

$|V_{td}/V_{ts}|$ from $B \rightarrow V\gamma$ PATRICIA BALL^{1,2*} AND ROMAN ZWICKY^{1†}¹ *IPPP, Department of Physics, University of Durham, Durham DH1 3LE, UK*² *CERN, CH-1211 Geneva 23, Switzerland***Abstract**

The dominant theoretical uncertainty in extracting $|V_{td}/V_{ts}|$ from the ratio of branching ratios $R \equiv \mathcal{B}(B \rightarrow (\rho, \omega)\gamma)/\mathcal{B}(B \rightarrow K^*\gamma)$ is given by the ratio of form factors $\xi \equiv T_1^{B \rightarrow K^*}(0)/T_1^{B \rightarrow \rho}(0)$. We re-examine ξ in the framework of QCD sum rules on the light-cone, taking into account hitherto neglected SU(3)-breaking effects. We find $\xi = 1.17 \pm 0.09$. Using QCD factorisation for the branching ratios, and the current experimental average for R quoted by HFAG, this translates into $|V_{td}/V_{ts}|_{B \rightarrow V\gamma}^{\text{HFAG}} = 0.192 \pm 0.014(\text{th}) \pm 0.016(\text{exp})$. This result agrees, within errors, with that obtained from the Standard Model unitarity triangle, $|V_{td}/V_{ts}|_{\text{SM}} = 0.216 \pm 0.029$, based on tree-level-only processes, and with $|V_{td}/V_{ts}|_{\Delta m} = 0.2060^{+0.0081}_{-0.0060}(\text{th}) \pm 0.0007(\text{exp})$, from the CDF measurement of B_s oscillations.

This version differs from the original version of the paper, published as JHEP 04 (2006) 046, by the inclusion of the new BaBar measurement of $B \rightarrow \rho(\omega)\gamma$ presented at ICHEP 2006, which significantly shifts the results for $|V_{td}/V_{ts}|$.

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1 Introduction

Recently, the Belle collaboration measured the $b \rightarrow d$ penguin-dominated decay $B \rightarrow (\rho, \omega)\gamma$ [1], whereas BaBar obtained an upper bound in 2004 [2] and presented a measurement at ICHEP 2006 [3]. Assuming the Standard Model (SM) to be valid, this process offers the possibility to extract the CKM matrix element $|V_{td}|$, in complementarity to the determination from B_d mixing and the SM unitarity triangle based on $|V_{ub}/V_{cb}|$ and the angle γ . In order to extract $|V_{td}|$ from the measured rate, one needs to know both short-distance weak and strong interaction effects and long-distance QCD effects. Whereas the former can, at least in principle, be calculated to any desired precision in the framework of effective field theories, and actually are currently known to next-to-leading order in QCD [4], the assessment of long-distance QCD effects is notoriously difficult. After a long history of phenomenologically or $1/N_c$ -motivated factorisation formulas, QCD factorisation [5, 6] has provided a consistent framework allowing one to write the relevant hadronic matrix elements as

$$\langle V\gamma|Q_i|B\rangle = \left[T_1^{B\rightarrow V}(0) T_i^I + \int_0^1 d\xi du T_i^{II}(\xi, u) \phi_B(\xi) \phi_{V;\perp}(v) \right] \cdot \epsilon. \quad (1)$$

Here ϵ is the photon polarisation 4-vector, Q_i is one of the operators in the effective Hamiltonian, $T_1^{B\rightarrow V}$ is a $B \rightarrow V$ transition form factor, and $\phi_B, \phi_{V;\perp}$ are leading-twist light-cone distribution amplitudes of the B meson and the vector meson V , respectively. These quantities are universal non-perturbative objects and describe the long-distance dynamics of the matrix elements, which is factorised from the perturbative short-distance interactions included in the hard-scattering kernels T_i^I and T_i^{II} . The above QCD factorisation formula is valid in the heavy-quark limit $m_b \rightarrow \infty$ and is subject to corrections of order Λ_{QCD}/m_b . Although it is possible to determine $|V_{td}|$ from the branching ratio of $B \rightarrow (\rho, \omega)\gamma$ itself, the associated theoretical uncertainties get greatly reduced when one considers the ratio of branching ratios for $B \rightarrow K^*\gamma$ and $B \rightarrow (\rho, \omega)\gamma$ instead. One then can extract $|V_{td}/V_{ts}|$ from

$$\frac{\mathcal{B}(B \rightarrow (\rho, \omega)\gamma)}{\mathcal{B}(B \rightarrow K^*\gamma)} = \left| \frac{V_{td}}{V_{ts}} \right|^2 \left(\frac{1 - m_{\rho, \omega}^2/m_B^2}{1 - m_{K^*}^2/m_B^2} \right)^3 \left(\frac{T_1^{\rho, \omega}(0)}{T_1^{K^*}(0)} \right)^2 [1 + \Delta R], \quad (2)$$

where the estimates of ΔR available in the literature lie between, approximately, 0 and 0.2 [5, 6]. ΔR contains all non-factorisable effects induced by $T_i^{I, II}$ in (1). As $|V_{ts}| = |V_{cb}|$ in the SM, up to a small correction $\sim 2\%$, and $|V_{cb}|$ is known with a precision of 2% [7], $|V_{td}|$ follows immediately from $|V_{td}/V_{ts}|$. The theoretical uncertainty of this determination is governed by both the ratio of form factors $T_1^{K^*}(0)/T_1^{\rho, \omega}(0)$ and the value of ΔR , which parametrises not only SU(3)-breaking effects, but also power-suppressed corrections to the QCD factorisation formula. The aim of the present paper is to re-examine the size of SU(3)-breaking corrections to $T_1^{K^*}(0)/T_1^{\rho, \omega}(0)$ from QCD sum rules on the light-cone and to determine a value of $|V_{td}/V_{ts}|$ from (2), evaluating ΔR in QCD factorisation; we will address the issue of power-suppressed corrections to ΔR in a separate publication [8].

Our paper is organised as follows: in Section 2 we discuss the QCD sum rule for the ratio of form factors $T_1^{K^*}(0)/T_1^{\rho, \omega}(0)$ and its dependence on SU(3)-breaking parameters.

In Section 3 we extract a value of $|V_{td}/V_{ts}|$ from the experimental branching ratio, using QCD factorisation for the calculation of ΔR . We summarise in Section 4. The appendix contains some formulas relevant to the calculation of NLO evolution of twist-2 vector-meson light-cone distribution amplitudes.

2 The Form-factor Ratio $T_1^{B \rightarrow K^*} / T_1^{B \rightarrow \rho}$

In this section, we present a concise formula for the form factor $T_1^{B \rightarrow V}(0)$, as obtained from QCD sum rules on the light-cone, and discuss the hadronic quantities that enter this expression. We do not discuss the technique of QCD sum rules itself, or that of QCD sum rules on the light-cone, for which we refer to the literature [9]. Suffice it to say that the light-cone sum rule for T_1 is based on the light-cone expansion of the correlation function of the chromomagnetic dipole operator Q_7 and the interpolating field $\bar{q}i\gamma_5 b$ of the B meson. The expansion is in terms of the convolution of process-specific perturbative kernels and universal meson light-cone distribution amplitudes (DAs) of the final-state vector meson, which are ordered in terms of increasing twist. These DAs have been studied in Refs. [10, 11], mostly for the ρ meson, including two- and three-particle Fock states up to twist 4. An extension to the K^* meson is in preparation [12]. The light-cone expansion is matched to the description of the correlation function in terms of hadrons by analytic continuation into the physical regime and the application of a Borel transformation, introducing the Borel parameter M^2 and exponentially suppressing contributions from higher-mass states. In order to extract the contribution of the B meson, one describes that of other hadron states by a continuum model, which introduces a second model parameter, the continuum threshold s_0 . The sum rule then yields the form factor in question, T_1 , multiplied by the coupling of the B meson to its interpolating field, i.e. the B meson's leptonic decay constant f_B . At tree level, the sum rule for $T_1^{B \rightarrow V}(0)$ then reads, to twist-4 accuracy:

$$\begin{aligned}
\frac{m_B^2 f_B}{m_b} T_1^{B \rightarrow V}(0) e^{-m_B^2/M^2} &= f_V^\perp m_b \int_{u_0}^1 du e^{-m_b^2/(uM^2)} \frac{\phi_\perp(u)}{2u} \\
&+ f_V^\parallel m_V \int_{u_0}^1 du e^{-m_b^2/(uM^2)} \left[\frac{\Phi(u)}{2u} + \frac{1}{2} g_\perp^{(v)}(u) + \frac{1}{8u} \left(1 - u \frac{d}{du} \right) g_\perp^{(a)}(u) \right. \\
&\quad \left. - \frac{1}{u} \frac{d}{du} \int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_2 \frac{u - \alpha_1}{2\alpha_3^2} \left(\mathcal{A}(\underline{\alpha}) + \mathcal{V}(\underline{\alpha}) \right) \right] \\
&+ f_V^\perp m_b \frac{m_V^2}{m_b^2} \int_{u_0}^1 du e^{-m_b^2/(uM^2)} \left[\frac{1}{2} \frac{d}{du} \left\{ u \bar{u} \phi_\perp(u) + 2I_L(u) + uH_3(u) \right. \right. \\
&\quad \left. \left. - \int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_2 \frac{1}{\alpha_3} \left(S(\underline{\alpha}) - \tilde{S}(\underline{\alpha}) + T_1^{(4)}(\underline{\alpha}) - T_2^{(4)}(\underline{\alpha}) + T_3^{(4)}(\underline{\alpha}) - T_4^{(4)}(\underline{\alpha}) \right) \right\} \right. \\
&\quad \left. - \frac{1}{8} u \frac{d^2}{du^2} \mathbb{A}_\perp(u) \right], \tag{3}
\end{aligned}$$

$$\equiv m_b \int_{u_0}^1 du e^{-m_b^2/(uM^2)} \left[f_V^\perp R_1(u) + f_V^\parallel \frac{m_V}{m_b} R_2(u) + f_V^\perp \left(\frac{m_V}{m_b} \right)^2 R_3(u) \right], \quad (4)$$

where u_0 is given by m_b^2/s_0 . f_V^\parallel and f_V^\perp are the decay constants of, respectively, longitudinally and transversely polarised vector mesons. ϕ_\perp , Φ , $g_\perp^{(v,a)}$, I_L and H_3 are two-particle distribution amplitudes and integrals thereof, as defined in Ref. [13]. \mathcal{A} , \mathcal{V} , S , \tilde{S} and $T_i^{(4)}$ are three-particle DAs. u is the longitudinal momentum fraction of the quark in a two-particle Fock state of the final-state vector meson, whereas $\alpha_{1,2,3}$, with $\sum \alpha_i = 1$, are the momentum fractions of the partons in a three-particle state. The light-cone expansion is accurate up to terms of order $(m_V/m_b)^3$. Although we only write down the tree-level expression for the form factor, radiative corrections are known for R_1 [14] and the two-particle contributions to R_2 [13], and will be included in the numerical analysis. All scale-dependent quantities are calculated at the (infra-red) factorisation scale $\mu_F^2 = m_B^2 - m_b^2$. The form factor itself carries an ultra-violet scale dependence, which however cancels in the ratio.

It is clearly visible from the above formula that the respective weight of various contributions is controlled by the parameter m_V/m_b ; the next term in the light-cone expansion contains twist-3, -4 and -5 DAs and is of order $(m_V/m_b)^3$. Nonetheless, (4) cannot be interpreted as $1/m_b$ expansion: for $m_b \rightarrow \infty$, the support of the integrals in u also becomes of $\mathcal{O}(1/m_b)$, as $1 - u_0 \sim 1 - m_b^2/s_0 \sim \omega_0/m_b$, with $\omega_0 \sim 1$ GeV a hadronic quantity [15]. In this case, the scaling of the various terms in m_b is controlled by the behaviour of the DAs near the end-point $u \rightarrow 1$. For finite m_b , however, the sum rules are not sensitive to the details of the end-point behaviour, as we shall see below. Numerically, the expansion in terms of m_V/m_b works very well and is a reformulation of the ordering of contributions in terms of the parameter δ introduced in Ref. [13].

We have already discussed T_1 in Refs. [14, 13]; in the present paper we focus on the ratio

$$\xi \equiv \frac{T_1^{B \rightarrow K^*}(0)}{T_1^{B \rightarrow \rho}(0)}, \quad (5)$$

which governs the extraction of $|V_{ts}/V_{td}|$ from $B \rightarrow V\gamma$ decays. Our sum rules can of course be used to determine each form factor separately, but we expect the ratio to be more accurate, because ξ is independent of the B -meson decay constant f_B and also, to very good accuracy, of m_b and the sum rule parameters M^2 and s_0 ; we shall come back to that point below. Hence, in this paper, we will not re-analyse the absolute values of $T_1^{B \rightarrow (\rho, K^*)}(0)$ nor, consequently, the branching ratios themselves. However, ξ is very sensitive to SU(3)-breaking effects in the DAs, and it is precisely these effects we shall focus on in this paper. A similar analysis for the ratio of the $D \rightarrow K$ and $D \rightarrow \pi$ form factors was carried out in Ref. [16].

Compared with our previous results of Refs. [14, 13], in this paper we implement the following improvements:

- updated values of SU(3)-breaking in twist-2 parameters;
- complete account of SU(3)-breaking in twist-3 and -4 DAs;

- estimate of higher-order conformal contributions to twist-4 DAs, using the renormalon model of Ref. [17];
- NLO evolution for twist-2 parameters.

Before presenting numerical results for ξ , let us first discuss the values of the hadronic input parameters collected in Table 1. First of all, we would like to mention that we will not distinguish between the form factors of ρ and ω . Their difference is mainly caused by different values of the decay constants, $f_\rho^{\parallel(\perp)} \neq f_\omega^{\parallel(\perp)}$, whose precise determination, e.g from experimental data for $\omega \rightarrow e^+e^-$, is complicated by mixing with the ϕ meson. In the present paper we take the view that the uncertainty induced by letting $T_1^{B \rightarrow \rho} = T_1^{B \rightarrow \omega}$ is negligible compared to other uncertainties.

The longitudinal decay constants $f_{\rho, K^*}^{\parallel}$ can be extracted from the experimental decay rates $\tau^- \rightarrow (\rho^-, K^{*-})\nu_\tau$ as [22]

$$f_\rho^{\parallel} = (0.209 \pm 0.002) \text{ GeV}, \quad f_{K^*}^{\parallel} = (0.217 \pm 0.005) \text{ GeV}.$$

There is no direct experimental measurement of the tensor decay constants f_{ρ, K^*}^{\perp} , which instead have to be determined from non-perturbative methods such as lattice calculations [23, 24] or QCD sum rules [25, 18]. Lattice results are available in the quenched approximation with a chirally improved lattice Dirac operator, which allows one to reach small quark masses, and for the ratio of decay constants $f_V^{\perp}/f_V^{\parallel}$ [24]; a first study for the ρ with dynamical fermions was reported in Ref. [26]. One result of these calculations is that the ratio of decay constants only weakly depends on the quark masses. For the ρ , Ref. [24] quotes

$$\left(\frac{f_\rho^{\perp}}{f_\rho^{\parallel}} \right)_{\text{latt}} (2 \text{ GeV}) = 0.72 \pm 0.02,$$

obtained for the lattice spacing $a = 0.15 \text{ fm}$. As for QCD sum rules, the value $f_\rho^{\perp}(1 \text{ GeV}) = (0.160 \pm 0.010) \text{ GeV}$ was obtained in Ref. [27]. For the present paper, we have re-analysed the corresponding sum rules, using updated values of α_s and NLO evolution of f_ρ^{\perp} , and find

$$f_\rho^{\perp}(1 \text{ GeV}) = (0.165 \pm 0.009) \text{ GeV}. \quad (6)$$

Also f_ρ^{\parallel} can be calculated from sum rules, yielding $(0.206 \pm 0.007) \text{ GeV}$. If one calculates the ratio directly from QCD sum rules, one finds¹

$$\left(\frac{f_\rho^{\perp}}{f_\rho^{\parallel}} \right)_{\text{SR}} (2 \text{ GeV}) = 0.69 \pm 0.04,$$

in agreement with the lattice result.

The determination of $f_{K^*}^{\perp}$ is less straightforward, see Ref. [18], where

$$f_{K^*}^{\perp}(1 \text{ GeV}) = (0.185 \pm 0.010) \text{ GeV} \quad (7)$$

¹The NLO scaling factor $f^{\perp}(2 \text{ GeV})/f^{\perp}(1 \text{ GeV})$ is 0.876.

	ρ	$\mu = 1 \text{ GeV}$	Ref.	K^*	$\mu = 1 \text{ GeV}$	Ref.	Order	Remarks
R_1	f_ρ^\perp	0.165 ± 0.009	TP	$f_{K^*}^\perp$	0.185 ± 0.010	[18]	twist-2	in units of GeV
	$a_1^\perp(\rho)$	0		$a_1^\perp(K^*)$	0.04 ± 0.03	[18]	twist-2	G-odd
	$a_2^\perp(\rho)$	0.15 ± 0.07	TP	$a_2^\perp(K^*)$	0.11 ± 0.09	TP	twist-2	$a_2^\perp(K^*) - a_2^\perp(\rho)$ constrained
	Δ_ρ^\perp	1.24 ± 0.11	TP	$\Delta_{K^*}^\perp$	1.18 ± 0.14	TP	twist-2	BT model [19]
	p_ρ^\perp	3		$p_{K^*}^\perp$	3		twist-2	for ϕ_\perp
R_2	f_ρ^\parallel	0.209 ± 0.002	exp.	$f_{K^*}^\parallel$	0.217 ± 0.005	exp.	twist-2	in units of GeV
	$a_1^\parallel(\rho)$	0		$a_1^\parallel(K^*)$	0.03 ± 0.02	[18]	twist-2	G-odd
	$a_2^\parallel(\rho)$	$= a_2^\perp(\rho)$	TP	$a_2^\parallel(K^*)$	$= a_2^\perp(K^*)$	TP	twist-2	$a_2^\parallel(K^*) - a_2^\parallel(\rho)$ constrained
	Δ_ρ^\parallel	$= \Delta_\rho^\perp$		$\Delta_{K^*}^\parallel$	$= \Delta_{K^*}^\perp$		twist-2	BT model [19]
	p_ρ^\parallel	$= p_\rho^\perp$		$p_{K^*}^\parallel$	$= p_{K^*}^\perp$		twist-2	for ϕ_\parallel
	$\zeta_{3\rho}^A$	0.032 ± 0.010	[20]	$\zeta_{3K^*}^A$	$(1.0 \pm 0.1)\zeta_{3\rho}^A$	TP	LO twist-3	UR
	$\kappa_{3\rho}^\parallel$	0		$\kappa_{3K^*}^\parallel$	0.001 ± 0.001	TP	LO twist-3	G-odd, UR
	$\omega_{3\rho}^A$	-2 ± 2	[20]	$\omega_{3K^*}^A$	$= \omega_{3\rho}^A$		NLO twist-3	UR
	$\omega_{3\rho}^V$	4 ± 2	[20]	$\omega_{3K^*}^V$	$= \omega_{3\rho}^V$		NLO twist-3	UR
	$\lambda_{3\rho}^A$	0		$\lambda_{3K^*}^A$	0 ± 2	TP	NLO twist-3	G-odd, UR
	$\lambda_{3\rho}^V$	0		$\lambda_{3K^*}^V$	0 ± 2	TP	NLO twist-3	G-odd, UR
R_3	$\kappa_{3\rho}^\perp$	0		$\kappa_{3K^*}^\perp$	0 ± 0.01	TP	LO twist-3	G-odd, UR
	$\omega_{3\rho}^T$	7 ± 7	[10]	$\omega_{3K^*}^T$	$= \omega_{3\rho}^T$		NLO twist-3	UR
	$\lambda_{3\rho}^T$	0		$\lambda_{3K^*}^T$	0 ± 2	TP	NLO twist-3	G-odd, UR
	$\zeta_{4\rho}^T$	0.10 ± 0.05	[21]	$\zeta_{4K^*}^T$	$= \zeta_{4\rho}^T$		LO twist-4	UR

Table 1: Hadronic parameters entering $R_{1,2,3}$ in the sum rule for T_1 , Eq. (4). For twist-2 DAs, we use both the conformal expansion Eq. (8), truncated after $n = 2$, and the model of Ball and Talbot (BT) [19] given in terms of two parameters, Δ and p . The values of $a_2(\rho)$ and $a_2(K^*)$ are highly correlated; we fix $a_2(K^*) - a_2(\rho) = -0.04 \pm 0.02$ for both longitudinal and transverse DAs. The twist-3 and -4 G-odd parameters have never been considered before; all twist-3 and -4 parameters are under revision (UR) and will be considered in full detail in Ref. [12]. In this paper (TP), twist-3 SU(3)-breaking effects are only taken into account at LO in the conformal expansion. Higher-orders in the conformal expansion of twist-4 DAs are calculated in the renormalon model of Ref. [17], see text.

was obtained. Evaluating the ratio $f_{K^*}^\perp/f_{K^*}^\parallel$ directly from sum rules, we find

$$\left(\frac{f_{K^*}^\perp}{f_{K^*}^\parallel}\right)_{\text{SR}}(2\text{ GeV}) = 0.73 \pm 0.04,$$

which agrees with the interpolation between the corresponding results for ρ and ϕ obtained from lattice [24].

Summarising, it is probably fair to say that the present status of f_V^\perp decay constants is not entirely satisfactory. The accuracy of the QCD sum rule estimates is unlikely to improve, so any significant reduction of uncertainty has to come from lattice. For the moment, however, all existing lattice results still come with considerable uncertainty (no continuum limit, no results for K^* with chirally improved Dirac operator), so that in the numerical analysis of ξ we will use the experimental results for f_{ρ,K^*}^\parallel and the QCD sum rule results (6) and (7) for f_{ρ,K^*}^\perp .

As for twist-2 DAs, the standard approach is to parametrise them in terms of a few parameters which are the leading-order terms in the conformal expansion

$$\phi(u, \mu^2) = 6u(1-u) \left(1 + \sum_{n=1}^{\infty} a_n(\mu^2) C_n^{3/2}(2u-1)\right). \quad (8)$$

To leading-logarithmic accuracy the (non-perturbative) Gegenbauer moments a_n renormalize multiplicatively. This feature is due to the conformal symmetry of massless QCD at one-loop, the a_n start to mix at next-to-leading order, see appendix. Although (8) is not an expansion in any obvious small parameter, the contribution of terms with large n to physical amplitudes is suppressed by the fact that the Gegenbauer polynomials oscillate rapidly and hence are “washed out” upon integration over u with a “smooth” (i.e. not too singular) perturbative hard-scattering kernel. For vector mesons, one usually takes into account the terms with $n = 1, 2$; the a_n are estimated from QCD sum rules which are known to become less reliable for larger n . As an alternative, one can build models for ϕ based on an assumed fall-off behaviour of a_n for large n . The model of Ball and Talbot (BT) [19], for instance, assumes that, at a certain reference scale, e.g. $\mu = 1\text{ GeV}$, the a_n fall off as powers of n :

$$a_{2n} \propto \frac{1}{(n+1)^p}.$$

BT fix the absolute normalisation of the Gegenbauer moments by the first inverse moment:

$$\int_0^1 \frac{du}{2u} (\phi(u) + \phi(1-u)) \equiv 3\Delta = 3 \left(1 + \sum_{n=1}^{\infty} a_{2n}\right),$$

which can be viewed as a convolution with the singular hard-scattering kernel $1/u$ and gives all a_n the same (maximum) weight 1. The rationale of this model is that the DA is given in terms of only two parameters, p and Δ , and allows one to estimate the effect of higher order terms in the conformal expansion on observables. In this paper, we calculate the form factor using both conformal expansion, truncated after $n = 2$, and the BT model,

normalised by a_2 and taking into account terms up to $n = 8$. We shall see below that the effect of terms with $n > 2$ is very small.

For the ρ , $a_2^{\perp,\parallel}$ have been determined in Ref. [27]. In the present study we have re-examined the corresponding sum rules and find, at the scale $\mu = 1 \text{ GeV}$, $a_2^\perp(\rho) = 0.15 \pm 0.07$ and $a_2^\parallel(\rho) = 0.14 \pm 0.06$, which is slightly smaller than the results quoted in Ref. [27]. As both values are nearly equal, we shall use a common value

$$a_2^\perp(\rho) = 0.15 \pm 0.07 = a_2^\parallel(\rho). \quad (9)$$

The corresponding value of Δ is

$$\Delta_\rho^\perp = 1.24 \pm 0.11 = \Delta_\rho^\parallel,$$

with a central value slightly larger than that used in Ref. [13]. The value of $a_2(K^*)$ has been determined in Ref. [25]. Again, we re-examine these sum rules for the present paper. We find $a_2^\perp(K^*) = 0.11 \pm 0.09$ and $a_2^\parallel(K^*) = 0.10 \pm 0.08$, which is more conveniently presented by the difference between $a_2(K^*)$ and $a_2(\rho)$:

$$\begin{aligned} a_2^\perp(K^*) - a_2^\perp(\rho) &= -0.04 \pm 0.02, \\ a_2^\parallel(K^*) - a_2^\parallel(\rho) &= -0.03 \pm 0.02. \end{aligned}$$

As both differences are nearly equal, we shall use

$$a_2(K^*) - a_2(\rho) = -0.04 \pm 0.03 \quad (10)$$

for both polarisations. This translates into $\Delta_{K^*}^\perp = 1.18 \pm 0.14$, with errors largely correlated with those of Δ_ρ^\perp .

The value of $a_1(\rho)$ vanishes by G-parity. The values of $a_1^{\parallel,\perp}(K^*)$ have been subject to some controversy over the recent years, which was settled only very recently; in this work, we use the values obtained in Ref. [18]:

$$a_1^\perp(K^*) = 0.04 \pm 0.03, \quad a_1^\parallel(K^*) = 0.03 \pm 0.02. \quad (11)$$

All odd Gegenbauer moments, i.e. the antisymmetric contribution to $\phi(u)$, can be resummed using the same power-like behaviour of large moments as in the BT model. This model is also discussed in Ref. [19] and normalised to a_1 ; we include terms up to $n = 9$.

Twist-3 and -4 DAs of vector mesons have been studied in Refs. [10, 11]. The results are complete for mesons with definite G-parity (with equal-mass quarks), but miss certain G-parity-breaking corrections. A complete analysis of all these corrections is in preparation [12]; here, we include those results that are already available [28]. In Ref. [10], the two-particle twist-3 DAs $g_\perp^{(v,a)}$ and $h_\parallel^{(s,t)}$ have been expressed in terms of integrals over the twist-2 DAs $\phi_{\perp,\parallel}$ and the three-particle twist-3 DAs $\mathcal{A}, \mathcal{V}, \mathcal{T}$. These integral relations are complete, but the explicit expressions for the three-particle twist-3 DAs given in [10] have

to be extended to include G-parity-breaking corrections as follows:

$$\begin{aligned}
\mathcal{A}(\underline{\alpha}) &= 360\alpha_1\alpha_2\alpha_3^2\zeta_3^A \left\{ 1 + \lambda_3^A(\alpha_1 - \alpha_2) + \omega_3^A \left(\frac{7}{2}\alpha_3 - \frac{3}{2} \right) \right\}, \\
\mathcal{V}(\underline{\alpha}) &= 360\alpha_1\alpha_2\alpha_3^2 \left\{ \kappa_3^\parallel + \frac{3}{2}\zeta_3^A\omega_3^V(\alpha_1 - \alpha_2) + \kappa_3^\parallel\lambda_3^V \left(\frac{7}{2}\alpha_3 - \frac{3}{2} \right) \right\}, \\
\mathcal{T}(\underline{\alpha}) &= 360\alpha_1\alpha_2\alpha_3^2 \left\{ \kappa_3^\perp + \frac{3}{2}\zeta_3^A\omega_3^T(\alpha_1 - \alpha_2) + \kappa_3^\perp\lambda_3^T \left(\frac{7}{2}\alpha_3 - \frac{3}{2} \right) \right\}. \tag{12}
\end{aligned}$$

Here ζ_3^A and ω_3 are G-parity conserving quantities, whereas κ_3 and λ_3 are G-parity breaking. As κ_3^\perp contributes to the form factor only at $\mathcal{O}(m_V^2/m_b^2)$, and the λ_3 parameters are of non-leading conformal spin, we set, in the present analysis, the central values of all these parameters to zero and only take into account κ_3^\parallel . A QCD sum rule estimate of this parameter yields [12, 28]

$$\kappa_3^\parallel(1 \text{ GeV}) = 0.001 \pm 0.001. \tag{13}$$

The effect of non-zero values of κ_3^\perp and λ_3 is taken into account by the variation of these parameters around zero within the range given in Table 1; the dependence of $T_1^{K^*}$ on NLO G-parity breaking parameters is very small, as expected.

The two-particle twist-4 DAs h_3 and \mathbb{A}_\perp have been discussed in Ref. [11]; they are given by integrals over chiral-odd twist-4 three-particle DAs. The determination of the conformal-expansion coefficients of the latter is complicated by the fact that they contain ‘‘kinetic’’ mass-correction terms given by twist-2 matrix elements, which, to date, have not been obtained in a closed form, but have to be unravelled order by order in the conformal expansion. In addition, the direct determination of the ‘‘genuine’’ twist-4 corrections from QCD sum rules becomes increasingly complicated at higher-order conformal spin. For that reason, we invoke an alternative estimate of these corrections based on the renormalon-model developed in Ref. [17]. The general idea of this technique is to estimate matrix elements of ‘‘genuine’’ twist-4 operators by the quadratically divergent contributions that appear when the matrix elements are defined using a hard UV cut-off. In this way, three-particle twist-4 DAs can be expressed in terms of the leading-twist DA ϕ_\perp [17]:

$$\begin{aligned}
T_1(\underline{\alpha}) &= -T_3(\underline{\alpha}) = \zeta_4^T \left[\frac{\alpha_2\phi_\perp(\alpha_1)}{(1-\alpha_1)^2} - \frac{\alpha_1\phi_\perp(1-\alpha_2)}{(1-\alpha_2)^2} \right], \\
T_2(\underline{\alpha}) &= T_4(\underline{\alpha}) = -\frac{1}{2}\zeta_4^T \left[\frac{\phi_\perp(\alpha_1)}{1-\alpha_1} - \frac{\phi_\perp(1-\alpha_2)}{1-\alpha_2} \right], \\
S(\underline{\alpha}) &= -\tilde{S}(\underline{\alpha}) = \frac{1}{2}\zeta_4^T \left[\frac{\phi_\perp(\alpha_1)}{1-\alpha_1} + \frac{\phi_\perp(1-\alpha_2)}{1-\alpha_2} \right]. \tag{14}
\end{aligned}$$

The above formulas differ from those given in Ref. [17] by the change of argument $\alpha_2 \rightarrow 1 - \alpha_2$ in the second terms on the right-hand side; this is to properly account for G-parity-breaking effects [29]. One prediction of the renormalon model is that the two independent LO twist-4 couplings ζ_4^T and $\tilde{\zeta}_4^T$ add up to 0, which is consistent with the direct calculation

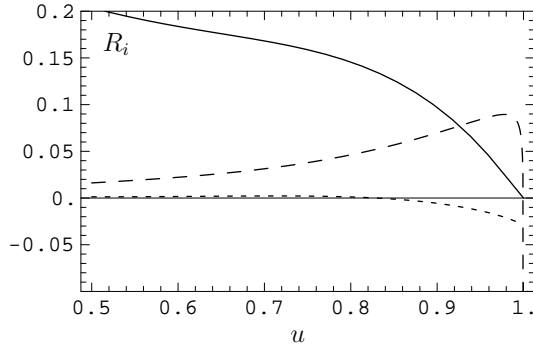


Figure 1: Contribution of R_i to the convolution integral in (4) as a function of u . Solid line: $f_\rho^\perp R_1(u)$, long dashes: $f_\rho^\parallel(m_\rho/m_b)R_2(u)$, short dashes: $f_\rho^\perp(m_\rho/m_b)^2 R_3(u)$.

from QCD sum rules [21]. The above formulas also allow one to estimate the “genuine” twist-4 G-parity breaking contributions to T_i and S , \tilde{S} ; we refrain from giving explicit formulas in this paper, but refer to Ref. [12]. For the calculation of the contribution of twist-4 terms to ξ , we use two methods: firstly the full renormalon model (14), and the corresponding expression for \mathbb{A}_\perp as given by the equations of motion [11] ($h_3 = 0$ in this model). This accounts for the genuine twist-4 corrections; the “kinetic” corrections, as far as they are known, are added using truncated conformal expansion. Secondly, we use truncated expansion for all twist-4 DAs, describing G-parity-breaking terms by the values they assume in the renormalon model, see Refs. [29, 12] for more details. The predictions of both methods for the end-point behaviour of the DAs near $u = 0, 1$ differ quite drastically; nonetheless, both prescriptions given nearly the same result after integration over u .

One more parameter that enters the kinetic mass corrections to twist-3 and -4 DAs, induced by the equations of motion, are the quark masses $m_{s,u,d}$. We choose $\overline{m}_s(2 \text{ GeV}) = (0.10 \pm 0.02) \text{ GeV}$, which is in accordance with both lattice [30] and QCD sum rule calculations [31], and let $m_{u,d} = 0$.

With all DAs available, we can now assess the respective size of the contributions of the various R_i to the sum rule (4). To this end, we plot, in Fig. 1, the functions R_i for the ρ meson, multiplied by the corresponding weight factors, for $u > 0.5$ which is about the smallest value of u_0 . The plot clearly shows that R_1 is dominant. It also shows that $R_{2,3}$ exhibit (integrable) end-point singularities for $u \rightarrow 1$. Based on these results, we expect the impact of the first neglected term in the light-cone expansion, which is $\mathcal{O}(m_V^3/m_b^3)$, to be very small.

Before we can evaluate the sum rule for ξ , we also have to discuss the choice of m_b and the sum-rule-specific parameters M^2 and s_0 . The good news is that, although numerator and denominator of (5) both depend on m_b , s_0 and M^2 , this dependence cancels to a large extent in the ratio. The reason hereof is quite evident from (4): M^2 controls the respective weights of contributions of different u ; as these contributions are nearly equal in numerator and denominator of (5), except for moderately sized SU(3) breaking, it follows that one can choose M^2 equal in $T_1^{B \rightarrow (\rho, K^*)}$ and that the resulting dependence on M^2 should be very small. This is borne out by the left panel of Fig. 2, where we plot ξ as function of M^2 for central values of the input parameters and $s_0 = 35 \text{ GeV}^2$. For comparison, we also show

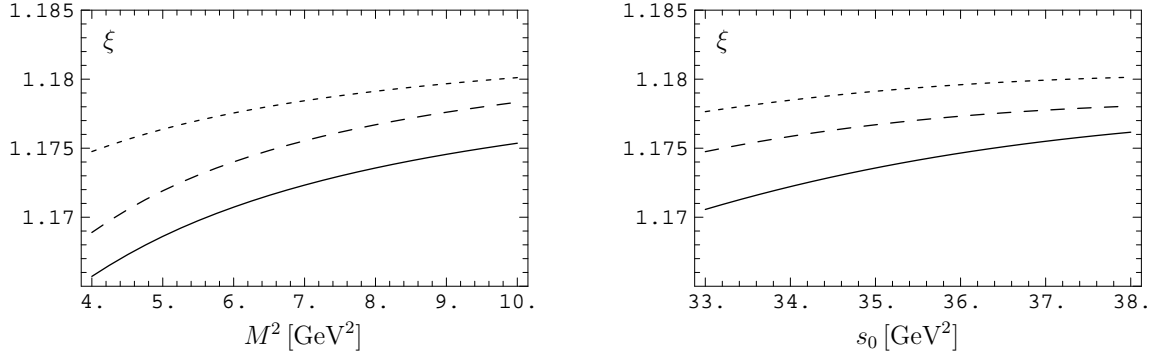


Figure 2: Left panel: ξ as a function of the Borel parameter M^2 for $s_0 = 35 \text{ GeV}^2$ and central values of the input parameters. Right panel: ξ as a function of the continuum threshold s_0 for $M^2 = 8 \text{ GeV}^2$ and central values of the input parameters. Solid lines: DAs in conformal expansion; long dashes: BT model [19] for twist-2 DAs; short dashes: BT model for twist-2 DAs and renormalon model for twist-4 DAs [17].

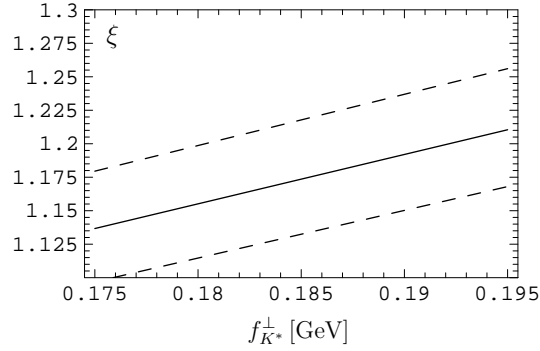


Figure 3: ξ as a function of $f_{K^*}^\perp(1 \text{ GeV})$. Solid line: $f_\rho^\perp(1 \text{ GeV}) = 0.165 \text{ GeV}$, dashed lines: f_ρ^\perp shifted by $\pm 0.009 \text{ GeV}$.

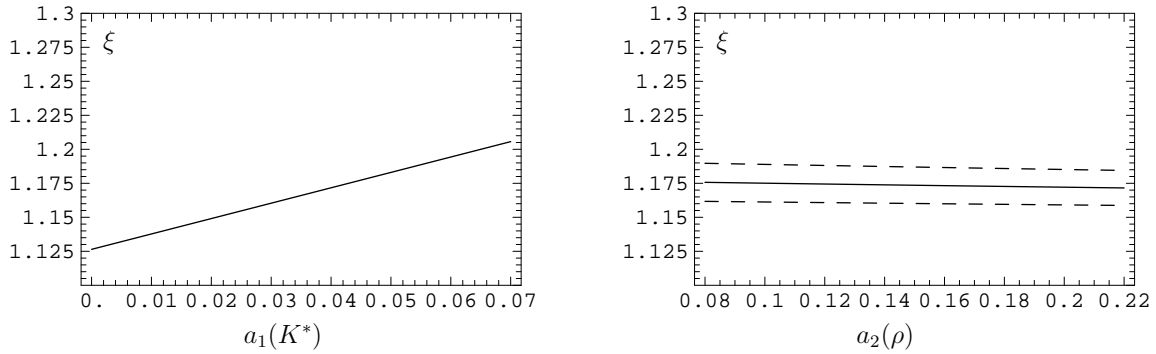


Figure 4: Left panel: ξ as a function of $a_1(K^*)$ at 1 GeV. Right panel: ξ as a function of $a_2(\rho)$ at 1 GeV. Solid line: $a_2(K^*) = a_2(\rho) - 0.04$; dashed lines: $a_2(K^*)$ shifted by ± 0.02 . Longitudinal and transverse parameters a_i^\parallel and a_i^\perp are set equal.

ξ calculated using the BT model for the twist-2 DAs (long dashes) and, in addition, the renormalon-model for twist-4 DAs (short dashes). All three calculations agree with one another very well. The fact that the impact of the BT model is only minor shows that the sum rules are sensitive only to a few gross characteristics of the twist-2 DAs, but not to the details of their behaviour near the end-point $u = 1$. As for the renormalon-model DAs, the difference to the truncated conformal expansion is most marked for small values of M^2 , which can be easily understood from the fact that for smaller M^2 the weight of contributions from u close to 1 gets enhanced and hence the difference between the end-point behaviour of conformally expanded DAs and renormalon-modelled DAs becomes more visible. The right panel of Fig. 2 illustrates the effect of a variation of s_0 for fixed M^2 , which is also small. The value of s_0 sets the lower limit of the integral over u , and again the dependence on s_0 largely cancels in the ratio as the integrands are equal up to SU(3)-breaking effects. As s_0 itself is nearly independent of the final-state meson, it is natural to choose the same value in both numerator and denominator. As for m_b , it only enters in the ratio m_V/m_b , which controls the respective contributions of $R_{1,2,3}$, the lower limit of integration $u_0 = m_b^2/s_0$ and the Borel exponential $\exp(-m_b^2/(uM^2))$. In the latter two parameters, a change of m_b is effectively compensated by a change of s_0 or M^2 , which, as we have just discussed, induces only very small variations of the sum-rule result. The ratio m_V/m_b changes from 0.185 for K^* and $m_b = 4.8$ GeV to 0.193 for $m_b = 4.6$ GeV, which also has only very minor impact. Based on these observations, we choose to evaluate ξ for fixed $m_b = 4.8$ GeV, $M^2 = 8$ GeV² and $s_0 = 35$ GeV² and attach to ξ a corresponding uncertainty of ± 0.005 .

We are now in a position to obtain a result for ξ and estimate its uncertainty. The dominant uncertainty is due to the dependence of ξ on the chiral-odd twist-2 parameters. In Fig. 3 we plot ξ as a function of $f_{K^*}^\perp(1 \text{ GeV})$, for various values of $f_\rho^\perp(1 \text{ GeV})$. The uncertainty in both parameters causes an uncertainty in ξ of ± 0.08 . In Fig. 4, left panel, we show the dependence of ξ on $a_1(K^*)$, which induces a change in ξ by ± 0.03 ; the variation of $a_1^\perp(K^*)$ and $a_1^\parallel(K^*)$ as separate quantities induces the same change. The right panel shows the dependence on a_2 which is rather mild and causes ξ to change by ± 0.02 . The variation of the remaining parameters within the limits specified in Table 1 causes another ± 0.02 shift in ξ , so that we arrive at the following result:

$$\begin{aligned} \xi &= \frac{T_1^{B \rightarrow K^*}(0)}{T_1^{B \rightarrow \rho}(0)} = 1.17 \pm 0.08(f_{\rho, K^*}^\perp) \pm 0.03(a_1) \pm 0.02(a_2) \pm 0.02(\text{twist-3 and -4}) \\ &\quad \pm 0.01(\text{sum-rule parameters, } m_b \text{ and twist-2 and -4 models}) \\ &= 1.17 \pm 0.09. \end{aligned} \tag{15}$$

The total uncertainty of ± 0.09 is obtained by adding the individual terms in quadrature. Let us stress again that the error of this result is dominated by far by parameter uncertainties, and is nearly independent of the sum rule specific parameters; it is also independent of f_B .

3 Determination of $|V_{td}/V_{ts}|$

Let us now turn to the calculation of the ratio of branching ratios and the determination of $|V_{td}/V_{ts}|$. The Belle collaboration has measured the quantity

$$R_{\text{exp}} \equiv \frac{\overline{\mathcal{B}}(B \rightarrow (\rho, \omega)\gamma)}{\overline{\mathcal{B}}(B \rightarrow K^*\gamma)},$$

where $\overline{\mathcal{B}}(B \rightarrow (\rho, \omega)\gamma)$ is defined as the CP-average $\frac{1}{2}[\mathcal{B}(B \rightarrow (\rho, \omega)\gamma) + \mathcal{B}(\bar{B} \rightarrow (\bar{\rho}, \omega)\gamma)]$ of

$$\mathcal{B}(B \rightarrow (\rho, \omega)\gamma) = \frac{1}{2} \left\{ \mathcal{B}(B^+ \rightarrow \rho^+\gamma) + \frac{\tau_{B^+}}{\tau_{B^0}} [\mathcal{B}(B^0 \rightarrow \rho^0\gamma) + \mathcal{B}(B^0 \rightarrow \omega\gamma)] \right\},$$

and $\overline{\mathcal{B}}(B \rightarrow K^*\gamma)$ is the isospin- and CP-averaged branching ratio of the $B \rightarrow K^*\gamma$ channels. In 2005, Belle reported a 5.1σ measurement [1],

$$R_{\text{exp}}^{\text{Belle}} = 0.032 \pm 0.008(\text{stat}) \pm 0.002(\text{syst}), \quad (16)$$

followed by a 5.2σ measurement by BaBar in 2006 [3]:

$$R_{\text{exp}}^{\text{BaBar}} = 0.024 \pm 0.005, \quad (17)$$

where the statistical and systematical uncertainty are added in quadrature. HFAG combines both results into the average [32]

$$R_{\text{exp}}^{\text{HFAG}} = 0.028 \pm 0.005. \quad (18)$$

Within QCD factorisation, and using the notations of Ref. [6], the amplitude for $B \rightarrow V\gamma$ can be written as

$$A(\bar{B} \rightarrow V\gamma) = \frac{G_F}{\sqrt{2}} [\lambda_u a_7^u(V\gamma) + \lambda_c a_7^c(V\gamma)] \langle V\gamma | Q_7 | \bar{B} \rangle,$$

where λ_q are products of CKM matrix elements and the factorisation coefficients $a_7^{u,c}$ consist of Wilson coefficients and non-factorisable corrections from hard scattering and annihilation; explicit expressions can be found in Ref. [6]. $a_7^{u,c}$ depends in particular on the form factor T_1 and the twist-2 DA $\phi_{V;\perp}$. The theoretical expression for R is then given by

$$R_{\text{th}} = \left| \frac{V_{td}}{V_{ts}} \right|^2 \frac{1}{\xi^2} \left(\frac{1 - m_\rho^2/m_B^2}{1 - m_{K^*}^2/m_B^2} \right)^3 \left| \frac{a_7^c(\rho\gamma)}{a_7^c(K^*\gamma)} \right|^2 \left(1 + \text{Re}(\delta a_\pm + \delta a_0) \left[\frac{R_b^2 - R_b \cos \gamma}{1 - 2R_b \cos \gamma + R_b^2} \right] \right. \\ \left. + \frac{1}{2} (|\delta a_\pm|^2 + |\delta a_0|^2) \left\{ \frac{R_b^2}{1 - 2R_b \cos \gamma + R_b^2} \right\} \right) \quad (19)$$

with $\delta a_{0,\pm} = a_7^u(\rho^{0,\pm}\gamma)/a_7^c(\rho^{0,\pm}\gamma) - 1$. Here, γ is one angle of the CKM unitarity triangle and R_b one of its sides:

$$R_b = \left(1 - \frac{\lambda^2}{2} \right) \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right|.$$

Equation (19) differs from the expression given in Ref. [6] by the terms in $|\delta a|^2$ which were neglected in that paper. It is obtained in the SM, assuming that $\bar{\mathcal{B}}(B^0 \rightarrow \rho^0 \gamma) \equiv \bar{\mathcal{B}}(B^0 \rightarrow \omega \gamma)$, which indeed should be the case up to a small difference in the decay constants, a tiny difference in phase space and the sign of the contribution of weak annihilation (WA) diagrams, which is also small numerically. Equation (19) is also valid in extensions of the SM where the CKM matrix is still unitary and the a_7 do not carry a new weak phase, for instance Minimal Flavour Violation; in this case new physics could change the values of $\delta a_{0,\pm}$.

Let us first discuss the dependence of (19) on CKM parameters, described by the terms in square and curly brackets. The up-to-date value of $|V_{ub}/V_{cb}|$, as provided by the heavy flavour averaging group HFAG in March 2006, is, adding errors in quadrature [32]:

$$\left| \frac{V_{ub}}{V_{cb}} \right| = 0.106 \pm 0.008.$$

In Fig. 5 we plot the CKM factors

$$f_{\text{CKM}} = \frac{R_b^2 - R_b \cos \gamma}{1 - 2R_b \cos \gamma + R_b^2}, \quad g_{\text{CKM}} = \frac{R_b^2}{1 - 2R_b \cos \gamma + R_b^2}$$

as functions of γ ; the uncertainty induced by R_b is small. What is the currently preferred value of γ ? HFAG is yet to provide averages of the individual results obtained by BaBar and Belle, so we use the value quoted by the UTfit collaboration in March 2006 [33]:

$$\gamma_{\text{UTfit}} = (71 \pm 16)^\circ, \quad (20)$$

which is obtained from tree processes only and hence can be assumed to be free of new physics. We then obtain

$$f_{\text{CKM}} = 0.07 \pm 0.12, \quad g_{\text{CKM}} = 0.23 \pm 0.07. \quad (21)$$

As f_{CKM} is rather small for the angle γ_{UTfit} , Eq. (20), the contribution of the corresponding non-factorisable contributions to (19), collected in $\text{Re } \delta a_{0,\pm}$, is heavily suppressed.

The parameters $|a_7^c(\rho\gamma)|$ and $|a_7^c(K^*\gamma)|$ are almost exactly equal, so we set $|a_7^c(\rho\gamma)/a_7^c(K^*\gamma)| = 1$. Is that value likely to be changed by non-factorisable corrections? One type of such corrections, soft-gluon emission from charm loops, has been calculated in Refs. [34, 28]; it amounts to a contribution to $|a_7^c|$ of $\mathcal{O}(1/(m_b m_c^2))$ and is small by itself ($\sim 2\%$), and even smaller is its SU(3)-breaking that changes $|a_7^c(\rho\gamma)/a_7^c(K^*\gamma)|$ by less than 1%. Another source of corrections comes from terms in $(a_7^u(K^*\gamma) - a_7^c(K^*\gamma))/a_7^c(K^*\gamma)$, multiplied by the CKM factor $\sim \lambda^2 f_{\text{CKM}}$, which is tiny indeed. Hence, we do not see any obvious source of significant corrections to $|a_7^c(\rho\gamma)/a_7^c(K^*\gamma)| = 1$.

The value of $\delta a_{0,\pm}$ is calculated in QCD factorisation, which is accurate to $\mathcal{O}(\alpha_s)$, but misses, in general, terms that are suppressed by inverse powers of $m_{b,c}$. The most important of these power-suppressed corrections is weak annihilation, which can actually be calculated in QCD factorisation, at least at tree level. WA is CKM suppressed in $B \rightarrow K^*\gamma$ and mainly affects $a_7^u(\rho^\pm\gamma)$, but it is small for $a_7^u((\rho^0, \omega)\gamma)$ because of a suppression by Wilson

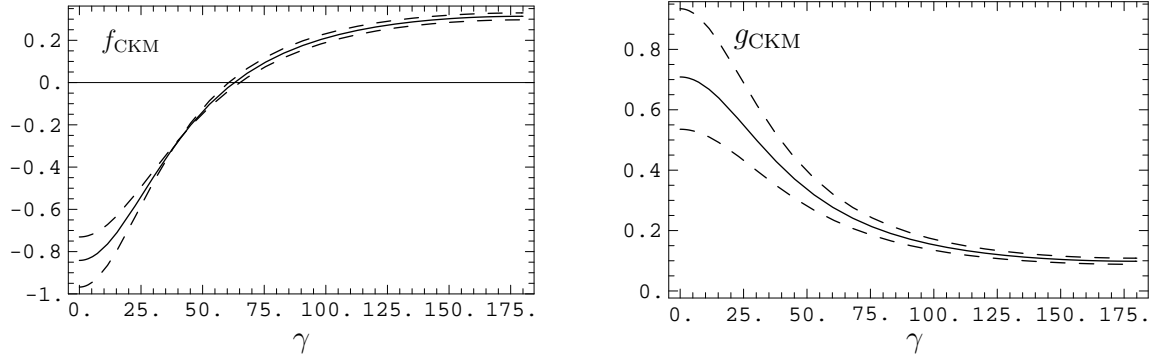


Figure 5: Left panel: the CKM factor f_{CKM} in Eq. (19) (square brackets) as a function of γ for $|V_{ub}/V_{cb}| = 0.106$ (solid line) and $|V_{ub}/V_{cb}| = 0.106 \pm 0.008$ (dashed lines). Right panel: ditto for g_{CKM} (curly brackets).

coefficients and the fact that WA is proportional to the electric charge of the quark involved, namely the u quark for $a_7^u(\rho^\pm\gamma)$ and the d quark for $a_7^d((\rho^0, \omega)\gamma)$. Although WA is formally power-suppressed, it gets enhanced in $a_7^u(\rho^\pm\gamma)$ by large Wilson coefficients and the absence of $\mathcal{O}(\alpha_s)$ suppression that affects other non-factorisable corrections. Numerically, WA is actually as large as the leading (in $1/m_b$) non-factorisable terms. In view of the importance of this contribution, we treat WA in two different ways: firstly, by using the QCD-factorised expression given in Ref. [6]; and secondly, by using the results obtained from QCD sum rules on the light-cone [35, 36, 8].

The WA contribution to the amplitude of e.g. $B^- \rightarrow \rho^- \gamma$ can be written in the following way:

$$A(B^- \rightarrow \rho^- \gamma)_{\text{WA}} = \frac{G_F}{\sqrt{2}} \lambda_u \left(C_1 + \frac{1}{3} C_2 \right) \langle \rho^- \gamma | (\bar{d}u)_{V-A} (\bar{u}b)_{V-A} | B^- \rangle.$$

In naive factorisation, the matrix element on the r.h.s. can be written as

$$\begin{aligned} \langle \rho^- \gamma | (\bar{d}u)_{V-A} (\bar{u}b)_{V-A} | B^- \rangle &= \langle \rho^- | (\bar{d}u)_{V-A} | 0 \rangle \langle \gamma | (\bar{u}b)_{V-A} | B^- \rangle \\ &+ \langle \rho^- \gamma | (\bar{d}u)_{V-A} | 0 \rangle \langle 0 | (\bar{u}b)_{V-A} | B^- \rangle. \end{aligned}$$

The second term on the r.h.s. has been shown to vanish in the chiral limit, see Ref. [37] for more details, so we will focus on the first term. Corrections to naive factorisation are of $\mathcal{O}(\alpha_s)$, which may also relax the chiral suppression of the second term. Neglecting the latter, we have, in the notations of Ref. [36]:²

$$\begin{aligned} \langle \rho^-(p)\gamma(q) | (\bar{d}u)_{V-A} (\bar{u}b)_{V-A} | B^-(p_B) \rangle &= \sqrt{4\pi\alpha} \frac{m_\rho f_\rho^\parallel}{m_B} \epsilon_\mu^{(\rho)} \{ F_V \epsilon^{\mu\nu\rho\sigma} \epsilon_\nu^{(\gamma)} p_{B\rho} q_\sigma \\ &- i F_A [\epsilon^{\mu(\gamma)} (p_B \cdot q) - q^\mu (\epsilon^{(\gamma)} \cdot p_B)] \}. \quad (22) \end{aligned}$$

²Equation (22) differs from the definition given in Ref. [36] by an overall sign. The reason is that in [36] the covariant derivative $D_\mu = \partial_\mu - ieA_\mu$ was used, corresponding to a negative value of e . In order to keep $F_{V,A}$ positive, we change the sign of the definition.

The form factors $F_{A,V}$ can be calculated in QCD factorisation themselves; both $F_{A,V}$ are then equal, and to LO accuracy one has

$$F_{\text{WA}}^{\text{QCDF}} \equiv F_{A,V}^{\text{QCDF}} = \frac{Q_u f_B}{\lambda_B} \quad (23)$$

with $Q_u = 2/3$ the electric charge of the u quark and λ_B the first inverse moment of the B -meson DA ϕ_B :

$$\int_0^1 d\xi \frac{\phi_B(\xi)}{\xi} = \frac{m_B}{\lambda_B}.$$

Equation (23) agrees with the result obtained in Ref. [6] by direct calculation of the WA diagram. Corrections are either of $\mathcal{O}(\alpha_s)$, and have been calculated in Ref. [38], or they are suppressed by powers of $1/m_b$. The dominant source of the latter comes from photon-emission from a soft u quark and has been calculated in Refs. [35, 36], together with the perturbative photon emission giving rise to (23). The emission of photons from a soft quark line is governed by the parameter χ , the so-called magnetic susceptibility of the quark condensate, $\langle 0|\bar{q}\sigma_{\alpha\beta}q|0\rangle_F = \sqrt{4\pi\alpha} Q_q \chi \langle \bar{q}q \rangle F_{\alpha\beta}$, which has been discussed in detail in Refs. [21, 39], together with higher-twist DAs of the photon. Its contribution is, in the heavy quark limit, suppressed by one power of $1/m_b$ with respect to (23), but at finite quark mass its size is set by the dimensionless parameter $\chi \langle \bar{u}u \rangle / f_B \approx 0.2$, with $\chi(1 \text{ GeV}) = (3.15 \pm 0.3) \text{ GeV}^{-2}$ [39], which is not really small. In calculating the WA contribution to δa_{\pm} , we will use both expressions for $F_{V,A}$: $F_{\text{WA}}^{\text{QCDF}}$, Eq. (23), and $F_{\text{WA}}^{\text{QCDSR}}$ from the QCD sum rule calculation, see Ref. [8] for details.

Let us first discuss δa_0 , where WA is suppressed and can be neglected. Its dependence on hadronic parameters is controlled by the factor $f_B/(T_1^{B \rightarrow \rho} \lambda_B)$; it also depends, to a lesser extent, on f_ρ^\perp and the twist-2 DA $\phi_{\rho,\perp}$. To estimate the uncertainty of $\text{Re } \delta a_0$ and $|\delta a_0|^2$, we set $f_B = (0.205 \pm 0.025) \text{ GeV}$, which is an average of quenched and unquenched lattice calculations [40, 41] and QCD sum rule determinations [42]. We also use $T_1^\rho = 0.27 \pm 0.03$ from light-cone sum rules,³ and $\lambda_B(1 \text{ GeV}) = (0.46 \pm 0.11) \text{ GeV}$, obtained in Ref. [43]. This value supersedes the guesstimate $\lambda_B = (0.35 \pm 0.15) \text{ GeV}$ [44] used in previous calculations and agrees with the value $(0.46 \pm 0.16) \text{ GeV}$ found in Ref. [45]. We evaluate all spectator-interaction contributions, that is those involving λ_B , at the scale $\mu_h^2 = m_B^2 - m_b^2$, which is of order $\sim \Lambda_{\text{QCD}} m_b$ as advocated in Ref. [6], but by a factor 2 larger; this is motivated, in part, by the fact that the anomalous dimensions governing the renormalisation-group running of the Wilson coefficients are given for 5 flavours only in Ref. [4] and hence should not be used at scales as small as $(\Lambda_{\text{QCD}} m_b)^{1/2} \sim 1.5 \text{ GeV}$. We then need to evolve λ_B from 1 GeV to μ_h , which can be done using the following evolution relation [46]:

$$\lambda_B^{-1}(\mu) = \lambda_B^{-1}(\mu_0) \left\{ 1 + \frac{\alpha_s}{3\pi} \ln \frac{\mu^2}{\mu_0^2} (1 - 2\sigma_B(\mu_0)) \right\}, \quad (24)$$

where $\sigma_B(1 \text{ GeV}) = 1.4 \pm 0.4$ is given by an integral over the B -meson DA ϕ_B and was estimated in Ref. [43]. We then have

$$\lambda_B(\mu_h) = (0.51 \pm 0.12) \text{ GeV}.$$

³This value, and in particular its error, is quoted from our previous paper in Ref. [13], but is in agreement with the evaluation of Eq. (4).

We can now cast most of the dependence of δa_0 on hadronic input parameters into a dependence on $\lambda_B(\mu_h)$ only, varying it in the interval $\lambda_B = (0.51_{-0.11}^{+0.20})$ GeV. We also allow for 20% power-suppressed corrections to the leading (in $1/m_b$) non-factorisable corrections and find

$$\begin{aligned} \text{Re } \delta a_0 &= 0.06 \pm 0.02(\lambda_B, f_B, T_1) \pm 0.06(\mathcal{O}(1/m_b)), \\ |\delta a_0|^2 &= 0.014 \pm 0.004(\lambda_B, f_B, T_1)_{-0.009}^{+0.017}(\mathcal{O}(1/m_b)). \end{aligned} \quad (25)$$

Let us now turn to δa_{\pm} . Neglecting the effect of WA, one has $\delta a_{\pm} = \delta a_0$. Varying λ_B , f_B and T_1 as before, and allowing for 20% power-suppressed corrections to leading non-factorisable contributions, we find

$$\begin{aligned} \text{Re } \delta a_{\pm}^{\text{QCDF}} &= -0.19 \pm 0.09(\lambda_B, f_B, T_1) \pm 0.06(\mathcal{O}(1/m_b)), \\ |\delta a_{\pm}^{\text{QCDF}}|^2 &= 0.05_{-0.03}^{+0.04}(\lambda_B, f_B, T_1)_{-0.01}^{+0.02}(\mathcal{O}(1/m_b)), \end{aligned} \quad (26)$$

in QCD factorisation and

$$\begin{aligned} \text{Re } \delta a_{\pm}^{\text{QCDSR}} &= -0.06 \pm 0.04(\text{SR}) \pm 0.06(\mathcal{O}(1/m_b)), \\ |\delta a_{\pm}^{\text{QCDSR}}|^2 &= 0.02 \pm 0.01(\text{SR})_{-0.02}^{+0.03}(\mathcal{O}(1/m_b)), \end{aligned} \quad (27)$$

using QCD sum rules for the WA contribution. The SR error reflects the dependence of the result on the QCD sum rule specific parameters M^2 and s_0 and the value of χ .

Taking everything together, we have

$$\begin{aligned} R_{\text{th}}^{\text{QCDF}} &= \left| \frac{V_{td}}{V_{ts}} \right|^2 [0.75 \pm 0.11(\xi) \pm 0.03(a_7^{u,c}, \gamma, R_b)], \\ R_{\text{th}}^{\text{QCDSR}} &= \left| \frac{V_{td}}{V_{ts}} \right|^2 [0.75 \pm 0.11(\xi) \pm 0.02(a_7^{u,c}, \gamma, R_b)]. \end{aligned} \quad (28)$$

This result makes it clear that the theoretical uncertainty associated with δa is small and that the error is dominated by that of ξ — the reduction of which is mostly a matter of more accurate (lattice and QCD sum rule) calculations, but is not affected by uncalculable $1/m_b$ corrections. Within the quoted accuracy, the two different methods to calculate the WA contribution agree. We would like to stress here that it is precisely the CKM suppression of $\delta a_{0,\pm}$ which also suppresses their uncertainties and renders the application of QCD factorisation to $B \rightarrow V\gamma$ viable.

We are now in a position to obtain values for $|V_{td}/V_{ts}|$. Comparing (28) with the experimental results (16), (17) and (18), we get

$$\begin{aligned} \left| \frac{V_{td}}{V_{ts}} \right|_{B \rightarrow V\gamma}^{\text{Belle}} &= 0.207 \pm 0.016(\text{th}) \pm 0.027(\text{exp}), \\ \left| \frac{V_{td}}{V_{ts}} \right|_{B \rightarrow V\gamma}^{\text{BaBar}} &= 0.179 \pm 0.014(\text{th}) \pm 0.020(\text{exp}), \end{aligned}$$

$$\left| \frac{V_{td}}{V_{ts}} \right|_{B \rightarrow V\gamma}^{\text{HFAG}} = 0.192 \pm 0.014(\text{th}) \pm 0.016(\text{exp}). \quad (29)$$

These values can be compared with that following from R_b , γ and the unitarity of the CKM matrix:

$$\left| \frac{V_{td}}{V_{ts}} \right|_{\text{SM}} = \lambda(1 + R_b^2 - 2R_b \cos \gamma)^{1/2} = 0.216 \pm 0.029. \quad (30)$$

Both results agree well within errors. As (30) is obtained from tree-level processes only, it represents the “true” value of $|V_{td}/V_{ts}|$ in the SM.

A third determination of $|V_{td}/V_{ts}|$ can be obtained from B mixing. In the SM, we have

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}} \left| \frac{V_{ts}}{V_{td}} \right|^2. \quad (31)$$

The current world average for Δm_d is $\Delta m_d = (0.507 \pm 0.005) \text{ps}^{-1}$ [22]. Δm_s has recently been measured by the CDF collaboration [47],

$$\Delta m_s = 17.77 \pm 0.10(\text{stat}) \pm 0.07(\text{syst}) \text{ps}^{-1}, \quad (32)$$

with an accuracy that exceeds 5σ significance. D0 provided a two-sided bound at 90% CL [48]:

$$17 \text{ps}^{-1} < \Delta m_s < 21 \text{ps}^{-1}. \quad (33)$$

The hadronic matrix elements in (31) are obtained from lattice simulations. The most up-to-date results for the decay constants have been obtained by the HPQCD group, using unquenched $n_f = (2 + 1)$ configurations [49]:

$$f_{B_s}/f_{B_d} = 1.20(3)(1),$$

where the first error is statistical and from chiral extrapolation and the second comes from “other uncertainties” [49]. The particular strength of this calculation is that light quark masses as small as $m_s/8$ could be reached, which implies that only a moderate extrapolation to the physical chiral limit is required. As for the ratio of $B_{B_{d,s}}$, the currently best result is obtained from unquenched $n_f = 2$ calculations (JLQCD collaboration [50]):

$$B_{B_s}/B_{B_d} = 1.017(16)^{(+56)}_{(-17)},$$

where the first error is statistical and the second systematic. In this calculation, the minimal light quark mass was $m_q = 0.5m_s$, which requires a more substantial extrapolation to the physical limit and is responsible for the large systematic uncertainty. A combination of both results yields [41]:

$$\frac{f_{B_s} B_{B_s}^{1/2}}{f_{B_d} B_{B_d}^{1/2}} = 1.210 \left(\begin{smallmatrix} +47 \\ -35 \end{smallmatrix} \right), \quad (34)$$

where the errors have been added in quadrature. This procedure may be problematic as it combines results with different systematic effects, but yields the most reliable unquenched result to date.⁴ From this, one finds

$$\left| \frac{V_{td}}{V_{ts}} \right|_{\Delta m} = 0.2060^{+0.0081}_{-0.0060}(\text{th}) \pm 0.0007(\text{exp}), \quad (35)$$

which is the result obtained by the CDF collaboration [47]. This value, too, agrees with the two previous determinations. Finally, one can compare our result also to the results of global fits of the unitarity triangle. The UTfit collaboration quotes, in September 2006, [33]

$$\left| \frac{V_{td}}{V_{ts}} \right|_{\text{UTfit}} = 0.202 \pm 0.008,$$

whereas CKMfitter gets [52]

$$\left| \frac{V_{td}}{V_{ts}} \right|_{\text{CKMfitter}} = 0.201^{+0.008}_{-0.007}.$$

Again, all values agree within errors.

4 Summary and Conclusions

In this paper we have presented a new analysis of the form-factor ratio $\xi \equiv T_1^{B \rightarrow K^*} / T_1^{B \rightarrow \rho}$ from QCD sum rules on the light-cone, paying particular attention to the size of SU(3)-breaking effects. We have obtained

$$\xi = 1.17 \pm 0.09;$$

this value is nearly independent of QCD sum-rule-specific parameters and the error is dominated by that of the tensor decay constants f_{ρ, K^*}^\perp . A reduction of these errors by a factor of two would reduce the total uncertainty to ± 0.06 . The numerical values of these constants come mainly from QCD sum rules, partly from quenched lattice calculations. A determination from unquenched lattice calculations with reduced errors would be very desirable indeed.

We then have analysed the non-factorisable corrections to $R \equiv \bar{\mathcal{B}}(B \rightarrow (\rho, \omega)\gamma) / \bar{\mathcal{B}}(B \rightarrow K^*\gamma)$ in the framework of QCD factorisation. The dominant power-suppressed correction comes from weak annihilation diagrams that mostly affect $B^\pm \rightarrow \rho^\pm \gamma$. We have estimated these corrections both in QCD factorisation and using QCD sum rules, and find that the results agree within errors; we will present a more detailed discussion of power-suppressed corrections in a separate publication [8]. Our present best estimate of R_{th} is given in Eq. (28). We then extracted the ratio of CKM matrix elements $|V_{td}/V_{ts}|_{B \rightarrow V\gamma}$ from R_{exp} obtained by BaBar and Belle, respectively, and averaged by HFAG, and find the values

⁴A critical discussion of these lattice results, and their impact on the constraints on new physics from B mixing, can be found in Ref. [51].

given in Eq. (29). Our results for this parameter agree well with all other determinations available from various sources as summarised in the previous section. They also agree with the value extracted from B mixing, using the new measurement of Δm_s reported by the CDF collaboration. Presently, there is no indication for new physics to be inferred from these results.

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Addendum to v3

Please note that in the arXiv version v2 of this paper, which is identical with the published version JHEP 04 (2006) 046, we used the BaBar bound quoted in Ref. [2], $R_{\text{exp}}^{\text{BaBar}} < 0.029$ at 90% CL, which was combined, by HFAG, with the Belle measurement to $R_{\text{exp}}^{\text{HFAG}} = 0.024 \pm 0.006$ and resulted in $|V_{td}/V_{ts}|_{B \rightarrow V\gamma}^{\text{HFAG}} = 0.179 \pm 0.014(\text{th}) \pm 0.022(\text{exp})$. These values have changed with the BaBar measurement of $B(B \rightarrow (\rho, \omega)\gamma)$ reported in Ref. [3]; the corresponding new result for $|V_{td}/V_{ts}|$ is given in (29).

Appendix: NLO Evolution of Twist-2 DAs

To leading-logarithmic accuracy, the (non-perturbative) Gegenbauer moments a_n in Eq. (8) renormalize multiplicatively as

$$a_n^{\text{LO}}(\mu^2) = L^{\gamma_n^{(0)}/(2\beta_0)} a_n(\mu_0^2), \quad (\text{A.1})$$

where $L = \alpha_s(\mu^2)/\alpha_s(\mu_0^2)$, $\beta_0 = (33 - 2N_f)/3$, and the anomalous dimensions $\gamma_n^{(0)}$ are given by

$$\begin{aligned} \gamma_n^{\parallel(0)} &= 8C_F \left(\psi(n+2) + \gamma_E - \frac{3}{4} - \frac{1}{2(n+1)(n+2)} \right), \\ \gamma_n^{\perp(0)} &= 8C_F \left(\psi(n+2) + \gamma_E - \frac{3}{4} \right). \end{aligned}$$

To next-to-leading order accuracy, the scale dependence of the Gegenbauer moments is more complicated and reads [53]

$$a_n^{\text{NLO}}(\mu^2) = a_n(\mu_0^2) E_n^{\text{NLO}} + \frac{\alpha_s(\mu^2)}{4\pi} \sum_{k=0}^{n-2} a_k(\mu_0^2) L^{\gamma_k^{(0)}/(2\beta_0)} d_{nk}^{(1)}, \quad (\text{A.2})$$

where

$$E_n^{\text{NLO}} = L^{\gamma_n^{(0)}/(2\beta_0)} \left\{ 1 + \frac{\gamma_n^{(1)}\beta_0 - \gamma_n^{(0)}\beta_1}{8\pi\beta_0^2} \left[\alpha_s(\mu^2) - \alpha_s(\mu_0^2) \right] \right\}$$

with $L = \alpha_s(\mu)/\alpha_s(\mu_0)$, $\beta_1 = 102 - (38/3)N_f$; $\gamma_n^{(1)}$ are the diagonal two-loop anomalous dimensions, which have been calculated, for the vector current, in Ref. [54], and, for the tensor current, in Ref. [55]. The mixing coefficients $d_{nk}^{(1)}$, $k \leq n - 2$, are given in closed form in Ref. [53]; these formulas are valid for arbitrary currents upon substitution of the corresponding one-loop anomalous dimension.⁵

For the lowest moments $n = 0, 1, 2$ one has, explicitly:

$$\begin{aligned} \gamma_0^{\parallel(1)} &= 0, & \gamma_1^{\parallel(1)} &= \frac{23110}{243} - \frac{512}{81} N_f, & \gamma_2^{\parallel(1)} &= \frac{34072}{243} - \frac{830}{81} N_f, \\ \gamma_0^{\perp(1)} &= \frac{724}{9} - \frac{104}{27} N_f, & \gamma_1^{\perp(1)} &= 124 - 8N_f, & \gamma_2^{\perp(1)} &= \frac{38044}{243} - \frac{904}{81} N_f, \end{aligned} \quad (\text{A.3})$$

and

$$\begin{aligned} d_{20}^{\parallel(1)} &= \frac{35}{9} \frac{20 - 3\beta_0}{50 - 9\beta_0} (1 - L^{50/(9\beta_0)-1}), \\ d_{20}^{\perp(1)} &= \frac{28}{9} \frac{16 - 3\beta_0}{40 - 9\beta_0} (1 - L^{40/(9\beta_0)-1}). \end{aligned} \quad (\text{A.4})$$

References

- [1] K. Abe *et al.* [Belle coll.], Phys. Rev. Lett. **96** (2006) 221601 [arXiv:hep-ex/0506079].
- [2] B. Aubert *et al.* [BaBar coll.], Phys. Rev. Lett. **94** (2005) 011801 [arXiv:hep-ex/0408034].
- [3] B. Aubert *et al.* [BaBar coll.], arXiv:hep-ex/0607099.
- [4] K. G. Chetyrkin, M. Misiak and M. Münz, Phys. Lett. B **400** (1997) 206 [Erratum, *ibid.* B **425** (1998) 414] [arXiv:hep-ph/9612313].
- [5] A. Ali and A. Y. Parkhomenko, Eur. Phys. J. C **23** (2002) 89 [arXiv:hep-ph/0105302];
A. Ali, E. Lunghi and A. Y. Parkhomenko, Phys. Lett. B **595** (2004) 323 [arXiv:hep-ph/0405075];
M. Beneke, T. Feldmann and D. Seidel, Eur. Phys. J. C **41** (2005) 173 [arXiv:hep-ph/0412400];
T. Becher, R. J. Hill and M. Neubert, Phys. Rev. D **72** (2005) 094017 [arXiv:hep-ph/0503263].
- [6] S. W. Bosch and G. Buchalla, Nucl. Phys. B **621** (2002) 459 [arXiv:hep-ph/0106081] and JHEP **0501** (2005) 035 [arXiv:hep-ph/0408231].
- [7] Result from inclusive semileptonic decays as obtained at CKM 05, <http://ckm2005.ucsd.edu/>; see also O. Buchmüller and H. Flächer, Phys. Rev. D **73** (2006) 073008 [arXiv:hep-ph/0507253].

⁵We thank D. Mueller for correspondence on this point.

- [8] P. Ball and R. Zwicky, in preparation.
- [9] See P. Colangelo and A. Khodjamirian, arXiv:hep-ph/0010175, and references therein.
- [10] P. Ball *et al.*, Nucl. Phys. B **529** (1998) 323 [arXiv:hep-ph/9802299].
- [11] P. Ball and V. M. Braun, Nucl. Phys. B **543** (1999) 201 [arXiv:hep-ph/9810475].
- [12] P. Ball, V. M. Braun, G. W. Jones and A. Lenz, preprint IPPP/06/62 (in preparation).
- [13] P. Ball and R. Zwicky, Phys. Rev. D **71** (2005) 014029 [arXiv:hep-ph/0412079].
- [14] P. Ball and V. M. Braun, Phys. Rev. D **58** (1998) 094016 [arXiv:hep-ph/9805422].
- [15] E. Bagan, P. Ball and V. M. Braun, Phys. Lett. B **417** (1998) 154 [arXiv:hep-ph/9709243].
- [16] P. Ball, Phys. Lett. B **641** (2006) 50 [arXiv:hep-ph/0608116].
- [17] V. M. Braun, E. Gardi and S. Gottwald, Nucl. Phys. B **685** (2004) 171 [arXiv:hep-ph/0401158].
- [18] P. Ball and R. Zwicky, Phys. Lett. B **633** (2006) 289 [arXiv:hep-ph/0510338] and JHEP **0206** (2006) 034 [arXiv:hep-ph/0601086].
- [19] P. Ball and A. N. Talbot, JHEP **0506** (2005) 063 [arXiv:hep-ph/0502115].
- [20] A. R. Zhitnitsky, I. R. Zhitnitsky and V. L. Chernyak, Sov. J. Nucl. Phys. **41** (1985) 127 [Yad. Fiz. **41** (1985) 199].
- [21] I. I. Balitsky, V. M. Braun and A. V. Kolesnichenko, Nucl. Phys. B **312** (1989) 509.
- [22] W. M. N. Yao *et al.* [Particle Data Group], J. Phys. G **33**, 1 (2006).
- [23] D. Becirevic *et al.*, JHEP **0305** (2003) 007 [arXiv:hep-lat/0301020].
- [24] V. M. Braun *et al.*, Phys. Rev. D **68** (2003) 054501 [arXiv:hep-lat/0306006].
- [25] P. Ball and M. Boggione, Phys. Rev. D **68** (2003) 094006 [arXiv:hep-ph/0307337].
- [26] M. Göckeler *et al.*, PoS **LAT2005** (2005) 063 [arXiv:hep-lat/0509196].
- [27] P. Ball and V. M. Braun, Phys. Rev. D **54** (1996) 2182 [arXiv:hep-ph/9602323].
- [28] P. Ball and R. Zwicky, arXiv:hep-ph/0609037.
- [29] P. Ball, V. M. Braun and A. Lenz, JHEP **0605** (2006) 004 [arXiv:hep-ph/0603063].
- [30] F. Knechtli, Acta Phys. Polon. B **36** (2005) 3377 [arXiv:hep-ph/0511033];
M. Göckeler *et al.*, Nucl. Phys. Proc. Suppl. **153** (2006) 154 [arXiv:hep-lat/0602028].

- [31] E. Gamiz *et al.*, Phys. Rev. Lett. **94** (2005) 011803 [arXiv:hep-ph/0408044];
S. Narison, arXiv:hep-ph/0510108;
K. G. Chetyrkin and A. Khodjamirian, arXiv:hep-ph/0512295.
- [32] E. Barberio *et al.* [HFAG], arXiv:hep-ex/0603003; updated results available at
<http://www.slac.stanford.edu/xorg/hfag/>.
- [33] M. Bona *et al.* [UTfit coll.], JHEP **0507** (2005) 028 [arXiv:hep-ph/0501199]; updated
results available at <http://www.utfit.org/>.
- [34] A. Khodjamirian *et al.*, Phys. Lett. B **402** (1997) 167 [arXiv:hep-ph/9702318].
- [35] A. Khodjamirian, G. Stoll and D. Wyler, Phys. Lett. B **358** (1995) 129 [arXiv:hep-
ph/9506242];
A. Ali and V. M. Braun, Phys. Lett. B **359** (1995) 223 [arXiv:hep-ph/9506248].
- [36] P. Ball and E. Kou, JHEP **0304** (2003) 029 [arXiv:hep-ph/0301135].
- [37] B. Grinstein and D. Pirjol, Phys. Rev. D **62** (2000) 093002 [arXiv:hep-ph/0002216];
A. Khodjamirian and D. Wyler, arXiv:hep-ph/0111249.
- [38] S. Descotes-Genon and C. T. Sachrajda, Nucl. Phys. B **650** (2003) 356 [arXiv:hep-
ph/0209216];
E. Lunghi, D. Pirjol and D. Wyler, Nucl. Phys. B **649** (2003) 349 [arXiv:hep-
ph/0210091].
- [39] P. Ball, V. M. Braun and N. Kivel, Nucl. Phys. B **649** (2003) 263 [arXiv:hep-
ph/0207307].
- [40] S. Hashimoto, Int. J. Mod. Phys. A **20** (2005) 5133 [arXiv:hep-ph/0411126].
- [41] M. Okamoto, PoS **LAT2005** (2005) 013 [arXiv:hep-lat/0510113].
- [42] A. A. Penin and M. Steinhauser, Phys. Rev. D **65** (2002) 054006 [arXiv:hep-
ph/0108110];
M. Jamin and B. O. Lange, Phys. Rev. D **65** (2002) 056005 [arXiv:hep-ph/0108135].
- [43] V. M. Braun, D. Y. Ivanov and G. P. Korchemsky, Phys. Rev. D **69** (2004) 034014
[arXiv:hep-ph/0309330].
- [44] M. Beneke *et al.*, Phys. Rev. Lett. **83** (1999) 1914 [arXiv:hep-ph/9905312].
- [45] A. Khodjamirian, T. Mannel and N. Offen, Phys. Lett. B **620** (2005) 52 [arXiv:hep-
ph/0504091].
- [46] B. O. Lange and M. Neubert, Phys. Rev. Lett. **91** (2003) 102001 [arXiv:hep-
ph/0303082];
P. Ball, arXiv:hep-ph/0308249.

- [47] A. Abulencia *et al.* [CDF Coll.], Phys. Rev. Lett. **97** (2006) 062003 [arXiv:hep-ex/0606027]; arXiv:hep-ex/0609040.
- [48] V. Abazov *et al.* [D0 coll.], Phys. Rev. Lett. **97** (2006) 021802 [arXiv:hep-ex/0603029].
- [49] A. Gray *et al.* [HPQCD coll.], Phys. Rev. Lett. **95** (2005) 212001 [arXiv:hep-lat/0507015].
- [50] S. Aoki *et al.* [JLQCD coll.], Phys. Rev. Lett. **91** (2003) 212001 [arXiv:hep-ph/0307039].
- [51] P. Ball and R. Fleischer, arXiv:hep-ph/0604249.
- [52] J. Charles *et al.* [CKMfitter group], Eur. Phys. J. C **41** (2005) 1 [arXiv:hep-ph/0406184]; updated results and plots available at <http://ckmfitter.in2p3.fr>.
- [53] D. Mueller, Phys. Rev. D **49** (1994) 2525 and D **51** (1995) 3855.
- [54] E. G. Floratos, D. A. Ross and C. T. Sachrajda, Nucl. Phys. B **129** (1977) 66 [Erratum, *ibid.* B **139** (1978) 545];
A. Gonzalez-Arroyo, C. Lopez and F. J. Yndurain, Nucl. Phys. B **153** (1979) 161.
- [55] W. Vogelsang, Phys. Rev. D **57** (1998) 1886 [arXiv:hep-ph/9706511];
A. Hayashigaki, Y. Kanazawa and Y. Koike, Phys. Rev. D **56** (1997) 7350 [arXiv:hep-ph/9707208].