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Hadronic fluctuations at the QCD phase transition

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Abstract

We discuss the properties of hadronic fluctuations, i.e. fluctuations of net quark and isospin numbers as well as electric charge, in the vicinity of the QCD transition in isospin-symmetric matter at vanishing quark chemical potential. We analyse second- and fourth-order cumulants of these fluctuations and argue that the ratio of quartic and quadratic cumulants reflects the relevant degrees of freedom that carry the quark number, isospin or charge, respectively. In the hadronic phase we find that an enhancement of charge fluctuations arises from contributions of doubly charged hadrons to the thermodynamics. The rapid suppression of fluctuations seen in the high-temperature phase suggests that in the QGP, net quark number and electric charge are predominantly carried by quasi-particles, with the quantum numbers of quarks.

1 Introduction

In a medium of strongly interacting elementary particles, described by equilibrium thermodynamics of Quantum Chromodynamics (QCD), the transition between the low-temperature hadronic and high-temperature quark–gluon phase occurs in a small temperature interval and leads to a rapid change in entropy and energy density. Although it is, most likely, that this transition is not a true phase transition, but rather a smooth but rapid crossover, it is expected that this transition leads to large fluctuations of energy and net quark number densities. In fact, lattice studies of various hadronic susceptibilities, such as net quark number, isospin or charge susceptibilities, show that quadratic fluctuations of these quantum numbers increase rapidly in the transition region [1, 2]. These susceptibilities continue to grow also above the transition temperature, T_c, and come close to the ideal-gas value at temperatures $T \simeq 2T_c$. Below T_c bulk thermodynamics as well as the generic structure of susceptibilities is found to be in good agreement with properties of a hadronic resonance gas [3].

Recent lattice studies of 2-flavour QCD at non-zero quark (μ_q) and isospin (μ_I) chemical potential [4–7] have sustain that higher-order derivatives of the QCD partition function (generalized susceptibilities), show even more pronounced variations with temperature in the vicinity of the QCD transition temperature. In particular, the fourth–order derivatives with respect to $\mu_{q,I}$, or equivalently with respect to the corresponding up and down quark chemical potentials (μ_u, μ_d), have pronounced peaks at the transition temperature. As a consequence quark number fluctuations calculated at non-zero μ_q increase with increasing values of μ_q and develop a cusp along the transition line. This is in agreement with expectations based on the (mean-field) analysis of quark number fluctuations in sigma models [8] as well as NJL models [9]. These models share the relevant $O(4)$ symmetry with 2-flavour QCD, and give some hint for the existence of a second-order phase transition point in the QCD phase diagram at some non-zero value of the quark chemical potential. In this paper we will, however, concentrate on a discussion of hadronic fluctuations at vanishing quark chemical potential. This is a regime of the QCD phase diagram most relevant to current heavy-ion experiments at RHIC as well as future experiments at the LHC.

Lattice calculations of bulk thermodynamics, e.g. of the equation of state, as well as studies of generalized susceptibilities, show significant deviations from the ideal-gas behaviour for a rather large temperature range above

T_c. In particular, the equation of state shows sizeable deviations from the ideal-gas relation, $\epsilon = 3p$, up to $T \simeq (3\text{–}4)T_c$. This is in line with recent experimental findings that hint at the formation of a strongly interacting medium in heavy-ion collisions at RHIC energies [10].

It has been recently argued that this strongly interacting medium may, in fact, consist of a large set of heavy coloured bound states, which could exist in a temperature interval $T_c \leq T \lesssim (1\text{–}2)T_c$ [11, 12]. The so-called sQGP model suggests that these bound states can contribute as much as 20% to the pressure in the high-temperature phase of QCD. Such states clearly will also contribute to hadronic fluctuations. We will discuss here to what extent fluctuations of such states are consistent with lattice results on quark number and charge fluctuations in the QGP.

In the next section we will give a definition of the observables we are going to analyse and present the lattice results we are going to discuss in the following sections. In section 3 we discuss the properties of quadratic and quartic fluctuations in the vicinity of the QCD transition temperature. Section 4 is devoted to a discussion of hadronic fluctuations in the low- and high-temperature phases of QCD. Section 5 contains our conclusions.

2 Hadronic fluctuations at $\mu_B = 0$

Quark number as well as charge and isospin fluctuations are obtained from the grand canonical partition function of QCD through appropriate combinations of derivatives with respect to quark chemical potentials. As we will concentrate here on a discussion of their properties at finite temperature and vanishing quark chemical potential in 2-flavour QCD, we will use the QCD partition function with non-vanishing up and down quark chemical potentials only as a tool to generate the derivatives of interest. After having done so the chemical potentials will be set to zero.

The pressure of a thermodynamic system is directly related to the grand canonical partition function:

$$P(V,T,\mu_u,\mu_d) = \frac{T}{V} \ln Z(V,T,\mu_u,\mu_d) \quad , \tag{1}$$

where μ_u and μ_d are the chemical potentials for up and down quarks, respectively.

The fluctuations of quark number, isospin and charge as well as expectation values of higher-moments of these observables can be obtained from

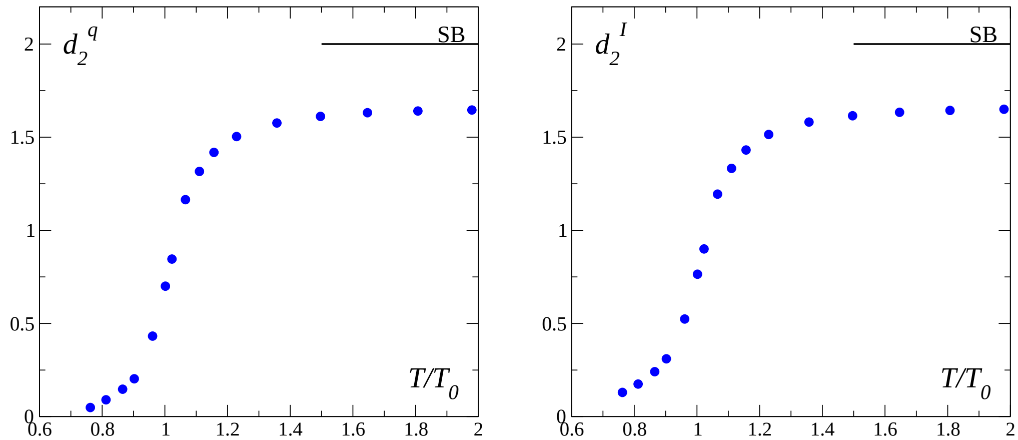

Figure 1: Quadratic fluctuations of the quark (left) and isospin numbers.

generalized susceptibilities: [6, 7, 13]

$$\chi^{n,m} = \frac{\partial^{n+m} P}{\partial \mu_u^n \, \partial \mu_d^m} \quad . \tag{2}$$

These susceptibilities, evaluated for vanishing quark chemical potentials, define the coefficients of a Taylor series for the pressure expanded simultaneously in terms of μ_u and μ_d. In the following we will consider only the mass-degenerate case ($m_u = m_d$) for which the susceptibilities are symmetric, $\chi^{n,m} = \chi^{m,n}$. Recent lattice calculations have shown that the fourth derivatives, $\chi^{4,0}$, as well as the off-diagonal susceptibilities, $\chi^{2,2}$, become large in the vicinity of the transition temperature [6]. Quartic fluctuations thus seem to be particularly sensitive to details of the transition to the high-temperature phase.

Fluctuations of net quark ($N_q = \Delta N_u + \Delta N_d$), isospin ($N_I = \Delta N_u - \Delta N_d$) and charge ($Q = \frac{2}{3}\Delta N_u - \frac{1}{3}\Delta N_d$) numbers, with $\Delta N_{u,d} = N_{u,d} - \bar{N}_{u.d}$ denoting the difference of quarks and antiquarks, are conveniently discussed in terms of quark and isospin chemical potentials:

$$\mu_q = \frac{\mu_u + \mu_d}{2} \quad , \quad \mu_I = \frac{\mu_u - \mu_d}{2} \quad . \tag{3}$$

Charge fluctuations can then be obtained from appropriate combinations of derivatives with respect to $\mu_{u,d}$ or $\mu_{q,I}$, respectively:

$$\frac{\partial}{\mu_Q} \equiv \frac{2}{3}\frac{\partial}{\partial \mu_u} - \frac{1}{3}\frac{\partial}{\partial \mu_d} = \frac{1}{6}\frac{\partial}{\partial \mu_q} + \frac{1}{2}\frac{\partial}{\partial \mu_I} \quad . \tag{4}$$

The first two, non-vanishing derivatives with respect to $\mu_{q,I,Q}$ then yield

$$\begin{aligned} d_2^x \equiv \frac{1}{VT^3}\frac{\partial^2 \ln Z}{\partial(\mu_x/T)^2} &= \frac{1}{VT^3}\langle(\delta N_x)^2\rangle \quad , \quad x = q,\ I,\ Q \\ d_4^x \equiv \frac{1}{VT^3}\frac{\partial^4 \ln Z}{\partial(\mu_x/T)^4} &= \frac{1}{VT^3}\left(\langle(\delta N_x)^4\rangle - 3\langle(\delta N_x)^2\rangle^2\right) \quad . \end{aligned} \tag{5}$$

We note that the derivatives of the partition function with respect to μ_q are directly related to the expansion coefficients c_k calculated in Refs. [5, 6]. Furthermore, d_2^I is related to the expansion coefficient c_2^I, while d_2^Q can be obtained from a combination of c_2 and c_2^I, which were obtained also in these references.

As is evident from Eq. (4) charge fluctuations can easily be expressed in terms of corresponding fluctuations in quark and isospin (or net up and down quark) numbers. While the quadratic charge fluctuations are entirely determined by the corresponding quark number and isospin fluctuations, the quartic charge fluctuations are also sensitive to correlations between δN_q and δN_I. They thus provide independent information:

$$\begin{aligned} d_2^Q &= \frac{1}{6^2}\left(d_2^q + 9d_2^I\right) \ , \\ d_4^Q &= \frac{1}{6^4}\left(d_4^q + 81d_4^I + 54\left(\langle(\delta N_q)^2(\delta N_I)^2\rangle - 3\langle\delta N_q\delta N_I\rangle^2\right)\right) \ . \end{aligned} \tag{6}$$

In Fig. 1 we show the two independent second-order cumulants d_2^q and d_2^I. They are directly proportional to the coefficients c_2 and c_2^I calculated in Refs. [5, 6]. The three fourth-order cumulants[1] are shown in Fig. 2.

While the second-order cumulants show the well known continuous increase towards the high-temperature ideal-gas value, the fourth-order cumulants clearly have a pronounced peak at the transition temperature and exceed the ideal-gas value by more than a factor 4. Here it is worthwhile to note that the lattice results shown in Figs. 1 and 2 are based on calculations that were performed with still rather heavy quarks, corresponding to a pion mass of about 770 MeV. Although we still no have satisfactory understanding of the universal behaviour of the QCD transition in 2-flavour QCD, the studies of the quark mass dependence of, for instance, the chiral susceptibility [14–17] suggest that the peaks in susceptibilities increase with decreasing

[1] These results are based on fourth-order derivatives of the pressure calculated in Refs. [5, 6].

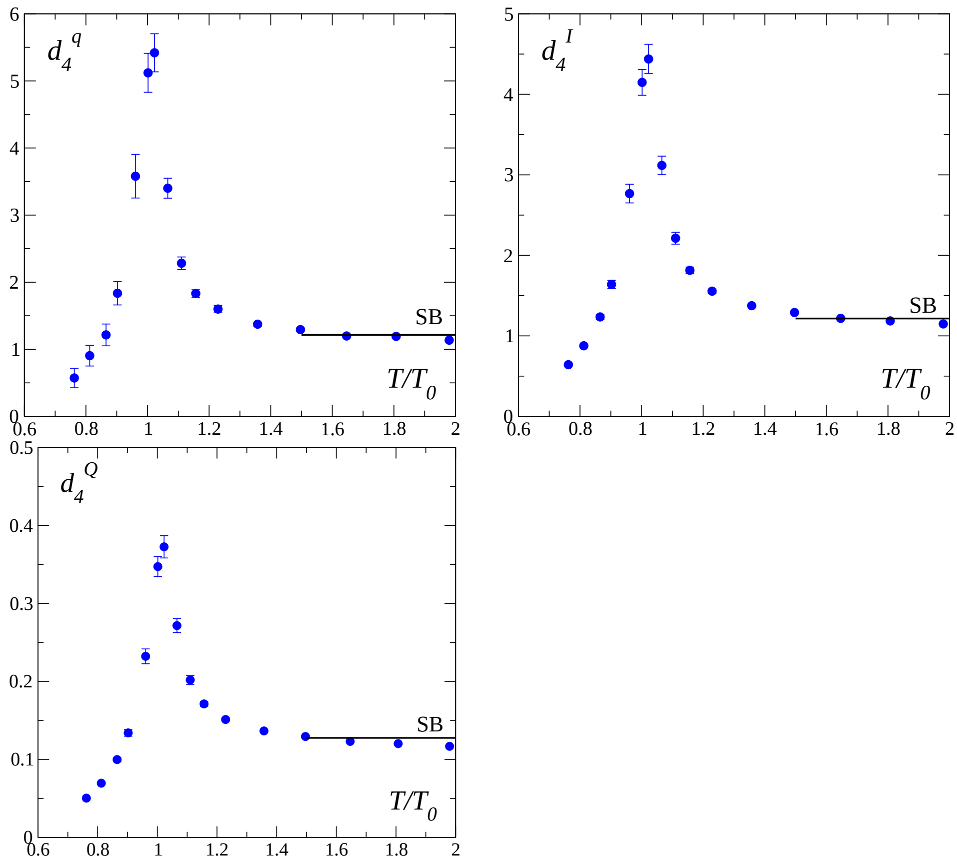

Figure 2: Fourth cumulant of the quark (top, left) and isospin (top, right) number fluctuations as well as the fourth cumulant of the charge fluctuations (bottom, left).

quark mass. We thus also expect that the peaks found in fourth derivatives of the pressure will increase with decreasing quark mass. In the next section we briefly discuss the generic structure of second- and fourth-order cumulants expected to be found in the chiral limit.

3 Chiral symmetry restoration and hadronic fluctuations

In the vicinity of the chiral transition temperature, and for small values of the quark chemical potentials, the pressure receives contributions from a regular and a singular part:

$$P(T,\mu_u,\mu_d) = P_r(T,\mu_u,\mu_d) + P_s(\bar{t},\bar{\mu}_u,\bar{\mu}_d) \quad , \tag{7}$$

where we have introduced in the singular part the reduced temperature $\bar{t} = |T - T_c|/T_c$ and the dimensionless chemical potentials $\bar{\mu}_{u,d} = \mu_{u,d}/T$. Deviations from criticality are controlled by a generalized version of the reduced temperature variable, which takes into account that the temperature-like direction at non-vanishing quark chemical potential in general will be a mixture of the couplings $\bar{t}$ and μ_q/T. This generalized reduced temperature has to respect the invariance of the QCD partition function under exchange of particles and antiparticles ($\mu_q \rightarrow -\mu_q$):

$$\begin{aligned} t &\equiv \bar{t} + A(\bar{\mu}_u^2 + \bar{\mu}_d^2) + B\bar{\mu}_u\bar{\mu}_d \\ &= \bar{t} + (2A+B)\bar{\mu}_q^2 + (2A-B)\bar{\mu}_I^2 \quad . \end{aligned} \tag{8}$$

The singular part of the pressure may then be written as

$$P_s(\bar{t},\bar{\mu}_u,\bar{\mu}_d) \sim b^{-1} f_s(b^{1/y_t} t) \sim t^{2-\alpha} \quad , \tag{9}$$

where we have introduced an arbitrary scale parameter b and where $y_t = 2-\alpha$ is the usual thermal critical exponent, which is related to the critical exponent α. Equation (9) shows that the critical behaviour of hadronic fluctuations $\mu_u = \mu_d = 0$ is controlled by the critical exponent of the specific heat, α [8]. The contribution of the singular part to second- and fourth-order cumulants of quark ($k \equiv q$) and isospin ($k \equiv I$) is then given by

$$\begin{aligned} \left(\partial^2 P_s/\partial\mu_k^2\right)_{\mu_u=\mu_d=0} &\sim t^{1-\alpha} \;, \\ \left(\partial^4 P_s/\partial\mu_k^4\right)_{\mu_u=\mu_d=0} &\sim t^{-\alpha} \;. \end{aligned} \tag{10}$$

For a conventional second-order phase transition with a critical exponent $\alpha > 0$ we would thus expect that the fourth-order cumulants diverge in the chiral limit. However, if the chiral transition is indeed of second-order we expect it to belong to the universality class of three-dimensional $O(4)$ symmetric spin models. In this case the exponent α will be negative, $\alpha \simeq -0.21$ [18]. The contribution of the singular part will then vanish at $t = 0$ and the value of the cumulants at $t = 0$ will thus be determined by the regular part alone. In the vicinity of $t = 0$ the temperature dependence of the second-order cumulant will then be dominated by the regular part. This is in accordance with the rapid but smooth increase of second cumulants across the transition temperature. The temperature dependence of the fourth-order cumulant, however, will be dominated by the singular part and as α is small the temperature variation will result in a sharp cusp. This is in accordance with the behaviour seen in Fig. 2.

If the chiral transition in 2-flavour QCD is of second-order, we thus expect the second- and fourth-order cumulants to stay finite at the critical point. This, however, only holds for the critical point at $\mu_q = 0$, which we have discussed above. If the transition in 2-flavour QCD at $\mu_q = 0$ indeed belongs to the $O(4)$ universality class, this transition point will be part of a line of second-order phase transitions at non-zero quark chemical potential. At a transition point with $\mu_q > 0$, one would then expect that already the second-order cumulant of quark number fluctuations develops a cusp [8] and that the fourth-order cumulant would diverge.

4 Hadronic fluctuations in a thermal medium

In the previous section we have discussed the qualitative structure of second- and fourth-order cumulants in the vicinity of the QCD transition temperature. Here we want to look in somewhat more detail into the temperature dependence of these cumulants in the low and high-temperature phases of QCD. In particular, we want to argue that the ratios of fourth- and second-order cumulants of quark number and charge fluctuations,

$$R^q_{4,2} = \frac{d^q_4}{d^q_2} \quad , \quad R^Q_{4,2} = \frac{d^Q_4}{d^Q_2} \quad , \tag{11}$$

are sensitive observables that allow the identification of the carriers of quark number and electric charge in a thermal medium. We will first discuss this for a hadron gas at low-temperature and then move on to a discussion of the relevant degrees of freedom in the high-temperature phase. This discussion can easily be extended to the case of isospin fluctuations, which, however, we will not consider here any further.

4.1 Fluctuations in hadronic matter

We describe the low-temperature phase of QCD in terms of a resonance gas with free particle dispersion relations for all constituents [3, 19]. The resulting pressure, however, also accounts for interactions between hadrons, to the extent that the thermodynamics of an interacting system of elementary hadrons can effectively be approximated by that mixture of ideal-gases of stable particles and resonances [19, 20]. The pressure P is then expressed as a sum over all mesonic and baryonic degrees of freedom:

$$P(T, \mu_u, \mu_d) = P^M(T, \mu_u, \mu_d) + P^B(T, \mu_u, \mu_d) \quad . \tag{12}$$

Using Eq. (3) to replace the up and down quark chemical potentials by μ_q and μ_I, the contribution of mesons, baryons and their resonances belonging to fixed baryon and isovector quantum numbers is obtained from Eq. (12) in compact form. With the exception of the pion, all hadrons are heavy, with respect to the temperatures of interest. For these particles quantum statistics plays no role, and the Boltzmann approximation of the partition functions is appropriate. This yields, in the meson sector,

$$\frac{P^M(T, \mu_I)}{T^4} = \sum_i F(T, m_i) \cosh \frac{2I_i^{(3)} \mu_I}{T} \quad , \tag{13}$$

where the sum is taken over all non-strange mesons[2], $I_i^{(3)}$ being the third component of the isospin of the species i. The function $F(T, m_i)$ is the contribution of particles and the corresponding antiparticles of mass m_i with spin degeneracy factor d_i to the pressure at vanishing baryon and isospin chemical potential

$$F(T, m_i) = \frac{p_i}{T^4} = \frac{d_i}{\pi^2} \left(\frac{m_i}{T}\right)^2 K_2 \left(\frac{m_i}{T}\right) \quad . \tag{14}$$

[2]In order to be able to compare with lattice calculations performed for 2-flavour QCD, we have neglected the contribution of strange particles to the hadronic pressure.

Neglecting the mass difference between isospin partners, the meson contribution arises from isospin singlet $G^{(1)}$ ($I_i^{(3)} = 0$) and triplet $G^{(3)}$ ($I_i^{(3)} = 0,\ \pm 1$) mesons, respectively. Thus,

$$\frac{P^M(T,\mu_q,\mu_I)}{T^4} = G^{(1)}(T) + \frac{1}{3}G^{(3)}(T)[2\cosh(\frac{2\mu_I}{T}) + 1] \quad . \tag{15}$$

The contribution to the pressure arising from non-strange baryons and their resonances is given by

$$\frac{P^B(T,\mu_q,\mu_I)}{T^4} = \sum_i F(T,m_i)\cosh\frac{3\mu_q + 2I_i^{(3)}\mu_I}{T} \quad . \tag{16}$$

The baryonic part of the pressure receives contributions from isospin doublet $F^{(2)}$ ($I_i^{(3)} = \pm 1/2$) and quartet $F^{(4)}$ ($I_i^{(3)} = (\pm 1/2, \pm 3/2)$) baryons, respectively. Again ignoring the mass differences between isospin partners, the total baryonic contribution to the pressure is given by

$$\begin{aligned} \frac{P^B(T,\mu_q,\mu_I)}{T^4} &= F^{(2)}(T)\cosh(\frac{3\mu_q}{T})\cosh(\frac{\mu_I}{T}) \\ &+ \frac{1}{2}F^{(4)}(T)\cosh(\frac{3\mu_q}{T})[\cosh(\frac{\mu_I}{T}) + \cosh(\frac{3\mu_I}{T})] \quad . \end{aligned} \tag{17}$$

We note that only the isospin quartet baryons contain doubly charged hadrons, which is reflected in the appearance of a term proportional to $\cosh(3\mu_I/T)$.

Within this resonance gas formulation we now can evaluate cumulants of the quark number, isospin and charge in the confined phase of QCD. In fact, by examining the cumulants introduced in section 2, we can find the contributions of different hadronic quantum number channels and can establish relations between the various cumulants. Within the Boltzmann approximation for the hadron resonance gas, which we have introduced above, we find

$$\begin{aligned} d_2^q(T) &= 9\left(F^{(2)}(T) + F^{(4)}(T)\right) \quad , \\ d_4^q(T) &= 81\left(F^{(2)}(T) + F^{(4)}(T)\right) \quad , \end{aligned} \tag{18}$$

$$
\begin{aligned}
d_2^Q(T) &= \frac{2}{3}G^{(3)}(T) + \frac{1}{2}F^{(2)}(T) + \frac{3}{2}F^{(4)}(T) \quad , \\
d_4^Q(T) &= \frac{2}{3}G^{(3)}(T) + \frac{1}{2}F^{(2)}(T) + \frac{9}{2}F^{(4)}(T) \quad .
\end{aligned} \tag{19}
$$

These equations are valid for isospin-symmetric systems at vanishing baryon number density. The corresponding derivatives of the contributions to the total pressure (Eqs. (13–17)) were thus taken at $\mu_I = \mu_q = 0$. The resonance gas model leads to simple relations for the ratios of cumulants of quark number and charge fluctuations. In particular, we found

$$
R_{4,2}^q \equiv \frac{d_4^q}{d_2^q} = \frac{\langle(\delta N_q)^4\rangle}{\langle(\delta N_q)^2\rangle} - 3\langle(\delta N_q)^2\rangle = 9 \quad , \tag{20}
$$

which is shown to agree well with the lattice results below T_c [5, 6].

The ratio $R_{4,2}^q$ is completely insensitive to details of the hadron spectrum contributing to the resonance gas. In particular, it reflects the fact that the quark number is carried only by baryons, i.e. the relevant degrees of freedom either carry net quark number 0, in which case they do not contribute to $R_{4,2}^q$, or 3, in which case they contribute to $R_{4,2}^q$ with a relative factor of 3^2.

This is similar for the ratio of cumulants of charge fluctuations, although we here have to take into account that hadrons with charge 0, 1 and 2 can contribute to the thermodynamics. The ratio $R_{4,2}^Q$ become sensitive to the relative abundance of doubly charged hadrons:

$$
\begin{aligned}
R_{4,2}^Q \equiv \frac{d_4^Q}{d_2^Q} &= \frac{\langle(\delta Q)^4\rangle}{\langle(\delta Q)^2\rangle} - 3\langle(\delta Q)^2\rangle \\
&= \frac{4G^{(3)} + 3F^{(2)} + 27F^{(4)}}{4G^{(3)} + 3F^{(2)} + 9F^{(4)}} \quad .
\end{aligned} \tag{21}
$$

At low-temperature, this ratio is dominated by charge fluctuations in the pion sector, which contributes to $G^{(3)}$. The ratio will thus approach unity at low-temperatures and monotonically increase with temperature. This indicates that in the low-temperature limit all charged degrees of freedom carry one unit of charge. The ratio $R_{4,2}^Q$ does increase only from contributions arising from isospin quartet baryons, which can carry charge $Q = 2$. In fact, denoting the contribution of all hadrons carrying charge $Q = \pm n$ by $F_H^{Q=n}(T)$, one can verify that Eq. (21) can be rewritten as[3]

$$R_{4,2}^Q = \frac{F_H^{Q=1}(T) + 4F_H^{Q=2}(T)}{F_H^{Q=1}(T) + 16F_H^{Q=2}(T)} \quad . \tag{22}$$

Results from lattice calculations for the ratios defined in Eqs. (20) and (21), are shown in Fig. 3. In the low-temperature phase these ratios clearly show the basic features expected from the analysis of a hadron resonance gas. Above T_c, however, they rapidly drop and strongly deviate from the resonance gas behaviour. In fact, already at $T \simeq 1.5T_c$ the ratios are close to the corresponding ideal-gas values. In high-temperature perturbation theory the leading $\mathcal{O}(g^2)$ corrections to quadratic and quartic cumulants cancel in the corresponding ratios:

$$R_{4,2}^q(T)_{|\text{pert.th.}} = \frac{6}{\pi^2} + \mathrm{O}(g^3) \ , \tag{23}$$

$$R_{4,2}^Q(T)_{|\text{pert.th.}} = \frac{34}{15\pi^2} + \mathrm{O}(g^3) \ . \tag{24}$$

These values are also shown as solid lines in Fig. 3.

4.2 Fluctuations in the QGP

The considerations presented in the previous section for hadrons below T_c carry over to any thermodynamic system containing heavy quasi-particle states that are build up from quarks and gluons. As long as their masses are significantly larger than T_c, their contribution to the thermodynamics can be treated within the Boltzmann approximation. States with mass m_i containing a net number of k_u up quarks and k_d down quarks will contribute to the pressure of the system with

$$\frac{p_i}{T^4} = F(T, m_i) \cosh\left(k_u\mu_u + k_d\mu_d\right) \quad , \tag{25}$$

where $F(T, m_i)$ is given by Eq. (14), i.e. it again combines the contribution of a particle and its antiparticle.

[3]This becomes obvious when one reconstructs the origin of the algebraic factors appearing in Eq. (19). For instance, the factor 3/2 in front of $G^{(3)}$ indicates that, out of 3 triplet states, 2 carry charge $|Q| = 1$ and the 3rd state does not contribute as it carries charge $Q = 0$. Similarly the factors 3/2 and 9/2 appearing in front of the quartet contribution $F^{(4)}$ arise from the fact that, out of the 4 states, 2 carry charge $|Q| = 1$, 1 state carries charge $Q = 2$ and the 4th state does not contribute as it carries charge $Q = 0$.

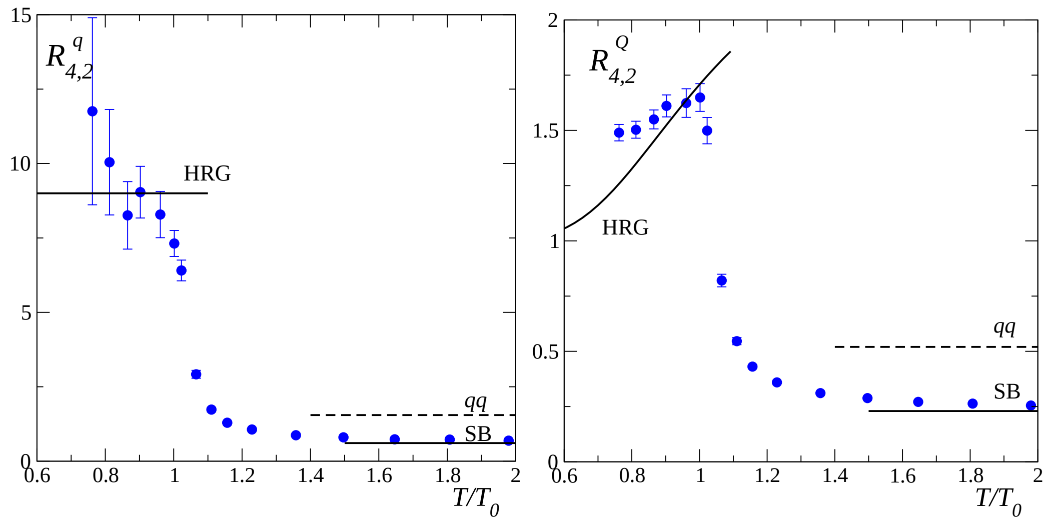

Figure 3: The ratios of fourth and second cumulants of quark number (left) and charge (right) fluctuations. At low-temperatures the ratios $R^q_{4,2}$ and $R^Q_{4,2}$ are compared to resonance gas model predictions. At high-temperature the asymptotic ideal-gas value is indicated by a horizontal line. The dashed line represents the estimate for the value of the ratios at $T = 1.4T_c$ in the presence of coloured qq and qg states asssuming that the qq states contribute with half their statistical weight as discussed in the text.

A model containing heavy coloured bound states above T_c has been put forward by E. Shuryak and I. Zahed [11] to explain the experimental evidence for strong interactions in the hot and dense medium of quarks and gluons created in heavy-ion collisions at RHIC. In the strongly coupled QGP model (sQGP), it is assumed that, in addition to heavy quasi-particles (quarks and gluons with large thermal masses), also a large number of coloured bound states contribute to the thermodynamics in the high-temperature phase of QCD. The model suggests, that aside from states carrying net quark number 1 and charge $Q = 1/3$ and $2/3$ there will also be states with net quark number 2 and charge $Q = 1$ and $4/3$ contributing to the thermodynamics in the QGP. From the discussion given in the previous section it should be obvious that this must have consequences for quark number and charge fluctuations in the QGP and the ratios $R^q_{4,2}$ and $R^Q_{4,2}$ should be sensitive to these new states. The lattice results shown in Fig. 3, can thus put bounds on their relative abundance.

For our discussion of baryon number and charge fluctuations, the quark–quark and quark–gluon bound states are of particular interest. The sQGP model suggests that the latter melt at $T_{\rm melt} \equiv T_{qg} \simeq 2T_c$ while the former are only weakly bound and melt already at $T_{\rm melt} \equiv T_{qq} \simeq 1.4T_c$ [11, 12]. It has been argued in Ref. [11] that the gradual disappearance of these states could be taken into account by reducing their contribution to the thermodynamic potential with increasing temperature. For the corresponding weight factor, we will use here the parameterization introduced in Ref. [11]:

$$R_C(T, T_{melt}) = \frac{1}{1+\exp\left(C(T-T_{\rm melt})/T_c\right)} \quad , \tag{26}$$

where C controls the temperature interval over which the contribution of a given bound state to the partition function gradually disappears.

In order to estimate the contribution of coloured states to the fluctuations of baryon number and charge, we consider the contribution of these states to the density-, (chemical potential)-dependent part of the pressure,

$$\begin{aligned} \frac{\Delta p}{T^4} &= \frac{1}{2}\left(F_q(T) + R_2(T,2)F_{qg}(T)\right)\left(\cosh(\mu_u/T) + \cosh(\mu_d/T)\right) \\ &\quad + \frac{1}{3}R_C(T,1.4)F_{qq}(T)(\cosh(2\mu_u/T) + \cosh(2\mu_d/T) \\ &\quad + \cosh((\mu_u+\mu_d)/T)) \ , \end{aligned} \tag{27}$$

where we again used the Boltzmann approximation as all states are expected to have masses $m_i \gtrsim 10T_c$ [11]. We will approximate the masses of quark–gluon and quark–quark bound states as twice the quasi-particle mass of quarks, and will thus neglect mass shifts arising from a non-vanishing binding energy. Moreover, we used $C = 2$ for the quark–gluon bound states [11] and will here only discuss the contribution of quark–quark bound states as a function of R_C. With this we can estimate the contribution of quark–quark states to the quadratic and quartic quark number and charge fluctuations introduced in Eq. (5):

$$\begin{aligned} d_2^q &= F_q(T) + (R_2(T,2)F_{qg}(T) + 4R_C(T,1.4)F_{qq}(T)) \\ d_4^q &= F_q(T) + (R_2(T,2)F_{qg}(T) + 16R_C(T,1.4)F_{qq}(T)) \ . \end{aligned} \tag{28}$$

and

$$d_2^Q = \frac{1}{9}\Big(\frac{5}{2}F_q(T) + \Big(\frac{5}{2}R_2(T,2)F_{qg}(T) + \frac{11}{2}R_C(T,1.4)F_{qq}(T)\Big)\Big)$$

$$d_4^Q \;=\; \frac{1}{81}\Big(\frac{17}{2}F_q(T) + \Big(\frac{17}{2}R_2(T,2)F_{qg}(T) + \frac{137}{2}R_C(T,1.4)F_{qq}(T)\Big)\Big) \quad (29)$$

Apparently both ratios $R_{4,2}^{q,Q}$ are enhanced over the corresponding ratios that would only contain states with the quantum number of quarks[4] owing to the presence of states that carry two units of quark number. Assuming that the coloured bound state masses are about twice as large as the quasi-particle quark masses, we find at the melting temperature of the quark–quark states $R_{4,2}^q = 1.55$ and $R_{4,2}^Q = 0.52$. These values are shown as the dashed lines in Fig. 3. How quickly this enhancement disappears at larger temperature depends on the precise choice of the suppression factor R_C. At $T_{\rm melt} \simeq 1.4T_c$, the contribution of quark–quark states should be suppressed below 10% of that of an ordinary bound state ($R_C \lesssim 0.1$) in order to be compatible with the lattice results. This also could mean that the melting temperature is substantially smaller ($T_{\rm melt} \lesssim 1.1T_c$).

We also note that the perturbative values given in Eq. (24) are obtained for a massless quarks–antiquarks (fermion) gas. These ratios are smaller by about 50% than the corresponding values in the Boltzmann approximation. This indicates the uncertainties inherent in the estimates given above and suggests that if coloured bound states exist in the QGP and contribute to the thermodynamics, this presumably has to be discussed in terms of a more complex interaction pattern, which also takes into account quantum effects.

5 Conclusions

We have discussed some generic features of quark number, isospin and charge fluctuations in 2-flavour QCD. We argue that the ratio of quartic and quadratic fluctuations are sensitive observables that can directly provide information on the constituents of the thermal medium that carry net quark number and electric charge, respectively. We have shown that these ratios, below the QCD transition temperatures, are in reasonable agreement with a hadronic resonance gas. Above T_c the ratios rapidly drop and approach the high-temperature ideal-gas values. This suggests that, already for $T \gtrsim 1.5T_c$ quark number and charge are predominantly carried by states with the quantum number of quarks. This leaves little room for the contribution of quark–quark

[4] Note that this latter ratio could also include a contribution from quark–gluon bound states. Moreover, the observables discussed here are insensitive to the presence of coloured quark–antiquark bound states, which could exist in the QGP [11].

bound states. It is conceivable that these constraints would be even more stringent in 3-flavour QCD calculations, as the relevant states in the sQGP model are proportional to n_f^2 while the relevant degrees of freedom in the (perturbative) high-temperature limit of QCD are only proportional to n_f. A similar conclusion on the incompatibility of the sQGP model with lattice results on hadronic fluctuations has been drawn recently in [21].

It would, of course, be interesting to verify these results in experimental studies of event-by-event fluctuations [22, 23] of baryon number or charge. However, if indeed the medium created in a heavy-ion collision is strongly interacting and rapidly equilibrates there is little hope to find evidence for fluctuations originate from the QGP. At best, one may hope to study the quadratic and quartic fluctuations at freeze-out. Also this, however, would be interesting, as the observation of the importance of doubly charged hadrons leading to $R_{4,2}^{Q} \simeq 1.5$ could give further support to the validity of the resonance gas model.

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References

[1] S. Gottlieb et al., Phys. Rev. Lett. **59** 2247 (1987) and Phys. Rev. D **38** 2888 (1988)

[2] R. V. Gavai, J. Potvin and S. Sanielevici, Phys. Rev. D **40** 2743 (1989)

[3] F. Karsch, A. Tawfik and K. Redlich, Phys. Lett. B **571** 67 (2003) and Eur. Phys. J. C **29** 549 (2003)

[4] S. Ejiri, Ch. R. Allton, S. J. Hands, O. Kaczmarek, F. Karsch, E. Laermann and Ch. Schmidt, Phys. Rev. D **68** 014507 (2003)

[5] C.R. Allton, S. Ejiri, S.J. Hands, O. Kaczmarek, F. Karsch, E. Laermann, C. Schmidt and L. Scorzato, Phys. Rev. D **66** 074507 (2002)

[6] C. R. Allton, M. Döring, S. Ejiri, S. J. Hands, O. Kaczmarek, F. Karsch, E. Laermann and K. Redlich, Phys. Rev. D **71** 054508 (2005)

[7] R. Gavai and S. Gupta, *Simple patterns for non-linear susceptibilities near* T_c, hep-lat/0507023

[8] Y. Hatta and T. Ikeda, Phys. Rev. D **67** 014028 (2003)

[9] H. Fuji and M. Ohtani, Phys. Rev. D **70** 014016 (2004)

[10] *First Three Years of Operation of RHIC*, Nucl. Phys. A **757** 1 (2005)

[11] E.V. Shuryak and I. Zahed, Phys. Rev. D **70** 054507 (2004)

[12] J. Liao and E. V. Shuryak, *Polymer chains and baryons in a strongly coupled quark-gluon plasma*, hep-ph/0508035

[13] R. Gavai and S. Gupta, Phys. Rev. D **68** 034506 (2003)

[14] F. Karsch and E. Laermann, Phys. Rev. D **50** 6954 (1994)

[15] S. Aoki et al. (JLQCD Collaboration), Phys. Rev. D **57** 3910 (1998)

[16] M. D'Elia, A. Di Giacomo and C. Pica, *Two flavour QCD and Confinement*, hep-lat/0503030

[17] Y. Iwasaki, K. Kanaya, S. Kaya and T. Yoshié, Phys. Rev. Lett. **78** 179 (1997)

[18] J. Engels, L. Fromme and M. Seniuch, Nucl. Phys. B **572** 289 (2000)

[19] For a recent review, see P. Braun-Munzinger, K. Redlich and J. Stachel, *Quark Gluon Plasma 3*, R. C. Hwa and X.-N. Wang (Eds.), World Scientific, **491** (2004), hep-ph/0402291 and references therein; A. Andronic and P. Braun-Munzinger, hep-ph/0402291

[20] R. Hagedorn, Nuovo Cimento **35** 395 (1965)

[21] V. Koch, A. Majumder and J. Randrup, *Baryon-strangeness correlations: A diagnostic of strongly interacting matter*, nucl-th/0505052

[22] For a recent review, see S. Jeon and V. Koch, *Quark Gluon Plasma 3*, R. C. Hwa and X.-N. Wang (Eds.), World Scientific, **430** (2004) and references therein

[23] M. Gaździcki and S. Mrówczyński, Z. Phys. C **54** 127 (1992)
M. Asakawa, U. Heinz and B. Müller, Phys. Rev. Lett. **85** 2072 (2000).
M. A. Stephanov, K. Rajagopal and E. Shuryak, Phys. Rev. Lett. **81** 4816 (1998). S. Jeon and V. Koch, Phys. Rev. Lett. **85** 2076 (2000).
Zi-wei Lin and C.M. Ko, Phys. C **64** 041901 (2001). E.V. Shuryak and M. A. Stephanov, Phys. Rev. C **63** 064903 (2001)