Published by Institute of Physics Publishing for SISSA

Received: *March 9, 2006*
Accepted: *April 19, 2006*
Published: *May 3, 2006*

Higher-twist distribution amplitudes of the K meson in QCD

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Abstract: We present a systematic study of twist-3 and twist-4 light-cone distribution amplitudes of the K meson in QCD. The structure of SU(3)-breaking corrections is studied in detail. Non-perturbative input parameters are estimated from QCD sum rules and renormalons. As a by-product, we give a complete reanalysis of the twist-3 and -4 parameters of the π-meson distribution amplitudes; some of the results differ from those usually quoted in the literature.

Keywords: Kaon Physics, Sum Rules, QCD, Phenomenological Models.

JHEP05(2006)004

Contents

1. Introduction

The discovery of an approximate SU(3) flavour symmetry of strong interactions [1, 2] predates the quark model and has been of paramount importance in all subsequent developments. This symmetry has its origin in the smallness of up, down and strange quark masses with respect to the QCD scale $\Lambda_{\rm QCD}$. In static hadron properties, such as masses, magnetic moments, decay constants, etc., it is accurate to about 1 to 3% for quantities related by isospin and to about 20% for those related by U- and V-spin. The breaking of SU(3) in dynamical observables can be larger and up to now is not fully understood. One particularly striking example is the weak radiative decay $\Sigma \to p\gamma$: the experimental azimuthal asymmetry in this decay is $\alpha_\gamma = -0.76 \pm 0.08$ [3], which according to Hara's theorem [4] implies 100% SU(3) symmetry violation.

In recent years SU(3)-symmetry-breaking effects in heavy-meson decays have attracted increasing interest. These processes can be treated in heavy-quark expansion, which has proved a very powerful theoretical tool, so that in some cases, for instance weak radiative decays, $B \to \rho\gamma$ vs. $B \to K^*\gamma$, the uncertainty in SU(3) breaking is actually the dominant source of theoretical error. The particular challenge of such processes is that, in the presence of a hard scale, hard exclusive reactions are dominated by rare configurations of the hadrons' constituents: either only valence-quark configurations contribute and all quarks

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have small transverse separation (hard mechanism), or one of the partons carries most of the hadron momentum (soft or Feynman mechanism) [5]. The size of SU(3)-breaking effects in such rare configurations cannot be deduced from the symmetry breaking in static properties, where the bulk of the wave-function contributes.

Hard contributions are simpler to treat than their soft counterparts and their structure is well understood, see ref. [6]. They can be calculated in terms of the hadron distribution amplitudes (DAs) which describe the momentum-fraction distribution of partons at zero transverse separation in a particular Fock state, with a fixed number of constituents. DAs are ordered by increasing twist; the leading-twist-2 meson DA $\phi_{2;P}$, which describes the momentum distribution of the valence quarks in the meson P, is related to the meson's Bethe-Salpeter wave function $\phi_{P,BS}$ by an integral over transverse momenta:

$$\phi_{2;P}(u,\mu) = Z_2(\mu) \int^{|k_\perp|<\mu} d^2k_\perp \, \phi_{P,BS}(u,k_\perp).$$

Here u is the quark momentum fraction, Z_2 is the renormalization factor (in the light-cone gauge) for the quark-field operators in the wave function, and μ denotes the renormalization scale. The study of the leading-twist DA of the pion has attracted much attention in the literature, whereas the status of SU(3)-breaking effects that are responsible for the difference between the kaon and the pion DAs has been controversial for a while [7]. These corrections have been recently reconsidered in the framework of QCD sum rules in refs. [8 – 11], with a consistent picture finally emerging. We will give a short review of these developments below.

Higher-twist DAs are much more numerous and describe either contributions of "bad" components in the wave function, or contributions of transverse motion of quarks (antiquarks) in the leading-twist components, or contributions of higher Fock states with additional gluons and/or quark-antiquark pairs. Within the hard-rescattering picture, the corresponding contributions to the hard exclusive reactions are suppressed by a power (or powers) of the large momentum Q, which explains why they have received less attention.

In turn, soft contributions are intrinsically non-perturbative and cannot be further reduced (or factorized) in terms of simpler quantities without additional assumptions. At present, they can only be estimated using light-cone sum rules [12 – 14], see refs. [15, 16] for sample applications to heavy quark decays. In this technique soft contributions are extracted from the dispersion relations for suitable correlation functions, by introducing an auxiliary "semi-hard" scale (Borel parameter) at which the two different representations of the correlation function, in terms of quarks and in terms of hadronic states, are matched. In calculations of this kind, the necessary non-perturbative input again reduces to DAs, and the higher-twist DAs play a very important role, since they are only suppressed by powers of the Borel parameter and not by powers of the hard scale Q. The crucial point and main technical difficulty in the construction of higher-twist DAs is the necessity to satisfy the exact equations of motion (EOM), which yield relations between physical effects of different origins: for example, using EOM, the contributions of orbital angular momentum in the valence component of the wave function can be expressed (for mesons) in terms of contributions of higher Fock states. An appropriate framework for implementing these

constraints was developed in ref. [17]: it is based on the derivation of EOM relations for non-local light-ray operators [18], which are solved order by order in the conformal expansion; see ref. [19] for a review and further references. In this way it is possible to construct self-consistent approximations for the DAs, which involve a minimum number of hadronic parameters. Another approach, based on the study of renormalons, was suggested for twist 4 in refs. [20, 21]: this technique is appealing as it allows one to obtain an estimate of high-order contributions to the conformal expansion which are usually omitted. In this paper, we generalize this approach to include SU(3)-breaking corrections and show how to combine renormalon-based estimates of "genuine" twist-4 effects with meson mass corrections.

Pion DAs of twist 3 and 4 have already been studied in ref. [17]. In ref. [22], these results were extended to the pseudoscalar octet; they include those meson-mass corrections that break chiral symmetry, while still preserving G-parity. SU(3)-breaking in the normalization of the twist-4 DA was estimated in ref. [23]. In this paper we continue the analysis of twist-3 and 4 DAs of the K meson and present, for the first time, the complete set of DAs, including also G-parity-breaking terms that vanish in the limit of quarks with equal mass. The results are of direct relevance to the discussion of, for instance, B-meson decays into light mesons using light-cone sum rules and also in the SCET framework. We refrain from an analysis of the η DAs, as the inclusion of the singlet part is crucial for phenomenological applications, but goes beyond the scope of this paper, and in fact deserves a separate study.

Our paper is organized as follows: in section 2 we explain notation and review the existing information on leading-twist DAs. Section 3 is devoted to twist-3 DAs: we give their general classification, calculate meson-mass corrections, perform a conformal expansion and formulate models in terms of a few non-perturbative parameters. A similar programme is carried out for twist-4 DAs in section 4. In section 5 we present numerical results for all DAs and conclude in section 6 with a short summary and outlook. The appendices contain a collection of relevant formulas, in particular the QCD sum rules for the relevant twist-2, -3 and -4 matrix elements.

2. General framework and twist-2 DAs

Light-cone meson DAs are defined in terms of matrix elements of non-local light-ray operators stretched along a certain light-like direction z_μ, $z^2 = 0$, and sandwiched between the vacuum and the meson state. We adopt the generic notation

$$\phi_{t;M}(u),\ \psi_{t;M}(u),\ \dots \tag{2.1}$$

and

$$\Phi_{t;M}(\underline{\alpha}),\ \Psi_{t;M}(\underline{\alpha}),\ \dots \tag{2.2}$$

for two-particle and three-particle DAs, respectively. The first subscript $t = 2, 3, 4$ stands for the twist; the second one, $M = \pi, K, \dots$, specifies the meson. For definiteness, we will write most of the expressions for K mesons, i.e. $s\bar{q}$ bound states with $q = u, d$. The variable u in the definition of two-particle DAs always refers to the momentum fraction carried by the quark, $u = u_s$; $\bar{u} \equiv 1 - u = u_{\bar{q}}$ is the antiquark momentum fraction. The

set of variables in the three-particle DAs, $\underline{\alpha} = \{\alpha_1, \alpha_2, \alpha_3\} = \{\alpha_s, \alpha_{\bar{q}}, \alpha_g\}$, corresponds to the momentum fractions carried by the quark, antiquark and gluon, respectively.

To facilitate the light-cone expansion, it is convenient to introduce a second light-like vector p_μ such that

$$p_\mu = P_\mu - \frac{1}{2} z_\mu \frac{m_M^2}{pz}, \tag{2.3}$$

where P_μ is the meson momentum, $P^2 = m_M^2$. We also need the projector onto the directions orthogonal to p and z,

$$g_{\mu\nu}^\perp = g_{\mu\nu} - \frac{1}{pz}(p_\mu z_\nu + p_\nu z_\mu), \tag{2.4}$$

and use the notation

$$a_z \equiv a_\mu z^\mu, \qquad a_p \equiv a_\mu p^\mu, \qquad b_{\mu z} \equiv b_{\mu\nu} z^\nu, \qquad \text{etc.} \tag{2.5}$$

for arbitrary Lorentz vectors a_μ and tensors $b_{\mu\nu}$. $a_\perp$ denotes the generic component of a_μ orthogonal to z and p.

We use the standard Bjorken-Drell convention [24] for the metric and the Dirac matrices; in particular, $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, and the Levi-Civita tensor $\epsilon_{\mu\nu\lambda\sigma}$ is defined as the totally antisymmetric tensor with $\epsilon_{0123} = 1$. The covariant derivative is defined as $D_\mu = \partial_\mu - igA_\mu$ and the dual gluon-field-strength tensor as $\widetilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}$.

The leading twist-2 DA of the K meson is defined as[1]

$$\langle 0|\bar{q}(z)[z,-z]\gamma_z\gamma_5 s(-z)|K(P)\rangle = if_K(pz)\int_0^1 du\, e^{i\xi pz}\,\phi_{2;K}(u,\mu^2)\,. \tag{2.6}$$

Here $[x,y]$ stands for the path-ordered gauge factor along the straight line connecting the points x and y:

$$[x,y] = \mathrm{P}\exp\left[ig\int_0^1 dt\,(x-y)_\mu A_\mu(tx+(1-t)y)\right], \tag{2.7}$$

and μ is the renormalization (factorization) scale. We also use the short-hand notation

$$\xi = 2u - 1. \tag{2.8}$$

The decay constant f_K is defined, as usual, as

$$\langle 0|\bar{q}(0)\gamma_\mu\gamma_5 s(0)|K(P)\rangle = if_K P_\mu, \tag{2.9}$$

with $f_K = 160\,\mathrm{MeV}$ [3]. The normalization follows from the requirement that the local limit $z \to 0$ of (2.6) reproduce (2.9), so that

$$\int_0^1 du\,\phi_{2;K}(u) = 1\,. \tag{2.10}$$

[1]The leading-twist DA of a $\bar{K}$ meson is given by $\phi_{2;\bar{K}}(u) = \phi_{2;K}(1-u)$.

A convenient tool to study DAs is provided by conformal expansion [19].[2] The underlying idea is similar to partial-wave decomposition in quantum mechanics and allows one to separate transverse and longitudinal variables in the Bethe-Salpeter wave-function. The dependence on transverse coordinates is formulated as scale dependence of the relevant operators and is governed by renormalization-group equations, the dependence on the longitudinal momentum fractions is described in terms of irreducible representations of the corresponding symmetry group, the collinear conformal group SL(2,$\mathbb{R}$). The conformal partial-wave expansion is explicitly consistent with the equations of motion since the latter are not renormalized. It thus makes maximum use of the symmetry of the theory to simplify the dynamics.

To construct the conformal expansion for an arbitrary multiparticle distribution, one first has to decompose each constituent field into components with fixed Lorentz-spin projection onto the light-cone. Each such component has conformal spin

$$j = \frac{1}{2}\,(l+s),$$

where l is the canonical dimension and s the (Lorentz-) spin projection. In particular, $l = 3/2$ for quarks and $l = 2$ for gluons. The quark field is decomposed as $\psi_+ \equiv \Lambda_+\psi$ and $\psi_- = \Lambda_-\psi$ with spin projection operators $\Lambda_+ = \not{p}\not{z}/(2pz)$ and $\Lambda_- = \not{z}\not{p}/(2pz)$, corresponding to $s = +1/2$ and $s = -1/2$, respectively. For the gluon field strength there are three possibilities: $G_{z\perp}$ corresponds to $s = +1$, $G_{p\perp}$ to $s = -1$, and both $G_{\perp\perp}$ and G_{zp} correspond to $s = 0$. Multiparticle states built of fields with definite Lorentz-spin projection can be expanded in irreducible representations of SL(2,$\mathbb{R}$) with increasing conformal spin. The explicit expression for the DA of an m-particle state with the lowest possible conformal spin $j = j_1 + \cdots + j_m$, the so-called asymptotic DA, is [17]

$$\phi_{as}(\alpha_1,\alpha_2,\ldots,\alpha_m) = \frac{\Gamma(2j_1+\cdots+2j_m)}{\Gamma(2j_1)\cdots\Gamma(2j_m)}\,\alpha_1^{2j_1-1}\alpha_2^{2j_2-1}\ldots\alpha_m^{2j_m-1}. \tag{2.11}$$

Multiparticle irreducible representations with higher spin $j+n, n = 1,2,\ldots$, are given by polynomials of m variables (with the constraint $\sum_{k=1}^m \alpha_k = 1$), which are orthogonal over the weight function (2.11).

In particular, for the leading-twist DA $\phi_{K;2}$ defined in (2.6), the expansion goes in Gegenbauer polynomials:

$$\phi_{K;2}(u,\mu^2) = 6u(1-u)\left(1+\sum_{n=1}^{\infty} a_n^K(\mu^2)C_n^{3/2}(2u-1)\right). \tag{2.12}$$

To leading-logarithmic accuracy (LO), the (non-perturbative) Gegenbauer moments a_n renormalize multiplicatively with

$$a_n^{\rm LO}(\mu^2) = L^{\gamma_n^{(0)}/(2\beta_0)}\,a_n(\mu_0^2), \tag{2.13}$$

[2] See ref. [25] for an alternative approach not based on conformal expansion.

where $L \equiv \alpha_s(\mu^2)/\alpha_s(\mu_0^2)$, $\beta_0 = (11N_c - 2N_f)/3$, and the anomalous dimensions $\gamma_n^{(0)}$ are given by

$$\gamma_n^{(0)} = 8C_F \left(\psi(n+2) + \gamma_E - \frac{3}{4} - \frac{1}{2(n+1)(n+2)} \right). \tag{2.14}$$

The reason why leading-order renormalization respects the (anomalous) conformal symmetry is that it is driven by tree-level counterterms that retain the symmetry properties of the Lagrangian. More technically, the Callan-Symanzik equation that governs the renormalization-scale dependence coincides to this accuracy with the Ward identity for the dilatation operator, which is an element of the collinear conformal group [19].

To next-to-leading order (NLO) accuracy, the scale dependence of the Gegenbauer moments is more complicated and reads [26, 27]

$$a_n^{\rm NLO}(\mu^2) = a_n(\mu_0^2) E_n^{\rm NLO} + \frac{\alpha_s(\mu^2)}{4\pi} \sum_{k=0}^{n-2} a_n(\mu_0^2)\, E_k^{\rm NLO}\, d_{nk}^{(1)}, \tag{2.15}$$

where

$$E_n^{\rm NLO} = L^{\gamma_n^{(0)}/(2\beta_0)} \left[1 + \frac{\gamma_n^{(1)}\beta_0 - \gamma_n^{(0)}\beta_1}{8\pi\beta_0^2} \Big[\alpha_s(\mu^2) - \alpha_s(\mu_0^2) \Big] \right], \tag{2.16}$$

$\gamma_n^{(1)}$ are the diagonal two-loop anomalous dimensions [28], $\beta_1 = 102 - (38/3)N_f$, and the mixing coefficients $d_{nk}^{(1)}$, $k \le n-2$, are given in closed form in ref. [27], see also, for instance, ref. [29] for a recent compilation. For the lowest moments $n = 0, 1, 2$ one needs

$$\gamma_0^{(1)} = 0\,, \qquad \gamma_1^{(1)} = \frac{23096}{243} - \frac{512}{81}\, N_f\,, \qquad \gamma_2^{(1)} = \frac{34450}{243} - \frac{830}{81}\, N_f \tag{2.17}$$

and

$$d_{20}^{(1)} = \frac{7}{30}(5C_F - \beta_0) \frac{\gamma_2^{(0)}}{\gamma_2^{(0)} - 2\beta_0} \left[1 - L^{-1+\gamma_2^{(0)}/(2\beta_0)} \right]. \tag{2.18}$$

The odd Gegenbauer moments a_{2n+1} are first order in SU(3)-symmetry breaking for the kaon and vanish for the pion by virtue of G-parity. The numerical value of a_1^K was the subject of significant controversy until recently. The existing estimates are all obtained using QCD sum rules. The first calculation of a_1^K by Chernyak and Zhitnitsky yielded $a_1^K \approx 0.1$ [30, 31], but unfortunately suffers from a sign mistake in the perturbative contribution [7]. After the error is corrected, one finds that the two numerically leading contributions come with different sign and cancel to a large extent, so that the sum rule becomes unstable and numerically unreliable. This problem was reanalysed in refs. [8 – 11] using a different set of sum rules, where it was also checked that the results are consistent with the equations of motion for the relevant operators [9, 11]. The results are given in table 1.

As our best estimate, we take

$$a_1^K(1\,\text{GeV}) = 0.06 \pm 0.03. \tag{2.19}$$

Calculations of the second Gegenbauer moment for the pion DA, a_2^π, have attracted quite a bit of attention and have a long history. Three different approaches have been used: direct calculations using QCD sum rules, pioneered by Chernyak and Zhitnitsky; analysis

Method	$\mu = 1\,\text{GeV}$	$\mu = 2\,\text{GeV}$	Reference
QCDSR,D	0.05 ± 0.02	0.04 ± 0.02	[8]
QCDSR,ND;EOM	0.10 ± 0.12	0.08 ± 0.09	[9]
QCDSR,D;EOM	0.06 ± 0.03	0.05 ± 0.02	[10, 11]

Table 1: The Gegenbauer moment $a_1^K(\mu^2)$ from QCD sum rules. The abbreviations stand for: QCDSR: QCD sum rules; D and ND: diagonal and non-diagonal correlation function, respectively; EOM: equations of motion. The error estimates should be taken with some caution, as there is no systematic approach to estimate uncalculated higher-order terms in the OPE.

Method	$\mu = 1\,\text{GeV}$	$\mu = 2\,\text{GeV}$	Reference
CZ model	0.56	0.38	[32, 31]
QCDSR	$0.26^{+0.21}_{-0.09}$	$0.17^{+0.14}_{-0.06}$	[8]
QCDSR	0.28 ± 0.08	0.19 ± 0.05	this paper
QCDSR,NLC	0.19 ± 0.06	0.13 ± 0.04	[33 – 35]
$F_{\pi\gamma\gamma^*}$,LCSR	0.19 ± 0.05	$0.12 \pm 0.03\,(\mu = 2.4)$	[36]
$F_{\pi\gamma\gamma^*}$,LCSR	0.32	$0.20\,(\mu = 2.4)$	[37]
$F_{\pi\gamma\gamma^*}$,LCSR,R	0.44	0.30	[38]
$F_{\pi\gamma\gamma^*}$,LCSR,R	0.27	0.18	[39]
F_π^{em},LCSR	$0.24 \pm 0.14 \pm 0.08$	$0.16 \pm 0.09 \pm 0.05$	[40, 41]
F_π^{em},LCSR,R	0.20 ± 0.03	0.13 ± 0.02	[42]
$F_{B\to\pi\ell\nu}$,LCSR	0.19 ± 0.19	0.13 ± 0.13	[16]
LQCD, quenched, W/CW	$0.381 \pm 0.234^{+0.114}_{-0.062}$	$0.233 \pm 0.143^{+0.088}_{-0.038}$ $(\mu = 2.67)$	UKQCD [43]
LQCD, $N_f = 2$, W/CW	0.364 ± 0.126	$0.236 \pm 0.082\,(\mu^2 = 5)$	QCDSF/UKQCD [44]

Table 2: The Gegenbauer moment $a_2^\pi(\mu^2)$. The CZ model involves $a_2^\pi = 2/3$ at the low scale $\mu = 500\,\text{MeV}$; for the discussion of the extrapolation to higher scales, see ref. [37]. The abbreviations stand for: QCDSR: QCD sum rules; NLC: non-local condensates; LCSR: light-cone sum rules; R: renormalon model for twist-4 corrections; LQCD: lattice calculation; $N_f = 2$: calculation using $N_f = 2$ dynamical quarks; W/WC: Wilson glue and non-perturbatively $\mathcal{O}(a)$ improved Clover-Wilson fermion action.

of the experimental data on the pion electromagnetic and transition form factors and the B weak decay form factor, using light-cone sum rules; and lattice calculations. The summary of these results is presented in table 2; see also, for instance, refs. [37, 29] for another recent compilation.

Our conclusion from table 2 is rather pessimistic: a_2^π can only be determined with large errors, whatever approach is chosen. A fair quote is probably

$$a_2^\pi(1\,\text{GeV}) = 0.25 \pm 0.15\,. \tag{2.20}$$

The K-meson DA has attracted comparatively less attention. The old estimate by Chernyak and Zhitnitsky, $\langle\xi^2\rangle_K/\langle\xi^2\rangle_\pi = 0.8 \pm 0.02$ [31], translates to

$$a_2^K/a_2^\pi = 0.59 \pm 0.04 \quad \leftrightarrow \quad (a_2^K)_{\text{CZ}}(1\,\text{GeV}) = 0.33 \tag{2.21}$$

for the CZ model. A recent calculation, ref. [8], including radiative corrections to the sum rules gives, however

$$a_2^K/a_2^\pi \simeq 1\,, \qquad a_2^K(1\,\text{GeV}) = 0.27^{+0.37}_{-0.12}. \tag{2.22}$$

This result is consistent with the most recent lattice calculation, using $N_f = 2$ dynamical fermions [44], which shows that $\langle\xi^2\rangle_\pi$ stays practically constant under a variation of the pion mass. For the purpose of the present paper we have done an update of the QCD sum-rule calculation [8], using the corrected $\mathcal{O}(\alpha_s)$ quark-condensate contribution given in ref. [10], see appendix B, which yields

$$a_2^K/a_2^\pi = 1.05 \pm 0.15\,, \qquad a_2^K(1\,\text{GeV}) = 0.30 \pm 0.15\,. \tag{2.23}$$

The difference with [8] is small and mainly due to the larger value of the strong coupling that we are using in this work. We conclude that the existing evidence points towards a very small SU(3) violation in the second coefficient in the Gegenbauer expansion, so that we accept $a_2^K = a_2^\pi$ in the range given in eq. (2.20) as our final estimate.

Estimates of yet higher-order Gegenbauer coefficients are rather uncertain. The light-cone sum-rule calculations of the transition form factor $F_{\pi\gamma\gamma^*}$ in refs. [36 – 39] suggest a negative value for a_4^π, which is consistent with the result $a_4^\pi(1\,\text{GeV}) > -0.07$ obtained in ref. [16]. However, this conclusion may be premature because of the omission of yet higher moments and absence of any convincing method to estimate systematic errors involved in the analysis. For this reason we adopt, in this paper, a model for the leading-twist DA, which is given by the Gegenbauer expansion (2.12) truncated after the second term.

Last but not least we have to specify the value of the strange-quark mass. In the last year several lattice calculations with dynamical fermions have been published; see refs. [45, 46] for a summary and short review. In all these calculations the physical kaon mass is used as an input to fix the strange-quark mass. There is good agreement between data sets obtained using different non-perturbative renormalization procedures and, in fact, also with earlier quenched calculations. On the other hand, the data still show considerable dependence on the lattice spacing, so that it is clear that simulations on finer lattices are needed for a systematic continuum extrapolation. In a different approach, the strange-quark mass can be extracted from the e^+e^- annihilation cross section and/or hadronic τ-decay data using QCD sum rules. These calculations have reached a certain degree of maturity and yield results that are in reasonable agreement with lattice determinations; see ref. [47] for a recent summary and further references. In this paper we use

$$\overline{m}_s(2\,\text{GeV}) = (100 \pm 20)\,\text{MeV}, \tag{2.24}$$

which corresponds to $\overline{m}_s = (137 \pm 27)\,\text{MeV}$ at 1 GeV.

3. Twist-3 distributions

In this section we define all the twist-3 DAs of the kaon and derive models for them to next-to-leading order in conformal expansion, which fulfil the QCD equations of motion. We also work out the leading-order scale-dependence of the corresponding hadronic parameters. Numerical values for the parameters and the corresponding models are given in section 5.

To twist-3 accuracy, there are two two-particle DAs defined as

$$\langle 0|\bar{q}(z)i\gamma_5 s(-z)|K(P)\rangle = \frac{f_K m_K^2}{m_s+m_q}\int_0^1 du\, e^{i(2u-1)pz}\,\phi_{3;K}^p(u), \tag{3.1}$$

$$\langle 0|\bar{q}(z)\sigma_{\alpha\beta}\gamma_5 s(-z)|K(P)\rangle = -\frac{i}{3}\frac{f_K m_K^2}{m_s+m_q}(P_\alpha z_\beta - P_\beta z_\alpha)\int_0^1 du\, e^{i(2u-1)pz}\,\phi_{3;K}^\sigma(u). \tag{3.2}$$

In addition, there is also one three-particle DA:

$$\begin{aligned}\langle 0|\bar{q}(z)\sigma_{\mu\nu}\gamma_5 gG_{\alpha\beta}(vz)s(-z)|K(P)\rangle &= \\ = i\,f_{3K}\left(p_\alpha p_\mu g_{\nu\beta}^\perp - p_\alpha p_\nu g_{\mu\beta}^\perp - (\alpha\leftrightarrow\beta)\right)&\int \mathcal{D}\underline{\alpha}\, e^{-ipz(\alpha_2-\alpha_1+v\alpha_3)}\Phi_{3;K}(\alpha_1,\alpha_2,\alpha_3)+\cdots,\end{aligned} \tag{3.3}$$

where the integration measure is defined as

$$\int \mathcal{D}\underline{\alpha} = \int_0^1 d\alpha_1\, d\alpha_2\, d\alpha_3\,\delta(1-\alpha_1-\alpha_2-\alpha_3) \tag{3.4}$$

and the dots stand for Lorentz structures of twist 5 and higher.

To next-to-leading order in conformal spin, $\Phi_{3;K}$ is given by

$$\Phi_{3;K}(\underline{\alpha}) = 360\alpha_1\alpha_2\alpha_3^2\left\{1+\lambda_{3K}(\alpha_1-\alpha_2)+\omega_{3K}\,\frac{1}{2}\,(7\alpha_3-3)\right\}. \tag{3.5}$$

The three parameters f_{3K}, λ_{3K}, and ω_{3K} can be defined in terms of matrix elements of local twist-3 operators as follows:

$$\begin{aligned}\langle 0|\bar{q}\sigma_{z\xi}\gamma_5 gG_{z\xi}s|K\rangle &= 2if_{3K}(pz)^2,\\ \langle 0|\bar{q}\sigma_{z\xi}\gamma_5[iD_z,gG_{z\xi}]s-\frac{3}{7}\,i\partial_z\bar{q}\sigma_{z\xi}\gamma_5 gG_{z\xi}s|K\rangle &= 2if_{3K}(pz)^3\,\frac{3}{28}\,\omega_{K3},\\ \langle 0|\bar{q}i\overleftarrow{D}_z\,\sigma_{z\xi}\gamma_5 gG_{z\xi}s-\bar{q}\sigma_{z\xi}\gamma_5 gG_{z\xi}i\overrightarrow{D}_z\,s|K\rangle &= 2if_{3K}(pz)^3\,\frac{1}{14}\,\lambda_{K3}.\end{aligned} \tag{3.6}$$

Numerical values for these parameters can be obtained from QCD sum rules and will be discussed in section 5.

The operators in (3.6) renormalize multiplicatively in the chiral limit, with one-loop anomalous dimensions [48]

$$\begin{aligned}\gamma_{3;f}^{(0)} &= 2C_A+\frac{14}{3}\,C_F = \frac{110}{9},\\ \gamma_{3;\omega}^{(0)} &= \frac{20}{3}\,C_A+\frac{7}{3}\,C_F = \frac{208}{9},\\ \gamma_{3;\lambda}^{(0)} &= \frac{5}{3}C_A+\frac{47}{6}C_F = \frac{139}{9}\,.\end{aligned} \tag{3.7}$$

For a massive strange quark, the operators in (3.6) mix with twist-2 ones. Using the light-ray-operator technique of ref. [18], this mixing can be expressed in compact form as

$$O_3(z,vz,0)^{\mu^2} = O_3(z,vz,0)^{\mu_0^2} - im_s\frac{C_F\alpha_s}{2\pi}\ln\frac{\mu^2}{\mu_0^2}\frac{1}{v}\int_0^1 dt\,\Big[O_2(z,vz) - 2tO_2(z,tvz)\Big], \quad (3.8)$$

where

$$O_3(z,vz,0)^{\mu^2} = [\bar q(z)\sigma_{z\nu}\gamma_5 gG_{z\nu}(vz)s(0)]^{\mu^2} \quad (3.9)$$

and

$$O_2(az,bz)^{\mu^2} = [\bar q(az)\gamma_z\gamma_5 s(bz)]^{\mu^2}; \quad (3.10)$$

μ^2 stands for the normalization point. Sandwiching (3.8) between the K state and the vacuum, and expanding in powers of pz, one can easily derive the mixing for local operators with an arbitrary number of derivatives. We find that f_{3K} mixes with $f_K m_s$ and with $f_K m_s a_1^K$, whereas λ_{3K} and ω_{3K} mix in addition with $f_K m_s a_2^K$. The corresponding LO renormalization-group-improved expressions read

$$\begin{aligned}
f_{3K}(\mu^2) &= L^{55/(9\beta_0)} f_{3K}(\mu_0^2) + \frac{2}{19}\Big(L^{4/(\beta_0)} - L^{55/(9\beta_0)}\Big) f_K m_s(\mu_0^2) \\
&\quad + \frac{6}{65}\Big(L^{55/(9\beta_0)} - L^{68/(9\beta_0)}\Big) f_K[m_s a_1^K](\mu_0^2), \\
[f_{3K}\omega_{3K}](\mu^2) &= L^{104/(9\beta_0)}[f_{3K}\omega_{3K}](\mu_0^2) + \frac{1}{170}\Big(L^{4/(\beta_0)} - L^{104/(9\beta_0)}\Big) f_K m_s(\mu_0^2) \\
&\quad + \frac{1}{10}\Big(L^{68/(9\beta_0)} - L^{104/(9\beta_0)}\Big) f_K[m_s a_1^K](\mu_0^2) \\
&\quad + \frac{2}{15}\Big(L^{86/(9\beta_0)} - L^{104/(9\beta_0)}\Big) f_K[m_s a_2^K](\mu_0^2), \\
[f_{3K}\lambda_{3K}](\mu^2) &= L^{139/(18\beta_0)}[f_{3K}\lambda_{3K}](\mu_0^2) - \frac{14}{67}\Big(L^{4/(\beta_0)} - L^{139/(18\beta_0)}\Big) f_K m_s(\mu_0^2) \\
&\quad + \frac{14}{5}\Big(L^{68/(9\beta_0)} - L^{139/(18\beta_0)}\Big) f_K[m_s a_1^K](\mu_0^2) \\
&\quad - \frac{4}{11}\Big(L^{86/(9\beta_0)} - L^{139/(18\beta_0)}\Big) f_K[m_s a_2^K](\mu_0^2),
\end{aligned} \quad (3.11)$$

where L is the leading-log scaling factor: $L = \alpha_s(\mu^2)/\alpha_s(\mu_0^2)$.

The two-particle twist-3 DAs (3.1) and (3.2) are not independent, but related to the three-particle DA $\Phi_{3;K}$ by EOM [17, 22]. The EOM relations contain terms that depend on quark masses and can conveniently be expressed in terms of two dimensionless parameters $\rho_\pm^K$:

$$\rho_+^K = \frac{(m_s+m_q)^2}{m_K^2}, \qquad \rho_-^K = \frac{m_s^2 - m_q^2}{m_K^2}; \quad (3.12)$$

numerically $\rho_+^K \simeq \rho_-^K$. The rationale for introducing two parameters is that ρ_-^K changes sign when switching from K mesons to $\bar K$ mesons, i.e. $\rho_+^{\bar K} = \rho_+^K$, but $\rho_-^{\bar K} = -\rho_-^K$. In the analysis of twist-3 DAs given in ref. [22], only terms in ρ_+^K have been included. Here we complete these studies by taking into account also the terms in ρ_-^K.

From the non-local operator identities (A.3) and (A.4), one obtains the following relations for moments of the DAs, dropping the index K:

$$\begin{aligned}
M_n^{\phi_3^p} &= \delta_{n0} + \frac{n-1}{n+1}\, M_{n-2}^{\phi_3^p} + 2(n-1) M_{n-2}^{\varphi_3^{(1)}} + \frac{2(n-1)(n-2)}{n+1}\, M_{n-3}^{\varphi_3^{(2)}} \\
&\quad - \rho_+ \frac{n-1}{n+1}\, M_{n-2}^{\phi_2} + \rho_- M_{n-1}^{\phi_2}\,, \\
M_n^{\phi_3^\sigma} &= \delta_{n0} + \frac{n-1}{n+3}\, M_{n-2}^{\phi_3^\sigma} + \frac{6(n-1)}{n+3}\, M_{n-2}^{\varphi_3^{(1)}} + \frac{6n}{n+3}\, M_{n-1}^{\varphi_3^{(2)}} \\
&\quad - \rho_+ \frac{3}{n+3}\, M_n^{\phi_2} + \rho_- \frac{3}{n+3}\, M_{n-1}^{\phi_2}\,,
\end{aligned} \tag{3.13}$$

where we use the notation

$$M_n^\phi = \int_0^1 du\, (2u-1)^n \phi(u)$$

and introduce the auxiliary functions

$$\varphi_3^{(1)}(u) = \int_0^u d\alpha_1 \int_0^{\bar u} d\alpha_2\, \frac{2}{1-\alpha_1-\alpha_2}\, \Phi_3(\underline{\alpha})\,, \tag{3.14}$$

$$\varphi_3^{(2)}(u) = \int_0^u d\alpha_1 \int_0^{\bar u} d\alpha_2\, \frac{2}{(1-\alpha_1-\alpha_2)^2}\, (\alpha_1-\alpha_2-(2u-1))\, \Phi_3(\underline{\alpha})\,. \tag{3.15}$$

The normalization is chosen in such a way that

$$M_0^{\phi_3^p} = \int_0^1 du\, \phi_3^p(u) = 1, \qquad M_0^{\phi_3^\sigma} = \int_0^1 du\, \phi_3^\sigma(u) = 1-\rho_+. \tag{3.16}$$

Except for the new terms in ρ_-, these moment relations agree with those obtained in refs. [17, 22].

The relations (3.13) can be solved exactly: separating the contributions of quark-antiquark-gluon operators and the terms in $\rho_\pm$,

$$\begin{aligned}
\phi_3^p(u) &= 1 + \phi_{3,g}^p(u) + \rho_+\, \phi_{3,+}^p(u) + \rho_-\, \phi_{3,-}^p(u)\,, \\
\phi_3^\sigma(u) &= 6u\bar u + \phi_{3,g}^\sigma(u) + \rho_+\, \phi_{3,+}^\sigma(u) + \rho_-\, \phi_{3,-}^\sigma(u)\,,
\end{aligned} \tag{3.17}$$

we obtain the integral representations (cf. ref. [49])

$$\begin{aligned}
\phi_{3,g}^p(u) &= \frac14 \int_0^u \frac{dv}{\bar v} \Big[(2v-1)\,(\varphi_3^{(1)})''(v) - 2\,(\varphi_3^{(1)})'(v) + (\varphi_3^{(2)})''(v)\Big] \\
&\quad - \frac14 \int_u^1 \frac{dv}{v} \Big[(2v-1)\,(\varphi_3^{(1)})''(v) - 2\,(\varphi_3^{(1)})'(v) + (\varphi_3^{(2)})''(v)\Big]\,,
\end{aligned} \tag{3.18}$$

$$\phi_{3,+}^p(u) = \frac14 \int_0^u \frac{dv}{\bar v}\, \phi_2'(v) - \frac14 \int_u^1 \frac{dv}{v}\, \phi_2'(v)\,, \tag{3.19}$$

$$\phi_{3,-}^p(u) = \frac14 \int_0^u \frac{dv}{\bar v}\, \big[2\phi_2(v) - \phi_2'(v)\big] - \frac14 \int_u^1 \frac{dv}{v}\, \big[2\phi_2(v) + \phi_2'(v)\big]\,, \tag{3.20}$$

where primes denote the derivatives in v: $\phi'(v) = (d/dv)\phi(v)$ etc.

For the second twist-3 DA the solutions of the moment relations read, in the same notation:

$$\phi^{\sigma}_{3,g}(u) = -\frac{3}{2}\,u\bar u\left\{\int_0^u dv\left(\frac{1}{\bar v^2}+\frac{2}{\bar v}\right)\Big((\varphi_3^{(1)})'(v)+(2v-1)\,(\varphi_3^{(2)})'(v)\Big)\right. \tag{3.21}$$

$$\left. -\int_u^1 dv\left(\frac{1}{v^2}+\frac{2}{v}\right)\Big((\varphi_3^{(1)})'(v)+(2v-1)\,(\varphi_3^{(2)})'(v)\Big)\right\}, \tag{3.22}$$

$$\phi^{\sigma}_{3,+}(u) = -\frac{3}{2}\,u\bar u\left(\int_0^u dv\,\frac{1}{\bar v^2}\,\phi_2(v)+\int_u^1 dv\,\frac{1}{v^2}\,\phi_2(v)\right),$$

$$\phi^{\sigma}_{3,-}(u) = \frac{3}{2}\,u\bar u\left\{\int_0^u dv\,\left(\frac{1}{\bar v^2}+\frac{2}{\bar v}\right)\phi_2(v)-\int_u^1 dv\,\left(\frac{1}{v^2}+\frac{2}{v}\right)\phi_2(v)\right\}. \tag{3.23}$$

We stress that the relations (3.18) to (3.23) are valid in full QCD and involve no approximation whatsoever. One consequence of these relations is that quark-mass corrections to $\phi_3^{p,\sigma}$ contain logarithmic end-point singularities. In particular for the asymptotic leading-twist DA $\phi_{2;K}(u)=6u(1-u)$ we obtain

$$\phi^{p}_{3;K}(u)\big|_{\text{no gluons, asymptotic }\phi_{2;K}} = 1+\rho_+^K\,\frac{3}{2}\,(2+\ln u\bar u)+\rho_-^K\,\frac{3}{2}\left(1-2u+\ln\,\frac{u}{\bar u}\right). \tag{3.24}$$

To NLO in conformal spin we obtain, using the truncated conformal expansions (2.12) for $\phi_{2;K}$ and (3.5) for $\Phi_{3;K}$:

$$\begin{aligned}\phi^{p}_{3;K}(u) &= 1+3\rho_+^K(1+6a_2^K)-9\rho_-^K a_1^K+C_1^{1/2}(2u-1)\left[\frac{27}{2}\,\rho_+^K a_1^K-\rho_-^K\left(\frac{3}{2}+27a_2^K\right)\right]\\ &+C_2^{1/2}(2u-1)\left(30\eta_{3K}+15\rho_+^K a_2^K-3\rho_-^K a_1^K\right)+C_3^{1/2}(2u-1)\left(10\eta_{3K}\lambda_{3K}-\frac{9}{2}\,\rho_-^K a_2^K\right)\\ &-3\eta_{3K}\omega_{3K}C_4^{1/2}(2u-1)+\frac{3}{2}\,(\rho_+^K+\rho_-^K)(1-3a_1^K+6a_2^K)\ln u\\ &+\frac{3}{2}\,(\rho_+^K-\rho_-^K)(1+3a_1^K+6a_2^K)\ln\bar u,\end{aligned} \tag{3.25}$$

$$\begin{aligned}\phi^{\sigma}_{3;K}(u) &= 6u\bar u\left[1+\frac{3}{2}\,\rho_+^K+15\rho_+^K a_2^K-\frac{15}{2}\,\rho_-^K a_1^K+\left(3\rho_+^K a_1^K-\frac{15}{2}\,\rho_-^K a_2^K\right)C_1^{3/2}(2u-1)\right.\\ &\left.+\left(5\eta_{3K}-\frac{1}{2}\,\eta_{3K}\omega_{3K}+\frac{3}{2}\,\rho_+^K a_2^K\right)C_2^{3/2}(2u-1)+\eta_{3K}\lambda_{3K}C_3^{3/2}(2u-1)\right]\\ &+9u\bar u(\rho_+^K+\rho_-^K)(1-3a_1^K+6a_2^K)\ln u+9u\bar u(\rho_+^K-\rho_-^K)(1+3a_1^K+6a_2^K)\ln\bar u\,,\end{aligned} \tag{3.26}$$

where, to simplify notations, we have introduced the parameter

$$\eta_{3K}=\frac{f_{3K}}{f_K}\,\frac{m_q+m_s}{m_K^2}. \tag{3.27}$$

These expressions are our final results for the two-particle twist-3 DAs and supersede those given in refs. [17, 22]. The terms multiplying $\ln u$ and $\ln\bar u$ are the first three terms in the conformal expansion of $\phi'_{2;K}(0)$ and $\phi'_{2;K}(1)$, respectively. Numerical values for the hadronic parameters are given in table 3. The leading-order scale-dependence follows from (3.11) and the scale dependence of the quark masses in $\rho_\pm^K$ and η_{3K}.

We note in passing that the EOM relations

$$\frac{u}{2}\left\{\phi_{3;\pi}^{p}(u)+\frac{1}{6}\left(\phi_{3;\pi}^{\sigma}(u)\right)'\right\}_{\text{no gluons}} = \frac{1}{6}\,\phi_{3;\pi}^{\sigma}(u)\Big|_{\text{no gluons}},$$
$$\frac{1-u}{2}\left\{\phi_{3;\pi}^{p}(u)-\frac{1}{6}\left(\phi_{3;\pi}^{\sigma}(u)\right)'\right\}_{\text{no gluons}} = \frac{1}{6}\,\phi_{3;\pi}^{\sigma}(u)\Big|_{\text{no gluons}},$$

are no longer fulfilled for $\phi_{3;K}^{p,\sigma}$, but violated by mass corrections in $\rho_{\pm}^{K}$.

4. Twist-4 distributions

In this section we derive models for the two- and three-particle twist-4 DAs to NLO in the conformal expansion. There are four K-meson three-particle DAs of twist 4, defined as [17, 22][3]

$$\langle 0|\bar{q}(z)\gamma_\mu\gamma_5 gG_{\alpha\beta}(vz)s(-z)|K(P)\rangle = \\ = p_\mu(p_\alpha z_\beta - p_\beta z_\alpha)\,\frac{1}{pz}\,f_K\Phi_{4;K}(v,pz) + (p_\beta g_{\alpha\mu}^{\perp} - p_\alpha g_{\beta\mu}^{\perp})f_K\Psi_{4;K}(v,pz)+\cdots, \quad (4.1)$$

$$\langle 0|\bar{q}(z)\gamma_\mu ig\widetilde{G}_{\alpha\beta}(vz)s(-z)|K(P)\rangle = \\ = p_\mu(p_\alpha z_\beta - p_\beta z_\alpha)\,\frac{1}{pz}\,f_K\widetilde{\Phi}_{4;K}(v,pz) + (p_\beta g_{\alpha\mu}^{\perp} - p_\alpha g_{\beta\mu}^{\perp})f_K\widetilde{\Psi}_{4;K}(v,pz)+\cdots, \quad (4.2)$$

with the short-hand notation

$$\mathcal{F}(v,pz) = \int \mathcal{D}\underline{\alpha}\, e^{-ipz(\alpha_2-\alpha_1+v\alpha_3)}\mathcal{F}(\underline{\alpha}).$$

The integration measure $\mathcal{D}\underline{\alpha}$ is defined in (3.4), and the dots denote terms of twist 5 and higher. For massless quarks and, more generally, for two equal-mass quarks, G-parity implies that the DAs Φ and Ψ are antisymmetric under the interchange of the quark momenta, $\alpha_1 \leftrightarrow \alpha_2$, whereas $\widetilde{\Phi}$ and $\widetilde{\Psi}$ are symmetric [17, 22]. Note that unlike twist-2 and twist-3 DAs, which are dimensionless, the twist-4 DAs have mass dimension 2 (GeV^2). The corresponding contributions to hard exclusive processes are suppressed by two powers of the hard scale with respect to leading twist.

The distribution amplitudes $\Phi_{4;K}$ and $\widetilde{\Phi}_{4;K}$ correspond to the light-cone projection $\gamma_z G_{zp}$, which picks up the $s=+1/2$ components of both quark and antiquark field and the $s=0$ component of the gluon field. The conformal expansion reads:

$$\Phi_{4;K}(\underline{\alpha}) = 120\alpha_1\alpha_2\alpha_3[\phi_0^K + \phi_1^K(\alpha_1-\alpha_2) + \phi_2^K(3\alpha_3-1)+\cdots],$$
$$\widetilde{\Phi}_{4;K}(\underline{\alpha}) = 120\alpha_1\alpha_2\alpha_3[\widetilde{\phi}_0^K + \widetilde{\phi}_1^K(\alpha_1-\alpha_2) + \widetilde{\phi}_2^K(3\alpha_3-1)+\cdots]. \quad (4.3)$$

G-parity implies that, for the π meson, $\phi_0^\pi = \phi_2^\pi = \widetilde{\phi}_1^\pi = 0$, whereas ϕ_0^K, ϕ_2^K and $\widetilde{\phi}_1^K$ are $\mathcal{O}(m_s - m_q)$.

In turn, the DAs $\Psi_{4;K}$ and $\widetilde{\Psi}_{4;K}$ correspond to the light-cone projection $\gamma_\perp G_{z\perp}$, which is a mixture of different quark-spin states with $s_q = +1/2, s_{\bar{q}} = -1/2$ and $s_q = -1/2, s_{\bar{q}} =$

[3] In the notation of ref. [22], $\Phi_{4;K} = m_K^2\mathcal{A}_\parallel$, $\Psi_{4;K} = m_K^2\mathcal{A}_\perp$, $\widetilde{\Phi}_{4;K} = m_K^2\mathcal{V}_\parallel$ and $\widetilde{\Psi}_{4;K} = m_K^2\mathcal{V}_\perp$.

$+1/2$, respectively. In both cases $s = +1$ for the gluon. We separate the different quark-spin projections by introducing the auxiliary amplitudes $\Psi^{\uparrow\downarrow}$ and $\Psi^{\downarrow\uparrow}$, defined as

$$\begin{aligned}\langle 0|\bar{q}(z)ig\widetilde{G}_{\alpha\beta}(vz)\gamma_z\gamma_\mu\gamma_p s(-z)|K(P)\rangle &= f_K\left(p_\beta g^\perp_{\alpha\mu} - p_\alpha g^\perp_{\beta\mu}\right)\Psi^{\uparrow\downarrow}(v,pz),\\ \langle 0|\bar{q}(z)ig\widetilde{G}_{\alpha\beta}(vz)\gamma_p\gamma_\mu\gamma_z s(-z)|K(P)\rangle &= f_K\left(p_\beta g^\perp_{\alpha\mu} - p_\alpha g^\perp_{\beta\mu}\right)\Psi^{\downarrow\uparrow}(v,pz).\end{aligned} \tag{4.4}$$

The original distributions $\Psi_{4;K}$ and $\widetilde{\Psi}_{4;K}$ are given by

$$\widetilde{\Psi}(\underline{\alpha}) = -\frac{1}{2}\Big[\Psi^{\uparrow\downarrow}(\underline{\alpha}) + \Psi^{\downarrow\uparrow}(\underline{\alpha})\Big], \quad \Psi(\underline{\alpha}) = \frac{1}{2}\Big[\Psi^{\uparrow\downarrow}(\underline{\alpha}) - \Psi^{\downarrow\uparrow}(\underline{\alpha})\Big]. \tag{4.5}$$

$\Psi^{\uparrow\downarrow}$ and $\Psi^{\downarrow\uparrow}$ have a regular expansion in terms of conformal polynomials, to wit:

$$\begin{aligned}\Psi^{\uparrow\downarrow}(\underline{\alpha}) &= 60\alpha_2\alpha_3^2\left[\psi_0^{\uparrow\downarrow} + \psi_1^{\uparrow\downarrow}(\alpha_3 - 3\alpha_1) + \psi_2^{\uparrow\downarrow}\left(\alpha_3 - \frac{3}{2}\alpha_2\right)\right],\\ \Psi^{\downarrow\uparrow}(\underline{\alpha}) &= 60\alpha_1\alpha_3^2\left[\psi_0^{\downarrow\uparrow} + \psi_1^{\downarrow\uparrow}(\alpha_3 - 3\alpha_2) + \psi_2^{\downarrow\uparrow}\left(\alpha_3 - \frac{3}{2}\alpha_1\right)\right].\end{aligned} \tag{4.6}$$

For the π meson, thanks to G-parity,

$$\Psi^{\uparrow\downarrow}_{4;\pi}(\alpha_1,\alpha_2) = \Psi^{\downarrow\uparrow}_{4;\pi}(\alpha_2,\alpha_1), \tag{4.7}$$

so that $\psi_i^{\uparrow\downarrow} \equiv \psi_i^{\downarrow\uparrow}$.[4] For K, we write

$$\psi_i^{\uparrow\downarrow} = \psi_i^K + \theta_i^K, \qquad \psi_i^{\downarrow\uparrow} = \psi_i^K - \theta_i^K, \tag{4.8}$$

where the θ_i correspond to SU(3)-breaking corrections that also violate G-parity. From (4.5), the following representations can readily be derived:

$$\begin{aligned}\widetilde{\Psi}_{4;K}(\underline{\alpha}) = -30\alpha_3^2\Big\{&\psi_0^K(1-\alpha_3) + \psi_1^K\Big[\alpha_3(1-\alpha_3) - 6\alpha_1\alpha_2\Big] \\ &+\psi_2^K\Big[\alpha_3(1-\alpha_3) - \frac{3}{2}(\alpha_1^2+\alpha_2^2)\Big] - (\alpha_1-\alpha_2)\Big[\theta_0^K + \alpha_3\theta_1^K + \frac{1}{2}(5\alpha_3-3)\theta_2^K\Big]\Big\},\end{aligned} \tag{4.9}$$

$$\begin{aligned}\Psi_{4;K}(\underline{\alpha}) = 30\alpha_3^2\Big\{&\theta_0^K(1-\alpha_3) + \theta_1^K\Big[\alpha_3(1-\alpha_3) - 6\alpha_1\alpha_2\Big] \\ &+\theta_2^K\Big[\alpha_3(1-\alpha_3) - \frac{3}{2}(\alpha_1^2+\alpha_2^2)\Big] - (\alpha_1-\alpha_2)\Big[\psi_0^K + \alpha_3\psi_1^K + \frac{1}{2}(5\alpha_3-3)\psi_2^K\Big]\Big\}.\end{aligned} \tag{4.10}$$

In addition, we introduce one more three-particle DA $\Xi_4(\underline{\alpha})$ [21]:

$$\left\langle 0\left|\bar{q}(z)\gamma_\mu\gamma_5 g D^\alpha G_{\alpha\beta}(vz)s(-z)\right|K^+(p)\right\rangle = if_K p_\mu p_\beta \int \mathcal{D}\underline{\alpha}\; \mathrm{e}^{-ipz(\alpha_2-\alpha_1+v\alpha_3)}\,\Xi_{4;K}(\underline{\alpha}). \tag{4.11}$$

The Lorentz structure $p_\mu p_\beta$ is the only one relevant at twist 4. Because of the EOM, $D^\alpha G^A_{\alpha\beta} = -g\sum_q \bar{q}t^A\gamma_\beta q$, where the summation goes over all light flavors, $\Xi_{4;K}(\underline{\alpha})$ can be viewed as describing either a quark-antiquark-gluon or a specific four-quark component of

[4] This implies, in particular, that only one of the DAs Ψ and $\widetilde{\Psi}$ is dynamically independent.

the pion, with the quark-antiquark pair in a color-octet state and at the same space-time point. The conformal expansion of $\Xi_{4;K}(\underline{\alpha})$ starts with $J = 4$ and reads

$$\Xi_{4;K}(\underline{\alpha}) = 840\alpha_1\alpha_2\alpha_3^3\Big[\Xi_0^K + \cdots\Big], \tag{4.12}$$

where Ξ_0^K has mass dimension 2. The dots stand for terms with higher conformal spin $J = 5, 6, \ldots$, which are beyond our accuracy. This DA was not considered in refs. [17, 22] because $\Xi_0^K = \mathcal{O}(m_s - m_q)$ and vanishes for mesons built of quark and antiquark with equal mass.

The expressions in eqs. (4.3), (4.9), (4.10), (4.11) represent the most general parametrization of the twist-4 DAs to NLO in the conformal-spin expansion and involve 13 non-perturbative parameters. Not all of them are independent, though. In the following, we shall establish their mutual relations and also express the expansion coefficients in terms of matrix elements of local operators.

The asymptotic three-particle DAs correspond to contributions of the lowest conformal spin $J = j_s + j_{\bar q} + j_g = 3$. The parameters ϕ_0^K, $\widetilde\phi_0^K$, ψ_0^K and θ_0^K describing these DAs can be expressed in terms of local matrix elements as

$$\begin{aligned}\langle 0|\bar q\gamma_\alpha g\widetilde G_{\mu\alpha}s|K(P)\rangle &= iP_\mu f_K\delta_K^2,\\ \langle 0|\bar q\gamma_\alpha\gamma_5 igG_{\mu\alpha}s|K(P)\rangle &= iP_\mu f_K m_K^2\kappa_{4K}.\end{aligned} \tag{4.13}$$

These are the only two local twist-4 operators of dimension 5. Note that the second matrix element vanishes for equal-mass quarks, because of G-parity. It also vanishes in the chiral limit $m_q, m_s \to 0$ because of the factor m_K^2. Moreover, in this limit κ_{4K} can be calculated exactly to leading order in m_s [9]:

$$\kappa_{4K} = -\frac{1}{8} + \mathcal{O}(m_s); \tag{4.14}$$

numerical estimates of the corrections can be obtained from QCD sum rules and will be discussed below.

Taking the local limit of eqs. (4.1), (4.2), one obtains

$$\psi_0^K = \widetilde\phi_0^K = -\frac{1}{3}\,\delta_K^2, \qquad \phi_0^K = -\theta_0^K = \frac{1}{3}\,m_K^2\kappa_{4K}. \tag{4.15}$$

What about the scale-dependence of these parameters? Like f_{3K}, δ_K^2 renormalizes multiplicatively for massless quarks, but mixes with operators of lower twist for $m_s \neq 0$. At the operator level, neglecting $\mathcal{O}(m_s^2)$ corrections, the mixing is given by

$$(\bar q\gamma_\alpha g\widetilde G_{\mu\alpha}s)^{\mu^2} = (\bar q\gamma_\alpha g\widetilde G_{\mu\alpha}s)^{\mu_0^2}\left(1 - \frac{8}{9}\,\frac{\alpha_s}{\pi}\,\ln\frac{\mu^2}{\mu_0^2}\right) - \frac{1}{9}\,\frac{\alpha_s}{\pi}\,\ln\frac{\mu^2}{\mu_0^2}\,m_s\,[\partial_\mu(\bar q i\gamma_5 s)]^{\mu_0^2}. \tag{4.16}$$

Taking matrix elements and resumming the logarithms, we find

$$[\delta_K^2](\mu^2) = L^{32/(9\beta_0)}[\delta_K^2](\mu_0^2) + \frac{1}{8}\,\left(1 - L^{32/(9\beta_0)}\right)m_K^2, \tag{4.17}$$

with, as before, $L = \alpha_s(\mu^2)/\alpha_s(\mu_0^2)$.

The scale dependence of κ_{4K} can most easily be derived by observing that this parameter is related to a_1^K and quark masses by the equations of motion [9]:

$$\kappa_{4K} = -\frac{1}{8}\,\frac{m_s - m_q}{m_s + m_q} - \frac{9}{40}\,a_1^K + \frac{m_s^2 - m_q^2}{2m_K^2}. \tag{4.18}$$

Taking into account the known scale dependence of a_1^K and $m_{s,q}$, one obtains

$$\kappa_{4K}(\mu^2) = \kappa_{4K}(\mu_0^2) - \frac{9}{40}\,\left(L^{32/(9\beta_0)} - 1\right) a_1^K(\mu_0^2) + \left(L^{8/\beta_0} - 1\right) \frac{[m_s^2 - m_q^2](\mu_0^2)}{2m_K^2}\,. \tag{4.19}$$

To NLO in conformal spin, the discussion becomes more involved. As explained in ref. [17], for massless quarks the corresponding contributions can be expressed in terms of matrix elements of the three existing G-parity-even local quark-antiquark-gluon operators of twist-4. These three operators are not independent, however, but related by the QCD equations of motion. One is left with one new non-perturbative parameter only, call it ω_{4K},[5] which can be defined as

$$\begin{aligned}\langle 0|\bar{q}[iD_\mu, ig\widetilde{G}_{\nu\xi}]\gamma_\xi s - \frac{4}{9}\,i\partial_\mu\bar{q}ig\widetilde{G}_{\nu\xi}\gamma_\xi s|K(P)\rangle \;&=\\ = f_K\delta_K^2\omega_{4K}\left(P_\mu P_\nu - \frac{1}{4}\,m_K^2 g_{\mu\nu}\right) &+ \mathcal{O}(\text{twist } 5).\end{aligned} \tag{4.20}$$

The scale dependence of ω_{4K}, for massless quarks, is given by

$$[\delta_K^2\omega_{4K}](\mu^2) = L^{10/\beta_0}\,[\delta_K^2\omega_{4K}](\mu_0^2)\,.$$

For massive quarks, a distinction must be made between G-parity-conserving and G-parity-breaking contributions. G-parity-conserving corrections do not involve new operators, and the difference to the massless case is mainly due to corrections proportional to the meson mass. This case is described in detail in refs. [50, 22]. Here we just quote the results obtained in ref. [22]:

$$\begin{aligned}\phi_1^K &= \frac{21}{8}\,\delta_K^2\omega_{4K} - \frac{9}{20}\,m_K^2 a_2^K, \qquad & \widetilde{\phi}_2^K &= \frac{21}{8}\,\delta_K^2\omega_{4K},\\ \psi_1^K &= \frac{7}{4}\,\delta_K^2\omega_{4K} - \frac{3}{20}\,m_K^2 a_2^K, \qquad & \psi_2^K &= \frac{7}{2}\,\delta_K^2\omega_{4K} + \frac{3}{20}\,m_K^2 a_2^K.\end{aligned} \tag{4.21}$$

The G-parity-breaking contributions, on the other hand, involve a different set of local operators and in particular

$$\bar{q}\gamma_z\gamma_5 D_\xi g G^{\xi z} s = -g^2 \sum_{\psi=u,d,s} (\bar{q}\gamma_z\gamma_5 t^a s)(\bar{\psi}\gamma_z t^a \psi)$$

which determines the normalization and the leading conformal spin contribution to the DA $\Xi_{4;K}(\underline{\alpha})$ defined in eq. (4.11). Hence, a complete treatment of G-parity-breaking corrections to twist-4 DAs requires also the inclusion of $\Xi_{4;K}$.

[5] In the notation of ref. [17] $\omega_4 = (8/21)\epsilon$.

It is beyond the scope of this paper to work out the corresponding relations between the matrix elements of local operators and expansion coefficients. For this reason, and also because QCD sum-rule estimates of matrix elements of large mass dimension are not very reliable, we adopt a different approach and estimate G-parity-breaking corrections of spin $J = 4$ using the renormalon model of ref. [21]. The general idea of this technique is to estimate matrix elements of "genuine" twist-4 operators by the quadratically divergent contributions that appear when the matrix elements are defined using a hard UV cut-off, see ref. [21] for details and further references. In this way, three-particle twist-4 DAs can be expressed in terms of the leading-twist DA ϕ_2:

$$\begin{aligned}
\Psi^{\rm ren}_{4;K}(\alpha_1,\alpha_2,\alpha_3) &= \frac{\delta_K^2}{6}\left[\frac{\phi_{2;K}(\alpha_1)}{1-\alpha_1}-\frac{\phi_{2;K}(\bar\alpha_2)}{1-\alpha_2}\right],\\
\Phi^{\rm ren}_{4;K}(\alpha_1,\alpha_2,\alpha_3) &= \frac{\delta_K^2}{3}\left[\frac{\alpha_2\phi_{2;K}(\alpha_1)}{(1-\alpha_1)^2}-\frac{\alpha_1\phi_{2;K}(\bar\alpha_2)}{(1-\alpha_2)^2}\right],\\
\widetilde\Psi^{\rm ren}_{4;K}(\alpha_1,\alpha_2,\alpha_3) &= \frac{\delta_K^2}{6}\left[\frac{\phi_{2;K}(\alpha_1)}{1-\alpha_1}+\frac{\phi_{2;K}(\bar\alpha_2)}{1-\alpha_2}\right],\\
\widetilde\Phi^{\rm ren}_{4;K}(\alpha_1,\alpha_2,\alpha_3) &= -\frac{\delta_K^2}{3}\left[\frac{\alpha_2\phi_{2;K}(\alpha_1)}{(1-\alpha_1)^2}+\frac{\alpha_1\phi_{2;K}(\bar\alpha_2)}{(1-\alpha_2)^2}\right],\\
\Xi^{\rm ren}_{2;K}(\alpha_1,\alpha_2,\alpha_3) &= -\frac{2\delta_K^2}{3}\left[\frac{\alpha_2\,\phi_{2;K}(\alpha_1)}{1-\alpha_1}-\frac{\alpha_1\,\phi_{2;K}(\bar\alpha_2)}{1-\alpha_2}\right],
\end{aligned}\tag{4.22}$$

where, in difference to [21], we do not assume that $\phi_2(u)$ is symmetric under the exchange $u \leftrightarrow 1-u$.

The expressions in (4.22) do not rely on conformal expansion and contain the contributions of all conformal partial waves. Projecting onto the contributions with the lowest spin $J = 3, 4$, we obtain

$$\begin{aligned}
&\phi_0^K = 0 && \phi_1^K = \frac{7}{12}\delta_K^2 && \phi_2^K = -\frac{7}{20}a_1^K\delta_K^2,\\
&\widetilde\phi_0^K = -\frac{1}{3}\delta_K^2, && \widetilde\phi_1^K = -\frac{7}{4}a_1^K\delta_K^2, && \widetilde\phi_2^K = \frac{7}{12}\delta_K^2,\\
&\psi_0^K = -\frac{1}{3}\delta_K^2, && \psi_1^K = \frac{7}{18}\delta_K^2, && \psi_2^K = \frac{7}{9}\delta_K^2,\\
&\theta_0^K = 0, && \theta_1^K = \frac{7}{10}a_1^K\delta_K^2, && \theta_2^K = -\frac{7}{5}a_1^K\delta_K^2\,.
\end{aligned}\tag{4.23}$$

It follows that in the renormalon model

$$\omega_{4K} = \omega_{4\pi} = \frac{2}{9},\tag{4.24}$$

which is in good agreement with direct QCD sum-rule calculations [17]. We also find

$$\Xi_0^K = \frac{1}{5}a_1^K\delta_K^2\,.\tag{4.25}$$

Note that in the renormalon model $\theta_0^K = 0$. This is due to the fact that the contribution in κ_{4K} in eq. (4.15) is obtained as the matrix element of the operator (4.13) which vanishes

by the EOM (up to a total derivative), see eq. (4.18). Therefore, against appearances, this contribution has to be interpreted as "kinematic" power correction induced by the non-vanishing K-meson mass rather than a "genuine" twist-4 effect.

We are now in the position to derive expressions for the two-particle DAs of twist 4. They are defined as

$$\begin{aligned}\langle 0|\bar{q}(x)[x,-x]\gamma_\mu\gamma_5 s(-x)|K(P)\rangle \;=\; & if_K P_\mu \int_0^1 du\, e^{i\xi Px}\left(\phi_{2;K}(u)+\frac{1}{4}\,x^2\phi_{4;K}(u)\right)\\ & +\frac{i}{2}\,f_K\,\frac{1}{Px}\,x_\mu\int_0^1 du\, e^{i\xi Px}\psi_{4;K}(u),\end{aligned} \tag{4.26}$$

which is the extension of eq. (2.6) to twist-4 accuracy.[6] From the operator relations (A.1) and (A.2), we obtain

$$\psi_{K;4}(u) \;=\; m_K^2\{2\phi^p_{3;K}(u)-\phi_{2;K}(u)\}+\frac{d}{du}\int_0^u d\alpha_1\int_0^{\bar u} d\alpha_2\frac{2(\Phi_{4;K}(\underline{\alpha})-2\Psi_{4;K}(\underline{\alpha}))}{1-\alpha_1-\alpha_2}, \tag{4.27}$$

$$\begin{aligned}\frac{d^2\phi_{4;K}(u)}{du^2} \;=\; & 12\psi_{4;K}(u)-12m_K^2\phi_{2;K}(u)-2\frac{d}{du}\left[(2u-1)(m_K^2\phi_{2;K}(u)+\psi_{4;K}(u))\right]\\ & +\frac{d^2}{du^2}\int_0^u d\alpha_1\int_0^{\bar u} d\alpha_2\,\frac{4(2\Psi_{4;K}(\underline{\alpha})-\Phi_{4;K}(\underline{\alpha}))}{(1-\alpha_1-\alpha_2)^2}\,(\alpha_1-\alpha_2-(2u-1))\\ & +4\,\frac{m_s-m_q}{m_s+m_q}\,m_K^2\,\frac{d\phi^p_{3;K}(u)}{du}\end{aligned} \tag{4.28}$$

with the boundary condition $\phi_{4;K}(0)=\phi_{4;K}(1)=0$.

We solve this relations splitting the result in "genuine" twist-4 contributions $\psi^{T4}_{4;K}$ and Wandzura-Wilczek-type mass corrections $\psi^{WW}_{4;K}$ as

$$\psi_{4;K}(u)=\psi^{T4}_{4;K}(u)+\psi^{WW}_{4;K}(u) \tag{4.29}$$

with

$$\psi^{T4}_{4;K}(u) \;=\; \frac{20}{3}\,\delta_K^2 C_2^{1/2}(2u-1)+5\left\{5\theta_1^K-\theta_2^K\right\}C_3^{1/2}(2u-1)\,, \tag{4.30}$$

$$\begin{aligned}\psi^{WW}_{4;K}(u) \;=\; & m_K^2\left\{1+6\rho_+^K(1+6a_2^K)-18\rho_-^K a_1^K\right\}C_0^{1/2}(2u-1)\\ & +m_K^2\left\{-12\kappa_{4K}-\frac{9}{5}a_1^K+27\rho_+^K a_1^K-3\rho_-^K(1+18a_2^K)\right\}C_1^{1/2}(2u-1)\\ & +\left\{m_K^2\left(1+\frac{18}{7}a_2^K+30\rho_+^K a_2^K-6\rho_-^K a_1^K\right)+60\frac{f_{3K}}{f_K}\,(m_s+m_q)\right\}C_2^{1/2}(2u-1)\\ & +\left\{m_K^2\left(\frac{9}{5}a_1^K+\frac{16}{3}\kappa_{4K}-9\rho_-^K a_2^K\right)+20\,\frac{f_{3K}}{f_K}\,(m_s+m_q)\lambda_{3K}\right\}C_3^{1/2}(2u-1)\\ & +\left\{-\frac{9}{28}\,m_K^2 a_2^K-6\,\frac{f_{3K}}{f_K}\,(m_s+m_q)\omega_{3K}\right\}C_4^{1/2}(2u-1)\\ & +6m_q(m_s+m_q)(1+3a_1^K+6a_2^K)\ln\bar u+6m_s(m_s+m_q)(1-3a_1^K+6a_2^K)\ln u\,,\end{aligned} \tag{4.31}$$

where $\xi=2u-1$, see eq. (2.8). $\psi^{WW}_{4;K}$ vanishes for $m_K\to 0$ and $m_{s,q}\to 0$.

[6] $\psi_{4;K}$ and $\phi_{4;K}$ are related to the DAs defined in ref. [22] by $\phi_{4;K}=m_K^2 g_K$ and $\psi_{4;K}=m_K^2\mathbb{A}_K$.

The complete expression for

$$\phi_{4;K}(u) = \phi_{4;K}^{T4}(u) + \phi_{4;K}^{WW}(u) \tag{4.32}$$

is rather lengthy. We find for the "genuine" twist-4 part:

$$\begin{aligned}\phi_{4;K}^{T4}(u) = {} & \frac{200}{3}\,\delta_K^2 u^2\bar{u}^2 + 20u^2\bar{u}^2\xi\left\{4\theta_1^K - 5\theta_2^K\right\} \\ & + 21\delta_K^2\omega_{4K}\Big\{u\bar{u}(2+13u\bar{u}) + \big[2u^3(6u^2-15u+10)\ln u\big] + [u\leftrightarrow\bar{u}]\Big\} \\ & + 40\phi_2^K\left\{u\bar{u}\xi(2-3u\bar{u}) - \big[2u^3(u-2)\ln u\big] + [u\leftrightarrow\bar{u}]\right\},\end{aligned} \tag{4.33}$$

and for the mass-corrections, neglecting numerically small terms of order m_s^2:

$$\begin{aligned}\phi_{4;K}^{WW}(u) = {} & \frac{16}{3}\,m_K^2\kappa_{4K}\Big\{u\bar{u}\xi(1-2u\bar{u}) + \big[5(u-2)u^3\ln u\big] - [u\leftrightarrow\bar{u}]\Big\} \\ & + 4\,\frac{f_{3K}}{f_K}\,(m_s+m_q)u\bar{u}\left\{30\left(1-\xi\,\frac{m_s-m_q}{m_s+m_q}\right)\right. \\ & \quad + 10\lambda_{3K}\left(\xi\,[1-u\bar{u}] - \frac{m_s-m_q}{m_s+m_q}\,[1-5u\bar{u}]\right) \\ & \quad \left. -\omega_{3K}\left(3-21u\bar{u}+28u^2\bar{u}^2 + 3\xi\frac{m_s-m_q}{m_s+m_q}[1-7u\bar{u}]\right)\right\} \\ & - \frac{36}{5}m_K^2 a_2^K\left\{\frac{1}{4}u\bar{u}(4-9u\bar{u}+110u^2\bar{u}^2) + [u^3(10-15u+6u^2)\ln u] + [u\leftrightarrow\bar{u}]\right\} \\ & + 4m_K^2\,u\bar{u}\,(1+3u\bar{u})\left(1+\frac{9}{5}a_1^K\,\xi\right).\end{aligned} \tag{4.34}$$

The DAs for $\bar{K}$ mesons are obtained by replacing u by $1-u$. Note that $\psi_{4;K}$ has logarithmic end-point singularities for finite quark mass, whereas $\phi_{4;K}$ has no such singularities, so that one can safely neglect the $\mathcal{O}(m_s^2)$ terms.

The expressions given above provide a self-consistent model of the twist-4 DAs which includes the first three terms of the conformal expansion.[7] An estimate of the contribution of higher orders can be obtained using the renormalon model. In this case, the "genuine" twist-4 contributions to the two-particle DAs given in eqs. (4.29) and (4.32) have to be replaced by

$$\begin{aligned}\phi_{4;K}^{T4,\mathrm{ren}}(u) = {} & \frac{8}{3}\,\delta_K^2\int_0^1 dv\,\phi_{2;K}(v)\left\{\frac{1}{v^2}\left[u^2+u+(v-u)\ln\left(1-\frac{u}{v}\right)\right]\theta(v-u)\right. \\ & \qquad \left. + \frac{1}{\bar{v}^2}\left[\bar{u}^2+\bar{u}+(u-v)\ln\left(1-\frac{\bar{u}}{\bar{v}}\right)\right]\theta(u-v)\right\}, \\ \psi_{4;K}^{T4,\mathrm{ren}}(u) = {} & \frac{\delta_K^2}{3}\,\frac{d^2}{du^2}\int_0^1 dv\,\phi_{2;K}(v)\left\{\left(\frac{u}{v}\right)^2\theta(v-u) + \left(\frac{\bar{u}}{\bar{v}}\right)^2\theta(u-v)\right\}\end{aligned} \tag{4.35}$$

and used in combination with the complete renormalon-model expressions for the three-particle DAs given in eq. (4.22). As explained in ref. [21], the renormalon model does not

[7] One shortcoming of the model is that G-parity-breaking meson mass corrections of spin $J = 4$ are missing and we only include the "genuine" G-parity-breaking twist-4 corrections estimated in the renormalon model. Numerically, both effects may be of the same order.

take into account the damping of higher conformal-spin contributions by the increasing anomalous dimensions and, therefore, provides an upper bound for their contribution. The effect of these corrections is, most importantly, to significantly enhance the end-point behaviour of higher-twist DAs in some cases, which can be important in phenomenological applications.

5. Models for distribution amplitudes

In this section we compile the numerical estimates of all necessary parameters and present explicit models of the twist-3 and -4 two-particle distribution amplitudes that we introduced in sections 3 and 4. The important point is that these DAs are related to three-particle ones by exact QCD equations of motion and have to be used together; this guarantees the consistency of the approximation. Our approximation thus introduces a minimum number of non-perturbative parameters, which are defined as matrix elements of certain local operators between the vacuum and the meson state, and which we estimate using QCD sum rules. More sophisticated models can be constructed in a systematic way by adding contributions of higher conformal partial waves when estimates of the relevant non-perturbative matrix elements will become available.

Our approach involves the implicit assumption that the conformal partial wave expansion is well convergent. This can be justified rigorously at large scales, since the anomalous dimensions of all involved operators increase logarithmically with the conformal spin J, but is non-trivial at relatively low scales of order $\mu \sim (1\text{–}2)\,\text{GeV}$ which we choose as reference scale. An upper bound for the contribution of higher partial waves can be obtained from the renormalon model.

Since orthogonal polynomials of high orders are rapidly oscillating functions, a truncated expansion in conformal partial waves is, almost necessarily, oscillatory as well. Such a behaviour is clearly unphysical, but this does not constitute a real problem since physical observables are given by convolution integrals of distribution amplitudes with smooth coefficient functions. A classical example for this feature is the $\gamma\gamma^*$-meson form factor, which is governed by the quantity

$$\int du\, \frac{1}{u}\, \phi(u) \sim \sum a_i,$$

where the coefficients a_i are exactly the "reduced matrix elements" in the conformal expansion. The oscillating terms are averaged over and strongly suppressed. Stated otherwise: models of distribution amplitudes should generally be understood as distributions (in the mathematical sense).

We give all relevant numerical input parameters for our model DAs in table 3, at the scale $\mu = 1\,\text{GeV}$, which is appropriate for QCD sum-rule results, and, using the LO and NLO scaling relations given in section 3 and 4, at the scale $\mu = 2\,\text{GeV}$, in order to facilitate the comparison with future lattice determinations of these quantities. The mixing of K-meson parameters with operators of lower twist depending on m_s is numerically small.

The parameters related to twist-2 matrix elements have been determined using various methods; see the discussion in section 2. Matrix elements of twist-3 and 4 operators for the

K	$\mu = 1\,\text{GeV}$	$\mu = 2\,\text{GeV}$	π	$\mu = 1\,\text{GeV}$	$\mu = 2\,\text{GeV}$	Remarks
$\overline{m}_s$	137 ± 27	100 ± 20	$\overline{m}_q$	5.6 ± 1.6	4.1 ± 1.1	in units of MeV; see section 2
a_1^K	0.06 ± 0.03	0.05 ± 0.02	a_1^π	0	0	taken from ref. [10]; G-odd
a_2^K	0.25 ± 0.15	0.17 ± 0.10	a_2^π	0.25 ± 0.15	0.17 ± 0.10	SU(3) breaking small; average over various refs.
f_{3K}	0.45 ± 0.15	0.33 ± 0.11	$f_{3\pi}$	0.45 ± 0.15	0.31 ± 0.10	in units of $10^{-2}\,\text{GeV}^2$
ω_{3K}	-1.2 ± 0.7	-0.9 ± 0.5	$\omega_{3\pi}$	-1.5 ± 0.7	-1.1 ± 0.5	
λ_{3K}	1.6 ± 0.4	1.45 ± 0.35	$\lambda_{3\pi}$	0	0	G-odd
δ_K^2	0.20 ± 0.06	0.17 ± 0.05	δ_π^2	0.18 ± 0.06	0.14 ± 0.05	in units of GeV^2
κ_{4K}	-0.09 ± 0.02	-0.10 ± 0.02	$\kappa_{4\pi}$	0	0	taken from ref. [11]; G-odd
ω_{4K}	0.2 ± 0.1	0.13 ± 0.07	$\omega_{4\pi}$	0.2 ± 0.1	0.13 ± 0.07	taken from ref. [17]; SU(3) breaking not incl.

Table 3: Hadronic parameters for the K DAs. We also give the corresponding parameters for the π, which are a by-product of our calculations. All parameters have been calculated in this paper at $\mu = 1\,\text{GeV}$, unless stated otherwise. The evolution between 1 and 2 GeV is done at NLO accuracy for $\overline{m}_{q,s}$ and $a_{1,2}^{\pi,K}$, and at LO accuracy for the other parameters. The twist-4 parameters θ_i^K, ϕ_i^K etc. are given by eq. (4.23), based on the renormalon model.

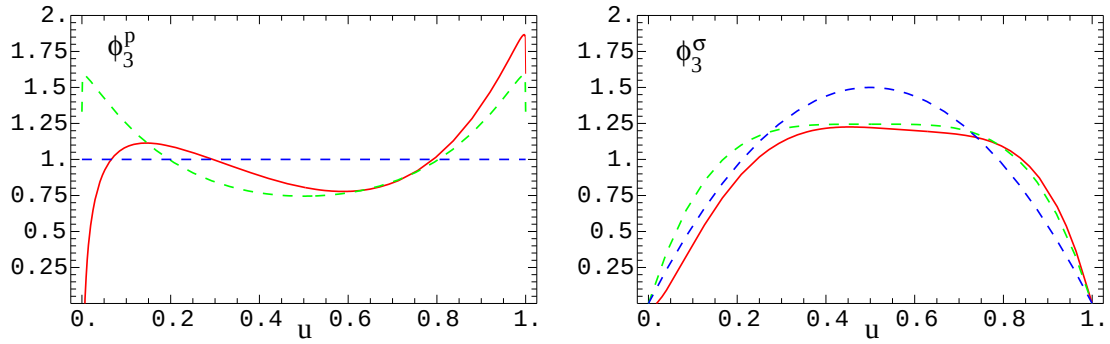

Figure 1: Left panel: ϕ_3^p as a function of u for the central value of the hadronic parameters, for $\mu = 1\,\text{GeV}$. Red (solid) line: $\phi_{3;K}^p$, green (long dashed): $\phi_{3;\pi}^\sigma$, blue (short dashed): asymptotic DA. Right panel: same for ϕ_3^σ.

π meson were calculated a long time ago from QCD sum rules [51, 52, 17]. In this paper, we perform a complete reanalysis of these sum rules and also include SU(3)-breaking effects relevant for the K meson. The corresponding sum rules and plots are given in the appendices. One important result is that we cannot confirm the sum rule for $f_{3\pi}$ derived in ref. [52] and that our numerical value is considerably larger than that found in this paper. On the other hand, our central value for δ_π^2 is similar to the one obtained in ref. [51], see also ref. [37].

Finally, in figure 1 we plot the twist-3 and -4 two-particle DAs for the π meson, assuming massless quarks, and for the K meson, together with the corresponding asymptotic DAs. The figures show that quark-mass corrections significantly modify the end-point behaviour of ϕ_3^p, where they induce a logarithmic end-point divergency, even if the contributions of

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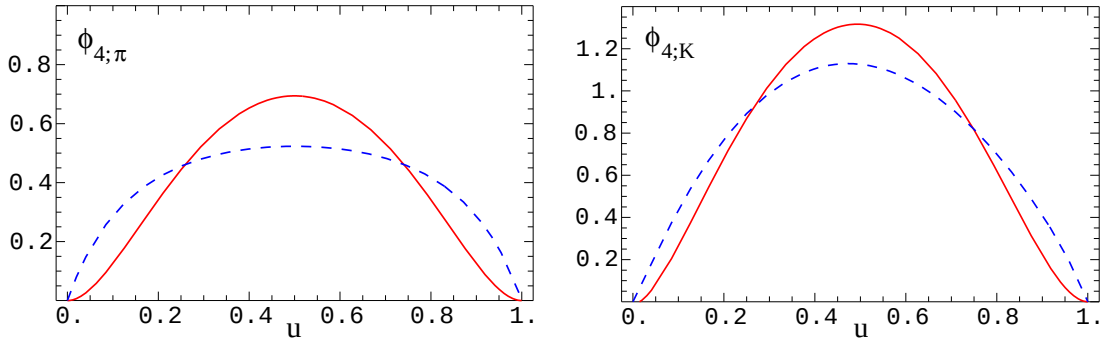

Figure 2: Left panel: $\phi_{4;\pi}$ as a function of u for the central value of the hadronic parameters, for $\mu = 1\,\text{GeV}$. Red (solid) line: $\phi_{4;\pi}$ in conformal expansion, blue (dashed): $\phi_{4;\pi}$ using the renormalon model $\phi_{4;\pi}^{T4,\text{ren}}$ for the genuine twist-4 corrections. Right panel: same for $\phi_{4;K}$.

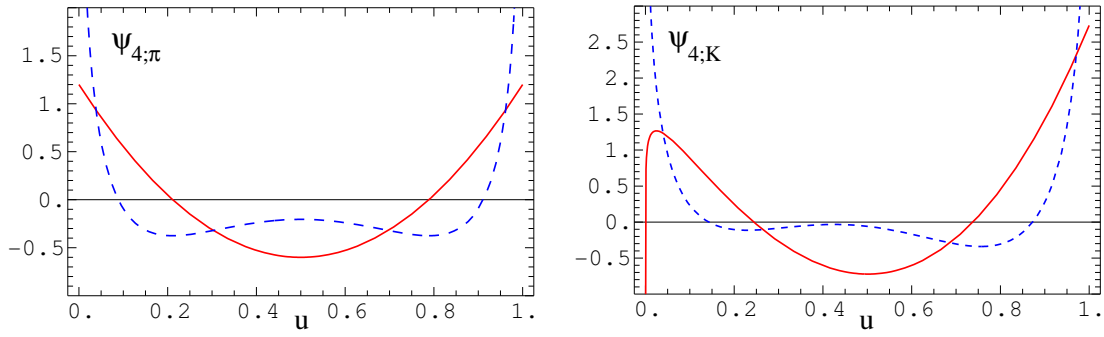

Figure 3: Same as figure 2 for ψ_4.

gluonic operators are neglected. This is not a problem because, as mentioned above, the DAs themselves need not be finite, it is only their convolution with perturbative scattering amplitudes that is meaningful. In figures 2 and 3 we show the twist-4 two-particle DAs ϕ_4 and ψ_4, also for the π (left panels) and the K (right panels). The solid (red) curve in figure 2 is obtained from eq. (4.32) using the conformal expansion (4.33) to NLO in the conformal spin, whereas the dashed (blue) curve includes the higher-spin contributions to the genuine twist-4 corrections as given by the renormalon model (4.35). The mass corrections $\phi_{4;\pi}^{WW}$ vanish for the pion. It is clear that the higher-order contributions induced by (4.35) modify both the end-point behaviour of $\phi_{4;\pi}$ and the size of the DA away from the end-points. For the K, the absolute difference between both curves at, say, $u = \frac{1}{2}$, is very nearly the same as for π, but the relative difference is much reduced because of large SU(3)-breaking effects induced by the mass-dependent contribution $\phi_{4;K}^{WW}$. Also note that the asymmetry of the curves induced by the non-vanishing value of a_1^K is not very pronounced, which is due to the smallness of that parameter as compared to $a_0^K = 1$ and a_2^K. In figure 3 we plot ψ_4, with the same meaning of the curves as in figure 2. Also here it is obvious that the renormalon model modifies the end-point behaviour of the DA, in particular for $\psi_{4;K}$, where it changes the sign of the logarithmic divergence at $u = 0$.

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6. Summary and conclusions

In this paper we have studied the twist-3 and -4 two- and three-particle distribution amplitudes of K-mesons in QCD and expressed them in a model-independent way by a minimal number of non-perturbative parameters. The work presented here is an extension of refs. [17, 22, 21] and completes the analysis of SU(3)-breaking corrections by also including G-parity-breaking corrections in $m_s - m_q$. Our approach consists of two components. One is the use of the QCD equations of motion, which allow dynamically dependent DAs to be expressed in terms of independent ones. The other ingredient is conformal expansion, which makes it possible to separate transverse and longitudinal variables in the wave functions, the former ones being governed by renormalization-group equations, the latter ones being described in terms of irreducible representations of the corresponding symmetry group. We have derived expressions for all twist-3 and -4 two- and three-particle distribution amplitudes to next-to-leading order in the conformal expansion, including both chiral corrections $\mathcal{O}(m_s + m_q)$ and G-parity-breaking corrections $\mathcal{O}(m_s - m_q)$; the corresponding formulas are given in sections 3 and 4. We have also generalized the renormalon model of ref. [21] to describe SU(3)-breaking contributions to high-order conformal partial waves.

We have done a complete reanalysis of the numerical values of the relevant higher-twist hadronic parameters from QCD sum rules. Our sum rules can be compared, in the chiral limit, with existing calculations for the π [51, 52]. We confirm the sum rule for the twist-4 matrix element δ_π^2 quoted in ref. [51], but obtain different results for the twist-3 matrix elements given in ref. [52], which lead to a 50% increase in the numerical value of the coupling $f_{3\pi}$. Whenever possible, we have aimed at determining these matrix elements from more than one sum rule; we find mutually consistent results, which provides a consistency check of the approach. We have also studied the scale-dependence of all parameters to leading-logarithmic, or, if possible, next-to-leading-logarithmic accuracy, taking into account the mixing with operators depending on the strange-quark mass m_s. Our final numerical results, at the scales 1 and 2 GeV, are collected in table 3.

We hope that our results will contribute to a better understanding of SU(3)-breaking effects in hard exclusive processes and in particular in the decays of B and B_s mesons into final states containing K mesons.

Acknowledgments

P.B. is grateful to the University of Regensburg for hospitality.

A. Non-local operator identities

For completeness, we quote the following non-local operator identities from ref. [50]:

$$\frac{\partial}{\partial x_\mu}\,\bar q(x)\gamma_\mu\gamma_5 s(-x) = -i\int_{-1}^{1} dv\, v\bar q(x)x_\alpha g G_{\alpha\mu}(vx)\gamma_\mu\gamma_5 s(-x) + (m_q - m_s)\bar q(x) i\gamma_5 s(-x), \tag{A.1}$$

$$\partial_\mu\{\bar q(x)\gamma_\mu\gamma_5 s(-x)\} = -i\int_{-1}^{1} dv\,\bar q(x)x_\alpha g G_{\alpha\mu}(vx)\gamma_\mu\gamma_5 s(-x) + (m_s+m_q)\bar q(x) i\gamma_5 s(-x), \tag{A.2}$$

$$\partial_\mu \bar q(x)\sigma_{\mu\nu}\gamma_5 s(-x) = -i\frac{\partial}{\partial x_\nu}\bar q(x)\gamma_5 s(-x) + \int_{-1}^{1} dv\, v\bar q(x)x_\rho g G_{\rho\nu}(vx)\gamma_5 s(-x) - i\int_{-1}^{1} dv\,\bar q(x)x_\rho g G_{\rho\mu}(vx)\sigma_{\mu\nu}\gamma_5 s(-x) + (m_s-m_q)\bar q(x)\gamma_\nu\gamma_5 s(-x), \tag{A.3}$$

$$\frac{\partial}{\partial x_\mu}\bar q(x)\sigma_{\mu\nu}\gamma_5 s(-x) = -i\partial_\nu\bar q(x)\gamma_5 s(-x) + \int_{-1}^{1} dv\,\bar q(x)x_\rho g G_{\rho\nu}(vx)\gamma_5 s(-x) - i\int_{-1}^{1} dv\, v\bar q(x)x_\rho g G_{\rho\mu}(vx)\sigma_{\mu\nu}\gamma_5 s(-x) - (m_s+m_q)\bar q(x)\gamma_\nu\gamma_5 s(-x). \tag{A.4}$$

Here ∂_μ is the total derivative defined as

$$\partial_\mu\{\bar q(x)\Gamma s(-x)\} \equiv \frac{\partial}{\partial y_\mu}\{\bar q(x+y)[x+y,-x+y]\Gamma s(-x+y)\}\Big|_{y\to 0}.$$

By taking matrix elements of the above relations between the vacuum and the meson state, one obtains exact integral representations for those DAs that are not dynamically independent.

B. Sum rules for twist-2 matrix elements

In this appendix we list and evaluate the QCD sum rules for twist-2 matrix elements of the K. The sum rule for f_K, including SU(3)-breaking corrections, was calculated in refs. [53, 10], that for a_1^K in refs. [8, 10], and that for a_2^K in ref. [7], apart from the perturbative terms in m_s^2 and the radiative corrections to the quark condensate, which are new. The sum rules read:

$$f_K^2 e^{-m_K^2/M^2} = \frac{1}{4\pi^2}\int_{m_s^2}^{s_0} ds\, e^{-s/M^2}\,\frac{(s-m_s^2)^2(s+2m_s^2)}{s^3} + \frac{\alpha_s}{\pi}\frac{M^2}{4\pi^2}\left(1-e^{-s_0/M^2}\right) + \frac{m_s\langle\bar s s\rangle}{M^2}\left(1+\frac{m_s^2}{3M^2}+\frac{13}{9}\frac{\alpha_s}{\pi}\right) + \frac{1}{12M^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\left(1+\frac{1}{3}\frac{m_s^2}{M^2}\right) + \frac{4}{3}\frac{\alpha_s}{\pi}\frac{m_s\langle\bar q q\rangle}{M^2} + \frac{16\pi\alpha_s}{9M^4}\langle\bar q q\rangle\langle\bar s s\rangle + \frac{16\pi\alpha_s}{81M^4}\left(\langle\bar q q\rangle^2+\langle\bar s s\rangle^2\right), \tag{B.1}$$

$$a_1^K f_K^2 e^{-m_K^2/M^2} = \frac{5}{4\pi^2}m_s^4\int_{m_s^2}^{s_0} ds\, e^{-s/M^2}\,\frac{(s-m_s^2)^2}{s^4} + \frac{5m_s^2}{18M^4}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\left(-\frac{1}{2}+\gamma_E-\mathrm{Ei}\left(-\frac{s_0}{M^2}\right)+\ln\frac{m_s^2}{M^2}+\frac{M^2}{s_0}\left(\frac{M^2}{s_0}-1\right)e^{-s_0/M^2}\right)$$

$\langle\bar{q}q\rangle = (-0.24 \pm 0.01)^3\,\text{GeV}^3$	$\langle\bar{s}s\rangle = (1-\delta_3)\,\langle\bar{q}q\rangle$
$\langle\bar{q}\sigma gGq\rangle = m_0^2\,\langle\bar{q}q\rangle$	$\langle\bar{s}\sigma gGs\rangle = (1-\delta_5)\langle\bar{q}\sigma gGq\rangle$
$\left\langle\frac{\alpha_s}{\pi}\,G^2\right\rangle = (0.012 \pm 0.006)\,\text{GeV}^4$	
$m_0^2 = (0.8 \pm 0.1)\,\text{GeV}^2, \quad \delta_3 = 0.2 \pm 0.2, \quad \delta_5 = 0.2 \pm 0.2$	
$\overline{m}_s(2\,\text{GeV}) = (100 \pm 20)\,\text{MeV} \quad \longleftrightarrow \quad \overline{m}_s(1\,\text{GeV}) = (137 \pm 27)\,\text{MeV}$	
$\alpha_s(m_Z) = 0.1187 \pm 0.002 \;\longleftrightarrow\; \alpha_s(1\,\text{GeV}) = 0.53^{+0.06}_{-0.05}$	

Table 4: Input parameters for sum rules at the renormalization scale $\mu = 1\,\text{GeV}$. The value of m_s is obtained from unquenched lattice calculations with $N_f = 2$ flavours as summarized in ref. [45], which agrees with the results from QCD sum-rule calculations [47]. $\alpha_s(m_Z)$ is the PDG average [3].

$$
\begin{aligned}
&-\frac{5}{3}\,\frac{m_s\langle\bar{s}s\rangle}{M^2}\left\{1+\frac{\alpha_s}{\pi}\left[-\frac{124}{27}+\frac{8}{9}\left(1-\gamma_E+\ln\frac{M^2}{\mu^2}+\frac{M^2}{s_0}\,e^{-s_0/M^2}+\text{Ei}\left(-\frac{s_0}{M^2}\right)\right)\right]\right\}\\
&-\frac{5}{3}\,\frac{m_s^3\langle\bar{s}s\rangle}{M^4}-\frac{20}{27}\,\frac{\alpha_s}{\pi}\,\frac{m_s\langle\bar{q}q\rangle}{M^2}+\frac{5}{9}\,\frac{m_s\langle\bar{s}\sigma gGs\rangle}{M^4}+\frac{80\pi\alpha_s}{81M^4}\left(\langle\bar{q}q\rangle^2-\langle\bar{s}s\rangle^2\right), \qquad \text{(B.2)}
\end{aligned}
$$

$$
\begin{aligned}
&a_2^K f_K^2 e^{-m_K^2/M^2} =\\
&\frac{7}{4\pi^2}\,m_s^4\int_{m_s^2}^{s_0} ds\,e^{-s/M^2}\,\frac{(s-m_s^2)^2(2m_s^2-s)}{s^5}+\frac{7}{72\pi^2}\,\frac{\alpha_s}{\pi}\,M^2(1-e^{-s_0/M^2})+\frac{7}{36M^2}\left\langle\frac{\alpha_s}{\pi}\,G^2\right\rangle\\
&+\frac{7}{3}\,\frac{m_s\langle\bar{s}s\rangle}{M^2}\left\{1+\frac{\alpha_s}{\pi}\left[-\frac{184}{27}+\frac{25}{18}\left(1-\gamma_E+\ln\frac{M^2}{\mu^2}+\frac{M^2}{s_0}\,e^{-s_0/M^2}+\text{Ei}\left(-\frac{s_0}{M^2}\right)\right)\right]\right\}\\
&-\frac{49}{27}\,\frac{\alpha_s}{\pi}\,\frac{m_s\langle\bar{q}q\rangle}{M^2}-\frac{35}{18}\,\frac{m_s\langle\bar{s}\sigma gGs\rangle}{M^4}+\frac{224\pi\alpha_s}{81M^4}\left(\langle\bar{q}q\rangle^2+\langle\bar{s}s\rangle^2\right)+\frac{112\pi\alpha_s}{81M^4}\,\langle\bar{q}q\rangle\langle\bar{s}s\rangle. \qquad \text{(B.3)}
\end{aligned}
$$

We evaluate the sum rules using the input given in table 4. The results for f_K and a_2^K are shown in figure 4; f_K depends rather sensitively on the choice of s_0. In order to reproduce the experimental result $f_K = 160\,\text{MeV}$, one has to choose $s_0 = 1.1\,\text{GeV}^2$. This is the value we will use also for all other sum rules for K matrix elements. For a_2^K, we then find

$$a_2^K(1\,\text{GeV}) = 0.30 \pm 0.15, \qquad \text{(B.4)}$$

which is slightly larger than the result obtained in ref. [7] and agrees with that obtained in ref. [8]. For a_1^K, we obtain the same result as refs. [10, 11]:

$$a_1^K(1\,\text{GeV}) = 0.06 \pm 0.03. \qquad \text{(B.5)}$$

C. Sum rules for twist-3 matrix elements

In this appendix we estimate the parameters of the twist-3 distribution amplitudes f_{3K}, λ_{3K} and ω_{3K} from QCD sum rules. Our approach is similar to that of ref. [52], where $f_{3\pi}$

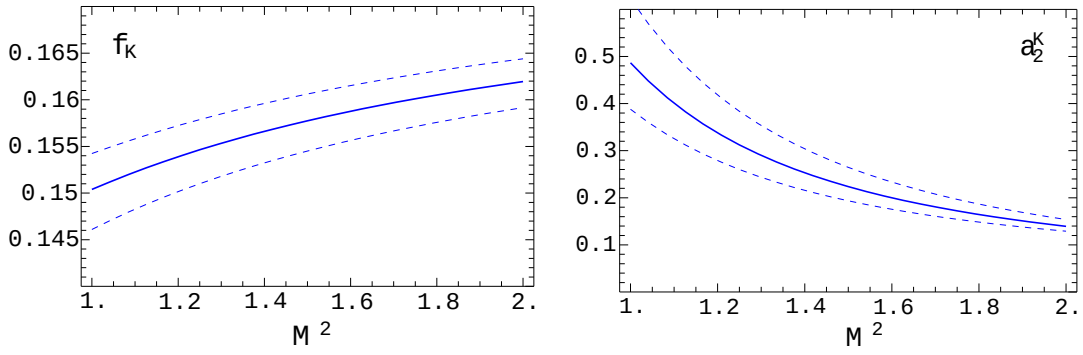

Figure 4: Left panel: f_K as function of the Borel parameter M^2 for $s_0 = 1.1\,\text{GeV}^2$. Solid line: central values of input parameters, dashed lines: variation of f_K within the allowed range of input parameters. Figure taken from ref. [10]. Right panel: same for a_2^K at the scale $\mu = 1\,\text{GeV}$. The results for a_2^K are new.

and $\omega_{3\pi}$ have been determined, and based on the calculation of the correlation function of a non-local light-ray operator, which enters the definition of the three-particle distribution amplitude (3.3), with the corresponding local operator:

$$\begin{aligned}\Pi_D &= i\int d^4y\, e^{-ipy}\langle 0|T\bar q(z)i\sigma_{\mu z}\gamma_5 gG_{\mu z}(vz)s(0)\bar s(y)i\sigma_{\nu z}\gamma_5 gG_{\nu z}(y)q(y)|0\rangle \\ &\equiv (pz)^4\int \mathcal{D}\underline{\alpha}\, e^{-ipz(\alpha_2+v\alpha_3)}\,\pi_D(\underline{\alpha})\,. \end{aligned} \tag{C.1}$$

We also study the correlation function of that operator with the pseudoscalar current:[8]

$$\begin{aligned}\Pi_{ND} &= i\int d^4y\, e^{-ipy}\langle 0|T\bar q(z)\sigma_{\mu z}\gamma_5 gG_{\mu z}(vz)s(0)\bar s(y)\gamma_5 q(y)|0\rangle \\ &\equiv (pz)^2\int \mathcal{D}\underline{\alpha}\, e^{-ipz(\alpha_2+v\alpha_3)}\,\pi_{ND}^{(1)}(\underline{\alpha})\,; \end{aligned} \tag{C.2}$$

for brevity, we do not show the Wilson lines in the non-local operators. Our calculation goes beyond that done in ref. [52] by including SU(3)-breaking corrections, and by also studying sum rules based on the non-diagonal correlation function, which allows a non-trivial consistency check of the results.

Somewhat imprecisely, we will refer to Π_D and Π_{ND} as "diagonal" and "non-diagonal" correlation functions, respectively. The hadronic representation of the non-diagonal correlation function Π_{ND} only contains pseudoscalar $J^P = 0^-$ contributions, whereas the diagonal correlation function Π_D also contains contributions of states with higher spin, $J^P = 2^-$ and $J^P = 1^+$. This is not a disadvantage, since such states all have considerably higher masses than the K meson, and can effectively be thought of as parts of the continuum contribution. For reasons that will become clear below, we have also calculated a

[8] Note that the currents in Π_{ND} contain no factors i, in contrast to Π_D. This is so as to obtain a positive spectral density.

correlation function similar to (C.1), but with operators of opposite parity:

$$\begin{aligned}\bar\Pi_D &= i\int d^4y\, e^{-ipy}\langle 0|T\bar q(z)\sigma_{\mu z}gG_{\mu z}(vz)s(0)\bar s(y)\sigma_{\nu z}gG_{\nu z}(y)q(y)|0\rangle\\ &\equiv (pz)^4\int\mathcal{D}\underline{\alpha}\, e^{-ipz(\alpha_2+v\alpha_3)}\,\bar\pi_D(\underline{\alpha})\,.\end{aligned}\tag{C.3}$$

For the diagonal correlation function we find, using factorization approximation for the four-quark condensates and dropping terms that vanish after Borel transformation:

$$\begin{aligned}\pi_D(\underline{\alpha}) = {}& \frac{\alpha_s}{\pi^3}\,\alpha_1\alpha_2\alpha_3^2\,p^2\ln\frac{\mu^2}{-p^2} - \frac{2}{3}\,\frac{\alpha_s}{\pi}\,\frac{m_s\langle\bar ss\rangle}{p^2}\,\alpha_2\alpha_3^2\delta(\alpha_1) - \frac{2}{3}\,\frac{\alpha_s}{\pi}\,\frac{m_q\langle\bar qq\rangle}{p^2}\,\alpha_1\alpha_3^2\delta(\alpha_2)\\ &+\frac{\alpha_s}{\pi}\,\frac{m_s\langle\bar s\sigma gGs\rangle}{p^4}\left(-\frac{7}{72}\,\alpha_3^2+\frac{1}{4}\,\alpha_2\alpha_3+\frac{1}{9}\,i(pz)\alpha_2\alpha_3^2\right)\delta(\alpha_1)\\ &+\frac{\alpha_s}{\pi}\,\frac{m_q\langle\bar q\sigma gGq\rangle}{p^4}\left(-\frac{7}{72}\,\alpha_3^2+\frac{1}{4}\,\alpha_1\alpha_3-\frac{1}{9}\,i(pz)\alpha_1\alpha_3^2\right)\delta(\alpha_2)\\ &+\frac{\alpha_s^2\langle\bar ss\rangle^2}{p^4}\left(\frac{44}{243}\,\alpha_3^2+\frac{2}{9}\,\alpha_2\alpha_3-\frac{32}{243}\,i(pz)\alpha_2\alpha_3^2\right)\delta(\alpha_1)\\ &+\frac{\alpha_s^2\langle\bar qq\rangle^2}{p^4}\left(\frac{44}{243}\,\alpha_3^2+\frac{2}{9}\,\alpha_1\alpha_3+\frac{32}{243}\,i(pz)\alpha_1\alpha_3^2\right)\delta(\alpha_2)\\ &+\frac{32}{27}\,\frac{\alpha_s^2\langle\bar ss\rangle\langle\bar qq\rangle}{p^4}\Big(\alpha_1\alpha_3\,\delta(\alpha_2)+\alpha_2\alpha_3\,\delta(\alpha_1)\Big).\end{aligned}\tag{C.4}$$

To this accuracy, the expressions for π_D and $\bar\pi_D$ are almost the same, the only difference being that in $\bar\pi_D$ the last term in (C.4), the contribution of $\langle\bar qq\rangle\langle\bar ss\rangle$, comes with a minus sign. In the chiral limit, we can compare the above result with that obtained in ref. [52]: we find agreement for the perturbative contribution, but a different answer for the contribution of the four-quark condensates. The leading-order contribution $\mathcal{O}(\alpha_s)$ of the gluon condensate as well as that of the dimension-6 triple-gluon condensate $\langle g^3fG^3\rangle$ both vanish. We also have calculated the contribution of the gluon condensate in the local limit, $e^{-ipz(\alpha_2+v\alpha_3)}\to 1$, and find

$$\Pi_D\big|_{\langle\frac{\alpha_s}{\pi}G^2\rangle} = \bar\Pi_D\big|_{\langle\frac{\alpha_s}{\pi}G^2\rangle} = -\frac{89}{5184}\,\frac{\alpha_s}{\pi}\,\left\langle\frac{\alpha_s}{\pi}\,G^2\right\rangle\,\frac{(pz)^4}{p^2},\tag{C.5}$$

which differs from the result obtained in ref. [52]. In particular, we do not reproduce the logarithmic term quoted in [52].

For the non-diagonal correlation function we find

$$\begin{aligned}\pi_{ND}(\underline{\alpha}) = {}& \frac{\alpha_s}{2\pi^3}\,\alpha_1\alpha_2\alpha_3\left(\frac{1}{1-\alpha_1}+\frac{1}{1-\alpha_2}\right)p^2\ln\frac{\mu^2}{-p^2}\\ &+\frac{1}{12}\left\langle\frac{\alpha_s}{\pi}\,G^2\right\rangle\,\frac{\alpha_1\alpha_2\delta(\alpha_3)}{\alpha_1m_q^2+\alpha_2m_s^2-\alpha_1\alpha_2p^2}\\ &+\frac{\alpha_s}{3\pi}\frac{1}{p^2}\Big[m_q\langle\bar qq\rangle\alpha_1^2\delta(\alpha_2)+m_s\langle\bar ss\rangle\alpha_2^2\delta(\alpha_1)\Big]\\ &+\frac{2\alpha_s}{3\pi}\frac{1}{p^2}\,\left[\alpha_3+\alpha_3^2\left(\ln\frac{\mu^2}{-p^2}-\ln(\bar\alpha_3\alpha_3)-1\right)\right]\Big[m_s\langle\bar qq\rangle\delta(\alpha_2)+m_q\langle\bar ss\rangle\delta(\alpha_1)\Big]\end{aligned}$$

$$
\begin{aligned}
&+\left[\frac{16}{27}\pi\alpha_s\langle\bar{s}s\rangle^2+\frac{1}{6}m_s\langle\bar{s}\sigma gGs\rangle\right]\frac{1}{p^4}\delta(\alpha_1)\delta(\alpha_3)\\
&+\left[\frac{16}{27}\pi\alpha_s\langle\bar{q}q\rangle^2+\frac{1}{6}m_q\langle\bar{q}\sigma gGq\rangle\right]\frac{1}{p^4}\delta(\alpha_2)\delta(\alpha_3)\\
&+\frac{16\pi\alpha_s}{9p^4}\langle\bar{q}q\rangle\langle\bar{s}s\rangle\delta(\alpha_1)\delta(\alpha_2). \qquad \text{(C.6)}
\end{aligned}
$$

The sum rules for the couplings f_{3K}, λ_{3K} and ω_{3K} are derived by expanding the correlation functions in powers of (pz):

$$
\begin{aligned}
\Pi_D &= (pz)^4\left\{\Pi_D^{(0)}+i(pz)\left[\Pi_D^{(\lambda)}+(2v-1)\Pi_D^{(\omega)}\right]+\mathcal{O}((pz)^2)\right\},\\
\Pi_{ND} &= (pz)^2\left\{\Pi_{ND}^{(0)}+i(pz)\left[\Pi_{ND}^{(\lambda)}+(2v-1)\Pi_{ND}^{(\omega)}\right]+\mathcal{O}((pz)^2)\right\}. \qquad \text{(C.7)}
\end{aligned}
$$

Comparing these expressions with the corresponding expansion of the K contribution to the correlation functions expressed in terms of the DA (3.5), one obtains

$$
\begin{aligned}
4f_{3K}^2e^{-m_K^2/M^2} &= \mathcal{B}\left[\Pi_D^{(0)}\right](M^2),\\
\frac{1}{7}\,f_{3K}^2\lambda_{3K}e^{-m_K^2/M^2} &= \mathcal{B}\left[\Pi_D^{(\lambda)}+\frac{1}{2}\Pi_D^{(0)}\right](M^2),\\
-\frac{3}{14}\,f_{3K}^2\omega_{3K}e^{-m_K^2/M^2} &= \mathcal{B}\left[\Pi_D^{(\omega)}+\frac{3}{14}\Pi_D^{(0)}\right](M^2), \qquad \text{(C.8)}
\end{aligned}
$$

and similarly

$$
\begin{aligned}
2f_{3K}\frac{f_Km_K^2}{m_s+m_q}e^{-m_K^2/M^2} &= \mathcal{B}\left[\Pi_{ND}^{(0)}\right](M^2),\\
\frac{1}{14}\,f_{3K}\lambda_{3K}\,\frac{f_Km_K^2}{m_s+m_q}\,e^{-m_K^2/M^2} &= \mathcal{B}\left[\Pi_{ND}^{(\lambda)}+\frac{1}{2}\Pi_{ND}^{(0)}\right](M^2),\\
-\frac{3}{28}\,\omega_{3K}f_{3K}\,\frac{f_Km_K^2}{m_s+m_q}\,e^{-m_K^2/M^2} &= \mathcal{B}\left[\Pi_{ND}^{(\omega)}+\frac{3}{14}\Pi_{ND}^{(0)}\right](M^2), \qquad \text{(C.9)}
\end{aligned}
$$

from the diagonal and non-diagonal correlation functions, respectively. Here and below $\mathcal{B}[\ldots](M^2)$ stands for the Borel transformation with respect to p^2; M^2 is the Borel parameter.

From Π_D, we obtain the following sum rule for f_{3K}:

$$
\begin{aligned}
4\,f_{3K}^2\big|_D\,e^{-m_K^2/M^2} &= \frac{\alpha_s}{360\pi^3}\int_0^{s_0}ds s e^{-s/M^2}+\frac{\alpha_s}{18\pi}\left(m_s\langle\bar{s}s\rangle+m_q\langle\bar{q}q\rangle\right)\\
&\quad+\frac{89}{5184}\,\frac{\alpha_s}{\pi}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle+\frac{\alpha_s}{108\pi}\,\frac{1}{M^2}\left(m_s\langle\bar{s}\sigma gGs\rangle+m_q\langle\bar{q}\sigma gGq\rangle\right)\\
&\quad+\frac{71}{729}\,\frac{\alpha_s^2}{M^2}\left(\langle\bar{q}q\rangle^2+\langle\bar{s}s\rangle^2\right)+\frac{32}{81}\,\frac{\alpha_s^2}{M^2}\,\langle\bar{q}q\rangle\langle\bar{s}s\rangle, \qquad \text{(C.10)}
\end{aligned}
$$

where the subscript D indicates that this sum rule is derived from the correlation function Π_D. The last term on the right-hand side comes from the factorisation of the four-quark condensate $(\bar{q}\sigma_{\mu\nu}t^Aq)(\bar{q}\sigma_{\mu\nu}t^Aq)$. In ref. [52], the authors have argued that this term, which

induces a large power correction in their sum rule for $f_{3\pi}$, is unreliable because of a potential breakdown of the factorisation approximation for that particular condensate; they suggested to determine $f_{3\pi}$ from a sum rule derived from the sum of the correlation functions $\Pi_D+\bar\Pi_D$ instead, where these large contributions cancel. Indeed, the Dirac structures $\sigma_{\mu\nu}$ and $i\sigma_{\mu\nu}\gamma_5$ are not independent, but related by $i\sigma_{\mu\nu}\gamma_5 = -\frac{1}{2}\,\epsilon_{\mu\nu\rho\sigma}\sigma_{\rho\sigma}$, which induces the relation

$$\begin{aligned}\bar\Pi_D &= i\int d^4y\, e^{-ipy}\langle 0|T\bar q(0)i\sigma_{\mu z}\gamma_5 gG_{\nu z}(0)s(0)\bar s(y)i\sigma_{\mu z}\gamma_5 gG_{\nu z}(y)q(y)|0\rangle \\ &\quad - i\int d^4y\, e^{-ipy}\langle 0|T\bar q(0)i\sigma_{\mu z}\gamma_5 gG_{\nu z}(0)s(0)\bar s(y)i\sigma_{\nu z}\gamma_5 gG_{\mu z}(y)q(y)|0\rangle\,.\end{aligned}$$

$\bar\Pi_D$ receives no contributions from 0^- states because their contributions to the two correlation functions on the right-hand side are equal and cancel in the difference; the same applies to 1^+ states, so that the lowest resonance contributing to $\bar\Pi_D$ is 1^-. These states can safely be included in the continuum so that it is possible to extract f_{3K} from the sum of correlation functions $\Pi_D+\bar\Pi_D$. On the other hand, our sum rule (C.10), derived from Π_D only, with the correct coefficients for gluon and four-quark condensates, is actually not very sensitive to the term in $\langle\bar qq\rangle\langle\bar ss\rangle$, but dominated by the gluon condensate. As there is no strong theoretical argument in favour or disfavour of either diagonal sum rule, the one based on Π_D and the one based on $\Pi_D+\bar\Pi_D$, we decide to use both. We also determine f_{3K} from a third sum rule based on the non-diagonal correlation function Π_{ND}; the difference between these three results will be interpreted as theoretical uncertainty.

Explicitly, we obtain, in addition to (C.10), the following sum rules for f_{3K}, with the index indicating the underlying correlation function:

$$\begin{aligned}4\,f_{3K}^2\big|_{D+\bar D}\,e^{-m_K^2/M^2} &= \frac{\alpha_s}{180\pi^3}\int_0^{s_0} ds s e^{-s/M^2} + \frac{\alpha_s}{9\pi}\left(m_s\langle\bar ss\rangle + m_q\langle\bar qq\rangle\right) \\ &\quad + \frac{89}{2592}\,\frac{\alpha_s}{\pi}\left\langle\frac{\alpha_s}{\pi}\,G^2\right\rangle + \frac{\alpha_s}{54\pi}\,\frac{1}{M^2}\left(m_s\langle\bar s\sigma gGs\rangle + m_q\langle\bar q\sigma gGq\rangle\right) \\ &\quad + \frac{142}{729}\,\frac{\alpha_s^2}{M^2}\left(\langle\bar qq\rangle^2 + \langle\bar ss\rangle^2\right), \qquad \text{(C.11)}\end{aligned}$$

$$\begin{aligned}2f_{3K}|_{ND}\frac{f_K m_K^2}{m_s+m_q}e^{-m_K^2/M^2} &= \frac{\alpha_s}{72\pi^3}\int_0^{s_0} ds s e^{-s/M^2} + \frac{1}{12}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle - \frac{\alpha_s}{9\pi}(m_q\langle\bar qq\rangle + m_s\langle\bar ss\rangle) \\ &\quad - \frac{2}{9}\,\frac{\alpha_s}{\pi}(m_s\langle\bar qq\rangle + m_q\langle\bar ss\rangle)\left(\frac{8}{3} + \gamma_E - \ln\frac{M^2}{\mu^2} + \int_{s_0}^\infty \frac{ds}{s}\,e^{-s/M^2}\right) \\ &\quad + \frac{1}{6M^2}(m_s\langle\bar s\sigma gGs\rangle + m_q\langle\bar q\sigma gGq\rangle) \\ &\quad + \frac{16}{27}\,\frac{\pi\alpha_s}{M^2}\left(\langle\bar qq\rangle^2 + \langle\bar ss\rangle^2\right) + \frac{16}{9}\,\frac{\pi\alpha_s}{M^2}\,\langle\bar qq\rangle\langle\bar ss\rangle\,. \qquad \text{(C.12)}\end{aligned}$$

The sum rules for $f_{3\pi}$ are obtained by taking the chiral limit of the above expressions.

As for ω_{3K}, we have not calculated the gluon-condensate contribution to the diagonal sum rule, which is expected to be dominant, so we cannot use the diagonal sum rule and

only consider the non-diagonal one:

$$
\begin{aligned}
2(f_{3K}\omega_{3K})|_{ND}\frac{f_K m_K^2}{m_s+m_q}e^{-m_K^2/M^2} &= -\frac{\alpha_s}{60\pi^3}\int_0^{s_0} ds s e^{-s/M^2} + \frac{5}{27}\frac{\alpha_s}{\pi}(m_q\langle\bar{q}q\rangle + m_s\langle\bar{s}s\rangle) \\
&- \frac{2}{3}\frac{\alpha_s}{\pi}(m_s\langle\bar{q}q\rangle + m_q\langle\bar{s}s\rangle)\left(\frac{8}{3}+\gamma_E - \ln\frac{M^2}{\mu^2} + \int_{s_0}^{\infty}\frac{ds}{s}\,e^{-s/M^2}\right) \\
&- \frac{1}{3}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle + \frac{2}{3M^2}(m_s\langle\bar{s}\sigma gGs\rangle + m_q\langle\bar{q}\sigma gGq\rangle) \\
&- \frac{64}{27}\frac{\pi\alpha_s}{M^2}\left(\langle\bar{q}q\rangle^2 + \langle\bar{s}s\rangle^2\right) + \frac{256}{27}\frac{\pi\alpha_s}{M^2}\langle\bar{q}q\rangle\langle\bar{s}s\rangle\,. \qquad \text{(C.13)}
\end{aligned}
$$

In evaluating this sum rule, we replace f_{3K} by the expression obtained from (C.12).

As for λ_{3K}, the gluon-condensate contribution is suppressed by a factor $m_s^2 - m_q^2$ by virtue of G-parity and can safely be neglected in the diagonal sum rule. We did calculate this contribution for the non-diagonal sum rule, though, where indeed it gives only a small contribution. On the other hand, the $\langle\bar{q}q\rangle\langle\bar{s}s\rangle$ contribution is also absent because of G-parity, so that the two diagonal sum rules for $f_{3K}^2\lambda_{3K}\big|_D$ and $f_{3K}^2\lambda_{3K}\big|_{D+\bar{D}}$ differ by a global factor 2. As the values of $f_{3K}^2\big|_D$ and $f_{3K}^2\big|_{D+\bar{D}}$ also differ by a factor of approximately 2, this theoretical uncertainty cancels to a large extent. The sum rules read:

$$
\begin{aligned}
4\,(f_{3K}^2\lambda_{3K})\big|_D\, e^{-m_K^2/M^2} &= -\frac{14}{45}\frac{\alpha_s}{\pi}\left(m_s\langle\bar{s}s\rangle - m_q\langle\bar{q}q\rangle\right) \\
&+ \frac{35}{512}\frac{\alpha_s}{\pi}\frac{1}{M^2}\left(m_s\langle\bar{s}\sigma gGs\rangle - m_q\langle\bar{q}\sigma gGq\rangle\right) + \frac{7}{9}\frac{\alpha_s^2}{M^2}\left(\langle\bar{q}q\rangle^2 - \langle\bar{s}s\rangle^2\right), \qquad \text{(C.14)}
\end{aligned}
$$

$$
\begin{aligned}
4\,(f_{3K}^2\lambda_{3K})\big|_{D+\bar{D}}\, e^{-m_K^2/M^2} &= -\frac{28}{45}\frac{\alpha_s}{\pi}\left(m_s\langle\bar{s}s\rangle - m_q\langle\bar{q}q\rangle\right) \\
&+ \frac{35}{216}\frac{\alpha_s}{\pi}\frac{1}{M^2}\left(m_s\langle\bar{s}\sigma gGs\rangle - m_q\langle\bar{q}\sigma gGq\rangle\right) + \frac{14}{9}\frac{\alpha_s^2}{M^2}\left(\langle\bar{q}q\rangle^2 - \langle\bar{s}s\rangle^2\right), \qquad \text{(C.15)}
\end{aligned}
$$

$$
\begin{aligned}
2\,(f_{3K}\lambda_{3K})|_{ND}\,\frac{f_K m_K^2}{m_s+m_q}\,e^{-m_K^2/M^2} &= \frac{7}{6}\frac{\alpha_s}{\pi}\left(m_s\langle\bar{s}s\rangle - m_q\langle\bar{q}q\rangle\right) \\
&- \frac{7}{9}\frac{\alpha_s}{\pi}(m_s\langle\bar{q}q\rangle - m_q\langle\bar{s}s\rangle)\left(\frac{8}{3}+\gamma_E - \ln\frac{M^2}{\mu^2} + \int_{s_0}^{\infty}\frac{ds}{s}\,e^{-s/M^2}\right) \\
&- \frac{7}{6M^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle(m_s^2 - m_q^2)\left(1+\gamma_E - \ln\frac{M^2}{\mu^2} - M^2\int_{s_0}^{\infty}\frac{ds}{s^2}\,e^{-s/M^2}\right) \\
&+ \frac{7}{3M^2}(m_q\langle\bar{q}\sigma gGq\rangle - m_s\langle\bar{s}\sigma gGs\rangle) + \frac{224}{27}\frac{\pi\alpha_s}{M^2}\left(\langle\bar{q}q\rangle^2 - \langle\bar{s}s\rangle^2\right). \qquad \text{(C.16)}
\end{aligned}
$$

Again, when evaluating these sum rules, we replace f_{3K} by the corresponding expressions obtained from (C.10), (C.11), and (C.12).

The numerical results from all these sum rules are shown in figures 5 to 7. As for $f_{3\pi}$ and f_{3K}, all three sum rules yield very similar results, which is a strong indication for the consistency of the approach. The diagonal sum rules are very stable in M^2, the non-diagonal ones less so. Taking into account the uncertainties of the input parameters

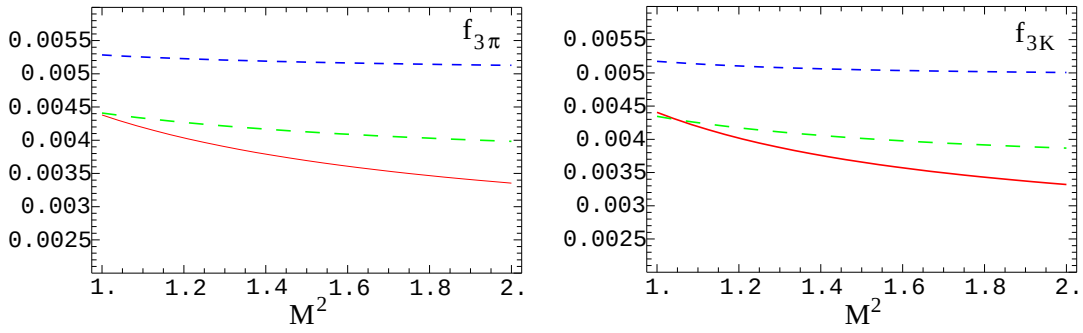

Figure 5: Left panel: $f_{3\pi}$ as a function of the Borel parameter, calculated from the non-diagonal sum rule (C.12) (red, solid line), the pure-parity diagonal sum rule (C.10) (green, long dashes) and the mixed-parity diagonal sum rule (C.11) (blue, short dashes); $s_0 = 0.8\,\mathrm{GeV}^2$. Right panel: same for f_{3K}; $s_0 = 1.1\,\mathrm{GeV}^2$.

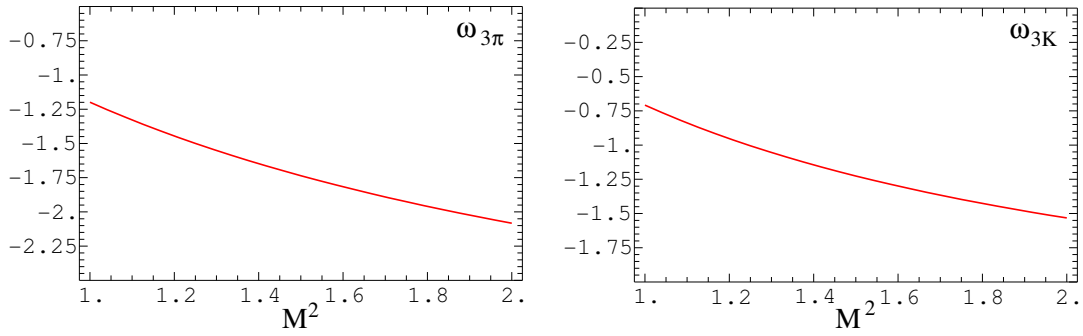

Figure 6: Left panel: $\omega_{3\pi}$ as a function of the Borel parameter from the non-diagonal sum rule (C.13); $s_0 = 0.8\,\mathrm{GeV}^2$. Right panel: same for ω_{3K}; $s_0 = 1.1\,\mathrm{GeV}^2$. The results from diagonal sum rules are not shown because the gluon-condensate contribution is unknown.

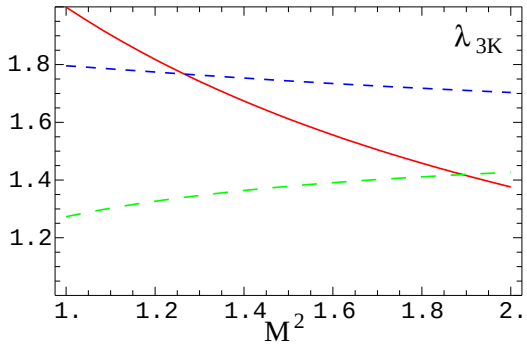

Figure 7: λ_{3K} as a function of the Borel parameter, calculated from the non-diagonal sum rule (C.16) (red, solid line), the pure-parity diagonal sum rule (C.14) (green, long dashes) and the mixed-parity diagonal sum rule (C.15) (blue, short dashes); $s_0 = 1.1\,\mathrm{GeV}^2$; $\lambda_{3\pi} = 0$ by virtue of G-parity.

as given in table 4 and the difference in the results from the different sum rules, we obtain the estimates

$$f_{3\pi}(1\,\text{GeV}) = (0.0045 \pm 0.0015)\,\text{GeV}^2, \quad f_{3K}(1\,\text{GeV}) = (0.0045 \pm 0.0015)\,\text{GeV}^2. \quad \text{(C.17)}$$

The effect of SU(3) breaking is very small,

$$f_{3K}/f_{3\pi} = 0.98 \pm 0.03\,, \quad \text{(C.18)}$$

as all sum rules are dominated by the contribution of the gluon condensate. Note that our value for $f_{3\pi}$ is about 50% larger than the one obtained in ref. [52], which is due to, as we believe, the incorrect results for the contributions of the gluon and four-quark condensate contributions obtained in this paper. As for ω_3, as explained above, we only evaluate the non-diagonal sum rule. We find that the sum rules are less stable in M^2, as with f_3, and that now the effect of SU(3) breaking is more prominent. Our final estimate is

$$\omega_{3\pi}(1\,\text{GeV}) = -1.5 \pm 0.7\,, \qquad \omega_{3K}(1\,\text{GeV}) = -1.2 \pm 0.7\,, \quad \text{(C.19)}$$

where the error reflects in particular the uncertainty of the value of the gluon condensate. Our result is to be compared with that of ref. [52], $\omega_{3\pi} \approx -3$. Finally, λ_{3K} can be determined from three sum rules, as the gluon-condensate contribution is suppressed by a factor m_s^2. All three sum rules yield perfectly consistent values, despite the fact that the two diagonal sum rules (C.14) and (C.15) differ by an overall factor of 2, which, as expected, is largely cancelled by the different values of $f_{3K}^2|_D$ and $f_{3K}^2|_{D+\bar{D}}$. We obtain

$$\lambda_{3K}(1\,\text{GeV}) = 1.6 \pm 0.4; \quad \text{(C.20)}$$

the error is smaller than for ω_{3K} because the gluon condensate is suppressed. This result is new.

D. Sum rules for twist-4 matrix elements

The aim of this section is to estimate the decay constant δ_K^2 that determines the normalization of twist-4 distribution amplitudes. To this end we define the currents

$$J_\mu^A = \bar{q}\, g\widetilde{G}_{\mu\alpha}\gamma_\alpha s\,, \qquad J_\mu^V = \bar{q}\, g\widetilde{G}_{\mu\alpha}\gamma_\alpha\gamma_5 s\,, \quad \text{(D.1)}$$

with quantum numbers $J^P = 1^+$ and 1^-, respectively, and calculate the correlation functions

$$\Pi_{\mu\nu}^{A,V} = i\int d^4x\, e^{ipx}\,\langle 0|TJ_\mu^{A,V}(x)(J_\nu^{A,V})^\dagger(0)|0\rangle = p_\mu p_\nu\,\Pi_0^{A,V}(p^2) - g_{\mu\nu}\,\Pi_1^{A,V}(p^2)\,, \quad \text{(D.2)}$$

taking into account contributions of operators with dimension up to eight. Note that the relative sign between f_K and δ_K^2 can be fixed from the non-diagonal correlation function of J_μ^A and the axial vector current. This calculation was done in ref. [51] and will not be repeated here; the result is that δ_K^2 is positive.

Similar correlation functions have been considered in the past, mainly in connection with searches for exotic quark-antiquark-gluon mesons [54]. We obtain

$$\begin{aligned}
\Pi_0^{A,V} &= \frac{\alpha_s}{160\pi^3}p^4\ln\frac{\mu^2}{-p^2} + \frac{1}{72}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\ln\frac{\mu^2}{-p^2} \\
&+ \frac{\alpha_s}{6\pi}[m_q\langle\bar{q}q\rangle + m_s\langle\bar{s}s\rangle]\ln\frac{\mu^2}{-p^2} \mp \frac{2\alpha_s}{9\pi}[m_s\langle\bar{q}q\rangle + m_q\langle\bar{s}s\rangle]\ln\frac{\mu^2}{-p^2} \\
&\mp \frac{8\pi\alpha_s}{9p^2}\langle\bar{q}q\rangle\langle\bar{s}s\rangle + 0\cdot\langle g^3fG^3\rangle \\
&+ \frac{5}{108}\frac{\alpha_s}{\pi}\frac{1}{p^2}[m_q\langle\bar{q}\sigma gGq\rangle + m_s\langle\bar{s}\sigma gGs\rangle] \\
&\pm \Big[\frac{1}{9}\ln\frac{\mu^2}{-p^2} + \frac{2}{27}\Big]\frac{\alpha_s}{\pi}\frac{1}{p^2}[m_s\langle\bar{q}\sigma gGq\rangle + m_q\langle\bar{s}\sigma gGs\rangle] \\
&- \frac{25\pi\alpha_s}{324p^4}m_0^2[\langle\bar{q}q\rangle^2 + \langle\bar{s}s\rangle^2] \pm \frac{143\pi\alpha_s}{162p^4}m_0^2\langle\bar{q}q\rangle\langle\bar{s}s\rangle \\
&+ \frac{\pi}{18p^4}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle[m_q\langle\bar{q}q\rangle + m_s\langle\bar{s}s\rangle]\,, \qquad \text{(D.3)}
\end{aligned}$$

$$\begin{aligned}
\Pi_1^{A,V} &= \frac{\alpha_s}{240\pi^3}p^6\ln\frac{\mu^2}{-p^2} - \frac{1}{36}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle p^2\ln\frac{\mu^2}{-p^2} \\
&+ \frac{\alpha_s}{6\pi}[m_q\langle\bar{q}q\rangle + m_s\langle\bar{s}s\rangle]p^2\ln\frac{\mu^2}{-p^2} \mp \frac{\alpha_s}{18\pi}[m_s\langle\bar{q}q\rangle + m_q\langle\bar{s}s\rangle]p^2\ln\frac{\mu^2}{-p^2} \\
&\mp \frac{8\pi\alpha_s}{9}\langle\bar{q}q\rangle\langle\bar{s}s\rangle - \frac{1}{192\pi^2}\cdot\langle g^3fG^3\rangle \\
&- \frac{19}{144}\frac{\alpha_s}{\pi}[m_q\langle\bar{q}\sigma gGq\rangle + m_s\langle\bar{s}\sigma gGs\rangle]\ln\frac{\mu^2}{-p^2} \\
&\pm \frac{19}{144}\frac{\alpha_s}{\pi}[m_s\langle\bar{q}\sigma gGq\rangle + m_q\langle\bar{s}\sigma gGs\rangle]\ln\frac{\mu^2}{-p^2} \\
&+ \frac{25\pi\alpha_s}{162p^2}m_0^2[\langle\bar{q}q\rangle^2 + \langle\bar{s}s\rangle^2] \pm \frac{181\pi\alpha_s}{162p^2}m_0^2\langle\bar{q}q\rangle\langle\bar{s}s\rangle \\
&+ \frac{\pi}{18p^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle[m_q\langle\bar{q}q\rangle + m_s\langle\bar{s}s\rangle] \mp \frac{\pi}{6p^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle[m_s\langle\bar{q}q\rangle + m_q\langle\bar{s}s\rangle]\,. \qquad \text{(D.4)}
\end{aligned}$$

In both cases the upper sign refers to the axial and the lower sign to the vector correlation function, respectively; Π_0^A has been calculated, in the chiral limit, in ref. [51]. The quark mass corrections and the expression for Π_1^A are new.

In this work we follow the procedure proposed in ref. [51] and write the sum rule directly for the correlation function Π_0^A:

$$f_K^2\delta_K^4 e^{-m_K^2/M^2} = \mathcal{B}[\Pi_0^A](M^2)\,. \qquad \text{(D.5)}$$

The results for δ_π^2 and δ_K^2 are shown in figure 8. We find

$$\delta_\pi^2 = (0.18\pm0.06)\,\text{GeV}^2, \qquad \delta_K^2 = (0.20\pm0.06)\,\text{GeV}^2 \qquad \text{(D.6)}$$

and

$$\delta_K^2/\delta_\pi^2 = 1.10\pm0.05\,, \qquad \text{(D.7)}$$

which can be compared to the estimate $(f_K\delta_K^2)/(f_\pi\delta_\pi^2) = 1.07^{+0.14}_{-0.13}$ obtained in ref. [23].

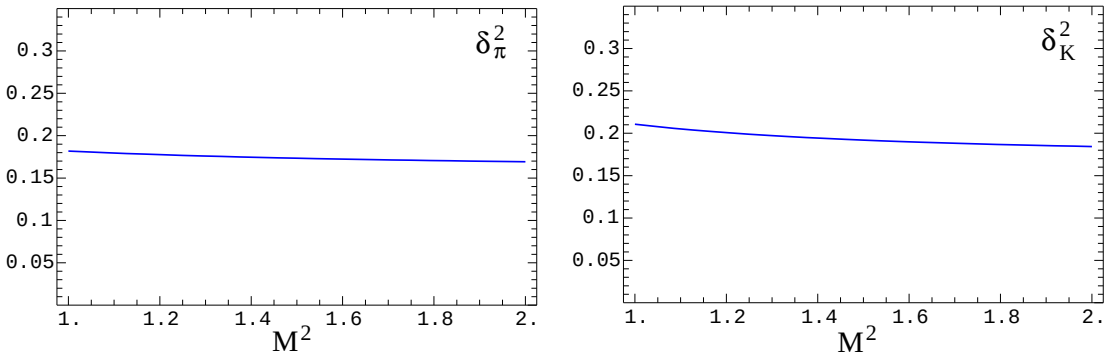

Figure 8: Left panel: δ_π^2 as a function of the Borel parameter from the sum rule (D.5); $s_0 = 0.8\,\mathrm{GeV}^2$. Right panel: the same for δ_K^2; $s_0 = 1.1\,\mathrm{GeV}^2$.

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