

SPLIT EXTENDED SUPERSYMMETRY FROM
INTERSECTING BRANESIgnatios Antoniadis ^{a,*}, Karim Benakli ^b, Antonio Delgado ^a,
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Universités de Paris VI et VII, France*^c *Institució Catalana de Recerca i Estudis Avançats (ICREA)**Theoretical Physics Group, IFAE/UAB, E-08193 Bellaterra, Barcelona, Spain*^d *Institut für Theoretische Physik, ETH Hönggerberg, 8093 Zürich, Switzerland***Abstract**

We study string realizations of split extended supersymmetry, recently proposed in hep-ph/0507192. Supersymmetry is broken by small (ϵ) deformations of intersection angles of D -branes giving tree-level masses of order $m_0^2 \sim \epsilon M_s^2$, where M_s is the string scale, to localized scalars. We show through an explicit one-loop string amplitude computation that gauginos acquire hierarchically smaller Dirac masses $m_{1/2}^D \sim m_0^2/M_s$. We also evaluate the one-loop Higgsino mass, μ , and show that, in the absence of tree-level contributions, it behaves as $\mu \sim m_0^4/M_s^3$. Finally we discuss an alternative suppression of scales using large extra dimensions. The latter is illustrated, for the case where the gauge bosons appear in $N = 4$ representations, by an explicit string model with Standard Model gauge group, three generations of quarks and leptons and gauge coupling unification.

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1 INTRODUCTION

Implementing the idea of split supersymmetry [1] in string theory is straightforward [2]. The appropriate framework is type I theory [3] compactified in four dimensions in the presence of constant internal magnetic fields [4, 5], or equivalently D -branes intersecting at angles [6, 7] in the T -dual picture. However, in simple brane constructions the gauge group sector comes in multiplets of extended supersymmetry, while matter states are in $N = 1$ representations. In Ref. [8] we showed that these economical string-inspired brane constructions reconcile with unification of gauge couplings at scales safe from proton decay problems, and provide a natural Dark Matter candidate.

Indeed a simple way to break supersymmetry in the above context is by deforming the intersection angles of the Standard Model branes from their special values corresponding to a supersymmetric configuration. A small deformation of these angles by ϵ breaks supersymmetry via a D -term vacuum expectation value (VEV), associated to a magnetized abelian gauge group factor in the T -dual picture, $\langle D \rangle = \epsilon M_s^2$ with M_s the string scale [4, 9]. This leads to mass shifts of order $m_0^2 \sim \epsilon M_s^2$ for all charged scalar fields localized at the intersections, such as squarks and sleptons, while gauginos (and Higgsinos) remain massless. Alternatively supersymmetry breaking can be communicated to the scalar observable sector by radiative corrections from a supersymmetric messenger sector, with D -breaking triggered by a magnetized abelian subgroup, or a non-supersymmetric sector with large extra dimensions. In all cases all previously massless scalars in the observable sector are expected to acquire large masses by radiative corrections and a fine-tuning is needed in the Higgs sector in order to keep the hierarchy between the electroweak scale and m_0 , as required in split supersymmetry.

On the other hand fermion (gaugino and Higgsino) masses are protected by a chiral R -symmetry and the magnitude of radiative corrections depends on the mechanism of its breaking. In fact R -symmetry is in general broken in the gravitational sector by the gravitino mass but its value, as well as the mediation of the breaking to the brane (Standard Model) sector, is model dependent and brings further uncertainties. Here we will restrict ourselves to possible sources of fermion mass generation due to brane effects described by open string propagation within only global supersymmetry, assuming that gravitational (closed strings) corrections are negligible. Indeed, R -symmetry is in general broken by α' -string corrections and gaugino Majorana masses can be induced by a dimension-seven effective operator which is the chiral F -term [10]: $\int d^2\theta \mathcal{W}^2 \text{Tr} W^2$, where \mathcal{W} and W are the magnetic $U(1)$ and non-abelian gauge

superfields, respectively. Its moduli dependent coefficient is given by the topological partition function $F_{(0,3)}$ on a world-sheet with no handles and three boundaries ¹. From the effective field theory point of view, it corresponds to a two-loop correction involving massive open string states. Upon a VEV $\langle \mathcal{W} \rangle = \theta \langle D \rangle$, the above F -term generates Majorana gaugino masses that are hierarchically smaller than the scalar masses and behave as $m_{1/2}^M \sim m_0^4/M_s^3$.

In models where the gauge bosons come in multiplets of extended supersymmetry, there exists the possibility of generating Dirac gaugino masses that do not require the breaking of R -symmetry. Such a mass can be induced at one-loop via the effective chiral dimension-five operator [11, 8]: $\int d^2\theta \mathcal{W} \text{Tr}(WA)$, where A denotes the $N = 1$ chiral superfield(s) containing the additional gaugino(s). Upon the D -auxiliary VEV $\langle \mathcal{W} \rangle$ this term generates Dirac gaugino masses that scale as $m_{1/2}^D \sim m_0^2/M_s$, and are thus much higher than the Majorana masses $m_{1/2}^M$. In Ref. [8], we studied the renormalization group evolution and showed that this scenario is compatible with one-loop gauge coupling unification at high scale for both cases where the gauge sector is $N = 2$ and $N = 4$ supersymmetric.

The low energy sector of these models contains, besides the Standard Model fields, just some fermion doublets (Higgsinos) and eventually two singlets, the Binos, if the corresponding corrections to their Dirac mass vanish. In fact, the Higgsinos must acquire a mass, μ , of order the electroweak scale in order to provide a Dark Matter candidate. This can be induced by the following dimension-seven operator, generated at one loop level [10, 8]: $\int d^2\theta \mathcal{W}^2 \bar{D}^2 \bar{H}_1 \bar{H}_2$, where $H_{1,2}$ are the two $N = 1$ Higgs supermultiplets. It follows that the induced Higgsino mass is of the same order as the gaugino Majorana masses, $\mu \sim m_{1/2}^M \sim m_0^4/M_s^3$.

The appearance of gauginos in multiplets of extended supersymmetry is common in previous attempts to embed the Standard Model in intersecting brane constructions [12]. There are several examples in the literature of such models with gauge sectors forming multiplets of either $N = 4$ [13] or $N = 2$ extended supersymmetry [14].

In this work we describe the general string framework realizing the above scenario of split supersymmetry with extended supersymmetric gauge sector and perform an explicit one-loop computation of the dimension-five and seven effective operators needed to produce Dirac gaugino and Higgsino masses, respectively. The relevant world-sheet diagram is the annulus involving two D -brane stacks on its boundaries (or three in the case of Higgsinos). We find that both Dirac gaugino and Higgsino masses are in general non-vanishing when the two brane-stacks are parallel in one of the three internal compactification planes. The leading behavior in the

¹Two of them correspond to \mathcal{W} and W gauge groups while the third one can be an orientifold.

supersymmetric limit, $m_0/M_s \rightarrow 0$, gives the coefficient of the corresponding effective operator, which thus receives non-trivial contributions only from $N = 2$ supersymmetric sectors. Moreover we find that in this limit the result simplifies and becomes topological; the non-zero mode determinants cancel and the effective couplings depend only on the momentum lattice of the plane where the two brane-stacks are parallel. In the Higgsino case the two fermions should come from an $N = 2$ supersymmetry preserving intersection, localized in the remaining two internal planes.

Finally, for concreteness, we present an explicit string construction with the Standard Model gauge group, precisely three generations of quarks and leptons, and sharing the desired features described above. Moreover, it emerges from an $SU(5)$ grand unified group and thus satisfies gauge coupling unification, realizing a particular D -brane configuration proposed in Ref. [2]. Standard Model particles live in the intersection of supersymmetric branes while there is a non-supersymmetric brane such that particles living in its intersection with the observable branes are non-chiral and act as messengers of gauge mediated supersymmetry breaking. In this case the hierarchy between the string scale and the masses of supersymmetric partners can be triggered by extra dimensions hierarchically larger than the string length.

Our paper is organized as follows. In Section 2, we describe the general framework of supersymmetry breaking in D -brane models intersecting at angles. In Section 3, we perform the one-loop computation of the induced Dirac gaugino masses and extract the coefficient of the relevant dimension-five effective operator by evaluating the behavior in the supersymmetric limit. In Section 4, we perform a similar computation for the Higgsino masses and the corresponding dimension-seven effective operator. In Section 5, we present an explicit construction of the Standard Model spectrum in this framework with the desired features. Our conclusions are drawn in Section 6 and finally some relevant formulae are presented in Appendix A and some technical computational details about the bosonic correlation function of Higgsinos in Appendix B.

2 SUPERSYMMETRY BREAKING FROM INTERSECTING BRANES

We will consider a set of intersecting stacks of branes that can be divided into two subsets: the first one, denoted as \mathcal{O} (for observable), gives rise in its light spectrum to the observable sector, i.e. a supersymmetric version of the Standard Model with the gauge sector in $N = 2$ or $N = 4$ representations. The second subset, which we denote as \mathcal{M} (for messenger), provides

the supersymmetry breaking messengers through its intersections with the branes in \mathcal{O} .

In order to perform explicit computations, we consider a compactification on a six torus factorizable as $T_1^2 \otimes T_2^2 \otimes T_3^2$ with appropriate projections and orientifold planes to provide the desired supersymmetric framework. A basis cycles $[\mathbf{a}^{(i)}]$ and $[\mathbf{b}^{(i)}]$ of the corresponding homology classes is defined for every torus T_i^2 . Every stack a of $D6$ -branes in type IIA wraps a 3-cycle $[\Pi_a]$ factorizable into the product of 1-cycles:

$$[\Pi_a] = \bigotimes_i [\Pi_a^{(i)}] = \bigotimes_i (n_a^{(i)}[\mathbf{a}^{(i)}] + m_a^{(i)}[\mathbf{b}^{(i)}]) \quad (2.1)$$

and forms angles ² with the cycles $[\mathbf{a}^{(i)}]$ given by:

$$\tan \theta_a^{(i)} = \frac{m_a^{(i)} R_2^{(i)} / R_1^{(i)} + n_a^{(i)} \cot \varphi^{(i)}}{n_a^{(i)}} \quad (2.2)$$

where $R_1^{(i)}$ and $R_2^{(i)}$ are the radii along the horizontal $X^{(i)}$ ($\mathbf{a}^{(i)}$ -cycles) and vertical $Y^{(i)}$ ($\mathbf{b}^{(i)}$ -cycles) axes, respectively, and $\varphi^{(i)}$ is the angle of the tilted torus T_i^2 .

In the presence of an orientifold plane along the $X^{(i)}$ -axis, the angle $\varphi^{(i)}$ is fixed to either $\cot \varphi^{(i)} = 0$, which corresponds to rectangular tori, or to $\cot \varphi^{(i)} = R_2^{(i)} / 2R_1^{(i)}$ which corresponds to tilted tori (see Fig. 1). In these cases, we can write the angles $\theta_a^{(i)}$ as

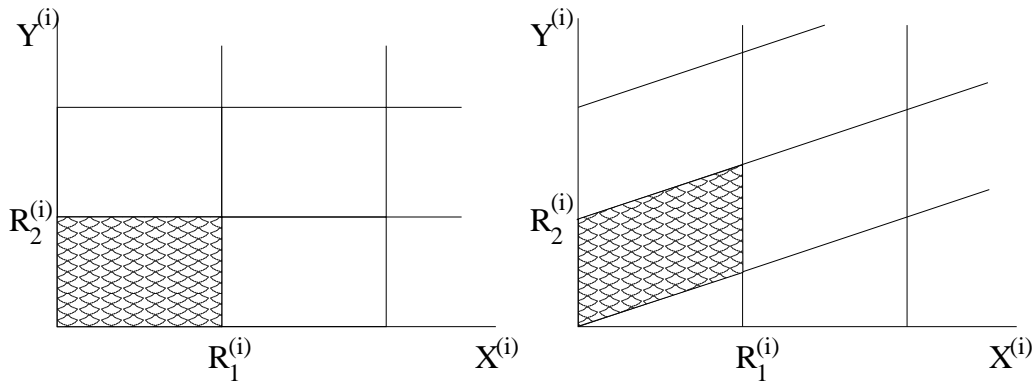


Figure 1: *Rectangular (left panel) and tilted (right panel) tori.*

$$\tan \theta_a^{(i)} = \frac{\tilde{m}_a^{(i)} R_2^{(i)}}{n_a^{(i)} R_1^{(i)}} \quad (2.3)$$

²Here we choose $-\pi/2 \leq \theta_a^{(i)} \leq \pi/2$.

where we define $\tilde{m}_a^{(i)} = m_a^{(i)} + b_i n_a^{(i)}$ and $b_i = 0$ ($b_i = 1/2$) for rectangular (tilted) tori. From here on and for simplicity we will remove the tilde from $\tilde{m}_a^{(i)}$. Given a generic couple (a, b) of stacks of branes they intersect with angles $\pi\theta_{ab}^{(i)}$ in the i -th torus:

$$\theta_{ab}^{(i)} = \frac{\theta_a^{(i)} - \theta_b^{(i)}}{\pi} \quad (2.4)$$

Of special importance is the number of such intersections as it measures the number of chiral fermions. It is given by

$$I_{ab} = [\Pi_a] \cdot [\Pi_b] = \prod_i \left(n_a^{(i)} m_b^{(i)} - m_a^{(i)} n_b^{(i)} \right) \quad (2.5)$$

and it corresponds in the T -dual picture to the index theorem for compactification with internal magnetic fields. In the special case where the brane stack b is the image of a under the orientifold action ($b = a^*$), the chiral states given by this formula fall in two categories: they transform either in the antisymmetric (A) or in the symmetric (S) representation of the gauge group, due to the orientifold projection. Their respective multiplicities are given by:

$$I_{aa^*}^{A,S} = \left(\prod_i 2m_a^{(i)} \right) \frac{1}{2} \left(\prod_j n_a^{(j)} \mp 1 \right). \quad (2.6)$$

In simple toroidal compactifications, states arising from open strings with both ends on the same stack of branes form representations of $N = 4$ supersymmetry. For more complicated compactifications, part of these states can be projected out and one remains with representations of lower supersymmetry. In this work, we will focus on the cases where these states are in $N = 4$ or $N = 2$ multiplets as already stated.

Open strings stretching between two non-parallel stacks of N_a and N_b branes give rise to states in the bifundamental representation (N_a, \bar{N}_b) of $U(N_a) \otimes U(N_b)$. The lightest modes contain, in addition to massless fermions, scalars with masses:

$$\begin{aligned} m_{ab,1}^2 &= \frac{-|\theta_{ab}^{(1)}| + |\theta_{ab}^{(2)}| + |\theta_{ab}^{(3)}|}{2} M_s^2 \\ m_{ab,2}^2 &= \frac{|\theta_{ab}^{(1)}| - |\theta_{ab}^{(2)}| + |\theta_{ab}^{(3)}|}{2} M_s^2 \\ m_{ab,3}^2 &= \frac{|\theta_{ab}^{(1)}| + |\theta_{ab}^{(2)}| - |\theta_{ab}^{(3)}|}{2} M_s^2 \\ m_{ab,4}^2 &= \left(1 - \frac{|\theta_{ab}^{(1)}| + |\theta_{ab}^{(2)}| + |\theta_{ab}^{(3)}|}{2} \right) M_s^2 \end{aligned} \quad (2.7)$$

For parallel branes the first three states become the $N = 4$ scalar partners of the gauge vector bosons. On the other hand, if some of them are massless there is a boson–fermion degeneracy that indicates that part of the original supersymmetry is preserved. Moreover, if any of the $\theta_{ab}^{(i)}$ angles vanishes, a supersymmetric mass $\ell_i M_s$ can be generated through separation of the branes by a distance $\ell_i M_s^{-1}$ in the corresponding torus. Finally, the last mass in (2.7) contains the contribution of a massive string oscillator that could become the lightest state for some values of the angles.

There are three cases corresponding to three different kinds of intersections that must be discussed:

- The case where branes a and b are in the observable set \mathcal{O} with intersections giving rise to chiral $N = 1$ multiplets. The lightest states ϕ_{ab} are identified with quarks, leptons and their supersymmetric partners. This can be achieved with the choice

$$\begin{aligned}
m_{ab,1}^2 &= |\gamma_2| M_s^2 \\
m_{ab,2}^2 &= |\gamma_1| M_s^2 \\
m_{ab,3}^2 &= 0 \\
m_{ab,4}^2 &= (1 - |\gamma_3|) M_s^2
\end{aligned} \tag{2.8}$$

which is such that for generic values of γ_1 and γ_2 the system only preserves one supersymmetry. For simplicity, we are using here the notation $\theta_{ab}^{(i)} = \gamma_i$ and choosing $|\gamma_3| = |\gamma_1| + |\gamma_2|$.

- The case where branes a and b are in the observable set \mathcal{O} , and states in their intersection give rise to $N = 2$ supermultiplets which are identified with supersymmetric pairs of Higgs doublets. This corresponds to Eq. (2.8) with $\gamma_1 = 0$, i.e.

$$\begin{aligned}
m_{ab,1}^2 &= |\gamma_2| M_s^2 \\
m_{ab,2}^2 &= 0 \\
m_{ab,3}^2 &= 0 \\
m_{ab,4}^2 &= (1 - |\gamma_2|) M_s^2 .
\end{aligned} \tag{2.9}$$

We assume that n_H Higgs chiral multiplets remain light while the other $N = 2$ multiplets get large ($\sim M_s$) supersymmetric masses. Actually, this is not the only way to obtain Higgs multiplets. One of the two doublets may emerge from a chiral intersection together with the

leptons, say between a $U(2)$ and a $U(1)$ brane, while the other one could arise as a chiral “anti-doublet” from the intersection of $U(2)$ with the mirror of $U(1)$ brane.

- The case where brane a is in the observable set \mathcal{O} and brane b in sector \mathcal{M} . The states ϕ_{ab} living at such intersections are assumed to be non-chiral and we will call them “messengers” for reasons that will be apparent below. They are non-supersymmetric because of mass splitting between fermionic and bosonic modes. Through loop effects, they will induce supersymmetry breaking to the observable sector. We will use here the notation $\theta_{ab}^{(i)} = \alpha_i$ and choose again $|\alpha_3| = |\alpha_1| + |\alpha_2|$. Two different kinds of models can arise at this level. They correspond to the following two possibilities:

i) The first possibility is that the states in the intersection ϕ_{ab} originate as a perturbation around an $N = 2$ supersymmetric solution. In fact, in order to have a scale of supersymmetry breaking hierarchically small compared to the string scale we perform a tiny deformation of the intersection angles (preserving $N = 2$ supersymmetry):

$$|\alpha_i| \rightarrow |\alpha_i| + \epsilon_i. \quad (2.10)$$

In one of the tori, chosen to be the first one, the branes should remain parallel and separated³ by a distance $\ell_1 M_s^{-1}$, i.e.

$$\begin{aligned} \alpha_1 &= \epsilon_1 = 0 \\ \alpha_2 + \alpha_3 &= \epsilon \neq 0 \end{aligned} \quad (2.11)$$

leading to the localized scalar masses:

$$\begin{aligned} m_{ab,1}^2 &\simeq (|\alpha_2| + \ell_1^2) M_s^2 \\ m_{ab,2}^2 &\simeq (-\epsilon + \ell_1^2) M_s^2 \\ m_{ab,3}^2 &\simeq (\epsilon + \ell_1^2) M_s^2 \\ m_{ab,4}^2 &\simeq (1 - |\alpha_2| + \ell_1^2) M_s^2 \end{aligned} \quad (2.12)$$

These states will induce at one-loop Dirac masses for gauginos and Higgsinos, as we will show in next sections. For instance the Dirac gaugino masses turn out to be

$$m_{1/2}^D \simeq \frac{\alpha}{4\pi} \epsilon M_s \quad (2.13)$$

³Note that this supersymmetric mass will prevent the appearance of tachyons for scalars in (2.12) and will make the configuration stable.

where α is the gauge coupling. This provides a stringy realization of gauge mediated supersymmetry breaking in the absence of R -symmetry breaking.

Scalars localized at supersymmetric intersections, as those discussed in (2.8), can be given a small supersymmetry breaking through another tiny deformation of the corresponding angles by ϵ . This can be described at the effective field theory level as a Fayet-Iliopoulos D -term breaking corresponding to the presence of an anomalous $U(1)$ factor $\langle D \rangle \sim \epsilon M_s^2$ [4, 9]. As a result, scalar masses in the observable sector can acquire different masses depending on whether matter is charged under this $U(1)$ or not. In particular:

- If the observable sector is charged, bosons of the matter multiplets acquire tree-level masses, $m_0^2 \sim \epsilon M_s^2$.
- Otherwise the bosons of the observable sector receive masses at the two-loop level [15]

$$m_0^2 \simeq \left(\frac{\alpha}{4\pi} \epsilon M_s \right)^2 . \quad (2.14)$$

In both cases the transmission of supersymmetry breaking to the gaugino and Higgsino sector is mediated through the “messengers” since the quark and lepton sectors are chiral (and do not contribute at this order) while the Higgs sector (with masses tuned to remain in the TeV range) will contribute negligibly. Note that the brane b does not have to be necessarily in sector \mathcal{M} . It could also be part of the “observable” sector. Consider for instance the example where chiral quark doublets come from the intersection of a $U(3)$ and a $U(2)$ brane stack. Non chiral antiquark doublets, coming from the intersection of $U(3)$ with the orientifold image of $U(2)$ can play the role of messengers generating Dirac gaugino masses for both $U(3)$ and $U(2)$ gauge groups. Moreover, the image of $U(2)$ may have non trivial intersection with another $U(1)$ stack producing leptons, and thus it is part of sector \mathcal{O} .

ii) The second possibility corresponds to the case where a supersymmetry is preserved by the subset of branes in \mathcal{O} for some choice of the compactification moduli, but will never be conserved by the whole set of branes $\mathcal{O} \oplus \mathcal{M}$. This is for instance the case for the toroidal compactification discussed in Section 5 below. Let us denote by $c_i = \pm 1$ the relevant coefficients that define in (2.7) the supersymmetry preserved by the observable sector. The intersection between the \mathcal{O} and \mathcal{M} branes leads to states ϕ_{ab} with a supersymmetry breaking mass

$$m_{ab}^2 \simeq \frac{1}{2\pi} \sum_i c_i \left| \arctan \left\{ \frac{m_a^{(i)} R_2^{(i)}}{n_a^{(i)} R_1^{(i)}} \right\} - \arctan \left\{ \frac{m_b^{(i)} R_2^{(i)}}{n_b^{(i)} R_1^{(i)}} \right\} \right| M_s^2 \equiv \epsilon M_s^2 \quad (2.15)$$

Keeping the branes (a, b) parallel in one of the tori allows to use again the states ϕ_{ab} at their intersection as messengers to produce gaugino and Higgsino Dirac masses, as in Sections 3 and 4. However now the desired suppression of ϵ will be generated by the presence of a large hierarchy between the size of the compact dimensions. More precisely for

$$\frac{R_2^{(i)}}{R_1^{(i)}} \ll 1 \quad (2.16)$$

one can obtain an ϵ parameter hierarchically smaller than one

$$\epsilon \simeq \frac{1}{2\pi} \sum_i c_i \left| \frac{m_a^{(i)}}{n_a^{(i)}} - \frac{m_b^{(i)}}{n_b^{(i)}} \right| \frac{R_2^{(i)}}{R_1^{(i)}} \quad (2.17)$$

where the sum obviously goes over the tori where branes intersect. In the absence of tree level supersymmetry breaking, superpartners in the observable sector will acquire two-loop mass splitting given by (2.14). A toy model based on these ideas will be presented in Section 5.

3 STRING COMPUTATION OF THE DIRAC MASS

As the gauginos lie in representations of extended supersymmetry ($N \geq 2$), they come in copies that can pair up to receive a Dirac mass at one-loop order in the string genus expansion. The corresponding world-sheet has the topology of a cylinder stretching between two stacks of N_a and N_b branes, respectively (see Fig. 2). The vertex operators $V^{(1)}$ and $V^{(2)}$ associated with the gauginos are inserted on one boundary, for example that corresponding to the N_a branes.

The mass is given by the integrated two-point correlation function of the relevant vertex operators:

$$A(1, 2) = \int dz \int dw \langle V^{(1)}(z) V^{(2)}(w) \rangle \quad (3.1)$$

where the integrals are along the boundary of the annulus.

The $N = 2$ supersymmetry which relates the gauge vector and gauginos is chosen to be associated with the supercharges that preserve the (undeformed) $N = 2$ preserving brane intersection (see Eq. (2.12) with $\epsilon = 0$). These supercharges are given by

$$(Q_{\alpha, -++}, Q_{, +--}^{\dot{\alpha}}) \quad (Q_{\alpha, ---}, Q_{, +++}^{\dot{\alpha}}) \quad (3.2)$$

where \pm denotes helicities in the internal directions and $\alpha, \dot{\alpha}$ are Weyl spinor indices in four dimensions. The supercharges have been grouped into Majorana spinors in four dimensions.

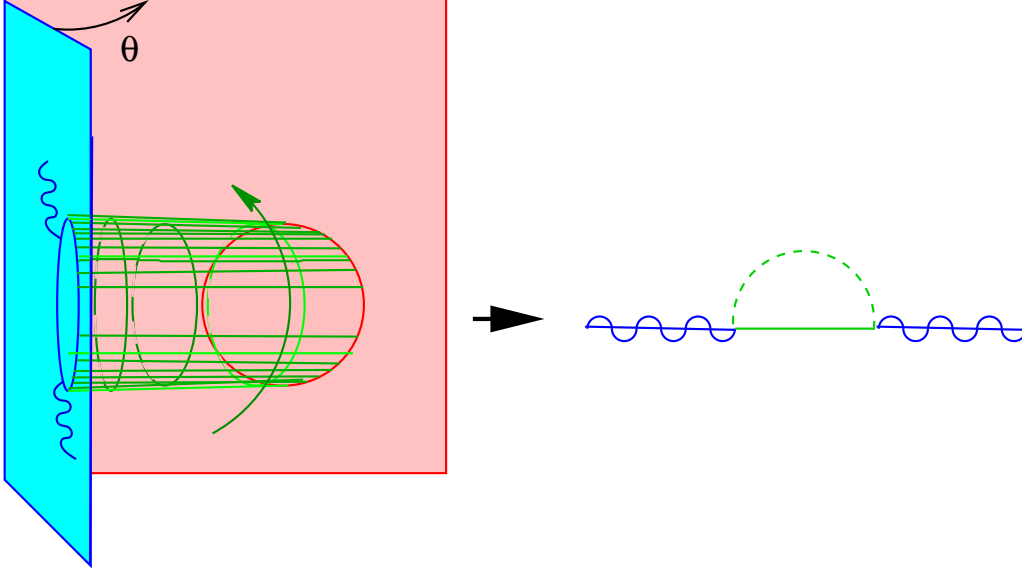


Figure 2: *Non-supersymmetric states stretching between the two-branes induce at one-loop masses for the gauginos on each brane.*

As the gauginos of $N = 2$ supersymmetry have the same internal helicities as the supercharges (3.2), the vertex operators in the $-\frac{1}{2}$ -ghost picture are given by:

$$\begin{aligned}
 V_{-\frac{1}{2}}^{(1)}(z) &= g_o \lambda^1 e^{-\frac{\phi}{2}} e^{\frac{i}{2} \vec{S} \cdot \vec{H}} e^{\frac{i}{2} (H_1 + H_2 + H_3)} e^{ik_1 \cdot X}(z) \\
 V_{-\frac{1}{2}}^{(2)}(z) &= g_o \lambda^2 e^{-\frac{\phi}{2}} e^{-\frac{i}{2} \vec{S} \cdot \vec{H}} e^{-\frac{i}{2} (-H_1 + H_2 + H_3)} e^{ik_2 \cdot X}(z),
 \end{aligned} \tag{3.3}$$

together with their CPT conjugates which are obtained by simply reversing helicities in the internal directions and flipping the space-time chirality. Here ϕ is the bosonized two-dimensional reparametrization ghost field, X^μ are the target space coordinates, $k_1 = k_2 = 0$ are the space-time momenta, H_i arise from bosonization of the spin fields and \vec{S} is the helicity vector in four dimensions which is constrained by the GSO projection to be either

$$\vec{S} = (+, +) \quad \text{or} \quad \vec{S} = (-, -) . \tag{3.4}$$

Also g_o is some normalization constant and λ^i are Chan-Paton factors of the associated gauge group factor.

On the cylinder, the total ϕ -charge must vanish. Therefore, we must use picture-changing to transform one of the vertex operators to the $+\frac{1}{2}$ picture. This is done by operating with the

BRST charge:

$$V_{\frac{1}{2}}^{(2)}(w) = \lim_{z \rightarrow w} e^\phi T_F(z) V_{-\frac{1}{2}}^{(2)}(w), \quad (3.5)$$

where $T_F(z)$ is the world-sheet supercurrent:

$$T_F(z) = i\sqrt{\frac{2}{\alpha'}}(\partial X^\mu \psi_\mu + \sum_{i=1,2,3} \frac{1}{\sqrt{2}}(\partial Z^i \bar{\Psi}^i + \partial \bar{Z}^i \Psi^i)). \quad (3.6)$$

The fields ψ^μ are real world-sheet fermions while in the second and third terms of (3.6) we have used complex coordinates for the internal directions:

$$Z^i = X^{(i)} + iY^{(i)} \quad \bar{Z}^i = X^{(i)} - iY^{(i)} \quad i = 1, 2, 3 \quad (3.7)$$

and analogously for Ψ^i and $\bar{\Psi}^i$. Note that $\Psi^i \bar{\Psi}^i = i\partial H_i$.

After picture-changing, the vertex operator $V_{-\frac{1}{2}}^{(2)}$ contains several terms, but only one of them contributes to the two-point function $\langle V^{(1)}(z)V^{(2)}(w) \rangle$. Indeed, the internal helicities must cancel between the two vertex operators to yield a non-vanishing correlator. For the choice made in (3.3), T_F must induce a flip of the sign of H_1 . We can therefore set

$$V_{\frac{1}{2}}^{(2)}(w) \rightarrow i\frac{1}{\sqrt{\alpha'}} g_0 \lambda^2 e^{\frac{\phi}{2}} e^{-\frac{i}{2}\vec{S}\cdot\vec{H}} e^{-\frac{i}{2}(H_1+H_2+H_3)} e^{ik_2 X} \partial Z^1 \quad (3.8)$$

Inserting these operators into a path integral on the cylinder, with $k_1 = k_2 = 0$, leads to

$$\langle V^{(1)}(z)V^{(2)}(w) \rangle = i\frac{1}{\sqrt{\alpha'}} g_o^2 \text{tr}(\lambda^1 \lambda^2) N_b [BC] \times [FC] \quad (3.9)$$

where the factor N_b comes from the trace over the Chan-Paton degrees of freedom of the second boundary of the cylinder. The factors $[BC]$ and $[FC]$ denote, respectively, correlation functions of world-sheet bosons and world-sheet fermions together with the associated ghosts of opposite statistics. They are given by

$$[BC] = \langle \mathbb{I} \rangle_{bc} \langle \mathbb{I} \rangle_{X^\mu} \langle \partial Z^1 \rangle_{Z^1} \langle \mathbb{I} \rangle_{Z^2} \langle \mathbb{I} \rangle_{Z^3} \quad (3.10)$$

and

$$[FC] = \sum_{a,b} C \begin{bmatrix} a \\ b \end{bmatrix} \langle e^{-\frac{\phi}{2}} e^{\frac{\phi}{2}} \rangle \langle e^{\frac{i}{2}\vec{S}\cdot\vec{H}} e^{-\frac{i}{2}\vec{S}\cdot H} \rangle \prod_{i=1,2,3} \langle e^{\frac{i}{2}H_i} e^{-\frac{i}{2}H_i} \rangle \quad (3.11)$$

where \mathbb{I} is the identity operator. In the bosonic correlator, $\langle \dots \rangle_{Z^i}$ denote bosonic path-integrals over the complex coordinates Z^i while $\langle \dots \rangle_{bc}$ is a path integral over the parametrization bc-ghosts. In the fermionic correlation function the sum is over the spin structures for which we use the same notation a, b as for brane stacks but the difference is obvious for the reader.

3.1 THE BOSONIC CORRELATOR

The one-point functions of identity operators are given by the appropriate piece of the partition function on the cylinder for the two stacks of intersecting $D6$ -branes. The bc-ghosts cancel exactly the path-integral over two of the four space-time coordinates X^μ , and we are left with the simple expression: ⁴

$$[BC] = \frac{1}{\eta(it)^2} \langle \partial Z^1(w) \rangle_{Z^1} \frac{\eta(it)}{\vartheta \left[\begin{smallmatrix} \frac{1}{2} + \alpha_2 \\ \frac{1}{2} \end{smallmatrix} \right]} \frac{\eta(it)}{\vartheta \left[\begin{smallmatrix} \frac{1}{2} + \alpha_3 \\ \frac{1}{2} \end{smallmatrix} \right]} \quad (3.12)$$

where α_2, α_3 are the non-vanishing intersection angles of the $D6$ -branes in the Z_2 and Z_3 planes. Here, t is the open string proper-time parametrizing the world-sheet annulus.

To compute the correlation function of $\partial Z^1(w)$, consider the coordinate domain of the annulus to be a square parametrized by two coordinates (τ, σ) with ranges $0 \leq \sigma \leq \pi, 0 \leq \tau \leq 2\pi t$ and the identification $\tau \sim \tau + 2\pi t$. The operator Z^1 is located on the boundary, thus

$$\langle \partial Z^1(w) \rangle = Tr \left(e^{-2\pi(t-\tau)H} \partial Z^1(\tau, 0) e^{-2\pi\tau H} \right) \quad (3.13)$$

where H is the hamiltonian for the Z^1 coordinate field. Only the zero-modes of the mode expansion of Z^1 contribute to the trace in (3.13) and, therefore, if the branes would intersect with a non-trivial angle in the Z^1 plane, the correlation function would vanish. We therefore need to impose

$$\alpha_1 = 0 \quad (3.14)$$

as anticipated in (2.11). This is also a necessary condition for the D -brane intersection to preserve the supercharges (3.2).

As the two stacks are parallel in the first torus, they can be separated at a distance $2\pi\ell$. For simplicity, this separation is taken along the $X^{(1)}$ direction. The mode expansions read:

$$\begin{aligned} X^{(1)} &= x_0^{(1)} + 2\alpha' p^{(1)} \tau - i\sqrt{2\alpha'} \sum_{n \neq 0} \frac{\alpha_{1,n}^{(1)}}{n} e^{in\tau} \cos n\sigma \\ Y^{(1)} &= y_0^{(1)} + (2nR_2^{(1)} + 2\ell)\sigma + \sqrt{2\alpha'} \sum_{n \neq 0} \frac{\alpha_{2,n}^{(1)}}{n} e^{in\tau} \sin n\sigma \end{aligned} \quad (3.15)$$

⁴The definitions of the various modular theta functions appearing in this section are given in the Appendix.

where n is the winding number. The momentum $p^{(1)}$ is quantized in units of $1/R_1^{(1)}$. Using the expression for the Hamiltonian,

$$H = \alpha'(p^{(1)})^2 + \frac{1}{4\alpha'}(2nR_2^{(1)} + 2\ell)^2 + \sum_{n>0, i=1,2} \alpha_{i,-n}^{(1)} \alpha_{i,n}^{(1)} - \frac{2}{24} \quad (3.16)$$

shows, as stated above, that the oscillators in (3.15) do not contribute to the trace:

$$\langle \partial Z^1(\tau, 0) \rangle = 2i\alpha' \sum_{m,n} \left(\frac{m}{R_1^{(1)}} + \left(\frac{\ell}{\alpha'} + \frac{nR_2^{(1)}}{\alpha'} \right) \right) e^{-2\pi\alpha't \left(\frac{m^2}{R_1^{(1)2}} + \left(\frac{nR_2^{(1)}}{\alpha'} + \frac{\ell}{\alpha'} \right)^2 \right)} \frac{1}{\eta(it)^2} \quad (3.17)$$

The sum over the quantized momenta does not contribute either, since the different terms cancel pairwise. However, the sum over windings is non-vanishing provided that the two stacks are non-coincident, i.e. $\ell \neq 0$. Inserting (3.17) into (3.12) yields the complete bosonic correlator:

$$[BC] = \frac{2i\alpha'}{\eta^2 \vartheta \left[\begin{smallmatrix} \frac{1}{2} + \alpha_2 \\ \frac{1}{2} \end{smallmatrix} \right] (0) \vartheta \left[\begin{smallmatrix} \frac{1}{2} + \alpha_3 \\ \frac{1}{2} \end{smallmatrix} \right] (0)} \sum_n \left(\frac{nR_2^{(1)}}{\alpha'} + \frac{\ell}{\alpha'} \right) e^{-2\pi\alpha't \left(\frac{nR_2^{(1)}}{\alpha'} + \frac{\ell}{\alpha'} \right)^2} \quad (3.18)$$

3.2 THE FERMIONIC CORRELATOR

To compute the fermionic correlator, we will make use of the elementary two-points functions:

$$\langle e^{\frac{\phi}{2}} e^{-\frac{\phi}{2}} \rangle = \left(\frac{\vartheta_1(z-w)}{\vartheta_1'(0)} \right)^{\frac{1}{4}} \frac{\eta}{\vartheta \left[\begin{smallmatrix} a \\ b \end{smallmatrix} \right] \left(\frac{z-w}{2} \right)} \quad (3.19)$$

$$\langle e^{\frac{i}{2}H_J} e^{-\frac{i}{2}H_J} \rangle = \left(\frac{\vartheta_1'(0)}{\vartheta_1(z-w)} \right)^{\frac{1}{4}} \vartheta \left[\begin{smallmatrix} a \\ b \end{smallmatrix} \right] \left(\frac{z-w}{2} \right) \frac{1}{\eta} \quad J = \xi, 1 \quad (3.20)$$

$$\langle e^{\frac{i}{2}H_i} e^{-\frac{i}{2}H_i} \rangle = \left(\frac{\vartheta_1'(0)}{\vartheta_1(z-w)} \right)^{\frac{1}{4}} \vartheta \left[\begin{smallmatrix} a + \alpha_i \\ b \end{smallmatrix} \right] \left(\frac{z-w}{2} \right) \frac{1}{\eta} \quad i = 2, 3 \quad (3.21)$$

where ξ labels the two non-compact complexified space-time dimensions and $a, b \in \{0, \frac{1}{2}\}$. The difference between (3.20) and (3.21) comes from the fact that in the Z^2 and Z^3 planes, the brane intersection angles α_2 and α_3 are non-vanishing. The $z-w$ dependence was determined for instance in Ref. [16] and the correct normalization is inferred by matching the short distance limit $z \rightarrow w$ on both sides. For example, from the operator product expansion (OPE) of bosonized fermions we have

$$\langle e^{\frac{i}{2}H_\xi} e^{-\frac{i}{2}H_\xi} \rangle \rightarrow \frac{1}{(z-w)^{\frac{1}{4}}} \langle \mathbb{I} \rangle = \frac{1}{(z-w)^{\frac{1}{4}}} \frac{\vartheta \left[\begin{smallmatrix} a \\ b \end{smallmatrix} \right] (0)}{\eta} \quad (3.22)$$

as the correlator of the identity operator is the spin-structure dependent partition function of a complex fermion. Taking the same limit on the right-hand side of (3.20) and comparing with (3.22) allows to check that the dependence on the cylinder modulus is correctly reproduced.

Inserting (3.19)-(3.21) into (3.11), we obtain:

$$[FC] = \frac{1}{\eta(it)^4} \frac{\vartheta_1'(0)}{\vartheta_1(z-w)} \Sigma \quad (3.23)$$

where

$$\Sigma = \sum_{a,b} C \begin{bmatrix} a \\ b \end{bmatrix} \vartheta \begin{bmatrix} a \\ b \end{bmatrix}^2 \left(\frac{z-w}{2} \right) \vartheta \begin{bmatrix} a + \alpha_2 \\ b \end{bmatrix} \left(\frac{z-w}{2} \right) \vartheta \begin{bmatrix} a + \alpha_3 \\ b \end{bmatrix} \left(\frac{z-w}{2} \right). \quad (3.24)$$

The coefficients $C \begin{bmatrix} a \\ b \end{bmatrix}$ can be obtained by the same method as the one used above to deduce the normalization of the elementary correlators. From the OPE we deduce that the fermionic contribution $[FC]$ has a pole in $z-w$ whose residue, being the correlator of the identity, should be the full fermionic partition function on the annulus:

$$\begin{aligned} [FC] &\rightarrow \frac{1}{(z-w)} Z_F = & (3.25) \\ &\frac{1}{z-w} \sum_{a,b} \eta_{a,b} e^{-2\pi i b(\alpha_2 + \alpha_3)} \frac{\vartheta \begin{bmatrix} a \\ b \end{bmatrix}^2(0)}{\eta^2} \frac{\vartheta \begin{bmatrix} a + \alpha_2 \\ b \end{bmatrix}(0)}{\eta} \frac{\vartheta \begin{bmatrix} a + \alpha_3 \\ b \end{bmatrix}(0)}{\eta} \end{aligned}$$

where as usual $\eta_{0,0} = \eta_{\frac{1}{2},\frac{1}{2}} = -\eta_{0,\frac{1}{2}} = -\eta_{\frac{1}{2},0} = 1$. Comparing with the limit $z \rightarrow w$ of the right-hand side of (3.23), we obtain

$$C \begin{bmatrix} a \\ b \end{bmatrix} = \eta_{a,b} e^{-2\pi i b(\alpha_2 + \alpha_3)} \quad (3.26)$$

The spin-structure sum in Σ can now be performed by using the Riemann theta-identity (A.10). To apply this identity we must first use the rearrangement

$$\vartheta \begin{bmatrix} a + \alpha \\ b \end{bmatrix} (z) = e^{2\pi i(b+z)\alpha} q^{\frac{\alpha^2}{2}} \vartheta \begin{bmatrix} a \\ b \end{bmatrix} (z + \alpha it) \quad (3.27)$$

to put the theta functions in the appropriate form. The α_i -dependent phase factor in $C \begin{bmatrix} a \\ b \end{bmatrix}$ then cancels out. After applying the Riemann identity, we can use (3.27) again to put α_2 and

α_3 back into the argument of the theta function. The result is:

$$\begin{aligned} \Sigma = & 2\vartheta \left[\begin{matrix} \frac{1}{2}(1 + \alpha_2 + \alpha_3) \\ \frac{1}{2} \end{matrix} \right] (z - w) \\ & \vartheta \left[\begin{matrix} \frac{1}{2}(1 - \alpha_2 + \alpha_3) \\ \frac{1}{2} \end{matrix} \right] (0) \vartheta \left[\begin{matrix} \frac{1}{2}(1 + \alpha_2 - \alpha_3) \\ \frac{1}{2} \end{matrix} \right] (0) \vartheta \left[\begin{matrix} \frac{1}{2}(1 - \alpha_2 - \alpha_3) \\ \frac{1}{2} \end{matrix} \right] (0) \end{aligned} \quad (3.28)$$

We can now expand this result around the $N = 2$ supersymmetric configuration of D -branes using $\epsilon = \alpha_2 + \alpha_3$. Expanding (3.28) to lowest order in ϵ gives

$$[FC] = -8\pi^2 \epsilon \eta^2 \vartheta \left[\begin{matrix} \frac{1}{2} + \alpha_2 \\ \frac{1}{2} \end{matrix} \right] (0) \vartheta \left[\begin{matrix} \frac{1}{2} + \alpha_3 \\ \frac{1}{2} \end{matrix} \right] (0) \quad (3.29)$$

where we made use of the identity $\vartheta'_1(0)^2 = 4\pi^2 \eta^6$.

3.3 THE TWO-GAUGINO AMPLITUDE

Inserting (3.18) and (3.29) into (3.9) we see that, to lowest order in ϵ , all contributions from the string oscillators cancel and only the classical part of the correlation function remains. This piece is independent of $z - w$ and the integrals over the locations of the vertex operators in (3.1) only contribute a factor $(2\pi t)^2$. The result is then simply

$$A(1, 2) = 64\pi^4 t^2 \sqrt{\alpha'} \epsilon g_0^2 \text{tr}(\lambda^1 \lambda^2) N_b \sum_n \left(\frac{nR_2^{(1)}}{\alpha'} + \frac{\ell}{\alpha'} \right) e^{-2\pi t \alpha' \left(\frac{nR_2^{(1)}}{\alpha'} + \frac{\ell}{\alpha'} \right)^2} \quad (3.30)$$

From this we can obtain the two-gaugino amplitude as

$$\mathcal{A}(1, 2) = \frac{1}{4} \frac{V_4}{(8\pi^2 \alpha')^2} \int_0^\infty \frac{dt}{t^3} A(1, 2) \quad (3.31)$$

where the integration measure has been obtained by comparing with the partition function for intersecting $D6$ -branes. The infinite volume factor V_4 arises from momentum conservation. Had we chosen $k_1, k_2 \neq 0$, it would be replaced by a δ -function, $V_4 \rightarrow (2\pi)^4 \delta^{(4)}(k_1 + k_2)$. The normalization g_0 can in principle be determined. However for our purposes we only need to know that g_0 is proportional to the open string coupling and therefore $g_0^2 \sim g_s$, where $g_s = e^\varphi$ is the closed string coupling, determined by the VEV of the string dilaton φ . For the Dirac mass of the gauginos we then obtain the final result:

$$m_{1/2}^D \sim g_s \epsilon N_2 \int_0^\infty \frac{dt}{t} \sum_n \left(\frac{nR_2^{(1)}}{\alpha'} + \frac{\ell}{\alpha'} \right) e^{-2\pi t \alpha' \left(\frac{nR_2^{(1)}}{\alpha'} + \frac{\ell}{\alpha'} \right)^2} \quad (3.32)$$

By a Poisson resummation, it can easily be checked that the integral is finite. Note also that in the limit where $\ell \rightarrow 0$, the mass vanishes.

To interpret the meaning of this result, we consider the mass spectrum of open strings which stretch between the two stacks of D -branes. In the limit where $\epsilon \rightarrow 0$, these string modes form $N = 2$ hypermultiplets. The mass of the lightest multiplet is given by $\alpha' m^2 = \frac{\ell^2}{\alpha'}$. When ϵ is non-vanishing, supersymmetry is completely broken and there is a mass splitting between fermions and scalars. The lightest fermions remain at $\alpha' m_F^2 = \frac{\ell^2}{\alpha'}$, while their scalar superpartners have their masses lifted to $\alpha' m_S^2 = \alpha' m_F^2 + \frac{1}{2}\epsilon$. The result (3.32) then implies the relation, in the limit $\ell \ll \sqrt{\alpha'}$:

$$m_{1/2}^D \sim g_s \epsilon \frac{\ell}{\alpha'} \sim g_s \frac{m_S^2 - m_F^2}{M_s^2} m_F, \quad (3.33)$$

where $M_s^2 = 1/\alpha'$ is the string scale. For string size brane separation, such that $\ell M_s \sim 1$, we recover the anticipated behavior (2.13).

4 COMPUTATION OF THE HIGGSINO MASS

Consider two stacks of $D6$ -branes: the first stack of N_a branes aligned with the horizontal $X^{(i)}$ axes ⁵ and the second stack of N_c branes intersecting the horizontal axes in the three tori at angles $\theta_c^{(i)} = \beta_i$, $i = 1, 2, 3$. For $\beta_1 = 0$ and $\beta_2 = \beta_3$, this configuration preserves $N = 2$ supersymmetry and the lightest string modes on the intersection describe hypermultiplets. They are made of two $N = 1$ chiral multiplets, identified as two Higgs bosons and their $N = 1$ superpartners, the Higgsinos.

A simple way to generate a mass term for these Higgsinos is to separate the branes in the first torus where they are parallel. This corresponds to switching on a vacuum expectation value for a particular scalar that parametrizes the location of the brane in the direction $X^{(1)}$. The mass is then given by the tension times the minimal length of the string, which is just the brane separation. This mass generation is not always possible, as for specific constructions such as orbifold models, the corresponding scalar is projected out of the light spectrum.

In this section we will investigate the possibility that the Higgsino mass is generated at one-loop in a way similar to the Dirac gaugino mass. The appropriate string amplitude corresponds to a world-sheet with the topology of an annulus. On the first boundary two vertex operators that create the Higgsinos are inserted. The Higgsinos arise from open strings with the two

⁵This can be achieved by a rotation of the subgroup $U(1)^3 \subset SO(6)$.

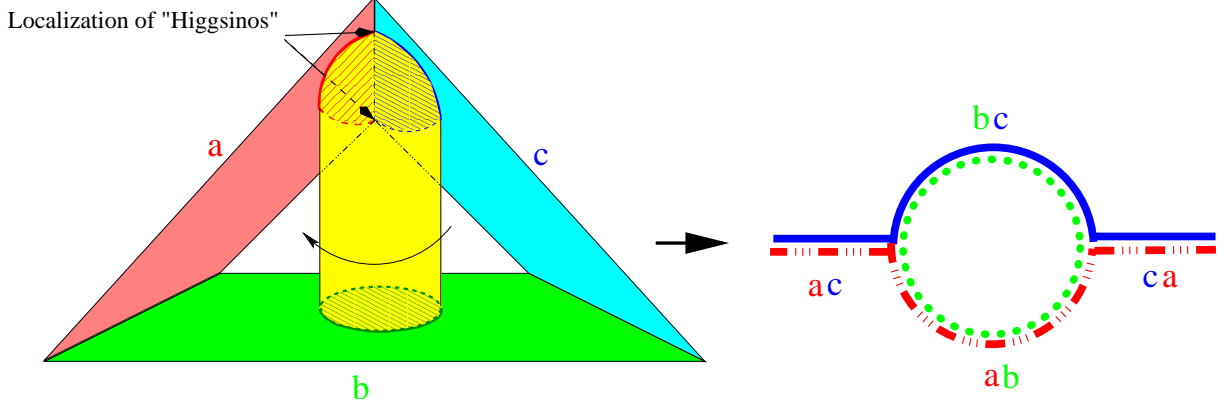


Figure 3: *Higgsinos are localized at the intersection of branes a and c. Non-supersymmetric states stretching between the branes (a, b) and (c, b) induce at one-loop masses for the Higgsinos.*

endpoints located on different stacks of D -branes. This means that part of the first boundary of the annulus lies on the stack of N_a branes and the other part lies on the other stack of N_c branes (see Fig. 3). This change of boundary conditions is implemented by the insertion of twist fields σ_{β_i} and $\sigma_{1-\beta_i}$. The other boundary of the annulus must be located on a third stack of N_b branes. The two stacks N_b and N_a intersect at angles α_i . As this additional stack must preserve the same supersymmetries as the other stacks, we will start from the simplest possibility $\alpha_i = 0$ and make a small deformation of the angles to break supersymmetry, as described in Section 2. The generalization to $\alpha_i \neq 0$ is straightforward.

The vertex operators of the Higgsinos in the $-\frac{1}{2}$ -ghost picture are:

$$\begin{aligned}
 V_{-\frac{1}{2}}^{(1)}(z) &= g_o \lambda^1 e^{-\frac{\phi}{2}} e^{\frac{i}{2} \vec{S} \cdot \vec{H}} e^{\frac{i}{2} H_1} e^{-i(\beta_2 - \frac{1}{2}) H_2} e^{-i(\beta_3 - \frac{1}{2}) H_3} \sigma_{1-\beta_2} \sigma_{1-\beta_3} e^{ik_1 \cdot X}(z) \\
 V_{-\frac{1}{2}}^{(2)}(z) &= g_o \lambda^2 e^{-\frac{\phi}{2}} e^{-\frac{i}{2} \vec{S} \cdot \vec{H}} e^{\frac{i}{2} H_1} e^{i(\beta_2 - \frac{1}{2}) H_2} e^{i(\beta_3 - \frac{1}{2}) H_3} \sigma_{\beta_2} \sigma_{\beta_3} e^{ik_2 \cdot X}(z)
 \end{aligned}$$

Note the similarities with the vertex operators of the gauginos (3.3). The helicities have been shifted by the intersection angles. In addition we have inserted the bosonic twist operators as world-sheet superpartners of the operators $e^{\pm i(\beta_i - \frac{1}{2}) H_i}$. In the limit where $\beta_i \rightarrow 0$, we recover precisely the gaugino vertices since in this limit the bosonic twists become the identity operators. Since the annulus has vanishing ghost-charge we need again to transform one of the operators to the $\frac{1}{2}$ -picture. This is done precisely in the same way as in Section 3. To obtain a non-vanishing correlator, the world-sheet supercurrent $T_F(z)$ must be used to flip the sign of

H_1 in one of the exponents. The result is

$$V_{\frac{1}{2}}^{(2)}(z) \rightarrow g_0 \lambda^2 e^{\frac{\phi}{2}} e^{-\frac{i}{2} \vec{S} \cdot \vec{H}} e^{-\frac{i}{2} H_1} e^{i(\beta_2 - \frac{1}{2}) H_2} e^{i(\beta_3 - \frac{1}{2}) H_3} \sigma_{\beta_2} \sigma_{\beta_3} e^{ik_2 X} \partial Z^1. \quad (4.1)$$

Inserting the vertex operators at zero momentum in the correlator leads to an expression of the same form as (3.9),

$$\langle V^{(1)}(z) V^{(2)}(w) \rangle = i \frac{1}{\sqrt{\alpha'}} g_o^2 \text{tr}(\lambda^1 \lambda^2) N_b [BC] \times [FC] \quad (4.2)$$

where now

$$[BC] = \langle \mathbb{I} \rangle_{bc} \langle \mathbb{I} \rangle_{X^\mu} \langle \partial Z^1 \rangle_{Z^1} \langle \sigma_{1-\beta_2} \sigma_{\beta_2} \rangle_{Z^2} \langle \sigma_{1-\beta_3} \sigma_{\beta_3} \rangle_{Z^3} \quad (4.3)$$

$$\begin{aligned} [FC] &= \sum_{a,b} C \begin{bmatrix} a \\ b \end{bmatrix} \langle e^{-\frac{\phi}{2}} e^{\frac{\phi}{2}} \rangle \langle e^{\frac{i}{2} \vec{S} \cdot \vec{H}} e^{-\frac{i}{2} \vec{S} \cdot \vec{H}} \rangle \langle e^{\frac{i}{2} H_1} e^{-\frac{i}{2} H_1} \rangle \\ &\times \prod_{i=2,3} \langle e^{-i(\beta_i - \frac{1}{2}) H_i} e^{i(\beta_i - \frac{1}{2}) H_i} \rangle \end{aligned} \quad (4.4)$$

Because of the factor $\langle \partial Z^1 \rangle_{Z^1}$ in $[BC]$ (which was computed in section 3), we must impose that $\alpha_1 = 0$ and that the branes are separated by a distance $\ell \neq 0$ in the $X^{(1)}$ direction in order to obtain a non-vanishing contribution.

4.1 THE FERMION CORRELATOR

The computation of the fermion correlator is similar to the one in Section 3.2. The correlator is computed for general α_2, α_3 then expanded around the ($N = 4$) supersymmetric configuration $\alpha_i = 0$. A slight generalization of (3.21) is given by:

$$\langle e^{iaH_k} e^{-iaH_k} \rangle = \left(\frac{\vartheta'_1(0)}{\vartheta_1(z-w)} \right)^{a^2} \vartheta \left[\begin{matrix} a + \alpha_k \\ b \end{matrix} \right] (a(z-w)) \frac{1}{\eta} \quad ; \quad k = 2, 3 \quad (4.5)$$

with $a, b \in \{0, \frac{1}{2}\}$. The remaining elementary correlators are unchanged and are given in (3.19) and (3.20). Inserting these in (4.4) we obtain

$$[FC] = \frac{1}{\eta^4} \left(\frac{\vartheta'_1(0)}{\vartheta_1(z-w)} \right)^{1-\beta_2(1-\beta_2)-\beta_3(1-\beta_3)} \times \Sigma \quad (4.6)$$

where

$$\Sigma = \sum_{a,b} C \begin{bmatrix} a \\ b \end{bmatrix} \vartheta^2 \begin{bmatrix} a \\ b \end{bmatrix} \left(\frac{z-w}{2} \right) \vartheta \left[\begin{matrix} a + \alpha_2 \\ b \end{matrix} \right] \left((\beta_2 - \frac{1}{2})(z-w) \right) \vartheta \left[\begin{matrix} a + \alpha_3 \\ b \end{matrix} \right] \left((\beta_3 - \frac{1}{2})(z-w) \right) \quad (4.7)$$

The coefficients $C \begin{bmatrix} a \\ b \end{bmatrix}$ are given as before by (3.26). Using the Riemann identity (A.10) allows to perform the spin-structure sum and put Σ into a simpler form. After imposing the supersymmetry condition $\beta_2 = \beta_3 \equiv \beta$, one finds:

$$\begin{aligned} \Sigma &= 2\vartheta \left[\frac{1}{2}(1 + \alpha_2 + \alpha_3) \right]_{\frac{1}{2}} (\beta(z-w)) \times \vartheta \left[\frac{1}{2}(1 + \alpha_2 - \alpha_3) \right]_{\frac{1}{2}} (0) \\ &\times \vartheta \left[\frac{1}{2}(1 - \alpha_2 + \alpha_3) \right]_{\frac{1}{2}} (0) \times \vartheta \left[\frac{1}{2}(1 - \alpha_2 - \alpha_3) \right]_{\frac{1}{2}} ((1-\beta)(z-w)). \end{aligned} \quad (4.8)$$

This can now be expanded around the supersymmetric configuration $\alpha_i = 0$. For simplicity, we take $\alpha_2 = 0$ and $\alpha_3 = \epsilon$, and expand in powers of ϵ :

$$\Sigma = -2\vartheta \left[\frac{1}{2} \right]_{\frac{1}{2}} (\beta(z-w)) \vartheta \left[\frac{1}{2} \right]_{\frac{1}{2}} ((1-\beta)(z-w)) \vartheta_1'(0)^2 \epsilon^2. \quad (4.9)$$

4.2 THE BOSONIC CORRELATOR

The correlator of world-sheet fermions, Eq. (4.9), is already of order ϵ^2 . Since we are interested only in the leading ϵ behavior, for the purpose of computing the bosonic correlation function $[BC]$, we can keep the leading order corresponding to $\alpha_i = 0$. Most pieces of $[BC]$ have already been computed in Section 3. The additional complication is the appearance of correlators of bosonic twist fields. These have been studied for instance for arbitrary genus and any number of twist fields in Ref. [17]. The complete calculation is presented in Appendix B where only the relevant results will be quoted. In fact using Eq. (B.23) in (4.3), leads to

$$\begin{aligned} [BC] &= i \frac{R_1^{(2)} R_1^{(3)}}{\alpha'} (2i\alpha') \left| \frac{\vartheta_1(z-w)}{\vartheta_1'(0)} \right|^{-2\beta(1-\beta)} \frac{1}{\det W(\beta)} \frac{1}{\eta^8} \\ &\times \sum_n \left(\frac{nR_2^{(1)}}{\alpha'} + \frac{\ell}{\alpha'} \right) e^{-2\pi t \alpha' \left(\frac{nR_2^{(1)}}{\alpha'} + \frac{\ell}{\alpha'} \right)^2} \\ &\times \sum_{v_1^{(2)}, v_2^{(2)}} e^{-S_{cl}(v_1^{(2)}, v_2^{(2)})} \times \sum_{v_1^{(3)}, v_2^{(3)}} e^{-S_{cl}(v_1^{(3)}, v_2^{(3)})}, \end{aligned} \quad (4.10)$$

where $W(\beta)$ is defined as in (B.5) for the twist β and $v_{1,2}^{(i)}$ are defined in Eq. (B.16).

4.3 THE HIGGSINO TWO-POINT CORRELATION FUNCTION

Finally inserting (4.6), (4.9) and (4.10) into (4.2) gives for the two-point function:

$$\begin{aligned} \langle V^{(1)}(z)V^{(2)}(w) \rangle &= 32i\pi^3\sqrt{\alpha'}\epsilon^2g_0^2tr(\lambda^1\lambda^2)N_b\frac{R_1^{(2)}R_2^{(3)}}{\alpha'} \\ &\sum_n\left(\frac{nR_2^{(1)}}{\alpha'}+\frac{\ell}{\alpha'}\right)e^{-2\pi t\alpha'\left(\frac{nR_2^{(1)}}{\alpha'}+\frac{\ell}{\alpha'}\right)^2}f(z-w) \end{aligned} \quad (4.11)$$

where

$$\begin{aligned} f(z-w) &= \frac{\vartheta_1(\beta(z-w))\vartheta_1((1-\beta)(z-w))}{\vartheta_1(z-w)\eta^3}\frac{1}{\det W(\beta)} \\ &\times\sum_{v_1^{(2)},v_2^{(2)}}e^{-S_{cl}(v_1^{(2)},v_2^{(2)})}\times\sum_{v_1^{(3)},v_2^{(3)}}e^{-S_{cl}(v_1^{(3)},v_2^{(3)})} \end{aligned} \quad (4.12)$$

The Higgsino mass μ is proportional to the two-Higgsino amplitude obtained upon integration of the correlation function (4.11) over the position of the vertex operators and inserting the result into (3.31). Since the correlator depends only on $z-w$, one of the integrals is trivial and contributes simply a factor $2\pi t$. The final result reads

$$\mu\sim g_s\epsilon^2N_1\int_0^\infty\frac{dt}{t}\sum_n\left(\frac{nR_2^{(1)}}{\alpha'}+\frac{\ell}{\alpha'}\right)e^{-2\pi t\alpha'\left(\frac{nR_2^{(1)}}{\alpha'}+\frac{\ell}{\alpha'}\right)^2}I \quad (4.13)$$

where

$$I=\frac{1}{2\pi t}\int_0^{2\pi t}dx f(ix) \quad (4.14)$$

and it is of order ϵ^2 as expected.

5 A TOY MODEL

In this section we will present a simple model as a framework to implement the realization of the extended split supersymmetry scenario described above with gauge bosons in representations of $N=4$ supersymmetry. It consists on a toroidal orientifold based on the factorized six-dimensional torus $\bigotimes_i T_i^2$ with an orientifold plane along the $X^{(i)}$ axes. This implies for each $D6_a$ -brane with wrapping numbers (n_a, m_a) the presence of the image brane $D6_{a^*}$ with wrapping numbers $(n_a, -m_a)$.

The minimal setup is given by three intersecting stacks of $D6$ -branes giving rise to the gauge group

$$U(N_{a_1}) \otimes U(N_{a_2}) \otimes U(N_b) \equiv U(5) \otimes U(1) \otimes U(N_b) \quad (5.1)$$

The branes $D6_{a_1}$, $D6_{a_2}$ and their images, with supersymmetric intersections, provide the observable set \mathcal{O} while the brane $D6_b$ and its image stand for the (messenger) sector \mathcal{M} . The Standard Model gauge group is embedded in $U(5)$ in the following way:

- Open strings localized at the intersection of $D6_{a_1}$ and $D6_{a_1^*}$ describe three generations transforming as $\mathbf{10}$ of $SU(5)$. Their massless modes transform according to the antisymmetric representation and come in three generations, i.e.

$$\prod_i n_{a_1}^{(i)} = 1, \quad I_{a_1 a_1^*} = 3 \quad (5.2)$$

It is easy to see that an odd number of generations requires the use of tilted tori as those described in Fig. 1.

- Open strings localized at the intersection of $D6_{a_1}$ and $D6_{a_2^*}$ provide three generations of $\bar{\mathbf{5}}$, i.e.

$$I_{a_1 a_2^*} = -3. \quad (5.3)$$

- The $D6_{a_2}$ brane being parallel to the $D6_{a_1}$ in one torus, the massless modes at their intersections will be identified with $N = 2$ hypermultiplets that contain the Supersymmetric Standard Model Higgs doublets.

The supersymmetry breaking messengers arise from strings stretching between brane $D6_{a_1}$ and $D6_{b^*}$ on one side and between $D6_{a_2}$ and $D6_b$ on the other side. These branes are parallel (and separated) along the second and first torus, respectively, and their intersections contain non-chiral matter with non-supersymmetric masses.

We will discuss here a simple example given in Table 1 where the wrapping numbers of different $D6_I$ -branes ($I = a_1, a_2, b$) in the three tori are listed. It is easy to see that the conditions for cancellation of RR-charge tadpole are satisfied.

The association of any stack of branes $D6_I$ ($I = a_1, a_2, b$) with its orientifold image $D6_{I^*}$ preserves one of the four supersymmetries, that we label as S_α with $\alpha = 1, \dots, 4$, if the angles $\theta_I^{(i)}$ satisfy one of the corresponding relations:

$$\begin{aligned} S_1 : \theta_I^{(1)} + \theta_I^{(2)} + \theta_I^{(3)} &= 0 & S_2 : -\theta_I^{(1)} + \theta_I^{(2)} + \theta_I^{(3)} &= 0 \\ S_3 : \theta_I^{(1)} - \theta_I^{(2)} + \theta_I^{(3)} &= 0 & S_4 : \theta_I^{(1)} + \theta_I^{(2)} - \theta_I^{(3)} &= 0 \end{aligned} \quad (5.4)$$

N_I	$(n_I^{(1)}, m_I^{(1)})$	$(n_I^{(2)}, m_I^{(2)})$	$(n_I^{(3)}, m_I^{(3)})$
$N_{a_1} = 5$	$(1, 1/2)$	$(1, -1/2)$	$(1, -3/2)$
$N_{a_2} = 1$	$(1, 1/2)$	$(1, -5/2)$	$(1, 5/2)$
$N_b n_b^{(1)} n_b^{(2)} n_b^{(3)} = 10$	$n_b^{(1)}(1, 1/2)$	$n_b^{(2)}(1, 1/2)$	$n_b^{(3)}(1, 1/2)$

Table 1: *Wrapping numbers for the three stacks in the model.*

It is easy to check that none of these equations can be simultaneously satisfied by the three set of branes. Instead, we will look for ratios of radii

$$A_i = \frac{R_2^{(i)}}{R_1^{(i)}}, \quad (i = 1, 2, 3) \quad (5.5)$$

that allow one of the relations, chosen to be S_1 , to be satisfied by the stacks a_1 , a_2 and their images. This fixes two of the ratios as functions of the third one. For instance, in the limit of small ratios $A_i \ll 1$, the supersymmetric conditions read as

$$A_1 \simeq \frac{5}{2}A_2 \quad (5.6)$$

$$A_3 \simeq \frac{1}{2}A_2 \quad (5.7)$$

As a result, the intersections $a_1 \cap a_1^*$, $a_2 \cap a_2^*$ and $a_1 \cap a_2^*$ preserve S_1 , the intersections $a_1 \cap a_2$ and $a_1^* \cap a_2^*$ preserve $S_1 \oplus S_2$, the intersections $a_1 \cap b$ preserve $S_3 \oplus S_4$, and finally the other intersections involving b and/or b^* do not preserve any supersymmetry. At these intersections the lightest states have supersymmetry breaking masses of order:

$$\epsilon M_s^2 \sim A_2 M_s^2 \quad (5.8)$$

In particular, the states at the intersection $a_1 \cap b^*$ could act as messengers (a number of $\mathbf{5} + \bar{\mathbf{5}}$ of $SU(5)$) to generate at one-loop level Dirac masses for fermions, gauginos and Higgsinos, and at two-loop supersymmetry breaking masses for bosons of the observable sector.

Note that the gauge symmetry $U(5)$ can be broken to the Standard Model one by a discrete or continuous Wilson line without introducing extra fields. Unification of gauge couplings is thus guaranteed in such particular constructions. Actually, this model realizes the particular D -brane configuration of Ref. [2], in which the $U(5)$ stack is replaced by two separate brane collections, $U(3)$ and $U(2)$, describing strong and weak interactions. Moreover, the hypercharge is the linear combination $Y = -Q_3/3 + Q_2/2$ of the two corresponding $U(1)$ factors Q_3 and Q_2 . The antisymmetric representation of $SU(5)$ is then decomposed in terms of the quark

doublets, the up antiquarks and the right-handed lepton, the latter arising as antisymmetric representations of $U(3)$ and $U(2)$, respectively. Imposing now $I_{32} = I_{33^*}^A = I_{22^*}^A = 3$, the absence of symmetric representations $\prod_i n^{(i)} = 1$ for the two groups, as well as the absence of antiquark doublets $I_{32^*} = 0$, one finds that the strong and weak branes are parallel in all three planes and thus correspond to a Wilson line breaking of the GUT group $U(5)$ studied above ⁶.

6 CONCLUSIONS

Small ϵ deformations of the brane intersection angles provide a simple mechanism to break supersymmetry, which in the T -dual picture corresponds to the introduction of appropriate combinations of magnetic fluxes. At the effective field theory level, this can be described as Fayet-Iliopoulos D -term breaking corresponding to the presence of anomalous $U(1)$ factor(s). Charged scalars localized at their intersections acquire tree-level masses of order $m_0^2 \sim \epsilon M_s^2$.

Scalars in non-chiral sectors can transmit supersymmetry breaking to the observable sector through gauge mediated loop corrections. However, since the D -term does not break R -symmetry no gaugino Majorana masses are expected (at least to the lowest order). Phenomenologically interesting models can appear when the gauge sector has an extended supersymmetry in which case gauginos can get Dirac masses. A phenomenological application of this scenario is given in Ref. [8] where a minimal model based on extended split supersymmetry was shown to be consistent with unification and the presence of Dark Matter. Dirac masses for gauginos and Higgsinos were obtained from the D -breaking assuming the presence of some effective higher-dimensional operators. Even if such operators are absent at tree-level, the present work shows how they arise at one-loop. The precise string calculation points out important features of the messenger sector in the simplest realization. In particular, the fact that the messengers arise from strings stretching between branes that are separated in one direction. They correspond to deformations of $N = 2$ sectors.

A simple toroidal compactification, with an orientifold along one direction, does not allow non-trivial supersymmetric intersecting branes. A vacuum to be considered as starting point for the above mentioned ϵ deformation is then missing, so we instead propose a different scenario. We identify a subset of supersymmetric intersecting branes with the observable sector. The other branes lead to some non-supersymmetric intersections. The matter living at such non-

⁶Other constructions of a similar model with or without extended supersymmetric gauge sector are given in Ref. [18].

chiral intersections will, as in the previous scenario, play the role of the supersymmetry breaking messenger sector. In order to obtain masses hierarchically smaller than the string scale, small ratios of radii are required and they make all intersection angles small. We have presented a GUT model to illustrate this possibility.

Finally note that for both scenarios there are two possibilities from the phenomenological point of view: (i) To introduce an ϵ deformation breaking supersymmetry in the observable sector at tree-level. (ii) To keep the observable sector supersymmetric at tree-level. Non-supersymmetric branes such that the matter living on their intersection with the observable branes are non-chiral will transmit supersymmetry breaking by gauge interactions. In case (ii) both gauginos, squarks, sleptons and Higgses have masses of the same order of magnitude (as in the usual gauge mediated models) opening up the possibility of having a supersymmetric spectrum in the TeV range. This will give rise to a rich phenomenology accessible at LHC energies.

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A APPENDIX A: THETA FUNCTIONS

In this short appendix, we establish our conventions for the modular theta functions and list a few useful properties. The theta functions are defined by:

$$\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (z, \tau) = \sum_{n=-\infty}^{\infty} e^{\pi i(n+a)^2 \tau + 2\pi i(n+a)(z+b)} \quad (\text{A.1})$$

where τ is the (complex) modular parameter of the torus, not to be confused with the world-sheet coordinate used in the text. On the cylinder, this parameter is purely imaginary and in the main text we use the definition $\tau = it$.

Alternatively, the theta functions can be defined as an infinite product:

$$\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (z, \tau) = e^{i2\pi a(b+z)} q^{\frac{1}{2}a^2} \prod_{n \geq 1} (1 + q^{n+a-\frac{1}{2}} e^{2\pi i(b+z)}) (1 + q^{n-a-\frac{1}{2}} e^{-2\pi i(b+z)}) (1 - q^n) \quad (\text{A.2})$$

where $q = e^{2\pi i\tau}$. They satisfy the following periodicity conditions:

$$\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (z + 1, \tau) = -e^{2\pi i(a-\frac{1}{2})} \vartheta \begin{bmatrix} a \\ b \end{bmatrix} (z, \tau) \quad (\text{A.3})$$

$$\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (z + \tau, \tau) = -e^{-2\pi i(b-\frac{1}{2})} e^{-\pi i\tau - 2\pi iz} \vartheta \begin{bmatrix} a \\ b \end{bmatrix} (z, \tau), \quad (\text{A.4})$$

which is the reason why they are well suited to describe correlators on the torus. Defining

$$\vartheta_1(z) \equiv \vartheta \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} (z, \tau) \quad (\text{A.5})$$

we can see from the product representation that

$$\vartheta_1(z) = -2e^{i\frac{\pi}{4}\tau} \sin(\pi z) \prod_{m=1}^{\infty} (1 - q^m)(1 - zq^m)(1 - z^{-1}q^m). \quad (\text{A.6})$$

In particular

$$\vartheta_1(z) = z\vartheta_1'(0) + \dots, \quad (\text{A.7})$$

a fact that was used repeatedly in the main text.

We also need the Dedekind eta function, which is defined as

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n \geq 1} (1 - q^n) \quad (\text{A.8})$$

It is related to the function $\theta_1(z)$ by the simple identity

$$\theta_1'(0) = -2\pi\eta(\tau)^3 \quad (\text{A.9})$$

Finally, the theta functions satisfy the following Riemann identity:

$$\sum_{a,b} \eta_{a,b} \vartheta \begin{bmatrix} a \\ b \end{bmatrix} (z_1) \vartheta \begin{bmatrix} a \\ b \end{bmatrix} (z_2) \vartheta \begin{bmatrix} a \\ b \end{bmatrix} (z_3) \vartheta \begin{bmatrix} a \\ b \end{bmatrix} (z_4) = 2\vartheta \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} (z'_1) \vartheta \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} (z'_2) \vartheta \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} (z'_3) \vartheta \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} (z'_4) \quad (\text{A.10})$$

with

$$\begin{aligned} z'_1 &= \frac{1}{2}(z_1 + z_2 + z_3 + z_4) & z'_2 &= \frac{1}{2}(z_1 - z_2 + z_3 - z_4) \\ z'_3 &= \frac{1}{2}(z_1 - z_2 - z_3 + z_4) & z'_4 &= \frac{1}{2}(z_1 + z_2 - z_3 - z_4) \end{aligned}$$

B APPENDIX B: BOSONIC CORRELATOR FOR HIGGSINO MASS

Let us consider the correlator on the torus with twists of an arbitrary angle θ . This correlator has a quantum piece \mathcal{Z}_{qu} and a classical piece which takes into account the contributions from world-sheet instantons:

$$\langle \sigma_{1-\theta}(z_1) \sigma_\theta(z_2) \rangle = \mathcal{Z}_{qu} \sum_i e^{-S_{cl}(i)}, \quad (\text{B.1})$$

where the sum is over classical solutions of the equations of motion. Both pieces are expressed in terms of the so-called cut-differentials, which are holomorphic one-forms on a branched covering of the punctured torus. When expressed as functions of the torus coordinate z , they become multi-valued with branch-cut singularities at the location of the punctures. In the present context, where the torus has only two punctures corresponding to the insertions of the two twist fields, these cut-differentials are simply given by:

$$\omega(z) = \vartheta_1(z - z_1)^{-\theta} \vartheta_1(z - z_2)^{-(1-\theta)} \vartheta_1(z - z_2 - \theta(z_1 - z_2)) \quad (\text{B.2})$$

$$\omega'(z) = \vartheta_1(z - z_1)^{-(1-\theta)} \vartheta_1(z - z_2)^{-\theta} \vartheta_1(z - z_1 + \theta(z_1 - z_2)) \quad (\text{B.3})$$

and satisfy

$$\omega(z+1) = \omega(z+\tau) = \omega(z) \quad \omega'(z+1) = \omega'(z+\tau) = \omega'(z) \quad (\text{B.4})$$

where τ is the complex modulus of the torus. The singularities at the insertions z_i depend on the twist-angle θ .

From these cut-differentials, we construct a ‘‘period matrix’’ W_a^i defined as

$$W_a^1 = \int_{\gamma_a} dz \omega(z) \quad W_a^2 = \int_{\gamma_a} d\bar{z} \bar{\omega}'(\bar{z}) \quad (\text{B.5})$$

where the paths γ_a , $a = 1, 2$, denote the canonical homology basis of the torus. From [17] we obtain the quantum part of the correlation function:

$$\mathcal{Z}_{qu} = \frac{1}{\det W} |\vartheta_1(z_1 - z_2)|^{-2\theta(1-\theta)} \quad (\text{B.6})$$

To determine the classical part of the correlator, we need to evaluate the action for all possible classical solutions of the equations of motion. Each contribution will have the form

$$S_{cl} = \frac{1}{4\pi\alpha'} \int d^2z (\partial Z_{cl} \bar{\partial} \bar{Z}_{cl} + \bar{\partial} Z_{cl} \partial \bar{Z}_{cl}) \quad (\text{B.7})$$

where Z_{cl} and \bar{Z}_{cl} are classical solutions, i.e. Z_{cl} solves $\partial\bar{\partial}Z_{cl} = 0$ on the punctured torus with appropriate boundary conditions (i.e. branch cut singularities) at the location of the punctures. In particular, under parallel transport along the canonical homology basis of the torus, Z_{cl} is expected to shift as

$$\Delta_a Z_{cl} = \int_{\gamma_a} dz \partial Z_{cl} + \int_{\gamma_a} d\bar{z} \bar{\partial} Z_{cl} = v_a, \quad (\text{B.8})$$

where v_a are given complex numbers. For instance for a toroidal compactification $Z = X^1 + iX^2$ with

$$X^1 \sim X^1 + 2\pi R_1 \quad X^2 \sim X^2 + 2\pi R_2 \quad (\text{B.9})$$

when Z_{cl} is transported along a closed loop on the world-sheet torus, it must return to its original value modulo a lattice vector of the space-time torus. This implies

$$v_1 = 2\pi(m_1 R_1 + in_1 R_2) \quad v_2 = 2\pi(m_2 R_1 + in_2 R_2), \quad (\text{B.10})$$

where the integers m_i and n_i correspond to the winding numbers of the closed string propagating in the loop.

The classical part of the correlator is written as

$$\mathcal{Z}_{cl} = \sum_{v_1, v_2} e^{-S_{cl}(v_1, v_2)} \quad (\text{B.11})$$

where $S_{cl}(v_1, v_2)$ is now expressed in terms of the period matrix (B.5):

$$S_{cl} = \frac{i}{4\pi\alpha'} v_a \bar{v}_b \left((\bar{W}^{-1})^b{}_1 \bar{W}^1{}_d M^{ad} + (W^{-1})^a{}_2 W^2{}_c M^{bc} \right) \quad (\text{B.12})$$

The two-point correlation function of twist fields on the torus is then given by

$$\langle \sigma_{1-\theta}(z_1) \sigma_\theta(z_2) \rangle_T = \frac{1}{\det W} |\vartheta_1(z_1 - z_2)|^{-2\theta(1-\theta)} \sum_{v_1, v_2} e^{-S_{cl}(v_1, v_2)} \quad (\text{B.13})$$

where the sum over the v_i corresponds to a sum over all possible winding modes.

The case we are interested here, the annulus two-point correlation function, is obtained from (B.13) by taking the ‘‘square root’’ and replacing the complex modulus τ by $\tau \rightarrow it$. As a result, the twist correlator on the annulus takes the form

$$\langle \sigma_{1-\theta}(z_1) \sigma_\theta(z_2) \rangle_A = K(\tau) \frac{1}{\det^{\frac{1}{2}} W} \left| \frac{\vartheta_1(z_1 - z_2)}{\vartheta_1(0)} \right|^{-\theta(1-\theta)} \sum_{v_1, v_2} e^{-S_{cl}(v_1, v_2)} \quad (\text{B.14})$$

where now

$$S_{cl} = \frac{i}{8\pi\alpha'} v_a \bar{v}_b \left((\bar{W}^{-1})^b{}_1 \bar{W}^1{}_d M^{ad} + (W^{-1})^a{}_2 W^2{}_c M^{bc} \right) \quad (\text{B.15})$$

and $K(\tau)$ is a normalization factor which can depend on the modulus. It is now understood that the twist fields are located on one of the boundaries of the cylinder, $z_j = iy_j$ with y_j real. The values of v_a must also be appropriately modified. Instead of (B.10), we now have

$$v_1 = 2\pi R_1 m \quad v_2 = i4\pi(R_2 n + \ell) \quad (\text{B.16})$$

where R_1, R_2 are the compactification radii of the space-time torus and ℓ is the separation between the D -branes along the X^2 direction.

Taking the limit $y_1 \rightarrow y_2$, allows to check that this expression is correct and to determine the normalization constant $K(\tau)$. In this limit, the operator product expansion of the twist fields behaves as:

$$\begin{aligned} \langle \sigma_{1-\theta}(z_1) \sigma_\theta(z_2) \rangle &\rightarrow \frac{1}{(z_1 - z_2)^{\theta(1-\theta)}} \langle \mathbb{I} \rangle \quad (\text{B.17}) \\ &= \frac{1}{(z_1 - z_2)^{\theta(1-\theta)}} \frac{1}{\eta^2} \sum_{m,n} \exp \left(-2\pi\alpha't \left(\frac{m^2}{R_1^2} + \left(\frac{nR_2}{\alpha'} + \frac{\ell}{\alpha'} \right)^2 \right) \right) \end{aligned}$$

where we have used that $\langle \mathbb{I} \rangle$ is the partition function on the annulus of the complex boson Z . The factor $1/\eta^2$ is the contribution of the oscillators while the sum is over the momentum and winding modes of the string. The same result should be obtained by taking the limit $z_1 \rightarrow z_2$ on the right-hand side of (B.14).

The expression (B.3) for the cut-differential implies that

$$\omega(z) \rightarrow 1 \quad \omega'(z) \rightarrow 1 \quad (\text{B.18})$$

In this limit the components of the period matrix and its inverse can be explicitly evaluated :

$$W = \begin{pmatrix} W_1^1 & W_1^2 \\ W_2^1 & W_2^2 \end{pmatrix} = \begin{pmatrix} it & 1 \\ -it & 1 \end{pmatrix} \quad W^{-1} = \frac{1}{2it} \begin{pmatrix} 1 & -1 \\ it & it \end{pmatrix} \quad (\text{B.19})$$

Inserting this in the action (B.15) leads to

$$S_{cl} = -\frac{\pi}{2\alpha't} R_1^2 m^2 - \frac{2\pi t}{\alpha'} (R_2 n + \ell)^2 \quad (\text{B.20})$$

so that the classical piece of the correlator becomes

$$\begin{aligned} \sum_{v_1, v_2} e^{-S_{cl}} &= \sum_m \exp \left(-\frac{\pi R_1^2}{2\alpha't} m^2 \right) \sum_n \exp \left(-2\pi t \alpha' \left(\frac{\ell}{\alpha'} + n \frac{R_2}{\alpha'} \right)^2 \right) \\ &= \left(\frac{2\alpha't}{R_1^2} \right)^{\frac{1}{2}} \sum_{m,n} \exp \left(-2\pi\alpha't \left(\frac{m^2}{R_1^2} + \left(\frac{nR_2}{\alpha'} + \frac{\ell}{\alpha'} \right)^2 \right) \right) \quad (\text{B.21}) \end{aligned}$$

where the second equality is obtained by a Poisson resummation.

Using (A.7) and (B.19), the limit $z_1 \rightarrow z_2$ of the quantum part is

$$\mathcal{Z}_{qu} = \frac{K(\tau)}{|z_1 - z_2|^{\theta(1-\theta)}} \left(\frac{1}{2it} \right)^{1/2}. \quad (\text{B.22})$$

Inserting (B.21) and (B.22) into (B.1) and comparing with (B.18) shows that both expressions agree to each other provided that $K(\tau) = (iR_1^2/\alpha')^{1/2} 1/\eta^2$.

The final result for the correlation function of twist-antitwist on the annulus is

$$\langle \sigma_{1-\theta}(z) \sigma_\theta(w) \rangle = \left(\frac{iR_1^2}{\alpha'} \right)^{\frac{1}{2}} \frac{1}{\eta(it)^2} \frac{1}{\det^{\frac{1}{2}} W} \left| \frac{\vartheta_1(z-w)}{\vartheta_1'(0)} \right|^{-\theta(1-\theta)} \sum_{v_1, v_2} e^{-S_{cl}(v_1, v_2)} \quad (\text{B.23})$$

where v_1 and v_2 are given in Eq. (B.16).

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