Ref.TH.1622-CERN

FURTHER EVIDENCE FOR A 70, L=2 BARYON MULTIPLET

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ABSTRACT

The recently discovered f wave dominance of the decay $F_{35}(1877) \rightarrow \Delta\pi$ is interpreted as requiring mixing between 56, L=2 and 70, L=2 baryon multiplets. This conclusion is drawn in the context of a broken version of $SU(6)_W$. Further predictions include a) the existence of a second prominent F_{15} πN resonance around 2 GeV in mass and b) the more probable dominance of the 1.9 GeV enhancement seen in $\pi^+ p \rightarrow \Sigma^+(1385)K^+$ by P_{33} and P_{31} rather than F_{37} or F_{35} resonances.

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Ref.TH.1622-CERN
28 February 1973

I. INTRODUCTION

The discovery in 1967 [1] of indications of an F_{17} πN resonance with a mass near 1983 MeV provided the first hints about the existence of a $\underline{70}$, L=2 supermultiplet [2] in $SU(6)_W \times O(3)$. Since then phase shift analyses have been extended to higher energies enabling a more detailed study to be made in this mass region. As a result, both the new CERN [3] and Saclay [4] analyses provide stronger evidence for this state. In Fig. 1 we have plotted the CERN F_{17} phases as tabulated in CERN preprint TH.1408, but in a manner designed to improve the signal-to-noise ratio. What we have done is to plot the average of each successive three entries in the tabulation, removing the lowest entry and adding one more to the average as we proceed from each point on Fig. 1 to the next. In general this would be a good way of wiping out any fine structure, but this is unimportant in the present case since our purpose is to see whether there is any large resonant structure present. The resulting Argand diagram and speed plot both show clear indications of a resonance. But this is all history and by now well known.

What is new is the remarkable finding of Mehtani et al. [5] that the $F_{35}(1877)$ decay into $\Delta\pi$ proceeds entirely via an f wave final state orbital angular momentum, whereas one might have expected the p wave to dominate from centrifugal barrier considerations. Such a state of affairs – confirmed now by Herndon et al. [6] – is not understandable if the F_{35} resonance is given its traditional [7] $SU(6)_W \times O(3)$ classification of $\underline{56}$, L=2. However, as we shall show in the present paper, the suppression of p waves is precisely the effect that would follow from a small amount of interference with a near-by $\underline{70}$, L=2 resonance having the same quantum numbers. Of course, the alternative is always open to abandon broken $SU(6)_W$ as a reasonable classification group for higher-lying resonances and their couplings. However, we should like to suggest that such a course of action is not called for by any of the presently available experimental data.

The mere existence of a $\underline{70}$, L=2 supermultiplet has important consequences for certain classification schemes based on the concept of duality. Indeed, with the assumption that particles can be classified in complete $SU(6)_W$ multiplets, the existence of an even L $\underline{70}$ supermultiplet is only required if one tries to bring $B\bar{B} \to MM$ processes within the duality scheme [8),9]. If duality can indeed be applied to such high threshold channels, one would certainly expect its applicability to $B\bar{B} \to B'\bar{B}'$ channels which, as is well known [10] leads to the prediction of exotic meson resonances. One arrives thus at a situation where evidence for a $\underline{70}$, L=2 considerably strengthens the theoretical arguments in favour of the existence of relatively low-lying (in mass) exotic mesons.

This paper is organized as follows. In Section II we review briefly how $SU(6)_W$ symmetry is applied to experimentally measured partial decay widths. Section III assumes that the established f wave πN resonances are pure unmixed members of a $\underline{56}$, L=2 multiplet. This leads to a gross contradiction with the experimental situation. In Section IV we ask what effects would follow if those members of this $\underline{56}$, L=2 which have quantum numbers in common with corresponding states in a $\underline{70}$, L=2 were allowed to mix with the $SU(6)_W$ partners of the F_{17}. Not only does this reconcile theory with experiment in a most satisfying manner, but it enables us to predict in Section V many experimental properties of these supermultiplets noteworthy among them being the expected appearance of a second F_{15} resonance not far from the F_{17}. Section VI summarizes our conclusions.

II. ℓ BROKEN $SU(6)_W$

As a tool for relating resonance partial widths to one another, we shall use a broken version of $SU(6)_W$. The motivation for this symmetry and the quality of its fits to the data have recently been discussed at some length [11)-15)]. Here we give only a brief description of the parameters involved.

In $SU(6)_W$ a single reduced matrix element characterizes all decays from one multiplet to a pair of others. For example, one would have a single amplitude for all decays of the type $\underline{56}$, L=2 $\rightarrow$ $\underline{56}$, L=0 $\otimes$ $\underline{35}$, L=0 [e.g., $F_{37} \rightarrow N\pi$, ΣK, $\Delta\pi$, $N\rho$, ...]. A single but different amplitude would also suffice for all decays in $\underline{70}$, L=2 $\rightarrow$ $\underline{56}$, L=0 $\otimes$ $\underline{35}$, L=0 [e.g., $F_{17} \rightarrow N\pi$, ΣK, $\Delta\pi$, $N\rho$, ...]. The constraints of angular momentum and parity conservation, as well as certain selection rules of $SU(6)_W$, dictate that the orbital angular momenta ℓ of the final states will be $\ell = L \pm 1$, i.e., $\ell = 1$ (p wave) or $\ell = 3$ (f wave). In "ℓ broken $SU(6)_W$" the p wave and f wave amplitudes are treated separately from one another, while the $SU(6)_W$ relations <u>among</u> p waves and <u>among</u> f waves continue to hold.

III. UNMIXED <u>56</u>, L=2 PREDICTIONS

As already stated, we shall attempt to prove that mixing occurs by starting from a no-mixing assumption. Thus, beginning with an unmixed <u>56</u>, L=2 we require three f wave and a like number of p wave πN resonances. There are unique well-established candidates for the former, namely, $F_{37}(1923)$, $F_{35}(1877)$ and $F_{15}(1684)$. These masses are the averages of the individual numbers quoted by CERN [3] and Saclay [4]. For the p waves, unfortunately, there is no agreement : CERN suggests $P_{11}(1900)$, $P_{13}(1850)$ but no P_{33}, whereas Saclay claims a $P_{33}(1890)$ but no suitable P_{11} or P_{13} states. Fortunately, we need not base our argument on these latter states. We parametrize the observed partial widths in the usual manner :

$$\Gamma_\ell = C_\ell^2 G_\ell^2 B_\ell(kr) \frac{k}{M^*} M_N \quad (1)$$

where C_ℓ is an $SU(6)_W \times O(2)_{L_Z}$ Clebsch-Gordan coefficient, and G_ℓ is one of two coupling constants : a p wave coupling which goes with the $\ell = 1$ barrier factor [16]

$$B_1(kr) = \frac{(kr)^2}{1+(kr)^2} \quad (2)$$

or an f wave constant which accompanies the $\ell = 3$ barrier factor [16]

$$B_3(kr) = \frac{(kr)^6}{225+45(kr)^2+6(kr)^4+(kr)^6} \quad (3)$$

M^* and k are, respectively, the decaying resonance's rest mass and the c.m. decay momentum, and M_N is the traditional proton mass scale factor.

To enter here into an elaborate discussion about r the radius of interaction would obfuscate our argument very much. For our purposes, it is sufficient to consider $r = 0$ and $r = 1$ fermi as typifying values popular in the literature and to remark that these yield respective $F_{37} \to \Sigma K/N\pi$ branching ratios of 0.026 and 0.163. Experimentally this ratio is found to be 0.042 [17] or less [18], so we shall construct our f wave argument around $r = 0$.

The CERN and Saclay elastic πN widths for this F_{37} resonance centre about 86 MeV and the Σ^+K^+ phase shift analysis of Kalmus et al. [17] yields a ΣK rate of 3.6 MeV. The nearest we can approach to these values for $r = 0$ are 112 MeV and 2.9 MeV, respectively, 30% discrepancies which are well within the spirit of SU(3).

The $F_{15}(1684) \rightarrow N\pi$ rate is then predicted at 57 MeV to be compared with the CERN-Saclay average of 86 MeV - still only a 30% discrepancy [19].

Now comes the first important prediction : the f wave $F_{35}(1877) \rightarrow \Delta\pi$ rate is predicted to be 20 MeV.

In order to predict the p wave contribution to this process, it would help to have two experimental pieces of input data - to fix both the p wave coupling and its interaction radius. However, owing to the absence of any well established p wave πN resonances, as discussed above, we can use only the $\Delta\pi$ decay rate of the $F_{15}(1684)$. This p wave partial width was measured by Brody et al. [20] to be 18 MeV, a value visibly compatible with the amplitude plots of Herndon et al. [6] (who have not yet published any partial widths). Taking this to fix our p wave coupling, and assuming typical interaction radii, we arrive at the second important prediction : the p wave rate for $F_{35}(1877) \rightarrow \Delta\pi$ takes the value 47 MeV or 28 MeV, depending on whether $r = 0$ or 1 fermi.

By comparing this with our f wave prediction the contradiction becomes apparent. The figures of Herndon et al. [6] show no hint of a p wave contribution, but a possible f wave effect. Mehtani et al. [5] are more emphatic claiming a large f wave $\Delta\pi$ rate for the F_{35} and a negligible p wave. [Their upper limit is $\Gamma^{(p)}/\Gamma^{(f)} < 4$.]

At this stage we might :

a) conclude that the experiments are wrong, or

b) abandon ℓ broken $SU(6)_W$ as a useful symmetry, or

c) bearing in mind that the established existence of $P_{11}(1448)$ and $P_{11}(1743)$ indicates the presence of at least something else besides a $\underline{56}$, L=2, look to some interference effect to save this symmetry.

We shall argue that approach c) is the most sensible alternative.

IV. <u>70</u>, L=2 INTERFERENCE

In seeking a second $SU(6)_W$ multiplet to explain the p wave suppression of $F_{35} \to \Delta\pi$ it is noteworthy that <u>70</u>, L=2 is a most natural and conservative choice. In the first place L = 2 is the simplest way of building a $J^P = 5/2^+$ resonance. Secondly, there is not the slightest amount of experimental evidence for a second <u>56</u>, L=2 – in any case, as will become apparent below, such a multiplet could not produce the desired effect. Thirdly, <u>56</u>, L=4 contains an F_{35} but its $H_{3,11}$ partner observed [4] at 2390 MeV makes it exceedingly unlikely that there could be any significant mixing with the $F_{35}(1877)$. Fourthly, <u>70</u>, L=2 is the simplest home for the $F_{17}(2024)$. In the fifth place the only remaining multiplets that could supply an F_{35} state would be <u>positive</u> parity multiplets of the kind : <u>56</u>, L=1, <u>56</u>, L = 3 or <u>70</u>, L=3 – all far more extreme assumptions in any three-quark picture. Lastly, both the harmonic oscillator quark model [21] and the unbroken duality scheme [8],[9] actually predict a <u>70</u>, L=2 in this mass region.

Thus the additional states that fall under our consideration are F_{17}, F_{15}, F_{35}, F_{15} f wave πN resonances and P_{13}, P_{11}, P_{33}, P_{13} p wave states. We consider first the f waves.

i) $F_{35}(1877)$

$F_{35}(1877)$ can now be some linear combination of <u>56</u> and <u>70</u>. To be more quantitative, we shall suppose that ℓ broken $SU(6)_W$ relates the couplings of all members of the <u>56</u>, L=2 and separately those of the <u>70</u>, L=2 states. In principle we could take the observed elastic width of $F_{17}(2024)$ to fix the f wave coupling constant for the <u>70</u>. Its p wave coupling could then be arranged so as to supply the necessary interference effects for the $F_{35}(1877)$. In practice, however, our argument becomes much more transparent if we make one more theoretical assumption. Since in the unbroken $SU(6)_W$ limit the couplings of these two multiplets would differ by an over-all constant factor, we shall assume that this still holds true in the ℓ broken limit, i.e., that the ratio of the f wave couplings for the two multiplets and that of their p wave couplings is the same.

Let R be the ratio of the <u>70</u> couplings to those of the <u>56</u> on some arbitrary scale. Then, defining the physical F_{35} by the mixture :

$$F_{35}(1877) = \underline{56}\cos\theta - \underline{70}\sin\theta \qquad (4)$$

we predict

$$\Gamma_{N\pi} = \Gamma^{56}_{N\pi} \times \left(\cos\Theta + \frac{\sqrt{7}}{4} R \sin\Theta\right)^2 \qquad (5)$$

$$\Gamma_{\Delta\pi(f)} = \Gamma^{56}_{\Delta\pi(f)} \times \left(\cos\Theta + \frac{\sqrt{7}}{4} R \sin\Theta\right)^2 \qquad (6)$$

$$\Gamma_{\Delta\pi(p)} = \Gamma^{56}_{\Delta\pi(p)} \times \left(\cos\Theta - \frac{2}{\sqrt{7}} R \sin\Theta\right)^2 \qquad (7)$$

[Note that any one of Eqs. (5)-(7) defines R.]

Thus, under our mixing assumptions, either the f waves are enhanced and the p wave suppressed or vice versa. We need to suppress the p wave. So what effect will the corresponding enhancement have on our f wave predictions ?

Recall that our f wave prediction for $F_{35}(1877) \to \Delta\pi$ was 20 MeV. Experimentally, Mehtani et al. [5] claim ~64 MeV, so this f wave prediction can certainly stand some enhancement. Moreover, the unmixed $F_{35} \to N\pi$ prediction is 19 MeV. Experimentally, CERN and Saclay claim 43 MeV so mixing improves this prediction too !

In order to say how much mixing is needed, we must know the value of R. From the elastic width of the $F_{17}(2024)$, this turns out to be $R = 0.9$. However, in view of the large error on the experimental value of this πN width, we shall set $R = 1$. This still provides an excellent fit ($\Gamma[F_{17} \to N\pi]$ = = 24 MeV, cf., 20±11 [3),4)]) but has the virtue of making our equations much tidier. $R = 1$ also has some important theoretical significance which we shall return to in the concluding section.

With the scale of the 70 decay rates thus fixed relative to those of the 56 we can answer the F_{35} mixing angle question. Figure 2 displays the f wave width enhancement and p wave width suppression factors as a function of the mixing angle Θ. From it we can see that the f wave rates achieve their maximum possible enhancement of ~1.4 anywhere in the range $\Theta = 25^\circ$ to 40°. However, in the same range the p wave rate is falling very rapidly in order to vanish at $\Theta = \arctan(\sqrt{7}/2)$. It is thus clear that some value close to $\Theta = 35^\circ$ provides a satisfactory mixing angle.

But what of the other F_{35} state ? Mixing implies the existence of the orthogonal combination

$$F'_{35} = \underline{56}\ \sin\theta + \underline{70}\ \cos\theta \qquad (8)$$

However by making the replacements $\sin\theta \to -\cos\theta$ and $\cos\theta \to \sin\theta$ in Eqs. (5) to (7), it is clear that this state has its p wave decay enhanced at the expense of the f waves. In particular, if we suppose this resonance to have the mixing parameters (8), a mixing angle of $\theta = \arctan(\sqrt{7}/4) \simeq 34^\circ$ implies a vanishing πN partial width. This resonance is consequently expected to be far too inelastic to be visible.

ii) $F_{15}(1684)$

At this stage the alert reader will be worried that our p wave coupling constant, which we have hitherto fixed from the $\Delta\pi$ decay rate of $F_{15}(1684)$, might be seriously wrong on account of possible mixing between the F_{15} member of the $\underline{56}$, L=2 and the two F_{15} states in the $\underline{70}$, L=2. The f wave coupling constant need cause no worry since it has been fixed with sole reference to $F_{37}(1923)$ and $F_{17}(2024)$ - the two states that have no partners with which mixing could occur. But since we must now admit the possibility of F_{15} mixing, the p wave situation must be re-examined.

We therefore define the mixture :

$$F_{15}(1684) = N_1\,[\underline{70}(8,2)] + N_2\,[\underline{70}(8,4)] + N_3\,[\underline{56}(8,2)] \qquad (9)$$

where the mixing parameters N_i, $i = 1,2,3$ obey the constraint $N_1^2+N_2^2+N_3^2 = 1$. Thus far we have assumed $N_3 = 1$, $N_1 = N_2 = 0$. Suppose, however, that this is no longer the case, and that we treat N_1 and N_2 as independent mixing parameters. For any pair of values in the range $-1 \leq N_{1,2} \leq +1$ but such that $N_1^2+N_2^2 \leq 1$, the unmixed $\underline{56}$ decay width predictions will now be increased or decreased by some factor. In Figs. 3a and 3b we display contours of this modification factor at 30% intervals for the $N\pi$ width and the p wave $\Delta\pi$ width, respectively. The points at the origin (pure $\underline{56}$) are normalized to one. Two possibilities are important.

Consider first Fig. 3a. Recalling that the unmixed $F_{15} \to N\pi$ prediction is already some 30% too low when compared with experiment, we see that mixing could not reasonably take us outside the 70% contour if theory and experiment are still to agree to within a factor of two - the maximum occasional violation that we should be prepared to accept based on previous ℓ broken $SU(6)_W$ studies. If the reader cares to trace the 70% contour from Fig. 3a and place it over Fig. 3b, he will find a very small area of overlap with the 170% contour for the $F_{15} \to \Delta\pi_p$ rate. This indicates that the worst overestimate we might have made in the p wave coupling (squared) is a factor of 1.7. If the $F_{35}(1877)$ unmixed p wave prediction is now scaled down by this factor, it is found to be comparable with the unmixed f wave $\Delta\pi$ prediction. It is thus just possible that the f wave prediction might be a factor of two too small, and the p wave too large by the same amount. In this extreme case the ratio $\Gamma^{(p)}_{\Delta\pi} / \Gamma^{(f)}_{\Delta\pi}$ would be just about compatible with the experimental limit of Mehtani et al. [5], and mixing would not be needed for the F_{35}. But we would have invoked a far more complicated mixing situation for the F_{15} and this only to achieve a poor fit.

The second possibility is that mixing (with $N_1 > 0$, $N_2 > 0$) could improve the $F_{15} \to N\pi$ prediction slightly (see Fig. 3a). This would imply that our estimate of the p wave coupling constant was in fact too small (see Fig. 3b), thus emphasizing once more the need for F_{35} mixing.

The amplitude $\pi N(\ell=3) \to F_{15} \to \pi\Delta(\ell=1)$ is directly measurable and its sign can be compared with that of $\pi N(\ell=2) \to D_{15} \to \pi\Delta(\ell=2)$ [6]. This comparison allows one to determine whether the sign of the product $G_f G_p$ is that of unbroken $SU(6)_W$ or the opposite [12]. The question was raised in Ref. 12) of the possibility that the F_{15} might be mixed. We can see from Fig. 3c that the sign of the amplitude $\pi N(\ell=3) \to F_{15}(1684) \to \pi\Delta(\ell=1)$ remains that of the unmixed case, as long as this state remains at least 50% <u>56</u>, i.e., as long as $N_1^2 = 1 - N_2^2 - N_3^2 \gtrsim \frac{1}{2}$. Hence we expect the test suggested in Ref. 12) to be insensitive to all but rather drastic mixing of the $F_{15}(1684)$, a circumstance we have shown is unlikely.

In brief, we must conclude that the experimental data available at present yield no evidence for any substantial $F_{15}(1684)$ mixing.

V. PARTIAL WIDTH PREDICTIONS

Having fixed the constants for f wave and p wave decays of all the L=2 baryons, we are in a position to predict a number of partial widths. These are presented in the Table for unmixed <u>56</u> and <u>70</u> decays. Aside from phase space effects, we expect the <u>sum</u> of predicted partial widths (into a given channel) for any set of states which may mix to be roughly invariant under mixing. That is to say, if mixing causes one state to have an enhanced partial width into some final state, this will be at the expense of its mixing partners. The specific phases associated with decay amplitudes for each state may be constructed using the method outlined in Section II.

We shall discuss some interesting features of these tables; the reader is invited to find others.

a) Importance of $\Sigma K/N\pi$ ratio, $F_{37}(1923)$

Our F_{35} mixing argument has been quantitatively built up assuming an f wave interaction radius of 0 rather than 1 fermi. Our sole justification for this assumption is the experimentally reported [17] ΣK branching ratio of $F_{37}(1923)$. Were future experiments to indicate an increased branching ratio by a factor of 5 or so, the $r=0$ assumption would have to be abandoned. However, it is important to realize that this would not alter our conclusion that there has to be F_{35} mixing. Our tables list the unmixed $r=1$ fermi predictions (based now on the $F_{37}\rightarrow N\pi$ rate only) so that the reader can, with the help of Figs. 2-3, verify this statement by reconstructing the appropriate Section IV argument for such an interaction radius.

b) Invariant nature of $N\rho/\Sigma K$ ratio, $F_{37}(1923)$

The phase space volumes for $F_{37}(1923)\rightarrow N\rho$ and ΣK are almost the same. The ratio $C^2(N\rho)/C^2(\Sigma K) = 4/3$ should thus be reflected in observed partial widths, whatever the radius of interaction r. A preliminary examination of the data of Ref. 6) shows that $\Gamma(N\rho) \gg \frac{4}{3}\Gamma(\Sigma K)$. If this is borne out by the final analysis, $\not{\!}$ broken $SU(6)_W$ will be in considerable trouble. See also a) above.

c) $\Sigma^* K$ bump around 1900 MeV

It has been noted previously [9] that the predicted f wave decay $F_{37}(1923)\rightarrow \Sigma^*(1385)K$ is very small. Such a prediction depends only on SU(3) and $F_{37}\rightarrow\Delta\pi$. On the other hand, the reaction $\pi^+p\rightarrow\Sigma(1385)^+K^+$ definitely shows an enhancement around 1900 MeV [22]. A rudimentary partial wave analysis [22] is compatible with the hypothesis that this bump is due to $F_{37}(1923)$, but also with many other possibilities. We have presented some other predictions of $\Sigma^*(1385)K$ partial widths for Δ states around 1900 MeV. We would regard $P_{33}(1890)$ and $P_{31}(1900)$ as rather more likely sources of the bump; $F_{35}(1877)$ would be less likely if its p wave $\Delta\pi$ decay is really inhibited by mixing as we suggest.

In any event, a partial wave analysis of $\pi^+ p \to \Sigma(1385)^+ K^+$ in the region from threshold ($p_L \cong 1.3$ GeV/c) to above the region of the prominent Δ resonances ($p_L \simeq 1.9$ GeV/c) is greatly needed. Not only would such an analysis provide a useful test of SU(3) when combined with Refs. 5) and 6), but it would be free from the problem of meson resonance production and would have a cleaner isobar signal than $\pi^+ p \to \pi^0 \Delta^{++}$ (where the Δ is relatively broad). (Some background due to $\pi^+ p \to \pi^+ N^{*+} \to \pi^+ \Lambda K^+$ would be present.)

d) Magnitude of $N\rho$ decay of $F_{35}(1877)$

Mixing (see Section IV) multiplies the predicted $N\rho$ width of $F_{35}(1877)$ by $(\cos\theta + 1/\sqrt{7}\,\sin\theta)^2$. Although for $\theta = 35^\circ$ this represents constructive interference between the 56 and 70, the net result is only a 7% enhancement of the predicted value. The Argand circle of Ref. 6), on the other hand, indicates a p wave $N\rho$ width many times that predicted. If borne out, this fact would indicate trouble for ℓ broken $SU(6)_W$.

e) Cautionary note regarding P_{33}, P_{31}, P_{11} and P_{13} predictions

Three observed πN resonances have been assigned to 56, L=2 in the Table, and indeed from their measured properties such an assignment is clearly most reasonable. However, it is difficult to assess what effect mixing might have on the predictions for their as-yet unobserved decay channels. The reason for this is that all of these resonances could in principle mix with the L=0 states also predicted by the harmonic oscillator quark model to be in this mass region [21], and for which there are already some experimental indications [2),23)]. Although it would require tensor forces to bring about this mixing, such effects have indeed been suggested in related circumstances [24)] and thus remain a distinct possibility. Better agreement on the properties of the p wave resonances is an important goal for future phase shift analyses, and one that will enable these questions to be settled. In the meantime the predicted properties of $P_{13}(1850)$, $P_{31}(1900)$ and $P_{33}(1890)$ should be regarded as less firm than those of their higher spin partners.

f) Predicted large f wave $\Delta\pi$ mode of $P_{13}(1850)$

This result depends on the assignment shown. The 70(8,2), $L^P = 2^+$ member also has both a large $N\pi$ and a large f wave $\Delta\pi$ width. The phases are such that mixing could suppress the f wave $\Delta\pi$ mode of one state and the $N\pi$ mode of the other. Hence the unmixed prediction cannot be regarded as firm. Ref. 6) sees no evidence for $P_{13}(1850) \to \Delta\pi$.

g) <u>Inelastic decay modes of $F_{17}(2024)$</u>

This resonance is predicted to have appreciable ΣK and $\Delta\pi$ partial widths [9]. The latter should be looked for by extending the analysis of Ref. 6) to slightly higher energies.

h) <u>States of $\underline{70}$, $L^P=2^+$ with large elastic widths</u>

Let us list in turn any $\underline{70}$, $L^P=2^+$ $N\pi$ resonance whose elastic width is predicted to exceed 15 MeV (for $r=0$). There are not many of them !

1) $F_{17}(2024)$: probably observed (see above);
2) $F_{35}(2000)$: mixing with $F_{35}(1877)$ likely (see above);
3) $F_{15}(2000)$: possibly observed in both Refs. 3) and 4);
4) $P_{13}(\underline{8},2)(2000)$: may mix with $P_{13}(1850)$ (see above).

The effects of mixing can easily lead to non-observance of the second state, as illustrated above for the F_{35} case. We are thus in a situation in which elastic πN phase shift analyses may have told us nearly all we can learn about the existence of the $\underline{70}$, $L^P=2^+$ multiplet. To learn more about the non-strange (N and Δ) states, it is thus imperative that one compares these elastic analyses with such reactions as

$$\begin{array}{lll} \pi^+ p & \rightarrow & \pi^0 \Delta^{++} \\ & \rightarrow & \rho^+ p \\ & \rightarrow & K^+ \Sigma^+ \\ & \rightarrow & K^+ \Sigma^{*+} \\ \pi^- p & \rightarrow & \pi^+ \Delta^- \\ & \rightarrow & K^+ \Sigma^- \\ & \rightarrow & K^0 \Lambda \\ & \rightarrow & \eta n \\ & \rightarrow & \rho N \\ & \rightarrow & \omega n \end{array}$$

We would anticipate that the resonances obtained in analyses of these reactions may not always correspond to the set listed in Table 1a. The analysis of the $\Delta\pi$ system has been an important step in this respect.

i) Apology to hyperons

In general the mixing problems encountered for N and Δ states become more severe when we turn to Λ and Σ resonances. (We do not discuss Ξ's and Ω's, which cannot be made in the direct channel.) The only hyperon whose existence would be clear-cut evidence for a 70, L=2 has been reported in at least one experiment : it is the $\Lambda(2020)$; $J^P = \frac{7}{2}^+$ [25]. It is predicted to decouple from $N\bar{K}$ and to have large $\Sigma\pi$ and $\Sigma^*\pi$ modes [9),26]. The phase shift analysis of $K^-p \rightarrow \Sigma^{*\pm}\pi^{\mp}$ would shed some light on such a state.

If the data on hyperon resonances were to improve by an order of magnitude in the region up to ~1800 MeV, one could begin at least to sort out the structure of the negative parity states, usually assigned to 70, $L^P=1^-$. A full discussion of all the predicted positive parity states seems prohibitive at present, but theoretical predictions of mixing and partial widths exist for the negative parity states [13], and these could be usefully tested.

VI. CONCLUSION

We have shown in this paper that large intermultiplet mixing is necessary if $SU(6)_W$ in its ℓ broken form is to remain a viable symmetry for the positive parity baryon resonances. Such a mixing scheme, which does after all occur among meson resonances [27], is in fact the only reasonable alternative short of abandoning the whole $SU(6)_W$ approach. This conclusion depends crucially on some recent experimental results [5] concerning the decay modes of the $F_{35}(1877)$ πN resonance. For this particle the relative contributions of p wave and f wave to its $\Delta\pi$ decay mode were found to disagree with a pure 56, L=2 assignment. To resolve this contradiction, we have proposed in this paper that, as predicted by duality [8),9] and by the harmonic oscillator quark model [21] and as suggested by experiment, there exists a 70, L=2 multiplet of $SU(6)_W \times O(3)$, not too far removed from the 56, L=2 and that, in at least one case, intermultiplet mixing does occur. This mixing then leads to a qualitative understanding of all the known partial decay widths of the F_{35} resonance. Our proposal can be further tested by a partial wave analysis of $\pi^+p \rightarrow \Sigma(1385)^+K^+$, where we predict that the bump observed near 1900 MeV [22] should be dominated by P_{31} and/or P_{33} resonances and not by F_{35} or F_{37}. Other testable predictions of this mixing scheme have been discussed in the previous section.

The best-known member of the 56, L=2 multiplet is perhaps the Regge recurrence of the nucleon, $F_{15}(1684)$. We have shown that this state is unlikely to be very strongly mixed with 70, L=2 members. Within the limits of allowable mixing parameters, we find that its contribution to the amplitude for $\pi N \to \pi\Delta$ is of the same sign as in the unmixed case. This conclusion means that a test suggested previously [12] for distinguishing between $SU(6)_W$ and its ℓ broken version is not likely to be affected by mixing between 56, L=2 and 70, L=2.

It is probable that little further knowledge about the 70, L=2 multiplet per se (apart from the confirmation of at least one more prominent F_{15} resonance as predicted here) is to be gained by pursuing elastic πN phase shift analyses. Not only are many of the predicted partial widths of the unseen states small, but the p wave πN resonance properties may be affected by 56 and 70, L=0 multiplets (for which there is some scanty experimental evidence). Such analyses should nevertheless not be abandoned before the present rather disturbing lack of agreement regarding p wave resonances is sorted out. It is, however, the inelastic channels : $\pi N \to K\Lambda$, $K\Sigma$, ηN, $\pi\Delta$, $K\Sigma^*$, ρN, ωN, ..., which, if studied in detail in the mass range up to 2.2 GeV, will shed most light on the properties of these L=0 and L=2 resonances.

It is remarkable that not only is a 70, L=2 multiplet predicted at this mass by the harmonic oscillator quark model, but also the ratio of the couplings for [56, L=2 → ground state] to [70, L=2 → ground state] decays is found to lie very close to the value predicted by this model [2]. If this success continues for the 56, L=0 and 70, L=0 multiplets - and there are indications (not reported here) that this is the case - it would be extremely interesting to use this naïve model to reconstruct the pure resonant part of such scattering amplitudes as $\pi N \to \pi N$, $\pi\Delta$, ρN, etc. By imposing on these amplitudes the self-consistency conditions based on a semi-local dual description of such processes [28], one might be led to a better understanding of their direct and crossed channel helicity structure.

The one remaining multiplet predicted by the harmonic oscillator quark model to lie in the mass region around 2 GeV is the 20, $L^P = 1^+$. Since its members have no coupling to $56 \otimes 35$ it is no surprise that there are no experimental candidates for this multiplet. Such states could, however, exist as $70 \otimes 35$ resonances and might be observable in a suitably designed production experiment [29].

Finally, the mere existence of a $\underline{70}$, L=2 multiplet of resonances as close in mass to the $\underline{56}$, L=2 as this one seems to be has profound implications from the standpoint of duality. It means indeed that $B\bar{B} \to MM'$ channels are not significantly less reliable than those of $MM' \to M''M'''$ and $MB \to M'B'$ from which the well-known duality successes derive [8),9)]. This considerably strengthens the theoretical arguments [10)] leading to the prediction of exotic mesons.

Table

a) Unmixed 56, $L^P = 2^+$ predictions

State	Mode	$C_\ell^2(\underline{56})$		Γ_{pred} (MeV)				Γ_{expt} a) (MeV)
				p-wave		f-wave		
		p-wave	f-wave	r = 0	r = 1f	r = 0	r = 1f	
$F_{37}(1923)$	$N\pi$	-	$\frac{4}{525}$	-	-	112 b)	86 c)	86
	ΣK	-	$\frac{4}{525}$	-	-	2.8 b)	14	3.6 d), 0.86 o)
	$\Delta\pi$	-	$\frac{1}{70}$	-	-	31	74	e)
	$\Sigma^* K$	-	$\frac{1}{175}$	-	-	0.004	0.06	f)
	$N\rho$	-	$\frac{16}{1575}$	-	-	4.9	22	large g)
$F_{35}(1877)$	$N\pi$	-	$\frac{8}{4725}$	-	-	19	18	42
	ΣK	-	$\frac{8}{4725}$	-	-	0.28	1.8	5.7 o)
	$\Delta\pi$	$\frac{7}{450}$	$\frac{64}{4725}$	49	28	20	57	∿ 64 (f-wave) p) < 16 (p-wave) p)
	$\Sigma^* K$	$\frac{7}{1125}$	$\frac{128}{23625}$	0.23 q)	0.76 q)	0.0003 q)	0.003 q)	
	$N\rho$	$\frac{28}{3375}$	$\frac{184}{23625}$	11	10	1.6	9.6	large p-wave h)
$F_{33}(1890)$	$N\pi$	$\frac{4}{675}$	-	46	15	-	-	22 i)
	ΣK	$\frac{4}{675}$	-	8.2	7.3	-	-	4.9 o)
	$\Delta\pi$	$\frac{8}{675}$	$\frac{1}{150}$	39	22	11	30	unseen j)
	$\Sigma^* K$	$\frac{16}{3375}$	$\frac{1}{375}$	0.18 q)	0.58 q)	0.0001 q)	0.002 q)	
	$N\rho$	$\frac{28}{3375}$	$\frac{4}{1125}$	13	11	0.94	5.2	unseen j)
$P_{31}(1900)$	$N\pi$	$\frac{8}{675}$	-	95	31	-	-	66 k)
	ΣK	$\frac{8}{675}$	-	18	15	-	-	7.3 o)
	$\Delta\pi$	$\frac{1}{270}$	-	13	6.8	-	-	not seen r) possible n)
	$\Sigma^* K$	$\frac{1}{675}$	-	0.06	0.18	-	-	
	$N\rho$	$\frac{4}{675}$	-	9.7	7.8	-	-	possible n)
$F_{15}(1684)$	$N\pi$	-	$\frac{1}{54}$	-	-	58	121	85
	ΛK	-	$\frac{1}{150}$	-	-	0.03	0.39	< 0.28 s)
	$N\eta$	-	$\frac{1}{1350}$	-	-	0.15	0.91	< 0.7 s)
	$\Delta\pi$	$\frac{16}{1125}$	$\frac{32}{3375}$	18 c)	18 c)	1.4	9.5	18 l) (p-wave)
	$N\rho$	$\frac{64}{3375}$	$\frac{167}{6750}$	(not estimated-below threshold)				large p-wave r)
$P_{13}(1850)$	$N\pi$	$\frac{1}{54}$	-	132	46	-	-	75 m)
	ΛK	$\frac{1}{150}$	-	12	9.5	-	-	∿ 15 s)
	$N\eta$	$\frac{1}{1350}$	-	2.6	1.4	-	-	∿ 12 s)
	$\Delta\pi$	$\frac{8}{3375}$	$\frac{8}{375}$	6.7	4.1	24	79	unseen n)
	$N\rho$	$\frac{103}{6750}$	$\frac{32}{1125}$	16	17	3.2	22	large p-wave n)

Table

b) Unmixed 70, $L^P = 2^+$ predictions

State [t]	Mode	$C_\ell^2(\underline{70})$		Γ_{pred} (MeV) [u]				Γ_{expt} [a] (MeV)
				p-wave		f-wave		
		p-wave	f-wave	r = 0	r = 1f	r = 0	r = 1f	
$F_{17}(2024)$ (8,4)	Nπ	-	$\frac{1}{560}$	-	-	24	12	20
	ΣK	-	$\frac{1}{140}$	-	-	5.5	15	7 [o]
	Δπ	-	$\frac{3}{340}$	-	-	56	83	unseen [w]
	Nρ	-	$\frac{1}{420}$	-	-	2.4	5.9	unseen [w]
$F_{15}[2000]$ (8,4)	Nπ	-	$\frac{1}{2520}$	-	-	4.7	2.7	
	Δπ	$\frac{7}{300}$	$\frac{32}{1575}$	59	25	45	73	
	Nρ	$\frac{7}{3600}$	$\frac{23}{12600}$	3.1	1.8	1.4	4.0	
$P_{13}[2000]$ (8,4)	Nπ	$\frac{1}{720}$	-	7.2	2.0	-	-	
	Δπ	$\frac{4}{225}$	$\frac{1}{100}$	45	19	22	36	
	Nρ	$\frac{7}{3600}$	$\frac{1}{1200}$	3.1	1.8	0.63	1.8	
$P_{11}[2000]$ (8,4)	Nπ	$\frac{1}{360}$	-	14	4.0	-	-	
	Δπ	$\frac{1}{180}$	-	14	6.0	-	-	
	Nρ	$\frac{1}{720}$	-	2.2	1.3	-	-	
$F_{35}[2000]$ (10,2)	Nπ	-	$\frac{1}{720}$	-	-	16	9.3	
	Δπ	$\frac{1}{60}$	$\frac{1}{90}$	42	18	24	40	
	Nρ	$\frac{1}{450}$	$\frac{47}{3600}$	3.6	2.0	9.9	28	
$P_{33}[2000]$ (10,2)	Nπ	$\frac{1}{720}$	-	7.2	2.0	-	-	
	Δπ	$\frac{1}{360}$	$\frac{1}{40}$	7.0	3.0	55	90	
	Nρ	$\frac{43}{3600}$	$\frac{1}{300}$	19	11	2.5	7.2	
$F_{15}[2000]$ (8,2)	Nπ	-	$\frac{1}{90}$	-	-	131	74	possible [v]
	Δπ	$\frac{1}{75}$	$\frac{2}{225}$	34	15	20	32	
	Nρ	$\frac{1}{36}$	$\frac{7}{360}$	45	25	15	42	
$P_{13}[2000]$ (8,2)	Nπ	$\frac{1}{90}$	-	58	16	-	-	
	Δπ	$\frac{1}{450}$	$\frac{1}{50}$	5.6	2.4	44	72	
	Nρ	$\frac{1}{180}$	$\frac{1}{24}$	8.9	5.0	32	90	

FOOTNOTES TO THE TABLE

a) All masses, total widths and elasticities based on Refs. 3) and 4). Inelastic widths derived using these values together with inelastic amplitudes from Refs. cited.

b) Based on best fit to $F_{37} \rightarrow N\pi$, $\Sigma^* K$ as explained in the text.

c) Input value.

d) Amplitude taken from Ref. 17).

e) The amplitude in Ref. 5) implies $\Gamma_{\Delta\pi} \approx 100$ MeV, whereas that in 6) suggests a much smaller partial width : probably less than 35 MeV.

f) Ref. 30) implies a partial width of ~2.7 MeV. See text.

g) From the Argand circle of Ref. 6), this number could be as large as ~30 MeV.

h) Visual estimates based on Ref. 6) indicate that this number might be many times larger than the theoretical prediction. See text.

i) Ref. 4). Not seen in Ref. 3).

j) The Argand circle of Ref. 6) might be consistent with a $P_{33}(1890) \rightarrow \Delta\pi$ superposed on a large (possibly non-resonant) background peaking at a lower mass. No $N\rho$ signal seen.

k) Ref. 3). The P_{31} resonance in Ref. 4) is quoted at 1797 MeV.

l) Ref. 20).

m) Ref. 3). The P_{13} resonance in Ref. 4) is quoted at 1691 MeV.

n) Ref. 6).

o) Amplitude taken from Ref. 18).

p) Amplitude taken from Ref. 5).

q) For $\Sigma^* K$ predictions this resonance was given a mass of 1900 MeV.

r) Ref. 5).

s) Branching ratio taken from Ref. 30).

t) Masses of all unseen states set at 2000 MeV (in square brackets) for purposes of phase space calculations. Numbers in parentheses below states indicate SU(3) × SU(2) classification.

u) Numerical predictions for $\underline{70}$, $L^P = 2^+$ states based on setting $G_p(\underline{70}) = \sqrt{8/15}\, G_p(\underline{56})$ and $G_f(\underline{70}) = \sqrt{8/15}\, G_f(\underline{56})$. This is the harmonic oscillator quark model prediction and corresponds to our choice of scale factor $R = 1$. See text.

v) Refs. 3) and 4).

w) This state lies at the end of the energy range covered by Ref. 6).

REFERENCES

1) A. Donnachie, R. Kirsopp and C. Lovelace, Phys.Letters 26B, 161 (1968).

2) D. Faiman and A. Hendry, Phys.Rev. 173, 1720 (1968).

3) S. Almehed and C. Lovelace, Nuclear Phys. B40, 157 (1972).

4) R. Ayed, P. Bareyre and Y. Lemoigne, 'Zero strangeness baryon states below 2.5 GeV mass', CEN-Saclay preprint (1972).

5) U. Mehtani, S. Fung, A. Kernan, T. Schalk, Y. Williamson, R. Birge, G. Kalmus and W. Michael, Phys.Rev.Letters 29, 1634 (1973).

6) D. Herndon, R. Longacre, L. Miller, A. Rosenfeld, G. Smadja, P. Soding, R. Cashmore and D. Leith, 'A partial wave analysis of the reaction $\pi N \rightarrow \pi\pi N$ in the c.m. energy range 1300-2000 MeV', LBL report 1065 (1972).

7) R. Dalitz in Proceedings of the Second Topical Conference in Particle Physics, p.325, edited by S. Pakvasa and S. Tuan, University of Hawaii Press, Honolulu (1967).

8) J. Mandula, C. Rebbi, R. Slansky, J. Weyers and G. Zweig, Phys.Rev. Letters 23, 266 (1969); and Ann.Rev.Nuclear Sci. 20, 289 (1970).

9) J. Rosner, Phys.Rev. D7, 172 (1973).

10) J. Rosner, Phys.Rev.Letters 21, 950 (1968); 1422(E).

11) E. Colglazier and J. Rosner, Nuclear Phys. B27, 349 (1971). This paper contains also a large bibliography of work using unbroken $SU(6)_W$.

12) W. Petersen and J. Rosner, Phys.Rev. D6, 820 (1972); and to appear in Phys.Rev. D7 (1973).

13) D. Faiman and D. Plane, Nuclear Phys. B50, 379 (1972).

14) W. Petersen, Ph.D. Thesis, University of Minnesota (1973) - unpublished.

15) J. Rosner in Proceedings of the XVI International Conference on High Energy Physics, Batavia, Ill. (1972) - to be published.

16) J. Blatt and V.F. Weisskopf, Theoretical Nuclear Physics (Wiley, N.Y. 1952), p.390. See also F. von Hippel and C. Quigg, Phys.Rev. D5, 624 (1972).

17) G. Kalmus, G. Borreani and J. Louie, Phys.Rev. D2, 1824 (1970).

18) W. Langbein and F. Wagner, 'Phase shift analysis of $\pi N \rightarrow K\Sigma$' - to be published.

19) P. Freund, A. Maheshwari and E. Schonberg, Phys.Rev. 159, 1232 (1967).

20) A. Brody, R. Cashmore, A. Kernan, D. Leith, B. Levi, B. Shen, D. Herndon, L. Price, A. Rosenfeld and P. Soding, Phys.Letters 34B, 665 (1971).

21) O. Greenberg, Phys.Rev.Letters 13, 598 (1964).

22) W. Chinowsky, P. Condon, R. Kinsey, S. Klein, M. Mandelkern, P. Schmidt, J. Schultz, F. Martin, M. Perl and T. Tan, Phys.Rev. 171, 1421 (1968).

23) C. Heusch and F. Ravndal, Phys.Rev.Letters 25, 253 (1970).

24) H. Lipkin, Phys.Letters 35B, 534 (1971).

25) A. Barbaro-Galtieri in "Hyperon Resonances - 70", edited by E. Fowler, p.161 (Moore, Durham, N.Carolina).

26) D. Faiman, Nuclear Phys. B32, 573 (1971).

27) H. Harari, D. Horn, M. Kugler, H. Lipkin and S. Meshkov, Phys.Rev. 146, 1052 (1966).

28) Y. Gell, D. Horn, M. Jacob and J. Weyers, Nuclear Phys. B33, 379 (1971).

29) D. Faiman, Nuovo Cimento Letters, Series I 4, 1231 (1970).

30) Particle Data Group, Phys.Letters 39B, 1 (1972).

FIGURE CAPTIONS

Figure 1 : Argand diagram and speed plot for the πN F_{17} phase shift, after Ref. 3) with apologies to the authors.

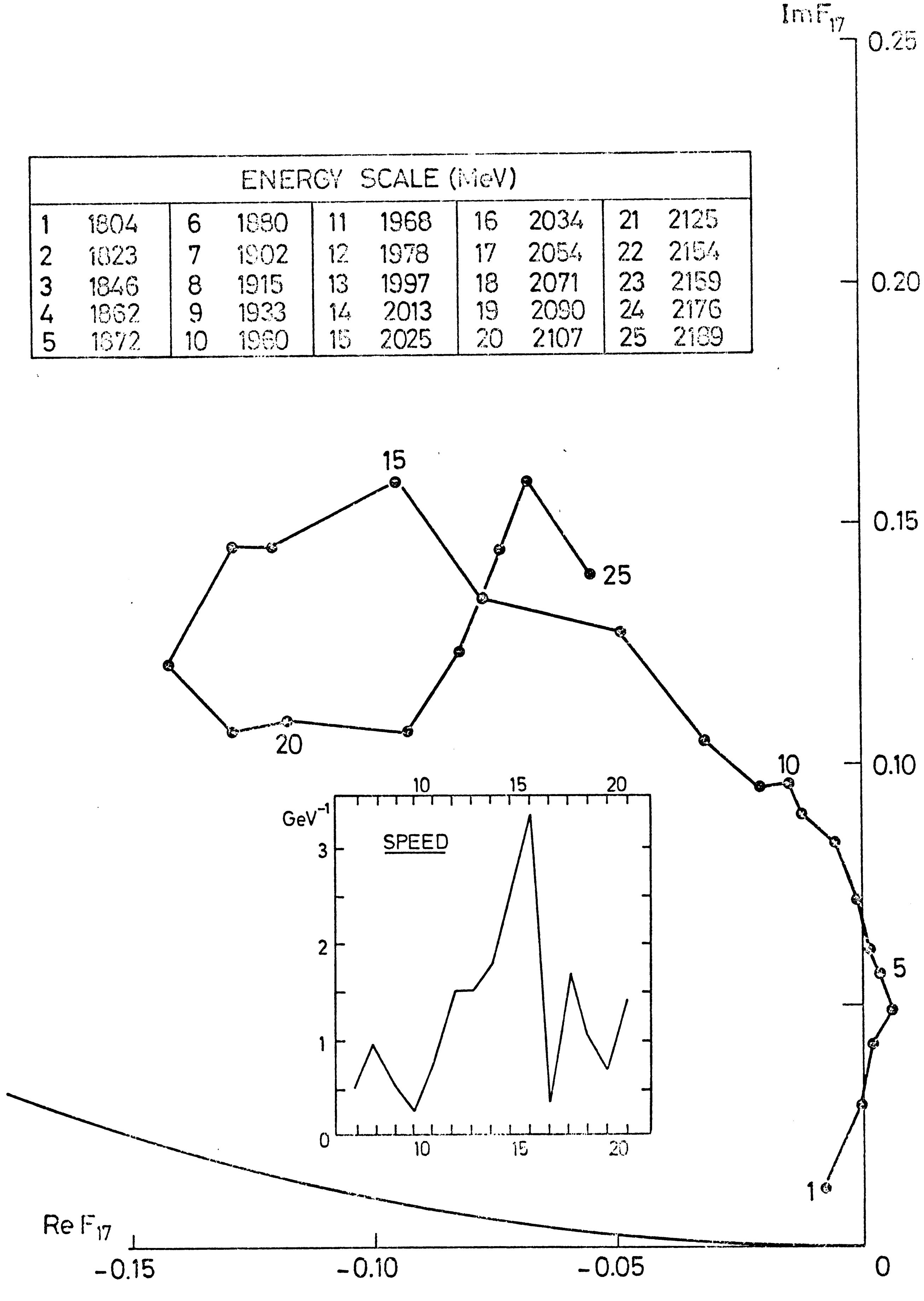
Figure 2 : f wave enhancement and p wave suppression factors as a function of the mixing angle for $F_{35}(1877)$. The scale of the 70 contribution relative to the 56 is based on the harmonic oscillator quark model.

Figure 3 : Contour plots of partial width enhancement or suppression factors for a 56, L=2 F_{15} πN resonance allowed to mix with the (8,2) and (8,4) members of a 70, L=2. The scale of the 70 contribution relative to the 56 is based on the harmonic oscillator quark model. N_1 and N_2 are the respective direction cosines of the F_{15} with respect to the 70(8,2) and 70(8,4) bases.

(a) $F_{15} \rightarrow N\pi$;

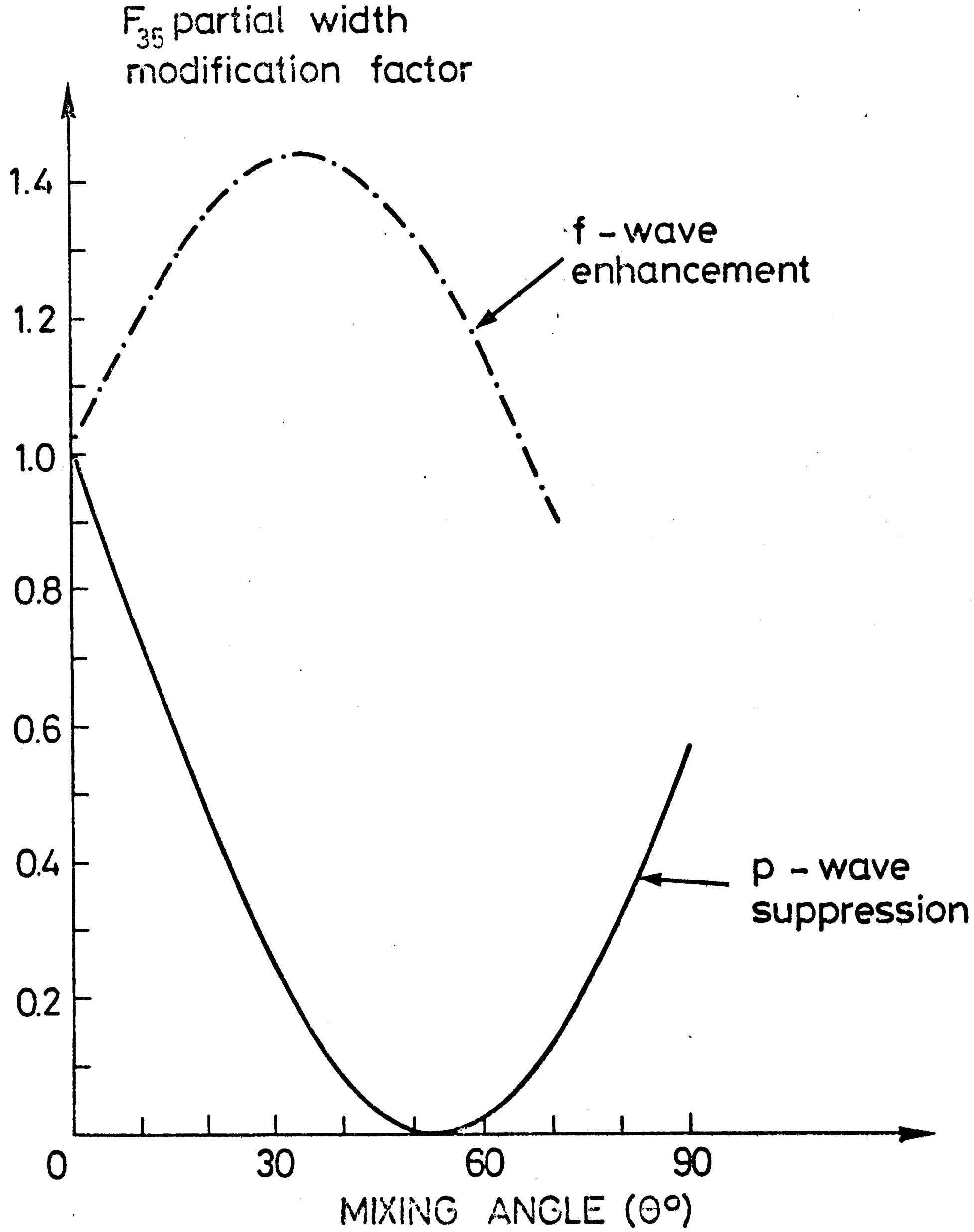
(b) $F_{15} \rightarrow \Delta\pi$ (p wave).

The shaded area on Fig. 3c represents mixing configurations that would cause the $(\ell=3)\,\pi N \rightarrow F_{15} \rightarrow \pi\Delta(\ell=1)$ amplitude to have the opposite sign from the pure 56, L=2 case.



ENERGY SCALE (MeV)									
1	1804	6	1880	11	1968	16	2034	21	2125
2	1823	7	1902	12	1978	17	2054	22	2154
3	1846	8	1915	13	1997	18	2071	23	2159
4	1862	9	1933	14	2013	19	2090	24	2176
5	1872	10	1960	15	2025	20	2107	25	2189

FIG. 1



F_{35} partial width
modification factor
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
30
60
90
MIXING ANGLE (Θ°)
f - wave
enhancement
p - wave
suppression

FIG. 2

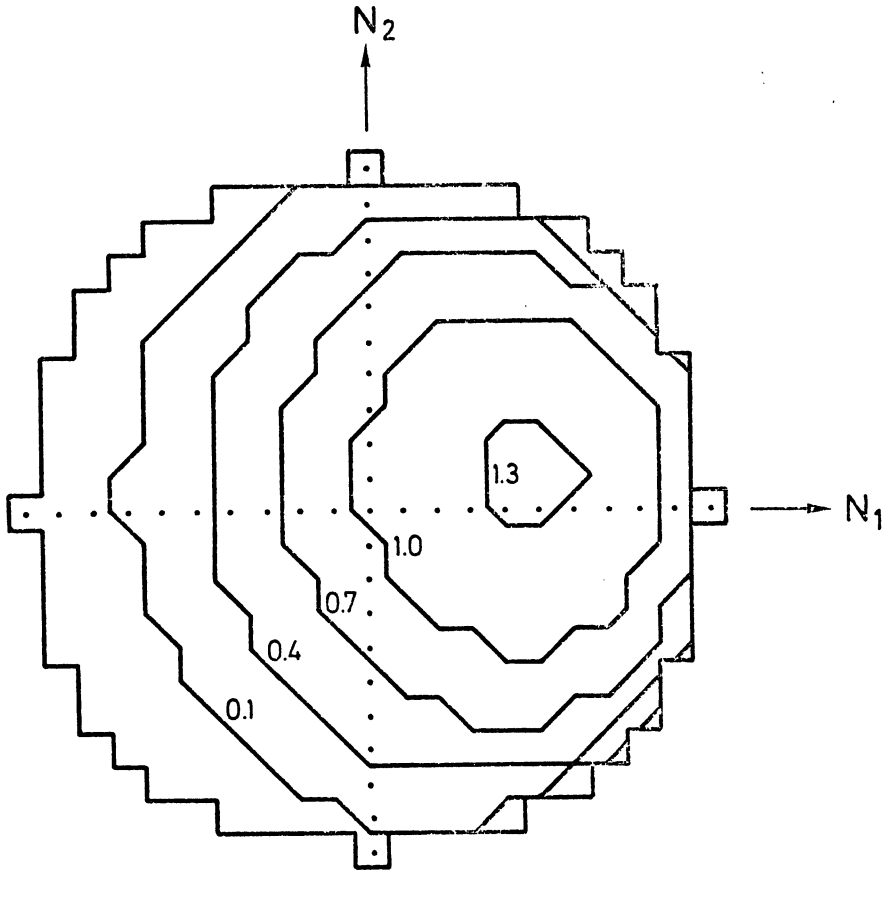


FIG.3a

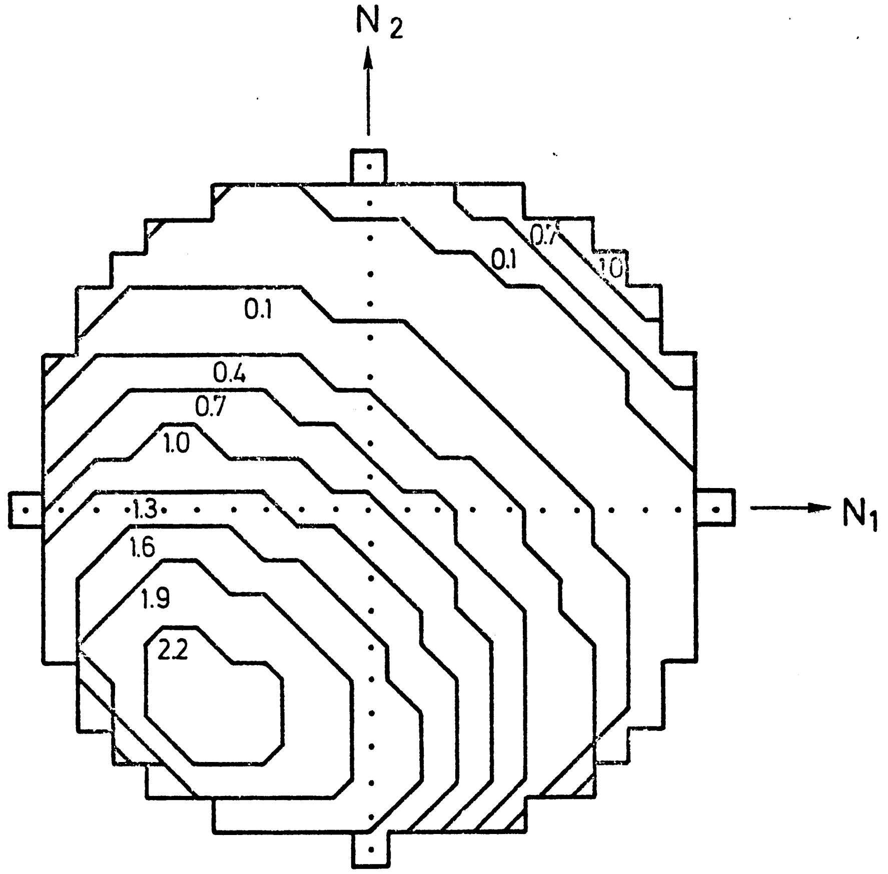


FIG. 3b

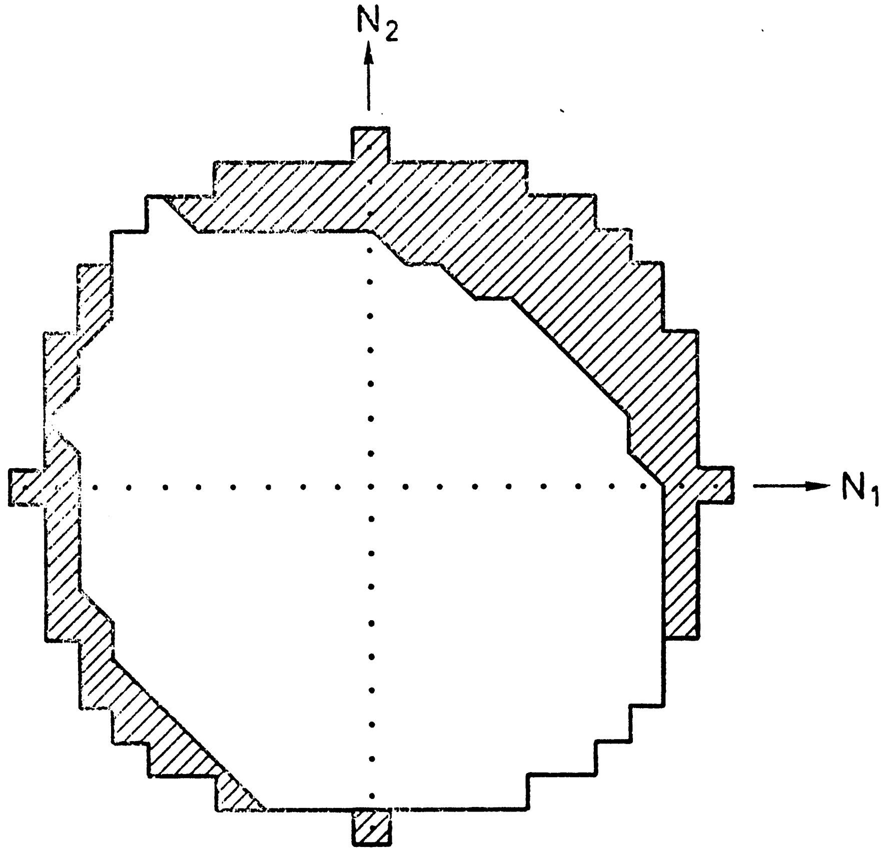


FIG. 3c