. \tag{2.11}$$

The O's can be obtained from the canonical evaluation of the equal-time commutators or, more simply, from the free field expansions with the covariant derivatives (2.10). In practice, we will only make use of the first few O's in the expansion.

Another expansion of interest is for the product of (conserved) electromagnetic currents

$$J_\mu = V_\mu^3 + \frac{1}{\sqrt{3}} V_\mu^8 \,. \tag{2.12}$$

The manifestly conserved form of the expansion is [3]

$$\begin{aligned} J_\mu(x)\, J_\nu(0) \xrightarrow[x^2 \to 0]{} & (\partial_\mu \partial_\nu - g_{\mu\nu}\Box)\, x^{-2} \sum_n x^{\alpha_1} \cdots x^{\alpha_n} R_{(0)\alpha_1 \cdots \alpha_n}(0) \\ & + i\varepsilon_{\mu\nu\alpha\beta}\, \partial^\alpha x^{-2} \sum_n x^{\alpha_1} \cdots x^{\alpha_n} R^\beta_{(1)\alpha_1 \cdots \alpha_n}(0) \\ & + \left[g_{\mu\nu} \partial_\alpha \partial_\beta - g_{\alpha\nu} \partial_\beta \partial_\mu - g_{\alpha\mu} \partial_\beta \partial_\nu + g_{\alpha\mu} g_{\beta\nu} \Box \right] (\log x^2) \sum_n x^{\alpha_1} \cdots x^{\alpha_n} R^{\alpha\beta}_{(2)\alpha_1 \cdots \alpha_n}(0). \end{aligned} \tag{2.13}$$

3. LARGE MASS BEHAVIOUR

In this section, we will recall how the LC expansions of Section 2 can be used to determine the behaviour of scattering amplitudes in the limit of large virtual external mass. Knowledge of this behaviour will be crucial for our use of mass dispersion relations in the following sections. We consider first the scaling limit and then show how, in composite particle theories, the large mass limit of interest arises as a special case. The treatment here is an expanded version of the discussion of Refs. 2),4),6),16).

The main points can be illustrated by consideration of the vertex function

$$A(p^2, q^2) = -i \int dx\, e^{i(p+q)\cdot x/2} \langle 0 | T\left[j\left(\tfrac{x}{2}\right) j\left(-\tfrac{x}{2}\right)\right] | k \rangle \tag{3.1}$$

between two scalar currents and the one scalar particle state $|k\rangle$. We take the scalar currents to have dimension two, as in (2.1). The currents carry momentum p and q and the scalar (on-shell) particle has momentum $k = p - q$ and mass $k^2 = m^2$. In addition to the (virtual) mass variables p^2 and q^2, we will use the energy variable

$$\nu = p \cdot k \tag{3.2}$$

and the scaling variable

$$\omega = \frac{p^2}{2\nu} = \frac{p^2}{p^2 - q^2 + m^2} \quad . \tag{3.3}$$

The kinematics are illustrated in Fig. 1. If there is a particle in the spectrum of $j(x)$ with squared mass $q^2 = m^2$, then the on-shell vertex function

$$A(p^2) = \langle q | j(0) | k \rangle \tag{3.4}$$

is given by

$$A(p^2) = \lim_{q^2 \to m^2} (q^2 - m^2) A(p^2, q^2) \quad ; \tag{3.5}$$

i.e., $A(p^2)$ is the residue of the particle pole. We will always assume that the particles $\langle q|$ and $|k\rangle$ are composite ones or that the form factor (3.4) decreases rapidly for $p^2 \to \infty$:

$$A(p^2) \xrightarrow[p^2 \to \infty]{} 0 \qquad \text{fast} \; . \tag{3.6}$$

This is the observed behaviour of all measured hadronic form factors, presumably corresponding to the composite nature of the low-lying hadrons (e.g., they lie on Regge trajectories). This rapid fall off occurs in all composite models [18]. This is to be contrasted with the form factors of elementary particles (as in finite orders of perturbation theory for renormalizable field theories), which approach constants (within log s) for $p^2 \to \infty$. It will not be necessary for our purposes to specify the actual rate of decrease of on-shell form factors. We need only assume a faster decrease than $1/p^2$. Recall that the nucleon electromagnetic form factors decrease like $1/(p^2)^2$, possibly corresponding to the three-quark structure of the nucleon, or perhaps even decrease exponentially, as would be expected in an "infinitely composite" picture of the nucleon.

By the usual arguments [3),4)], the behaviour of $A(p^2,q^2)$ in the scaling limit $p^2 \to \infty$, ω fixed is determined by the leading LC singularity (2.1). The matrix element behaviour

$$\langle 0 | T[j(\tfrac{x}{2}) j(-\tfrac{x}{2})] | k \rangle \xrightarrow[x^2 \to 0]{} \frac{1}{x^2 - i\varepsilon} f(x \cdot k) \tag{3.7}$$

is determined by the matrix elements

$$\langle 0|O_{\alpha_1\cdots\alpha_n}(0)|k\rangle = c_n k_{\alpha_1}\cdots k_{\alpha_n} + g_{\alpha_i\alpha_j}\text{-terms} \tag{3.8}$$

via the formal power series

$$f(\lambda) = \sum_n c_n \lambda^n \ . \tag{3.9}$$

The series (3.9) is assumed to be everywhere convergent so that $f(\lambda)$ is entire. (This is true in perturbation theory and follows if the only singularities are on the LC.) Substitution of (3.7) in (3.1) gives

$$A(p^2, q^2) \xrightarrow[\omega\ \text{fixed}]{p^2\to\infty} \frac{1}{p^2} G(\omega) \tag{3.10}$$

where

$$G(\omega) = (4\pi^2)\,\omega \int_{-1}^{1} d\eta \, \frac{F(\eta)}{\omega-\eta} \ . \tag{3.11}$$

Here $F(\eta)$ is the Fourier transform

$$F(\eta) = \frac{1}{2\pi}\int d\lambda \, e^{-i\lambda\eta} f(\lambda) \tag{3.12}$$

and the support property exhibited in (3.11) is a consequence of the usual analyticity.

A special case is the old Bjorken [19] limit $p_0 \to \infty$ with $\underset{\sim}{p}$ fixed, in which

$$\begin{aligned} A &\longrightarrow \frac{1}{p_0^2}\int dx\, \delta(x_0) \langle 0|[\partial_0 j(\tfrac{x}{2}), j(-\tfrac{x}{2})]|k\rangle \\ &= \frac{1}{p_0^2}\, 4\langle 0|j(0)|k\rangle \ . \end{aligned} \tag{3.13}$$

In this limit, $p^2 \to p_o^2$ and $\omega \to p_o/2k_o \to \infty$ so that (3.10) and (3.11) give

$$A \longrightarrow \frac{1}{p_o^2}(4\pi^2)\int_{-1}^{1} d\eta\, F(\eta) \; . \tag{3.14}$$

Comparison of (3.13) and (3.14) gives

$$4\pi^2 \int_{-1}^{1} d\eta\, F(\eta) \;=\; 4 \langle 0 | j(0) | k \rangle . \tag{3.15}$$

This result could have been immediately derived from (3.12) and (3.7), which give respectively

$$\int d\eta\, F(\eta) \;=\; f(0) \tag{3.16}$$

and

$$4\pi^2 f(0) \;=\; 4 \langle 0 | j(0) | k \rangle \; . \tag{3.17}$$

The actual limit of interest to us here is the fixed mass limit $p^2 \to \infty$ with q^2 fixed. This is a special case of (3.10) in which, according to (3.3), $\omega \to +1$. Let us for the moment assume that $G(1)$ exists and that our fixed mass limit can be obtained by first taking the scaling limit (3.10) and then letting $\omega \to 1$. We will afterwards verify this assumption for composite particles. We thus obtain

$$A(p^2, q^2) \xrightarrow[q^2 \text{ fixed}]{p^2 \to \infty} \frac{1}{p^2} G(1) \; . \tag{3.18}$$

The same result can be obtained directly from (3.1) and (3.7) provided the LC dominates this fixed mass limit. It follows immediately from (3.18), since the right side is independent of q^2, that the on-shell form factor (3.5) decreases faster than $1/p^2$ for $p^2 \to \infty$. This is precisely the behaviour we expect for composite particles and the behaviour we, of course, want to have. We emphasize, however, that we have not derived this good result from the LC behaviour. We have essentially assumed it via our assumption that $G(1)$ exists. The point is that we want on-shell form factors to decrease rapidly and we incorporate this requirement into our LC formalism by assuming the existence of $G(1)$.

Of course, in models containing elementary particles, the form factors involving these particles need not decrease and, correspondingly, the appropriate $G(1)$ will not exist. Examples of this are provided by the amplitudes given by finite order Feynman diagrams. Consider, for example, the low order diagram of Fig. 2, in which the current of momentum q couples to an elementary scalar meson of mass m^2. Taking non-derivative couplings, the amplitude is essentially

$$A_0(p^2,q^2) = \frac{1}{q^2-m^2} = \frac{1}{p^2}\frac{\omega}{\omega-1} \quad . \tag{3.19}$$

Comparison with (3.10) gives the corresponding scaling function to be

$$G_0(\omega) = \frac{\omega}{\omega-1} \quad . \tag{3.20}$$

Also, by (3.11),

$$F_0(\eta) = \frac{1}{4\pi^2}\,\delta(\eta-1) \quad . \tag{3.21}$$

Finally, the corresponding on-shell form factor (3.5) is

$$A_0(p^2) = 1 \quad . \tag{3.22}$$

The first thing we notice is that $G_0(\omega)$ has a pole at $\omega = 1$ so that $G_0(1)$ does not exist and Eq. (3.18) is meaningless. The correct behaviour in the limit (3.18) is $A_0(p^2,q^2) \to 1/q^2-m^2$. Roughly speaking, the linear vanishing of (3.18) for $p^2 \to \infty$ is made up for by the linear divergence of $G(1)$ so that the resultant asymptotic limit is a constant in p^2. For the same reason, the form factor behaviour (3.6) is not obtained, but rather (3.22) gives $A_0(p^2) \to 1$. (The diagram for the form factor is shown in Fig. 3.) We must conclude that the leading LC singularity does not dominate the fixed mass limit in this case. This is accompanied, as it must be, by the bad asymptotic behaviour of the form factor. The culprit is, of course, the elementary particle in Fig. 2 which simultaneously ruins LC dominance of the fixed mass off-shell limit <u>and</u> fast decrease of the on-shell form factor. As we have stressed above, we rule out this unphysical situation by fiat. We assume composite particles and we get LC dominance and rapid decrease of form factors. If the elementary particle in Fig. 2 is replaced by a composite particle, as in Fig. 4 (and likewise for the particle of momentum k), we then obtain a falling form factor and can show that the LC dominates the fixed mass limit.

Let us now return to the general case of composite particle theories where (3.6) and (3.18) are valid. Reference to the representation (3.11) provides an alternative description of our composite particle assumption. Since

$$G(1) = (4\pi^2)\int_{-1}^{1} d\eta \, \frac{F(\eta)}{1-\eta} \,, \tag{3.23}$$

the existence of $G(1)$ is essentially equivalent to the vanishing of $F(1)$. In composite models, the vanishing of $F(\eta)$ at the threshold point $\eta = 1$ is expected since the only contribution to $F(\eta)$ at threshold is from the single particle intermediate state so that $F(1)$ is controlled by the elastic form factor. Thus, a rapidly decreasing form factor insures the vanishing of $F(1)$. This is another way of seeing that a rapidly decreasing form factor requires the existence of $G(1)$. From either point of view, we conclude that $F(1) = 0$. $F(\eta)$ is, in fact, expected to vanish rapidly near $\eta = 1$. In composite models, the rate of decrease of $A(p^2)$ for $p^2 \to \infty$ is directly correlated to the rate of vanishing of $F(\eta)$ near $\eta = 1$.

The rapid vanishing of $F(\eta)$ near $\eta = 1$ and, similarly, near $\eta = -1$, does more than guarantee the existence of $G(1)$ and the LC dominance of the fixed mass limit. It also provides a means of estimating $G(1)$ and hence of using the limit (3.18) in a quantitative way. If $F(\eta)$ is strongly peaked near $\eta = 0$, then from (3.23) and (3.15) we obtain the approximate relation

$$G(1) \simeq (4\pi^2)\int_{-1}^{1} d\eta \, F(\eta) = 4\langle 0|j(0)|k\rangle \,. \tag{3.24}$$

[The same approximation works quite well for the SLAC-MIT structure function $F_2(\eta)$.] Since decay constants of the form $\langle 0|j(0)|k\rangle$ are in general known, (3.24) gives an estimate of $G(1)$ and hence of (3.18). Such estimates will be useful to us in later sections.

We emphasize that, in spite of its appearance, (3.18) with (3.24) does not in any way constitute an assumption that the fixed mass limit (3.18) is the same as the Bjorken limit (3.13). If this were exactly true, then only the first term $[1/x^2 O(0) = 1/x^2(-1/4\pi^2)\, j(0)]$ would be present in (2.1) so that (3.9) would become $f(\lambda) = c_0 = f(0) = (1/4\pi^2)4\langle 0|j|k\rangle$ and (3.12) would become $F(\eta) = c_0\,\delta(\eta)$. This form of $F(\eta)$ is unacceptable both physically and theoretically (e.g., it violates causality [3]). Our assumption is rather that $F(\eta)$ is peaked near $\eta = 0$ so that $G(1)$ is of the same order as $(4\pi^2) f(0)$.

We conclude this section by giving a more complete discussion of the on-shell form factor and a more exact statement of the condition for LC dominance of the fixed mass limit. To do this, we return to the scaling limit (3.10). The form (3.10) is the asymptotic form of the contribution of the leading LC singularity (3.7). Taking into account the contributions of the leading and non-leading contributions of the leading and non-leading LC singularities, assuming always canonical dimensions, we obtain an expansion of the form

$$A(p^2, q^2) \xrightarrow[\omega\ \text{fixed}]{p^2\to\infty} \frac{1}{p^2} G_1(\omega) + \frac{1}{(p^2)^2} G_2(\omega) + \dots + \frac{1}{(p^2)^r} G_r(\omega) + \dots \tag{3.25}$$

Here $G_1(\omega) = G(\omega)$ gives the leading contribution of the leading LC singularity, $G_2(\omega)$ gives the leading contribution of the next-leading LC singularity $(\log x^2)$ as well as the next-leading contribution of the leading LC singularity, etc. For example,

$$\begin{aligned} A(p^2, q^2) &\longrightarrow \int_{-1}^{1} d\eta \left[F_1(\eta)(p^2 - 2\eta\nu + \eta^2 m^2)^{-1} + F_2(\eta)(p^2 - 2\eta\nu + \eta^2 m^2)^{-2} + \dots \right] \\ &= \frac{\omega}{p^2} \int d\eta\, F_1(\eta) \left(\omega - \eta + \frac{\eta^2 m^2 \omega}{p^2}\right)^{-1} + \frac{\omega^2}{p^4} \int d\eta\, F_2(\eta) \left(\omega - \eta + \frac{\eta^2 m^2 \omega^2}{p^2}\right)^{-2} + \dots \\ &= \frac{1}{p^2}\left[\omega \int d\eta\, F_1(\eta) \frac{1}{\omega - \eta}\right] + \frac{1}{p^4}\left[m^2\omega^2 \int d\eta\, F_1(\eta) \frac{\eta^2}{\omega - \eta} + \omega^2 \int d\eta\, F_2(\eta) \frac{1}{(\omega-\eta)^2}\right] \\ &\quad + \dots \end{aligned} \tag{3.26}$$

If each $G_r(\omega)$ exists at $\omega = 1$, then

$$A(p^2, q^2) \xrightarrow[q^2\ \text{fixed}]{p^2\to\infty} \frac{1}{p^2} G_1(1) + \frac{1}{p^4} G_2(1) + \dots , \tag{3.27}$$

and the on-shell form factor (3.4) decreases faster than any power for $p^2 \to \infty$. Suppose, on the contrary, that $G_\ell(\omega)$ is the first G_r to have a pole at $\omega = 1$. (The usual analyticity implies that the only possible singularities of the G_r's are simple poles.) Thus,

$$G_\ell(\omega) \sim \frac{a}{\omega - 1} = a\left(\frac{p^2}{q^2 - m^2} - 1\right) , \tag{3.28}$$

and we can only conclude from (3.25) that

$$\begin{aligned} A(p^2, q^2) \xrightarrow[q^2\ \text{fixed}]{p^2\to\infty} &\frac{1}{p^2} G_1(1) + \dots + \frac{1}{(p^2)^{\ell-1}} G_{\ell-1}(1) \\ &+ \frac{1}{(p^2)^{\ell-1}} \frac{a}{q^2 - m^2} + \dots \end{aligned} \tag{3.29}$$

Furthermore, in this case, we have

$$A(p^2) \xrightarrow[p^2\to\infty]{} \frac{a}{(p^2)^{\ell-1}} \quad . \tag{3.30}$$

As long as $\ell > 2$, we have a reasonable rate of decrease for the form factor and LC dominance of the fixed mass limit. This, in fact, is the precise condition for LC dominance of the fixed mass limit in our framework.

The above discussion makes it clear that the LC formalism can accommodate any rate of decrease (3.30) of the form factor, but that it cannot predict what this rate is. From an ascetic point of view, perhaps the exponential decrease implied by the existence of each $G_r(1)$ is to be preferred.

4. MASS DISPERSION RELATIONS

In this section we continue to work with the scalar vertex function (3.1). For subsequent applications, however, we allow a fixed mass asymptotic behaviour more general than (3.18). We take

$$A(p^2, q^2) \xrightarrow[\substack{p^2\to\infty \\ q^2 \text{ fixed}}]{} (p^2)^{r-2} F_r \tag{4.1}$$

with r unspecified. The behaviour (4.1) corresponds to an LC singularity $(1/x^2)^r$ [Refs. 2),4)]. At this point, we introduce an assumption which we have abstracted from the SLAC-MIT deep inelastic electroproduction experiments. Namely, we assume that the leading asymptotic term (4.1) completely dominates A already for p^2 as low as $2.5\ \mathrm{GeV}^2$. The experimental support for this assumption, which we call "precocious asymptopia", is reviewed in Ref. 4).

We now wish to exploit the fact that, for fixed q^2, $A(p^2,q^2)$ is an analytic function of p^2 in the complex p^2 plane cut along the positive p^2 axis. This enables us to write the relations

$$A(p^2, q^2) = \frac{1}{\pi}\int_0^\Lambda dz \frac{a(z,q^2)}{z-p^2} + \frac{1}{2\pi i}\oint_{C_\Lambda} dz \frac{A(z,q^2)}{z-p^2} \ , \tag{4.2}$$

$$0 = \frac{1}{\pi}\int_0^\Lambda dz\, a(z,q^2) + \frac{1}{2\pi i}\oint_{C_\Lambda} dz\, A(z,q^2) \ , \tag{4.3}$$

where $a = \text{abs } A$ and C_Λ is the circular contour $|z| = \Lambda$. Choosing $\Lambda = 2.5\ \text{GeV}^2$, (4.1) can be used to evaluate the contour integrals $(p^2 \leq \Lambda)$:

$$\frac{F_r}{2\pi i} \oint_{C_\Lambda} dz \frac{z^{r-2}}{z-p^2} = (p^2)^{r-2} A_r \tag{4.4}$$

$$\frac{F_r}{2\pi i} \oint_{C_\Lambda} dz\, z^{r-2} = B_r \,. \tag{4.5}$$

Suppose there is a single low-lying particle P of mass μ with the quantum numbers of $A(x)$ so that

$$a(z,q^2) = \pi\delta(z-\mu^2)\, a_P(q^2) + a_N(z,q^2)\,. \tag{4.6}$$

Then $a_N(z,q^2)$ should not oscillate in the short integration range (this is the great virtue of precocious asymptopia) so that the mean value theorem can be used to conclude that for small p^2

$$\int_0^\Lambda dz \frac{a_N(z,q^2)}{z-p^2} = \frac{1}{M^2-p^2}\int_0^\Lambda dz\, a_N(z,q^2), \qquad 0 \leq M^2 \leq \Lambda\,. \tag{4.7}$$

We have always found that $1.5 \leq M^2 \leq 2$. This intermediate value is expected since $a_N(z)$ should be very small for $z < 1$, where the particle contribution dominates, and for $z > 2.5$, where (4.1) dominates so that $a_N \to 0$ since F_r is real. Putting all this into (4.2) and (4.3), we obtain [2),4),16)]

$$A(p^2;q^2) \cong a_P(q^2)\left(\frac{1}{\mu^2-p^2} - \frac{1}{M^2-p^2}\right) - \frac{B_r}{M^2-p^2} + (p^2)^{r-2}A_r \,. \tag{4.8}$$

Eq. (4.8) is our master equation which we shall use repeatedly in the following sections. It contains essentially no unknowns since A_r and B_r can be approximately determined from decay amplitudes as in (3.24) and $a_P(q^2)$ is determined from on-shell data. It, therefore, provides an approximate description of the off-shell behaviour of the amplitude. As we discussed in Section 1, this information is what we have been seeking.

Although (4.8) has been derived and discussed for scalar vertex functions, it should be clear that analogous results can be obtained when spin is included and for scattering amplitudes. Examples will be given in subsequent sections.

5. DEVIATIONS FROM VECTOR MESON DOMINANCE

Before applying the relation (4.8) to discuss the off-shell extrapolations relevant to symmetry breaking calculations, in this section we will review some of the applications we have made of it in the realm of vector meson dominance (VMD) [16]. Since in these applications the left side of (4.8) will be evaluated at a physical (photon) point, the effect of the off-shell extrapolation predicted by (4.8) can be directly compared with experiment. These applications thus serve as an important check of (4.8) and of the assumptions invoked in its derivation.

In this section, we always take $p^2 = 0$ in (4.8) and take $j(x)$ to be the electromagnetic current $J_\mu(x)$ in (3.1). Then the low-lying particle P is the appropriate vector meson. Equation (4.8) then relates the photon amplitude $A(0)$ to the vector meson amplitude a_P and the decay amplitudes B_r and A_r. The ordinary VMD result is the $M^2 \to \infty$, $A_r \to 0$ limit of (4.8) :

$$A(0) = a_P/\mu^2 \qquad (\mathrm{VMD}). \tag{5.1}$$

Our relation (4.8) is thus seen to supply corrections to this VMD result.

Let us first use (4.8) to compute the $\pi^0 \to \gamma\gamma$ decay amplitude. The off-shell amplitude for $\pi^0(k) \to \gamma^3(p) + \gamma^8(q)$,

$$(2\sqrt{3})\, F(p,q) = \varepsilon_{\mu\nu\alpha\beta}\, \varepsilon^\mu(p)\, \varepsilon^\nu(q)\, p^\alpha q^\beta A(p^2, q^2), \tag{5.2}$$

is given by (3.1) with $j = eJ_\mu$, eJ_ν and $|k> = |\pi^0(k)>$. The relevant operator product expansion is (2.13), where only the second sum contributes. The fixed mass asymptotic behaviour is

$$A(p^2, q^2) \xrightarrow[q^2 \text{ fixed}]{p^2 \to \infty} \frac{1}{p^2} G(1) \simeq \frac{1}{p^2} \frac{\sqrt{2} f_\pi e^2}{3}, \tag{5.3}$$

where we have used the approximation analogous to (3.24). The relevant equal-time commutation relation is

$$[J_i^3(0,\underline{x}),\ J_j^8(0)] = i \frac{1}{\sqrt{3}} \varepsilon_{ijk} A_k^3(0)\, \delta(\underline{x}), \tag{5.4}$$

and we have used the definition $< 0|A^3_\mu(0)|k > = (1/\sqrt{2})k_\mu f_\pi$ of the pion decay constant f_π. Thus, in (4.4) and (4.5), $A_r = 0$ and $B_r = \sqrt{2}f_\pi e^2/3$. The residue of the ω pole in $A(p^2,0)$ is essentially the $\omega \to \pi\gamma$ decay amplitude :

$$a_\omega = (em_\omega^2/2\gamma_\omega)A(\omega \longrightarrow \pi\gamma), \tag{5.5}$$

in the usual notation. Equation (4.8), evaluated at $p^2 = q^2 = 0$, thus becomes [16]

$$A(0,0) \cong \frac{e}{\gamma_\omega}\left(1-\frac{m_\omega^2}{M^2}\right)A(\omega \longrightarrow \pi\gamma) + \sqrt{2}f_\pi e^2/3M^2 . \tag{5.6}$$

It is even simpler to use our methods to estimate $A(0) \equiv A(\omega \to \pi\gamma)$ itself. As we have already said, $A(0)$ is essentially the residue of the ω pole of $A(p^2,0)$. The off-shell invariant amplitude $A(p^2)$ is the "on-shell" form factor

$$\langle \omega(q)| J_\mu(0) | \pi(k)\rangle \sim A(p^2)\,\varepsilon_{\mu\nu\alpha\beta}\,\varepsilon^\nu(q)\,q^\alpha p^\beta$$

and, therefore, decreases rapidly (faster than $1/p^2$) for $p^2 \to \infty$. Therefore, applying our formalism, we have a relation of the form (4.8) for $A(p^2)$ but with $A_r = B_r = 0$, as given by (4.4) and (4.5). The residue of the ρ pole in $A(p^2)$ is essentially the $\omega \to \rho\pi$ amplitude : $a_\rho = g_{\omega\rho\pi}(e/2\gamma_\rho)$ in the usual notation. Thus (4.8) gives [16]

$$A(0) \cong g_{\omega\rho\pi}\frac{e}{2\gamma_\rho}\left(1-\frac{m_\rho^2}{M^2}\right) . \tag{5.7}$$

Let us now compare our predictions (5.6) and (5.7) with experiment. We begin with (5.6). The latest data [20] indicate that the usual VMD result $A(0) \sim g_{\omega\rho\pi}\, e/2\gamma_\rho$ is wrong by about a factor of two in the rates. With $M^2 \simeq 2\mathrm{GeV}^2$, the correction factor $(1 - m_\rho^2/M^2)$ implied by our analysis nicely resolves this discrepancy. Consider next (5.6). Here the usual VMD result $A(0,0) \sim (e/\gamma_\omega)A(0)$ is wrong by about a factor of three in the rates [20]. Our result (5.6), with $M^2 \simeq 2$, is again in agreement with the data. Our formalism is thus seen to supply important and numerically accurate corrections to VMD.

For our present purposes, the precise numerical results (5.6) and (5.7) need not be taken too seriously. What is important is that the comparison of (5.6) and (5.7) with experiment has revealed no gross failures of our assumptions and calculations. On the contrary, the reliability of our methods is strongly suggested [21]. We, therefore, proceed with some confidence to apply (4.8) to discuss symmetry breaking effects where, unlike in the present situation, the left side will not be directly known.

6. SYMMETRY BREAKING FOR VERTEX FUNCTIONS

The formalism of Section 4 can be directly applied in order to relate symmetry breaking parameters in Lagrangians to breaking effects in states and deduce approximate relations between physical S matrix elements from operator algebras. An example of this is our analysis [1] in which some $SU(3) \times SU(3)$ symmetry breaking effects were estimated. We used the mass DR's to provide algebraic relations between the values of amplitudes at zero mass (given by equal-time commutation relations), at physical points (given by experiment), and at mass $\simeq 2.5$ GeV^2 (given by the LC expansions). Since use of the smooth threshold assumption relates the LC behaviour back to the equal-time behaviour, we end up with coupled equations for various physical parameters which can be solved simultaneously. This is how our algebraic equations for operators lead to algebraic equations for physical parameters and how the (exact) operator symmetry embodied in Eqs. (1.1)-(1.6) leads to (broken) symmetry for the physical states. This is, in fact, a different way of saying that we are calculating corrections to pseudoscalar meson pole dominance. In the exact $SU(3) \times SU(3)$ limit ($\alpha_0 = \alpha_8 = 0$), these mesons are massless and so current algebra gives on-shell $SU(3)$ symmetric predictions. In the real world with $\alpha_8 \neq 0$ and $\alpha_0 \neq 0$, the effects of both off-shell extrapolations and $SU(3)$ violations must be taken into account, and that is what our formalism attempts to accomplish. This, incidentally, puts pion PCAC and kaon PCAC on the same footing.

The vertex functions of interest are the (vaccum) - (one-pseudoscalar meson) matrix elements of A^a_μ and P^a. We define the usual pion and kaon decay constants by

$$\langle 0|A^{\pi^-}_\mu|\pi^+\rangle = ip_\mu f_\pi , \qquad \langle 0|A^{K^-}_\mu|K^+\rangle = ik_\mu f_K , \tag{6.1}$$

and "renormalization" constants by

$$\langle 0|P^{\pi^-}|\pi^+\rangle = Z_\pi , \qquad \langle 0|P^{K^-}|K^+\rangle = Z_K . \tag{6.2}$$

We always label (on or off-shell) pions (kaons) with momentum p_μ (k_μ). On-shell, $p^2 = m_\pi^2 = \mu^2$, $k^2 = m_K^2 = m^2$. The divergence equations (1.8) immediately give the relations

$$\mu^2 f_\pi = \frac{2}{\sqrt{3}}(\sqrt{2}\alpha_0 + \alpha_8)Z_\pi , \qquad m^2 f_K = \frac{2}{\sqrt{3}}(\sqrt{2}\alpha_0 - \tfrac{1}{2}\alpha_8)Z_K . \tag{6.3}$$

The analysis of Ref. 1) goes as follows. We define the two vertex functions ($k = p+q$)

$$\int dx\, e^{i(p-q)\cdot x/2}\langle 0|T\, D^a(\tfrac{x}{2})\mathcal{D}^K(-\tfrac{x}{2})|b\rangle , \qquad a = \pi, K . \tag{6.4}$$

Each of these satisfies two exact zero mass theorems, one at $q = 0$ $(p^2 = m^2$ or $k^2 = \mu^2)$ and one at $p = 0$ $(q^2 = m^2)$ or $k = 0$ $(q^2 = \mu^2)$. We define two more vertex functions

$$\int dx\, e^{i(p-q)\cdot x/2} \langle 0 | T\, D^a(\tfrac{x}{2}) V^K_\mu(-\tfrac{x}{2}) | b \rangle \quad , \quad a = \pi, K \; . \tag{6.5}$$

For $a = \pi$ we get a zero mass theorem at $p = 0$ $(q^2 = m^2)$ and for $a = K$ we get one at $k = 0$ $(q^2 = \mu^2)$. We next write mass DR's for these vertex functions and obtain equations of the form (4.8). The left sides are known in terms of the zero mass theorems, the pole contributions are known in terms of the on-shell $K_{\ell 3}$ amplitudes defined by

$$\langle \pi^0(p) | V^K_\mu(0) | K^+(k) \rangle = \frac{1}{\sqrt{2}} \left[(k+p)_\mu f_+(q^2) + (k-p)_\mu f_-(q^2) \right] , \tag{6.6}$$

and the LC contributions are known from the operator product expansions (if the leading contribution does not contribute, we must keep the next leading one). Equation (2.11) is a typical LC expansion used. In this way, using the relations (6.3), we end up with six equations [given in Ref. 1)] in the six unknowns

$$f_+(0) , \quad \xi(0) \equiv f_-(0)/f_+(0) , \quad f_+(m^2) + f_-(m^2) , \quad \alpha_8 , \quad A , \quad B \; .$$

A and B are uninteresting parameters defined in Ref. 1).

The solution to these equations depends on the choice of the parameter C defined in Eq. (1.9). Let us first take the non-chiral value (1.11b). The solution is then

$$f_+(0) \simeq 0.94 \tag{6.7a}$$

$$\xi(0) \simeq -0.7 \tag{6.7b}$$

$$f_+(m^2) + f_-(m^2) \simeq \frac{f_K}{f_\pi}(1.45) \tag{6.7c}$$

$$\alpha_8 \simeq -140 \text{ MeV} \; . \tag{6.7d}$$

These values were obtained using the usual value $M^2 = 1.5$ [22)] but they are quite insensitive to M^2 in the range $1.5 \leq M^2 \leq 2$.

Let us comment on these results [23]. Equation (6.7) is in good agreement with the Ademollo-Gatto theorem. Equation (6.7) is in good agreement with the recent $K_{\ell 3}$ data [24]. Note that it was obtained without making any assumptions about the q^2 dependence of the form factors. Equation (6.7) represents a 50% violation of the Callan-Treiman relation. Equation (6.7) is, as expected, a small SU(3) violating parameter. The chiral SU(2) × SU(2) symmetry breaking parameter is thus

$$\varepsilon_2 = \sqrt{2}\,\alpha_0 + \alpha_8 \simeq 580 \text{ MeV}. \tag{6.8}$$

We see that this formalism leads to a completely consistent picture of SU(3) × SU(3) symmetry breaking.

With the value (1.11b), Eq. (6.3) gives

$$\frac{\mu^2}{m^2}\frac{f_\pi}{f_K} \simeq (0.77)\frac{Z_\pi}{Z_K} . \tag{6.9}$$

Thus, since $f_\pi/f_K \sim 1$, we have $Z_\pi/Z_K \sim 1/12$. The above formalism clearly exhibits the mechanism which is responsible for the large difference between Z_K/Z_π and f_K/f_π in the non-chiral framework. The $M^2 \to \infty$ limits of four of the above-mentioned six equations are

$$Z_K \sim Z_\pi f_+(0) - \varepsilon_3(f_K + A) \tag{6.10a}$$

$$Z_\pi \sim Z_K f_+(0) + \varepsilon_3(f_\pi - A) \tag{6.10b}$$

$$f_K \sim f_\pi\left[f_+(m^2) + f_-(m^2)\right] \tag{6.11a}$$

$$f_\pi \sim f_K\left[f_+(\mu^2) - f_-(\mu^2)\right] . \tag{6.11b}$$

The important difference between (6.10) and (6.11) is the presence of the $\mp\,\varepsilon_3 f_{K,\pi}$ terms in (6.10) which come from the LC. These terms are numerically important and their occurrence with opposite signs in (6.10a) and (6.10b) gives rise to the difference between Z_K and Z_π. We thus see that this difference can be completely understood as a dynamical consequence of a formalism with a small symmetry breaking parameter at the Lagrangian level. We conclude that a large value of Z_K/Z_π is, contrary to what might be naively expected, completely consistent with the usual picture of a small breaking parameter. It is clearly very dangerous to assume that there is a simple connection between Lagrangian and state symmetry breaking, especially when the pseudoscalar octet is involved.

As is clear from the solution (6.7), the contributions form the order $1/M^2$ terms are also important. These terms constitute the corrections to pion and kaon pole dominance.

Let us now solve our six equations using the chiral value (1.11a) of C. We then find that $\xi(0)$ us small and $f_+(m_K^2) + f_-(m_K^2)$ is near f_K/f_π. These are the familiar chiral results [9] obtained by assuming the off-shell $K_{\ell 3}$ amplitudes are pion-pole dominated or, equivalently, that the chiral symmetry breaking parameter ε_π is small enough to justify a perturbative treatment. Actually, we have learned more since it is possible to obtain a larger (negative) $\xi(0)$ in the chiral framework provided the slope parameter λ_+, defined by

$$f_+(t) = f_+(0)\left[1 + \lambda_+ t/\mu^2\right] \quad , \tag{6.12}$$

is much larger than the value $\lambda_+ \simeq 0.023$ predicted by K* dominance [25]. The most recent experiment [26], which has achieved the highest degree of accuracy to date, gives $\lambda_+ = 0.023 \pm 0.005$. The latest X2-collaboration result is $\lambda_+ = 0.027 \pm 0.010$ [27]. Thus it appears that the chiral prediction for $\xi(0)$ is too small. Further experimental clarification is, of course, needed and should come in the near future. It seems fair to say, however, that the present experimental information favours (1.11b) over (1.11a).

We conclude this section with a brief discussion of the chiral symmetry breaking effects expected for on-shell form factors. Consider, as a well-known example, the nucleon matrix element of (1.8a) :

$$\langle N | D^\pi(0) | N \rangle = \bar{u}\gamma_5 u \, d(p^2) \quad . \tag{6.13}$$

Keeping only the pion pole in the p^2 dispersion relation gives the Goldberger-Treiman relation

$$2m_N g_A = d(0) \simeq \sqrt{2} f_\pi g_r \quad , \tag{6.14}$$

in conventional notation. It should be clear that our formalism gives instead

$$2m_N g_A \cong \sqrt{2} f_\pi g_r \left(1 - \frac{\mu^2}{M^2}\right) \quad . \tag{6.15}$$

Since the correction factor $(1 - \mu^2/M^2)$ is very near to unity in this case, we predict that (6.14) should be satisfied to within a few per-cent. This is consistent with experiment. The deviation from pion-pole dominance should be similarly small for all form factors. The general statement is [16]

$$\langle\alpha|D^2|\beta\rangle\Big|_{p^2=0} \cong \frac{\sqrt{2}}{f_\pi}\langle\alpha\pi^2|\beta\rangle\left(1-\frac{\mu^2}{M^2}\right) \tag{6.16}$$

for arbitrary low-lying states $\langle\alpha|, |\beta\rangle$.

7. - SYMMETRY BREAKING FOR SCATTERING AMPLITUDES

The formalism which we used in previous sections for vertex functions can be easily extended to scattering amplitudes [2]. Consider an off-shell forward scattering amplitude $T(\varkappa, \nu)$ for

$$\sigma(p) + j(q) \longrightarrow \sigma(p) + j(q),$$

where $\sigma(p)$ is a scalar particle of momentum p and mass $p^2 = 1$ and $j(q)$ is a scalar current of dimension two, momentum q, and mass $q^2 = \varkappa$. $T(\varkappa, \nu)$ is defined to have its $\varkappa$ poles removed so that the on-shell amplitude is $T(\mu^2, \nu)$. We assume the usual Regge behaviour

$$T(\varkappa,\nu) \xrightarrow[\varkappa\ \text{fixed}]{\nu\to\infty} \beta(\varkappa)\,\nu^{\alpha}, \tag{7.1}$$

the usual scaling behaviour

$$T(\varkappa,\nu) \xrightarrow[\omega=\frac{\varkappa}{2\nu}\ \text{fixed}]{\nu\to\infty} \nu^{-1}F(\omega), \tag{7.2}$$

and the usual connection between these limits [4] :

$$\beta(\varkappa) \xrightarrow[\varkappa\to\infty]{} \beta\,\varkappa^{-\alpha-1}, \tag{7.3}$$

$$F(\omega) \xrightarrow[\omega\to 0]{} \beta\cdot(2\omega)^{-\alpha-1}. \tag{7.4}$$

We cannot apply the analysis of Section 4 directly to the mass DR's satisfied by $T(\varkappa, \nu)$ since, for finite ν, the $\varkappa$ discontinuities come from intermediate states in both the current and the current-particle channels. In the limit $\nu \to \infty$, however, only the pure current discontinuities remain. It is the absorptive part corresponding to these pure discontinuities which should not oscillate in the short $\varkappa$ integration range so that the mean value theorem can be applied. The $\nu \to \infty$ limit of the mass DR for $T(\varkappa, \nu)$ gives, via (7.1), a mass DR for $\beta(\varkappa)$. This leads to a relation of the form (4.8) for $\beta(\varkappa)$. The constants A_r and B_r in (4.4) and (4.5) can be calculated in terms of β by use of (7.3). With this then explicit representation for $\beta(\varkappa)$, we can calculate the effect

$$\Delta\beta = \beta(\mu^2) - \beta(0) \tag{7.5}$$

of the off-shell extrapolation.

The next step is to write the finite energy sum rule

$$T(\varkappa,\nu) = \frac{1}{\pi}\int_0^N d\nu' \frac{\mathrm{Im}\,T(\varkappa,\nu')}{\nu'-\nu} + \frac{1}{2\pi i}\oint_{C_N} d\nu' \frac{T(\varkappa,\nu')}{\nu'-\nu} . \tag{7.6}$$

We choose N large enough (say, $N \geq 2.5$ GeV^2) so that the leading Regge term (7.1) dominates the contour integral I_N. We obtain for $\nu \ll N$

$$I_N = \beta(\varkappa) c_\alpha N^\alpha , \tag{7.7}$$

where c_α is a constant. [Care must be taken in using the correct phase implicit in (7.1).] From the 0-N integration in (7.6), we separate out the Born and low lying resonance contributions and treat them explicitly. The remainder $\mathrm{Im}\bar{T}(\varkappa, \nu')$ can be expressed as a sum over low-lying (since $\nu' \leq N = 2.5$ GeV^2) intermediate states, for each term of which a relation of the form (6.16) is valid. Putting all this into (7.6) and evaluating at $\nu = 0$, we obtain [2)]

$$T(\mu^2, 0) - T(0,0) = \left(\frac{2\mu^2}{M^2}\right)\bar{T}(\mu^2, 0) + (\Delta\beta) c_\alpha N^\alpha \\ + \text{(Born and Resonance Contribution)} \tag{7.8}$$

The relation (7.8) expresses the off-shell amplitude $T(0,0)$ in terms of quantities which can be calculated from on-shell data. The new ingredient which we have supplied is the contribution of the high energy part of T to the off-shell extrapolation. We emphasize again that neither pole dominance nor small symmetry breaking assumptions were involved in the derivation of (7.8). Our mass DR's have enabled us to approximately <u>compute</u> the effects of the off-shell extrapolation.

We now follow Ref. 2) and apply (7.8) to π-N scattering. The off-shell amplitudes $T^{(\pm)}(\varkappa, \nu)$ satisfy the exact zero energy theorems [8)]

$$T^{(-)\prime}(0,0) = -\frac{2}{f_\pi^2}(g_A^2 - 1)\frac{1}{2m_N} , \tag{7.9}$$

$$T^{(+)}(0,0) = -\frac{2}{f_\pi^2}\sigma_N \; , \tag{7.10}$$

where

$$\sigma_N \equiv \langle N|\sigma(0)|N\rangle \tag{7.11}$$

is the nucleon expectation value of the σ term

$$\sigma(0) = \frac{2}{3}\varepsilon_2\left[\sqrt{2}\,S^0(0) + S^8(0)\right] . \tag{7.12}$$

The analogues of Eq. (7.8) can be used to check (7.9) and (7.10), by relating them to on-shell quantities. For this purpose, the generalizations $\beta^{(\pm)}$ of the constant in (7.3) must be estimated. This is done in Ref. 2) by relating $T^{(\pm)}$ to the Compton scattering amplitudes in the deep inelastic limit. (To obtain this relation, the canonical formalism of Section 2 must be used strongly.)

From the relation for the isospin odd amplitude $T^{(-)}$, using the non-chiral value (1.11b) of C, we obtain $g_A \sim 1.23$ [2]. The chiral value (1.11a) of C gives $g_A \sim 1.30$. The most recent experiment determination of g_A give about 1.24 [28]. The non-chiral value for C is thus favoured, although the theoretical uncertainties in the calculations are sufficiently large in the present case that this must be taken as just a suggestion. Since completely ignoring off-shell effects gives only $g_A \sim 1.16$, however, these effects are rather important and indicate that chiral symmetry is not too good.

From the relation for the isospin even amplitude $T^{(+)}$, the non-chiral value (1.11b) gives a huge σ term $\sigma_N \sim 430 \pm 150$ MeV [2),29]. All we can say for the result of using the chiral value (1.11a) is that $\sigma_N \sim 100 \pm 100$ MeV. There is, unfortunately, no independent non-perturbative determination of σ_N to check which result is better. It is clear from (7.12), however, that, unless $\langle N|S^0|N\rangle$ is anomalously large, σ_N should be quite small if ε_2 is small (chiral value) and (1.3) is the correct symmetry breaking term. Some recent determinations [30),31] of σ_N, based on the <u>assumption</u> that ε_2 is small, give too large values to be consistent with this. Therefore, if these determinations are correct and if ε_2 is really small, then one must either give up (1.3) or invent some mechanism (e.g., a new low-lying meson [32),33]) for making $\langle N|S^0|N\rangle$ very big.

It is unfortunate that, at the present time at least, we cannot tell for certain from π-N scattering which, if either, value of C is correct. Nor can consideration of other scattering processes give us this information at present. All of the other current algebra, PCAC results usually taken as indicating the smallness of

ϵ_2 (e.g., Adler consistency condition, Weinberg-Tomozawa scattering lengths, Fubini photoproduction sum rules, $K_{3\pi}$ decay) can also be derived with our methods without this assumption [15]. We conclude that these scattering processes at present give us little information on the magnitude of ϵ_2.

8. - DISCUSSION

In the above sections, we have shown how LC controlled mass DR's can be used to accurately calculate the deviations from pion (and kaon) pole dominance for both vertex functions and scattering amplitudes. Our mass DR's can be looked upon as a means of deducing observable (dynamical) consequences from an underlying operator (algebraic) structure. This operator structure has been extended from the original $SU(3) \times SU(3)$ equal-time algebra [7], to the $(3,\bar{3}) + (\bar{3},3)$ symmetry breaking relations [7], to the additional equal-time relations [14] and full LC expansions [3] abstracted from the canonical gluon model. The equal-time commutation relations imply exact zero energy and mass theorems which are at unphysical (zero) masses for the pseudoscalar mesons but physical for photons. The mass DR's relate the values of the amplitudes at these points to their values at the mass shell points of the pseudoscalar mesons or vector mesons and to their values at mass Λ which, by precocious asymptopia, are given by the LC. The smooth threshold assumption relates this LC behaviour to either equal-time relations or to other measurable quantities. We are thus provided with algebraic relations among physical quantities which are the observable reflections of the basic underlying operator algebra.

The present experimental situation is unfortunately not yet accurate enough for us to unequivocally conclude whether $C/\sqrt{2}$ is near zero (non-chiral world) or near -1 (chiral world). The $K_{\ell 3}$ decay provides the most decisive test. Here the present results definitely favour a small C, but further clarification is needed. Pion-nucleon scattering perhaps also indicates a slight preference for small C since off-shell effects seem important in the Adler-Weisberger relation and rather large σ terms may be present.

Some further support for a small C comes from consideration of the $\pi^0 \to \gamma\gamma$ and $\eta \to 3\pi$ decays, both of which vanish in the limit $\epsilon_2 \to 0$. [8]. The first problem can be got around if perturbative anomalies [34] are present in D^{π^0} in the presence of electromagnetism (although the rate or sign comes out wrong in the usual perturbative models [15]), but more drastic remedies, such as the presence of non-electromagnetic isospin violating interactions or additions to the usual electromagnetic current (2.12), are needed to get the η to decay in the chiral framework [35)-37]. Similarly, the Dashen [11] electromagnetic mass shift sum rule, valid to order ϵ_2 but in terrible agreement with experiment, suggests that conventional electromagnetism should be changed if ϵ_2 is really small.

It is clear that difficulties are encountered if one wants a small ε_2. Each difficulty can, of course, be argued away. Thus, the present $K_{\ell 3}$ experiments may be wrong, the $(3,\bar{3}) + (\bar{3},3)$ symmetry breaking model may be wrong or a new scalar meson may be around to enhance $< N|S^0|N >$, an anomaly may be present in $D\pi^0$, and our present understanding of electromagnetism and isospin violation may be wrong. These are all possibilities which may turn out to be correct. It seems far simpler, however, to simply give up the idea that ε_2 is small. Indeed, it seems that every time one goes beyond the good current algebraic results which are independent of the assumption that ε_2 is small and tries to do perturbation theory in ε_2, one encounters an enormous disagreement with experiment unless one simultaneously invents a new particle or a new interaction. The new predictive power of good chiral symmetry has, in fact, been non-existent. The beauty of the chiral framework of course makes these attempts to save it understandable. Experiment, however, must remain the final judge.

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REFERENCES

1) R. Brandt and G. Preparata - Phys.Rev.Letters 26, 1605 (1971).

2) R. Brandt and G. Preparata - New-York University Preprint 6/71, to be published.

3) R. Brandt and G. Preparata - Nuclear Phys. B27, 541 (1971).

4) For a review of the theory and a summary of applications, see :
R. Brandt and G. Preparata - New-York University Preprint 1/71, to be published in the Proceedings of the 1971 Coral Gables Conference.

5) A more complete review is contained in :
R. Brandt and G. Preparata - Proceedings of the Summer Institute in Theoretical Physics, Hamburg, DESY (1971).

6) See also the lectures by :
R. Brandt and G. Preparata - 1970 Brandeis, Erice and Heidelberg Summer Schools.

7) M. Gell-Mann - Phys.Rev. 125, 1067 (1962).

8) See, for example :
S. Adler and R. Dashen - "Current Algebra and Applications to Particle Physics", Benjamin, New York (1968).

9) S. Weinberg - Rapporteur talk at the 14th International Conference on High Energy Physics, Vienna (1968).

10) Y. Nambu and G. Jona-Lasinio - Phys.Rev. 122, 345 (1961) ; 124, 246 (1961).

11) R. Dashen - Phys.Rev. 183, 1245 (1968).

12) M. Gell-Mann, R. Oakes and B. Renner - Phys.Rev. 175, 2195 (1968).

13) S. Glashow and S. Weinberg - Phys.Rev.Letters 20, 224 (1968).

14) See :
R. Brandt and G. Preparata - Phys.Rev. D1, 2577 (1970), for a previous application of this extended algebra.

15) R. Brandt and G. Preparata - Ann.Phys.(USA) 61, 119 (1970).

16) R. Brandt and G. Preparata - Phys.Rev.Letters 25, 1530 (1970).

17) G. Altarelli, R. Brandt and G. Preparata - Phys.Rev.Letters 26, 42 (1970).

18) See, for example :
D. Amati et al. - Phys.Letters 27B, 38 (1968).

19) J.D. Bjorken - Phys.Rev. 148, 1469 (1966).

20) J.E. Augustin et al. - Phys.Letters 28B, 503 (1969).

21) For further application to VMD problems, see Ref. 4) and :
R. Brandt, W.C. Ng, P. Vinciarelli and G. Preparata - New York University Preprint 9/71, to be published ;
G. Preparata - Phys.Letters 36B, 53 (1971).

22) The smaller value $M^2 = 1.5$ is expected for the π case since $m^2_\pi = 0.02$ whereas $m^2_\rho = 0.5$. These mesons are expected to dominate near their masses so that the effective continuum is from 0 to 2.5 in the π case and from 1 to 2.5 in the ρ case.

23) See Ref. 9) for a discussion of some previous attempts to calculate these quantities.

24) For a review, see :
M.K. Gaillard and L. Chounet - CERN 70-14 (1970).

25) R. Dashen and M. Weinstein - Phys.Rev.Letters 22, 1337 (1969).

26) Aachen-CERN-Torino Collaboration, V. Bisi et al. - Phys.Letters, to be published.

27) X2-Collaboration, H.J. Steiner et al. - Phys.Letters, to be published.

28) C.J. Christenson, V.E. Krohn and C.R. Ringo - Phys.Letters 28B, 411 (1969), from polarized neutron β decay, find a radiatively corrected $(g_A/g_V)_0 = 1.26 \pm 0.02$. From the ratio of the neutron β decay rate of C.J. Christenson et al. [Phys.Letters 26B, 11 (1967)] and the rate of ^{26}Al one gets $(g_A/g_V)_0 = 1.226 \pm 0.011$, and from the rate of ^{14}O one obtains $(g_A/G_V)_0 = 1.232 \pm 0.012$.

29) See also :
W.C. Ng and P. Vinciarelli - Phys.Rev., to be published.

30) T. Cheng and R. Dashen - Phys.Rev.Letters 26, 594 (1971).

31) G. Höhler, H. Jakob and R. Strause - Phys.Letters 35B, 4451 (1971) ; the value here is somewhat smaller than that of Ref. 30).

32) R. Crewther - Phys.Rev. D3, 3152 (1971).

33) G. Altarelli, N. Cabibbo, L. Maiani - Phys.Letters 35B, 4 (1971).

34) S. Adler - Phys.Rev. 177, 2426 (1969).

35) M. Weinstein - New York University Preprint 5/71.

36) There is, moreover, experimental evidence against these remedies. See :
T. Cheng and R. Dashen - to be published.

37) In the non-chiral framework, on the other hand, not only is there no problem about the magnitude of the decay rate, but the good current algebraic prediction of the slope is maintained. See :
R. Brandt, M. Goldhaber, C. Orzalesi and G. Preparata - Phys.Rev.Letters 24, 1517 (1970).

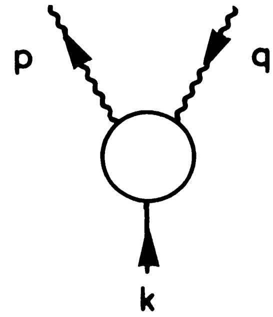

FIG. 1

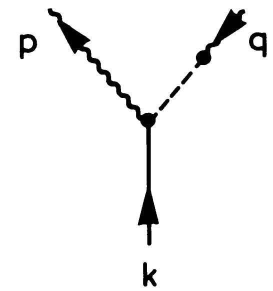

FIG. 2

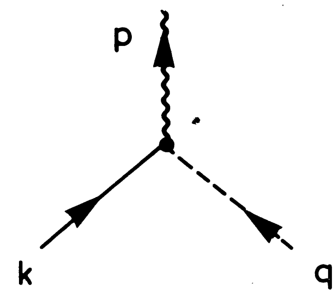

FIG. 3

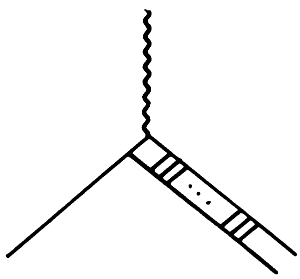

FIG. 4