Hadronic fluctuations in the QGP

F. Karsch[a*], S. Ejiri[b] and K. Redlich[c]

[a]Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA

[b]Department of Physics, The University of Tokyo, Tokyo 113-0033, Japan

[c]Physics Department, Theory Division, CERN, CH-1211 Geneva 23, Switzerland

We analyze fluctuations of quark number and electric charge, in 2-flavour QCD at finite temperature and vanishing net baryon number density. In the hadronic phase we find that an enhancement of charge fluctuations arises from contributions of doubly charged hadrons to the thermodynamics. The rapid suppression of fluctuations seen in the high temperature phase suggests that in the QGP quark number and electric charge are predominantly carried by quasi-particles with the quantum numbers of quarks.

1. Introduction

Lattice calculations of bulk thermodynamic observables, e.g. the energy density and pressure, clearly show that the transition to the high temperature phase of QCD is accompanied by the liberation of partonic degrees of freedom. Asymptotically, at infinite temperature, energy density and pressure approach the limit of an ideal gas of quarks and gluons. At $T \sim 3\ T_c$ the deviations from this asymptotic behaviour are still about 15% which is too large to be accounted for by ordinary high temperature perturbation theory. This shows the relevance of non-perturbative effects leading, for instance, to thermal quark and gluon masses. For temperatures close to T_c, *i.e.* $T \sim (1-2)\ T_c$, these non-perturbative features dominate bulk thermodynamic behaviour, leading to large deviations from the ideal gas relation $\epsilon = 3p$, a strong reduction of the velocity of sound and large screening lengths [1].

The observation of large elliptic flow in heavy ion collisions at RHIC, its successful description in terms of ideal hydrodynamics as well as the observed strong modification of jets also led to the conclusion that the medium produced in heavy ion collisions at RHIC, which is expected to be generated and thermalized at temperatures $T \sim (1-2)\ T_c$, is opaque and still strongly interacting [2]. It thus became of considerable interest to understand in more detail the structure of the interacting medium in the vicinity of the transition temperature. In this context it has been suggested that a large set of colored bound states of light quarks could exist above T_c and dominate the bulk thermodynamics for $T \simeq (1-2)\ T_c$ [3].

---

[*] This work was partly supported by the KBN under grant 2P03 (06925), the DFG under grant KA 1198/6-4 and the GSI collaboration grant BI-KAR. The work of FK has been partly supported by a contract DE-AC02-98CH1-886 with the U.S. Department of Energy.

Indeed, lattice calculations of in-medium properties of heavy quark bound states have led to the conclusions that correlation functions for some of these states, in particular J/ψ and η_c, are not significantly modified in the vicinity of T_c and that these states thus can persist to exist as bound states even at $T \sim (1.5-2)\ T_c$ [4]. On the other hand, there is ample evidence that correlation functions of light quarks are strongly modified immediately above T_c. For instance, scalar and pseudo-scalar correlation functions become degenerate as a consequence of chiral symmetry restoration; in the chiral limit the pseudo-scalar particle no longer is a Goldstone particle which is reflected in the structure of spectral functions where the temperature independent, δ-function like peak present at low temperature gets replaced by a broad temperature dependent "bump" at energies $\omega \sim 5T$ [5]. Moreover, the analysis of spatial correlation functions in the scalar and vector channels showed a strong sensitivity to modifications of temporal boundary conditions for the fermion (quark) fields above T_c while they are insensitive to this below T_c [6]. This suggests that in these quantum number channels a bosonic bound state does not give the dominant contribution to the correlation functions above T_c; the fermionic substructure of "independently" propagating quarks and anti-quarks becomes visible. If light quark bound states exist above T_c they thus must have quite peculiar quasi-particle properties.

In recent lattice studies of 2-flavour QCD at non-zero quark ($\mu_{u,d}$) chemical potential higher order derivatives of the QCD partition function (generalized susceptibilities) with respect to quark chemical potentials have been used to also analyze higher moments of fluctuations of quark number and electric charge [7]. In particular, the second order derivatives with respect to μ_q or μ_Q are monotonously rising across T_c while the fourth order derivatives have pronounced peaks at the transition temperature. We will show in the following that this reflects the transition from hadrons to quarks as the dominant degrees of freedom that carry baryon number and electric charge in the hot medium [8].

2. Quark number and charge fluctuations

Fluctuations of quark number and charge at vanishing net baryon number density (vanishing chemical potential) can be obtained from the QCD partition function through appropriate combinations of derivatives with respect to u, d-quark chemical potentials,

$$\begin{aligned}
d_2^x \equiv \frac{1}{VT^3}\frac{\partial^2 \ln Z}{\partial(\mu_x/T)^2}\Big|_{\mu_u=\mu_d=0} &= \frac{1}{VT^3}\langle(\delta N_x)^2\rangle \quad , \quad x=q,\ Q \\
d_4^x \equiv \frac{1}{VT^3}\frac{\partial^4 \ln Z}{\partial(\mu_x/T)^4}\Big|_{\mu_u=\mu_d=0} &= \frac{1}{VT^3}\left(\langle(\delta N_x)^4\rangle - 3\langle(\delta N_x)^2\rangle^2\right) \quad , \qquad (1)
\end{aligned}$$

where $\partial/\partial\mu_q = \partial/\partial\mu_u + \partial/\partial\mu_d$ and $\partial/\partial\mu_Q = \frac{2}{3}\partial/\partial\mu_u - \frac{1}{3}\partial/\partial\mu_d$. Results for these cumulants obtained from a calculation within 2-flavour QCD are shown in Fig. 1.

The ratios of the quark number and electric charge cumulants, $R^x_{4,2} \equiv d_4^x/d_2^x$, $x = q,\ Q$, are sensitive observables that allow to identify the carriers of quark number and electric charge in a thermal medium. This is quite apparent in the low temperature phase where a hadron resonance gas (in Boltzmann approximation) is known to give a good description of the bulk thermodynamics [9]. This immediately leads to the expectation that $R^q_{4,2} = 3^2 = 9$ for $T < T_c$. The analysis of electric charge fluctuations is a bit more subtle as doubly charged baryons start contributing to the thermodynamics significantly

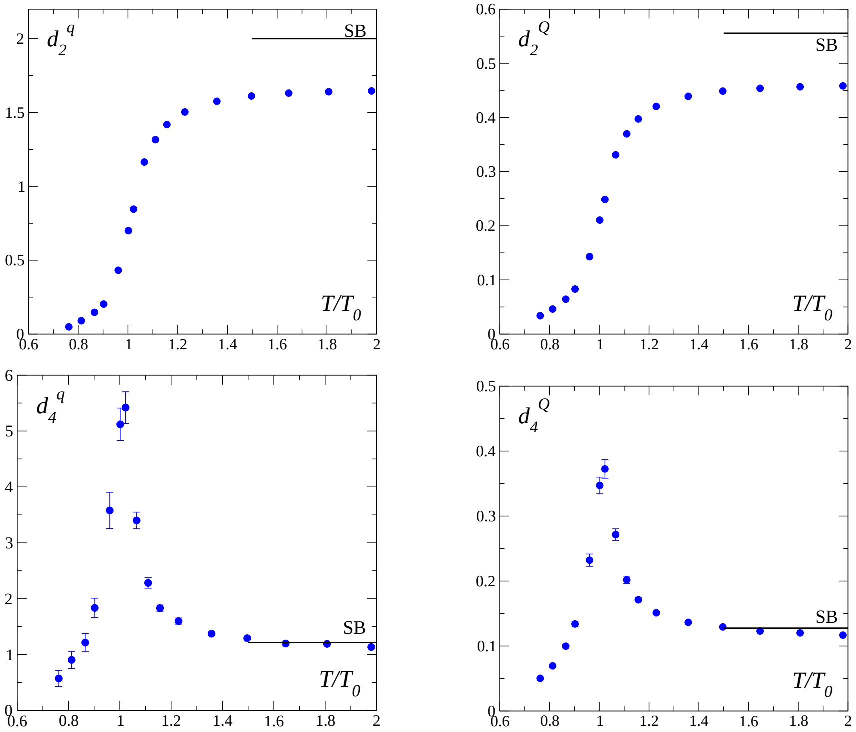

Figure 1. Second (upper) and fourth (lower) cumulant of the net quark number (left) and electric charge (right) in 2-flavour QCD calculated on lattices of size $16^3 \times 4$ with quark masses corresponding to a pseudo-scalar (pion) mass of about 770 MeV.

for $T \simeq T_c$. Taking into account separately the contribution from the mesonic isospin triplet sector ($G^{(3)}$) and the baryonic isospin doublet ($F^{(2)}$) and quartet ($F^{(4)}$) sectors one finds for the charge fluctuations [8],

$$R^Q_{4,2} \equiv \frac{d_4^Q}{d_2^Q} = \frac{\langle(\delta Q)^4\rangle}{\langle(\delta Q)^2\rangle} - 3\langle(\delta Q)^2\rangle = \frac{4G^{(3)} + 3F^{(2)} + 27F^{(4)}}{4G^{(3)} + 3F^{(2)} + 9F^{(4)}} \quad . \tag{2}$$

At low temperature this ratio is dominated by charge fluctuations in the pion sector, which contributes to $G^{(3)}$. The ratio will thus approach unity at low temperatures and is monotonically increasing with temperature. This indicates that in the low temperature limit all charged degrees of freedom carry one unit of charge; $R^Q_{4,2}$ increases only due to contributions arising from isospin quartet baryons which can carry charge $Q = 2$.

The generic features expected for quark number and charge fluctuations in a hadron gas are reproduced by the lattice results for $R^{q,Q}_{4,2}$ shown in Fig. 2. In the high temperature phase these ratios apparently change drastically. In fact, in the infinite temperature, ideal gas limit the ratios will approach $R^{q,\infty}_{4,2} = 6/\pi^2$ and $R^{Q,\infty}_{4,2} = 34/15\pi^2$, respectively. As can be seen from Fig. 2 these asymptotic values are almost reached for $T \gtrsim 1.5\, T_c$.

Like in the hadronic phase one would expect also for the high temperature phase that the presence of doubly charged states or states carrying more than one unit of quark number would lead to an increase of the ratios $R^{q,Q}_{4,2}$. The analysis of light quark bound states [3] indeed suggests that in addition to colored $\bar{q}q$ and gg states also qq-states could exist above T_c. The latter, however, would be only weakly bound and presumably would

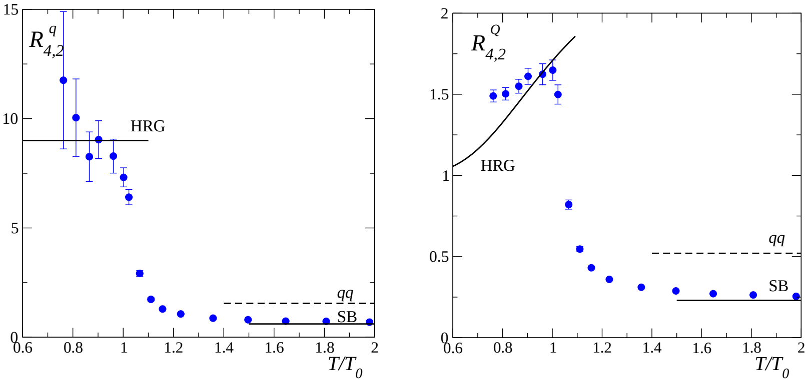

Figure 2. The ratios of fourth and second cumulants of quark number (left) and charge (right) fluctuations.

disappear already at $T \simeq 1.4T_c$. If such states would contribute to the thermodynamics above T_c they also would lead to an increase in charge and quark number fluctuations. The dashed lines shown in Fig. 2 indicate the increase in $R_{4,2}^{q,Q}$ at the presumed melting temperature of qq-states, $T \simeq 1.4T_c$ [3], assuming that at this temperature the qq-states contribute only with half their statistical weight to the QCD partition function. This clearly overestimates the fluctuations observed at high temperature in lattice calculations.

3. Conclusions

We have discussed some generic features of quark number and charge fluctuations in 2-flavour QCD. We argue that the ratio of quartic and quadratic fluctuations are sensitive observables that can directly provide information on the constituents of the thermal medium that carry net quark number and electric charge, respectively. We have shown that below the QCD transition temperature these ratios are in reasonable agreement with a hadronic resonance gas. Above T_c the ratios rapidly drop and approach the high temperature ideal gas values. This suggests that already for $T \gtrsim 1.5T_c$ quark number and charge are predominantly carried by states with the quantum numbers of quarks.

REFERENCES

1. F. Karsch, Lect. Notes Phys. **583** (2002) 209.
2. First Three Years of Operation of RHIC, Nucl. Phys. A **757** (2005) 1-283.
3. E.V. Shuryak and I. Zahed, Phys. Rev. D **70** (2004) 054507;
   J. Liao and E. V. Shuryak, hep-ph/0508035.
4. F. Karsch, Eur. Phys. J. C **43** (2005) 35.
5. F. Karsch et al., Phys. Lett. B **530** (2002) 147.
6. G. Boyd, S. Gupta, F. Karsch and E. Laermann, Z. Phys. C **64** (1994) 331.
7. C. R. Allton et al., Phys. Rev. D **71** (2005) 054508.
8. S. Ejiri, F. Karsch and K. Redlich, hep-ph/0509051.
9. F. Karsch, A. Tawfik and K. Redlich, Phys. Lett. B **571** (2003) 67 and Eur. Phys. J. C **29** (2003) 549.