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π EXCHANGE, Δ PRODUCTION AND MESON-MESON SCATTERING

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ABSTRACT

We propose a method of extracting amplitude components from charge exchange processes of the form $0^-N \to 0^-0^-\Delta$ or $0^-N \to 1^-\Delta$ by using the full decay distributions. The processes $\pi^+p \to \rho^0\Delta^{++}$, $\omega\,\Delta^{++}$ are analysed as a simple application and some properties of π and B exchange are discussed. The suitability of the method for extraction of meson-meson scattering amplitudes is emphasized.

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1. INTRODUCTION

The study of pseudoscalar exchange in high energy vector meson production processes has been of widespread interest. Detailed knowledge of the production amplitudes has so far been confined to $K^* \Lambda$ [1] $(\omega, \phi) \Lambda$ [2] and $\rho^o n$ [3] final states. Here we point out that the π exchange-dominated processes π (or K) $N \rightarrow \pi$ (or K) $\pi \Delta$ yield sufficient information in their decay correlations to enable the production amplitudes to be unravelled in a somewhat model-dependent way. In contrast to the processes involving nucleons in the final states (e.g., $\pi^- p \rightarrow \rho^o n$), pion exchange is here expected to occur principally through s channel helicity non-flip couplings so that a separate study of these amplitudes seems worthwhile.

The possibility of using helicity amplitudes extracted from $\pi N \rightarrow \pi\pi N, \pi\pi\Delta$ data to calculate $\pi\pi$ scattering amplitudes, is an appealing one [4),3)]. We propose a practical method of achieving this, for π(or K) $N \rightarrow \pi$(or K) $\pi\Delta$ which avoids the uncertainties encountered in moment extrapolation techniques [5)] by instead extrapolating the ratio of helicity amplitudes to the pion pole [3)]. The advantage of using the non-vanishing non-flip amplitudes of $\pi\pi\Delta$ (as compared to $\pi\pi N$) is partially offset at low momenta by the greater kinematical boundary ("t_{min}") problems in this process.

In the π(or K)π mass region below 1 GeV or so, only s and p wave dimeson states need be considered. Thus 12 p wave and four s wave helicity amplitudes are involved while the decay angular distribution of the dimeson and baryon systems involves 30 independent measurables supplying 24 independent pieces of information on the production processes, seven short of a complete determination [*)]. By making plausible assumptions about the baryon helicity couplings and about the factorizability of the dimeson and production systems we demonstrate the feasibility of a complete amplitude analysis in which the single discrete ambiguity is the sign of the s-p wave phase difference.

In Section 2 we define the amplitudes and measurables involved. The assumptions inherent in the model are discussed in Section 3. Next, a sample analysis of $\pi^+ p \rightarrow (\rho^o, \omega) \Delta^{++}$ data is presented and in Section 5 we make some concluding remarks

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*) We discount the unmeasurable over-all phase.

2. AMPLITUDES AND MEASURABLES

We define for the process

$$\pi N \rightarrow \pi\pi\Delta \tag{1}$$

the 16 (s or t channel) helicity amplitudes.

$$U^s_{3\pm} = H^s_{3\pm}, \quad U^0_{3\pm} = H^0_{3\pm}, \quad U^1_{3\pm} = \tfrac{1}{\sqrt{2}}\left(H^1_{3\pm} \mp H^1_{-3\mp}\right), \quad N^1_{3\pm} = \tfrac{1}{\sqrt{2}}\left(H^1_{3\pm} \pm H^1_{-3\mp}\right),$$

$$U^s_{1\pm} = H^s_{1\pm}, \quad U^0_{1\pm} = H^0_{1\pm}, \quad U^1_{1\pm} = \tfrac{1}{\sqrt{2}}\left(H^1_{1\pm} \pm H^1_{-1\mp}\right), \quad N^1_{1\pm} = \tfrac{1}{\sqrt{2}}\left(H^1_{1\pm} \mp H^1_{-1\mp}\right) \tag{2}$$

where we use the parity relations [6]

$$H^{\lambda_D}_{2\lambda_\Delta\lambda_N} = -(-)^{\lambda_D+\lambda_\Delta+\lambda_N} H^{-\lambda_D}_{-2\lambda_\Delta-\lambda_N} = -\tau(-)^{\lambda_\Delta-\lambda_N} H^{\lambda_D}_{-2\lambda_\Delta-\lambda_N} \tag{3}$$

λ_D is the helicity of the dimeson state of angular momentum ℓ (restricted to 0 or 1 in the following) and in the second equality when applicable, τ is the exchanged naturality which in this context is only defined at high energies. In terms of covariant couplings [7] g_s, g_p and G [*], the π exchange contribution to the s channel helicity vertices M^{λ_D} and $B_{2\lambda_\Delta\lambda_N}$ of the amplitudes $H^{\lambda_D}_{2\lambda_\Delta\lambda_N}$ are

$$M^s = g_s, \qquad M^1 = -\frac{g_p}{2\sqrt{2}}, \qquad M^0 = \frac{-g_p}{4m'}\left(m'^2 - m^2 + t\right),$$

$$B_{3+} = G(M'+M), \qquad B_{1+} = \frac{G}{\sqrt{3}}\left[t + \frac{M'+M}{M'}\left(M'^2 - M^2 + t\right)\right],$$

$$B_{3-} = G, \qquad B_{1-} = \frac{G}{\sqrt{3}}\left[(M'+M) + \frac{1}{M'}\left(M'^2 - M^2 + t\right)\right] \tag{4}$$

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*) Other covariant couplings are assumed to be small and certainly vanish at the pion pole.

where m, M, m' and M' are the masses of the initial pseudoscalar meson, nucleon target, dipion and delta respectively. The π pole exchange s channel amplitudes are then proportional to $(-t')^{(n+x)/2} M^{\lambda_D} B_{2 \lambda_\Delta \lambda_N}$ where

$$n = |\lambda_D - \lambda_\Delta + \lambda_N| ,$$
$$n + x = |\lambda_D| + |\lambda_\Delta - \lambda_N| . \tag{5}$$

The equivalent t channel amplitudes are all zero except $H_{1+}^{\lambda_D}$ for $\lambda_D = s, 0$. Identical B meson couplings are expected in an exchange-degenerate scheme. In $\pi N \to \pi\pi\Delta$ ($KN \to K\pi\Delta$) the expected exchanges are π, A_2 (π, B, A_2, ρ) and possibly A_1 although we shall ignore its disruptive influence (if any) on the coupling scheme of the next section.

The Stodolsky-Sakurai [8] model for ρ vector meson exchange predicts, in the t channel

$$B_{3-} = B_{1+} = (B_{3+} - \sqrt{3}\, B_{1-}) = 0 \tag{6}$$

and exchange degeneracy would suggest a similar coupling structure for A_2 exchange. Our model for pseudoscalar and vector or tensor meson exchange couplings is described in the following section.

The decay angular distribution for process (1) is written as

$$W(\theta_1, \phi_1; \theta_2, \phi_2) = \sum_{\substack{J_1 M_1 \\ J_2 M_2}} \mathcal{J}^{J_1 \, J_2}_{M_1 \, M_2} \, Y^{J_1}_{M_1}(\theta_1, \phi_1)^* \, Y^{J_2}_{M_2}(\theta_2, \phi_2)^* \tag{7}$$

where (θ_i, ϕ_i) are the polar angles of the dimeson $(i = 1)$ and delta $(i = 2)$ decay analysers in a helicity-type frame [1]. The 30 measurables are given in terms of density matrices and amplitudes in the Appendix.

3. MODEL ASSUMPTIONS

We assume (in the s channel for the moment) that for $|t| \lesssim 0.4$ GeV^2

$$B_{3-} = 0 \tag{8a}$$

$$B_{3+} = \sqrt{3}\, B_{1-} \tag{8b}$$

$$U^{s}_{2\lambda_\Delta \lambda_N} / U^{o}_{2\lambda_\Delta \lambda_N} = \Gamma e^{i\Delta} \quad (\text{real } \Gamma, \Delta \text{ independent of } \lambda_\Delta, \lambda_N) \tag{8c}$$

a) is motivated by angular momentum behaviour at small t;
b) is approximately true for π exchange [we neglect $(M'^2-M^2+t)/M'$ in the expression (4) for B_{1-}] and is suggested by the ρ exchange Stodolsky-Sakurai result [Eq. (6)] and the quasi-diagonal nature of the small t crossing matrix. It is then equally true for linearly absorbed π or ρ exchange and is also plausible for B and A_2 exchange by exchange degeneracy. The assumptions a) and b) for $0^- N \to 0^- \Delta$ charge exchange processes predict

$$\rho_{33} = \sqrt{3}\, \mathrm{Re}\, \rho_{1-3} \tag{9}$$

and this is found to be well satisfied in the s or t channel [9] *). It is emphasized that the full content of the Stodolsky-Sakurai result (6) is not used in the present model.
c) asserts (plausibly) that whether the dimeson helicity zero states are in s or p wave in angular momentum, is independent of the baryon helicity states. Such a statement is certainly true at $t = \mu^2$ but constitutes an assumption when made in the physical region in the presence of absorptive corrections which are dependent on the baryon helicities.

Although assumptions (8) may be made plausibly in s and t channels separately it may be shown that the results are not strictly equivalent, especially at larger t. Through the relation

$$T^{m_D}_{2m_\Delta m_N} = \sum_{\lambda_D \lambda_\Delta \lambda_N} \mathcal{D}^{\ell_D}_{\lambda_D m_D}(R)^{*}\, \mathcal{D}^{3/2}_{\lambda_\Delta m_\Delta}(R)^{*}\, \mathcal{D}^{1/2}_{\lambda_N m_N}(R)\, H^{\lambda_D}_{2\lambda_\Delta \lambda_N} \tag{10}$$

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*) In fact for these processes, assumptions (8a) and (8b) enable a complete analysis of the remaining amplitudes to be made.

where R is the Euler rotation $(-\pi/2, -\pi/2, 0)$ *), the assumptions (8) are equivalent to the transversity amplitude relations

$$T^{m_D}_{\pm 3 \pm} = \sqrt{3}\, T^{m_D}_{\pm 1 \mp}, \quad m_D = \pm 1, s \tag{11a}$$

$$T^{0}_{3-} = -T^{0}_{-3+} = \frac{\sqrt{3}}{2}\left(T^{0}_{1+} - T^{0}_{-1-}\right) \tag{11b}$$

$$T^{s}_{\pm 3 \pm} \Big/ \left(T^{1}_{\pm 3 \pm} - T^{-1}_{\pm 3 \pm}\right) = -\frac{1}{\sqrt{2}}\,\Gamma e^{i\Delta} \tag{11c}$$

By comparison, the quark model [10] which assumes only additivity (Class A relations), predicts in either s or t channel frames

$$T^{m_D}_{\pm 3 \pm} = \sqrt{3}\, T^{m_D}_{\pm 1 \mp}, \quad m_D = \pm 1 \tag{12a}$$

$$T^{0}_{3-} = -T^{0}_{-3+} = T^{0}_{1+} - T^{0}_{-1-} = 0 \tag{12b}$$

for p wave amplitudes, a result which is stronger than (11) by the forbidding of $m_N = -\frac{1}{2}$ to $m_\Delta = \frac{3}{2}$ transitions. This extra condition is a consequence of the transversity (m_s) of the two spectator baryon quarks being limited by one in modulus. In terms of helicity amplitudes, these quark model results (class A) are stronger than (8) by the extra condition

$$N^{1}_{1+} = 0 \tag{13}$$

which is expected to be invalid for ρ and absorbed π exchange **).

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*) Our choice of helicity and transversity axes is as in Ref. 1).

**) After completion of this work, details were received [11] of a Regge dipole model which predicts quark model class A and B results and is used to reduce the number of independent p wave amplitudes to four. We prefer to use the assumptions (8) which are weaker than class A results [Equation (13)] and allow for six independent p wave amplitudes.

From an exchange point of view, the factor $\sqrt{3}$ in Eqs. (8a), (11a), (12a) results from the kinematics of π exchange, the vector meson photon analogy and the crossing matrix properties at small t. In the quark model it is derived from the Clebsch-Gordan coefficients relating amplitudes with $m_s = 0$ and $m_s = \pm 1$. That such different approaches yield approximately the same result is remarkable.

To implement assumptions (8) we take (in s or t channel) U^0_{1+}, U^0_{3+}, U^1_{1+}, U^1_{3+}, N^1_{1+} and N^1_{3+} as independent p wave amplitudes which together with Γ and Δ constitute 13 parameters describing the 30 terms of the decay angular distribution. A subset of the 30 measured moments may then be used to solve first for the six moduli and Γ, then the five relative phases and Δ. In this manner it is seen that there are no discrete ambiguities apart from the (usually trivial) sign of the s-p phase difference, Δ. This method of solution, however, wastes the information contained in the 17 constraint equations and it is better to use maximum likelihood method on the event angles themselves.

4. SAMPLE ANALYSIS OF $\pi^+ p \rightarrow (\rho^0, \omega)\Delta^{++}$ AT 3.7 GeV/c [12)]

In the absence of information on the decay angles themselves, we present a sample analysis of the 20 published moments of

$$\pi^+ p \rightarrow \rho^0 \Delta^{++} \tag{14}$$

and

$$\pi^+ p \rightarrow \omega \Delta^{++} \tag{15}$$

at 3.7 GeV/c [12)]. To minimise the disadvantages of using moments we perform an eleven-parameter least-squares fit and use the chi-squared per fitted point to check how well the nine constraint equations are satisfied. Among the deficiencies of the published data and hence this sample analysis are the following.

i) The low beam momentum implies severe "t_{min}" problems. The data are presented versus $t' = t - t_{min}$ and a reliable estimate of the small t scale is difficult since t_{min} is not well defined.

ii) No s wave interference moments are available. Despite the authors' background correction [12)] the remaining s wave is likely to have some (not easily calculable) effect on the p wave joint decay density matrices.

For these reasons the results (as opposed to the method) of the sample analysis should be viewed with some caution.

Figures 1a and b show the s and t channel amplitude moduli and relative phases. The solutions shown are those which have lowest chi-squared, are continuous in t where possible and are consistent in the forward direction where s (helicity) and t channel (Gottfried-Jackson) frames coincide. For $\rho\Delta$ the average chi-squared of 10.5 and 9.5 per t' bin in each channel (13.8 and 13.1 for $\omega\Delta$) shows the nine implicit constraint equations to be adequately satisfied. All moduli except perhaps $|N^1_{1+}|$ are well determined. The small t dominance of over-all non-flip meson helicity zero amplitudes is clearly shown particularly for $\rho\Delta$ in the t channel as would be expected from Feynman diagram arguments. The phases of U^0_{3+}, U^1_{1+}, U^1_{3+} and N^1_{3+} relative to U^0_{1+} are reasonably well determined but that of N^1_{1+} (not shown in Fig. 1 for the sake of clarity) is not, partly because $|N^1_{1+}|$ is small (zero in the quark model). For pure π exchange the s channel $\rho\Delta$ amplitudes U^0_{3+} and U^0_{1+} should be equal in phase and approximately real over-all. The phase difference of Fig. 1a is consistent with this at $t'=0$ but grows rapidly with $|t'|$. At small t this quantity is determined principally by the measurables A7 and A3 (Re $\rho_{1\bar{3}}$ and Re ρ_{31}) which are listed in the Appendix. Since, in the absence of s wave information, the over-all sign of the phase differences is not measured we arbitrarily choose it for $\rho\Delta$ and $\omega\Delta$ to agree with simple absorption model expectations for π and B exchange assuming U^0_{1+} to have larger corrections than U^0_{3+}. Also shown in Figs. 1a and b are fits to the amplitudes of the forms

$$|U| = (-t')^{n/2} A e^{Bt}/(\mu^2 - t) \tag{16}$$

and

$$|N| = (-t')^{n/2} C e^{Dt} \tag{17}$$

where we have used $t_{min} = t - t' = -0.072$ GeV^2. These curves give a convenient estimate of the relative importance of each amplitude.

The quality of the present data merits no more than a brief discussion of some gross features of the amplitudes.

A. Exchange degeneracy

The expected Regge pole exchanges in $\pi^+ p \to \rho^0 \Delta^{++}$ and $\pi^+ p \to \omega \Delta^{++}$ are respectively π, A_2 and B, ρ. Pole dominance, exchange degeneracy and SU(3) symmetry with magic mixing implies

$$|U(\rho\Delta)/U(\omega\Delta)| = \left|\cot \frac{\pi\alpha_\pi}{2}\right| \tag{18}$$

and

$$|N(\rho\Delta)/N(\omega\Delta)| = \left|\cot \frac{\pi\alpha_{A_2}}{2}\right| \tag{19}$$

for EXD (π, B) and (A_2, ρ) Regge exchange respectively. Using the published normalisations [12),13)] (which are compatible with Fig. 1) and using $\alpha_\pi = t - \mu^2$ and $\alpha_{A_2} = 0.55 + t$ we find the left-hand side of Eq. (18) to be half the expected value for U^0_{1+}. However, Eq. (18) is very well satisfied for U^0_{3+}. For the so-called natural parity exchange amplitudes N^1_{1+} and N^1_{3+} (again in the s channel) Eq. (19) is satisfied only when $|t'| \lesssim 0.1$ GeV^2. Elsewhere, like U^0_{1+}, they show larger than expected $\omega\Delta$ production [*)]. We note that of all these amplitudes only U^0_{3+} is expected to be simple pole dominated (see the discussion in subsection B).

B. Absorptive corrections

Our knowledge of absorption systematics comes principally from vector and tensor meson exchange dominated reactions where it seems [14)] a good approximation to neglect all absorptive corrections (non-pole like amplitude contributions) in all but zero net helicity flip $(n = 0)$ amplitudes. In addition the imaginary parts apparently require larger corrections than do the real parts which seem well approximated by Regge pole exchange. For π exchange, however, the relative success of the Williams model [15)] for absorption suggests that evasive non-flip amplitudes [those with $n = 0$, $x \neq 0$ in Eq. (5)], while dominantly real, are also strongly absorbed.

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*) We discount possible π exchange contributions in $\omega\Delta$ through ρ-ω interference since they would presumably also contribute to U^0_{3+} where EXD is well satisfied.

Looking at the $\rho\Delta$ results of Fig. 1a, strong absorptive corrections must be present in at least two of the amplitudes U^0_{1+} $(n=0)$, U^0_{3+} $(n=1)$ and U^1_{1+} $(n=1)$ because of the well-defined phase differences observed. The $n=0$ amplitude is suspected of being one of these whereas U^0_{3+} and perhaps U^1_{1+} are expected to be pole-like and therefore approximately real. As working hypothesis, later to be justified, we select U^0_{3+} to be approximately real. The magnitudes and absolute phases of the remaining five independent $H_{2\lambda_\Delta\lambda_N}^{\lambda_D}$ of Eq. (2) may then be calculated from Fig. 1a. Of these amplitude components only $\mathrm{Im}\,H^1_{3+}$ and $\mathrm{Re}\,H^0_{1+}$ are found to have dip or zero structure. Both components exhibit cross-overs near $-t' = 0.1\ \mathrm{GeV}^2$ $(-t \sim 0.17\ \mathrm{GeV}^2)$ as shown in Fig. 2. H^0_{1+} and H^1_{3+} are the only pure $n=0$ amplitude combinations of our six chosen as independent, so that these results are, so far, in accordance with the geometrical view of absorption effects [16].

Assuming H^0_{3+} $(=U^0_{3+})$ to be pure π pole and the other π pole amplitudes to be given proportionally by Eqs. (4) and (5), we may calculate the effective absorptive ("cut") and natural parity pole (A_2) corrections at small t by simple subtraction of these calculated pole values from the total amplitudes [3]. Since a detailed numerical study would be inappropriate here, we merely state the general result i.e., under these assumptions all other amplitudes, including $n=1$ H^1_{1+} and H^1_{-1-}, are found to have strong <u>destructive</u> absorptive corrections. Thus, assuming U^1_{1+} to be real and pole-like would have led us to deduce <u>additive</u> corrections in H^0_{3+} at least. It is for this reason that H^0_{3+} is selected as the most likely $n=1$ amplitude to be pole-like.

In Regge pole absorption terminology, the t channel amplitude U^0_{3+} can only have contributions from π- cuts (through crossing from the s channel), while U^1_{1+} and U^1_{3+} can contain π- and A_2- cuts. Absorbed A_2 and π- cuts may contribute to N^1_{1+} and N^1_{3+}. Indeed, Fig. 1b shows $|U^{0t}_{3+}| < |U^{0s}_{3+}|$ although a large degree of absorption is obviously present. Similarly, $|U^{0t}_{1+}| > |U^{0s}_{1+}|$ is anticipated for non-zero t'. Although detailed study of the $\omega\Delta$ amplitudes is not possible, we note some points of interest. Unlike $\rho\Delta$ the dominance of U^0_{1+} over U^0_{3+} for $\omega\Delta$ is more marked in the s than in the t channel. The $\omega\Delta$ natural parity amplitudes are comparable in size with those of $\rho\Delta$ (see also subsection A). The phase difference between U^0_{3+} and U^1_{1+} for $\omega\Delta$ is very small in both channels showing that in this case it is not impossible that both the $n=1$ s channel amplitudes U^0_{3+} and U^1_{1+} are pole-like; however, there is then no reason to expect a zero phase difference in the t channel also.

5. CONCLUSIONS

We have demonstrated the theoretical plausibility and practical feasibility of an amplitude analysis of dimeson production reactions involving $\Delta(1236)$ final states. Some of these proposals were implemented in an analysis of the angular distribution moments of the reactions $\pi^+ p \to (\rho^0, \omega)\Delta^{++}$ in order to examine their exchange degeneracy properties and to isolate some features of absorbed pion exchange. π-B exchange degeneracy was found to be well satisfied only for the over-all helicity flip meson helicity zero amplitudes. A large degree of absorption in the dominant non-flip amplitude was tentatively deduced.

Finally we emphasize that the above methods, when applied to high statistics dimeson production data, suitably binned in mass, are highly suited to the extraction of meson-meson phase shifts. For example in the low mass region dominated by elastic s and p waves one may obtain the phase shifts δ_s and δ_p using

$$\Gamma = |\sin\delta_s / \sqrt{3}\sin\delta_p|$$

and

$$\Delta = \delta_s - \delta_p$$

where Γ and Δ are smooth extrapolations to $t = \mu^2$ (or the weighted means) of their observed values for $t' < 0$. The generalization to higher waves is straightforward and is made by allowing helicity couplings of higher ℓ states in Eq. (8) thus involving more amplitudes and more observables in the analysis.

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APPENDIX

We list the 30 measurable helicity statistical tensors $\mathcal{J}^{J_1J_2}_{M_1M_2}$ for process (1) in terms of the joint density matrix elements $\rho^{m_1m_1'}_{m_2m_2'}$ and the 16 helicity amplitudes of Eq. (2) of Section 2.

$$\mathcal{J}^{00}_{00} = \sigma = 2\left[\,|U^{\mu}_{\lambda'\lambda}|^2 + |N^{1}_{\lambda'\lambda}|^2\right] \tag{A.1}$$

$$-\sqrt{5}\,\mathcal{J}^{02}_{00} = \sigma\rho_{-} = 2\left[\,|U^{\mu}_{3\lambda}|^2 + |N^{1}_{3\lambda}|^2 - |U^{\mu}_{1\lambda}|^2 - |N^{1}_{1\lambda}|^2\right] \tag{A.2}$$

$$\frac{1}{2}\sqrt{\frac{5}{2}}\,\mathcal{J}^{02}_{01} = \sigma\,\mathrm{Re}\,\rho_{31} = \mathrm{Re}\left[N^{1}_{3\lambda}N^{1*}_{1\lambda} + U^{\mu}_{3\lambda}U^{\mu*}_{1\lambda}\right] \tag{A.3}$$

$$-\frac{1}{2}\sqrt{\frac{5}{2}}\,\mathcal{J}^{02}_{02} = \sigma\,\mathrm{Re}\,\rho_{1-3} = \lambda\,\mathrm{Re}\left[N^{1}_{1-\lambda}N^{1*}_{3\lambda} + U^{\mu}_{1\lambda}U^{\mu*}_{3-\lambda}\right] \tag{A.4}$$

$$-\sqrt{5}\,\mathcal{J}^{20}_{00} = \sigma\rho^{-} = 2\left[\,|N^{1}_{\lambda'\lambda}|^2 + |U^{1}_{\lambda'\lambda}|^2 - 2\,|U^{0}_{\lambda'\lambda}|^2\right] \tag{A.5}$$

$$5\,\mathcal{J}^{22}_{00} = \sigma\rho_{=} = 2\left[|N^{1}_{3\lambda}|^2 - |N^{1}_{1\lambda}|^2 + |U^{1}_{3\lambda}|^2 - |U^{1}_{1\lambda}|^2 - 2|U^{0}_{3\lambda}|^2 + 2|U^{0}_{1\lambda}|^2\right] \tag{A.6}$$

$$-\frac{5}{2\sqrt{2}}\,\mathcal{J}^{22}_{01} = \sigma\,\mathrm{Re}\,\rho_{\overline{13}} = \mathrm{Re}\left[U^{1}_{1\lambda}N^{1*}_{3\lambda} + N^{1}_{1\lambda}U^{1*}_{3\lambda} - 2\,U^{0}_{1\lambda}U^{0*}_{3\lambda}\right] \tag{A.7}$$

$$\frac{5}{2\sqrt{2}}\,\mathcal{J}^{22}_{02} = \sigma\,\mathrm{Re}\,\rho_{\overline{1-3}} = \lambda\,\mathrm{Re}\left[N^{1}_{1\lambda}U^{1*}_{3-\lambda} - N^{1}_{3-\lambda}U^{1*}_{1\lambda} - 2\,U^{0}_{1\lambda}U^{0*}_{3-\lambda}\right] \tag{A.8}$$

$$\frac{1}{2}\sqrt{\frac{5}{3}}\,\mathcal{J}^{20}_{10} = \sigma\,\mathrm{Re}\,\rho^{10} = \sqrt{2}\,\mathrm{Re}\left[U^{1}_{\lambda'\lambda}U^{0*}_{\lambda'\lambda}\right] \tag{A.9}$$

$$-\frac{5}{2\sqrt{3}}\,\mathcal{J}^{22}_{10} = \sigma\,\mathrm{Re}\,\rho^{10}_{-} = \sqrt{2}\,\mathrm{Re}\left[U^{1}_{3\lambda}U^{0*}_{3\lambda} - U^{1}_{1\lambda}U^{0*}_{1\lambda}\right] \tag{A.10}$$

$$\frac{5}{4}\sqrt{\frac{2}{3}}\,\mathcal{J}^{22}_{11} = \sigma\,\mathrm{Re}\left(\rho^{10}_{31} - \rho^{10}_{1-3}\right) = \frac{1}{\sqrt{2}}\mathrm{Re}\left[(N^{1}_{3\lambda} + U^{1}_{3\lambda})U^{0*}_{1\lambda} - (N^{1}_{1-\lambda} - U^{1}_{1-\lambda})U^{0*}_{3-\lambda}\right] \tag{A.11}$$

$$-\frac{5}{4}\sqrt{\frac{2}{3}}\, J^{22}_{12} = \sigma \mathrm{Re}(\rho^{10}_{3-1}+\rho^{10}_{1-3}) = \frac{\lambda}{\sqrt{2}} \mathrm{Re}\left[(N^1_{1\lambda}+U^1_{1\lambda})U^{0\,*}_{3-\lambda} - (N^1_{3\lambda}+U^1_{3\lambda})U^{0\,*}_{1-\lambda}\right] \tag{A.12}$$

$$-\sqrt{\frac{5}{6}}\, J^{20}_{20} = \sigma \rho^{1-1} = \left[|N^1_{\lambda'\lambda}|^2 - |U^1_{\lambda'\lambda}|^2\right] \tag{A.13}$$

$$\frac{5}{2}\sqrt{\frac{2}{3}}\, J^{22}_{20} = \sigma \underline{\rho^{1-1}} = \left[|N^1_{3\lambda}|^2 - |N^1_{1\lambda}|^2 - |U^1_{3\lambda}|^2 + |U^1_{1\lambda}|^2\right] \tag{A.14}$$

$$-\frac{5}{4\sqrt{3}}\, J^{22}_{21} = \sigma \mathrm{Re}\,\rho^{1-1}_{31} = -\frac{1}{2} \mathrm{Re}\left[(N^1_{3\lambda}+U^1_{3\lambda})(N^1_{1-\lambda}-U^1_{1-\lambda})^*\right] \tag{A.15}$$

$$\frac{5}{4\sqrt{3}}\, J^{22}_{22} = \sigma \mathrm{Re}\,\rho^{1-1}_{1-3} = -\frac{\lambda}{2} \mathrm{Re}\left[(N^1_{3-\lambda}+U^1_{3-\lambda})(N^1_{1\lambda}+U^1_{1\lambda})^*\right] \tag{A.16}$$

$$-\frac{5}{4}\sqrt{\frac{2}{3}}\, J^{22}_{-11} = \sigma \mathrm{Re}(\rho^{10}_{13}-\rho^{10}_{3-1}) = \frac{1}{\sqrt{2}} \mathrm{Re}\left[(N^1_{1\lambda}+U^1_{1\lambda})U^{0\,*}_{3\lambda} - (N^1_{3-\lambda}-U^1_{3-\lambda})U^{0\,*}_{1-\lambda}\right] \tag{A.17}$$

$$\frac{5}{4}\sqrt{\frac{2}{3}}\, J^{22}_{-12} = \sigma \mathrm{Re}(\rho^{10}_{-13}+\rho^{10}_{-31}) = \frac{\lambda}{\sqrt{2}} \mathrm{Re}\left[(N^1_{1-\lambda}-U^1_{1-\lambda})U^{0\,*}_{3\lambda} - (N^1_{3-\lambda}-U^1_{3-\lambda})U^{0\,*}_{1\lambda}\right] \tag{A.18}$$

$$-\frac{5}{4\sqrt{3}}\, J^{22}_{-21} = \sigma \mathrm{Re}\,\rho^{1-1}_{13} = \frac{1}{2} \mathrm{Re}\left[(N^1_{1\lambda}+U^1_{1\lambda})(N^1_{3\lambda}-U^1_{3\lambda})^*\right] \tag{A.19}$$

$$\frac{5}{4\sqrt{3}}\, J^{22}_{-22} = \sigma \mathrm{Re}\,\rho^{1-1}_{-13} = \frac{\lambda}{2} \mathrm{Re}\left[(N^1_{1-\lambda}-U^1_{1-\lambda})(N^1_{3\lambda}-U^1_{3\lambda})^*\right] \tag{A.20}$$

$$\frac{1}{2}\, J^{10}_{00} = \sigma \mathrm{Re}\,\rho^{0s} = 2\,\mathrm{Re}\left[U^0_{\lambda'\lambda}U^{s\,*}_{\lambda'\lambda}\right] \tag{A.21}$$

$$-\frac{\sqrt{5}}{2}\, J^{12}_{00} = \sigma \mathrm{Re}\,\underline{\rho^{0s}} = 2\,\mathrm{Re}\left[U^0_{3\lambda}U^{s\,*}_{3\lambda} - U^0_{1\lambda}U^{s\,*}_{1\lambda}\right] \tag{A.22}$$

$$-\frac{1}{2}\sqrt{\frac{5}{2}}\, J^{12}_{01} = \sigma \mathrm{Re}(\rho^{0s}_{13}-\rho^{0s}_{31}) = -\mathrm{Re}\left[U^0_{3\lambda}U^{s\,*}_{1\lambda} + U^0_{1-\lambda}U^{s\,*}_{3-\lambda}\right] \tag{A.23}$$

$$-\frac{1}{2}\sqrt{\frac{5}{2}}\, J^{12}_{02} = \sigma \mathrm{Re}(\rho^{0s}_{1-3}+\rho^{0s}_{3-1}) = \lambda\,\mathrm{Re}\left[U^0_{1\lambda}U^{s\,*}_{3-\lambda} - U^0_{3\lambda}U^{s\,*}_{1-\lambda}\right] \tag{A.24}$$

$$\frac{1}{2} J^{10}_{10} = \sigma \mathrm{Re}\, \rho^{1s} = \sqrt{2}\, \mathrm{Re}\left[U^{1}_{\lambda'\lambda} U^{s*}_{\lambda'\lambda} \right] \tag{A.25}$$

$$-\frac{\sqrt{5}}{2} J^{12}_{10} = \sigma \mathrm{Re}\, \rho^{1s}_{\underline{\quad}} = \sqrt{2}\, \mathrm{Re}\left[U^{1}_{3\lambda} U^{s*}_{3\lambda} - U^{1}_{1\lambda} U^{s*}_{1\lambda} \right] \tag{A.26}$$

$$\frac{1}{2}\sqrt{\frac{5}{2}} J^{12}_{11} = \sigma \mathrm{Re}\left(\rho^{1s}_{31} - \rho^{1s}_{-1-3}\right) = \frac{1}{\sqrt{2}} \mathrm{Re}\left[(N^{1}_{3\lambda} + U^{1}_{3\lambda}) U^{s*}_{1\lambda} - (N^{1}_{1-\lambda} - U^{1}_{1-\lambda}) U^{s*}_{3-\lambda} \right] \tag{A.27}$$

$$-\frac{1}{2}\sqrt{\frac{5}{2}} J^{12}_{12} = \sigma \mathrm{Re}\left(\rho^{1s}_{3-1} + \rho^{1s}_{1-3}\right) = \frac{\lambda}{\sqrt{2}} \mathrm{Re}\left[(N^{1}_{1\lambda} + U^{1}_{1\lambda}) U^{s*}_{3-\lambda} - (N^{1}_{3\lambda} + U^{1}_{3\lambda}) U^{s*}_{1-\lambda} \right] \tag{A.28}$$

$$-\frac{1}{2}\sqrt{\frac{5}{2}} J^{12}_{-11} = \sigma \mathrm{Re}\left(\rho^{1s}_{13} - \rho^{1s}_{-3-1}\right) = \frac{1}{\sqrt{2}} \mathrm{Re}\left[(N^{1}_{1\lambda} + U^{1}_{1\lambda}) U^{s*}_{3\lambda} - (N^{1}_{3-\lambda} - U^{1}_{3-\lambda}) U^{s*}_{1-\lambda} \right] \tag{A.29}$$

$$\frac{1}{2}\sqrt{\frac{5}{2}} J^{12}_{-12} = \sigma \mathrm{Re}\left(\rho^{1s}_{-13} + \rho^{1s}_{-31}\right) = \frac{\lambda}{\sqrt{2}} \mathrm{Re}\left[(N^{1}_{1-\lambda} - U^{1}_{1-\lambda}) U^{s*}_{3\lambda} - (N^{1}_{3-\lambda} - U^{1}_{3-\lambda}) U^{s*}_{1\lambda} \right] \tag{A.30}$$

In each case the implied summation over the free indices is made according to

$$\mu = s, 0, 1$$
$$\lambda' = 3, 1$$
$$\lambda = +, - \quad .$$

σ is the differential cross-section and

$$\rho_{\overline{m_2 m_2'}} = \rho^{11}_{m_2 m_2'} + \rho^{-1-1}_{m_2 m_2'} - 2\rho^{00}_{m_2 m_2'} \quad ,$$

$$\rho^{m_1 m_1'}_{\underline{\quad}} = \rho^{m_1 m_1'}_{33} + \rho^{m_1 m_1'}_{-3-3} - \rho^{m_1 m_1'}_{11} - \rho^{m_1 m_1'}_{-1-1} \quad .$$

Measurables (A.1) to (A.20), which are used in the sample analysis of Section 4, have incoherent s and p wave contributions. (A.21) to (A.30) are the s-p wave interference moments.

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FIGURE CAPTIONS

Figure 1 :

The moduli of helicity amplitudes $U_{2\lambda_D \lambda_\Delta \lambda_N}$, $N_{2\lambda_D \lambda_\Delta \lambda_N}$ of Eq. (2) for $\pi^+ p \to \rho^0 \Delta^{++}$ (solid symbols) and for $\pi^+ p \to \omega \Delta^{++}$ (open symbols) at 3.7 GeV/c in

a) s channel;

b) t channel.

In the lower parts of the figure are shown the corresponding amplitude phases taken as relative to that of U^0_{1+} in each of the four cases (s or t channel, $\rho\Delta^{++}$ or $\omega\Delta^{++}$). Since in each case the over-all sign of the phase differences is not measured, the differences shown were taken to correspond to simple absorption arguments for the sign of the U^0_{1+}, U^0_{3+} relative phase. For the sake of clarity the poorly measured phase of N^1_{1+} is not shown. The solid and broken lines are simple interpolating functions motivated by π exchange and whose exact form is specified by Eqs. (16) and (17) in the text.

Figure 2 :

The components $\mathrm{Re}\, H^0_{1+}$, $\mathrm{Im}\, H^0_{1+}$ and $\mathrm{Im}\, H^1_{3+}$ of the two independent $n = 0$ s channel amplitudes in $\pi^+ p \to \rho \Delta^{++}$. They are deduced from the amplitudes of Fig. 1a under the additional assumption that U^0_{3+} is purely real. The components $\mathrm{Re}\, H^0_{1+}$ and $\mathrm{Im}\, H^1_{3+}$ show (cross-over) zeros near $-t' = 0.1\ \mathrm{GeV}^2$.

The solid and broken lines are merely to guide the eye and are not related to those in Fig. 1.

S-CHANNEL AMPLITUDES FOR $\pi^+ p \rightarrow (\rho^0, \omega)\, \Delta^{++}$ AT 3.7 GeV/c

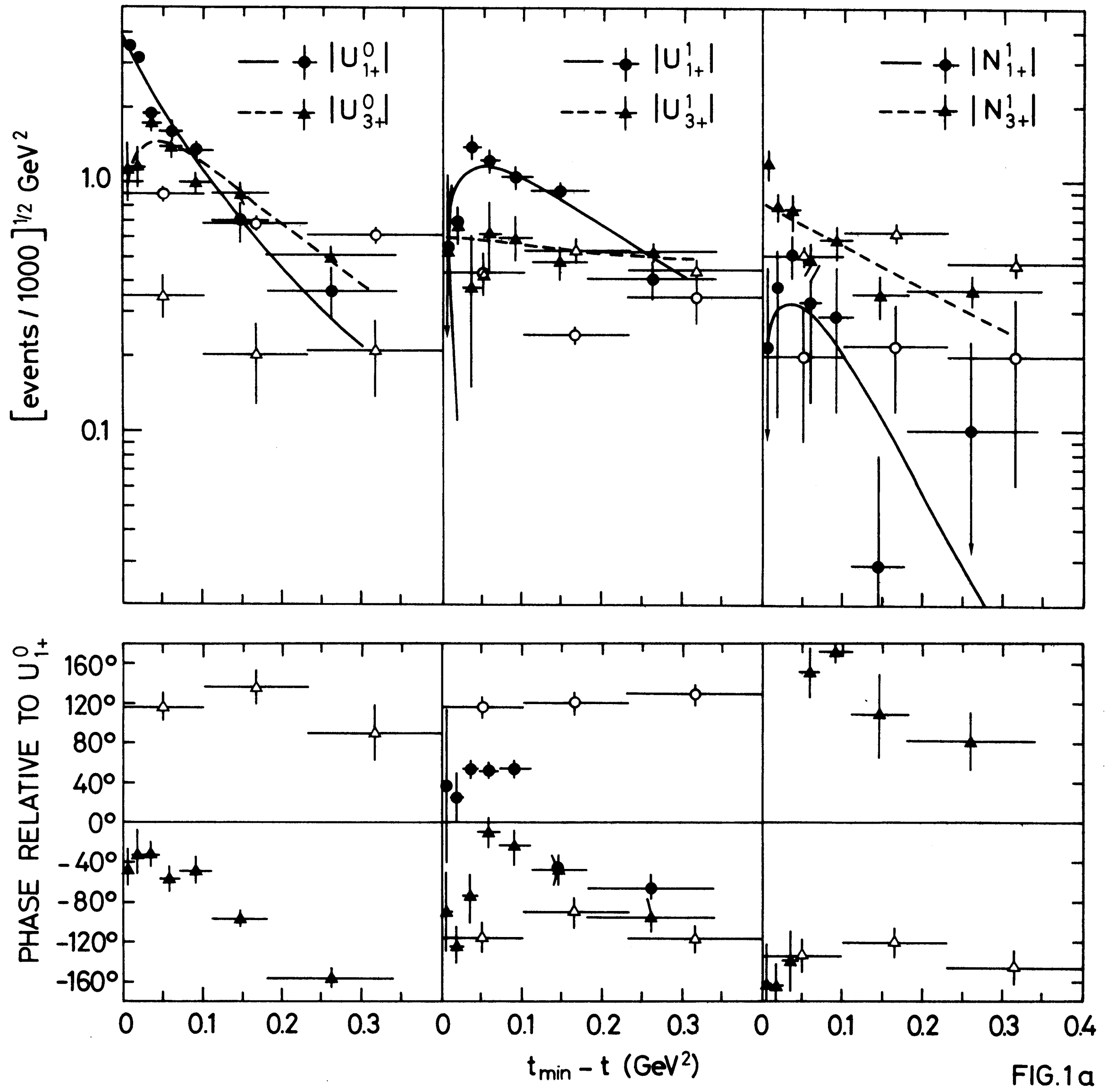


FIG.1a

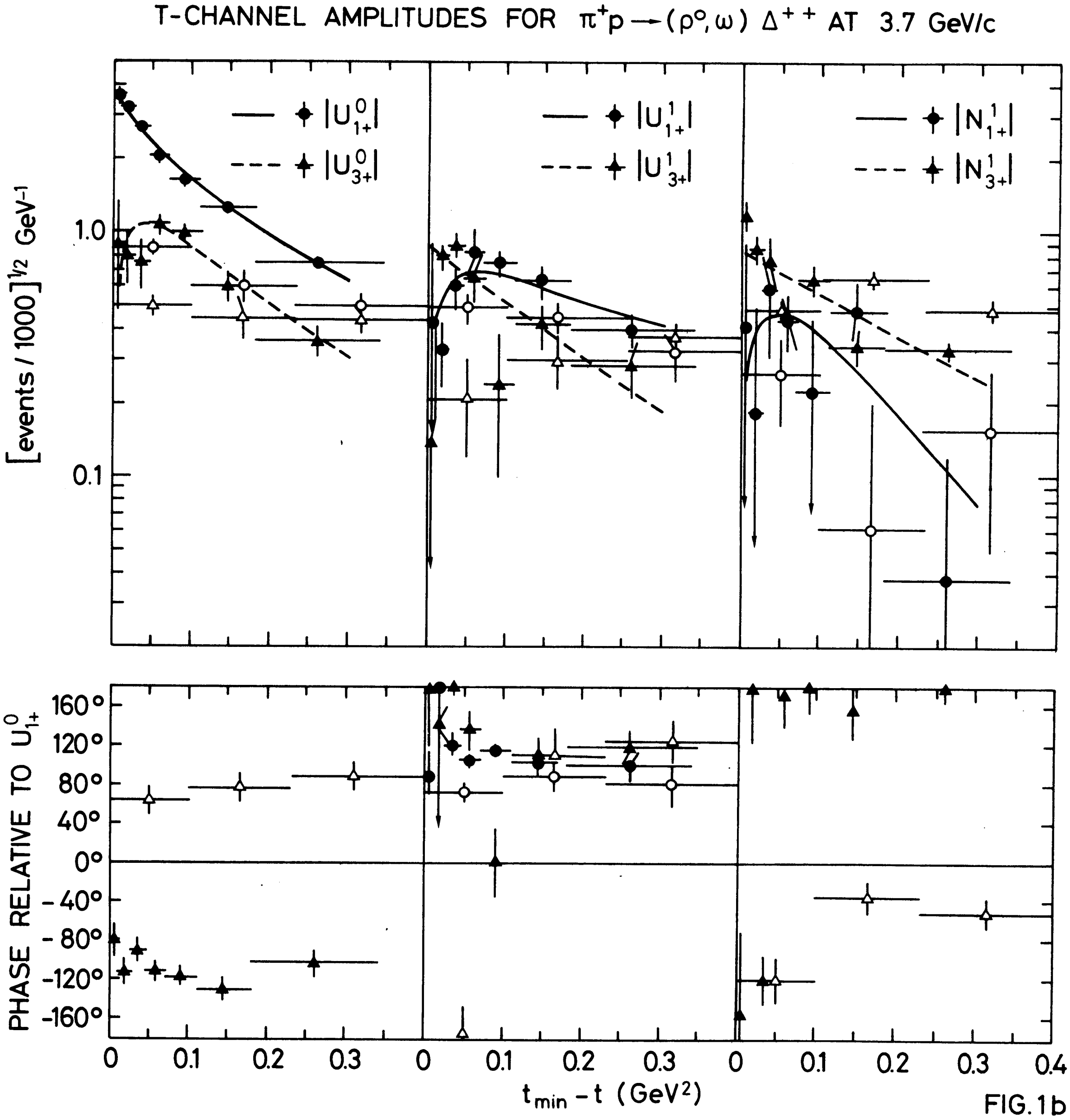
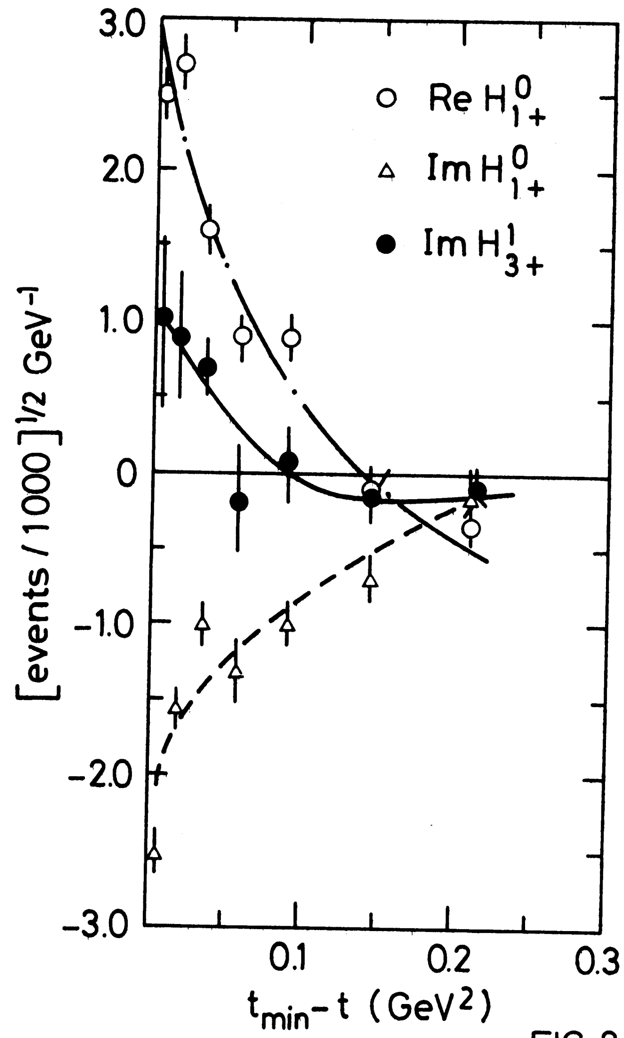

T-CHANNEL AMPLITUDES FOR π⁺p → (ρ°, ω) Δ⁺⁺ AT 3.7 GeV/c
$|U^0_{1+}|$
$|U^0_{3+}|$
$|U^1_{1+}|$
$|U^1_{3+}|$
$|N^1_{1+}|$
$|N^1_{3+}|$
[events / 1000]$^{1/2}$ GeV^{-1}
1.0
0.1
PHASE RELATIVE TO U^0_{1+}
160°
120°
80°
40°
0°
-40°
-80°
-120°
-160°
0
0.1
0.2
0.3
0.4
$t_{min} - t$ (GeV^2)

FIG. 1b



3.0
2.0
1.0
0
-1.0
-2.0
-3.0
0.1
0.2
0.3
Re H^0_{1+}
Im H^0_{1+}
Im H^1_{3+}
[events / 1000]$^{1/2}$ GeV^{-1}
$t_{min} - t$ (GeV^2)

FIG. 2