Ref. TH.993

VENEZIANO MODEL FOR KN SCATTERING

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CERN - Geneva

A B S T R A C T

A Veneziano-type model for kaon-nucleon scattering is shown to give a reasonable approximation to nature as regards :

i) elastic widths of the Y^* trajectories;

ii) forward scattering, and

iii) backward scattering at high energies.

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Ref. TH.993
31 January 1969

1. INTRODUCTION

A crossing symmetric, Regge behaved model of the Veneziano type [1] for the pion-nucleon system has recently been shown to provide a definite parametrization for both forward and backward scattering in terms of only a few parameters [2]. It predicted several characteristic features which were in good agreement with experiment [2]. However, as was remarked previously [3], the model suffers from a disadvantage which restricts its predictive power. One is forced to add satellite Veneziano terms in order to cancel unwanted terms in non-leading powers unless the condition $\alpha_{N_\alpha}(s) = \alpha_{\Delta_\delta}(s) = \alpha_{N_\gamma}(s)$ is satisfied. This complicates the issue in the low and intermediate energy region. Therefore, the discussion was confined to the leading terms which give a good approximation at high energy both in the t and u channels.

In this paper we present a Veneziano-type model for KN scattering, a process which is particularly interesting because of the absence of resonances in the KN channel. This implies that the t channel trajectories are exchange degenerate and also the u channel trajectories [4]. Thus our model takes a very simple form which provides a definite parametrization for not only the forward and backward KN and $\overline{\text{K}}$N amplitudes at high energy (excluding the Pomeranchuk contribution [5], of course), but also the elastic widths of the resonances on the Y^* Regge trajectories. Therefore it will be interesting to test if this simple Veneziano form is a good approximation to the real world at both high and low energies. It would also be helpful in the development of finite energy sum rules (FESR) bootstraps.

In Section 2 we construct the KN amplitudes of the Veneziano-type which satisfy the requirements of crossing symmetry, Regge asymptotic behaviour, and poles at points corresponding to resonances.

In Section 3 we examine the predictions of the model for elastic widths of the resonances on the Y_0^* and Y_1^* trajectories. We also derive the conditions for the elimination of the low spin parity doublets on the Y_0^* and Y_1^* trajectories.

In Section 4 we give the asymptotic forms for our amplitudes in both the forward and backward directions. We then compare the predictions with experiment. We fit the K^-p charge exchange (CEX) data.

2. VENEZIANO AMPLITUDES FOR KN SCATTERING

We begin by requiring of our amplitudes the following properties :

a) crossing symmetry under $s \leftrightarrow u$;

b) Regge asymptotic behaviour for t or u fixed;

c) simple poles at points corresponding to resonances on Regge trajectories;

d) they satisfy FESR [6] in all channels.

In the t channel we take into account the (ρ, A_2) trajectories which are required to be exchange degenerate (EXD) and the (ω, P') trajectories which are also required to be EXD. Since the $\pi\pi$ Veneziano formula forced the ρ and P' trajectories to be EXD [7], we assume

$$\alpha_\rho(t) = \alpha_{A_2}(t) = \alpha_\omega(t) = \alpha_{P'}(t) \equiv \alpha(t).$$

In the u channel the Y_0^* and Y_1^* trajectories, each of which we take to be EXD [4], are taken into account. Also following Schmid [4], we assume the absence of the particle with $j^P = \frac{1}{2}^-$ on the leading Y_1^* trajectory.

We then obtain the following representations for the usual invariant amplitudes A and B :

$$\begin{aligned} B_{\bar{K}p}(s,t,u) &= \sum_{I=0}^{1} \frac{\beta_{I0}^{(2)}}{\pi} B(1-\alpha_I(t), \tfrac{1}{2} - \alpha_{Y_0}(s)) + \sum_{I=0}^{1} \frac{\beta_{I1}^{(2)}}{\pi} B(1-\alpha_I(t), \tfrac{3}{2} - \alpha_{Y_1}(s)) \\ &= - B_{K^+p}(u,t,s), \end{aligned} \tag{1}$$

$$\begin{aligned} B_{\bar{K}n}(s,t,u) &= \sum_{I=0}^{1} \frac{2\beta_{I1}^{(2)}}{\pi} B(1-\alpha_I(t), \tfrac{3}{2} - \alpha_{Y_1}(s)) \\ &= - B_{K^+n}(u,t,s) \end{aligned} \tag{2}$$

where $B(X,Y) = \Gamma(X)\Gamma(Y)/\Gamma(X+Y)$,

$$\begin{aligned} A_{\bar{K}p}(s,t,u) &= \sum_{I,J=0}^{1} \frac{\beta_{IJ}^{(1)}}{\pi} C(1-\alpha_I(t), \tfrac{3}{2} - \alpha_{Y_J}(s)) + \frac{\gamma_0}{\pi} B(1-\alpha_I(t), \tfrac{1}{2} - \alpha_{Y_0}(s)) \\ &= A_{K^+p}(u,t,s), \end{aligned} \tag{3}$$

$$A_{\bar{K}n}(s,t,u) = \sum_{I=0}^{1} \frac{2\beta_{I1}^{(1)}}{\pi} C(1-\alpha_I(t), \tfrac{3}{2} - \alpha_{Y_1}(s))$$
$$= A_{K^+n}(u,t,s) \qquad (4)$$

where $C(X,Y) = \Gamma(X)\Gamma(Y)/\Gamma(X+Y-1)$.

In Eqs. (1) to (4) the indices I and J denote the isospin in the t channel and (s,u) channels, respectively. In (3) the term $\gamma_0 B(1-\alpha(t), \frac{1}{2}-\alpha_{Y_0}(s))$, which gives $A_{K^-p}(s,t,u)$ a pole at $s = m_\Lambda^2$, must vanish in the SU(3) limit. This implies that γ_0 is proportional to $(m_\Lambda - m)$ which is shown in Section 3. It is also worth noting that this term is of leading order for $s \to \infty$, u fixed, but not for $s \to \infty$, t fixed.

We can reduce the number of independent constants to five, since the isospin identification for bosons gives the four relations :

$$\left.\begin{aligned} \beta_{10}^{(k)} + 3\beta_{11}^{(k)} &= 0 \\ \beta_{00}^{(k)} - \beta_{01}^{(k)} &= 0 \end{aligned}\right\} k = 1, 2 \qquad (5)$$

among the nine constants $\beta_{IJ}^{(k)}$ and γ_0.

3. ELASTIC WIDTHS

The model provides a definite parametrization for the elastic widths of the Y^* resonances. For simplicity we confine our discussion to the B amplitude in K^-p scattering. Expanding Eq. (1) as a sum of poles and using the narrow width approximation we obtain

$$\mathrm{Im}\, B_{\bar{K}p}(s,t) = \sum_{I=0}^{1} \frac{\beta_{I0}^{(2)}}{a_0} \sum_{n=0}^{\infty} \binom{\alpha_I(t)+n-1}{n} \delta\left(s - \frac{1}{a_0}\left\{n + \frac{1}{2} - b_0\right\}\right)$$
$$+ \sum_{I=0}^{1} \frac{\beta_{I1}^{(2)}}{a_1} \sum_{n=0}^{\infty} \binom{\alpha_I(t)+n-1}{n} \delta\left(s - \frac{1}{a_1}\left\{n + \frac{3}{2} - b_1\right\}\right) \qquad (6)$$

where a_0, a_1 are the slopes of the Y_0^*, Y_1^* trajectories, respectively.

On the other hand [8])

$$\mathrm{Im}\, B_{K^-p}(s,t) = \sum_{\ell\pm, J} \pm\, C_J (4\pi^2) \Gamma^{el}_{\ell\pm}\, \delta(s - s_{\ell\pm}) \frac{1}{q^3_{\ell\pm}} \Big[\big\{ (\sqrt{s_{\ell\pm}} - m)^2 - m_K^2 \big\} P'_{\ell\pm 1}(z_{\ell\pm})$$

$$- \big\{ (\sqrt{s_{\ell\pm}} + m)^2 - m_K^2 \big\} P'_{\ell}(z_{\ell\pm}) \Big] \qquad (7)$$

where $C_J = \frac{1}{2}$ for $J = 0$ or 1. $\Gamma^{el}_{\ell\pm}$, $s_{\ell\pm}$, and $q_{\ell\pm}$ are the K^-p elastic widths, $(\mathrm{mass})^2$ and the centre-of-mass momentum of the resonance of total angular momentum $j = \ell \pm \frac{1}{2}$, $z_{\ell\pm} = 1 + t/2q^2_{\ell\pm}$. At the energy corresponding to a resonance [and taking the appropriate parity *)] we equate the leading powers of t, obtaining

$$2\Gamma^{el}_{\ell-} \frac{(\sqrt{s_{\ell-}} + m)^2 - m_K^2}{q^{2\ell+1}_{\ell-}} \frac{M(\ell)\Gamma(\ell)}{(2a)^{\ell-1}} = \frac{\beta^{(2)}_{00} - 3\beta^{(2)}_{11}}{\pi^2 a_0} \qquad (8)$$

for the Y^*_0 trajectory, and

$$2\Gamma^{el}_{\ell+} \frac{(\sqrt{s_{\ell+}} - m)^2 - m_K^2}{q^{2\ell+3}_{\ell+}} \frac{M(\ell+1)\Gamma(\ell+1)}{(2a)^{\ell}} = \frac{\beta^{(2)}_{00} + \beta^{(2)}_{11}}{\pi^2 a_1} \qquad (9)$$

for the Y^*_1 trajectory. The function $M(\ell)$ is defined by $P'_\ell(z) = M(\ell) z^{\ell-1} + \ldots$ and a is the slope of the boson trajectories. Thus our model predicts that the expressions on the left-hand sides of Eqs. (8) and (9) are constants. We assume a Chew-Frautschi plot for the Y^* trajectories

$$\alpha_{Y_0}(s) = -0.67 + 0.95\, s.$$

$$\alpha_{Y_1}(s) = -0.33 + 0.95\, s.$$

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*) In our convention the Y^*_0 resonances have masses $\sqrt{s} = -m_0$ and the Y^*_1 resonances $\sqrt{s} = m_1$, because we followed the convention of always dealing with the $\ell = J - \frac{1}{2}$ amplitude.

We also assume the same slope for the boson trajectory *). Using the experimental values of $\Gamma^{el}_{\ell\pm}$ for K^-p listed in the Rosenfeld tables [9], we give the values of the left-hand sides of Eqs. (8) and (9) in Table I. For convenience, we also give the values corresponding to $a = 1\ \mathrm{GeV}^{-2}$ in Table II. As can be seen, they are approximately constant, significantly so in view of the large variations of the elastic widths.

Our model appears to predict parity doubling of all resonances on the Y^* trajectories. Since the parity partners of the $\Lambda(\frac{1}{2}^+,1115)$, $\Lambda(\frac{3}{2}^-,1520)$ on the Y_0^* trajectory and the $\Sigma(\frac{3}{2}^+,1385)$ on the Y_1^* trajectory are not found in nature, we eliminate them. This is done by imposing the condition that the residue of

$$f_{\ell\pm} \equiv \frac{1}{2}\int_{-1}^{1} d(\cos\theta)\left\{ f_1 P_\ell(\cos\theta) + f_2 P_{\ell\pm 1}(\cos\theta)\right\}$$

vanishes when $\sqrt{s}$ takes the value corresponding to the resonance we want to kill. f_1 and f_2 are defined in terms of A and B in the conventional way. For the three resonances mentioned above we get the equations

$$\gamma_0 = -(m_\Lambda - m)\sum_{I=0}^{1}\beta_{I0}^{(2)}, \tag{10}$$

$$\sum_{I=0}^{1}\beta_{I0}^{(1)} = (1.520 - m_\Lambda)\sum_{I=0}^{1}\beta_{I0}^{(2)}, \tag{11}$$

$$\sum_{I=0}^{1}\beta_{I1}^{(1)} = (1.385 - m)\sum_{I=0}^{1}\beta_{I1}^{(2)}. \tag{12}$$

We point out here that both sides of Eq. (11) are equal to

$$-\frac{g_\Lambda^2}{4\pi}\cdot 4\pi^2 a_0 (m_\Lambda - m).$$

The residues of the parity partners of higher recurrences turn out to be negative. This difficulty can be circumvented [10] by adding a satellite of the form $\gamma_1 C(2-\alpha(t), \frac{3}{2}-\alpha_{Y_1}(s))$ to the B amplitude, where

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*) See I, or S. Mandelstam, Phys.Rev.Letters 21, 1724 (1968).

$$\gamma_1 \approx \frac{1}{8} \sum_{I=0}^{1} \beta_{I1}^{(2)} . \tag{13}$$

The effect of this additional term is small.

The Veneziano constants calculated from the elastic widths and Eqs. (10), (11) and (12) are listed in Table I for $a = 0.95\ \mathrm{GeV}^{-2}$ (in Table II we give the constants for $a = 1\ \mathrm{GeV}^{-2}$ for convenience).

Our model also gives a value for the Λ coupling constant. Taking $a = 0.95\ (1.0)\mathrm{GeV}^{-2}$ we find

$$\frac{g_\Lambda^2}{4\pi} = 11.3 \, {}^{+3.3}_{-2.5} \quad \left(10.0 \, {}^{+3}_{-2} \right). \tag{14}$$

This should be compared with the dispersion relation calculations [11) [Kim : 13.5±2.1, Zovko : 6.8±2.9, Martin-Poole : 4.6±1.3].

4. ASYMPTOTIC BEHAVIOUR

4.1 Forward scattering

By taking the limit $s \to \infty$, t fixed, we may extract the Regge amplitudes B_f, A_f, B_{A_2}, A_{A_2}, etc., from the relation

$$B_{A_2} = \frac{1}{4}\left(B_{\bar{K}p} + B_{K^+p} - B_{\bar{K}n} - B_{K^+n} \right)$$

and the seven other similar relations. The asymptotic forms of B and

$$A' = A + \frac{\nu_L + t/4m}{1 - t/4m^2} ,$$

the non-helicity flip amplitude [2), turn out to be

$$B_f(s,t) \cong - \frac{\beta_{11}^{(2)}}{\Gamma(\alpha(t))} \xi_f(t) \left[\alpha_Y(s) \right]^{\alpha(t)-1} . \tag{15}$$

$B_{A_2}(s,t) \cong$ same as in (15), with $\xi_\rho \to -\xi_{A_2}$.

$$A'_\rho(s,t) \cong \frac{1}{\Gamma(\alpha(t))}\left[\beta_{11}^{(1)} - \frac{2m\,\beta_{11}^{(2)}}{a_Y(4m^2-t)}\right]\xi_\rho(t)\left[\alpha_Y(s)\right]^{\alpha(t)}. \qquad (16)$$

$A'_{A_2}(s,t) \cong$ same as in (16), with $\xi_\rho \to -\xi_{A_2}$.

Similar expressions hold for A'_ω, B_ω, $A'_{P'}$, $B_{P'}$. Here $\xi(t)$ is the signature factor

$$\xi(t) = \frac{1+\tau e^{-i\pi\alpha}}{\sin \pi\alpha}.$$

Comparing these results with those of I, we see that the ratio

$$\frac{A'_R}{sB_R} \sim \text{constant} + \frac{2m}{4m^2-t}$$

for both πN and KN scattering where $R = \rho$ or P'. Therefore factorization is automatically satisfied for all t once the constants are fixed equal. Preliminary fits to the πN and KN CEX data indicate that this equality is consistent with experiment for the case of the ρ.

Using the above parametrization for the ρ and A_2 residue functions, we have fitted the K^-p CEX scattering data [13]. Since our model does not include the Pomeranchuk it does not give a definite parametrization for elastic scattering. The CEX fits are shown in the Figure. We stress that our fits have only two free parameters $\beta_{11}^{(1)}$ and $\beta_{11}^{(2)}$. The slopes of all the trajectories were fixed at 0.95 GeV^{-2}. The fit shown corresponds to

$$a = 0.95\ \text{GeV}^{-2},\ \beta_{11}^{(1)} = -30.0\ \text{mb GeV},\ \beta_{11}^{(2)} = -60.4\ \text{mb}. \qquad (17)$$

Here total cross-section data and factorization were used to fix the signs. Because of large error bars in the data, these numbers are not very well fixed, but even so the values of the β's shown and those obtained in Section 3 from the low energy data are in reasonable agreement. Another encouraging feature is the fact that Veneziano residues seem to be capable of reproducing the t dependence of the differential cross-section, which is usually parametrized by exponentials.

Our model implies that the K^-p and K^+n CEX differential cross-sections are identical. There is very little data on the latter reaction but what there is is consistent with this conclusion [14].

4.2 Backward scattering

Taking the limit $s \to \infty$, u fixed, we find that the u channel amplitudes for K^+p backward scattering is parametrized by

$$f_1^{I_u=0}(\sqrt{u}, s) = \frac{E_u + m}{\sqrt{u}} \frac{\gamma_{Y_0}(\sqrt{u})}{\Gamma(\frac{1}{2} + \alpha_{Y_0})} \frac{2}{\cos \pi\alpha_{Y_0}} s^{\alpha_{Y_0} - \frac{1}{2}} \tag{18}$$

where

$$\gamma_{Y_0}(\sqrt{u}) = -\frac{1}{8\pi}\left[\sum_{I=0}^{1} \beta_{I0}^{(1)}(\alpha_{Y_0} - \tfrac{1}{2}) - (\sqrt{u} - m_\Lambda)\sum_{I=0}^{1} \beta_{I0}^{(2)}\right] a^{\alpha_{Y_0} - \frac{1}{2}} . \tag{19}$$

We have used Eq. (10) to eliminate γ_o. Note that the effect of the parity partner of the Λ which has $\sqrt{u} = m_\Lambda$ is automatically zero. Also

$$f_1^{I_u=1}(\sqrt{u}, s) = \frac{E_u + m}{\sqrt{u}} \frac{\gamma_{Y_1}(\sqrt{u})}{\Gamma(\frac{1}{2} + \alpha_{Y_1})} \frac{2}{\cos \pi\alpha_{Y_1}} s^{\alpha_{Y_1} - \frac{1}{2}} \tag{20}$$

where

$$\gamma_{Y_1}(\sqrt{u}) = -\sum_{I=0}^{1} \beta_{I1}^{(1)}(\alpha_{Y_1} - \tfrac{1}{2})\, a^{\alpha_{Y_1} - \frac{1}{2}} . \tag{21}$$

Here $\alpha(\sqrt{u})$ and $\gamma(\sqrt{u})$ denote the Y^* trajectory and residue functions respectively. As can easily be seen, $f_1^{I_u=0,1}(\sqrt{u}, s)$ are both smoothly varying functions of $\sqrt{u}$ because of exchange degeneracy and thus we predict no dip at wrong signature nonsense points. Using values of the $\beta_{IJ}^{(k)}$ from Section 3 (see Table I), we find that

$$\gamma_{Y_0}(\sqrt{u} = 0) = -\left(20.8 \begin{smallmatrix} +15.9 \\ -14.3 \end{smallmatrix}\right) \text{GeV}^{-1} , \tag{22}$$

$$\gamma_{Y_1}(\sqrt{u} = 0) = 0.51 \begin{smallmatrix} +0.12 \\ -0.05 \end{smallmatrix} \text{GeV}^{-1} . \tag{23}$$

Thus, because of the smooth variation of the functions, we predict that the Y_0^* dominates backward scattering. This justifies Barger's assumption [15]. Also the value obtained in (23) for $\gamma_{Y_0}(\sqrt{u}=0)$ can be compared with the value Barger obtained from an analysis of backward scattering which was $|\gamma_{Y_0}(\sqrt{u}=0)| \approx 12.2\ \mathrm{GeV}^{-1}$. The preliminary analysis [16] of K^+p backward scattering within this formulation is also encouraging.

5. CONCLUSIONS

In conclusion, the Veneziano constants obtained from elastic widths of Y^* in Table I,

$$\beta_{00}^{(2)} = 94.1 \begin{array}{l} +27.3 \\ -19.6 \end{array} \text{ mb}, \qquad \beta_{11}^{(2)} = -\left(80.8 \begin{array}{l} +24.1 \\ -18.5 \end{array}\right) \text{mb}$$

$$\beta_{00}^{(1)} = 47.7 \begin{array}{l} +13.7 \\ -9.3 \end{array} \text{ mb GeV} \qquad \beta_{11}^{(1)} = -\left(34.4 \begin{array}{l} +10.4 \\ -8.2 \end{array}\right) \text{mb GeV}$$

should be compared with those obtained from the K^-p CEX scattering in Eq. (17),

$$\beta_{11}^{(2)} = -60.4 \text{ mb}, \qquad \beta_{11}^{(1)} = -30.0 \text{ mb GeV}.$$

For convenience, we also list the values of $\beta_{00}^{(2)}$ and $\beta_{00}^{(1)}$ obtained for the P' parameters from FESR under the Harari assumption [5]:

$$\beta_{00}^{(2)} \simeq 40 \text{ mb}, \qquad \beta_{00}^{(1)} \simeq 12 \text{ mb GeV}.$$

These values, obtained from different approaches, are in qualitative agreement. As was discussed in Section 4.2, these values are also consistent with the backward K^+p scattering.

In view of the qualitative agreement with experiment of a wide variety of predictions, we consider that this simple form in terms of only a few number of parameters for KN scattering is a reasonable approximation to nature, both at low and high energies.

However, there is still freedom to add non-leading Veneziano terms which affect the elastic widths of many daughters. Such a term could be a leading term either in the t channel or in the u channel.

More accurate experiments would obviously shed light on bounds of secondary Veneziano terms.

ACKNOWLEDGEMENTS

It is a pleasure to thank Dr. M. Jacob for discussions and reading of the manuscript. The authors are also thankful to Professor L. Van Hove, Dr. C. Lovelace and Dr. C. Schmid for comments, and to Professor W. Thirring and Professor J. Prentki for the hospitality extended to them in the CERN Theoretical Study Division.

TABLE I

Y_o^* trajectory					Y_1^* trajectory				
	J^P	ℓ^-	$\Gamma^{el}_{\ell^-}$ for K^-p (GeV)	l.h.s. of Eq. (8) Case : $a=0.95\ GeV^{-2}$		J^P	ℓ^+	$\Gamma^{el}_{\ell^+}$ for K^-p (GeV)	l.h.s. of Eq. (9) Case : $a=0.95\ GeV^{-2}$
$\Lambda(1115)$	$\frac{1}{2}^+$	1^-		$8(g_\Lambda^2/4\pi)$					
$\Lambda(1520)$	$\frac{3}{2}^-$	2^-	$(3.6\pm0.77)\times10^{-3}$	89.0 ± 18.9	$\Sigma(1385)$	$\frac{3}{2}^+$	1^+		
$\Lambda(1815)$	$\frac{5}{2}^+$	3^-	$(26.3\pm3.5)\times10^{-3}$	103.0 ± 13.75	$\Sigma(1770)$	$\frac{5}{2}^-$	2^+	$(22.5\pm3.38)\times10^{-3}$	3.80 ± 0.57
$\Lambda(2100)$	$\frac{7}{2}^-$	4^-	21×10^{-3}	78.5	$\Sigma(2030)$	$\frac{7}{2}^+$	3^+	6×10^{-3}	3.24
				Average value $=90.1\,^{+26.7}_{-20.0}$					Average value $=3.52\,^{+0.85}_{-0.29}$

$$(g_\Lambda^2/4\pi) = 11.3\,^{+3.34}_{-2.50}$$

$$\beta_{00}^{(2)} = 94.1\,^{+27.3}_{-19.6}\ \text{mb} \qquad \beta_{11}^{(2)} = -(80.8\,^{+24.1}_{-18.5})\ \text{mb}$$

$$\beta_{00}^{(1)} = 47.7\,^{+13.7}_{-9.3}\ \text{mb GeV} \qquad \beta_{11}^{(1)} = -(34.4\,^{+10.4}_{-8.2})\ \text{mb GeV}$$

TABLE II

Y_0^* trajectory					Y_1^* trajectory				
	J^P	ℓ^-	$\Gamma^{el}_{\ell^-}$ for K^-p (GeV)	l.h.s. of Eq. (8) Case : $a = 1\ \mathrm{GeV}^{-2}$		J^P	ℓ^+	$\Gamma^{el}_{\ell^+}$ for K^-p (GeV)	l.h.s. of Eq. (9) Case : $a = 1\ \mathrm{GeV}^{-2}$
$\Lambda(1115)$	$\frac{1}{2}^+$	1^-		$8(g_\Lambda^2/4\pi)$					
$\Lambda(1520)$	$\frac{3}{2}^-$	2^-	$(3.6\pm0.77)\times10^{-3}$	83.6 ± 17.8	$\Sigma(1385)$	$\frac{3}{2}^+$	1^+		
$\Lambda(1815)$	$\frac{5}{2}^+$	3^-	$(26.3\pm3.5)\times10^{-3}$	91.3 ± 12.2	$\Sigma(1770)$	$\frac{5}{2}^-$	2^+	$(22.5\pm3.38)\times10^{-3}$	3.36 ± 0.51
$\Lambda(2100)$	$\frac{7}{2}^-$	4^-	21×10^{-3}	65.3	$\Sigma(2030)$	$\frac{7}{2}^+$	3^+	6×10^{-3}	2.06
				Average value $=80.1^{+23.4}_{-14.8}$					Average value $=2.71^{+1.16}_{-0.65}$

$(g_\Lambda^2/4\pi) = 10^{+3}_{-2}$

$\beta^{(2)}_{00} = 82.6^{+25.0}_{-15.7}$ mb $\qquad \beta^{(2)}_{11} = -(72.4^{+20.6}_{-13.3})$ mb

$\beta^{(1)}_{00} = 32.8^{+10.0}_{-6.0}$ mb GeV $\qquad \beta^{(1)}_{11} = -(28.2^{+13.0}_{-8.4})$ mb GeV

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$$\left|\sum_I \beta_{I0}^{(1)}\right| = 306.0 \text{ GeV}^{-1} \quad \text{and} \quad \left|\sum_I \beta_{I0}^{(2)}\right| = 334.0 \text{ GeV}^{-2}.$$

These values should be compared with the following values calculated from Table I .

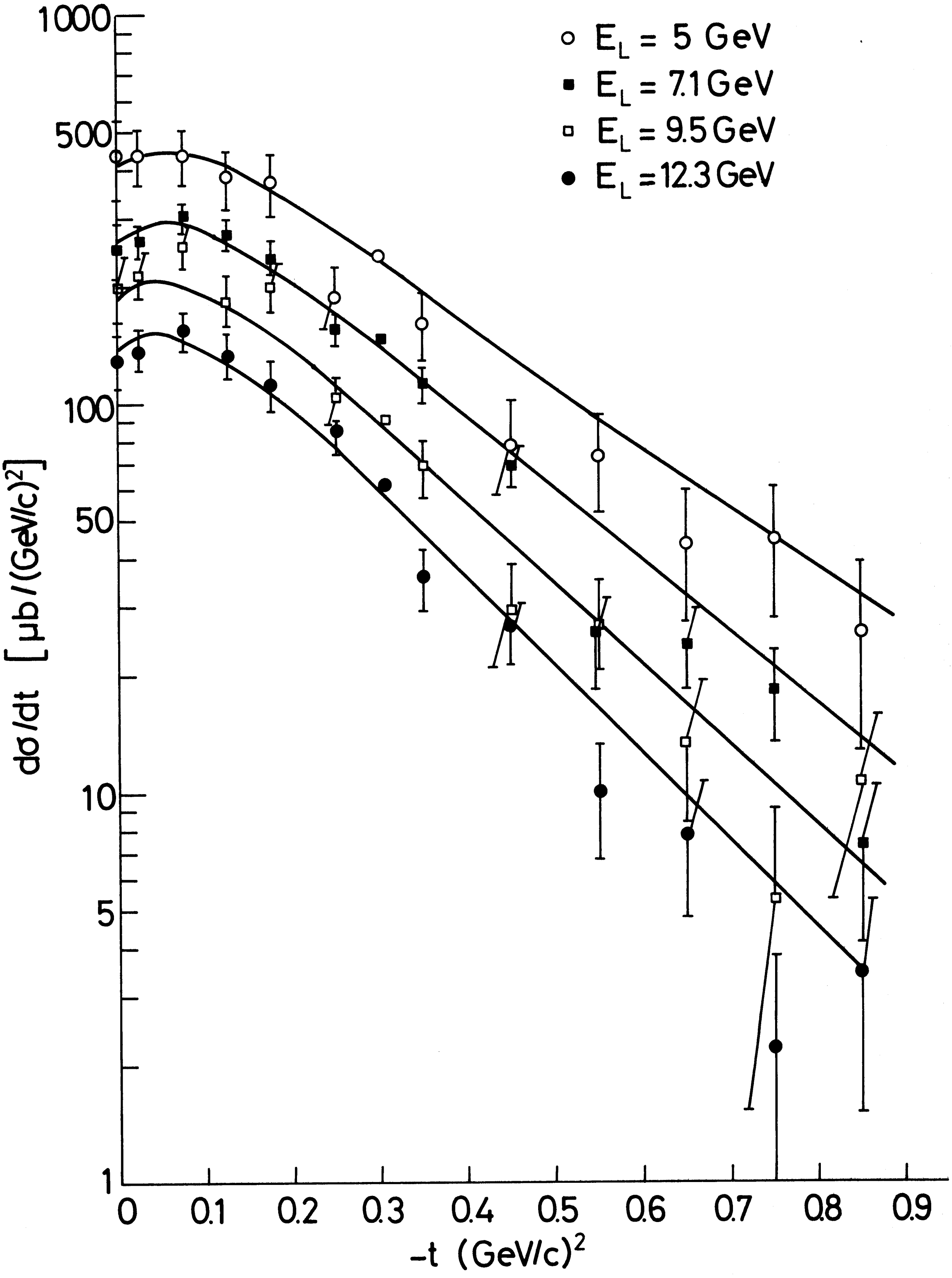
$$\sum_{I} \beta_{I0}^{(1)} = 378 \begin{smallmatrix} +112 \\ -84 \end{smallmatrix} \ \mathrm{GeV}^{-1} ,$$

$$\sum \beta_{I0}^{(2)} = 841 \begin{smallmatrix} +249 \\ -187 \end{smallmatrix} \ \mathrm{GeV}^{-2} .$$

* * *

FIGURE CAPTION

The $K^- p \rightarrow \bar{K}^0 n$ differential cross-section from Ref. 13) compared to our model with $\beta_{11}^{(1)} = -30$ mb GeV, $\beta_{11}^{(2)} = -60.4$ mb.



E_L = 5 GeV
E_L = 7.1 GeV
E_L = 9.5 GeV
E_L = 12.3 GeV
1000
500
100
50
10
5
1
$d\sigma/dt$ [$\mu b/(GeV/c)^2$]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
$-t$ $(GeV/c)^2$

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ERRATA AND ADDENDA

1) In Eq. (1), add the term $\frac{\gamma_1}{\pi} C\left(2-\alpha_I(t), \frac{3}{2}-\alpha_{Y_1}(s)\right)$.

   In Eq. (2), add the term $\frac{2\gamma_1}{\pi} C\left(2-\alpha_I(t), \frac{3}{2}-\alpha_{Y_1}(s)\right)$.

2) In Eq. (6), add the term

$$\frac{\gamma_1}{a_1}\left(\frac{5}{2}-\alpha_I(t)-\alpha_{Y_1}(s)\right)\sum_{n=0}^{\infty}\binom{\alpha_I(t)+n-2}{n}\delta\left(s-\frac{1}{a_1}\left\{n+\frac{3}{2}-b_1\right\}\right).$$

3) Eq. (9) should read

$$2\Gamma_{l+}^{el}\frac{(\sqrt{s_{l+}}+m)^2-m_K^2}{q_{l+}^{2l+1}}\frac{\Gamma(l)M(l)}{(2a)^{l-1}} = -\frac{\beta_{00}^{(2)}+\beta_{11}^{(2)}}{\pi^2 a_1} \qquad \text{(9a)}$$

$$2\Gamma_{l+}^{el}\frac{(\sqrt{s_{l+}}-m)^2-m_K^2}{q_{l+}^{2l+3}}\frac{\Gamma(l)M(l+1)}{(2a)^{l}} = -\frac{\gamma_1}{\pi^2 a_1}\ . \qquad \text{(9b)}$$

4) Eq. (12) should read $\sum_{I=0}^{1}\beta_{I1}^{(1)} = (1.385+m)\ \gamma_1$.

5) Eq. (21) should read

$$\gamma_{Y_1}(\sqrt{u}) = -\frac{1}{8\pi}\left(\alpha_{Y_1}-\frac{1}{2}\right)\gamma_1\left[\sqrt{u}+1.385\right]a^{\alpha_{Y_1}-\frac{1}{2}}$$

6) Eq. (22) should read

$$\gamma_{Y_0}(\sqrt{u}=0) = -\left(22.6 \begin{smallmatrix} +15.5 \\ -13.8 \end{smallmatrix}\right) \mathrm{GeV}^{-1}$$

Eq. (23) should read

$$\gamma_{Y_1}(\sqrt{u}=0) = -\left(2.1 \begin{smallmatrix} +1.8 \\ -1.3 \end{smallmatrix}\right) \mathrm{GeV}^{-1}$$

7) The values of the Veneziano constants obtained in the beginning of Section 5, p. 9, should be replaced by

$$\beta_{00}^{(2)} = 43 \begin{smallmatrix} +5.6 \\ -9.2 \end{smallmatrix} \ \mathrm{mb}, \quad \beta_{11}^{(2)} = -\left(97.5 \begin{smallmatrix} +36 \\ -26 \end{smallmatrix}\right) \mathrm{mb}$$

$$\beta_{00}^{(1)} = 3.6 \begin{smallmatrix} +7.2 \\ -5.6 \end{smallmatrix} \ \mathrm{mb\,GeV}, \quad \beta_{11}^{(1)} = -\left(44 \begin{smallmatrix} +11 \\ -8 \end{smallmatrix}\right) \mathrm{mb\,GeV}$$

$$\gamma_1 = -\left(17.5 \begin{smallmatrix} +15 \\ -11 \end{smallmatrix}\right) \mathrm{mb}$$

Therefore, the agreement between these values and those obtained from forward scattering becomes better.