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ELASTIC SCATTERING OF ELECTRONS FROM POLARIZED PROTONS AND INELASTIC ELECTRON SCATTERING EXPERIMENTS

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A B S T R A C T

Rigorous bounds, to order $\alpha \simeq 1/137$, are derived for the transverse polarization of protons in elastic electron-proton scattering. The bounds depend on the proton elastic form factors and on the proton inelastic structure functions. Numerical estimates of the bounds are made and the results are compared with the polarization data. We discuss their theoretical meaning and their implications upon future experiments.

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I. INTRODUCTION.

We shall be concerned with the tranverse polarization of the proton in elastic electron-proton scattering. To this effect, two kinds of experiments have been reported : measurements of the polarization of the recoiling proton [1-4], the target being unpolarized ; and measurements of the up-down asymmetry [5] using a polarized hydrogen target. At the same kinematical conditions the two types of observables have opposite values by time reversal invariance. In the region of energies and momentum transfers so far explored, the results of either type of experiments are compatible with zero.

The Born approximation to elastic electron-proton scattering predicts zero transverse polarization, even in the absence of time reversal invariance.An effect can first arise through the interference between the one and two photon exchange amplitudes.To this approximation and assuming time reversal invariance, assumptions which unless otherwise stated we shall adopt throughout this paper, the quantity which governs transverse polarization effects is the absorptive part of the Compton amplitude for off-shell photons scattering from nucleons. It is well known that the unitarity relation can be used to compute this absorptive amplitude in terms of physical on-mass-shell intermediate states. These include the elastic contribution, along with resonances and hadron continuum contributions. Calculations of the elastic part have been available for some time. They represent the whole answer to the transverse polarization [6,7,8] at electron energies below the one pion production threshold. The contribution from the lowest lying nucleon isobars has also been estimated in particular models [6,7]. The sum of elastic and resonances intermediate states is expected to give a good estimate of the whole polarization effect at low energies. However, at high energies, the effect of the hadron continuum contribution could be very important since inelastic electron-proton cross-sections are large. We know of no model independent calculations concerning this continuum contribution.

In this paper we have rigorously bounded, to order $\alpha \simeq 1/137$, the resonance and continuum contributions to the transverse proton polarization. A similar bound applied to elastic neutrino reactions has been previously discussed by two of the authors [9]. In the electron case, the bound is given in terms of the proton elastic form factors and inelastic structure functions. The latter are known in a sufficiently wide range to draw relevant conclusions. We have also made a recalculation of the elastic contribution.

The paper is organized as follows : the rest of this section included for the sake of completeness, lists a collection of well know facts on T-odd effects, time reversal invariance and unitarity, and their relevance to the calculation of the effect under discussion. Section II deals with the two photon exchange amplitude and its contribution to the up-down asymmetry. The restrictions imposed by parity and time reversal invariance are also studied in this section. In section III we give the results of the recalculation of the elastic contribution, and compare them to those obtained by other authors. Section IV includes the derivation of bounds on the resonance and continuum contributions to the transverse proton polarization. The results, their comparison with experiment and the conclusions are given in Section V.

I.1. Summary of results on T-odd effects, and application to the up-down asymmetry.

Let us call T_{if} the T-matrix element describing the transition from an initial asymptotic state i to a final asymptotic state f . If the relation between the S-matrix and T-matrix is defined as

$$S = 1 + i(2\pi)^4 \, \delta^{(4)}(\Sigma p_i - \Sigma p_f) T \quad , \qquad (1\text{-}1)$$

then the unitarity condition reads

$$T_{if} - T^{\dagger}_{if} = i\mathcal{A}_{if} \qquad (1\text{-}2)$$

where

$$\mathcal{A}_{if} = \sum_{\Gamma} T_{i\Gamma} T^{\dagger}_{\Gamma f} (2\pi)^4 \, \delta^{(4)}(\Sigma p_i - p_\Gamma) \quad , \qquad (1\text{-}3)$$

and the summation is extended to all possible on-shell intermediate states Γ . The amplitude $\mathcal{A}_{if}$ is the absorptive part of the amplitude T_{if} .

With $\tilde{i}$ and $\tilde{f}$ denoting the initial and final states i and f with spins and momenta reversed, time reversal invariance, implies

$$|T_{if}|^2 = |T_{\tilde{f}\tilde{i}}|^2 \quad . \qquad (1\text{-}4)$$

By T-odd effect in a transition $i \rightarrow f$ we mean any observable proportional to the difference of probabilities

$$|T_{if}|^2 - |T_{\tilde{i}\tilde{f}}|^2 \quad . \qquad (1.5)$$

From the definitions given above, it can be readily seen that, when time reversal invariance applies, the relation between T-odd effects and the absorptive part of the transition amplitude is

$$|T_{if}|^2 - |T_{\tilde{i}\tilde{f}}|^2 = 2\,\mathrm{Im}\,(T_{if}\mathcal{A}_{fi}) - |\mathcal{A}_{if}|^2 \quad . \tag{1.6}$$

If furthermore, the transition amplitude is proportional to some small coupling constant, like the fine structure constant $\alpha \simeq \frac{1}{137}$ in our case, then Eq.(1.6) can be expanded to various orders. To lowest order

$$|T_{if}|^2 - |T_{\tilde{i}\tilde{f}}|^2 = 0 \quad ,$$

and as is well known, T-odd effects are absent. In the next order, we get

$$|T_{if}|^2 - |T_{\tilde{i}\tilde{f}}|^2 = 2\,\mathrm{Im}\,(T_{if}\,\mathcal{A}_{fi}) \quad , \tag{1.7}$$

and T-odd effects appear as due to the interference between the Born approximation to the amplitude T_{if} (see Fig.1) and the lowest-order contribution to the absorptive amplitude $\mathcal{A}_{fi}$ which arises from two-photon exchange (see Fig.2) .

We discuss now the application to the up-down asymmetry in elastic electron proton scattering. For definiteness we shall consider the case where the proton target is polarized along the normal to the scattering plane which we take as the Z-axis and work in the center of mass (C.M.)system, at fixed energy and momentum transfer. The up-down asymmetry is defined as

$$\xi = \frac{N^{\uparrow} - N^{\downarrow}}{N^{\uparrow} + N^{\downarrow}} \quad , \tag{1.8}$$

where $N^{\uparrow}$ and $N^{\downarrow}$ are the number of events corresponding to the configurations shown in Figs. 3a and 3b. In terms of transition amplitudes we have (summation over unspecified spin indices is understood)

$$N^{\uparrow} \;\alpha\; | \, T_{\uparrow}(\vec{k},\vec{k}') \, |^2 \equiv |T_{if}|^2$$

and

$$N^{\downarrow} \;\alpha\; | \, T_{\downarrow}(\vec{k},\vec{k}') \, |^2 \quad .$$

Consider now the T-matrix describing the transition $\tilde{i} \to \tilde{f}$ (see Fig.4). We have

$$| \, T_{\downarrow} \, (-\vec{k},-\vec{k}') \, |^2 = |T_{\tilde{i}\tilde{f}}|^2 \quad .$$

Since $T_{\downarrow}(-\vec{k},-\vec{k}')$ and $T_{\downarrow}(\vec{k},\vec{k}')$ are related, up to a phase, by a rotation of π around the Z-axis, the asymmetry (Eq.1.8) is proportional to $|T_{if}|^2 - |T_{\tilde{i}\tilde{f}}|^2$ and thus is a T-odd effect. Applying Eq.(1.7) we have to first order in α

$$\xi = \frac{\mathrm{Im}\, T_{if}\, \mathcal{a}_{fi}}{|T_{if}|^2} \quad , \qquad (1.9)$$

i.e., <u>the up-down asymmetry is proportional to the absorptive part of the two-photon exchange amplitude</u>.

So far we have only considered the case where the target proton is polarized. One can also discuss the degree of polarization $\tilde{\xi}$ of the recoil proton with respect to the normal, when the target proton is not polarized. As is well known, time reversal invariance implies that in the same kinematic configurations

$$\xi = - \tilde{\xi} \quad . \qquad (1.10)$$

This constitutes an interesting test of time reversal invariance. Throughout this paper we shall however assume the validity of time reversal invariance.

I.2. Kinematics and Notation(*).

We are dealing with the process

$$e(k,\sigma) + p(p,\lambda) \rightarrow e(k',\sigma') + p(p',\lambda') \quad , \qquad (1.11)$$

where k,p,k' and p' denote the 4-momenta of the corresponding particles and σ,λ,σ' and λ' their spin projection along the normal to the scattering plane. The latter is defined as follows

$$\mathfrak{s}^{\mu} = \frac{1}{\sqrt{\Sigma}} \epsilon^{\mu\nu\rho\sigma} p_{\nu} k_{\rho} k'_{\sigma} \quad , \qquad (1.12)$$

where Σ is a normalization invariant, which is positive, and such that

$$\mathfrak{s}.\mathfrak{s} = -1 \quad .$$

In the laboratory frame

$$\vec{\mathfrak{s}} = \frac{\vec{k} \times \vec{k}'}{|k \times \vec{k}'|} \quad .$$

We shall denote by m and M the electron and proton masses

$$k^2 = k'^2 = m^2 \quad , \qquad p^2 = p'^2 = M^2 \quad ;$$

and by s and t the usual invariants

$$s = (p+k)^2 \quad , \qquad t = (k-k')^2 \quad . \qquad (1.13)$$

Often, we shall refer to the absorptive part of the two-photon exchange diagram

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(*) Our metric and Dirac matrices are those of J.D. Bjorken and S.D. Drell textbooks, ref.10 .

shown in Fig.2 and the following notation will be used :

$$q = k - k' \quad ; \quad q_1 = k - \ell \quad ; \quad q_2 = k' - \ell \quad , \tag{1.14}$$

where ℓ is the energy-momentum of the intermediate electron $(\ell^2 = m^2)$. With $\mathbb{P}$ denoting the total energy-momentum of the intermediate hadronic state, we define further invariants

$$\left.\begin{array}{c} \mathbb{P}^2 = W^2 \\ Q^2 = -q^2 = -t \quad , \quad Q_1^2 = -q_1^2 \quad , \quad Q_2^2 = -q_2^2 \\ p.q_1 = M\nu_1 \quad \text{and} \quad p.q_2 = M\nu_2 \end{array}\right\} \quad . \tag{1.15}$$

Useful relations between invariants and the various C.M. angles which we shall need are the following :

$$q^2 = -\frac{(s-M^2)^2}{2s}(1-\cos\theta) \tag{1.16}$$

$$q_{1,2}^2 = -\frac{(s-M^2)(s-W^2)}{2s}(1-\cos\theta_{1,2}) \quad . \tag{1.17}$$

$$\text{where} \quad \left.\begin{array}{l} \theta \text{ is the C.M. scattering angle in } k+p = k'+p' \quad ; \\ \theta_1 \text{ is the C.M. scattering angle in } k+p = \ell + \mathbb{P} \quad ; \\ \theta_2 \text{ is the C.M. scattering angle in } k'+p' = \ell + \mathbb{P} \quad ; \end{array}\right\} \tag{1.18}$$

and we have neglected the electron mass.

The angles θ, θ_1 and θ_2 , which are visualized in Fig.5, satisfy the triangular relation

$$\cos\theta_1 = \cos\theta\cos\theta_2 + \sin\theta\sin\theta_2\cos\varphi \quad . \tag{1.19}$$

We shall denote the matrix elements of the T-matrix describing the transition of Eq.(1.11)

$$T_{if} \equiv T_{\lambda\sigma,\lambda'\sigma'}(\vec{k},\vec{k}') \quad ,$$

where $\vec{k}$ and $\vec{k}'$ refer to the C.M. system. In this frame of reference it is useful to choose a system of axis as indicated in Fig.6. As we shall see this system of axis is very convenient to derive selection rales for the matrix elements. The asymmetry defined in Eq.(1.9) is then (with summation over σ,λ' and σ')

$$\xi(s,t) = \frac{\mathrm{Im}\ T_{\uparrow\sigma,\lambda'\sigma'}\ \mathcal{A}_{\lambda'\sigma',\uparrow\sigma}}{|T_{\uparrow\sigma,\lambda'\sigma'}|^2} = -\frac{\mathrm{Im}\ T_{\downarrow\sigma,\lambda'\sigma'}\ \mathcal{A}_{\lambda'\sigma',\downarrow\sigma}}{|T_{\downarrow\sigma,\lambda'\sigma'}|^2} \tag{1.20}$$

A convenient way to rewrite this expression is to use matrix notation for T and A in the spin space of the proton-electron system :

$$\lambda = \uparrow\ ,\ \sigma = \uparrow\ ;\quad \lambda = \uparrow\ ,\ \sigma = \downarrow\ ;\quad \lambda = \downarrow\ ,\ \sigma = \uparrow\ ;\quad \lambda = \downarrow\ ,\ \sigma = \uparrow\ .$$

Then

$$\xi(s,t) = \frac{\mathrm{Im\,Tr}\left\{\begin{pmatrix}1&&&\\&1&&\\&&-1&\\&&&-1\end{pmatrix} T\,(\vec{k},\vec{k}')\ \mathcal{A}\ (\vec{k}',\vec{k})\right\}}{\mathrm{tr}\left(T\,(\vec{k},\vec{k}')\ T^{\dagger}(\vec{k}\,,\vec{k}')\right)} \ . \tag{1.21}$$

The first matrix in the numerator contains the information of suming over the electron spins and subtracting the proton polarizations ; and

$$\mathcal{A}_{\lambda'\sigma',\lambda\sigma}(\vec{k}',\vec{k}) = \sum_{\Gamma} T_{\lambda'\sigma'}(p'+k' \to \Gamma)\ T^{*}_{\lambda\sigma}(p+k \to \Gamma)(2\pi)^4\delta^{(4)}(p+k-p'-k') \ . \tag{1.22}$$

II. THE CONTRIBUTION OF THE TWO-PHOTON EXCHANGE TO THE UP-DOWN ASYMMETRY IN THE ELASTIC SCATTERING OF ELECTRONS FROM POLARIZED PROTONS.

II.1. Selection Rules.

We can impose certain symmetry requirements to the matrix elements of T and $\mathcal{A}$.

The unitarity relation (Eq.(1.2)), to lowest order in the electromagnetic interactions, states that T is a hermitian operator i.e.,

$$T_{\lambda\sigma,\lambda'\sigma'}(\vec{k},\vec{k}') = \left(T_{\lambda'\sigma',\lambda\sigma}(\vec{k}',\vec{k})\right)^* = \left(T_{-\lambda'-\sigma',-\lambda-\sigma}(\vec{k},\vec{k}')\right)^* \quad , \qquad (2.1)$$

where the last equality is obtained applying a rotation of π around the X-axis (see Fig.6). The same hermiticity requirements apply to the matrix elements of $\mathcal{A}$ as can be seen from the definition given in Eq.(1.3) .

The operators T and $\mathcal{A}$ are also invariant under parity and time-reversal . Under the parity operation times a rotation of π around the Z-axis we have (see Fig.6)

$$T_{\lambda\sigma,\lambda'\sigma'}(\vec{k},\vec{k}') = e^{i\frac{\pi}{2}(\lambda+\sigma-\lambda'-\sigma')}\, T_{\lambda\sigma,\lambda'\sigma'}(\vec{k},\vec{k}') \qquad (2.2)$$

and similarly for the matrix elements of $\mathcal{A}$. Under time reversal times a rotation of π around the Y-axis we have

$$T_{\lambda\sigma,\lambda'\sigma'}(\vec{k},\vec{k}') = T_{\lambda'\sigma',\lambda\sigma}(\vec{k},\vec{k}') \qquad (2.3)$$

and similarly for the $\mathcal{A}$ matrix. Altogether, the selection rules given in Eqs.(2.1),(2.2) and (2.3) tell us that T , in the spin space of the proton-electron system is a matrix of the type

$$T = \begin{bmatrix} a & 0 & 0 & g \\ 0 & b & f & 0 \\ 0 & f & b^* & 0 \\ g & 0 & 0 & a^* \end{bmatrix} \qquad (2.4)$$

with g and f real. Similarly, we have that

$$G = \begin{bmatrix} A & 0 & 0 & G \\ 0 & B & F & 0 \\ 0 & F & B^* & 0 \\ G & 0 & 0 & A^* \end{bmatrix} , \qquad (2.5)$$

with E and F real. The expression for the numerator in Eq.(1.9) is

$$\mathrm{Im}\,(T_{if}\, G_{fi}) = \mathrm{Im}\,(a\, A + b\, B) \quad . \qquad (2.6)$$

The explicit calculation of a , b ; A and B will be carried out next within the framework of the Dirac formalism.

II.2. Explicit Calculations ; Dirac Formalism.

In the Dirac formalism, the T-matrix elements at the Born approximation are given by the expression

$$i\, T_{\lambda\sigma,\lambda'\sigma'}(pk,p'k') = (-ie)(ie)\,\bar{u}\,(k',\sigma')\gamma^{\mu}u(k,\sigma)\ \frac{-ig_{\mu\nu}}{q^2+i\epsilon}\ \bar{u}(p',\lambda')\Gamma^{\nu}(q^2)u(p,\lambda) \quad , \qquad (2.7)$$

where

$$\Gamma^{\mu}(q^2) = \left[F_1(q^2) + F_2(q^2)\right]\gamma^{\mu} - \frac{1}{2M}\, F_2(q^2)(p+p')^{\mu} \quad ; \qquad (2.8)$$

$F_1(q^2)$ and $F_2(q^2)$ are the proton electromagnetic form factors, with

$$F_1(0) = 1 \quad \text{and} \quad F_2(0) = \varkappa_p = 1.793 \tag{2.9}$$

where $\varkappa_p$ is the anomalous magnetic moment of the proton.

The corresponding matrix elements of the two-photon exchange absorptive amplitude are given by the expression (Eq.(1.22)) ,

$$\mathcal{A}_{\lambda'\sigma',\lambda\sigma}(\vec{k}',\vec{k}) = \frac{e^4}{(2\pi)^3} \int_{M^2}^{s} dW^2 \int d\Omega_\ell \, \frac{s-W^2}{8s} \, \frac{1}{q_1^2 q_2^2} \, \bar{u}(k,\sigma)\gamma^\mu(\not{\ell}+m)\gamma^\nu u(k',\sigma')W_{\nu\mu}(p,\lambda;p',\lambda'), \tag{2.10}$$

where $d\Omega_\ell$ is the element of solid angle of the intermediate electron in the overall C.M. system

$$d\Omega_\ell = d\cos\theta_2 \, d\varphi \quad , \tag{2.11}$$

and

$$W_{\nu\mu}(p,\lambda;p',\lambda') = \sum_\Gamma < p,\lambda|J_\mu(0)|\Gamma > < \Gamma|J_\nu(0)|p',\lambda'> (2\pi)^4\delta^{(4)}(p+q_1-\mathbb{P}) \, . \tag{2.12}$$

The hadronic tensor $W_{\nu\mu}(p,\lambda;p',\lambda')$ is the absorptive part of the Compton scattering of off-shell photons from polarized protons. The pole contribution, Γ = proton , can be explicitly calculated in terms of the electromagnetic proton form factors :

$$W^{(elas.)}_{\nu\mu}(p,\lambda;p',\lambda') = 2\pi \, \delta[(p+q_1)^2-M^2] \, \bar{u}(p,\lambda)\Gamma_\mu(q_1^2)(\not{p}+\not{q}_1+M)\Gamma_\nu(q_2^2) \, u(p',\lambda') \quad . \tag{2.13}$$

The great unknown in our problem is the inelastic $\Gamma \neq p$ term in $W_{\nu\mu}(p,\lambda;p',\lambda')$ In the forward direction, i.e., when $p = p'$, this term is directly related to the proton structure functions measurable in the inelastic electron proton scattering experiments. More precisely(*)

$$\frac{1}{4\pi M} W^{(inelas.)}_{\nu\mu}(p,\lambda;p,\lambda) = W_{\nu\mu}(p,q_1) + \text{spin dependent terms} \quad ; \qquad (2.14)$$

and

$$W_{\nu\mu}(p,q_1) = -\left(g_{\mu\nu} - \frac{q_{1\mu}q_{1\nu}}{q_1^2}\right) W_1(\nu_1,q_1^2) + \frac{1}{M^2}\left(p_\mu - \frac{p.q_1}{q_1^2} q_{1\mu}\right)\left(p_\nu - \frac{p.q_1}{q_1^2} q_{1\nu}\right) W_2(\nu_1,q_1^2) \qquad (2.15)$$

When $p \neq p'$ there is no such direct connection between $W_{\nu\mu}(p,\lambda;p',\lambda')$ and empirically known quantities. We shall see in section IV that it is however possible to bound the non-forward matrix elements $W^{(inelas.)}_{\nu\mu}(p,\lambda;p',\lambda)$ in terms of the proton structure functions W_1 and W_2 .

The connection between the spin-amplitudes introduced in Section II.1 and the Dirac formalism is now quite clear :

$$\left.\begin{aligned} a &= \frac{e^2}{-q^2} \bar{u}(k',\uparrow)\, \gamma^\mu\, u(k,\uparrow)\, \bar{u}\,(p',\uparrow)\, \Gamma_\mu(q^2)\, u(p,\uparrow) \\ b &= \frac{e^2}{-q^2} \bar{u}(k',\downarrow)\, \gamma^\mu\, u(k,\downarrow)\, \bar{u}\,(p',\uparrow)\, \Gamma_\mu(q^2)\, u(p,\uparrow) \end{aligned}\right\} \quad ; \qquad (2.16)$$

$$\left\{\begin{matrix} A \\ B \end{matrix}\right\} = \frac{e^4}{(2\pi)^3} \int_{M^2}^{s} dW^2 \int d\Omega_\ell \, \frac{s-W^2}{8s} \, \frac{1}{q_1^2 q_2^2} \, W_{\alpha\rho}(p,\uparrow;p',\uparrow) \left\{\begin{matrix} \bar{u}(k,\uparrow)\gamma^\rho(\not{\ell}+m)\gamma^\alpha u(k',\uparrow) \\ \bar{u}(k,\downarrow)\gamma^\rho(\not{\ell}+m)\gamma^\alpha u(k',\downarrow) \end{matrix}\right\} \qquad (2.17)$$

With these expressions, and the definition given in Eq.(1.20) we can write a rather compact expression for the asymmetry ,

$$\xi(s,t) = \frac{e^2}{(2\pi)^3} \frac{q^2}{D(s,t)} \int_{M^2}^{s} dW^2 \int d\Omega_\ell \, \frac{s-W^2}{s} \, \frac{i}{q_1^2 q_2^2} \, L^{\mu\nu\rho}(H^{\uparrow}_{\mu\nu\rho} - H^{\downarrow}_{\mu\nu\rho}) \quad , \qquad (2.18)$$

(*) Notice that in the forward direction $q = 0$ and $q_1 = q_2$.

where

$$D(s,t) = 8 \left\{ t^2(F_1+F_2)^2 + 2\left[(s-M^2)^2 + st\right]\left(F_1^2 - \frac{t}{4M^2} F_2^2\right) \right. \quad ; \qquad (2.19)$$

$$L^{\mu\nu\rho} = \mathrm{Tr} \quad (k'\gamma^\mu k \gamma^\rho \ell \gamma^\nu) + O(m^2) \left. \right\} \quad , \qquad (2.20)$$

and

$$H^{\uparrow}_{\mu\nu\rho} - H^{\downarrow}_{\mu\nu\rho} = \sum_{\text{spins}} \bar{u}(p',\lambda')\Gamma_\mu(q^2)\gamma_5 s\, u(p,\lambda)\; W_{\nu\rho}(p,\lambda;p',\lambda') \quad . \qquad (2.21)$$

In fact,, because of parity conservation and time reversal invariance, in the sum over spins only the diagonal terms $\lambda = \lambda'$ contribute to the asymmetry. We have thus, using the electric and magnetic proton form factors

$$G_M = F_1 + F_2 \quad ; \quad G_E = F_1 + \frac{t}{4M^2} F_2 \quad , \qquad (2.22)$$

$$H^{\uparrow}_{\mu\nu\rho} - H^{\downarrow}_{\mu\nu\rho} = \sqrt{\frac{2}{M^2+p.p'}} \left\{ i\, G_M(q^2)\epsilon^{\mu\alpha\beta\gamma} p'_\alpha\, p_\beta\, s_\gamma \left[W_{\nu\rho}(p,\uparrow;p',\uparrow) + \right.\right.$$

$$\left.\left. + \; W_{\nu\rho}(p,\downarrow;p',\downarrow)\right] + MG_E(q^2)(p+p')_\mu \left[W_{\nu\rho}(p,\uparrow;p',\uparrow) - W_{\nu\rho}(p,\downarrow;p',\downarrow)\right] \right\} \quad . (2.23)$$

From the expression of the leptonic tensor $L^{\mu\nu\rho}$ it can be seen that

$$k_\rho\, L^{\mu\nu\rho} = \ell_\rho\, L^{\mu\nu\rho} = k'_\nu\, L^{\mu\nu\rho} = \ell_\nu\, L^{\mu\nu\rho} = O(m^2) \quad ; \qquad (2.24)$$

therefore, to the approximation where terms proportional to the lepton mass are neglected, it seems worth while to expand the hadronic tensor $W_{\nu\rho}(p,\lambda;p',\lambda')$ in a basis of fourvectors containing k , ℓ and k' . More precisely, we shall expand each vertex $<\Gamma|J_\nu(0)|p',\lambda'>$ corresponding to a specific intermediate state Γ , in a basis of 4-vectors :

$$k' \;,\; \ell \;,\; n \;,\; m'$$

where n is the normal to the 4-plane defined by k, k' and ℓ ; i.e.,

$$n^{\mu} = \frac{1}{\sqrt{N}} \epsilon^{\mu\nu\rho\sigma} k_{\nu} k'_{\rho} \ell_{\sigma} \tag{2.25}$$

and

$$N = 2(k.\ell)(k'.\ell)(k.k') \quad ; \tag{2.26}$$

m'^{μ} is in the 2-plane orthogonal to k' and ℓ , and orthogonal to n , i.e.,

$$m'^{\mu} = \frac{1}{\sqrt{N}} \left[k^{\mu}(k'.\ell) - k'^{\mu}(k.\ell) - \ell^{\mu}(k.k') \right] \quad . \tag{2.27}$$

Similarly, we shall expand $< p,\lambda|J_{\ell}(0)|\Gamma >$ in a basis of 4-vectors :

$$k \; , \; \ell \; , \; n \; , \; m$$

where m is obtained from m' by the substitution $k \rightleftarrows k'$. After expansion of each vertex into these basis, and summation over all intermediate states Γ , we arrive at an expression of the type

$$W_{\nu\rho}(p,\lambda;\, p',\lambda\,) = n_{\nu}\, n_{\rho}\, Q_{\lambda} + n_{\nu}\, m_{\rho}\, R_{\lambda} +$$

$$+ \; m'_{\nu}\, n_{\rho}\, S_{\lambda} + m'_{\nu}\, m_{\rho}\, T_{\lambda} + \ldots \quad , \tag{2.28}$$

where the terms not explicitely written are such that when contracted with the leptonic tensor give contributions proportional to the lepton mass (See Eq.(2.24)] and we neglect them . Clearly

$$Q_{\lambda} = n^{\nu}\, n^{\rho}\, W_{\nu\rho}(p,\lambda;p',\lambda) \tag{2.29}$$

and the same thing for R_{λ} , S_{λ} and T_{λ} . After some algebra, the expression of the asymmetry we arrive at is as follows

$$\xi(s,t) = \frac{\alpha}{\pi} \frac{-t}{\sqrt{1-\frac{t}{4M^2}}} \frac{1}{D(s,t)} \int_{M^2}^{s} dW^2 \int_{-1}^{+1} d(\cos\theta_2) \int_0^{2\pi} d\varphi \; \frac{F(W^2;\theta_2,\varphi)}{(1-\cos\theta_1)(1-\cos\theta_2)} \tag{2.30}$$

where

$$F(W^2;\theta_2,\varphi) = M_1(Q_\uparrow + Q_\downarrow - T_\uparrow - T_\downarrow) +$$
$$M_2(R_\uparrow + R_\downarrow + S_\uparrow + S_\downarrow) + i\,E_1(Q_\uparrow - Q_\downarrow - T_\uparrow + T_\downarrow) +$$
$$i\,E_2(R_\uparrow - R_\downarrow + S_\uparrow - S_\downarrow) \quad . \tag{2.31}$$

The functions M_1, M_2, E_1 and E_2 can be explicitly calculated ; M_1 and M_2 are proportional to the magnetic form factor $G_M(t)$; E_1 and E_2 are proportional to the electric form factor $G_E(t)$. In terms of the variables θ_1, θ_2 and φ [see Eq.(1.19)] we have

$$M_1 = G_M(t) \cdot \frac{s-M^2}{4s} \left\{ \frac{s+M^2}{2\sqrt{s}} (1-\cos\theta) \left[\left(\frac{s-M^2}{s+M^2} + \cos\theta_2 \right) \sin\theta + \right.\right.$$
$$\left.\left. \sin\theta_2 \cos\varphi\,(1-\cos\theta) \right] - \sqrt{s}\,\sin\theta\,(1-\cos\theta_1 + 1-\cos\theta_2) \right\} \quad ; \tag{2.32a}$$

$$M_2 = G_M(t) \frac{\sqrt{s}}{2} \left[\left(\frac{s+M^2}{2s} \right)^2 \left(1+ \frac{s-M^2}{s+M^2} \cos\theta \right)^2 -1 \right] \sin\theta_2 \sin\varphi \quad ; \tag{2.32b}$$

$$E_1 = G_E(t)\, M \left[(1-\cos\theta_2)(1+\cos\theta) - \sin\theta\,\sin\theta_2\cos\varphi \right] \quad ; \tag{2.32c}$$

$$E_2 = G_E(t)\, M \sin\theta\,\sin\theta_2\,\sin\varphi \quad . \tag{2.32d}$$

As it can be seen from Eq.(2.28), to the approximation where terms proportional to the lepton mass are neglected, there are only eight components of the hadronic tensor $W_{\nu\rho}(p,\lambda;p',\lambda')$ which contribute to the asymmetry(*). Another reason

(*) In general, assuming parity conservation and gauge invariance, $W_{\nu\rho}(p,\lambda;p',\lambda')$ has eighteen invariant amplitudes.

to use the expansion indicated in Eq.(2.28) is, as we shall show in Section IV, that the components Q_λ, R_λ, S_λ and T_λ can be easily bounded in terms of the spin independent structure functions W_1 and W_2 , Eq.(2.15), of the proton.

III. CALCULATION OF THE ELASTIC CONTRIBUTION TO THE UP-DOWN ASYMMETRY.

With $W^{(elas.)}_{\nu\rho}(p,\lambda;p',\lambda')$ given by Eq.(2.13), the elastic contribution to the up-down asymmetry, Eq.(2.18), can be readily given in terms of the proton form factors. We have done the algebraic part of this calculation using the computer program SCHOONSCHIP developed by Veltman [11]. The integration were performed numerically, using the customary dipole fit for the form factors[12] :

$$\frac{G_M(q^2)}{1+\varkappa_p} = G_E(q^2) = \frac{1}{\left[1-q^2/0.71(\mathrm{GeV}/c)^2\right]^2} \qquad (3.1)$$

The results of the elastic contribution to the asymmetry at different energies and center of mass scattering angles are shown in Figure 7. They are in exellent agreement with a recent calculation by Hey [8], who performed the integrations analitically. The elastic contribution in the kinematical conditions of the different experiments are contained in Table 1 .

IV. BOUNDS TO THE UP-DOWN ASYMMETRY FROM INELASTIC ELECTRON SCATTERING EXPERIMENTS.

In this section we derive rigorous bounds, to order α , for the inelastic contribution to the up-down asymmetry. We present two types of bounds : in terms of cross sections and in terms of structure functions.

IV.1. Bound in terms of cross-sections.

Our starting point are the expressions given in Eq.(2.16). We have

$$\left\{ \begin{matrix} a \\ b \end{matrix} \right\} = \frac{e^2}{t} \frac{\mathrm{Tr}\left[(\not{k}+m)(\not{k}'+m)(1 \pm \gamma_5 \not{s})\gamma^\mu\right]}{2\sqrt{2(m^2+ k.k')}} \bar{u}(p',\uparrow)\Gamma_\mu(q^2)u(p,\uparrow) =$$

$$= \frac{e^2}{t} \frac{2}{\sqrt{2(m^2+k.k')}} \left[\pm\, i(k\,k'\,s\,\gamma^\mu) + m(k+k')^\mu\right]\bar{u}(p',\uparrow)\Gamma_\mu(q^2)u(p,\uparrow) \ , \qquad (4.1)$$

where we have used the short-hand notation

$$(p_1\ p_2\ p_3\ p_4) \equiv \epsilon^{\mu\nu\rho\sigma}\ p_{1\mu}\ p_{2\nu}\ p_{3\rho}\ p_{4\sigma}$$

To the approximation where terms of order $O(\frac{m}{\sqrt{-t}})$ are neglegted, we have that

$$a = -\,b + O(\frac{m}{\sqrt{-t}}) \quad ; \qquad (4.2)$$

and

$$\mathrm{Im}(aA + bB) = \mathrm{Im}\ a(A-B) + O(\frac{m}{\sqrt{-t}}) \quad , \qquad (4.3)$$

with

$$a = \frac{e^2}{t} \frac{4}{\sqrt{(-t)2(M^2+p.p')}} (k\,k'\,s\,\gamma^\mu) \left[iM\ G_E(p+p')_\mu - G_M(pp's\gamma_\mu)\right] \qquad (4.4)$$

and [see Eqs.(1.22) and (2.5)]

$$\left\{\begin{matrix} A \\ B \end{matrix}\right\} = \frac{1}{(2\pi)^3}\int_{(M+m_\pi)^2}^{s} dW^2 \int d\Omega_\ell \ \frac{s-W^2}{8s} \sum_\Gamma \left\{\begin{matrix} T_{\uparrow\uparrow}(k',p'\to\Gamma)T^*_{\uparrow\uparrow}(k,p\to\Gamma) \\ T_{\uparrow\downarrow}(k',p'\to\Gamma)T^*_{\uparrow\downarrow}(k,p\to\Gamma) \end{matrix}\right\} \quad . \tag{4.5}$$

Clearly

$$|\mathrm{Im}\ a(A-B)| \le |a|\ (|A|+|B|) \quad , \tag{4.6}$$

and, using Schwartz inequality,

$$\left|\sum_\Gamma T_{\lambda\sigma}(k',p'\to\Gamma)T^*_{\lambda'\sigma'}(k,p\to\Gamma)\right|^2 \le \sum_\Gamma \left|T_{\lambda\sigma}(k',p'\to\Gamma)\right|^2 \sum_\Gamma\left|T_{\lambda'\sigma'}(k,p\to\Gamma)\right|^2 \quad . \tag{4.7}$$

The relation between $|T|^2$ and the inelastic cross sections are

$$\frac{d\,\sigma^{\uparrow\uparrow}(k,p)}{dW^2\ d(\cos\ \theta_1)} = \frac{1}{(4\pi)^2}\ \frac{1}{4s}\ \frac{s-W^2}{s-M}\ \sum_\Gamma\ |T_{\uparrow\uparrow}(k,p\to\Gamma)|^2 \tag{4.8}$$

and similarly for the other spin configurations and $k'+p'\to\Gamma$. Again, as can be seen from the explicit dependence of these cross-sections on the leptonic tensor [see Eqs. (2.17) and (4.5) with k=k' and p = p'] the spin parallel and antiparallel cross-sections differ at most by terms of order $O(\frac{m}{\sqrt{-t}})$. Moreover, the cross-sections $\sigma^{\uparrow\uparrow}$ and $\sigma^{\downarrow\downarrow}$ are equal. This allows us to get a bound to the asymmetry in terms of unpolarized inelastic cross-sections. After some algebra we get the result

$$|\xi_{inel.}(s,t)| \le \frac{1}{2\pi\alpha}\ (-t)(s-M^2)\sqrt{\frac{\Phi(s,t)}{D(s,t)}}\ \times$$

$$\int_{(M+m_\pi)^2}^{s} dW^2 \int_{-1}^{+1} d(\cos\theta_2) \int_0^{2\pi}\frac{d\varphi}{2\pi}\left\{\frac{d\sigma\ (1)}{dW^2 d(\cos\theta_1)}\ \frac{d\sigma\ (2)}{dW^2 d(\cos\theta_2)}\right\}^{\frac{1}{2}} \quad , \tag{4.9}$$

where $D(s,t)$ has been defined in Eq.(2.19) ; and

$$\Phi(s,t) = \frac{\left[(s-M^2)^2 + st\right] G_E^2 - \frac{t}{16M^2}\left[2(s-M^2)+t\right]^2 G_M^2}{2\left[(s-M^2)^2 + st\right] G_E^2 + t\left[t\left(1-\frac{t}{4M^2}\right) - \frac{1}{2M^2}\left[(s-M^2)^2+st\right]\right] G_M^2} \quad ; \qquad (4.10)$$

The factor $\Phi(s,t)$ is very close to $\frac{1}{2}$. Had we not taken into account the symmetry properties of the amplitude $e+p \to e+p$ and ignored the relation between parallel and antiparallel cross-sections we would have found $\Phi(s,t) = 2$.

IV.2. Bound in terms of structure functions.

Another way to obtain a bound to the asymmetry is to start with Eq.(2.30) and bound the functions Q_λ, R_λ, S_λ and T_λ . For definiteness, let us consider the case of R_λ ; from Eqs.(2.12) and (2.28) we have that

$$R_\lambda = \sum_\Gamma m^\mu < p,\lambda|J_\mu(0)|\Gamma > < \Gamma|J_\nu(0)|p',\lambda > n^\nu (2\pi)^4 \delta^{(4)}(p+q-\mathbb{P}) . \quad (4.11)$$

Applying the Schwartz inequality in the same way as before we have that

$$|R_\lambda| \leq \sqrt{m^\mu m^\nu W_{\mu\nu}(p,\lambda;p,\lambda)} \sqrt{n^\mu n^\nu W_{\mu\nu}(p',\lambda;p',\lambda)} \quad . \qquad (4.12)$$

Furthermore, since $m^\mu m^\nu$ and $n^\mu n^\nu$ are symmetric tensors, only the spin independent structure functions of $W_{\mu\nu}(p,\lambda;p,\lambda)$ and $W_{\mu\nu}(p',\lambda;p',\lambda)$ will appear in the bound since their spin dependent parts are antisymmetric in μ , ν (*) . With

$$\left.\begin{aligned} W_i(1) &\equiv W_i(q^2,\nu_1) \\ \text{and} \qquad W_i(2) &\equiv W_i(q_2^2,\nu_2) \qquad i = 1,2 \end{aligned}\right\} ; \qquad (4.13)$$

(*) See e.g. Doncel and de Rafael, ref.13 .

$$|R_\lambda| \leq 4\pi M \sqrt{W_1(1) + \frac{1}{M^2}(p.m)^2 W_2(1)} \sqrt{W_1(2) + \frac{1}{M^2}(p'.n)^2 W_2(2)} \equiv \tilde{R} \qquad (4.14a)$$

and similarly

$$|Q_\lambda| \leq 4\pi M \sqrt{W_1(1) + \frac{1}{M^2}(p.n)^2 W_2(1)} \sqrt{W_1(2) + \frac{1}{M^2}(p'.n)^2 W_2(2)} \equiv \tilde{Q} \qquad (4.14b)$$

$$|S_\lambda| \leq 4\pi M \sqrt{W_1(1) + \frac{1}{M^2}(p.n)^2 W_2(1)} \sqrt{W_1(2) + \frac{1}{M^2}(p'.m)^2 W_2(2)} \equiv \tilde{S} \qquad (4.14c)$$

$$|T_\lambda| \leq 4\pi M \sqrt{W_1(1) + \frac{1}{M^2}(p.m)^2 W_2(1)} \sqrt{W_1(2) + \frac{1}{M^2}(p'.m)^2 W_2(2)} \equiv \tilde{T} \; ; \qquad (4.14d)$$

in this way the absorptive part of the non-forward off-shell Compton scattering amplitude from polarized nucleons can be bounded in terms of the proton structure functions W_1 and W_2 alone.

The expression of $F(W^2;\theta_2;\varphi)$, Eq.(2.31), is such that M_1, E_1 and M_2 and E_2 are real. With the definitions given in Eqs.(4.14) we have

$$|F(W^2;\theta_2;\varphi)| \leq 2 \left\{ \sqrt{M_1^2 + E_1^2}\,(\tilde{Q}+\tilde{T}) + \sqrt{M_2^2+E_2^2}\,(\tilde{R}+\tilde{S}) \right\} \equiv \tilde{F}(W^2;\theta_2,\varphi) \; ; \qquad (4.15)$$

and, from Eq.(2.30), the bound to the inelastic contribution in terms of the proton structure functions is

$$|\xi_{inel.}(s,t)| \leq \frac{\alpha}{8\pi} \frac{-t}{\sqrt{1-\frac{t}{4M^2}}} \frac{\int_{(M+m_\pi)^2}^{s} dW^2 \int_{-1}^{+1} d(\cos\theta_2) \int_0^{2\pi} d\varphi \, \frac{\tilde{F}(W^2;\theta_2,\varphi)}{(1-\cos\theta_1)(1-\cos\theta_2)}}{t^2(F_1+F_2)^2 + 2\left[(s-M^2)^2 + st\right]\left(F_1^2 - \frac{t}{4M^2}F_2^2\right)} \qquad (4.16)$$

The numerical estimates of this bound as well as on the bound in terms of cross sections Eq.(4.9), are discussed in the next section.

V. NUMERICAL RESULTS AND CONCLUSIONS.

In this section, we first sketch the way we use the experimental information on inelastic electron-proton scattering as input to our calculations. We then present the numerical results which we obtain, and compare them with the experimental situation. We also comment on the theoretical meaning of these results and their implications upon future experiments.

V.1. Data handling.

The input needed to evaluate our bounds is the inelastic electron-proton scattering cross section in a certain domain of the(*) ν, Q^2 plane. At each laboratory energy E of the incident electron, this domain is the inside of a triangle (see Fig.8) defined by two fixed straight lines : $W^2 = (M+m_\pi)^2$; $Q^2 = 0$, and a moving one : $Q^2 = 4E(E-\nu)$. In figure 8 we have drawn the moving edge (dotted line) for the energies at which ξ has been measured.

The region presently covered by inelastic scattering data[13,14] is the dashed-area of figure 8 . For $E = 15$ GeV and 18 GeV , it leaves unknown a large part of the triangle domain. However, at these energies, the photon propagator suppresses the high Q^2 contribution very strongly. Reasonnable extrapolations show, that to an accuracy of about 0.2 % , the contribution to the bound comes only from the part of the domain with $Q^2 < 7$ (GeV/c). Keeping only this part of the domain, the experimental region where extrapolation is needed exists only for $E = 18$ GeV and is small. Reliable estimates of

(*) In this section V.1 , ν and Q^2 refer either to ν_1, Q_1^2 or ν_2, Q_2^2 .

the bounds on ξ for $E > 18$ GeV should however avait new results on inelastic electron-proton scattering. In the resonance region we made a simple fit to the inelastic cross-section consisting on a succesion of linear expressions actually lying above the date. These slightly overestimate the resonance contribution which is relatively small at high electron energies. In the "deep inelastic region" ($W^2 > 4M^2$) we used the fit of Nachtmann [16]. The precise value of $R = \sigma_S/\sigma_T$ is of no importance as long as it is small, and we used $R = 0.2$ throughout. The main source of uncertainty are the errors on the experimental data[14,15]. We believe the precision of our bounds to be of the order of 10 %.

V.2. Comparison with experimental results and discussion.

For specific values of E and Q^2, we have computed the elastic contribution to the asymmetry and bounded the inelastic contribution. We can thus confine the asymmetry ξ between an upper and a lower limit. The results of the calculations and the nine experimental points so far reported are listed in Table I and plotted in figure 9. These results were obtained using the bound in terms of cross-sections discussed in Section IV.1 . It turns out that this simple bound is the best. The more sophisticated approach discussed in Section IV.2 leads to a sum of four terms proportional to M_1, M_2, E_1, and E_2 [see Eq.(2.31)] . The partial bounds on each of these terms are given in Table II . The corresponding total bound on ξ, which is also given in Table III, is smaller than the sum of the four partial ones because it has been obtained exploiting the reality of E_1, E_2, M_1, and M_2 [see Eq.(2.32)] . This total bound, however, comes out slightly larger than that obtained in terms of cross-sections.

At present, all experimental results are compatible with zero ; however their errors are often large compared to the intervals allowed by the bounds (see Fig.9). To be definite, let us adopt as a criterium for a measurement of the asymmetry to add usefully to our bounds, that its error bar be about three times smaller than the interval allowed by the bound. Within this criterium we must conclude that only the three SLAC experimental points and the Daresbury point at 3 GeV are definetely more stringent than the bounds. Considered in this way we believe that the bounds obtained in this paper give a hint to the standard of precision that future experiments on the up-down asymmetry should attain to be useful. For this reason we have drawn in figure 10 level curves of the bound in the range $0 < Q^2 < 2\ (\mathrm{GeV/c})^2$ and $0 < E < 18$ GeV . As mentioned before, at higher energies the needed experimental input is not yet available and for $Q^2 > 2\ (\mathrm{GeV/c})^2$ the bound is too large to be of interest anyway.

As can be seen in figure 10 the bound rises quickly with Q^2 . At first sight, one may think that this is due to the fact that the bound lacks the correct kinematical behaviour. More precisely, in the backward and forward directions the asymmetry should vanish, whereas the expression we get for the bound behaves only as Q^2 and consequently does not go to zero in the backward direction(*) . But in fact the lack of correct kinematical behaviour is certainly not the only reason for the quick rise of the bound with Q^2 . Indeed, E_2 in Eq.(2.32d) contains a factor $\sin\theta$ which survives the majorization procedure to bound the term proportional to E_2 in Eq.(2.30), yet this bound reaches more than 100 % at $E = 18$ GeV . It seems therefore that the quick rise of the bound on ξ with Q^2 is mostly due to the point-like structure of the inelastic electron-proton scattering cross-section.

(*) In that respect, we should like to point out that we have not been able either to get bounds with the proper kinematical constraints or to prove that this cannot be done.

On the other hand, experimental results point to low values of ξ . This, most probably, means that strong phase cancellations occur which the bound cannot reflect . There could be cancellations for exemple between the various terms in the expression of ξ given in Eq.(2.30) : one possibility is suggested by the near equality of the bounds to the terms proportional to M_1 and M_2 or E_1 and E_2 (see Table II) which could interfere destructively. Cancellations may also occur in the sum over intermediate states which would make the use of the Schwartz inequality a bad tool to obtain bounds.

An alternative possibility is that the bound is approximately saturated and that the two photon exchange contribution to ξ is indeed large. In this case one expects higher order terms in the perturbation expansion to become relevant and the smallness of the empirical asymmetries would then indicate that cancellations occur with these higher order terms.

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TABLE I. Experiment and results of calculations on the up-down asymmetry. The inelastic bound is the one in terms of cross-sections, Eq.(4.9) .

Experiment and Reference	E(GeV)	$Q^2(GeV^2/c^2)$	Experimental asymmetry	Elastic contr. in percent	Inelastic bound in percent	Lower limit in percent	Upper limit in percent
FRASCATI [1)]	.9	.8	$-2.4 \pm 5.$	.25	5.1	- 4.8	5.3
ORSAY [2)]	.95	.63	$-4. \pm 2.7$	.26	4.2	- 3.9	4.4
STANFORD [3)]	.9	.4	$-1.3 \pm 2.$	.16	2.5	- 2.4	2.7
DARESBURY [4)]	3.	1.26	$.6 \pm 3.$	.60	14.	- 13.	15.
DARESBURY [4)]	4.	1.5	-5.2 ± 5.5	.69	19.	- 18.	20.
DARESBURY [4)]	4.	1.9	-5.6 ± 8.7	.81	26.	- 25.	27.
SLAC [5)]	15.	.38	$-.4 \pm 1.4$	.19	5.8	- 5.6	6.0
SLAC [5)]	18.	.59	$-.5 \pm .9$	.27	9.4	- 9.1	9.7
SLAC [5)]	18.	.98	$-.3 \pm 1.8$	.62	16.	- 15	17.

TABLE II. Results of the bounds to terms proportional to M_1, M_2, E_1 and E_2 in Eq. (2.30). The total bound [Eq.(4.16)] is the one given in terms of structure functions.

Experiment and Reference	E(GeV)	$Q^2 (GeV/c)^2$	M_1	M_2	E_1	E_2	Total
FRASCATI [1]	.9	.8	2.7	2.7	1.4	1.4	6.1
ORSAY [2]	.95	.63	2.0	2.1	1.4	1.4	5.0
STANFORD [3]	.9	.4	1.0	1.1	1.0	1.1	3.0
DARESBURY [4]	3.	1.26	7.0	7.1	4.1	4.1	16.
DARESBURY [4]	4.	1.5	9.7	9.8	5.2	5.2	22.
DARESBURY [4]	4.	1.9	14.	14.	6.5	6.5	30.
SLAC [5]	15.	.38	2.2	2.2	2.4	2.4	6.6
SLAC [5]	18.	.59	3.9	4.0	3.4	3.6	10.
SLAC [5]	18.	.98	7.6	7.7	5.2	5.2	18.

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FIGURE CAPTIONS

Fig.1 One photon exchange contribution to elastic electron proton scattering.

Fig.2 Two photon exchange contribution to elastic electron proton scattering.

Fig.3 Center of mass configurations for elastic electron proton scattering:
(a) the target proton is polarized upwards,
(b) the target proton is polarized downwards.

Fig.4 Center of mass configurations corresponding to the transition amplitudes : $T_{\downarrow}(-\vec{k},-\vec{k}')$ in (a), and $T_{\downarrow}(\vec{k},\vec{k}')$ in (b) .

Fig.5 The center of mass angles θ , θ_1 and θ_2 defined in Eq.(1.16) .

Fig.6 System of axis in the center of mass system used to derive selection rules. The x and y axis bisect the angles defined by the particle momenta.

Fig.7 Elastic contribution to the up-down asymmetry in percent as a function of the C.M. angle θ . The numbers on the curves indicate the laboratory energy of the electron beam.

Fig.8 The triangle domain in the ν , Q^2 plane where inelastic electron scattering data are needed. The dotted lines : $Q^2 = 4E(E-\nu)$ represent the moving edge of the triangle ; the shaded area is the region presently covered by experimental points.

Fig.9 Comparison between experimental points and the allowed domain for ξ . The experimental results are presented from left to right in the same order as in Table I .

Fig.10 Level curves for ξ in the area $0 < E < 18$ GeV , $0 < Q^2 < 2(\mathrm{GeV}/c)^2$. The values of ξ are given in percent along the corresponding curve. The points where experiments have been made are indicated as follows :
□ Frascati, △ Orsay, O Stanford, ▼ Daresbury, ● Slac.

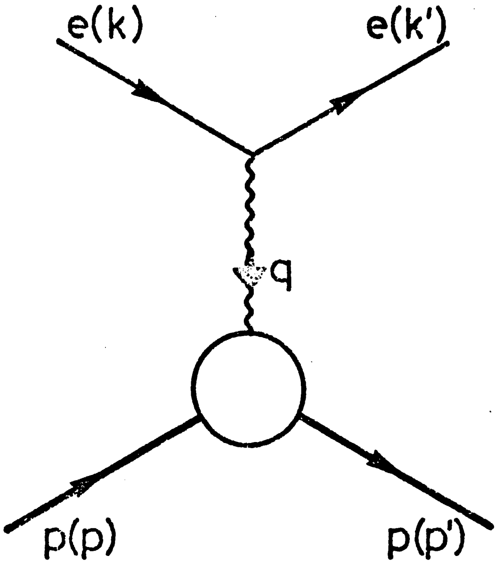


FIG.1

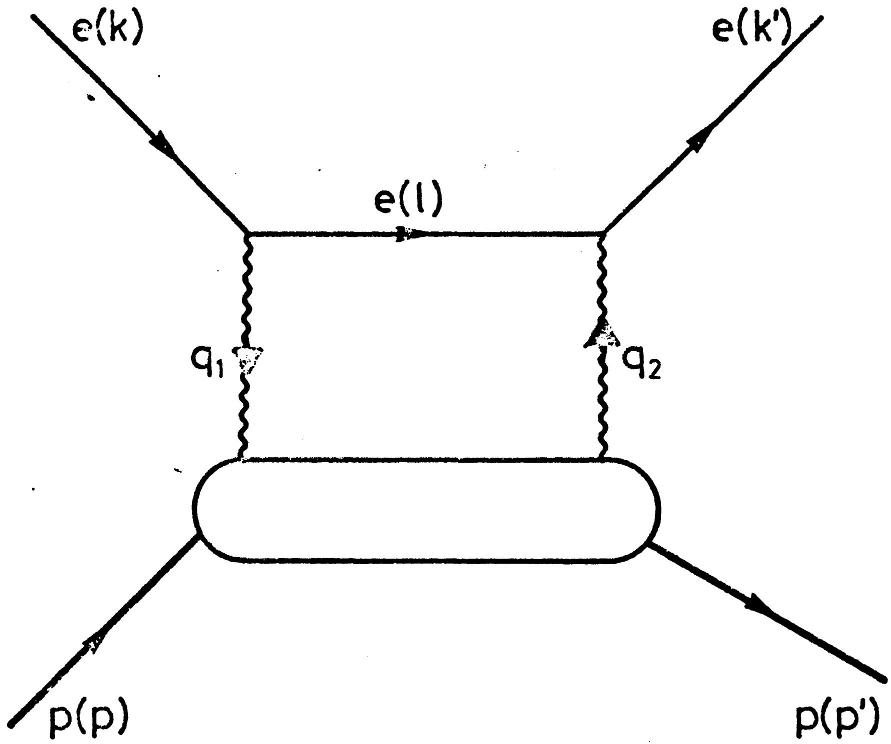


FIG. 2

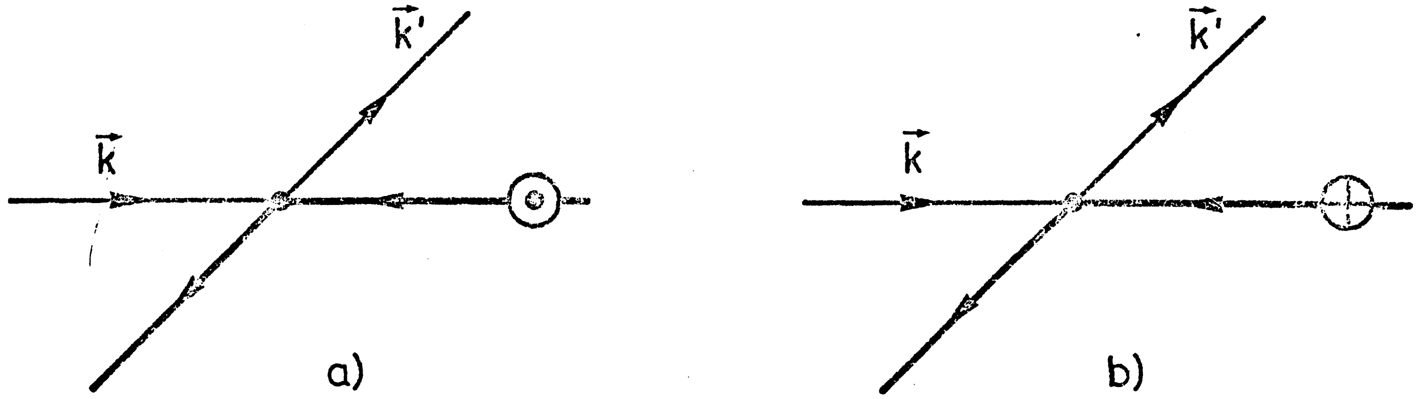
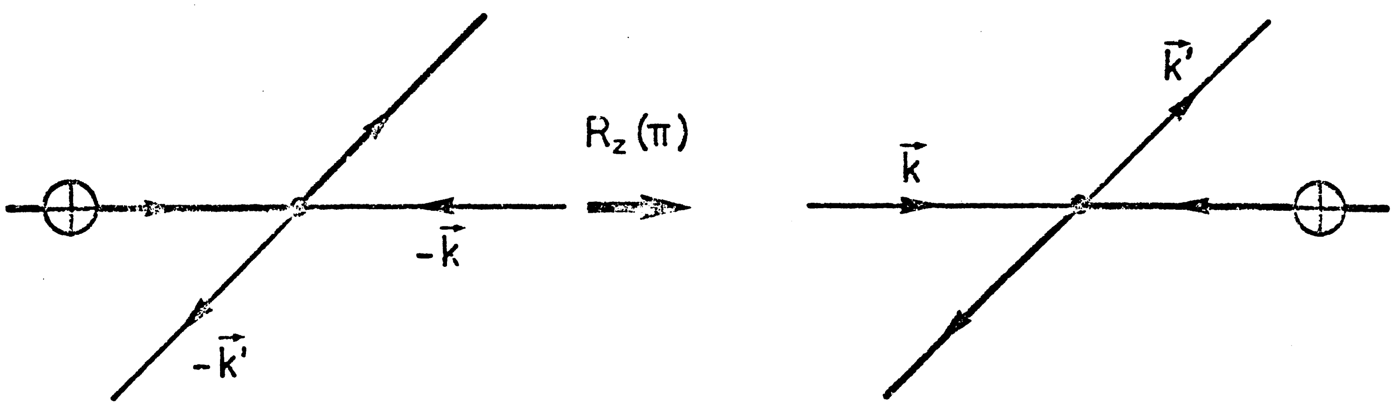


FIG. 3



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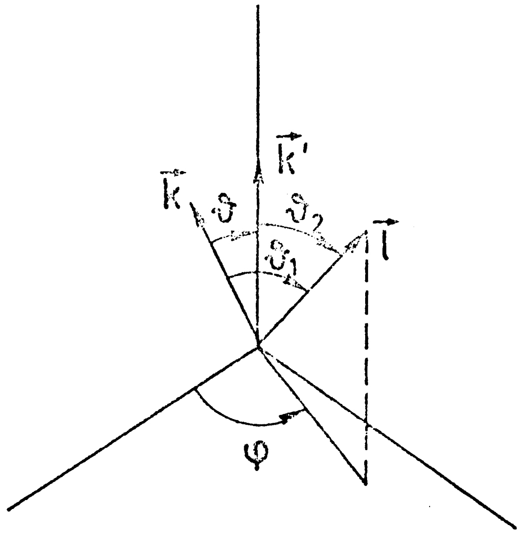
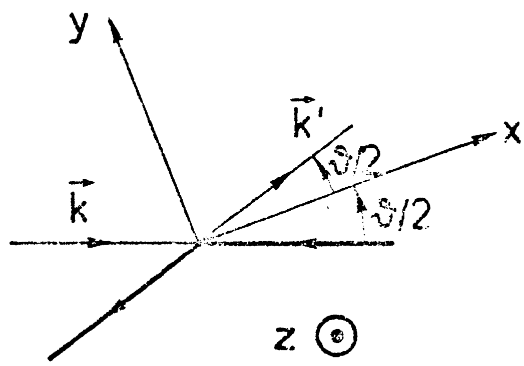


$\vec{k}'$
$\vec{k}$
ϑ
ϑ_2
$\vec{l}$
ϑ_1
φ

FIG. 5



y
$\vec{k}'$
x
$\vartheta/2$
$\vec{k}$
$\vartheta/2$
z

FIG.6

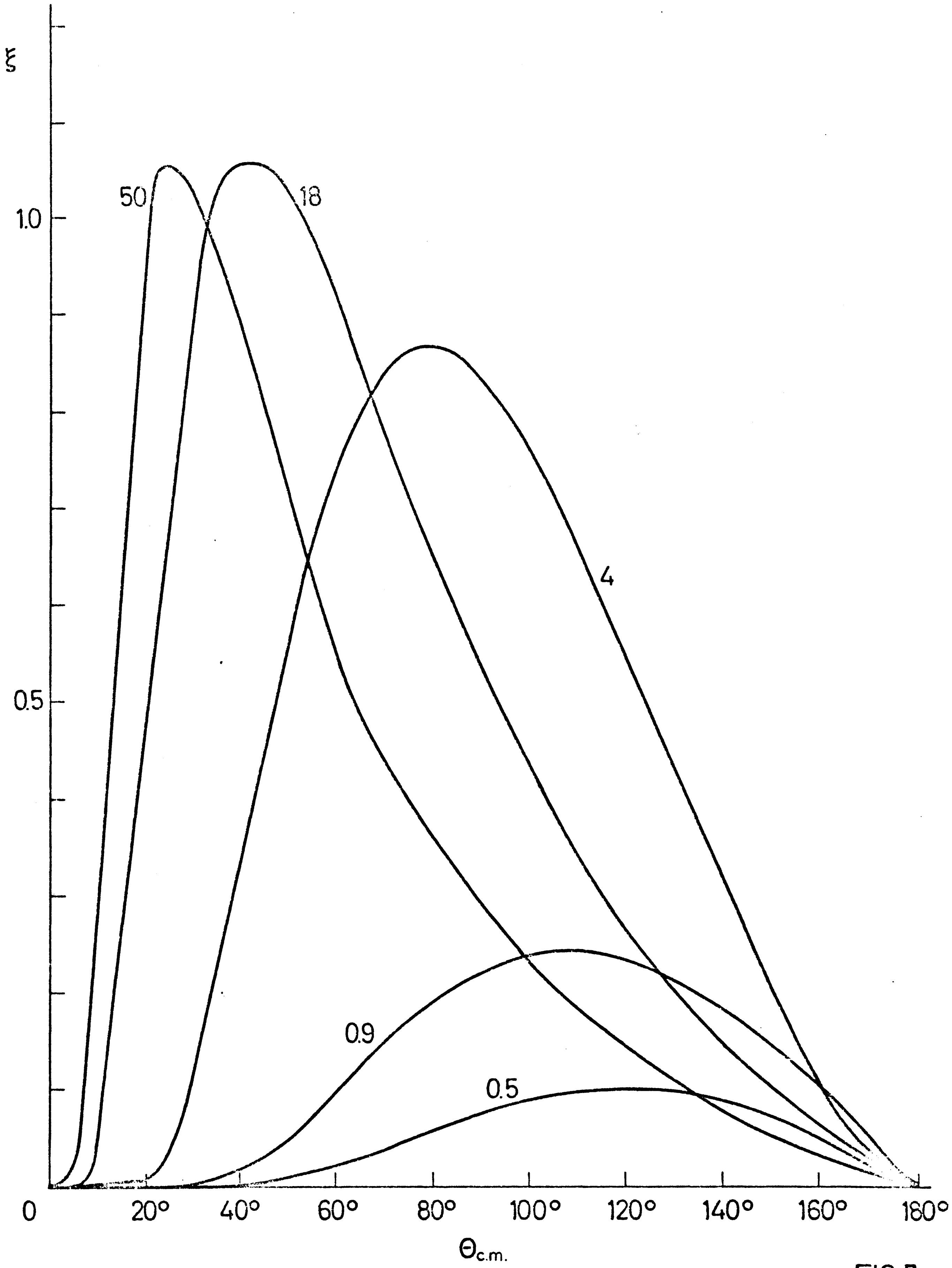
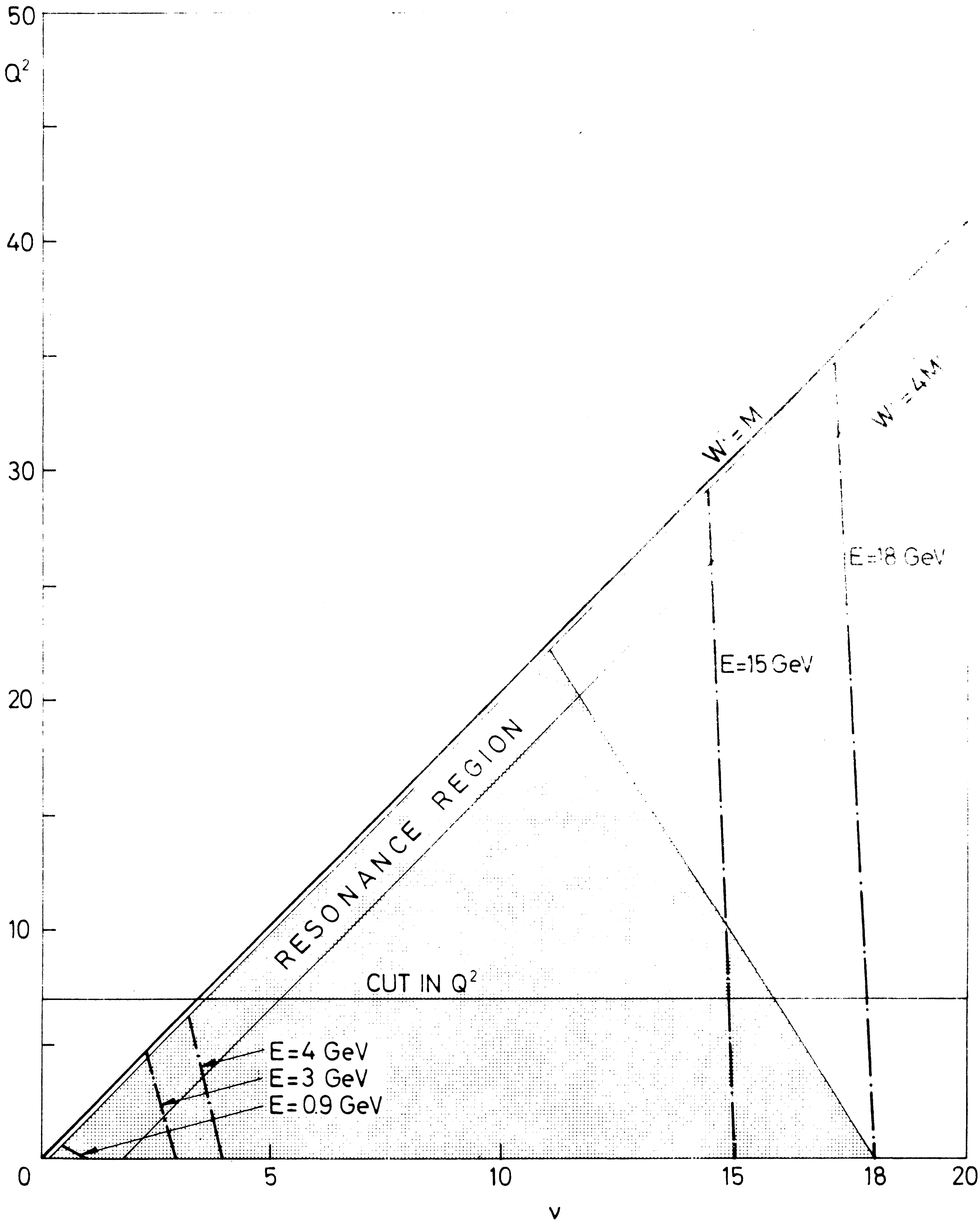


FIG. 7



50
Q²
40
30
20
10
0
5
10
15
18
20
ν
W = M
W = 4M
E=18 GeV
E=15 GeV
RESONANCE REGION
CUT IN Q²
E=4 GeV
E=3 GeV
E=0.9 GeV

FIG. 8

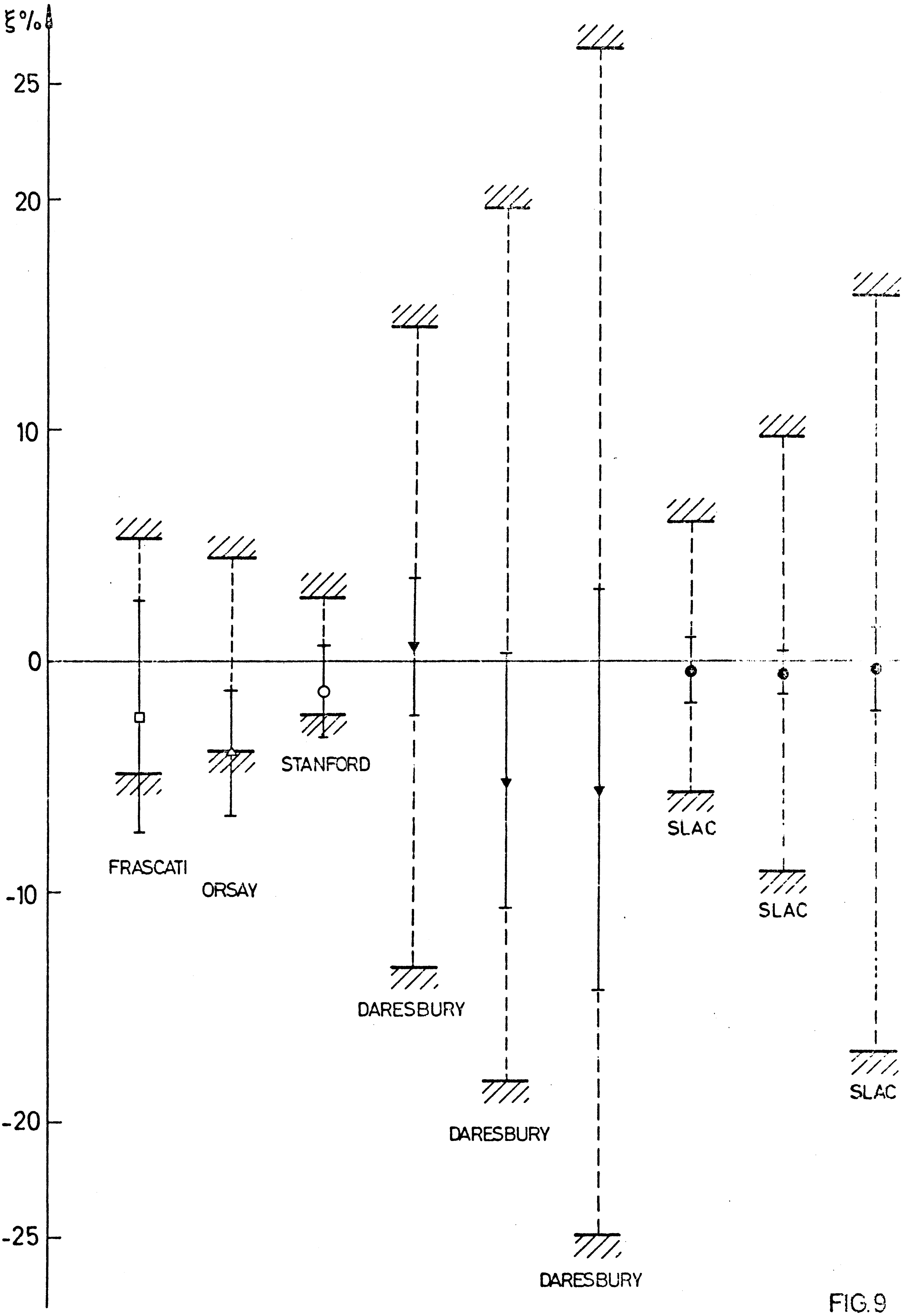

ξ%
25
20
10
0
-10
-20
-25
FRASCATI
ORSAY
STANFORD
DARESBURY
DARESBURY
DARESBURY
SLAC
SLAC
SLAC

FIG. 9

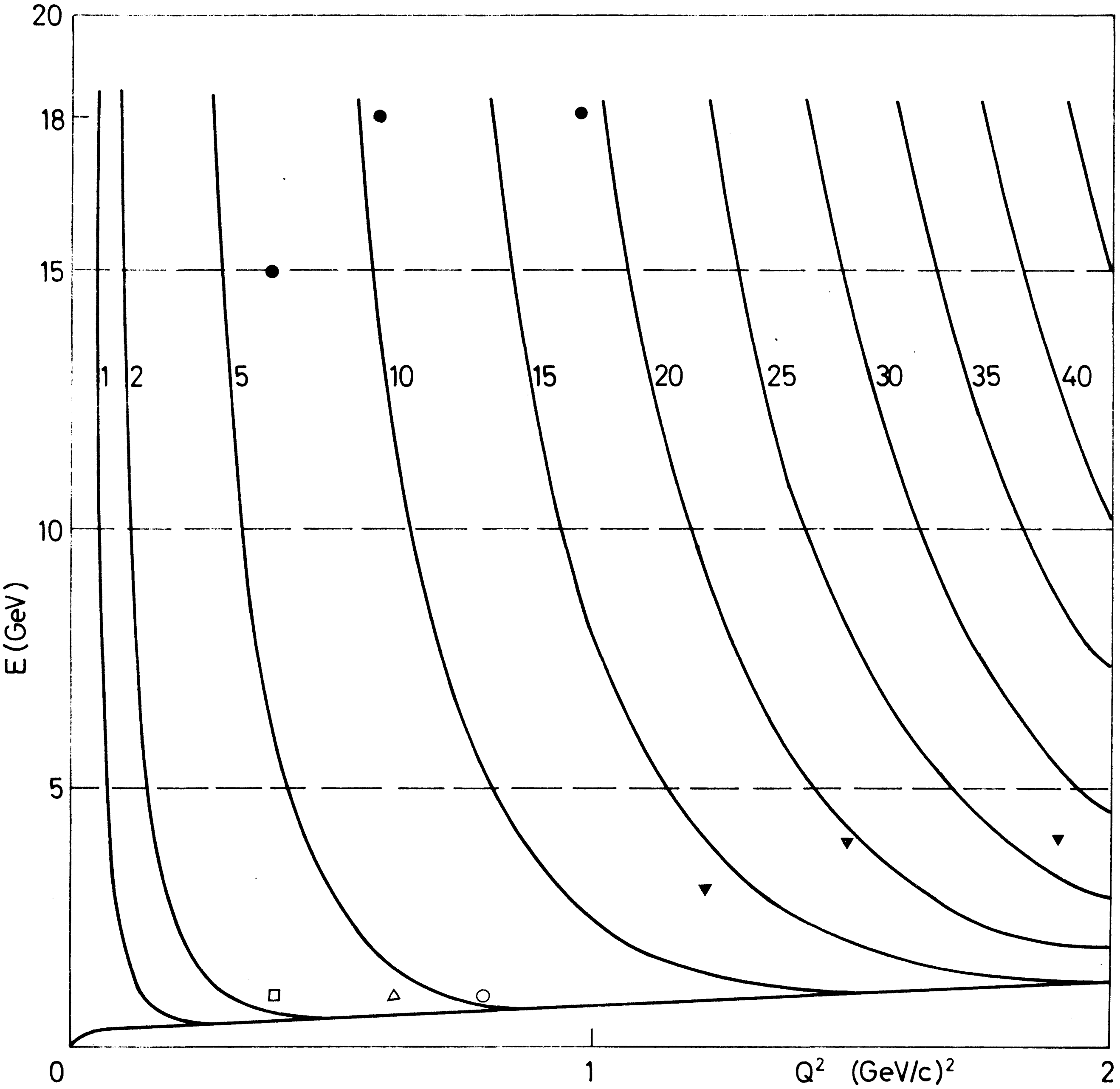


FIG.10