Minimum bunch length at the LHC

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Abstract

A luminosity upgrade of the LHC may be based on stronger focusing quadrupole magnets and/or a stronger focusing beam optics layout at the interaction regions, reducing the β-function from 55 cm to 25 cm. Due to the hourglass effect and the effects of the beam-beam interaction caused by the crossing angle, the bunch length becomes an important quantity for the luminosity. Within this scheme the reduction of the bunch length from nominal 0.31 m to 0.16 m leads to 43 % more luminosity [1].

The paper presents estimates of the minimum bunch lengths at which Landau Damping is lost for three scenarios: (i) without additional measures, (ii) applying RF amplitude modulation and (iii) using a higher harmonic RF system for bunch compression.

Coupled bunch instabilities arising in the case of loss of Landau Damping may also be suppressed by a coupled bunch feedback system preserving the longitudinal emittance. In this case the bunches are shorter than in the case where they blow-up due to instabilities. Estimates for the minimum required RF kick strengths for such a feedback system are given.

INTRODUCTION

In the present operation scenario for the LHC the longitudinal emittance is increased in a controlled way during acceleration by a factor of about 2.5 [2, 3], resulting in a full ($4\,\sigma$) bunch length of 0.31 m at top energy. Because of this blow up, there will always be sufficient Landau damping to stabilize the beam, in particular longitudinal coupled bunch instabilities are suppressed.

To obtain shorter bunches at top energy one may switch off the controlled emittance blow up (scenario i). By doing so, the beam may no longer be sufficiently stabilized and we have to consider additional measures for the suppression of longitudinal coupled bunch instabilities.

Conceivable measures for the beam stabilization are the increase of the bunch to bunch frequency spread by RF amplitude modulation (scenario ii) and a coupled bunch feedback system.

The additional voltage of a higher harmonic RF system leads to a bunch compression and supplies automatically more Landau damping due to the shorter buckets (scenario iii).

BUNCH LENGTH WITH COMPLETELY PRESERVED EMITTANCE

The longitudinal emittance ϵ depends approximately on the bunch length l as [4]

$$\epsilon = \pi\, \Delta t\, \Delta E$$

with [5]

$$\Delta t = \frac{l}{2\,c}$$

$$\Delta E = \beta \sqrt{\frac{2\,E_{\mathrm{s}}}{\eta}} \sqrt{\frac{e}{2\pi}\frac{V}{h}\left(1-\cos\left(\frac{\omega_{\mathrm{rf}}}{2\,c}\,l\right)\right)}$$

where c is the speed of light, ΔE the energy deviation, Δt the time deviation of a particle with phase deviation $\Delta\phi = \omega_{\mathrm{rf}}\, l/2$, $\beta = \frac{v}{c}$, E_{s} the energy of the synchronous particle and $\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$ the slip factor. The slip factor follows from γ-transition γ_t, the factor $\gamma = \frac{E_s}{E_0}$ and the proton rest energy $E_0 = m_{\mathrm{p}}\, c^2$.

In the case of LHC the slip factor is at top energy $\eta_{7\,\mathrm{TeV}} = 0.0003225$. Due to the low γ-transition of 55.678 it has approximately the same value at injection energy ($\eta_{450\,\mathrm{GeV}} = 0.0003182$). The RF parameters of LHC are a sum voltage of $V = 16$ MV and the frequency of $\omega_{\mathrm{rf}} = 2\pi\, 400.8$ MHz resulting in the harmonic number $h = 35640$.

At the injection energy of $E_{\mathrm{s}} = 450$ GeV the longitudinal ($4\,\sigma$) emittance will be $\epsilon = 1$ eVs. It is increased during acceleration to $\epsilon = 2.5$ eVs at top energy $E_{\mathrm{s}} = 7$ TeV corresponding to a full bunch length of 0.31 m. With a completely preserved longitudinal emittance of $\epsilon = 1$ eVs the bunch length at top energy would be

$$l_{1\,\mathrm{eVs},7\,\mathrm{TeV}} = 0.19\,\mathrm{m}.$$

MINIMUM BUNCH LENGTH STABILIZED BY LANDAU DAMPING

According to [6] the coherent frequency shift Δf_{imp} caused by the effective impedance Z_{L}/n is at the LHC given by

$$\Delta f_{\mathrm{imp}} = 0.0237\,\frac{\mathrm{Hz}}{\Omega}\left(\mathrm{Im}\,\frac{Z_{\mathrm{L}}}{n}\right)\left(\frac{N_{\mathrm{b}}}{10^{11}}\right)\left(\frac{l}{\mathrm{m}}\right)^{-3}.$$

N_b is the number of particles per bunch and l the bunch length. This relation is valid for the bunch spacing of 25 ns.

Landau damping will suppress instabilities up to a threshold value of

$$\Delta f_\mathrm{threshold,Landau} = 6.45\,\mathrm{Hz}\left(\frac{l}{\mathrm{m}}\right)^2 .$$

By setting $\Delta f_\mathrm{threshold,Landau} = \Delta f_\mathrm{imp}$ we can calculate the minimum bunch length at which Landau damping will be lost:

$$\frac{l_\mathrm{min}}{\mathrm{m}} = \sqrt[5]{\frac{3.57\cdot 10^{-3}}{\Omega}\left(\mathrm{Im}\,\frac{Z_\mathrm{L}}{n}\right)\left(\frac{N_\mathrm{b}}{10^{11}}\right)}$$

Table 1 shows the minimum bunch length for the most recent value of the longitudinal impedance of $\mathrm{Im}\,Z_\mathrm{L}/n = 0.08\,\Omega$ [3] and an older estimate $\mathrm{Im}\,Z_\mathrm{L}/n = 0.28\,\Omega$ [6] which may be viewed as a worst case value.

Table 1: Minimum bunch length at which Landau damping will be lost without additional measures.

l_min	$\mathrm{Im}\,\frac{Z_\mathrm{L}}{n} = 0.08\,\Omega$	$0.28\,\Omega$
$N_\mathrm{b} = 1.1\cdot 10^{11}$	0.20 m	0.26 m
$N_\mathrm{b} = 1.7\cdot 10^{11}$	0.22 m	0.28 m

For ultimate intensity $N_\mathrm{b} = 1.7\cdot 10^{11}$ and also for nominal intensity $N_\mathrm{b} = 1.1\cdot 10^{11}$ Landau damping will be lost by switching off the controlled emittance blow up (scenario i) because the minimum bunch length determined by the impedance exceeds the bunch length of $l_{1\,\mathrm{eVs,7\,TeV}} = 0.19\,\mathrm{m}$ following from the initial longitudinal emittance.

STABILIZATION BY RF AMPLITUDE MODULATION

The minimum bunch length values for $\frac{Z_\mathrm{L}}{n} = 0.08\,\Omega$ in Table 1 are not too far away from the bunch length value for an emittance of $\epsilon = 1\,\mathrm{eVs}$. Hence, a method to double the instability threshold $\Delta f_\mathrm{threshold}$ may be sufficient to stabilize the beam. Such a method is the modulation of the RF amplitude (scenario ii). For the LHC it increases the instability threshold by about [7]

$$\Delta f_\mathrm{threshold,AM} = 0.51\,\mathrm{Hz}$$

resulting in

$$\Delta f_\mathrm{threshold} = \Delta f_\mathrm{threshold,Landau} + \Delta f_\mathrm{threshold,AM}.$$

For $\Delta f_\mathrm{threshold} = \Delta f_\mathrm{imp}$ the minimum bunch length at which Landau damping will be lost is given in Table 2.

The modulation of RF amplitude should be sufficient for stabilizing the beam for $\mathrm{Im}\,\frac{Z_\mathrm{L}}{n} = 0.08\,\Omega$ and a shorter bunch length is possible. Dividing the minimum bunch length for a given emittance by the minimum bunch length for a given impedance results in the safety margins of 26 % and 12 % respectively. Those safety margins are smaller than the safety margins of 55% and 41% obtained at normal operation with artificial beam blow up.

Table 2: Minimum bunch length at which Landau damping will be lost with rf amplitude modulation.

l_min	$\mathrm{Im}\,\frac{Z_\mathrm{L}}{n} = 0.08\,\Omega$	$0.28\,\Omega$
$N_\mathrm{b} = 1.1\cdot 10^{11}$	0.15 m	0.21 m
$N_\mathrm{b} = 1.7\cdot 10^{11}$	0.17 m	0.23 m

In case the initial emittance is smaller or the longitudinal impedance larger, additional measures have to be taken.

HIGHER HARMONIC RF SYSTEM

A RF system with three times the fundamental RF frequency ($3\times 400\,\mathrm{MHz} = 1.2\,\mathrm{GHz}$) [1] for bunch compression is under investigation [9] (scenario iii). With the resulting double RF system one injects into the buckets of the 400 MHz system, whereas the 1.2 GHz system is operated such, that its contribution to the sum voltage is 1/4 or less. During acceleration the voltage of the 1.2 GHz system is increased such, that it completely builds up the buckets at top energy.

The energy deviation ΔE is in the non-accelerating case given by

$$\Delta E = \beta\sqrt{\frac{2\,E_\mathrm{s}}{\eta}}\left(\frac{e}{2\pi}\left(\frac{V_{400}}{h_{400}}\left(1-\cos\left(\frac{\omega_\mathrm{rf}}{2\,c}\,l\right)\right)\right.\right.$$
$$\left.\left.+\frac{V_{1200}}{3\,h_{400}}\left(1-\cos\left(\frac{3\,\omega_\mathrm{rf}}{2\,c}\,l\right)\right)\right)\right)^{-1/2}$$

resulting with the voltages $V_{400} = 16\,\mathrm{MV}$ and $V_{1200} = 43\,\mathrm{MV}$ in the bunch length of

$$l_{1\,\mathrm{eVs,7\,TeV}} = 0.12\,\mathrm{m},$$

in the case the emittance is completely preserved. In case some (controlled) blow up will take place one may obtain the in [1] proposed bunch length value of

$$l_{1.8\,\mathrm{eVs,7\,TeV}} = 0.16\,\mathrm{m}.$$

By operating the LHC at top energy with both RF systems the effective voltage in the bucket core becomes $V_{1200,\mathrm{eff}} = 48.3\,\mathrm{MV}$ resulting in a synchrotron frequency of $f_\mathrm{s} = 72\,\mathrm{Hz}$ ($f_\mathrm{s} = 23.9\,\mathrm{Hz}$ for single RF). These values are obtained from $f_{\mathrm{s}\,0} \propto \sqrt{h\,V}$ and $f_\mathrm{s} = \sqrt{1 + r_{12}\,h_{12}} \times f_{\mathrm{s}\,0}$, where $r_{12} = V_{400}/V_{1200}$ is the voltage ratio and $h_{12} = 1/3$ the ratio of the harmonic numbers [10, p. 76].

With $\Delta f_{\mathrm{imp}} \propto f_{\mathrm{s}}/(h\,V)$ [6] the coherent frequency shift obeys

$$\Delta f_{\mathrm{imp}} = 0.0079\,\frac{\mathrm{Hz}}{\Omega}\left(\mathrm{Im}\,\frac{Z_{\mathrm{L}}}{n}\right)\left(\frac{N_{\mathrm{b}}}{10^{11}}\right)\left(\frac{l}{\mathrm{m}}\right)^{-3}.$$

Considering $\Delta f_{\mathrm{threshold,Landau}} \propto f_{\mathrm{s}}$ an approximation for the Landau damping in a double RF system is for $r_{12} < 1/h_{12}$ given by [10, p. 77]

$$\Delta f_{\mathrm{threshold,Landau}} = 19.4\,\mathrm{Hz}\,\frac{1 + r_{12}\,h_{12}^3}{\sqrt{1 + r_{12}\,h_{12}}}\left(\frac{3\,l}{\mathrm{m}}\right)^2.$$

The fact that the higher harmonic RF system mainly defines the bucket area at top energy is taken into consideration by the choice of $r_{12} = V_{400}/V_{1200}$ and the factor three in front of the bunch length[1].

At top energy the condition $r_{12} < 1/h_{12}$ is fulfilled and we get

$$\Delta f_{\mathrm{threshold,Landau}} = 167\,\mathrm{Hz}\left(\frac{l}{\mathrm{m}}\right)^2.$$

Table 3: Minimum bunch length at which Landau damping will be lost for a double rf system.

l_{min}	$\mathrm{Im}\,\frac{Z_{\mathrm{L}}}{n} = 0.08\,\Omega$	$0.28\,\Omega$
$N_{\mathrm{b}} = 1.1 \cdot 10^{11}$	0.08 m	0.11 m
$N_{\mathrm{b}} = 1.7 \cdot 10^{11}$	0.09 m	0.12 m

Table 3 shows the minimum bunch length at which Landau damping will be lost at the double RF system. Only in the case of an unexpected large effective impedance, Landau damping may be lost for beams with ultimate intensity and completely preserved emittance.

The estimates presented in Table 3 contain some uncertainties: The higher harmonic RF system itself will contribute to the effective longitudinal impedance, its contribution is under investigation [9]. Furthermore, shorter bunches lead in general to slightly larger effective impedances, depending on the exact frequency distribution of the accelerator impedance. In the estimates presented here, this fact is not taken into account.

Even in the case of an unexpected high effective impedance the beam should be sufficiently stabilized by a controlled emittance blow up to 1.8 eVs.

LONGITUDINAL COUPLED BUNCH FEEDBACK

A coupled bunch feedback system has to damp instabilities faster than the beam blows up and self-stabilization by Landau damping takes place. Hence, the required kick voltage V_{FBK} of a longitudinal coupled bunch feedback depends on the maximum growth rate $\Delta f_{\mathrm{threshold,Landau}}/f_{\mathrm{s}}$ stabilized by Landau damping, the RF voltage V_{RF} and the minimum detectable phase oscillation $\Delta\phi_{\mathrm{det}}$ like [6]

$$V_{\mathrm{FBK}} > 2\,V_{\mathrm{RF}}\,\frac{\Delta f_{\mathrm{threshold,Landau}}}{f_{\mathrm{s}}}\,\Delta\phi_{\mathrm{det}}.$$

For the single harmonic RF system the synchrotron frequency is at top energy $f_{\mathrm{s}} = 23.9\,\mathrm{Hz}$. A reasonable choice for $\Delta f_{\mathrm{threshold,Landau}}$ is the value corresponding to the emittance where the beam is reliably stabilized by Landau damping only.

If the bunch phase oscillations are detected in the same way as in the HERA proton ring [10, p. 32] the minimum detectable phase oscillation is $\Delta\phi_{\mathrm{det}} \approx \frac{2\pi}{360°}\,0.2°$. Digital filters may decrease this value. Using $\Delta f_{\mathrm{threshold,Landau}}$ for an emittance of $\epsilon = 2.5\,\mathrm{eVs}$ results in the minimum required kick voltage for a single RF system of

$$V_{\mathrm{FBK}} > 3\,\mathrm{kV}.$$

In the case of the double RF system, the beam is expected to be stabilized by Landau damping only, for longitudinal emittances of 1 eVs.

This estimate does not take into account transient effects at injection or during acceleration. Fast damping of synchrotron oscillations caused by such effects may require a larger kick strength.

SUMMARY

With the given RF system, bunches shorter than about 25 cm are not feasible in the LHC by switching off the controlled emittance blow up used in standard operation (scenario i). For the most recent estimate of the effective longitudinal impedance $\mathrm{Im}\,\frac{Z_{\mathrm{L}}}{n} = 0.08\,\Omega$, a modulation of the RF amplitude may be sufficient to stabilize the beam for the case of no controlled emittance blow up (scenario ii). This is no longer the case for initial emittances smaller than 1 eVs or an impedance larger than $0.08\,\Omega$. An impedance larger than $0.08\,\Omega$ may be caused by collimators for machine protection and for the reduction of back ground events in the high energy experiments ob by other operational constraints. In such a case additional measures have to be taken.

One measure is a higher harmonic RF system for bunch compression (scenario iii). For $\mathrm{Im}\,\frac{Z_{\mathrm{L}}}{n} = 0.08\,\Omega$ the beam should be well stabilized by Landau damping. Investigations on the contribution of this RF system itself to the effective impedance are under way [9].

Alternatively one may set up a longitudinal coupled bunch feedback system. Then, the minimum bunch length obtained by suppressing coupled bunch oscillations depends on the initial emittance and the (noise) performance of the feedback system.

[1] In case one applies this expression for a higher harmonic ($3h$) Landau damping RF system one has to choose r_{12} inversely ($r_{12} = V_{3h}/V_h$ instead of $r_{12} = V_h/V_{3h}$) because the bucket area is then mainly defined by the lower harmonic RF system.

Table 4 gives an overview on the minimum bunch length in LHC for a single harmonic RF system, a double harmonic RF system and the measures required for suppressing longitudinal coupled bunch instabilities.

Table 4: Minimum bunch length in an upgraded LHC.

RF system	'low' Im $\frac{Z_L}{n}$	'high' Im $\frac{Z_L}{n}$	$l_{4\sigma}$
400 MHz	RF AM	feedback	$0.19\,\mathrm{m}$
400 MHz &1.2 GHz	blow up –	blow up RF AM feedback?	$0.16\,\mathrm{m}$ $0.12\,\mathrm{m}$?

A set up of fast multi bunch beam diagnostics and data logging, comparable to the system used at the HERA proton ring [8], may be required for the study of the instabilities and the supervision of the measures taken.

Longitudinal microwave instabilities are expected to appear for bunches shorter than about 5.5 cm [1]. This is less than 1/2 of the minimum bunch length discussed in this paper, resulting in a minimum safety margin of more than $100\,\%$. For that reason, longitudinal microwave instabilities have not been treated in this paper.

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