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We solve analytically for the $\pi^- p \to \pi^- \pi^+ n$ amplitudes in the ρ region at 17.2 GeV/c and find two acceptable solutions. We discuss the extrapolation of s and t channel amplitudes to the π pole and conclude that the former should be used for $\pi\pi$ phase shift analyses.

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ABSTRACT

We solve analytically for the $\pi^- p \rightarrow \pi^- \pi^+ n$ amplitudes in the ρ region at 17.2 GeV/c and find two acceptable solutions. We discuss the extrapolation of s and t channel amplitudes to the π pole and conclude that the former should be used for $\pi\pi$ phase shift analyses.

It has been noted [1] that an amplitude analysis of the reaction $\pi^- p \rightarrow \pi^- \pi^+ n$ (with S and P wave dipion production dominant) is possible under the assumption that exchanges with the quantum numbers of the A_1 can be neglected. Here we wish to point out that a twofold ambiguity is inherent in such an analysis. To see this, we express the observables in terms of the production amplitudes (S, P_0, $P_\pm$ in the notation of Ref. 2) as follows [1]:

$$\sigma \equiv \frac{d\sigma}{dt} = \left(|S|^2 + |P_0|^2\right) R + |P_+|^2 + |P_-|^2 \qquad (1)$$

$$\alpha \equiv (\rho_{00} - \rho_{11})\sigma = |P_0|^2 R - \tfrac{1}{2}\left(|P_+|^2 + |P_-|^2\right) \qquad (2)$$

$$\beta \equiv \rho_{1-1}\sigma = \tfrac{1}{2}\left(|P_+|^2 - |P_-|^2\right) \qquad (3)$$

$$\gamma_{10} \equiv \sqrt{2}\, \mathrm{Re}\,\rho_{10}\sigma = |P_-|\,|P_0| \cos\varphi \qquad (4)$$

$$\gamma_{0S} \equiv \mathrm{Re}\,\rho_{0S}\sigma = |P_0|\,|S|\, R \cos\Delta \qquad (5)$$

$$\gamma_{1S} \equiv \sqrt{2}\, \mathrm{Re}\,\rho_{1S}\sigma = |P_-|\,|S| \cos(\varphi - \Delta) \qquad (6)$$

where $R = 1$ for t channel quantities and $R = t/(t - t_{min})$ for s channel quantities. R accounts for the small π pole contribution to s channel nucleon helicity non-flip amplitudes so that S, P_0 denote the nucleon flip amplitudes.

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From these six observables we can solve for the six quantities $|P_0|$, $|P_\pm|$, $\gamma_s \equiv |S|/|P_0|$, φ and Δ. To do this we reduce the equations to a cubic equation in $x \equiv |P_0|^2$.

$$-3R^3x^3 + R^2(B+3A)x^2 + (3R\gamma_{10}^2 - R\gamma_{1s}^2 - \gamma_{0s}^2 - AB)Rx + (A\gamma_{0s}^2 - RB\gamma_{10}^2 + 2R\gamma_{10}\gamma_{0s}\gamma_{1s}) = 0, \tag{7}$$

where $A = \alpha + \beta$ and $B = \sigma + 2\alpha$. In Fig. 1 we show the amplitudes for the two allowed solutions as determined from the 17.2 GeV CERN-Munich [3] s channel density matrix elements in the ρ mass region ($700 \le M_{\pi\pi} \le 850$ MeV). The other solution is unphysical; $|P_0|^2 < 0$. The physical solutions have similar values of $|P_0|^2$, but solution 1 is characterized by $|\cos\varphi| \approx 1$ and solution 2 by $|\cos\Delta| \approx 1$. Only the relative sign of φ and Δ can be determined from Eq. (1) to (6). The values shown for Δ for solution 2 correspond to $\sin\varphi > 0$ while, for solution 1, $|\cos\varphi| \approx 1$ and so only $|\Delta|$ is shown. $\gamma_s \cos\Delta$ is remarkably constant in each solution. The more erratic behaviour of γ_s for solution 1 is mainly due to a sizeable $\gamma_s \sin\Delta$ component.

Similar properties are found [2] when the analysis is repeated in 20 MeV $\pi\pi$ mass bins in the range $500 \le M_{\pi\pi} \le 980$ MeV. Moreover, for both solutions, $|P_0|$, γ_s and Δ suitably extrapolated to the π exchange pole give $\pi\pi$ phase shifts consistent with elastic unitarity. However, only solution 1 is in agreement with the shape of the $\pi^0\pi^0$ mass spectrum observed [4] in the reaction $\pi^- p \to \pi^0\pi^0 n$.

For the 150 MeV mass bin about the ρ, solution 1 appears to have anomalously large average values of $|\Delta|$ and γ_s. This feature is understandable since, in this mass region, the P wave phase shift is rapidly varying while the S wave is approximately constant. Therefore, care must be taken in extracting $\pi\pi$ phase shifts from large mass bins across which the amplitudes are rapidly varying.

To investigate whether it is better to perform the phase shift analysis in the s or t channel, we have analysed the moments in both channels in the ρ mass region. In what follows, we study only solution 1 although the discussion applies equally well to solution 2. If P_0 were pure π pole exchange, we would expect

$$G_\pi \equiv \sqrt{\frac{2R}{-t}} \left(\frac{\mu^2 - t}{M_{\pi\pi}}\right) |P_0| \tag{8}$$

to show an exponential decrease in t. From Fig. 2, we see that this form is an appreciably better description in the s channel

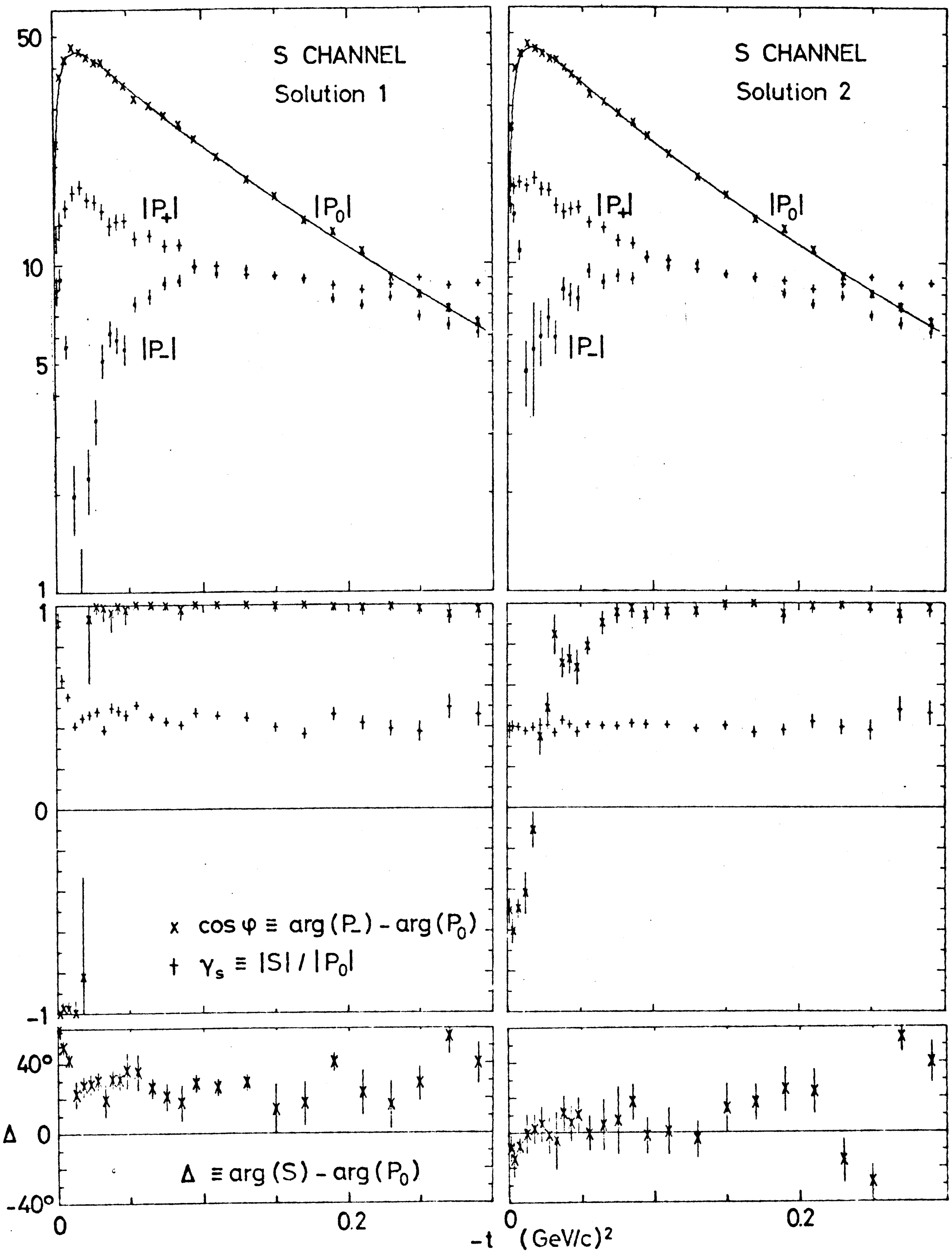

Figure 1 : Results of an s channel amplitude analysis of the 17.2 GeV $\pi^- p \rightarrow \pi^- \pi^+ n$ data [3]. The curves are the best fits to $|P_0|$ using an exponential form of G_π of Eq. (8) in the interval $0.005 \leq |t| \leq 0.2$ $(GeV/c)^2$.

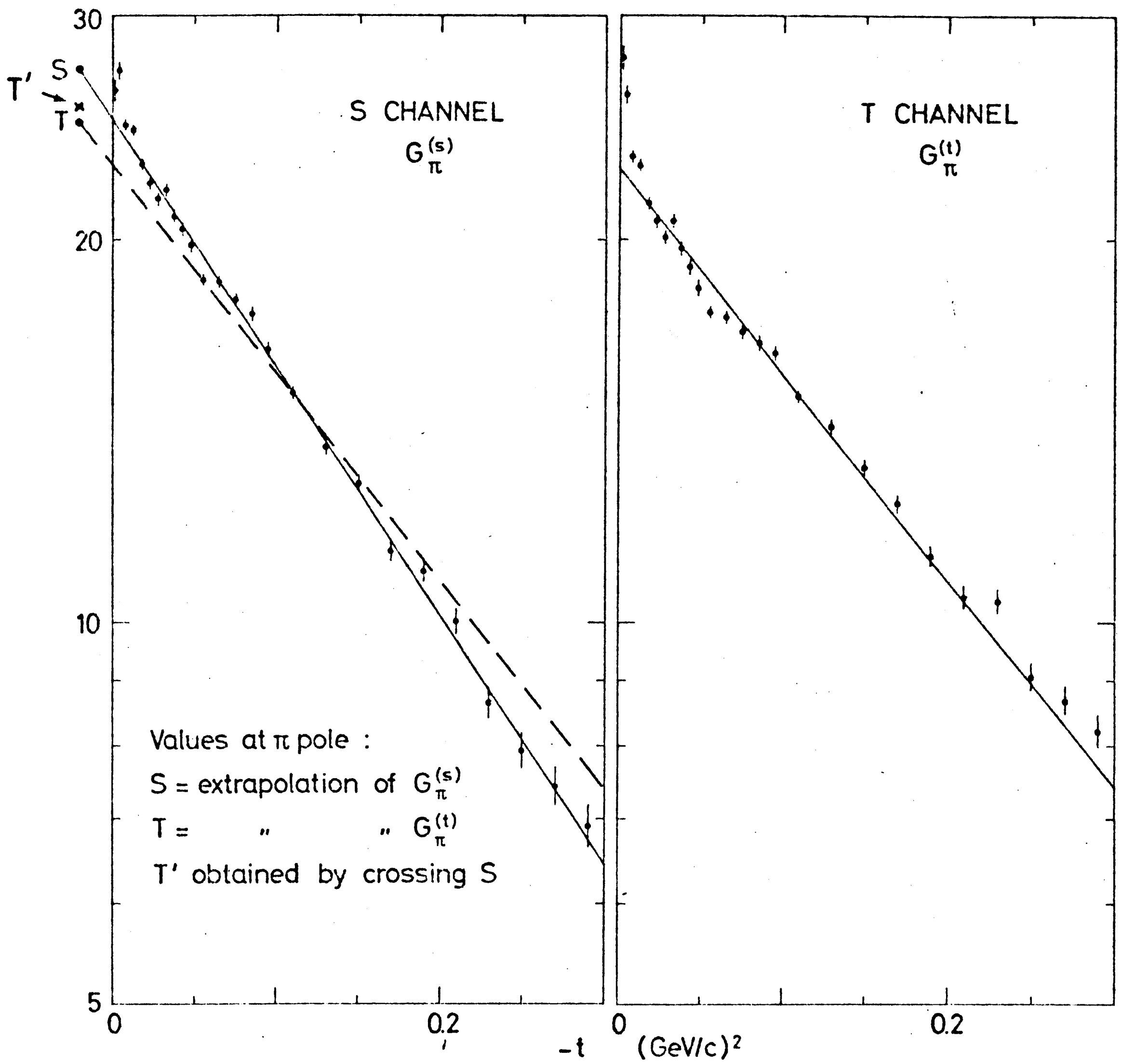

Figure 2 : t dependence of the π coupling, G_π of Eq. (8), as calculated from $P_0^{(s)}$ and $P_0^{(t)}$ of solution 1. The lines are the best fits to an exponential form of G_π in the interval $0.005 \leq |t| \leq 0.2$ GeV^2. For comparison, the t channel fit is shown as a dashed line on the s channel plot.

than in the t channel. We do not comment on the anomaly at very small $|t|$ ($|t| \leq 0.005$ GeV/c^2) as it has been discussed by W. Männer [5]. If there were no unnatural parity contributions other than π exchange, then, extrapolating to $t = \mu^2$ we would have

$$\left.\frac{G_\pi^{(s)}}{G_\pi^{(t)}}\right|_{t=\mu^2} = \cos\chi\Big|_{t=\mu^2} = \sqrt{\frac{M_{\pi\pi}^2}{M_{\pi\pi}^2 - 4\mu^2}}, \qquad (9)$$

as compared to the points S and T in Fig. 2. Here χ is the s-t crossing angle. From Eq. (9) and the point S, we would expect T to be at T'. The extrapolation of G_π is more stable in the s than in the t channel to changes of the t interval fitted. For instance, if data in the range $0.005 < |t| < 0.1$ GeV^2 are used, then S, and consequently T', are unchanged but T is raised to essentially T'. The difference between T and T', which indicates the presence of non π exchange contributions to P_0 can be understood in terms of the contamination of $P_0^{(t)}$ arising from the destructive non π contribution (C in the notation of Ref. 6) to $P_-^{(s)}$ since:

$$P_0^{(t)} = P_0^{(s)}\cos\chi + P_-^{(s)}\sin\chi. \qquad (10)$$

The existence of C is necessary to explain the sign change in $\mathrm{Re}\,\rho_{10}^{(s)}$ near $t = -\mu^2$. For small $|t|$, $\sin\chi \approx (2\sqrt{-t'})/M_{\pi\pi}$, so the contamination in $P_0^{(t)}$ increases rapidly with decreasing $M_{\pi\pi}$. The increase [6] of C with decreasing $M_{\pi\pi}$ further enhances the T-T' discrepancy at low values of $M_{\pi\pi}$.

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