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ANALYSIS OF $\pi^- p \to \pi^- \pi^+ n$ DATA AT 17.2 GeV/c

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A B S T R A C T

We propose an amplitude analysis for $\pi^- p \to \pi^- \pi^+ n$ data which allows for the possibility of A_2 exchange as well as absorbed pion exchange in a model independent way. This leads to an improved extrapolation method for extracting $\pi\pi$ phase shifts. Using the recent CERN-Munich data at 17.2 GeV/c we isolate the π and A_2 exchange contributions and determine the form of the absorptive corrections. No evidence for an exchange contribution with the quantum numbers of the A_1 is found.

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The continuing interest in the $\pi N \to \pi\pi N$ reaction is due to the possibility [1] of extracting the elastic $\pi\pi$ cross-section by extrapolation to the nearby π exchange pole at $t = \mu^2$. Since the original Chew-Low proposal, the extrapolation technique has been improved to allow for non-evasive contributions [2]. In its simplest form, often called the Williams model [3] (or Poor Man's Absorption [4]), the essential assumption is that under absorption the evasive s channel helicity amplitude $H^1_{+-} = t/(t-\mu^2)$ becomes $\mu^2/(t-\mu^2)$. This was shown [5] to give a good description *) of the density matrix elements in the interval $0 < -t \leq 0.15$ GeV^2 observed in the SLAC 15 GeV/c $\pi^- p \to \pi^- \pi^+ n$ experiment [6] with the $\pi^-\pi^+$ system in the ρ mass band.

In Fig. 1 we show the equivalent fit to the higher statistics 17.2 GeV/c CERN-Munich data [7]. Although this model gives a reasonable qualitative description of the data it is clear, with the improved statistics, that there are significant discrepancies (particularly in ρ_{1-1}) which suggest other exchange mechanisms besides simple absorbed π exchange.

In the first part of this letter we propose an amplitude analysis of the $\pi^- p \to \pi^- \pi^+ n$ data which allows for absorbed π and A_2 exchange in a model independent way. The extrapolation of these amplitude observables therefore gives a more reliable technique for determining $\pi\pi$ phase shifts. We then proceed, by assuming absorption only modifies the (over-all) non-flip amplitude, to isolate the various exchange mechanisms. We find evidence for a sizeable A_2 exchange contribution, even at small t, comparable to that found in an analysis [8] of pion photoproduction.

We consider the production of s and p wave dipions in the ρ mass band. The former is described by two s channel helicity amplitudes H^S_{++}, H^S_{+-} and the latter by six amplitudes $H^{1,0,-1}_{++}$ and $H^{1,0,-1}_{+-}$. We make only the assumption that the amplitudes with the quantum numbers of A_1 exchange are negligible. We do not assume either phase or spin coherence of the amplitudes. Then the observables can be expressed in terms of amplitudes as follows:

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*) This fit was improved by including a 12% s channel non-flip contribution to allow for a possible $\Delta^0(1236)$ contamination. Since the non-flip π exchange forward amplitude for $\pi N \to \pi\pi\Delta$ vanishes as $m_\Delta \to m_N$ this correction is questionable.

$$\sigma \equiv \frac{d\sigma}{dt} = |M_s|^2 + |M_o|^2 + |M_+|^2 + |M_-|^2 \tag{1}$$

$$(\rho_{00} - \rho_{11})\sigma = |M_o|^2 - \frac{1}{2}\left(|M_+|^2 + |M_-|^2\right) \tag{2}$$

$$\rho_{1-1}\sigma = \frac{1}{2}\left(|M_+|^2 - |M_-|^2\right) \tag{3}$$

$$\mathrm{Re}\,\rho_{10}\sigma = \frac{1}{\sqrt{2}} |M_-||M_o| \cos\varphi \tag{4}$$

$$\mathrm{Re}\,\rho_{0s}\sigma = |M_o||M_s| \cos\Delta \tag{5}$$

$$\mathrm{Re}\,\rho_{1s}\sigma = \frac{1}{\sqrt{2}} |M_-||M_s| \cos(\varphi - \Delta) \tag{6}$$

where the relative phases are defined as $\varphi = \arg(M_-) - \arg(M_o)$ and $\Delta = \arg(M_s) - \arg(M_o)$, and where

$$M_{o,s} = H^{o,s}_{+-}, \qquad M_- = \frac{1}{\sqrt{2}}\left(H^1_{+-} + H^1_{-+}\right) \tag{7}$$

$$|M_+|^2 = \frac{1}{2}\left|H^1_{++} + H^1_{--}\right|^2 + \frac{1}{2}\left|H^1_{+-} - H^1_{-+}\right|^2. \tag{8}$$

We use the Jacob and Wick convention for the helicity amplitudes $H^{\lambda_\rho}_{\lambda_n \lambda_p}$. The amplitudes which we neglect, $H^{o,s}_{++}$ and $(H^1_{++} - H^1_{--})$, only enter the observable expressions quadratically; that is, there are no interference terms between them and the amplitudes of Eq. (7). The solution of Eqs. (1)-(6) will thus be stable to small contaminations from these omitted amplitudes. At high energies $|M_\pm|^2$ are the cross-sections for helicity one ρ production via natural and unnatural parity exchange respectively. $|M_{o,s}|^2$ are the cross-sections for zero helicity p and s wave $\pi^-\pi^+$ production. These six equations, Eqs. (1)-(6), can be solved at each t value for the four magnitudes $|M_{\pm,o,s}|$ and the two relative phases φ, Δ of the amplitudes.

In Fig. 2 we show the results of such an amplitude analysis performed on the s channel (or helicity frame) density matrix elements of the 17.2 GeV/c CERN-Munich data [7]. For large $-t$ we see that the natural parity exchange amplitude M_+ dominates suggesting A_2 exchange [9]. Consider now the amplitude M_o which dominates for small t. If we cross a pure π exchange pole in the t channel (which occurs only in the t channel amplitude $\bar{H}^o_{++}$) into s channel amplitudes we have to leading order in s

$$M_o \equiv H^o_{+-} = G\left[\frac{t + m_{\pi\pi}^2 - \mu^2}{\sqrt{2}\; m_{\pi\pi}}\right]\frac{\sqrt{-t'}}{t-\mu^2} \tag{9}$$

$$M_- = -\sqrt{2}\, G\, \frac{t'}{t-\mu^2} \tag{10}$$

$$H^o_{++} = r\, H^o_{+-}, \qquad r = \sqrt{t_{min}/t'} \tag{11}$$

where $t = t_{min}$ is the forward direction and $t' \equiv t - t_{min}$. The non-flip contribution, Eq. (11), is usually neglected, but for <u>very</u> small t it is appreciable *), even at 17.2 GeV/c. To take account of this contribution we multiply $|M_{o,s}|$ in Eqs. (1), (2) and (5) by $\sqrt{(1 + r^2)}$ before solving Eqs. (1)-(6). In Fig. 2 the resulting values of $|M_o|$ are compared with the form $a\sqrt{-t'}e^{bt}/(t - \mu^2)$. The excellent agreement supports the assumption that zero helicity $\pi^+\pi^-$ production is dominated by π exchange and that there is negligible "A_1 type" exchange.

From Fig. 2 we also see, contrary to what is usually assumed [11], that M_o and M_- are not coherent in phase. The signs of φ and Δ are not determined by solving Eqs. (1)-(6). However, a knowledge of the sign of φ would allow us to determine the sign of Δ (that is, if studied as a function of $m_{\pi\pi}$, to resolve the so-called up-down ambiguity without requiring a normalized cross-section). We shall see in a moment that $\mathrm{Im}\, M_-/M_o$, and, therefore $\sin\varphi$, are found to be positive. The values shown for Δ correspond to this choice.

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*) See also a discussion by Ochs [10].

We emphasize that Δ and $\gamma_s (\equiv |H^s_{+-}|/|H^o_{+-}|)$ are the appropriate quantities to extrapolate to the π pole at $t = \mu^2$. When performed for different intervals of $m_{\pi\pi}$, these amplitude extrapolations (together with the parameter a introduced above) offer a more attractive and model independent way of determining the $\pi\pi$ phase shifts. Δ and γ_s will be functions of $m_{\pi\pi}$ but should be independent of s and also of t. Moreover, for normalized data there will be stringent consistency checks between the extrapolated values of Δ, γ_s and $(t-\mu^2)|M_o|/\sqrt{-t'}$. Here, of course, our amplitude components only give information on the $\pi\pi$ phases averaged over the ρ band. An idea of the expected size of γ_s can be obtained by assuming in this interval that the p wave phase is $\delta_p = \pi/2$ and that the $I=0$ and $I=2$ s wave phases are $\delta^o_s = \pi/2$ and $\delta^2_s = 0$, then $\gamma_s = 2/3\sqrt{3} = 0.385$ and $\Delta = 0$. Although the $\pi\pi$ phases are not expected to retain constant values over the ρ band, it is encouraging that this estimate is in such good accord with the values shown in Fig. 2.

We can use the above amplitude analysis to estimate the A_2 exchange contributions and also the "absorptive" corrections, $C(t)$, to the (π and A_2) pole contributions to the $n=0$ s channel helicity amplitude H^1_{+-}. We allow $C(t)$ to be complex. In order to do this we assume that the single and double (over-all) flip amplitudes are well represented by π and A_2 pole exchanges.

For the π exchange pole we use Eqs. (9) and (10) with $G = G(t)$ and replace the quantity in brackets in Eq. (9) by $m_{\pi\pi}/\sqrt{2}$. For the A_2 pole we use the "signature" factor $\mathcal{S}(t) \equiv 1 + \exp(-i\pi\alpha)$ with $\alpha(t) = 0.5 + t$. We specify its relative contribution to nucleon flip and non-flip amplitudes by a parameter R, namely $H^1_{++} = RH^1_{+-}/\sqrt{-t'}$ for the A_2 pole. We anticipate that $R \sim 0.25$ from studies, Ref. 12), of ρ and A_2 exchange in spin 0 - spin $\frac{1}{2}$ scattering. Then the observables for helicity one ρ production at a given incident energy can be expressed as

$$M_- = -\sqrt{2}\, G \frac{t'}{t-\mu^2} + C/\sqrt{2} \tag{12}$$

$$|M_+|^2 = \frac{1}{2}|2g_A t'\mathcal{S} + C|^2 - 2t' R^2 g_A^2 |\mathcal{S}|^2. \tag{13}$$

Here we have assumed real π exchange. We repeated the analysis for Regge π exchange but find that it does not make much difference in the interval $0 \leq -t \leq 0.2$ GeV^2. The quantities G, g_A, C are determined at each t value from $|M_o|$, $|M_\pm|$ and $\cos\varphi$. The procedure is as follows.

At each t we calculate G from the observable $|M_0|$. Using this together with the observables $|M_-|$ and $\cos\varphi$ we determine Re C and $|\mathrm{Im}\, C|$ from Eq. (12). Equation (13) then becomes a quadratic equation for g_A, the A_2 coupling strength. We solve this for a range of values of $|R|$. For a given $|R|$, there are in principle four solutions for g_A at each t value (since the sign of Im C is not determined by the data). However, in practice there are two factors which suggest a unique solution for g_A and severely limit the range of variation of $|R|$. First, for many choices the quadratic equation for g_A does not have real roots, and secondly we require reasonable continuity in t of the solution for g_A. The favoured solution has Im C negative, that is Im C interferes destructively with the Im A_2 pole (as would be expected in an absorption approach). Further we find that $|R|$ must lie in the range $0.2 \lesssim |R| \lesssim 0.3$. We show the resulting components of the H^1_{+-} amplitude in Fig. 3. For comparison we show (by a dashed line) the expectations for Re C from the William's model for absorbed pion exchange, that is Re C $= G(t)$. If this identification of Re C is correct we can conclude that whereas A_2 exchange suffers absorption in the imaginary part, it undergoes little or no absorption in the real part of the $(n=0)$ H^1_{+-} amplitude. This is analogous to the results found for ρ exchange in the s channel non-flip πN amplitude [13].

The amplitude components shown in Fig. 3 agree well with the corresponding results found in pion photoproduction [8], and again indicate the importance of A_2 exchange effects. We emphasize that in our analysis an A_2 (nucleon) non-flip amplitude is essential for continuity of g_A with t. In photoproduction this amplitude is also required [8],[14] to account for the observed (target) polarization in $\gamma p \to \pi^+ n$.

Since π and A_2 exchanges have different s dependences it will be illuminating to study $\pi N \to \pi\pi N$ data at different energies. We conclude that pion exchange can indeed be cleanly extracted from $\pi N \to \pi\pi N$ data and that the present high statistics experiments offer the exciting prospect of accurate and unambiguous $\pi\pi$ phase shifts.

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FIGURE CAPTIONS

Figure 1 :

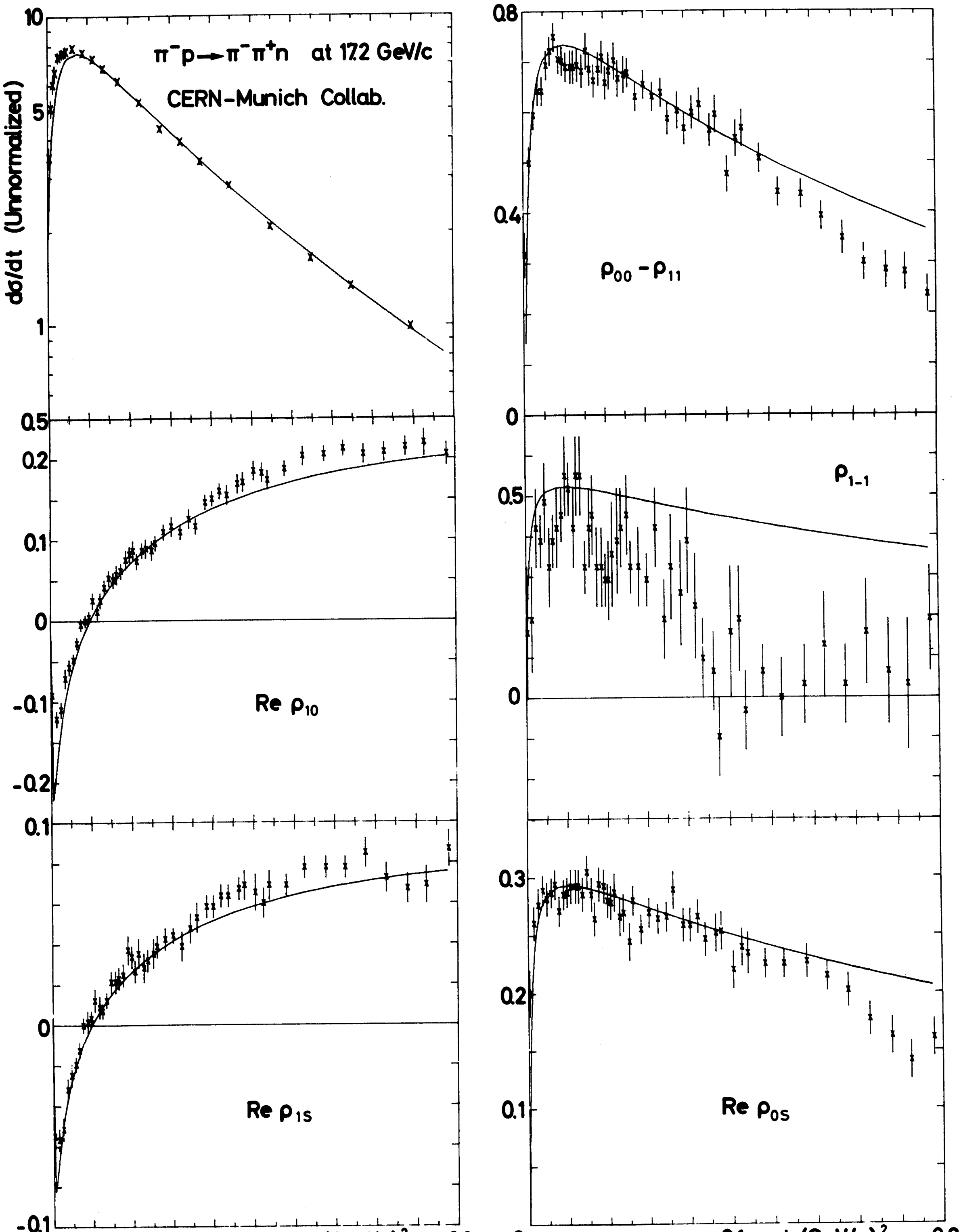
The curves are a Williams model fit to the differential cross-section, and s channel density matrix elements for $\pi^- p \to \pi^- \pi^+ n$ data [7] at 17.2 GeV/c with $0.71 < m_{\pi\pi} < 0.83$ GeV. The values of the parameters are $\gamma_s \equiv |H^s_{+-}|/|H^0_{+-}| = 0.373$ and $A = 4.13$, where H^0_{+-} is of the form $\sqrt{-t'}\,\exp(At)/(t-\mu^2)$.

Figure 2 :

An (s channel) amplitude analysis of the $\pi^- p \to \pi^- \pi^+ n$ data [7] at 17.2 GeV/c based on Eqs. (1)-(6), with $\gamma_s \equiv |M_s|/|M_0|$. The curve is the best fit of the form $a\sqrt{-t'}\,\exp(bt)/(t-\mu^2)$ to the values of $|M_0|$ in the interval $0 < -t < 0.2$ GeV^2; $b = 4.4$ GeV^{-2}. As discussed in the text the ambiguity in the sign of Δ is fixed by requiring $\sin\varphi > 0$. The corresponding plots obtained using the SLAC data [6] at 15 GeV/c show the same features.

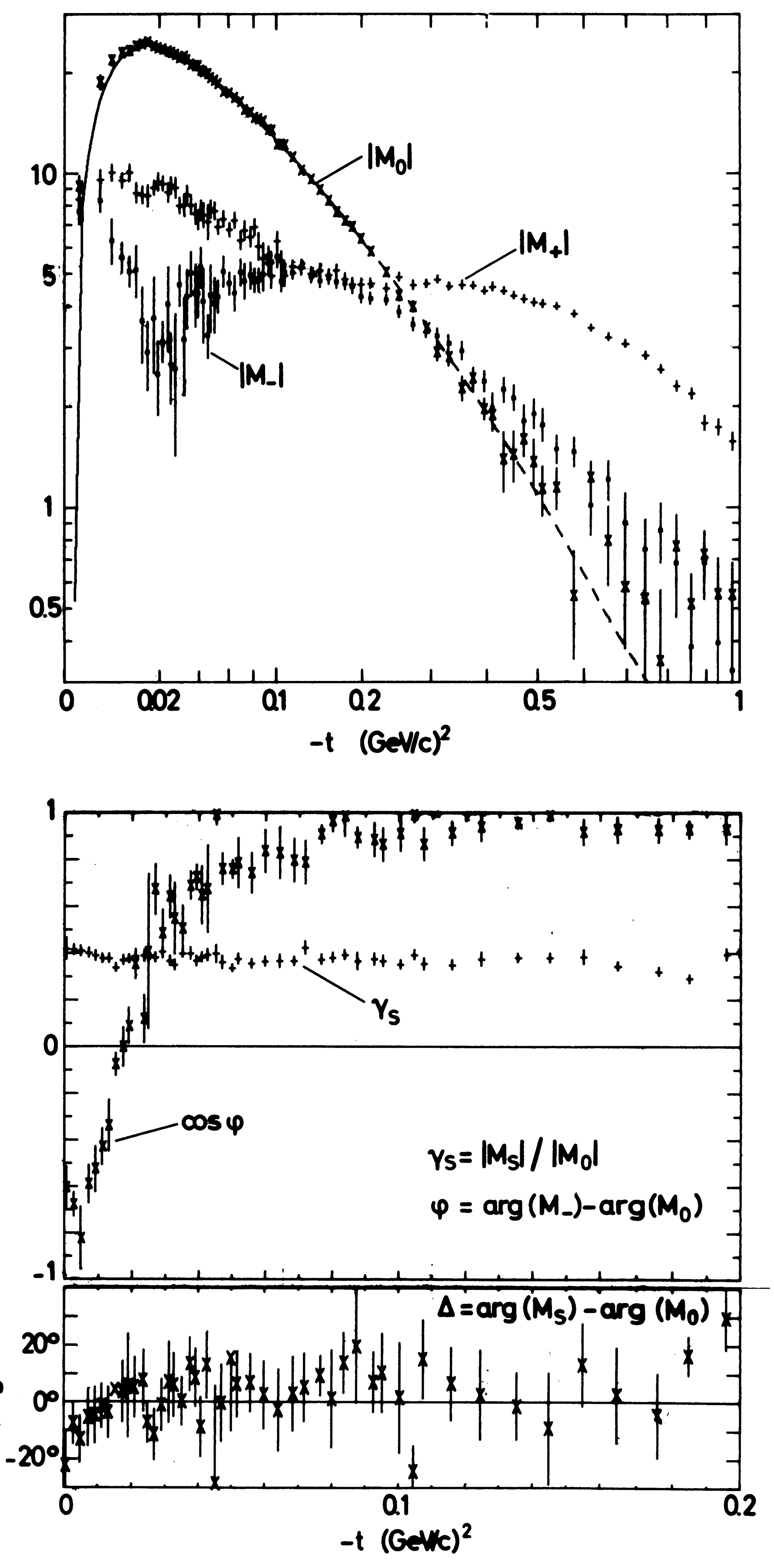
Figure 3 :

The components of the $(n = 0)$ s channel helicity amplitude H^1_{+-} for $|R| = 0.25$. For clarity we have joined the solutions at adjacent t values by straight lines; solid (dotted) lines for the real (imaginary) contributions. π, A_2 and C denote the π pole, A_2 pole and absorptive contributions respectively. The dashed line is the William's model prediction for Re C .



π⁻p → π⁻π⁺n at 17.2 GeV/c
CERN-Munich Collab.
dσ/dt (Unnormalized)
ρ00 − ρ11
Re ρ10
ρ1−1
Re ρ1S
Re ρ0S
−t (GeV/c)²

FIG 1



|M0|
|M+|
|M-|
10
5
1
0.5
0
0.02
0.1
0.2
0.5
1
-t (GeV/c)²
1
0
-1
γS
cos φ
γS = |MS| / |M0|
φ = arg(M−)−arg(M0)
Δ = arg(MS) − arg(M0)
Δ (degrees)
20°
0°
-20°
0
0.1
0.2
-t (GeV/c)²

FIG 2

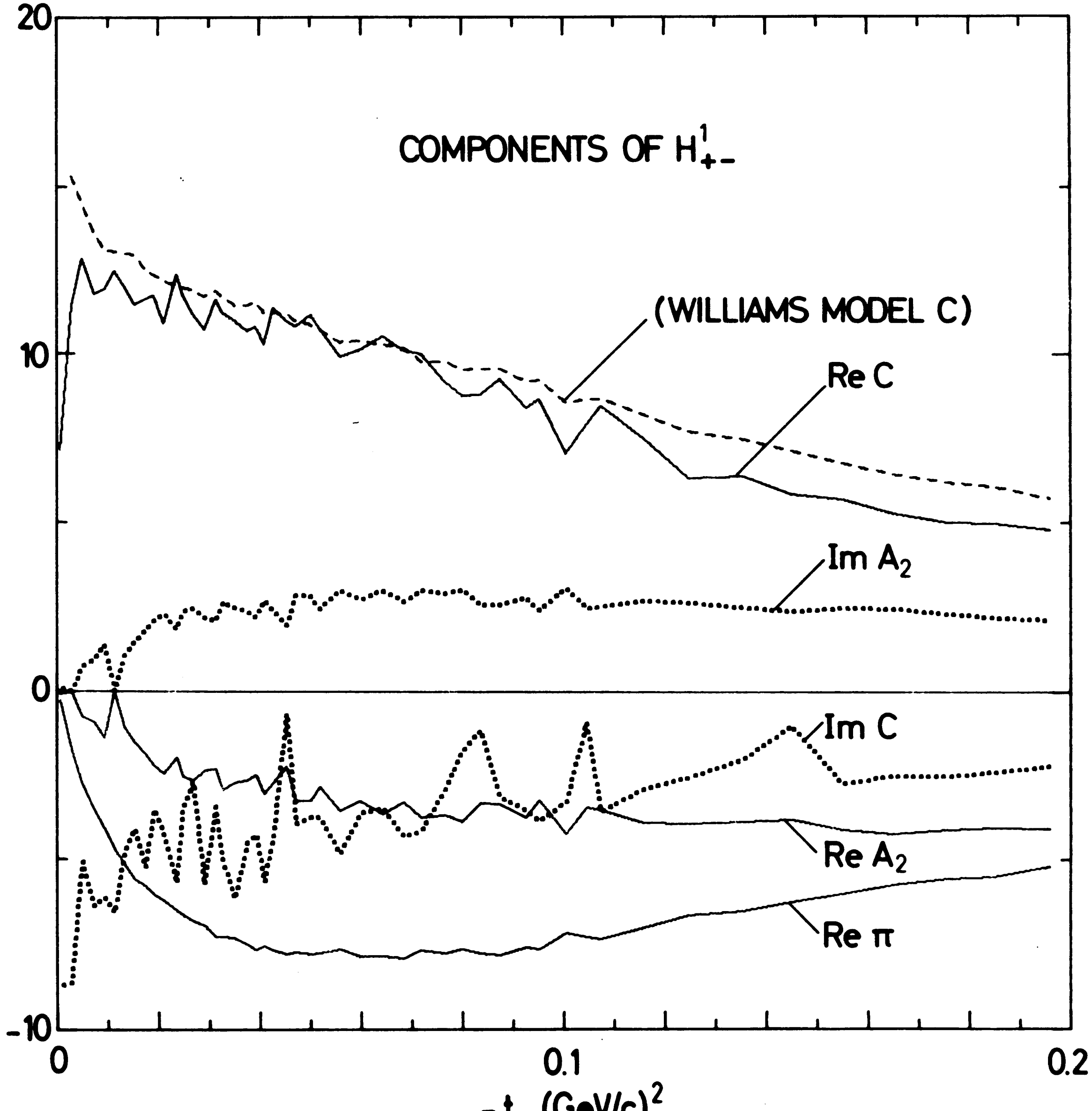

COMPONENTS OF H^{1}_{+-}
(WILLIAMS MODEL C)
Re C
Im A_2
Im C
Re A_2
Re π
20
10
0
-10
0
0.1
0.2
$-t \ (\mathrm{GeV}/c)^2$

FIG. 3