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Software Alignment of the CMS Tracker using MILLEPEDE II

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Abstract

The Alignment of the CMS tracker will require to determine about 10^5 alignment parameters. The MILLEPEDE program, a linear least-squares algorithm, is a promising candidate for this task, having been used successfully for alignment in several experiments. However, due to the inversion of a large matrix of linear equations, MILLEPEDE in its original form was limited to problems with about 10^4 parameters. A new version of the program, MILLEPEDE II, provides an iterative method to determine the solution of the matrix, which should work for systems with 10^5 parameters, if the matrix is sparse. This method is tested within the CMS object oriented reconstruction framework (ORCA). Its precision and CPU needs are studied and compared to the inversion method, using alignment scenarios of the CMS tracker with currently up to 12000 parameters.

Preliminary version

1 Introduction

Due to the huge number of independent silicon sensors (about 20000) and their excellent resolution (ranging from about $10\mu m$ to about $50\mu m$), the alignment of the CMS tracker is a complex and challenging task. It seems reasonable to require that our knowledge of the alignment uncertainties [1] should not lead to a significant degradation of the intrinsic tracker resolution. Therefore, the accuracy of the alignment has to be at least equal to, but ideally significantly better than, the ideal spatial resolution of fitted tracks. Certain physical requirements, such as the W mass measurement, place even more stringent constraints on the alignment precision. In order to achieve the desired systematic error of roughly 15 to 20 MeV on M_W, the momentum scale has to be known to an accuracy of about 0.020% to 0.025%. This translates into alignment requirements of e.g. $1\mu m$ uncertainty in the $r\phi$ plane. It seems clear that this kind of accuracy can only be reached with a track-based alignment procedure. However, other alignment methods, such as the Laser Alignment System (LAS) [1], exert an influence on the overall tracker alignment, as do the engineering and final assembly tolerances. The track-based alignment procedure will significantly benefit from a good prior knowledge of the positions of the tracker substructures. This knowledge should therefore be used as constraints in the track based alignment procedure. Tab. 1 lists the current estimates of important placement precisions for the individual substructures and elements of the silicon strip tracker that surround the pixel detector.

The CMS silicon strip tracker is composed of four mechanically independent sub-systems: The Tracker Inner Barrel (TIB), the Tracker Outer Barrel (TOB), and two Tracker End Caps (TEC). The barrel part consists of ten layers of silicon sensors. Each TEC comprises nine disks, each having silicon modules mounted on up to seven concentric rings. Individual silicon sensors are grouped together onto larger substructures. For the TIB and TOB, these are called half-shells and rods, respectively. Each rod holds six sensors, while the shells are much larger units. For the TEC, the substructures are petals, holding 17-28 sensors each.

Table 1: Current estimates of placement precisions for the tracker substructures (without PIXEL). For ranges, the highlighted number corresponds to the more probable mounting precision that could eventually be achieved.

Placement w.r.t.	TOB	TIB	TEC
Sensor vs. module	$\approx 10\mu m$	$\approx 10\mu m$	$\approx 10\mu m$
Module vs. rod/shell/petal	$\approx 100\mu m$	$\approx 200\mu m$	[**50** - 100] μm
Rod/shell/petal vs. disc/cylinder/disc	[**100** - 500] μm	$\approx 200\mu m$	[**100** - 200]μm
Disc/cylinder/disc vs. disc/cylinder/disc	[100 - **500**] μm	[100 - **500**] μm	[100 - **500**]μm

The position of each detector is described by six parameters: three position variables and three angles. However, for alignment of one-dimensional detectors, only five parameters are relevant since the position along the strip direction is not well measured and can be treated as fixed. With 20000 individual silicon sensors, resulting in 100000 positioning parameters, the alignment of the CMS silicon tracker poses formidable demands on a track based alignment algorithm. Several approaches are currently under investigation in CMS. The MILLEPEDE algorithm, described in this note, is capable of solving efficiently and accurately an alignment problem of this magnitude.

2 The MILLEPEDE Algorithm

MILLEPEDE [2, 3] is a well established and robust program package for alignment which has been used and tested successfully at several high energy physics experiments, for example at H1 [4], CDF [5], HERA-B [6], and others. Being a non-iterative method, it has been shown that it can improve the final alignment precision considerably compared to other algorithms.

MILLEPEDE is a linear least squares algorithm. Such algorithms have proven to be well suited for alignment problems since they are stable, fast and accurate and can take into account all correlations between parameters.

MILLEPEDE distinguishes between global parameters that are common to all data, namely the parameters describing the positions of the detectors, and local parameters, present only in a subset of the data. Track parameters are local parameters as they are specific to a single event. MILLEPEDE performs an overall least squares fit of the data, fitting all global and local parameters simultaneously. Making use of the special structure of the least squares matrix in such a fit, the problem is reduced to a matrix equation for the global parameters only. For N global parameters this amounts to an equation with a symmetric NxN matrix. In the previous version of MILLEPEDE, a solution was found by inverting the NxN matrix. However, due to CPU and memory constraints, this method can

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[1] Until stated otherwise, alignment uncertainties refer to the accuracy to which the position of individual detector elements are known.

only be used up to N = 5000-10000. Computing time increases with N^3 while memory goes with N^2. The alignment of the CMS tracker exceeds this limit by at least an order of magnitude, hence new methods had to be found to cope with the solution of such a system of linear equations. A new version, MILLEPEDE II, was developed, which offers different solution methods, and is applicable for N much larger than 10000.

3 MILLEPEDE II

In MILLEPEDE II, the two tasks of the program are split: Accumulation of track fit data (MILLE) and solving the set of linear equations (PEDE). The advantage of this procedure is that once a dataset has been defined for alignment, it can be stored permanently. This makes it possible to efficiently test the subsequent solution of the matrix equation under various conditions.

In addition to the matrix inversion and a diagonalization method, a new method for the solution of very large matrix equations is implemented. This minimum residual method applicable for sparse matrices (MINRES [7]) determines a good solution by iteration in acceptable time even for large N.

As mentioned before, to align the CMS tracker at the sensor level, the matrix is of the order 100000x100000 and matrix inversion is not a viable solution. Diagonalization has the advantage that the eigenvalues and eigenvectors lead to an improved understanding of the correlations between parameters. However, this method is even more CPU intensive. The third method, an iterative solution for sparse matrices, is most promising for the CMS tracker. Here the fact is used that the matrix containing the alignment parameters contains many zero-elements due to detector elements that are not linked to each other via common tracks. However, for comparison and testing of the robustness of the methods, it is useful to compare the results of the various methods. MILLEPEDE II has been successfully interfaced with the ORCA framework and the alignment of parts of the CMS tracker has been carried out using different scenarios.

Here it should be stressed again that MILLEPEDE is a non-iterative algorithm in that it does the determination of the local and global parameters simultaneously, while other algorithms decouple track fitting and finding of the alignment parameters. In the latter case, tracks need to be constantly refitted with new alignment parameters. In the context of this note, iterative merely means that the solution of the NxN matrix is found numerically.

MINRES

The alignment problem is described by the linear equation

$$Ax = b \tag{1}$$

where A is a symmetric N x N matrix with N being the number of global (alignment) parameters. The matrix is typically sparse due to the fact that sizable correlations only exist between certain parameters for specific detectors. For such a system with a large sparse, symmetric matrix, a solution can be found numerically by minimizing the residual

$$|r| = |(Ax - b)| \tag{2}$$

The method used here is the minimal residual algorithm implemented in the MINRES code [7] which utilizes the method of conjugate gradients to find a solution iteratively.

4 MILLEPEDE in the CMS Environment

MILLE

The MILLEPEDE alignment algorithms has been interfaced to the CMS object oriented reconstruction framework (ORCA [8]). The first part of the program, MILLE, writes an ntuple containing the “measurements”: global position variables, their errors, and local as well as global derivatives. Global parameters denote the alignment parameters of the detector modules, while local parameters are track parameters that are specific to each event.

PEDE

Building and solving the matrix is implemented in PEDE. Only non-zero elements are stored and double-precision is sufficient for the iterative method, which reduces the amount of memory space needed to manageable levels.

PEDE can read in multiple input files generated by MILLE which allows for parallel data accumulation. This feature is specially useful for large datasets.

A separate ASCII file is used to specify starting values of alignment parameters and information on whether these should be held fixed. If any of these parameters is changed, only PEDE needs to be rerun, while the ntuple with measurements written by MILLE is unchanged.

5 Alignment Study

5.1 The Coordinate System

In this note, both local and global variables are used. The coordinate systems are illustrated in Fig. 1. In the local coordinate system, the u-axis denotes the direction well measured, i.e. the sensor coordinate perpendicular to the strips, while v points in strip direction. The w-axis is perpendicular to the sensor surface. α, β, and γ are the angles for rotations around the u, v, and w-axis, respectively. For the barrel geometry, u corresponds to the global coordinate $r\phi$, v corresponds to global z and w corresponds to r, modulo sign changes for modules mounted back-to-back.

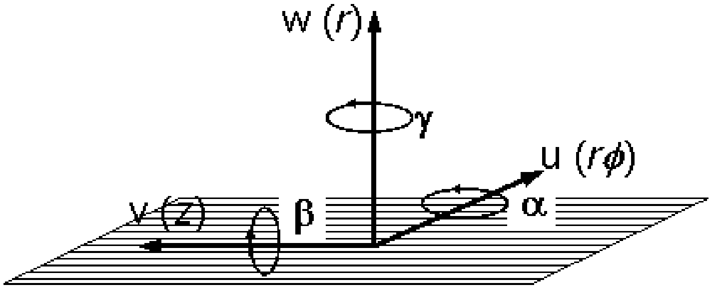

Figure 1: The coordinate system. u, v, w denote the local (detector) coordinates. The corresponding global coordinates are shown in parentheses for the barrel geometry. Also shown are the angles for rotations around the local axes.

5.2 Misalignment Scenarios

A separate ORCA package has been implemented to misalign the CMS detector [9]. Misalignment is implemented hierarchically. In the central part of the CMS tracking detector, this hierarchy is, from largest to smallest substructures:

- half barrels
- layers
- rods/ shells
- detectors

Since alignment of the full CMS tracker poses strong demands both on the alignment algorithm and on the datasets used, it makes sense to start with smaller alignment problems, gradually moving towards aligning the full detector. As can be seen below, a lot can be learned this way.

Misalignment Scenario A

In this study, the current default CMS misalignment scenario is used, with the following restriction for misalignment scenario A: Here we misalign the CMS tracker in the barrel region only up to the rod/shell level, that is: First

Table 2: Misalignment implemented in this study for global coordinates x, y, z, (rotation around z). All units are μm or μrad.

Placement w.r.t.	TOB	TIB
Half barrel vs. tracker	67, 67, 500, 59	105, 105, 500, 90
Layers vs. half barrel	0, 0, 0	0, 0, 0
Rods/ shells vs. layers	100, 100, 100	200, 200, 200
Module vs. rods/shells	100, 100, 100	200, 200, 200

the two half barrels are misaligned with respect to each other, then individual layers are misaligned, finally, rods and shells are misaligned with respect to each other, while the positions of individual detectors on rods/shells are kept fixed. Consistent with the default misalignment scenario, $r, r\phi, z$, and the rotation angle around the z-axis are misaligned, while the other angles are kept fixed. The amount of misalignment at each step is listed in Tab. 2.

Misalignment Scenario B

To go one step further than scenario A, misalignment of individual detectors is now turned on. This is currently the default misalignment scenario in CMS. However, since this introduces considerably more degrees of freedom, scenario A needs to be understood before moving to scenario B. Note that recently misalignment of all rotation angles of a detector with respect to the rod/shell became part of the default CMS misalignment. However, for this study these angles are not yet misaligned.

5.3 Data Samples

To successfully evaluate correlations between various detector substructures, it is necessary to exploit complementary data samples. $W \rightarrow \mu\nu$ and $Z \rightarrow \mu\mu$ events are valuable due to their clean signatures, large production rates and, in case of the Z, the possibility to use the invariant mass of the two tracks as an additional constraint. Tracks from pp-collisions can be supplemented by cosmic muons, which have the advantage that they correlate sub-detectors that would otherwise not be hit by the same track, due to the fact that cosmics do not originate in the interaction region. Finally, muons from beam halo events are useful, specially in the forward region.

For this study, the only dataset that was readily available was a sample of 1.8 million $Z \rightarrow \mu\mu$-events. This study is based on this sample exclusively. For alignment, tracks with $\mathrm{p_T} > 15$ GeV are selected. Currently, neither a beamspot nor a Z-mass constraint are used in the alignment procedure, although this can be added in a straightforward way if necessary.

5.4 Alignment

5.4.1 Scenario A: Alignment up to the Rod/ Shell Level

To evaluate the performance of the MILLEPEDE alignment algorithm, the CMS tracker is now aligned in a first step up to the rod/layer level, consistent with misalignment scenario A. To gain confidence in the iterative method to solve the set of linear equations introduced in MILLEPEDE II, results of this method are compared to the inversion method. The additional benefit of the inversion method is that the correlations and the errors of the alignment parameters are calculated, which allows for the calculation of pull-values.

At this stage of the study, certain valuable constraints have not been utilized yet: Tracking in the overlap region of detector modules is not by default turned on in the CMS software. Redoing the tracking allowing for more than one hit per layer in the overlap regions is possible but turned out to be too time consuming for this study. As mentioned above, cosmic muons have not been used yet. To obtain reliable results for this limited dataset, it turned out to be necessary to keep the three pixel layers and the outermost barrel layer fixed. This leads to 3480 free parameters. Fig. 2 shows hit residuals for $r\phi, z, r$, and γ, both for the inversion and the iterative method. Note that the angle γ is chosen since it is the only well-measured angle. It is the angle describing rotations around an axis perpendicular to the sensor (the local w-axis). However, the angle misaligned in the default CMS misalignment scenario corresponds to β for barrel modules, which is the angle describing rotations around the local v-axis. Fig. 3 shows pull distributions for the inversion method. Fig. 4 shows the global correlations for the inversion method. The global correlation for a given parameter is defined as the correlation-value for the linear combination of all other parameters that gives the largest correlation. A direct comparison of the results for the two methods is shown in Fig. 5. Plotted is the difference obtained for each parameter when the two alternative methods are used.

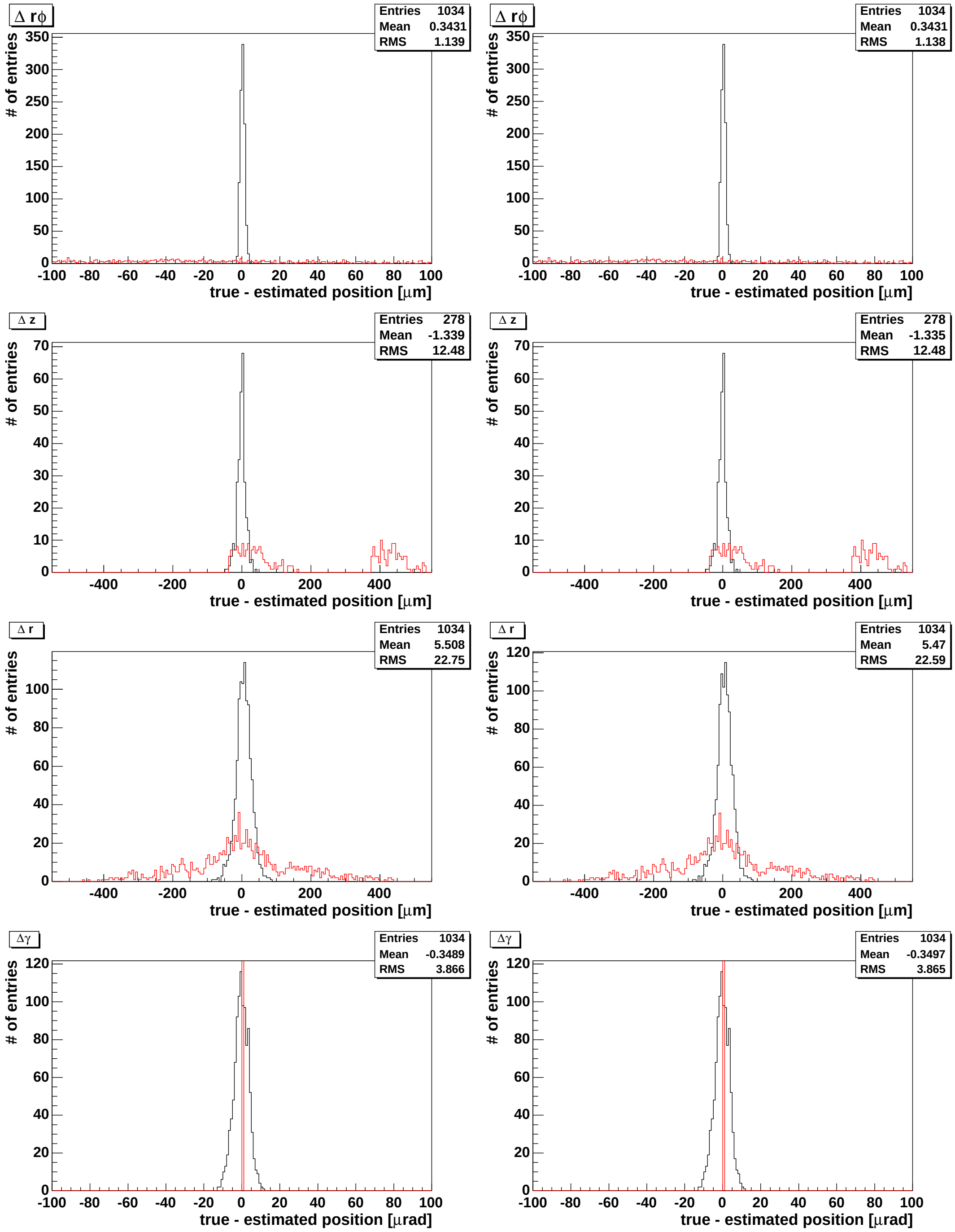

Figure 2: From top to bottom: Residuals in $r\phi, z, r$, and γ for alignment scenario A as a result of the inversion method (left) and the iterative method (right). The broad histograms show the residuals before alignment.

Results

The results of this alignment scenario look very encouraging. The residual distributions for the two methods to solve the set of linear equations are very similar (see Fig. 2). The residuals for $r\phi$ are around $1.1\mu m$, while they are around $12.5\mu m$ in z, $23\mu m$ in r, and 3.9 μrad for γ. The RMS-values of the pull distributions are close to one, underlining that the fit result is close to optimal and that the errors obtained by the inversion of the matrix are reasonable. The global correlation parameters shown in Fig. 4 are also reasonable, as they are much smaller than one. Fig. 5 underlines that both methods give equivalent results not only on average but on a parameter-by-parameter basis.

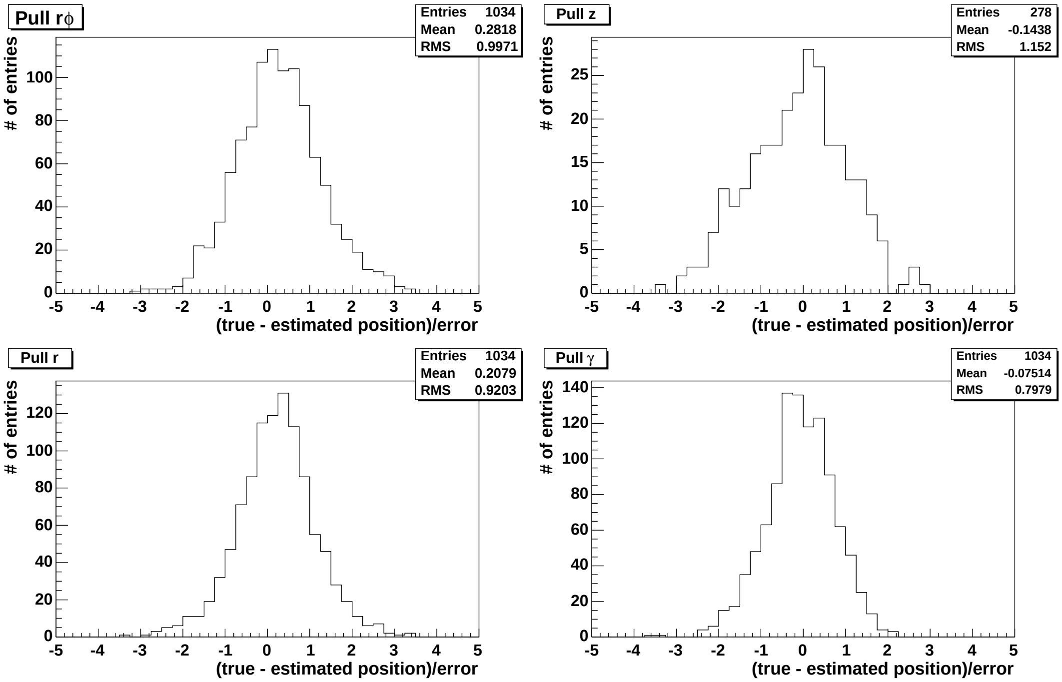

Figure 3: Pulls in $r\phi, z, r$, and γ for alignment scenario A as a result of the inversion method.

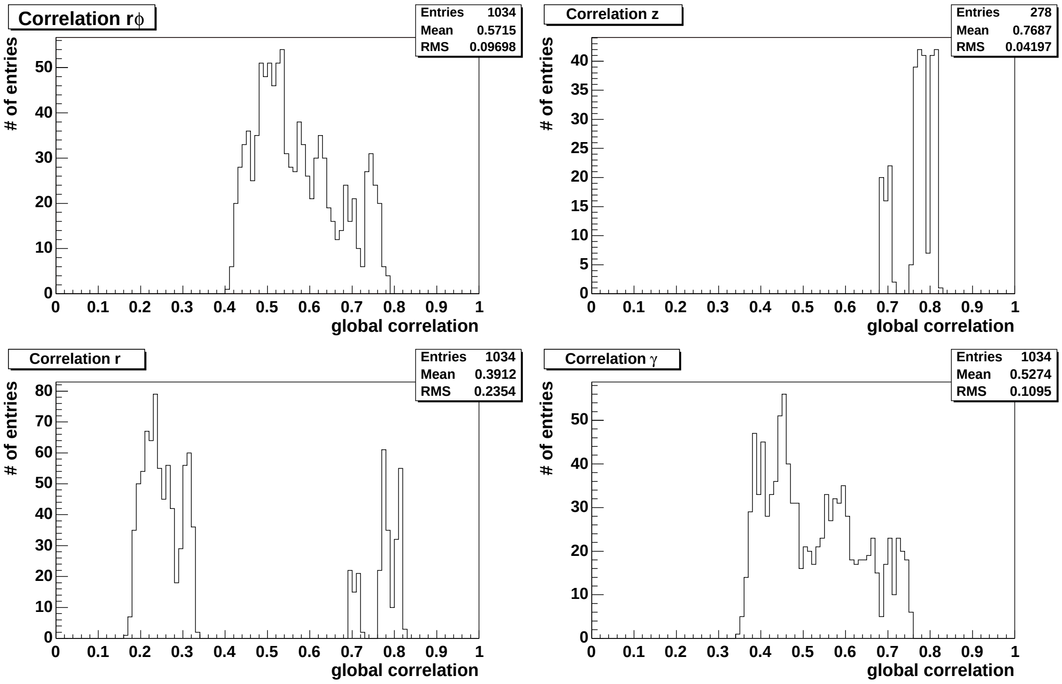

Figure 4: Correlations in $r\phi, z, r$, and γ for alignment scenario A as a result of the inversion method.

5.4.2 Scenario B: Alignment up to the Detector Level

Since the results for scenario A are encouraging, the misalignment and alignment procedures are now repeated with scenario B. Here the tracker is misaligned up to the detector level, where the misalignment again applies to the three space coordinates $r\phi, r, z$ and the angle β. Alignment is currently done in the central region ($|\eta| < 0.9$). Again, the three space coordinates are aligned as well as γ, resulting in 12015 alignment parameters. As before, the pixel layers and the outermost barrel layer are kept fixed. Fig. 6 shows the residual distributions for alignment at the detector level, again comparing the inversion and the iterative method. The residual distributions for the

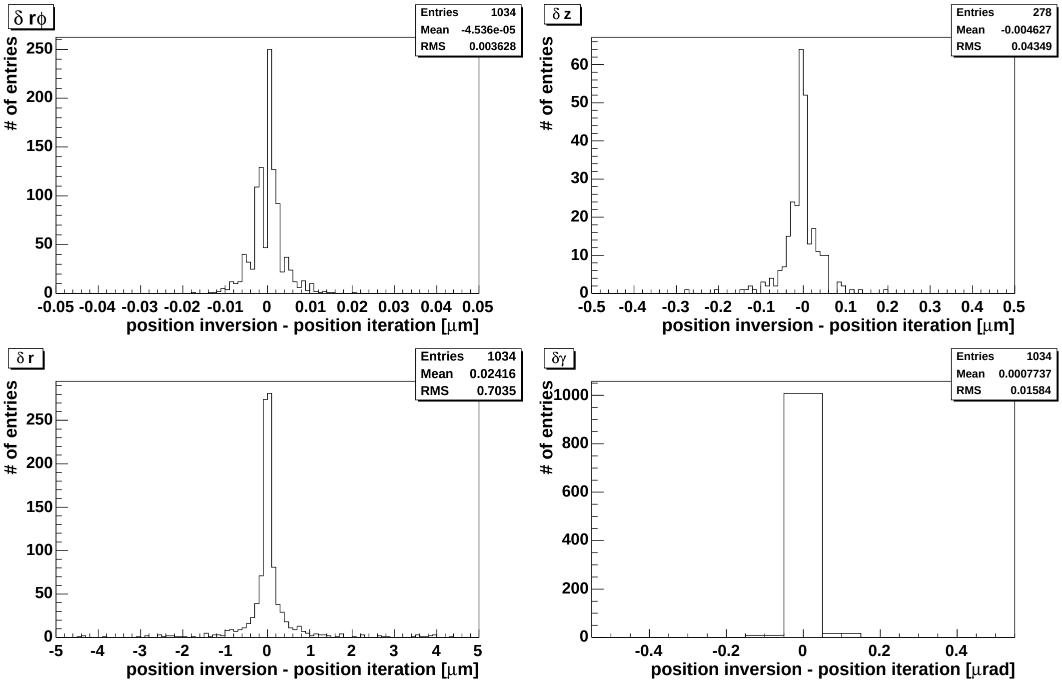

Figure 5: Direct comparison of the results for the inversion and iteration methods for alignment scenario A. Plotted is the difference obtained for each parameter when the two alternative methods are used.

four coordinates are as expected for a good fit. The width of the $r\phi$ distribution has increased to 4.6μm, which is expected from statistics since each individual detector is hit by less tracks than the larger substructures. Fig. 7 shows the pull distribution for this alignment scenario, while the global correlation parameters are shown in Fig. 8. Fig. 9 shows the parameter-by-parameter differences for the two methods, underlining again that both methods give very similar results.

5.4.3 CPU performance

The CPU times to perform the solutions of the matrix equation were determined for scenario B on a 3 GHz processor. The numbers quoted here are for solving the matrix only, not taking into account reading in of the data written by MILLE, which takes around 10 min. While inverting the 12015x12015 matrix took 12h 46 min 5 s, the iterative approach was much faster and took only 32 s.

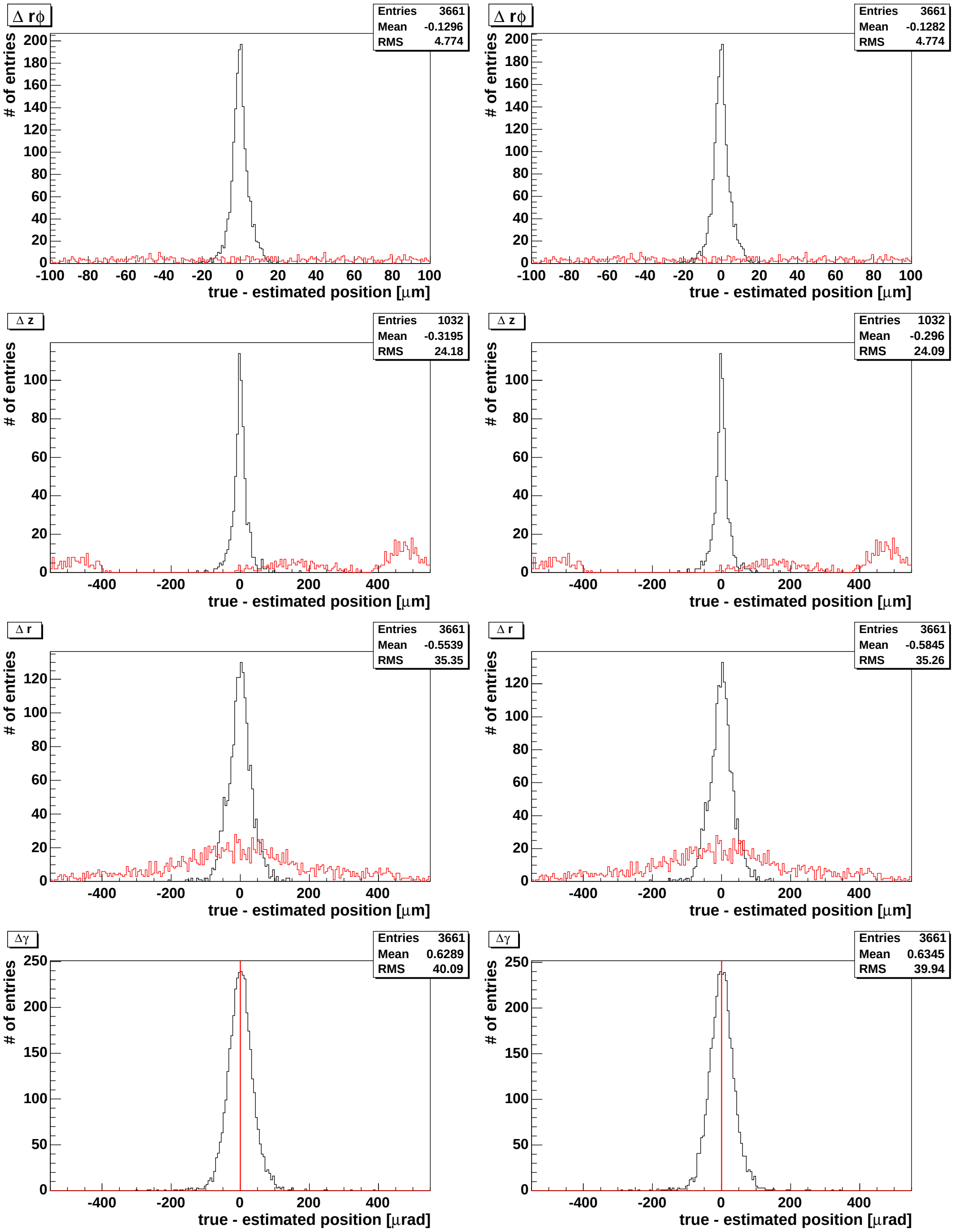

Figure 6: From top to bottom: Residuals in $r\phi, z, r$, and γ for alignment scenario B as a result of the inversion method (left) and the iterative method (right). The broad histograms show the residuals before alignment.

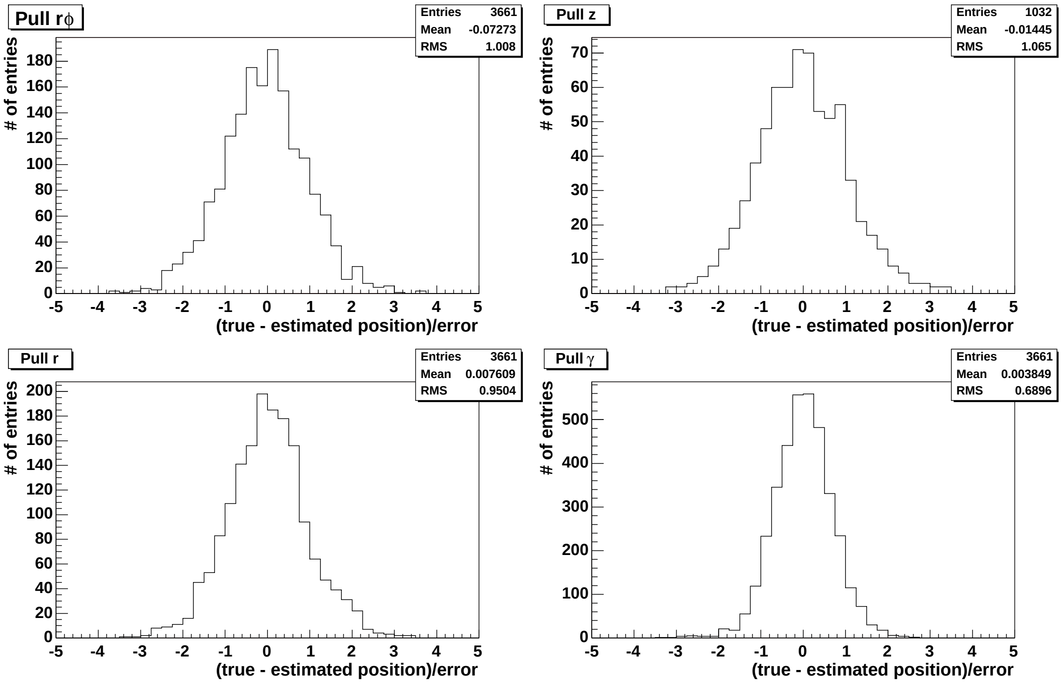

Figure 7: Pulls in $r\phi, z, r$, and γ for alignment scenario B as a result of the inversion method.

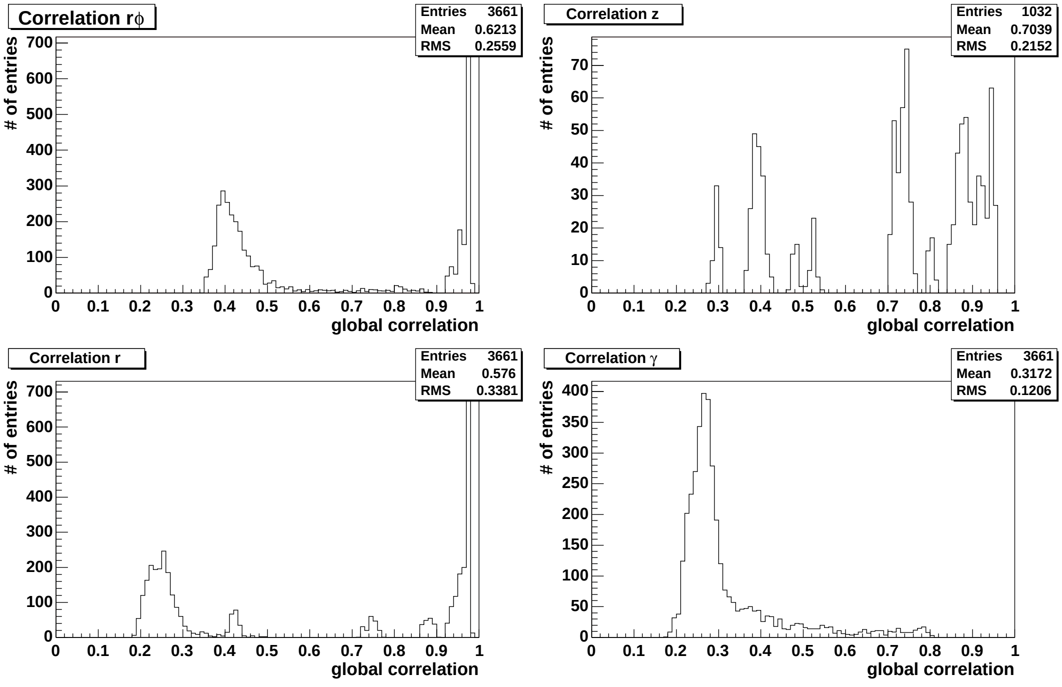

Figure 8: Correlations in $r\phi, z, r$, and γ for alignment scenario B as a result of the inversion method.

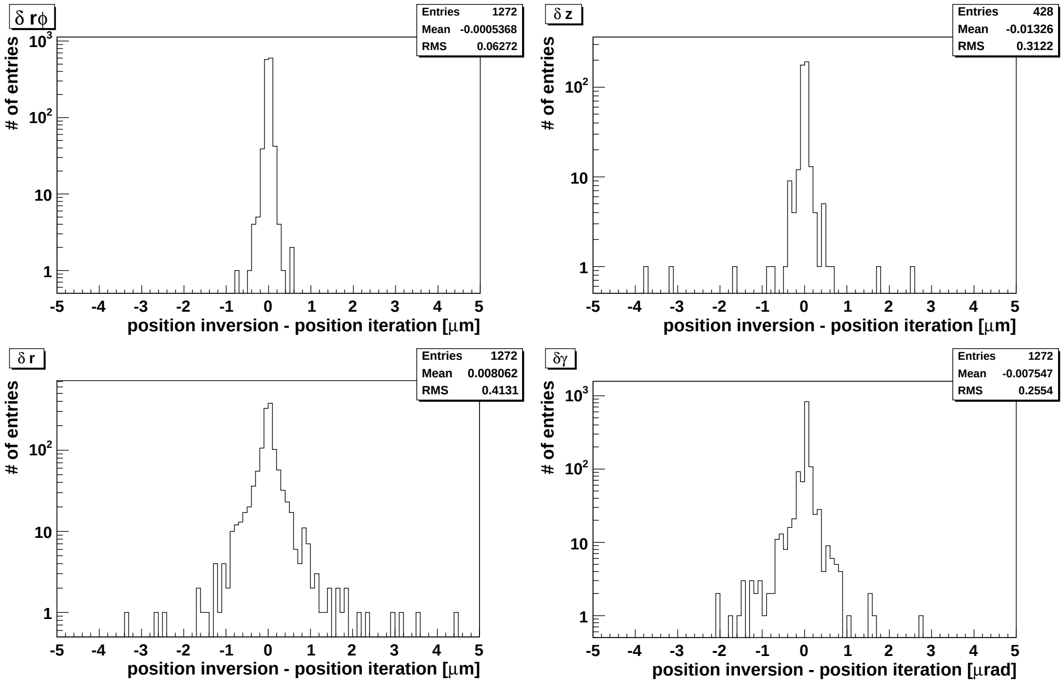

Figure 9: Direct comparison of the results for the inversion and iteration methods for alignment scenario B. Plotted is the difference obtained for each parameter when the two alternative methods are used.

6 Conclusion and Outlook

This note briefly reviews the MILLEPEDE II alignment algorithm and its implementation in the CMS environment. MILLEPEDE II has been used successfully to partially align the CMS silicon tracker in the barrel region up to rod/shell level, which amounts to 3480 free parameters, and up to the detector level, with 12015 free parameters. The results are very promising with resolutions in $r\phi$ of less than 5 μm and show that MILLEPEDE II should be well suited to align the full CMS tracker.

The next steps will be to extend the angular coverage to the full detector, to align all relevant angles, and to remove the constraint that the pixel tracker and the outermost barrel layer are kept fixed. This results in around 100000 parameters if all three angles are fitted. To obtain reliable results here, complementary datasets are needed to make optimal use of different correlations. In addition to $W \to \mu\nu$, $Z \to \mu\mu$, and other events from proton-proton collisions, it is foreseen to utilize beam-halo and cosmic events.

7 Acknowledgment

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