Electric Dipole Moments in Split Supersymmetry

G.F. Giudice[a] and A. Romanino[b]

[a] *CERN, Department of Physics, Theory Division*
CH–1211 Geneva 23, Switzerland
[b] *SISSA/ISAS and INFN, I–34013 Trieste, Italy*

Abstract

We perform a quantitative study of the neutron and electron electric dipole moments (EDM) in Supersymmetry, in the limit of heavy scalars. The leading contributions arise at two loops. We give the complete analytic result, including a new contribution associated with Z–Higgs exchange, which plays an important and often leading role in the neutron EDM. The predictions for the EDM are typically within the sensitivities of the next generation experiments. We also analyse the correlation between the electron and neutron EDM, which provides a robust test of Split Supersymmetry.

1   Introduction

The electric dipole moments (EDMs) of the Standard Model (SM) fermions are powerful probes of physics beyond the SM. Once the strong CP problem has been taken care of, the SM predictions for the EDMs of quarks and leptons are at least 7 orders of magnitudes below [1] the present experimental limits [2, 3, 4]. The situation is drastically different in supersymmetric extensions of the SM. The supersymmetry-breaking terms involve many new sources of CP-violation. Particularly worrisome are the phases associated, in the universal and flavour-diagonal case, to the invariants $\arg(A^* M_{\tilde{g}})$ and $\arg(A^* B)$. Such phases survive in the universal limit in which all the flavour structure originates from the SM Yukawas. If these phases are of order one, the electron and neutron EDMs induced at one-loop by gaugino-sfermion exchange are typically (barring accidental cancellations [5]) a couple of orders of magnitude above the limits [6, 7, 8], a difficulty which is known as the supersymmetric CP problem.

The Split limit of the MSSM [9, 10, 11] does not present a supersymmetric CP problem. Heavy sfermions suppress the dangerous one-loop contributions to a negligible level. Nevertheless, some phases survive below the sfermion mass scale and, if they do not vanish for an accidental or

a symmetry reason, they give rise to EDMs that are safely below the experimental limits, but sizeable enough to be well within the sensitivity of the next generation of experiments [11]. Such contributions only arise at the two-loop level, since the new phases appear in the gaugino-Higgsino sector, which is not directly coupled to the SM fermions.

In this paper, we perform a quantitative study of the neutron and electron EDMs in the limit of Split Supersymmetry. First, we compute the different contributions to the light quark and electron EDMs, the only relevant CP-violating operators. Indeed, quark chromoelectric dipoles and the gluon Weinberg operator [12] cannot be generated at two loops. For the EDM, the original CP-violation in the gaugino-Higgsino sector is communicated to the SM fermions by gauge boson and Higgs exchanges, specifically by i) γh , ii) WW, or iii) Zh exchange. No other possibilities are allowed at the two-loop level.

The γh exchange has been widely studied in the literature in several contexts [13, 14, 15, 16]. The case of Split Supersymmetry was considered in ref. [11]. The WW exchange has also been studied in different limits [17, 18, 19]. An exact 2-loop computation has been performed in the context of Split Supersymmetry in ref. [20] (see also ref. [21] for a computation in the context of a two-Higgs doublet model). Our results, for which we give explicit analytic expressions, differ from those in ref. [20]. Moreover, we identify a third, important contribution due to Zh exchange. The Zh contribution is suppressed in the case of the electron EDM by a $1 - 4\sin^2\theta_W$ factor, but it plays an important role in the neutron EDM. In fact, the Zh contribution is always comparable and often larger than the γh one (which in turn is tipically larger than the WW contribution). We have also recomputed the QCD renormalization effect, correcting a mistake present in the previous literature.

2   General expressions for the EDMs

CP-violating phases can enter the effective Lagrangian below the sfermion mass scale $\tilde{m}$ through the Yukawa couplings (which are irrelevant for our study), the μ-parameter, the gaugino masses M_i, $i = 1, 2, 3$, or the Higgs-Higgsino-gaugino couplings $\tilde{g}_u$, $\tilde{g}_d$, $\tilde{g}'_u$, $\tilde{g}'_d$ in

$$-\mathcal{L} = \sqrt{2}\left(\tilde{g}_u H^\dagger \tilde{W}^a T_a \tilde{H}_u + \tilde{g}'_u Y_{H_u} H^\dagger \tilde{B} \tilde{H}_u + \tilde{g}_d H_c^\dagger \tilde{W}^a T_a \tilde{H}_d + \tilde{g}'_d Y_{H_d} H_c^\dagger \tilde{B} \tilde{H}_d\right) + \text{h.c.}, \quad (1)$$

where $H_c = i\sigma_2 H^*$, T_a are the $SU(2)$ generators, and $Y_{H_u} = -Y_{H_d} = 1/2$. The Higgs vev is in its usual form $\langle H \rangle = (0, v)^T$, with $v \sim 174\,\text{GeV}$. The gaugino and Higgsino mass parameters $M_{1,2}$ and μ, and the couplings $\tilde{g}_u$, $\tilde{g}_d$, $\tilde{g}'_u$, $\tilde{g}'_d$ are in general complex. However, only three phases are independent and they are associated to the invariants $\phi_1 = \arg(\tilde{g}'^*_u \tilde{g}'^*_d M_1 \mu)$, $\phi_2 = \arg(\tilde{g}^*_u \tilde{g}^*_d M_2 \mu)$, $\xi = \arg(\tilde{g}_u \tilde{g}^*_d \tilde{g}'_d \tilde{g}'^*_u)$. The tree-level matching with the full theory above $\tilde{m}$ gives $\arg(\tilde{g}_u) = \arg(\tilde{g}'_u)$, $\arg(\tilde{g}_d) = \arg(\tilde{g}'_d)$, and therefore $\xi = 0$, thus leaving only two independent phases. Moreover, in most models of supersymmetry breaking the phases of M_1 and M_2 are equal, in which case there is actually only one CP-invariant.

In terms of mass eigenstates, the relevant interactions are

$$-\mathcal{L} = \frac{g}{c_W}\overline{\chi_i^+}\gamma^\mu(G_{ij}^R P_R + G_{ij}^L P_L)\chi_j^+ Z_\mu$$

$$+\left[g\overline{\chi_i^+}\gamma^\mu(C_{ij}^R P_R + C_{ij}^L P_L)\chi_j^0 W_\mu^+ + \frac{g}{\sqrt{2}}\overline{\chi_i^+}(D_{ij}^R P_R + D_{ij}^L P_L)\chi_j^+ h + \text{h.c.}\right], \quad (2)$$

where

$$G_{ij}^L = V_{iW^+}c_{W^+}V_{W^+j}^\dagger + V_{ih_u^+}c_{h_u^+}V_{h_u^+j}^\dagger \qquad -G_{ij}^{R^*} = U_{iW^-}c_{W^-}U_{W^-j}^\dagger + U_{ih_d^-}c_{h_d^-}U_{h_d^-j}^\dagger \qquad (3a)$$

$$C_{ij}^L = -V_{iW^+}N_{jW_3}^* + \frac{1}{\sqrt{2}}V_{ih_u^+}N_{jh_u^0}^* \qquad C_{ij}^R = -U_{iW^-}^*N_{jW_3} - \frac{1}{\sqrt{2}}U_{ih_d^-}^*N_{jh_d^0} \qquad (3b)$$

$$gD_{ij}^R = \tilde{g}_u^* V_{ih_u^+}U_{jW^-} + \tilde{g}_d^* V_{iW^+}U_{jh_d^-} \qquad D^L = (D^R)^\dagger. \qquad (3c)$$

In eq. (3a), $c_f = T_{3f} - s_W^2 Q_f$ ($s_W^2 \equiv \sin^2\theta_W$) is the neutral current coupling coefficient of the fermion $\tilde{f}$ and, accordingly, $c_{W^\pm} = \pm\cos^2\theta_W$, $c_{h_u^+, h_d^-} = \pm(1/2 - s_W^2)$. The matrices U, V, N diagonalize the complex chargino and neutralino mass matrices, $M_+ = U^T M_+^D V$, $M_0 = N^T N_0^D N$, where $M_+^D = \text{Diag}(M_1^+, M_2^+) \geq 0$, $M_0^D = \text{Diag}(M_1^0, \dots, M_4^0) \geq 0$ and

$$M_+ = \begin{pmatrix} M_2 & \tilde{g}_u v \\ \tilde{g}_d v & \mu \end{pmatrix}, \quad M_0 = \begin{pmatrix} M_1 & 0 & -\tilde{g}_d' v/\sqrt{2} & \tilde{g}_u' v/\sqrt{2} \\ 0 & M_2 & \tilde{g}_d v/\sqrt{2} & -\tilde{g}_u v/\sqrt{2} \\ -\tilde{g}_d' v/\sqrt{2} & \tilde{g}_d v/\sqrt{2} & 0 & -\mu \\ \tilde{g}_u' v/\sqrt{2} & -\tilde{g}_u v/\sqrt{2} & -\mu & 0 \end{pmatrix}. \quad (4)$$

In Split Supersymmetry, fermion EDMs are generated only at two loops, since charginos and neutralinos, which carry the information of CP violation, are only coupled to gauge and Higgs bosons. To identify all possible diagrams contributing to the EDM, let us first consider the case in which $M_{1,2}, \mu \gg M_W$. After we integrate out charginos and neutralinos at one-loop, we generate some effective couplings among SM bosons. These can be described in terms of gauge-invariant, CP-violating operators. There are 5 dimension-6 such operators: $\epsilon_{abc}\widetilde{W}_{\mu\nu}^a W^{b\nu\rho}W_\rho^{c\mu}$, $H^\dagger H\widetilde{W}_{\mu\nu}^a W^{a\mu\nu}$, $H^\dagger H\widetilde{B}_{\mu\nu}B^{\mu\nu}$, $D_\mu H^\dagger D_\nu H\widetilde{B}^{\mu\nu}$, $D_\mu H^\dagger T_a D_\nu H\widetilde{W}^{a\mu\nu}$, where $W_{\mu\nu}^a$ and $B_{\mu\nu}$ are the $SU(2)$ and $U(1)$ gauge strengths, and $\widetilde{W}_{\mu\nu}^a$ and $\widetilde{B}_{\mu\nu}$ are their duals. The effective couplings relevant to generate sizable two-loop contributions to the EDM must contain 3 fields, with at least one photon and at most one Higgs boson. The previously-listed operators induce only the effective couplings $\gamma\gamma h$, $\gamma Z h$, and γWW. Notice that CP-violating couplings of the kind $\gamma\gamma\gamma$, $\gamma\gamma Z$ and γZZ are not generated (in particular, the CP-violating operator $B_{\mu\nu}B^{\nu\rho}B_\rho^\mu$ identically vanishes unless there are three different abelian gauge fields). The absence of these couplings is also confirmed by an explicit one-loop calculation. Once we insert the effective couplings in a loop, we obtain 3 different diagrams contributing at the two-loop level to the EDM of the light SM fermion f, shown in Fig. 1. We therefore have

$$d_f = d_f^{\gamma H} + d_f^{ZH} + d_f^{WW}, \quad (5)$$

$$d_f^{\gamma H} = \frac{eQ_f\alpha^2}{4\sqrt{2}\pi^2 s_W^2} \, \text{Im}(D_{ii}^R) \frac{m_f M_i^+}{M_W m_H^2} f_{\gamma H}(r_{iH}^+) \tag{6a}$$

$$d_f^{ZH} = \frac{e\left(T_{3f_L} - 2s_W^2 Q_f\right)\alpha^2}{16\sqrt{2}\pi^2 c_W^2 s_W^4} \, \text{Im}\left(D_{ij}^R G_{ji}^R - D_{ij}^L G_{ji}^L\right) \frac{m_f M_i^+}{M_W m_H^2} f_{ZH}(r_{ZH}, r_{iH}^+, r_{jH}^+) \tag{6b}$$

$$d_f^{WW} = \frac{eT_{3f_L}\alpha^2}{8\pi^2 s_W^4} \, \text{Im}\left(C_{ij}^L C_{ij}^{R*}\right) \frac{m_f M_i^+ M_j^0}{M_W^4} f_{WW}(r_{iW}^+, r_{jW}^0), \tag{6c}$$

In eqs. (6) the sum over the indexes i, j is understood, Q_f is the charge of the fermion f, T_{3f_L} is the third component of the weak isospin of its left-handed component. Also, $r_{ZH} = (M_Z/m_H)^2$, $r_{iH}^+ = (M_i^+/m_H)^2$, $r_{iW}^+ = (M_i^+/M_W)^2$, $r_{iW}^0 = (M_i^0/M_W)^2$, where m_H is the Higgs mass, and the loop functions are given by

$$f_{\gamma H}(r) = \int_0^1 \frac{dx}{1-x} \, j\left(0, \frac{r}{x(1-x)}\right) \tag{7a}$$

$$f_{ZH}(r, r_1, r_2) = \frac{1}{2}\int_0^1 \frac{dx}{x(1-x)} \, j\left(r, \frac{xr_1 + (1-x)r_2}{x(1-x)}\right) \tag{7b}$$

$$f_{WW}(r_1, r_2) = \int_0^1 \frac{dx}{1-x} \, j\left(0, \frac{xr_1 + (1-x)r_2}{x(1-x)}\right). \tag{7c}$$

The symmetric loop function $j(r, s)$ is defined recursively by

$$j(r) = \frac{r\log r}{r - 1}, \quad j(r, s) = \frac{j(r) - j(s)}{r - s}. \tag{8}$$

In the determination of f_{ZH} we have used the symmetry of $\text{Im}(D_{ij}^R G_{ji}^R)M_i^+$ and $\text{Im}(D_{ij}^L G_{ji}^L)M_i^+$ under $i \leftrightarrow j$. Eq. (6) differs from the result in ref. [20]. Analytic expressions for the functions in eqs. (7) are given in the appendix.

The parameters entering the expression of the quark EDM in eqs. (6) have to be evaluated at the chargino mass scale M^+. The renormalization to the scale μ at which we evaluate the neutron EDM matrix element is determined by the anomalous dimension of the operator $\bar{q}\sigma_{\mu\nu}\gamma_5 q F^{\mu\nu}$. We find

$$d_q(\mu) = \eta_{\text{QCD}} \, d_q(M^+), \quad \eta_{\text{QCD}} = \left[\frac{\alpha_s(M^+)}{\alpha_s(\mu)}\right]^{\gamma/2b}, \tag{9}$$

where the β-function coefficient is $b = 11 - 2n_q/3$ and n_q is the number of effective light quarks. The anomalous-dimension coefficient is $\gamma = 8/3$. To eliminate the quark mass dependence in the short-distance contribution, it may be preferable to consider the ratio d_q/m_q. Its renormalization is given by

$$\frac{d_q}{m_q}(\mu) = \eta_{\text{QCD}}^{\gamma'/\gamma} \frac{d_q}{m_q}(M^+), \tag{10}$$

with $\gamma' = 32/3$. For $\alpha_s(M_Z) = 0.118 \pm 0.004$ and $\mu = 1\,\text{GeV}$ (the scale of the neutron mass), we find $\eta_{\text{QCD}} = 0.75$ for $M^+ = 1\,\text{TeV}$ and $\eta_{\text{QCD}} = 0.77$ for $M^+ = 200\,\text{GeV}$. The error in $\alpha_s(M_Z)$
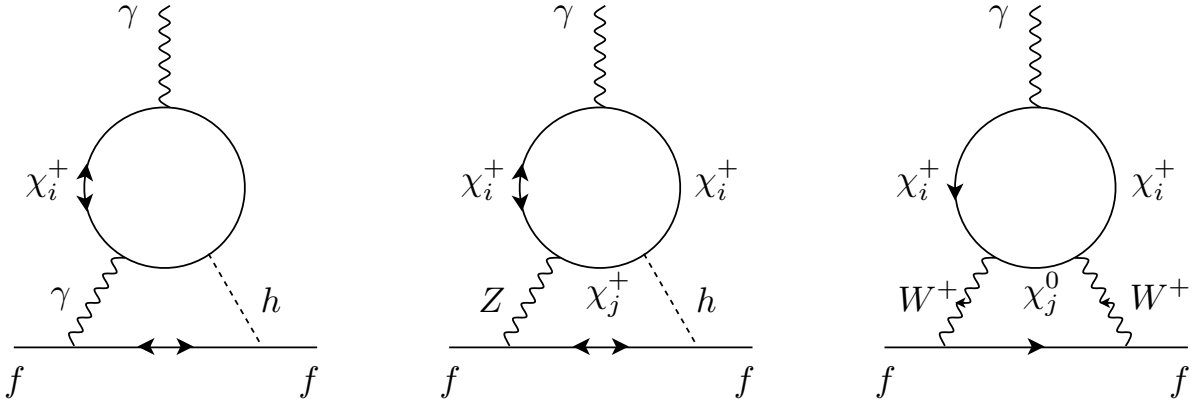
Figure 1: Two loop contributions to the light SM fermion EDMs. The third diagram is for a down-type fermion f.

gives an uncertainty on $\eta_{\rm QCD}$ of about 2%, while we expect an uncertainty of about 5% from next-to-leading order effects. Notice that the value of $\eta_{\rm QCD}$ obtained here is different than what computed in ref. [22] and generally used in the literature. Indeed, ref. [22] incorrectly uses the opposite sign for γ. Our result gives a QCD renormalization coefficient about a factor of 2 smaller than usually considered, and it agrees with the recent findings of ref. [23].

To express the neutron EDM in terms of the quark EDMs, we use the results of QCD sum-rule techniques [24, 25]:

$$d_n = (1 \pm 0.5) \left[\frac{f_\pi^2 m_\pi^2}{(m_u + m_d)(225\,{\rm MeV})^3} \right] \left(\frac{4}{3} d_d - \frac{1}{3} d_u \right), \tag{11}$$

where $f_\pi \approx 92\,{\rm MeV}$ and we have neglected the contribution of the quark chromoelectric dipoles, which does not arise at the two-loop level in the heavy-squark mass limit. Note that d_n depends on the light quark masses only through the ratio m_u/m_d, for which we take the value $m_u/m_d = 0.553 \pm 0.043$.

3   Expansions in the heavy-chargino limit

We now discuss the result in the limit in which the R-symmetry breaking scale, determining gaugino and Higgsino masses, is larger than M_Z and m_H. A leading-order perturbative expansion of eq. (6) in powers of $|M_{1,2}\mu|/M_Z^2$ and $|M_{1,2}\mu|/m_H^2$ (keeping all orders in $|M_{1,2}/\mu|$ and in M_Z/m_H)

gives

$$d_f^{\gamma H} \simeq \frac{e Q_f \alpha m_f}{8\pi^3} \frac{\tilde{g}_u \tilde{g}_d}{M_2 \mu} \sin\phi_2 F_{\gamma H} \left(\frac{M_2^2}{\mu^2}, \frac{M_2\mu}{m_H^2} \right) \tag{12a}$$

$$d_f^{ZH} \simeq \frac{e \left(T_{3f_L} - 2s_W^2 Q_f \right) \alpha m_f}{16\pi^3 c_W^2} \frac{\tilde{g}_u \tilde{g}_d}{M_2 \mu} \sin\phi_2 F_{ZH} \left(\frac{M_Z^2}{m_H^2}, \frac{M_2^2}{\mu^2}, \frac{M_2\mu}{m_H^2} \right) \tag{12b}$$

$$d_f^{WW} \simeq \frac{e T_{3f_L} \alpha m_f}{16\pi^3 s_W^2} \left[\frac{\tilde{g}_u \tilde{g}_d}{M_2 \mu} \sin\phi_2 F_{WW}^{(2)} \left(\frac{M_2^2}{\mu^2}, \frac{M_2\mu}{m_H^2} \right) + \frac{\tilde{g}_u' \tilde{g}_d'}{M_1 \mu} \sin\phi_1 F_{WW}^{(1)} \left(\frac{M_1^2}{\mu^2}, \frac{M_1\mu}{m_H^2} \right) \right], \tag{12c}$$

where $\tilde{g}_{u,d}$, $\tilde{g}_{u,d}'$, $M_{1,2}$ and μ now indicate the absolute value of the corresponding quantity and the functions $F_{\gamma H}$, F_{ZH}, F_{WW} are given in the appendix. As long as $M_1/M_2 < 1$ (as, for instance, in the case of gaugino masses unifying at the GUT scale), the second term in eq. (12c) is suppressed with respect to the first term and numerically it is not very significant. Notice that eqs. (12) explicitly exhibit the dependence on the two CP-violating invariants $|\tilde{g}_u \tilde{g}_d/M_2\mu| \sin\phi_2$ and $|\tilde{g}_u' \tilde{g}_d'/M_1\mu| \sin\phi_1$. Because of the suppression of the second term in eq. (12c), both the electron and neutron EDM are mostly characterized by a single invariant.

While eqs. (12a,b) can be obtained from an expansion at the first order in $v/M_{1,2}$, eq. (12c) arises only at the second order. This is because the origin of d_f^{WW} can be traced back to the vertices $\tilde{W}^{\mu\nu} D_\mu H D_\nu H$, $\tilde{B}^{\mu\nu} D_\mu H D_\nu H$ in the unbroken electroweak symmetry phase, requiring two insertions of the Higgs vev. The additional M/v factor in eq. (6c) leads to a contribution to the EDM which is parametrically of the same order of $d_f^{\gamma H}$ and d_f^{ZH} in the v/M expansion. Notice also that the coefficients of the potentially large logarithms of $|M_{1,2}\mu|/m_H^2$ correspond to the anomalous dimensions that mix the EDM operator with the CP-violating dimension-6 operators obtained from integrating out the supersymmetric particles well above the weak scale.

The relative importance of the three contributions to d_f in eq. (5) can be estimated from eqs. (12). Let us consider for definitess the case $M_2 = \mu$. By keeping only terms enhanced by a large $\log(M_2\mu/m_H^2)$ in the expressions (21) for $F_{\gamma H}$, F_{ZH}, $F_{WW}^{(2)}$, we obtain $F_{ZH} \approx F_{\gamma H}(-s_W^2 + 3/4)$ and $F_{WW}^{(2)} \approx -F_{\gamma H}/4$. As a consequence, we find

$$\frac{d_f^{ZH}}{d_f^{\gamma H}} \approx \frac{(T_{3f_L} - 2s_W^2 Q_f)(3 - 4s_W^2)}{8c_W^2 Q_f} \quad \frac{d_f^{WW}}{d_f^{\gamma H}} \approx -\frac{T_{3f_L}}{8s_W^2 Q_f} \quad (M_2 = \mu), \tag{13}$$

where in the expression for d_f^{WW} we have neglected the subleading second term in eq. (12c). Numerically, eq. (13) gives $d_e^{ZH} \approx 0.05\, d_e^{\gamma H}$, $d_e^{WW} \approx -0.3\, d_e^{\gamma H}$ and $d_n^{ZH} \approx d_n^{\gamma H}$, $d_n^{WW} \approx -0.7\, d_n^{\gamma H}$. These simple estimates show the importance of the ZH contribution to the neutron EDM. A detailed numerical analysis of the relative importance of the different contributions to the electron and neutron EDMs is given in the next Section. The qualitative estimates above are in a remarkably good agreement in the large $M_2 = \mu$ limit.

4   Numerical analysis

We now perform a numerical analysis of the full results for the EDM in eqs. (6). We consider a standard unified framework for the gaugino masses at the GUT scale. By using the RGEs

given in refs. [10, 26], the parameters in eqs. (6) can be expressed in terms of the single phase $\phi \equiv \phi_2$ and the four following positive parameters: M_2, μ (evaluated at the low-energy scale), $\tan \beta$, and the sfermion mass scale $\tilde{m}$. In first approximation, the dipoles depend on β and ϕ through an overall factor $\sin 2\beta \sin \phi$. Therefore, in order to maximize the effect, we choose $\tan \beta = 1$ and the phase ϕ evaluated at the low-energy scale such that $\sin \phi = 1$. Notice that if the ratio $|\mu/M_2|$ is much larger or much smaller than one, the renormalization effects which mixes μ and M_2 (peculiar of Split Supersymmetry) tend to suppress the effective phase. In this case, a maximal CP violation can only be achieved with very particular choices of the initial values for the higgsino and gaugino masses. Once we have fixed $\sin 2\beta \sin \phi = 1$, we are then left with the three dimensionful parameters M_2, μ, $\tilde{m}$. The overall sfermion scale $\tilde{m}$ enters only logarithmically through the RGE equations for $\tilde{g}_{u,d}$, $\tilde{g}'_{u,d}$. We choose to present the results as contour plots in the M_2–μ plane and set $\tilde{m} = 10^9 \, \text{GeV}$, which is consistent with the cosmological bounds given in ref. [27].

Fig. 2 shows the prediction for the electron EDM, the neutron EDM, and their ratio d_n/d_e. The red thick line corresponds to the present experimental limits $d_e < 1.6 \times 10^{-27} e \, \text{cm}$ [4], while the limit $d_n < 0.63 \times 10^{-27} e \, \text{cm}$ [2] does not pose a constraint on the parameters shown in Fig. 2. In the Split limit, and assuming gaugino mass unification, all EDMs are controlled by a single phase. The results for d_e and d_n shown in Fig. 2 scale approximately linearly with $\sin 2\beta \sin \phi$.

A robust test of Split Supersymmetry can be performed if both the electron and the neutron EDM are measured. Indeed, in the ratio d_n/d_e the dependence on $\sin \phi$, $\tan \beta$ and $\tilde{m}$ approximately cancels out. This can be easily understood from eqs. (12) which show that, as long as the chargino and neutralino masses are sufficiently larger than M_Z, the only dependence of d_n/d_e on $\tilde{g}_{u,d}$, $\tilde{g}'_{u,d}$, and $\sin \phi_{1,2}$ comes from the existence of the M_1-dependent term in d_f^{WW}, see eq. (12c). This term, as previously discussed, is numerically small. Nevertheless, because of the different loop functions associated to the different contributions, the ratio d_n/d_e varies by a $\mathcal{O}\,(100\%)$ factor when the M_2 and μ are varied in the range spanned in the Figures. Still, the variation of d_n/d_e is comparable with the theoretical uncertainty on the determination of d_n in terms of quark EDMs in eq. (11) due to the hadronic matrix element, and is significantly smaller than the variation in the ordinary MSSM prediction, even in the case of universal phases (see e.g. ref. [28][1]). On the other hand, the usual tight correlation between the electron and muon EDMs, $d_\mu/d_e = m_\mu/m_e$ persists.

Fig. 3 shows the relative importance of the different contributions to the EDMs. For the same reason explained above, the results shown in Fig. 3 are fairly independent of $\sin \phi$, $\tan \beta$ and $\tilde{m}$. As anticipated, the ZH contribution to the electron EDM is suppressed by the $T_{3f_L} - 2s_W^2 Q_f$ factor. On the other hand, the corresponding contribution to d_n is always important and represents the largest contribution in a significant portion of the parameter space shown in Fig. 3. The WW contribution is also sizable, especially in the case of the neutron EDM, but is typically smaller than the ZH or γH ones. While the ZH and γH contributions always add constructively, the WW contribution has an opposite sign. However, also due to the ZH contribution, its size is not large enough to flip the sign of the overall electron or neutron EDM.

---

[1]Note that the ZH contribution is missing in the analysis of the Split Supersymmetry case in ref. [28], which leads to a stronger correlation between d_e and d_n.
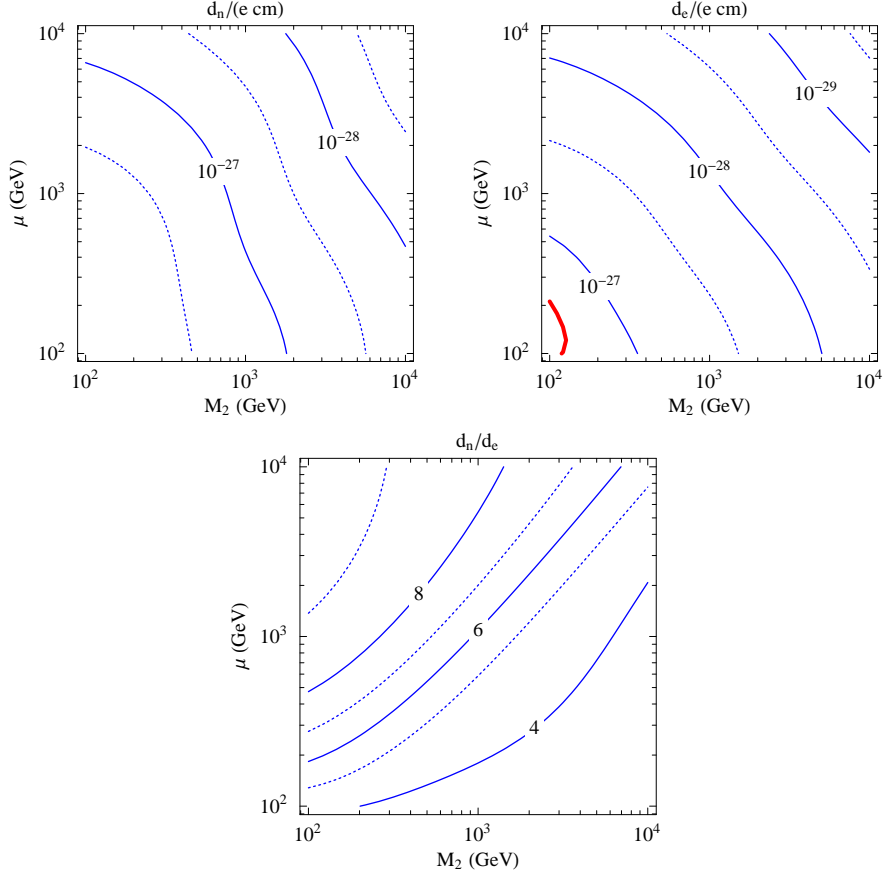
Figure 2: Prediction for d_n, d_e, and their ratio d_n/d_e. We have chosen $\tan\beta = 1$, $\sin\phi = 1$, and $\tilde{m} = 10^9$ GeV. The results for d_n and d_e scale approximately linearly with $\sin 2\beta \sin\phi$, while the ratio is fairly independent of $\tan\beta$, $\sin\phi$ and $\tilde{m}$. The red thick line corresponds to the present experimental limit $d_e < 1.6 \times 10^{-27} e$ cm [4].
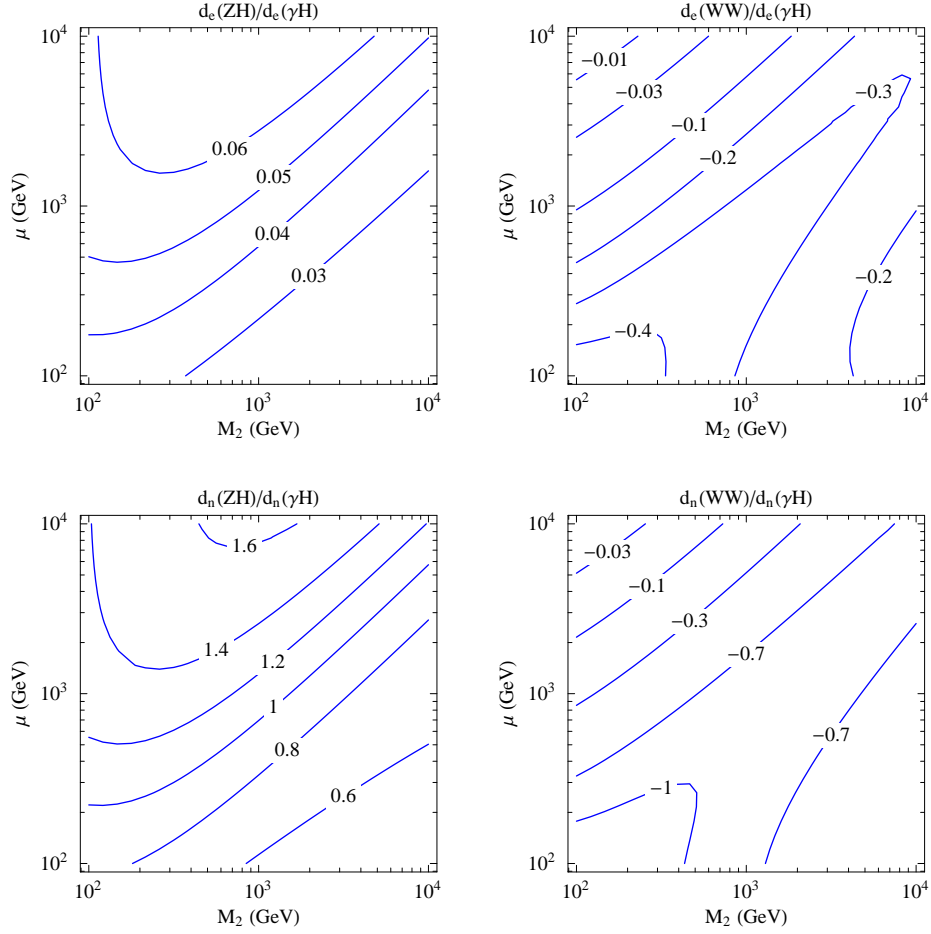
Figure 3: Relative importance of the different contributions to the EDMs. We have chosen $\tan\beta = 1$, $\sin\phi = 1$ and $\tilde{m} = 10^9$ GeV, but the result depends only very weakly on this choice.

5   Conclusions

In summary, we performed a quantitative study of the neutron and electron EDMs in Split Supersymmetry. Clearly, our results also apply to the MSSM in any limit in which the contributions involving sfermion exchange are suppressed. Our result for the WW exchange differs from the one in the literature. Moreover, we find a new contribution associated to Zh exchange, which plays an important and often leading role in the neutron EDM. We have also given the correct value for the QCD renormalization of the quark EDM. We performed an analytical and numerical analysis of our results, summarized in Figs. 2 and 3. The correlation between the electron and neutron EDMs is found to be stronger than in standard supersymmetric scenarios, and it may become a crucial experimental test for Split Supersymmetry. Still, we find an $\mathcal{O}\left(100\%\right)$ variation of d_n/d_e in the parameter space we considered, which is comparable with the hadronic uncertainty on the determination of d_n in terms of quark EDMs. The results summarized in Fig. 2 are quite promising in the light of the expected impressive improvement of the experimental sensitivities in the years to come and represent one of the most relevant windows on Split Supersymmetry.

Acknowledgments

Appendix

We write below the analytic expression of the loop functions in eqs. (6) and (7).

$$f_{\gamma H}(r) = \frac{1}{\sqrt{1-4r}}\left[\log r\ \log\frac{\sqrt{1-4r}-1}{\sqrt{1-4r}+1} + \mathrm{Li}_2\left(\frac{2}{1-\sqrt{1-4r}}\right) - \mathrm{Li}_2\left(\frac{2}{1+\sqrt{1-4r}}\right)\right] \quad (14)$$

$$f_{ZH}(r,r_1,r_2) = \frac{1}{r-1}\left\{g(r,r_1,r_2) - g(1,r_1,r_2)\right.$$
$$\left. -\frac{\log r}{2(x_1-x_2)}\left[\log\left(1-\frac{1}{x_1}\right) - \log\left(1-\frac{1}{x_2}\right)\right]\right\} \quad (15)$$

$$f_{WW}(r_1,r_2) = \frac{1}{\hat{y}_1-\hat{y}_2}\left\{(r_2-\hat{y}_1)\left[\log r_2\log\left(1-\frac{r_2}{\hat{y}_1}\right) - \log r_1\log\left(1-\frac{r_1}{\hat{y}_1}\right)\right.\right.$$
$$\left. +\mathrm{Li}_2\left(\frac{r_2}{\hat{y}_1}\right) - \mathrm{Li}_2\left(\frac{r_1}{\hat{y}_1}\right)\right] + (r_2-\hat{y}_2)\left[\log r_1\log\left(1-\frac{r_1}{\hat{y}_2}\right)\right.$$
$$\left.\left. -\log r_2\log\left(1-\frac{r_2}{\hat{y}_2}\right) + \mathrm{Li}_2\left(\frac{r_1}{\hat{y}_2}\right) - \mathrm{Li}_2\left(\frac{r_2}{\hat{y}_2}\right)\right]\right\}$$
$$+\frac{1}{\hat{x}_1-\hat{x}_2}\left\{\hat{x}_1\left[\mathrm{Li}_2\left(\frac{1}{1-\hat{x}_1}\right) - \mathrm{Li}_2\left(\frac{1}{\hat{x}_1}\right)\right] - \hat{x}_2\left[\mathrm{Li}_2\left(\frac{1}{1-\hat{x}_2}\right) - \mathrm{Li}_2\left(\frac{1}{\hat{x}_2}\right)\right]\right\} \quad (16)$$

$$g(r, r_1, r_2) = \frac{r_1 - r_2}{2(y_1 - y_2)} \left\{ \log r_1 \left[\log\left(1 - \frac{r_1}{y_1}\right) - \log\left(1 - \frac{r_1}{y_2}\right) \right] \right.$$

$$- \log r_2 \left[\log\left(1 - \frac{r_2}{y_1}\right) - \log\left(1 - \frac{r_2}{y_2}\right) \right] + \mathrm{Li}_2\left(\frac{r_1}{y_1}\right) - \mathrm{Li}_2\left(\frac{r_2}{y_1}\right) - \mathrm{Li}_2\left(\frac{r_1}{y_2}\right) + \mathrm{Li}_2\left(\frac{r_2}{y_2}\right) \right\}$$

$$+ \frac{1}{2(x_1 - x_2)} \left[\mathrm{Li}_2\left(\frac{1}{1 - x_1}\right) - \mathrm{Li}_2\left(\frac{1}{1 - x_2}\right) - \mathrm{Li}_2\left(\frac{1}{x_1}\right) + \mathrm{Li}_2\left(\frac{1}{x_2}\right) \right] \quad (17)$$

where

$$x_{1,2} = \frac{1}{2r} \left[r - r_1 + r_2 \mp \sqrt{(r - r_1 + r_2)^2 - 4rr_2} \right] \qquad \hat{x}_i = x_i|_{r=1} \qquad (18a)$$

$$y_{1,2} = r_2 + (r_1 - r_2)x_{1,2} \qquad \hat{y}_i = y_i|_{r=1}. \qquad (18b)$$

In the physical region, the functions $f_{\gamma H}$, f_{ZH} and f_{WW} do not develop imaginary parts.

In order to study the case of chargino masses larger than M_Z, or $r_1, r_2 \gg 1$, it is useful to switch to the variables $R = \sqrt{r_1 r_2}$, $\rho = r_1/r_2$ and expand in $1/R$. We then get

$$f_{\gamma H}(R) = \frac{2 + \log R}{2R} + \mathcal{O}\left(\frac{\log R}{R^2}\right) \qquad (19a)$$

$$f_{ZH}(r, r_1, r_2) = a_{ZH}(\rho)\frac{\log R}{R} + \frac{b_{ZH}(r, \rho)}{R} + \mathcal{O}\left(\frac{\log R}{R^2}\right) \qquad (19b)$$

$$f_{WW}(r_1, r_2) = a_{WW}(\rho)\frac{\log R}{R} + \frac{b_{WW}(\rho)}{R} + \mathcal{O}\left(\frac{\log R}{R^2}\right) \qquad (19c)$$

where

$$a_{ZH}(\rho) = \frac{\sqrt{\rho}}{2(\rho - 1)} \log \rho \qquad (20a)$$

$$b_{ZH}(r, \rho) = \frac{\sqrt{\rho}}{2(\rho - 1)} \left[\frac{r \log r \log \rho}{1 - r} - \mathrm{Li}_2(1 - \rho) + \mathrm{Li}_2(1 - 1/\rho) \right] \qquad (20b)$$

$$a_{WW}(\rho) = \frac{\sqrt{\rho}}{(\rho - 1)^2} (\rho - 1 - \log \rho) \qquad (20c)$$

$$b_{WW}(\rho) = \frac{\sqrt{\rho}}{(\rho - 1)^2} \left[\rho - 1 + \frac{(\rho + 1)}{2} \log \rho + \mathrm{Li}_2(1 - \rho) - \mathrm{Li}_2(1 - 1/\rho) \right]. \qquad (20d)$$

From these expansions, the expressions for the functions $F_{\gamma H}$, F_{ZH}, $F_{WW}^{(1)}$, $F_{WW}^{(2)}$ in eqs. (12) follow:

$$F_{\gamma H}(\rho, R) = -\frac{1}{2} \log R - 1 + \frac{(\rho + 1) \log \rho}{4(\rho - 1)} + \mathcal{O}\left(\frac{\log R}{R}\right) \qquad (21a)$$

$$F_{ZH}(r, \rho, R) = A_{ZH}(\rho) \log R + B_{ZH}(r, \rho) + \mathcal{O}\left(\frac{\log R}{R}\right) \qquad (21b)$$

$$F_{WW}^{(1)}(\rho, R) = A_{WW}^{(1)}(\rho) \log R + B_{WW}^{(1)}(\rho) + \mathcal{O}\left(\frac{\log R}{R}\right) \qquad (21c)$$

$$F_{WW}^{(2)}(\rho, R) = A_{WW}^{(2)}(\rho) \log R + B_{WW}^{(2)}(\rho) + \mathcal{O}\left(\frac{\log R}{R}\right), \qquad (21d)$$

where

$$A_{ZH}(\rho) = \frac{(\rho - 1)(2 - \rho) - \rho \log \rho}{4(\rho - 1)^2} + \frac{s_W^2}{2} \tag{22a}$$

$$\begin{aligned}
B_{ZH}(r, \rho) = \frac{1}{4(r - 1)(\rho - 1)^2} &\left\{ (2 - 2r + r \log r)(\rho - 1) \left[\rho - 2 - 2s_W^2(\rho - 1) \right] \right. \\
&+ (r - 1)(\rho - 1) \left[\frac{\rho}{2} + 1 - s_W^2(\rho + 1) \right] \log \rho + r\rho \log r \log \rho \\
&\left. + (r - 1)\rho \left[\mathrm{Li}_2(1 - \rho) - \mathrm{Li}_2(1 - 1/\rho) \right] \right\}
\end{aligned} \tag{22b}$$

$$A^{(1)}_{WW}(\rho) = \rho \, \frac{-\rho^2 + 1 + 2\rho \log \rho}{8(\rho - 1)^3} \tag{22c}$$

$$B^{(1)}_{WW}(\rho) = \rho \, \frac{-4\rho(\rho - 1) + (\rho^2 - 4\rho - 1) \log \rho - 4\rho \left[\mathrm{Li}_2(1 - \rho) - \mathrm{Li}_2(1 - 1/\rho) \right]}{16(\rho - 1)^3} \tag{22d}$$

$$A^{(2)}_{WW}(\rho) = \rho \, \frac{(\rho - 7)(\rho - 1) + 2(\rho + 2) \log \rho}{8(\rho - 1)^3} \tag{22e}$$

$$B^{(2)}_{WW}(\rho) = \rho \, \frac{4(\rho - 4)(\rho - 1) - (\rho^2 + 4\rho + 7) \log \rho - 4(\rho + 2) \left[\mathrm{Li}_2(1 - \rho) - \mathrm{Li}_2(1 - 1/\rho) \right]}{16(\rho - 1)^3}. \tag{22f}$$

References

[1] M. Pospelov and A. Ritz, Annals Phys. 318 (2005) 119, hep-ph/0504231.

[2] P.G. Harris et al., Phys. Rev. Lett. 82 (1999) 904.

[3] M.V. Romalis, W.C. Griffith and E.N. Fortson, Phys. Rev. Lett. 86 (2001) 2505, hep-ex/0012001.

[4] B.C. Regan et al., Phys. Rev. Lett. 88 (2002) 071805.

[5] M. Brhlik, G.J. Good and G.L. Kane, Phys. Rev. D59 (1999) 115004, hep-ph/9810457.

[6] S. Abel, S. Khalil and O. Lebedev, Nucl. Phys. B606 (2001) 151, hep-ph/0103320.

[7] D.A. Demir et al., Nucl. Phys. B680 (2004) 339, hep-ph/0311314.

[8] K.A. Olive et al., (2005), hep-ph/0506106.

[9] N. Arkani-Hamed and S. Dimopoulos, (2004), hep-th/0405159.

[10] G.F. Giudice and A. Romanino, Nucl. Phys. B699 (2004) 65, hep-ph/0406088.

[11] N. Arkani-Hamed et al., Nucl. Phys. B709 (2005) 3, hep-ph/0409232.

[12] S. Weinberg, Phys. Rev. Lett. 63 (1989) 2333.

[13] S.M. Barr and A. Zee, Phys. Rev. Lett. 65 (1990) 21.

[14] D. Chang, W.Y. Keung and A. Pilaftsis, Phys. Rev. Lett. 82 (1999) 900, hep-ph/9811202.

[15] D. Chang, W.F. Chang and W.Y. Keung, Phys. Rev. D66 (2002) 116008, hep-ph/0205084.

[16] A. Pilaftsis, Nucl. Phys. B644 (2002) 263, hep-ph/0207277.

[17] W.J. Marciano and A. Queijeiro, Phys. Rev. D33 (1986) 3449.

[18] T. Kadoyoshi and N. Oshimo, Phys. Rev. D55 (1997) 1481, hep-ph/9607301.

[19] N.G. Deshpande and J. Jiang, Phys. Lett. B615 (2005) 111, hep-ph/0503116.

[20] D. Chang, W.F. Chang and W.Y. Keung, Phys. Rev. D71 (2005) 076006, hep-ph/0503055.

[21] R. Lopez-Mobilia and T. H. West, Phys. Rev. D **51** (1995) 6495.

[22] R. Arnowitt, J. L. Lopez and D. V. Nanopoulos, Phys. Rev. D42 (1990) 2423.

[23] G. Degrassi, E. Franco, S. Marchetti and L. Silvestrini, (2005), hep-ph/0510137.

[24] M. Pospelov and A. Ritz, Phys. Rev. Lett. 83 (1999) 2526, hep-ph/9904483.

[25] M. Pospelov and A. Ritz, Phys. Rev. D63 (2001) 073015, hep-ph/0010037.

[26] A. Arvanitaki, C. Davis, P. W. Graham and J. G. Wacker, Phys. Rev. D **70** (2004) 117703, hep-ph/0406034.

[27] A. Arvanitaki et al., (2005), hep-ph/0504210.

[28] S. Abel and O. Lebedev, (2005), hep-ph/0508135.